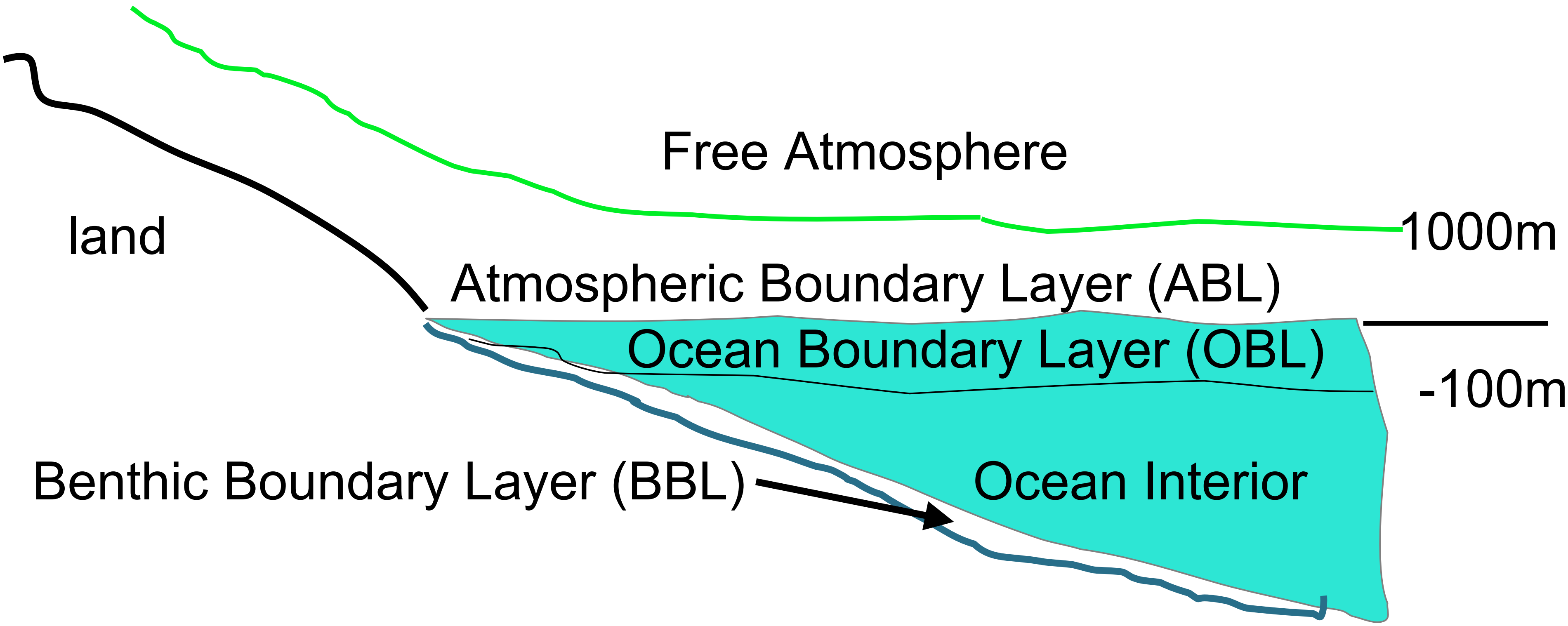
The Oceanic Boundary Layer (OBL)

- Planetary Boundary Layers
- The OBL
- Surface Forcing and Similarity Theory
- The Convective OBL
- Turbulence closures

Modeling and parameterizing ocean planetary boundary layers.
In OCEAN MODELING AND PARAMETERIZATION,
E.P. Chassignet and J. Verron (Eds.) Kluwer, 1998.

