# Syllabus for ATOC 0350:
# Mathematical Methods of Fluid and Solid Geophysics and Geology
# (a.k.a. GeoMath)

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## 1 Course Descriptions

### 1.1 GEOL0350 Mathematics of Fluid and Solid Geophysics and Geology

Intended for undergraduates conalphcentrating in geological and physical sciences or engineering, especially those interested in the quantitative study of Earth. Problem sets will cover common approaches to quantify the dynamics and chemistry of solids and fluids in nature. Mathematical topics to be introduced include linear algebra, vectors and tensors, differential equations, dynamical systems, eigenvalues and eigenvectors, empirical orthogonal functions, fractals, chaos, and statistics. Applications include waves in the oceans, atmosphere, and solid earth, convective and conductive heat flow, reaction rates, gravitational potential energy, Newton's laws on a rotating planet, measuring coastlines and ranges, and dating errors in stratigraphy. Prerequisites: GEOL 0220 and (MATH 0100, 0170, 0190, 0180, 0350 or 0200).

## 2 Contacts

Portions of the website are password-protected to ensure that fair use and copyrights are correctly obeyed as I share images from books, etc. You can access these by using:

```
username:  io
password:  ocean
```

## 3 Getting Help!

We are usually available by email. Baylor's office hours will be Monday 1:30-2:30 and Thursday 2-3 or by appointment (see my schedule at `http://fox-kemper.com/contact`). You can also drop into the Math Resource Center (MRC, `http://www.math.brown.edu/mrc/`) or sign up or drop in to a tutoring session (`http://www.brown.edu/academics/college/support/tutor`).

## 4 Meetings and Places

We will meet Monday, Wednesday, and Fridays from 10:00 to 10:50AM in GeoChem 0390. Baylor's office hours will be Monday 1:30-2:30 and Thursday 2-3 or by appointment (see my schedule at `http://fox-kemper.com/contact`) in his office (GeoChem 133) or lab (GeoChem 134). You should also have signed up for a lab/practicum/practice session for one of MTWR 3:00-3:50.

# 5 Website and Canvas

The primary resource for this class is the webpage: `http://fox-kemper.com/0350`. The class webpage is where all of your assignments will be announced, solution sets posted, links to additional reading will be posted, etc. Assignments should be turned in as pdfs using canvas. The copiers in GeoChem and elsewhere can be used to scan handwritten assignments (for free).

You will want to familiarize yourself with Wolfram Alpha (`http://www.wolframalpha.com`), it is a great resource for looking up math definitions. Wikipedia is also handy in a pinch (due to the armies of math & physics grad students who apparently have so very few social commitments that they punch in all the details of their dissertation appendices).

# 6 Required Course Activities, Expected Times, and Structure of Classtime

The regular class time will be presentation of new materials and discussion. This format requires buy-in from you, the student, however. You *must do the reading of the notes before class*, and preferably also *at least skim the associated chapters in the book* before class. If an individual student fails to do this, it will negatively influence her or his ability to follow, and if the class fails to do this I will have to expand the lecture mode–decreasing the problem-solving mode–which is not good for learning. In addition, you will visit the lab/practicum/practice sessions once a week (you signed up for one day from MTWR at 3-3:50), where you can work on homework problems in small groups, with input from the TA. The TA will be scheduling students from GEOL2300 to come to discuss your understanding and help with homework problems during this time. You should provide feedback to the TA or the professors about these interactions, because the GEOL2300 students are being assessed on how well they interact with you.

Magdalene Lampert, a researcher in math education, has shown that learning and retention in mathematical methods is improved by inverting the common classroom presentation order. Lecture, followed by discussion, followed by individual homework is not as effective as individual effort, group effort, full discussion. We will use the discussions and the practicum sessions to adhere to the latter format as best as possible.

Your individual effort begins with reading the notes and skimming the chapter before lectures. Then you will be challenged with questions throughout the class and practicum. In the practicum sessions, you will work individually for a few minutes, and then discuss in a group for a few minutes, and ask the leader for guidance or clarification. Finally, the whole class will discuss approaches to problems and the correct solutions. You will then review these problems again as you review the chapter reading and finalize your (related) homework problems, and study for exams.

