We've Got Navier-Stokes, So Why Aren't We Done?

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ATOC 6020 Oceanography Seminar
Inaugural Meeting
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Almost all photo credits are Wikipedia...

5 Fundamental Forces

- Electricity
- Magnetism
- Strong Nuclear
- Weak Nuclear
- Gravity

~~3~~ 4 Fundamental Forces

- Electromagnetism (photon)
- Strong Nuclear (gluon)
- Weak Nuclear (Z, W bosons)
- Gravity (graviton?)

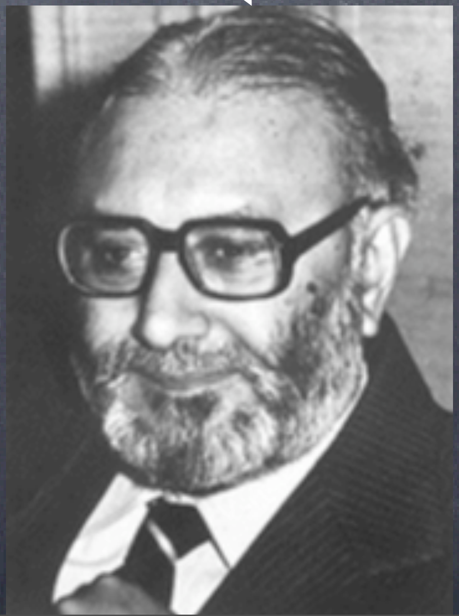
Thanks to Maxwell!
(Seen here with Santa)

3 ~~4~~ ~~5~~ Fundamental Forces

- Electroweak (photon, Z, W bosons)
- Strong Nuclear (gluon)
- Gravity (graviton?)

Maxwell

Thanks to Maxwell,
and Glashow, Weinberg, and Salam



The Pursuit of Elegance

- Many of Einstein's Princeton years were spent trying to unify Gravity and E&M.
- But, Einstein couldn't just ignore his Nobel-winning work: the photoelectric effect showed E&M was quantized...
- Gravity and Quantization don't play nice...
- So, he played a lot of tennis with Gödel, the man who proved knowing everything means nothing...

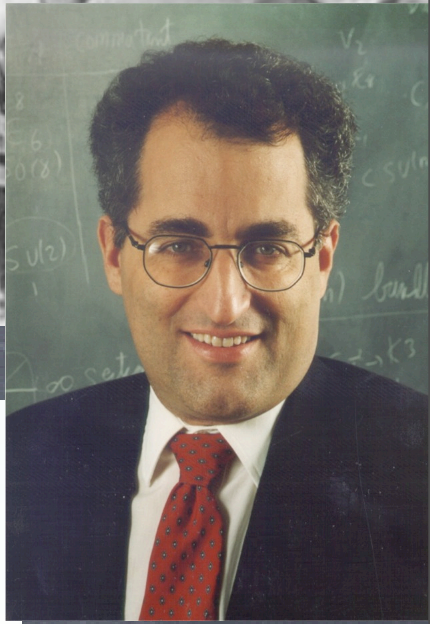
The Pursuit of Elegance

Coleman

- With Einstein in the game... Everyone wanted in.
- Quantum Electrodynamics
- Quantum Chromodynamics
- Supersymmetry
- Loop Quantum Gravity
- String Theory
- Twistor String Theory

Noether

Smolin



Witten

Gell-Mann

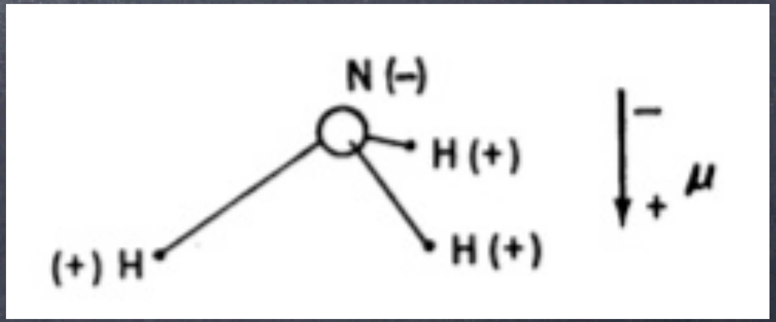
Feynman

Lisi

But, More is Different

- As we leave the microscopic, high-energy world, symmetries break and complexities arise.
- The 3 (or fewer) fundamental forces split into 5, and then the symmetries they possess degenerate, leaving pockets into which materials can sit unsymmetrically for long enough to fundamentally separate our world from the world of elementary particles.