1) Planetary Boundary Layers

-The portion of a geophysical fluid that is directly influenced (forced) by the boundary
-Geophysical fluids “feel” the earth’s rotation, $\Omega = 7.3 \times 10^{-5}\ s^{-1}$



The free atmosphere and ocean interior connect through the OBL and ABL

1) Turbulent (fully 3D) Boundary Layers

Reynolds' Decomposition of the state variables :

$X = \{U, V, W, T, S, P, \rho(T,S,P)\}$

$= X + x \quad : \quad \langle X \rangle = X \;; \quad \langle x \rangle = 0$

= Mean + fluctuation

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NUMERICAL MODELS :

X = Resolved + Unresolved (sub-grid-scale)

? Equivalent to Mean + fluctuation ?

Often assumed (implicitly), but NOT equivalent

$\langle$ Unresolved $\rangle \neq 0$, in general

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= Mean + fluctuation

Consider Advection of X by the total flow :

$$\boldsymbol{U} \bullet \nabla X = U\,\partial_x X + V\,\partial_y X + W\,\partial_z X$$

$$= \partial_x U X + \partial_y V X + \partial_z W X$$
$$- X\,[\partial_x U + \partial_y V + \partial_z W]$$

[ ] = 0 (uncompressible)

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1) Turbulent (fully 3D) Boundary Layers

Now Consider vertical advection of *U* velocity (*X* = *U*):

$\partial_z WU = \partial_z (W + w)(U + u) = \partial_z (W U + w U + u W + u w)$

Average , < > $= \partial_z (W U) + \partial_z (< u w >)$

NB: the divergence of the turbulent transport, $< u w >$

(Reynolds flux, turbulent flux / stress, kinematic flux)

The surface flux, $< u w >_o = u^{*2}$

where u^* is the turbulent velocity scale

SIM $<w x>_o = u^* x^*$; $x^* = \{ t^*, s^*, b^*, \text{etc.} \}$

1) Turbulent (fully 3D) Boundary Layers

-Consider mean U momentum equation

$\partial_t U$ =

- $- U\, \partial_x U - V\, \partial_y U - W\, \partial_z U$, Advection (non-linear)

- $+ f\, V$, Coriolis (earth's rotation)

- $- \partial_x P / \rho$, Pressure gradient

- $- \partial_z \langle wu \rangle$, Turbulent vertical mixing (NL)

- $- \partial_z \langle uu \rangle - \partial_z \langle vu \rangle$, Lateral mixing (NL)

- $+ \upsilon_m\, \partial_{zz} U$, Molecular viscosity (Damp)

1) Boundary Layer : REGIMES

-Distance, d from the boundary is the important length scale

$\partial_t U = - U \partial_x U - V \partial_x U - W \partial_z U$, Advection (non-linear)

$+ f V$, Coriolis (earth's rotation)

$- \partial_x P / \rho$, Pressure gradient

$- \partial_z \langle wu \rangle$, Turbulent mixing (non-linear)

$+ \upsilon_m \partial_{zz} U$, Molecular viscosity

Interior (small Rossby Number)

Rossby Number, $Ro = \dfrac{\text{non-linear}}{\text{Coriolis}} = \dfrac{U}{f d}$,

Where $f = 2 \Omega \sin(\text{latitude})$ is the vertical Coriolis parameter

$\approx 10^{-4} s^{-1}$

Therefore , far from the boundary there will be a geophysical fluid interior , characterized by $Ro << 1$

(geostrophic flow ===>> $fV = \partial_x P / \rho$)

1) Reynolds' Number, Re

$\partial_t U$ = $- U \partial_x U - V \partial_y U - W \partial_z U$, Advection (non-linear)

$+ f V$, Coriolis (earth's rotation)

$- \rho \partial_x P$, Pressure gradient

$- \partial_z \langle wu \rangle$, Vertical turbulent mixing (non-linear)

$+ \upsilon_m \partial_{zz} U$, Molecular viscosity

Viscous Surface Layer (small Re)

Reynolds Number, Re = $\frac{\text{non-linear}}{\text{viscous}} = \frac{d\, u^*}{\upsilon_m} = \frac{10^{-2}\ 10^{-4}}{10^{-6}} = 1$

At small d (< 1 cm) there is a viscous sub-layer !!!