## 6.1 Assignments, Exams, and Expected Time for Activities

- Scheduled class meetings, which will be suspended in the Reading Period (3 hours/week; 38 hours) and practicum meetings, which will continue in the Reading Period (1 hours/week; 13 hours) [Grading: 10% Attendance and participation.]
- Reading and reviewing class work (2 hours/week; 26 hours)
- Weekly assignments (6 hours/week for 12 weeks; 72 hours) [Grading: 50% Weekly homework]
- Weekly peer reviews (1 hours/week for 12 weeks; 12 hours) [Grading: 10% Reviews of other students' homework assignments.]
- Preparation for Midterm and Final (16 hours) [Grading: 20% Final, 10% Midterm]

- Final Exam (3 hours)
- Total: 180 hours [Grading: 100%]

What can I do to get a good grade? Turn in all of the assignments on time. For the format of the course to work, ON TIME matters, so that we can get to the reviewing. Also, *BONUS POINTS* are available on homework and exams for spotting typos in the notes, homework assignments, and exam problems. The more promptly you point them out (by email), and the more important they are, the more points you get!

The scheduling of the assignments are listed on the webpage, and other than the exceptional weeks around holidays will be as follows.

- Weekly assignment due by class time on Monday.
- Solution sets distributed by midnight Monday (assignments not accepted afterward).
- Peer reviewing and grading due by following Friday.
- *Iterum usque ad finem*

All of this will be charted out on the calendar on the website and in canvas.

### 6.1.1 Peer review

In addition to doing the problem sets, you will each be performing reviews of each others work. We will be using a rubric based on the AGU guidelines for review. A-F for presentation quality and 1-5 for science/math. Such a guide is useful to go by, and when you do reviews of your fellow students, I'll expect to get a A1 or B2 or B1 score, etc. An A1 will count for 100%, and presentation and accuracy will be equally weighted (an F5 will be 20%). There are a few lessons to be learned here, that will help you write your own papers and will help you provide effective and useful reviews in your career.

- Learning to spot unfounded claims
- Learning how to properly support claims
- Learning to distinguish poor writing/presentation from poor thinking
- Learning to label equations, graphs, and numerical information understandably
- Revisiting problems from a different perspective

You will have each of your homework assignments peer-reviewed by more than one person, and inconsistent results will be rechecked. The assignments for reviewers will rotate (ensuring fairness in grading by randomization). You should feel free to contact me with any concerns about the process or specific issues.

## 6.2 Calendar

The main webpage for the class `http://fox-kemper.com/0350` will have the calendar with all assignment deadlines, readings, etc. set up by the first class session. There will be weekly problem sets, one midterm, and a final exam.

# 7 Goals

In this class you will:

- Learn how to quantify some of the physical processes of the earth system.
- Learn how observations and budgets are quantified, evaluated and quality-controlled, and compared.
- Get practice solving diverse geophysical and geological problems using new mathematical techniques.
- Gain a broader perspective and more practice by peer reviewing and collaborating.

This class cannot possibly provide a complete understanding of all of the mathematical topics presented, instead the goal is to introduce the most basic ideas and give geophysical and geological examples where the mathematical tools are useful. A key goal is to introduce the mathematical language, so that students can better choose later mathematics classes and look up mathematical concepts on their own (e.g., using Wolfram Alpha).

# 8 Textbooks and Software

We will work from the course notes. There is not a required textbook, although for reference you should familiarize yourself with a copy of something like Boas (2006) or Arfken et al. (2013). You might also check out Wilks (2011), and Snieder (2004), which are in the library. Arfken et al. (2013), which is similar to Boas, is available electronically through the Brown Library.

We will solve problems drawn from many geophysics and geology textbooks (LeBlond and Mysak, 1978; Turcotte, 1997; Schubert et al., 2001; Turcotte and Schubert, 2002; Aki and Richards, 2002; Drazin and Reid, 2004; Holton, 2004; Snieder, 2004; McWilliams, 2006; Vallis, 2006; Marshall and Plumb, 2008; Cushman-Roisin and Beckers, 2010; Fowler, 2011; Kaper and Engler, 2013; Bourguignon et al., 2015), but these books are not required for the course. If electronic copies of them are available at Brown, I have added an url to the bibliography here and on the course website. Sufficient background will be provided along with each problem so that no further reading will be required. You may want to use software, which is allowed for homework (although not required and you must still be able to explain your work without the program). I strongly recommend Matlab and Mathematica, but there are lots of others.