P. W. Anderson

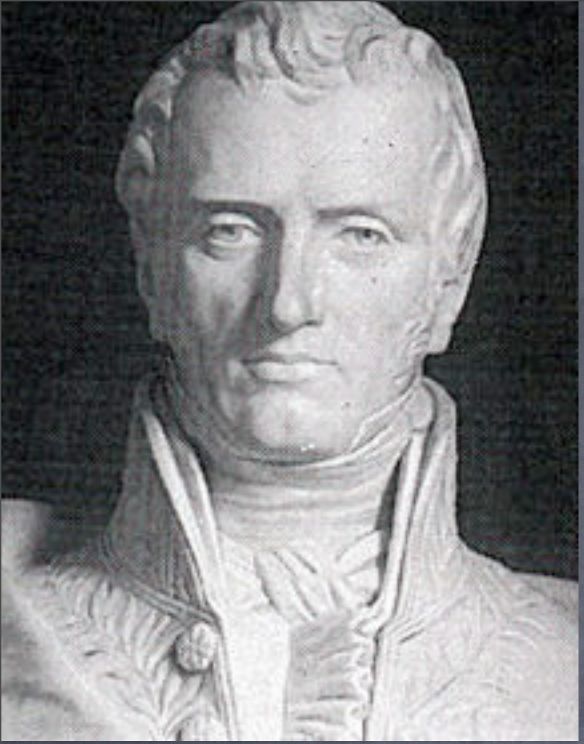


The Impossible Ammonia Dipole Moment

The Philosophers' Dilemma

(With input from S. Rand, Georgia State U.)

- The issue for philosophers is a lack of translation between scales
- The essence here is that translations of languages and concepts is philosopically hard
- For Example, the Twin Earths Scenario
- There is a real twin earths! Jade (nephrite) vs. Jade (jadeite) only since 1863 do we know the difference...



Navier-Stokes

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot \mathbb{P} + \rho \mathbf{f}$$

This equation relates the budget of momentum of a fluid. It is the F=ma.

It works over a big range of scales, from the molecular mean free path to galactic

We understand how to make couple it with E&M, (MHD) and relativity (e.g., Landau & Lifshitz)

Navier-Stokes

In many regimes & fluids, we understand how to address the pressure, the deviatoric stress tensor, and other forces.

For oceanography, we usually use

$$\overbrace{\rho\left(\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{\substack{\text{Unsteady}\\\text{acceleration}}} + \underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{\substack{\text{Convective}\\\text{acceleration}}}\right)}^{\text{Inertia}} = \underbrace{-\nabla p}_{\substack{\text{Pressure}\\\text{gradient}}} + \underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}} + \underbrace{\mathbf{f}}_{\substack{\text{Other}\\\text{forces}}}$$

For Incompressible, Newtonian Fluid

Navier-Stokes & Friends

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla \cdot \mathbb{P} + \rho \mathbf{f}$$

momentum

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

mass

Also, conservation laws for
Salt, Energy (Thermodynamics, Pot’l Temp),
chemical tracers, etc.

All of the equations are known, so what’s the problem?

- The Navier-Stokes equations are hard (largely) because of $\underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{\text{Convective acceleration}}$
- This term is nonlinear, so couples smallest scales to largest
- We cannot even prove or disprove existence or smoothness theorems of pointwise solutions of Navier-Stokes in 3d for all times.

In oceanography, it's not the theorems that fail, it's the solution methods!

Leading Edge Direct Numerical Simulations:
Thanks to Yukio Kaneda's TOY talk May 5, 2008

$N^3=4096^3$ DNS on Earth Simulator (#49): 40TFlops, 10TB -> 16.4 Tflops, 7.2 TB

For N=12,000
3.2Pflops, 270 TB, 1Pflops, 200TB

(Roadrunner@LANL can do 1Pflop, but not on this code...)