At greater d (Re>>1), a turbulent (3-d) boundary layer !!!

NB In OBL non-linear advection is turbulent mixing

1) 1-D (vertical) Boundary Layer Mixing

$\partial_t U = - U \ \partial_x U - W \ \partial_z U$, Advection (non-linear)

$+ f \ V$, Coriolis (earth's rotation)

$- \ \partial_x P / \rho$, Pressure gradient

$- \partial_z \langle wu \rangle$, Turbulent mixing (vertical)

$+ \upsilon_m \partial_{zz} U$, Molecular viscosity

TURBULENT BOUNDARY LAYER :

balance LHS tendency with vertical turbulent mixing

$\partial_t X = - \partial_z \langle wx \rangle \ =? \ -\partial_z \ (-K_x \ \ \partial_z X)$

down-gradient diffusion

$K_{[u,v]} = K_m$ is turbulent viscosity ;

$K_{[t,s,b]} = K_h$ is turbulent diffusivity

both have units !!! Length^2 / time (m^2/s)

1) 1-D (vertical) Boundary Layer Mixing

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TURBULENT BOUNDARY LAYER : Steady state balance of vertical turbulent mixing with Coriolis

$\partial_z \langle wu \rangle = f V = -\partial_z \left(K_m \; \partial_z U \right)$

$\partial_z \langle wv \rangle = -f U = -\partial_z \left(K_m \; \partial_z V \right)$

Ekman layer (spiral for viscosity K_m constant)

1) 1-D (vertical) Boundary Layer Mixing

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TURBULENT BOUNDARY LAYER :

balance LHS tendency with Coriolis

$$\partial_t U = f V + \partial_z \langle uw \rangle$$

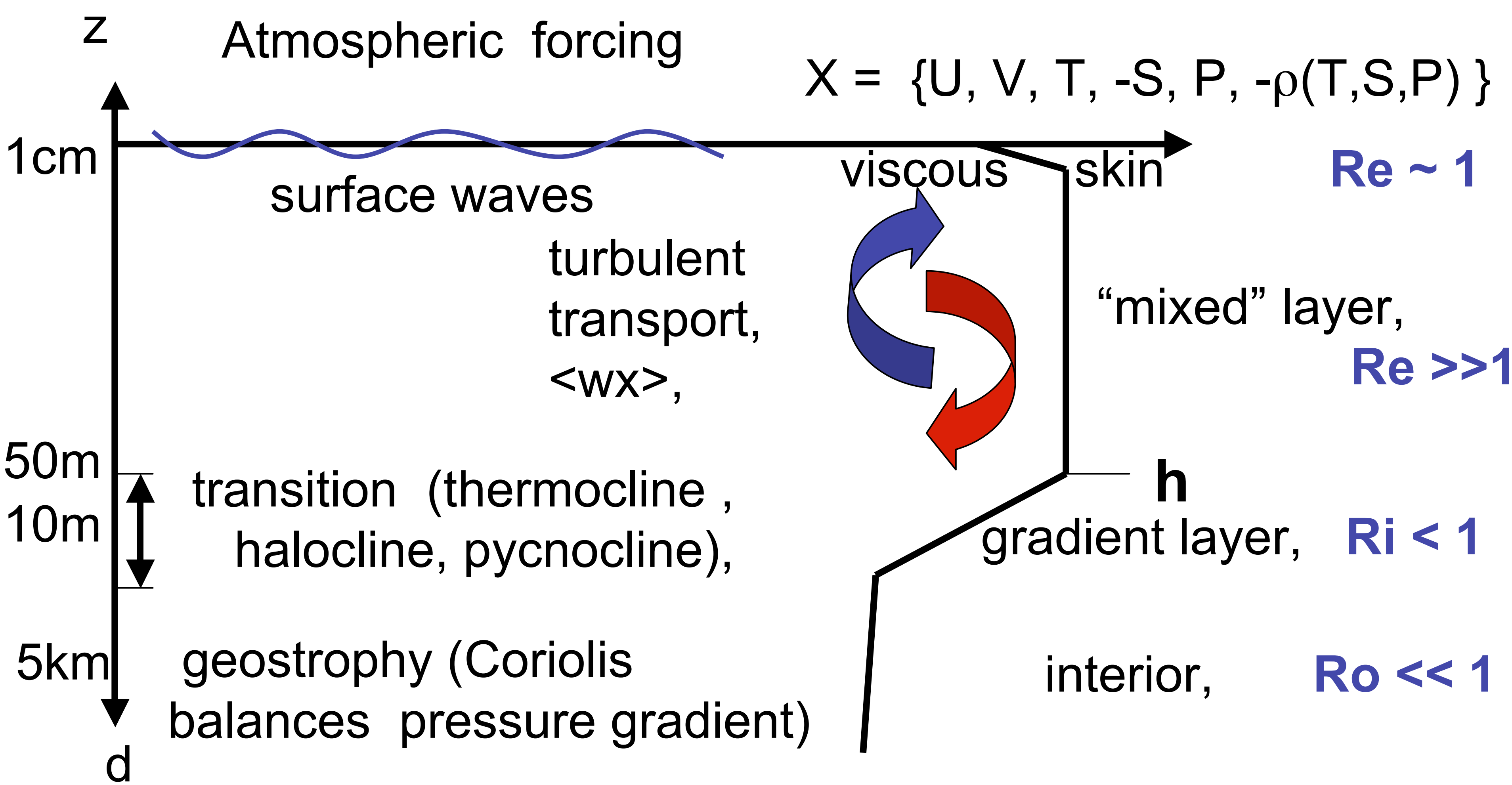
$$\partial_t V = - f U + \partial_z \langle vw \rangle$$

