Fall 2019 GEOL0350–GeoMath Midterm Exam

1 Sock it to me

Consider the areas and rings: A) The frisbee surface is bounded by a black line around its rim. B) Most sock surface area is bounded by the stylish stripe.

1.1 Curl Theorem

The air is flowing around underneath the frisbee (while in flight) and within and through the sock (while tumbling in a dryer) gives an equal counterclockwise circulation: $\oint \vec{v} \cdot \mathrm{d}\vec{\ell} = 1\ \mathrm{m}^2/\mathrm{s}$. What's true of the curl of the air velocity penetrating the frisbee and penetrating the sock?

The area integrals over the frisbee and the sock of the component of the curl penetrating in the normal direction are equal and upward.

1.2 Divergence in the Sock

What can you say about the divergence of the air velocity within the sock?

The volume integral of the divergence of the air velocity within the sock is equal to the area integral of the air velocity's normal component flowing through the sock and its opening.

1.3 Divergence in the Sock, Part Deux

What's the divergence of the curl of the air velocity within the sock?

zero

1.4 Stable Frisbee Rotation

Assuming the air directly in contact with the frisbee must co-rotate with the frisbee, does this say about the curl of the velocity of the air flow beneath and above a frisbee?

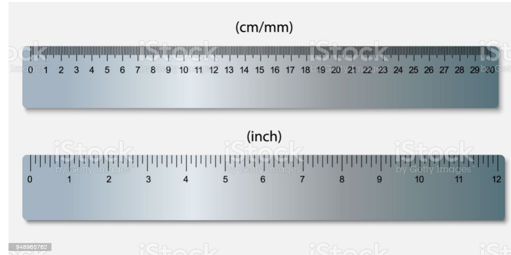
The curl of the air velocity above and below the frisbee must have a normal component through the frisbee that is continuous on average.

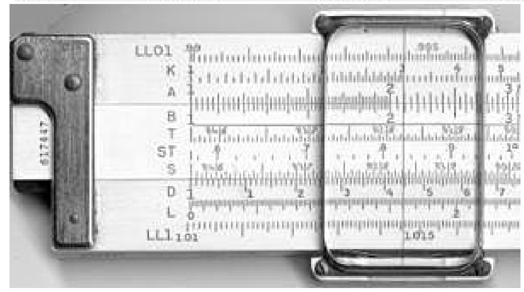
1.5 Stable Sock Situation

To keep a sock up, the elastic exerts a force inward from the stripe. What is the circulation of the force around the stripe and curl of this force through the sock?

The circulation is zero, because the force is perpendicular to the line elements. The curl is thus also zero. More generally, elastics are conservative and the curl of any conservative force is zero.

2 I forgot my slide rule!





We spent some time talking about linear functions and dimensional consistency and writing laws that are independent of units. Let's put these together!

2.1 Linear?

Is the function to transform from cm to mm linear?

Yes, the function is linear, as it is just multiplication by 10 and multiplication by a scalar is a linear function.

2.2 Try again

Show that our checks for linearity apply, namely $f(x_1 + x_2) = f(x_1) + f(x_2)$ and $f(ax) = af(x)$.

$10(x_1 + x_2) = 10x_1 + 10x_2$. $10(ax) = a(10x)$.

2.3 Inches?

Is the conversion from mm to inches linear?

Yes, the conversion to inches is linear as well, as well as the conversion to any unit of length.

2.4 What about on log-log paper?

Consider plotting distances on log-log paper in cm and mm and inches (e.g., draw a line from log(1) cm to log(10) cm, and then draw the line from log(10) mm to log(100) mm, etc.). Is the relationship between distances *as drawn on the log paper* still linear? What's the constant of proportionality between these lengths?

All of these lengths will be *equal*, just their starting point on the graph will be different, as $\log(a\Delta x) = \log a + \log \Delta x$.

3 The crookedness of the treacherous destroys them.

Solve the following sets of equations. Hint: the matrix inverse is

$$\mathbf{A} \equiv \begin{bmatrix} a & b \\ c & d \end{bmatrix}. \qquad \mathbf{A}^{-1} = \frac{1}{ad - cb} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}. \tag{1}$$

3.1 Straight

$$\begin{aligned} 2x + y &= 5, \\ -x + 2y &= 5 \end{aligned}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

3.2 Crooked

$$\begin{aligned} 5x - 10y &= -5, \\ -x + 2y &= 1 \end{aligned}$$

Here the determinant is zero and the equations are linearly proportional, so there are an infinite number of solutions where $-x + 2y = 1$ for any x.

3.3 Wicked

$$\begin{aligned} 5x - 10y &= -4, \\ -x + 2y &= 1 \end{aligned}$$

Here the determinant is zero and the equations are inconsistent, so there are no solutions.

4 Inconceivable!

Indicate why the following equations are "1. impossible to imagine or think of, 2. extremely unlikely" (Cambridge English Dictionary).

4.1 Q1

$$a = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots \longrightarrow \frac{-1}{a} = -1 - 2 - 4 - 8 + \dots$$

LHS is convergent. RHS is divergent, and not the inverse of convergent, because division does not distribute over addition in this way. Sign change is irrelevant.

4.2 Q2

$$\begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

LHS is a 1x3 matrix/vector, RHS is a scalar (dot product).

4.3 Q3

A fluid has positive curl ($\nabla \times \vec{v} > 0$) over a continuous area surrounded by negative curl ($\nabla \times \vec{v} < 0$) everywhere outside of that area. Thus, the circulation around any chosen contour is zero.

Just select either the region of positive curl only or negative curl only to find positive or negative, i.e., nonzero circulation.

4.4 Q4

Lincoln Chafee, former governor of RI, ran for president of the USA in 2016 on a platform that included converting to the metric system. Lincoln Chafee is 2 years tall and 68 centimeters old.

Dimensions! But also, the notion that anyone running on enthusiasm for the metric system winning in the USA.

4.5 Q5

Which are *inconceivable* for any three matrices of equal, square size, $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$:

$$\mathbf{AB} = \mathbf{BA} \qquad \mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{B} + \mathbf{C} + \mathbf{A} \qquad \mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$$

Matrice multiplication does not commute in general, so the first is extremely unlikely, i.e., inconceivable. Matrix addition does commute, so second is OK. Matrix multiplication does associate, so the last is true.