But, what did we gain? If viscous length is 1cm, then we're only at 40m-120m total domain size! How do we model the ocean?

On to the subgridscale and parameterizations

- What we do in order to model the global ocean is resolve what we can, parameterize the rest.
- Parameterization is akin to how we came up with Newtonian viscosity and Fickian diffusion
- Except, those theories apply to isotropic, homogeneous fluids, not a strongly heterogeneous, anistropic ocean. So instead of one form, we need many, and there aren't that many Newtons to go around...

Example: Eddy Viscosity & Mixing Lengths

- If there was a clean separation of scales between the eddies and the background...
- And if eddy stirring obeyed certain symmetry properties
- Then eddy stirring could be parameterized with a viscosity & diffusivity just as molecular mixing is. (due to Boussinesq)
- Instead of the mean free path of molecules, a ‘mixing length’ of eddies is used, along with typical velocity (e.g., sqrt(2 KE)). (due to Prandtl, see Tennekes and Lumley...)

Prandtl Form:

$$\nu_t = \kappa \left| \frac{\partial u}{\partial y} \right| l^2$$

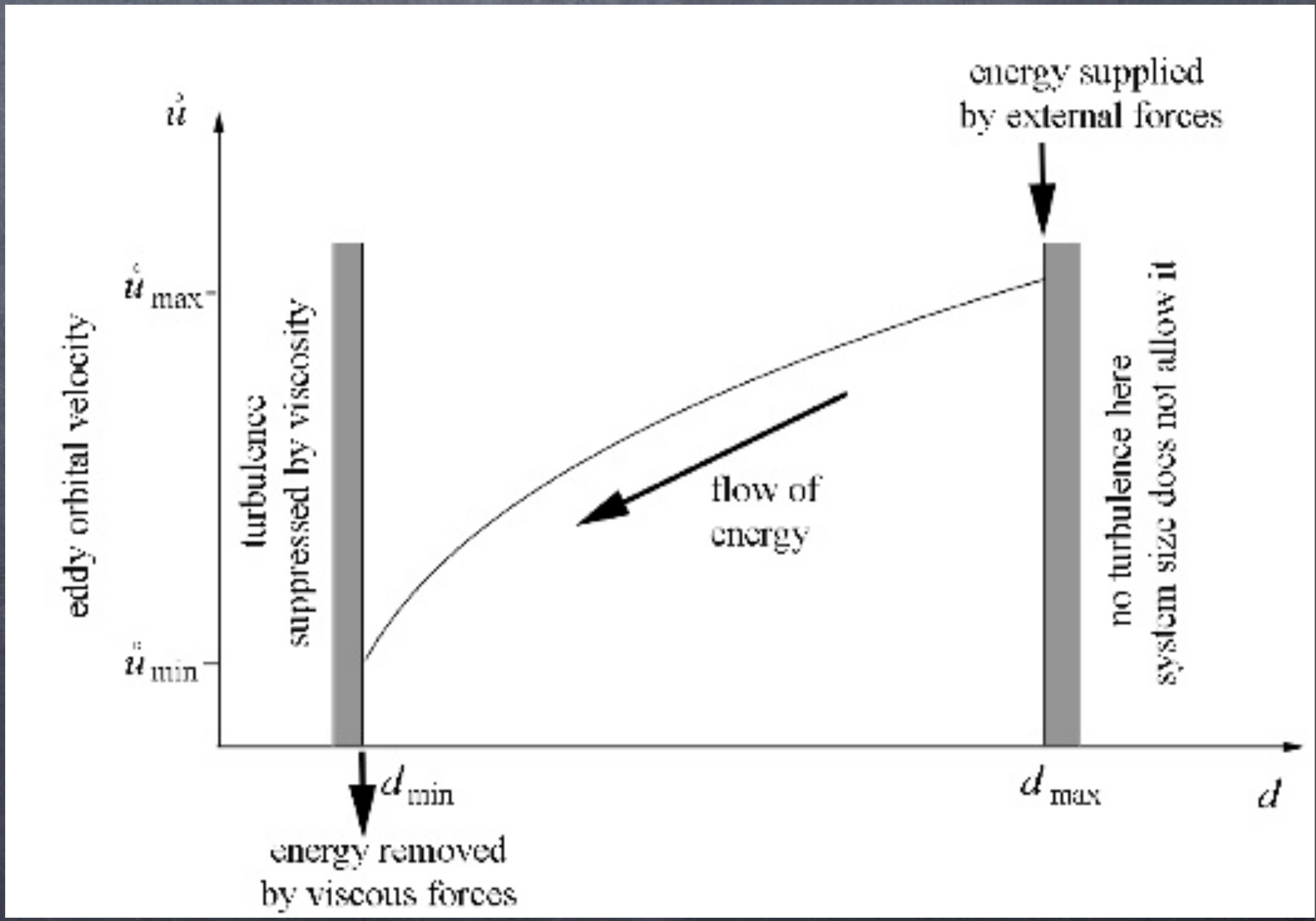
Example: Smagorinsky (1963)