Inertial Oscillations : wind (u* >0) forces and/or damps

NB : All terms and Resolved + unresolved

====>> Ocean General Circulation Model

2) The Ocean Boundary Layer (OBL)



Layered structure is a consequence of the changing balance of terms with increasing distance, d.

2) The Richardson Number, Ri

Stratified Shear Flow : Buoyancy $B(z) = -g\,\rho(z) / \rho_o$ [m/s^2]
Stratification is Buoyancy Frequency, N, $N^2 = \partial_z B > 0$ [s^{-2}]
Shear is $\partial_z \mathbf{V}$ [s^{-1}] for $\mathbf{V} = (U, V)$, high shear is unstable
lots of kinetic energy, KE
High stratification means stable (negative) potential energy, PE

$$Ri = \frac{N^2}{(\partial_z \mathbf{V})^2} = \frac{\text{stable PE}}{\text{available KE}} < 0.25 \text{ (empirical)} \implies \text{local turbulent mixing (K-H)}$$

(Kelvin-Helmholtz)

NON - DIMENSIONAL

3) Turbulent Surface Forcing

Wind Stress, $\boldsymbol{\tau}_o = (\tau_x , \tau_y)_o$

Freshwater flux, F_o = P , Precipitation, > 0
+ E , Evaporation, usually < 0

Surface heat flux, Q_o
= Q_{nsol} , non-solar heat fluxes < 0
+ SW_{net} (0) , net surface solar radiation > 0
- SW_{net} (d_s), , solar not driving the OBL

In limit of $d_s = 0$, solar radiation does not drive OBL,
Clearly d_s should not be beyond the OBL

3) Surface Kinematic Fluxes

$$| \langle \mathbf{v}\, w \rangle_o | = | \tau_o | / \rho_o = u^* u^* = u^{*2}$$

$$\langle w\, t \rangle_o = - Q_o / (\rho_o C_p) = u^* t^*$$

$$\langle w\, s \rangle_o = F_o S_o / \rho_o = u^* s^*$$

Surface buoyancy flux $B_o = -g (\alpha \langle w\, t \rangle_o - \beta \langle w\, s \rangle_o)$

