Spring 2026 EEPS1820
Homework 3, due Feb 23, 11:59PM

COVERING: Wyngaard Ch 5; Vallis Ch 3; Thorpe Ch 4

1 Wyngaard (2010) Problem 5.5

5.5 Derive the fluctuating vorticity equation.

a) Hint: begin from the following vorticity equation.

The vorticity ω_i is the curl of velocity; in tensor notation it is

$$\omega_i = \epsilon_{ijk} \frac{\partial u_k}{\partial x_j}. \tag{1.27}$$

Its conservation equation is

$$\frac{D\omega_i}{Dt} = \frac{\partial \omega_i}{\partial t} + u_j \frac{\partial \omega_i}{\partial x_j} = \omega_j \frac{\partial u_i}{\partial x_j} + \nu \frac{\partial^2 \omega_i}{\partial x_j \partial x_j}. \tag{1.28}$$

2 Wyngaard (2010) Problem 5.6

5.6 Use the result of Problem 5.5 to derive the conservation equation for the variance of fluctuating vorticity. Scale it; what are the two leading terms?

3 Vallis (2019) Problem 3.1

3.1 Consider two-dimensional, incompressible, fluid flow in a rotating frame of reference on the f-plane. Linearize the equations about a state of rest to obtain the momentum equations:

$$\frac{\partial u}{\partial t} - fv = -\frac{\partial \phi}{\partial x}, \qquad \frac{\partial v}{\partial t} + fu = -\frac{\partial \phi}{\partial y}. \tag{P3.1}$$

(a) Ignore the pressure term and determine the solution to the resulting equations. Show that the speed of fluid parcels is constant. Show that the trajectory of the fluid parcels is a circle with radius $|U|/f$, where $|U|$ is the fluid speed. (These solutions are inertial oscillations.)

(b) What is the period of oscillation of a fluid parcel?

(c) If parcels travel in straight lines in inertial frames, why is the answer to (b) not equal to the rotation period of the frame of reference? (See also Problem 4.3.)

a) Hint: Without loss of generality, align the coordinate system so that the initial velocity is in the x direction: $u_0 = |U|, v_0 = 0$.

4 Vallis (2019) Problem 3.3

3.3 Consider a rapidly rotating (i.e., in near geostrophic balance) Boussinesq fluid on the f-plane.

(a) Show that the pressure divided by the density scales as $\phi \sim fUL$.

(b) Show that the horizontal divergence of the geostrophic wind vanishes. Thus, argue that the scaling $W \sim UH/L$ is an *overestimate* for the magnitude of the vertical velocity. Obtain a scaling estimate for the magnitude of vertical velocity in rapidly rotating flow.

(c) Using these results, or otherwise, discuss whether hydrostatic balance is more or less likely to hold in a rotating flow than in non-rotating flow.

5 Vallis (2019) Problem 3.5

3.5 Using approximate but realistic values for the observed stratification, calculate the buoyancy period for (a) the midlatitude troposphere, (b) the stratosphere, (c) the oceanic thermocline, (d) the oceanic abyss.

6 Vallis (2019) Problem 3.7

3.7 (*a*) Estimate the magnitude of the zonal thermal wind 5 km above the surface in the midlatitude atmosphere in summer and winter using approximate values for the meridional temperature gradient in Earth's atmosphere.
(*b*) Repeat the exercise for Venus and Mars.
(*c*) Repeat the exercise for Earth's ocean, 1 km below the surface.

b) For Mars, you can take $T = -125°C$ at the pole and $T = 20°C$ at the equator (https://www.space.com/16907-what-is-the-temperature-of-mars.html). For Venus, you can use $T = 460°C$ at the pole and $T = 460°C$ at the equator (https://www.universetoday.com/14306/temperature-of-venus/). No need to do both summer & winter.

References

Vallis, G. K. (2019). *Essentials of Atmospheric and Oceanic Dynamics*. Cambridge University Press.

Wyngaard, J. C. (2010). *Turbulence in the Atmosphere*. Cambridge University Press.