- A parameterization of viscosity for turbulence.
- Relies on Kolmogorov Homogeneous Turbulence argument (spectrum of KE)
- Considers Truncation of spectrum at resolved wavelength, but preserving energy flux across wavelengths.
- Provides eddy viscosity scaling

$$\nu_* = \left(\frac{\Upsilon \Delta x}{\pi}\right)^2 |D_*|.$$

$$|D_*| \equiv \sqrt{S_*^{ik} S_{*ik}}$$

Figure from Cushman-Roisin & Beckers



Example: Fox-Kemper, Menemenlis (2008)

- A parameterization of mesoscale eddy viscosity/diffusivity scalings.
- Relies on Kolmogorov/Smagorinsky argument (spectrum of KE), but uses Charney/Kraichnan/Leith scalings for quasi-2d eddies instead of 3d homogeneous, isotropic turbulence

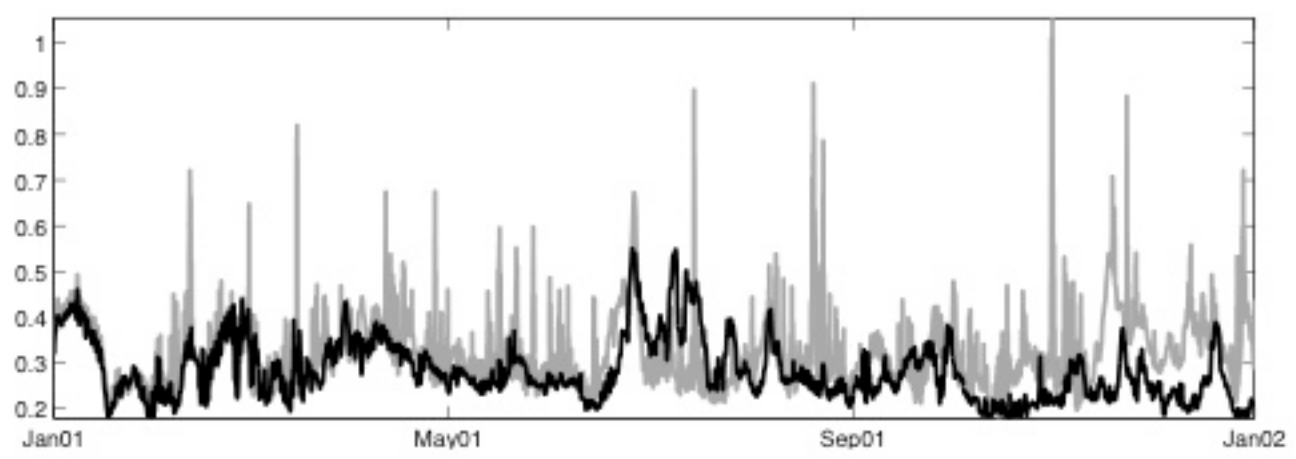


Figure 4. Maximum Courant number, $w\Delta t/\Delta z$, for vertical advection. Gray line is from the *LeithOnly* integration and black line is from the *LeithPlus* integration.