$\rho(T,S,P) \rightarrow \alpha = 2 - 4 \times 10^{-4}\ C^{-1}$; $\beta = 3.5 \times 10^{-4}\ (psu)^{-1}$

Monin-Obukhov Length, $L = u^{*3} / (\kappa B_o)$; < 0 unstable

Depth where wind power (= Force x Velocity = κu^{*3})
equals PE loss (gain) due to $B_o > 0$ ($B_o < 0$) = $B_o L$

3) Monin-Obukhov Similarity Theory

Near the surface of a boundary layer, but away from the surface roughness elements, the ONLY important turbulence parameters are the distance, d, and the surface kinematic fluxes.

$$|\langle \mathbf{v}\, w\rangle_o| = |\boldsymbol{\tau}_o| / \rho_o = u^*\, u^* = u^{*2}$$

$$\langle w\, t\rangle_o = -Q_o / (\rho_o\, C_p) = u^*\, t^*$$

$$\langle w\, s\rangle_o = F_o\, S_o / \rho_o = u^*\, s^*$$

Monin-Obukhov Length, $L = u^{*3} / (\kappa\, B_o)$; < 0 unstable

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3) Monin-Obukhov Similarity Theory

Near the surface of a boundary layer, but away from the surface roughness elements, the ONLY important turbulence parameters are the distance, d, and the surface kinematic fluxes.

KEY : Dimensional Analysis

5 parameters (u^*, t^*, s^*, d, L)

4 units (m, s, ºK, psu)

Non-dimensional groups are functions of (d/L), the stability parameter (< 0, unstable)

3) Dimensional Analysis (d = -z)

Non-dimensional gradients : $-\partial_z X \; d \,/\, x^* \;\propto\; \phi_x\,(d/L)$,

Empirically $\kappa = 0.4$, von Karman constant ,
makes $\phi_x\,(0) = 1$ in neutral (wind only) forcing ($B_o = 0$, $L \rightarrow \infty$)

$\kappa\, \partial_z X = x^* / z$ ===>> neutral logarithmic profiles, $X(z)$

Let : $\langle w\, x \rangle_o = u^*\, x^* = -K_x\, \partial_z X$, define diffusivity, K_x

Near the surface of a PBL similarity theory (MOS) says

$$K_x \rightarrow -\frac{u^*\, x^*}{\partial_z X} = \frac{\kappa\, d\, u^*}{\phi_x\,(d/L)} \xrightarrow{\text{neutral}} \kappa\, u^*\, d$$