## 8.1 Applications

Geophysical and geological applications touched on in this class are:

- Global Energy Balance

  Ice Ages

  Energy Balance Models

- Data constrained models and maps

  Climate Variability Patterns

  Stochastic versus Deterministic Variability

- Waves and Oscillations

  Ocean Waves, Tides, and Tsunamis

  Earthquakes and Seismic Waves

  Diurnal, Seasonal, and Orbital Variation Cycles

  Dispersive Wave Kinematics: Phase & Group velocity

- Transport Budgets

  Diffusion and Advection

  Heat transfer

  Tracers in Fluids

  Rheology

- Boundary Layers

- Landscape Evolution

- Flows

  Oceanic

  Atmospheric

  Groundwater

  Mantle Convection

- Chemical Reactions, Rates, and Equilibria

- Mechanics

  of Solids

  of Fluids

- Gravity

  Potential and Conservative Forces

- Stratigraphy

  Dating and errors

  Mapping

### 8.2 Math Tools & Critical Concepts

A list of the mathematical topics to be touched on in this class, and associated critical concepts:

- Review of Mathematical Preliminaries (1.5 Weeks)

  Series and Sequences

  Real, Imaginary, Complex

  Trigonometry

  Exponentials and Logarithms

  Units and Dimensions

  Derivatives and Integrals

- Linear Algebra (2 Weeks)
  - Vector Spaces
  - Matrices and Linear Equations
  - Bases and Orthogonality; Rank; Null Space and Span
  - Inverse Methods
  - Eigenvalues and Eigenvectors; Singular Value Decomposition/Empirical Orthogonal Functions/Principal Component Analysis
- Multivariate Calculus and Differential Geometry (2 Weeks)
  - Vectors
    - Coordinate Transformations
    - Rotation & Reflection, Angular Momentum, and Vorticity
    - Vector Differentiation and Integration: Div, Grad, Curl; Gauss, Green & Stokes
  - Tensors
    - Cartesian Tensors
    - Inner and Outer Products versus Matrix Multiplication
    - Symmetries: Principle of Tensor Covariance, Tensor Invariants, & Anisotropy
    - Curvilinear Coordinates and Transforms, especially Spherical Coordinates
    - Deriving Calculus Identities from Tensor Symmetries
- Differential Equations
  - Linear Ordinary Differential Equations and Dynamical Systems (2 Weeks)
    - Rate equations
    - First and Second Order Equations
    - Homogeneous and Inhomogeneous Equations
    - Linear and Nonlinear Equations
    - Series Solutions: Perturbation Analysis, Asymptotics, and Linearization
    - Sturm-Liouville Problems: Free Modes of Oscillation, Superposition
    - Time Series and Fourier Analysis
  - Linear Partial Differential Equations and Dynamical Systems (2 Weeks)
    - Boundary and Initial Value Problems
    - Separation of Variables
    - Laplace and Poisson Equations (applications of Elliptic PDEs)
    - Heat Flow and Wave Equations (applications of Elliptic and Hyberbolic PDEs)
    - Separation of variables in linear wave problems: Cramer's Rule and Oscillation Modes
    - Decompositions: Helmholtz Streamfunction and Potential, Toroidal and Polloidal, Polarization
- Chaos and Nonlinear Dynamics (1 Week)
- Probability and Statistics (1.5 Weeks)

# 9 Policies

## 9.1 Deadlines

Because of the reviewing process, the scheduling of assignments is tight. Thus, I will have to insist that all problem sets be turned in on time. If they are late, they will drop a letter grade. If they are really late (so that they mess up the next step in the reviewing process) they will be counted as missed and can not be made up. If you foresee that there are big problems coming up (medical, family, etc.) let me know *before* an assignment is due and we can figure something out.

## 9.2 Collaboration

I encourage you to work together, and I do not mind at all if you have similar problem sets or share figures or computer code. However, in this case, I want you to list all of your study group on each homework assignment (so I can avoid you peer-reviewing your group). You are all required to submit a version of each assignment as first author (that is, one that you wrote yourself), so don't submit identical versions of a problem. You need to be careful to cite your colleagues or the textbooks, websites, or papers you might be working from.

## 9.3 Miscellany

- Attendance is expected. If you will miss a class, please let me know when and why so I can be sure you'll get any announcements, etc.
- Clothing and behavior (e.g., cell & laptop use) should be appropriate for a learning environment.
- Discrimination and harassment will not be tolerated.
- Please contact me if you have any disabilities that require accommodation.

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