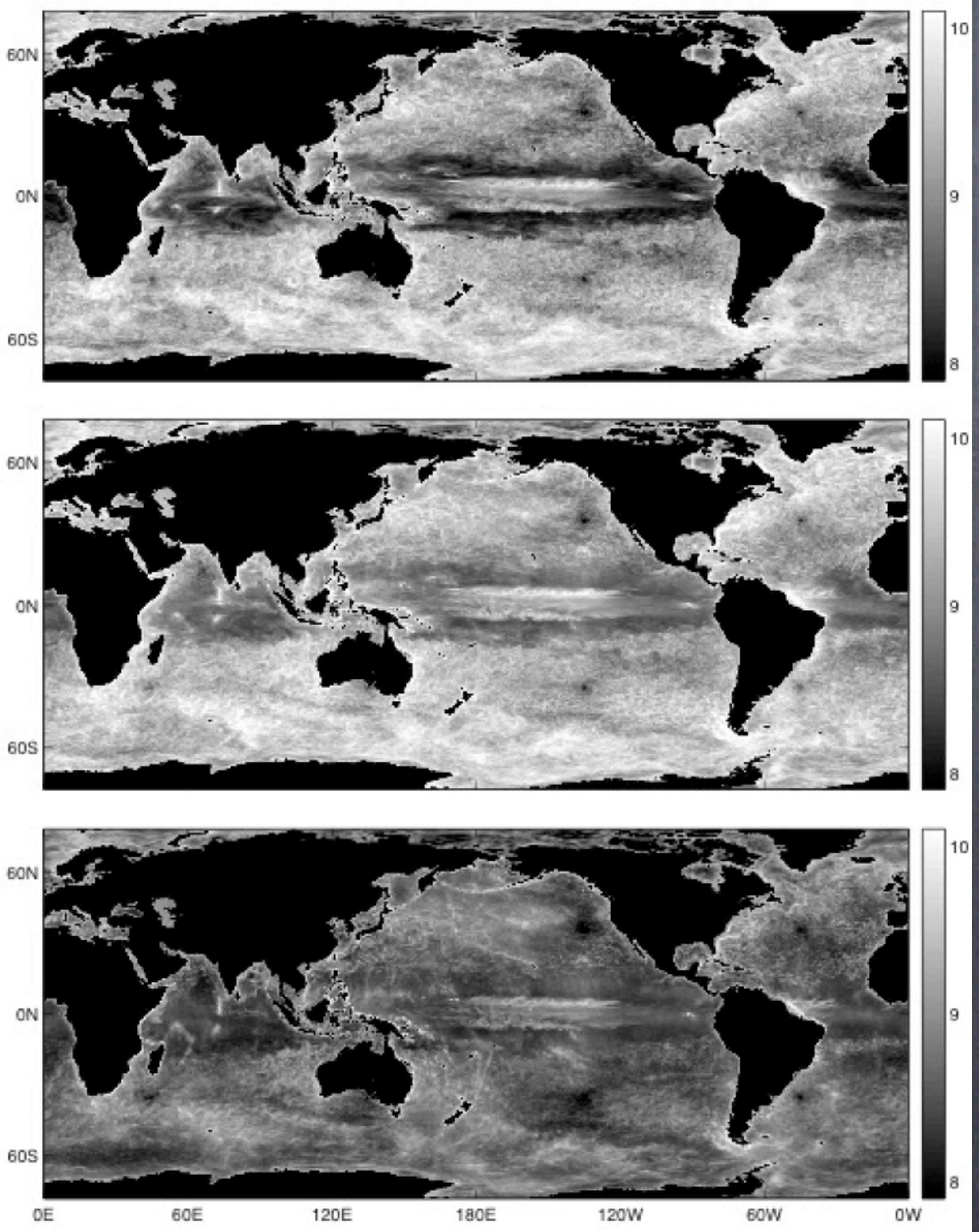


Figure 3. Monthly mean biharmonic viscosity, ν_4, in the model's surface level for December 2001. Units are $\mathrm{m}^4\ \mathrm{s}^{-1}$ and color scale displays $\log_{10}(\nu_4)$. Top panel is from the *LeithOnly* integration. Middle panel is from the *LeithPlus* integration. Bottom panel shows the divergent modification of the *LeithPlus* integration.