4) The convective OBL $(B_o < 0)$

Surface buoyancy flux, $B_o = -\langle wb \rangle_o < 0$

Wind stress, $\tau_o = 0$; $u^* = 0$

Convective Velocity Scale , $w^* = (-B_o\ h)^{1/3}$,

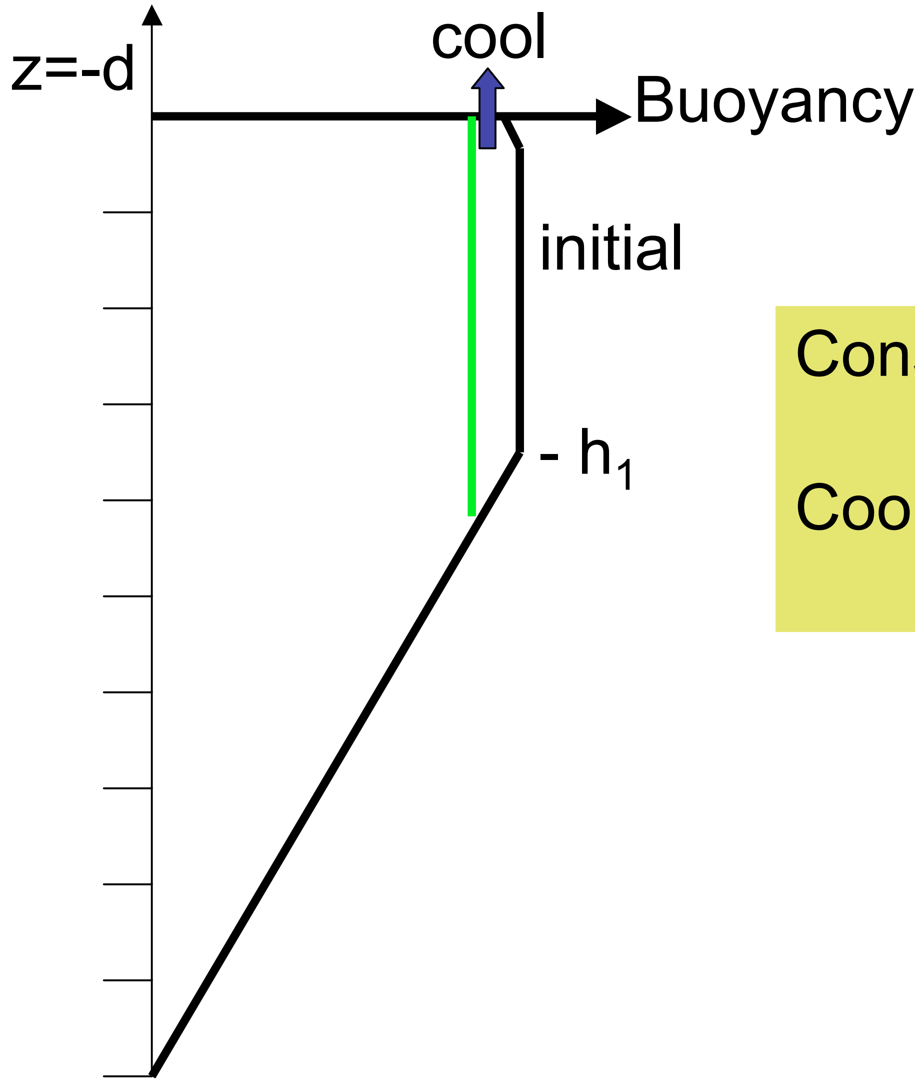
Where h is boundary layer depth : $\sigma = d / h$

$d/L = \kappa\ d\ B_o / u^{*3} \longrightarrow -\infty$

$\phi_x (d/L) \longrightarrow (1 - c\ d/L)^{-1/3} = (1 - c\,\sigma\, h/L)^{-1/3}$

$u^*/\phi_x \longrightarrow (u^{*3} - c\,\kappa\, d\, B_o)^{1/3} \longrightarrow (c\,\kappa\,\sigma)^{1/3}\ w^*$

4) The convective OBL : $\Delta B = \Delta t \ \partial_z - \langle wb \rangle$



Conservation :

Cool = Initial-Green

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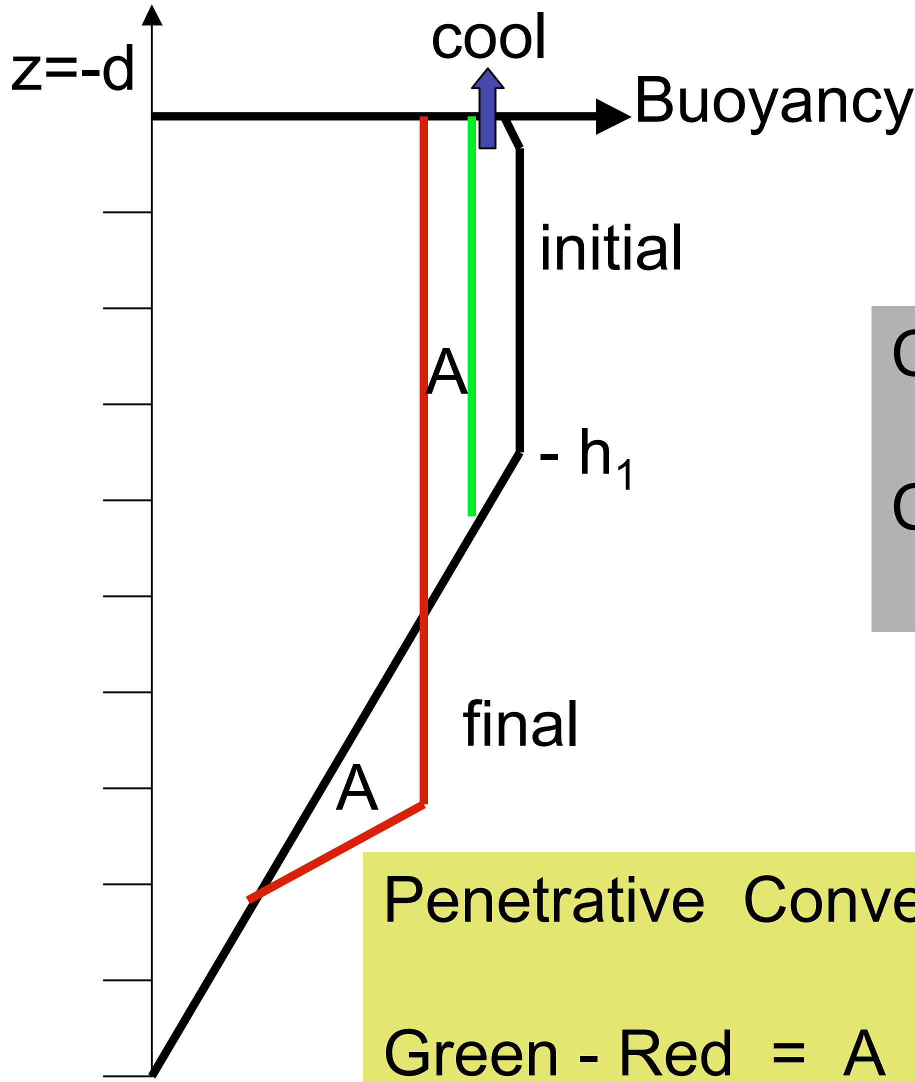

Conservation :