Example: Gent-McWilliams (1990)

- A parameterization of mesoscale eddy restratification/front slumping. (10-100km eddies).
- Relies on energetic argument (extraction of PE), some locality
- Unlike eddy diffusivity, it doesn't spuriously mix water types
- But, has an eddy transfer coeff., or GM 'Kappa' that is unspecified.
- Also, big problems at boundaries.

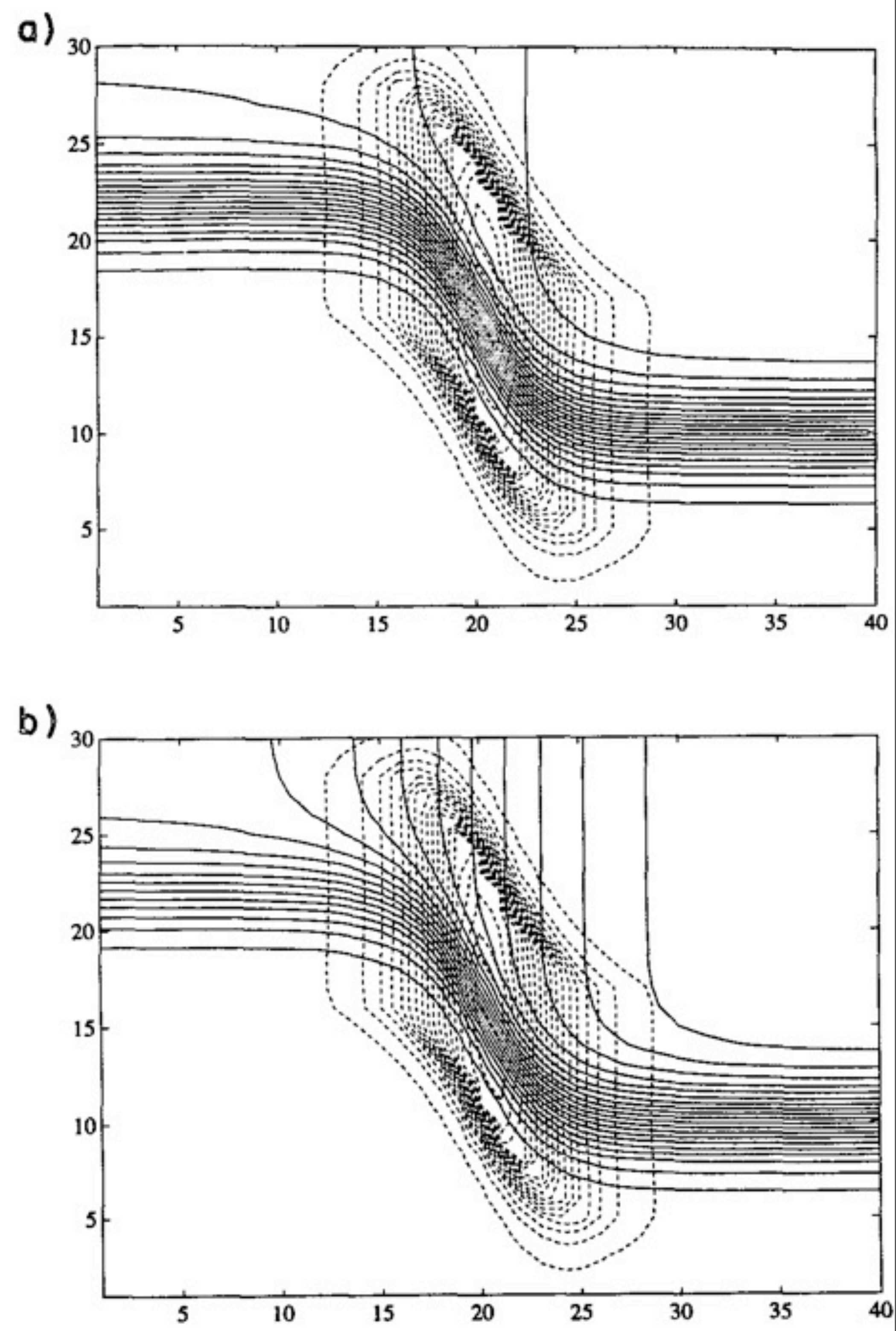
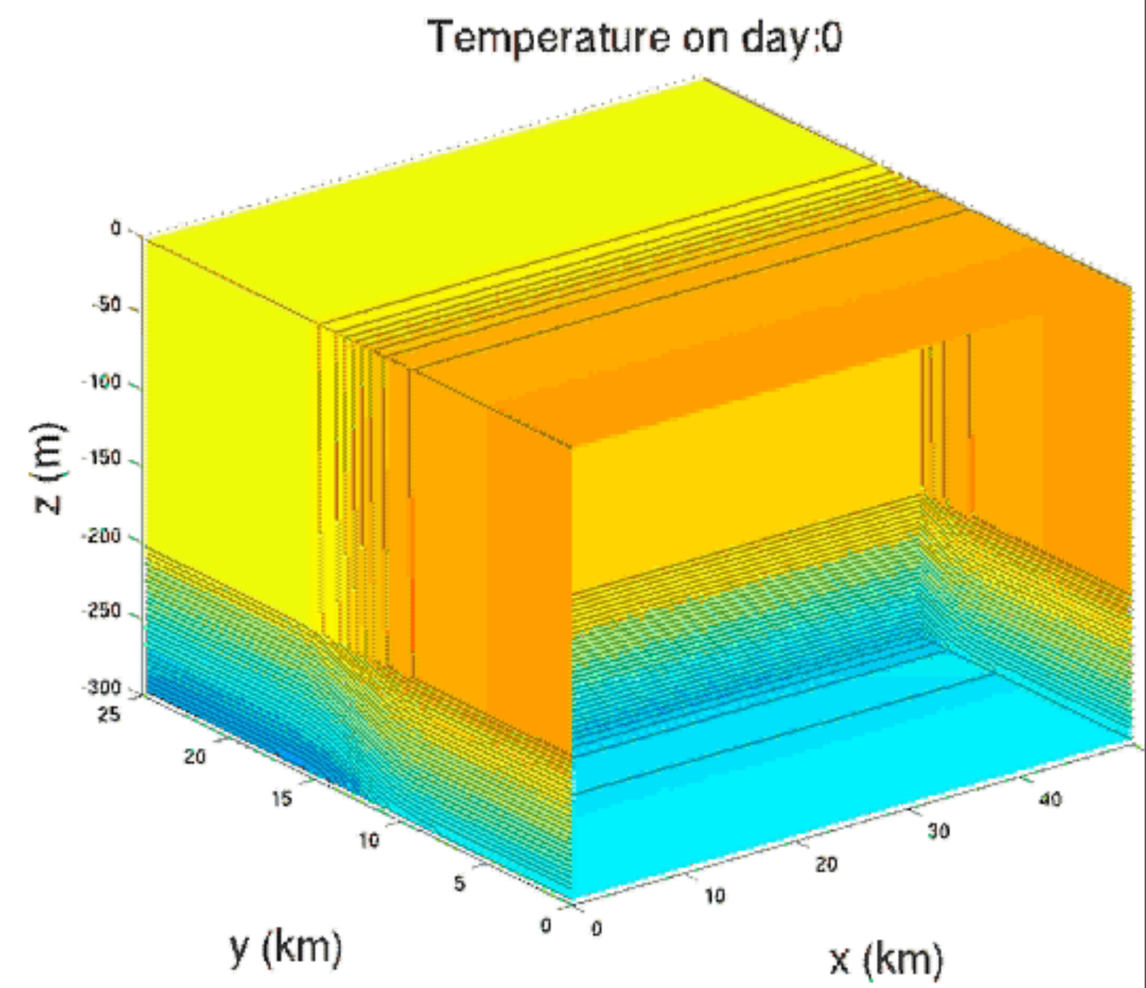