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Penetrative Convection

Green - Red = A

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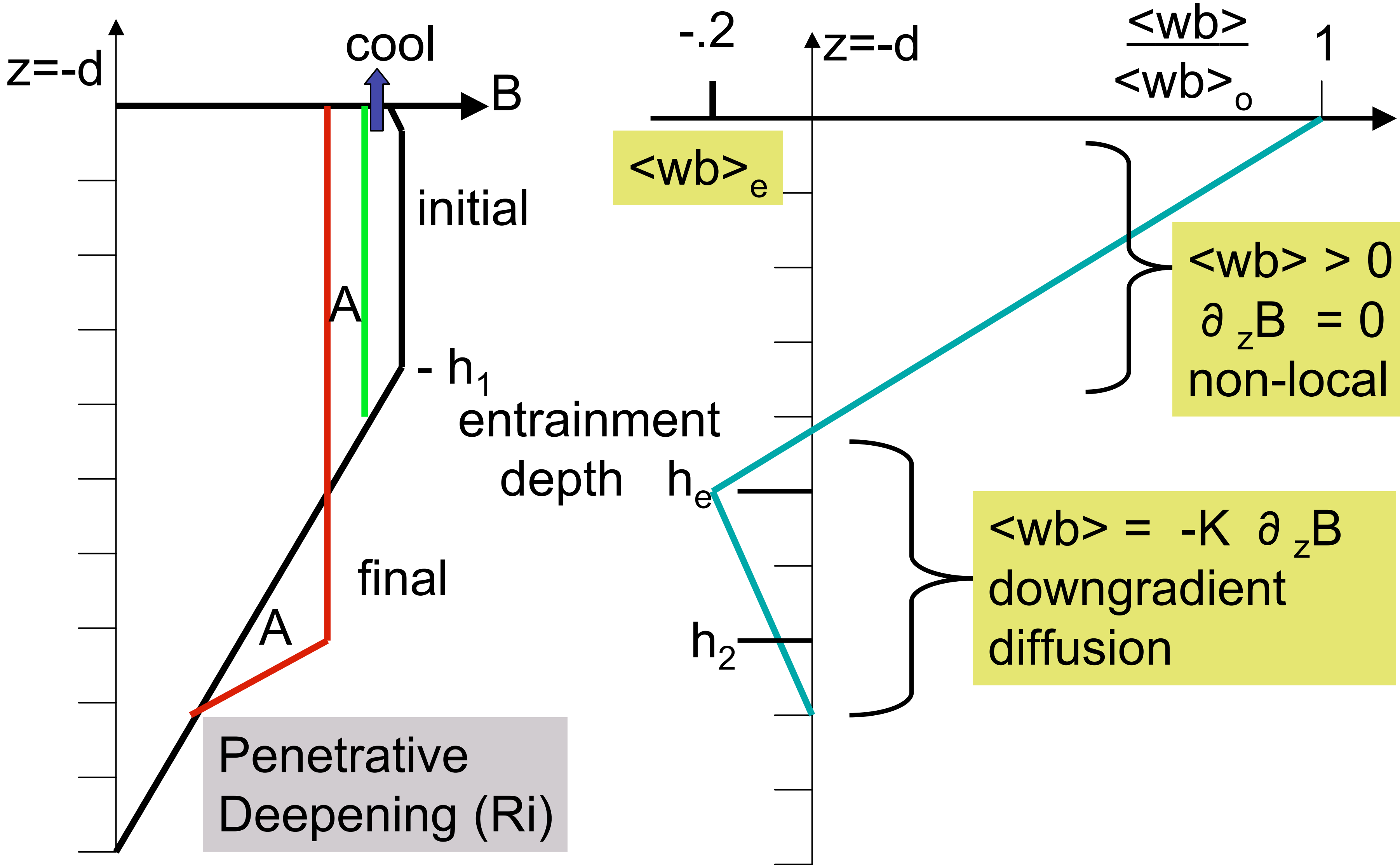

5) OBL Schemes

- Mixed-Layer Models
  - Prognostic, $\partial_t h$
    - TKE budget
      - Kraus-Turner (1984)
      - Niiler (1975)
      - Garwood (1978)
      - Gaspar (1988)
  - Diagnostic, h
    - Price-Weller-Pinkel
- Turbulence Closure Models
  - 1st order
    - Ekman
    - Pacanowski-Philander
    - K-Profile (KPP)
  - 2nd order
    - Mellor-Yamada (1984)
    - Gaspar et al. (1990)

4) 1st order closures (local)

ASSUME : analogy to molecular diffusion $\langle wx \rangle = -K_x \partial_z X$

e.g. Ekman : $K_u = K_v = K_m =$ CONSTANT

BUT non-zero fluxes are **observed** in regions of zero local gradient.

Therefore, the analogy is known to be wrong in a PBL, and can't be corrected by any choice of K_X, but may be good enough for some problems.

K (Ri) formulations are popular despite this problem (Pakanowski and Philander (1981)

Trick : A 50m thick upper grid level is like a OBL with infinite K_x

5) 1st order closures (non-local K-Profile)

Temperature variance equation says

$$\langle wx \rangle = - K_x \left(\partial_z X - \gamma_x \right)$$

Non-local - K_x knows about h, σ, and the surface forcing
- γ_x gives non-zero flux for $\partial_z X = 0$, as observed