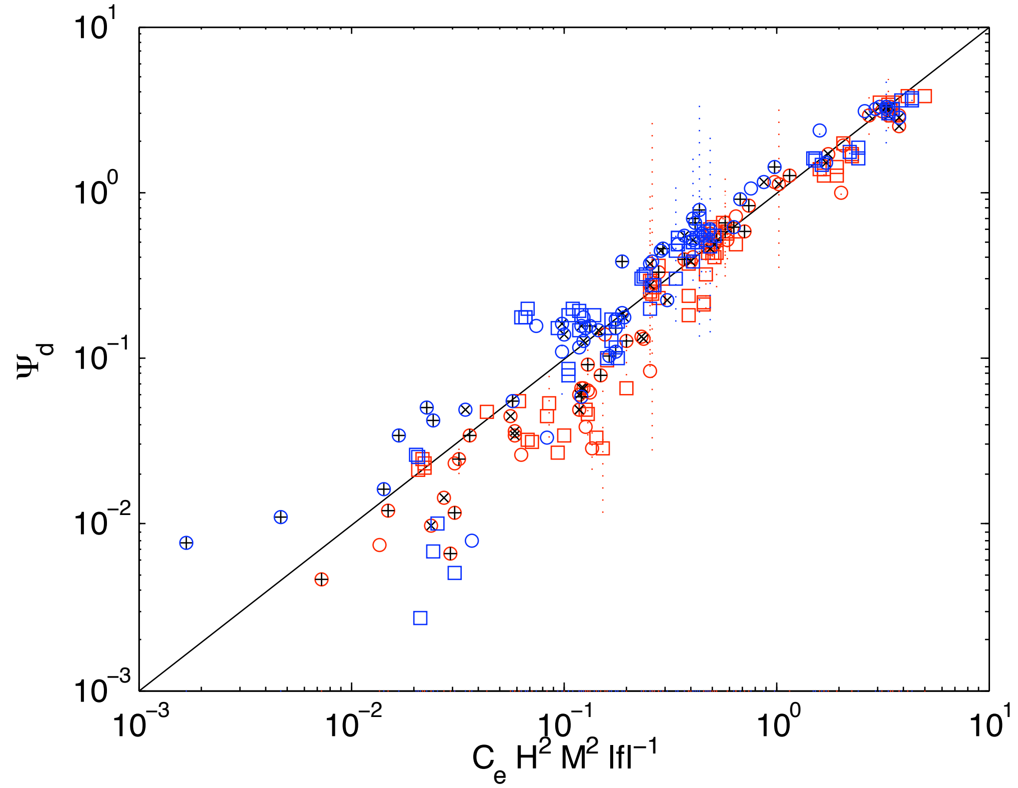
figure from Gent et al. 1995

FIG. 3. Initial states of (a) temperature and (b) salt [contour interval one-quarter that of (a)]. Both panels also show the streamfunction $\kappa\rho_x/\rho_z$ for the parameterized eddy-induced transport velocity.

Example: Fox-Kemper, Ferrari, Hallberg (2008)

- A parameterization of submesoscale eddy restratification/front slumping. (0.1-10km eddies in the mixed layer).
- Relies on energetic argument (extraction of PE), less locality than GM (nonlocal in vertical)
- Unlike eddy diffusivity, it doesn't spuriously mix water types
- Has no dimensional eddy transfer coeff., only a simulation-trained nondimensional efficiency factor.
- Some problems interfacing with GM.





Discussion

- The intensive research to find fundamental equations is not sufficient to understand our world
- In material science, nuclear physics, & physical chemistry, there is spontaneous symmetry breaking
- In fluids, this problem is manifest by the complexity of studying Navier-Stokes, esp. scale to scale interactions
- In fluid dynamics, we have DNS, LES, RANS tools. Subgridscale models of homogeneous & wall-bounded turbulence. Turbulence is seen as a uniform problem.
- In oceanography, we have a greater diversity of scale couplings. Demonstrably important parameterizations include: deep convection, mesoscale eddies, submesoscale eddies, Langmuir cells, 3d mixing, breaking waves (surface & internal), tidal mixing, etc.