Spring 2026 EEPS1820
Homework 7, due Apr 3, 11:59PM

COVERING: Vallis Ch 8, 10; Wyngaard 5, 6, 7, 8, 9; Thorpe Ch 3

1 Vallis (2019) Problem 8.3

8.3 Following the same procedure used in Sections 8.3 and 8.7, obtain the necessary conditions for instability in the two-level quasi-geostrophic model in the case with uniform shear. Show that these conditions are consistent with the conditions for instability calculated directly with a normal-mode approach.

2 Vallis (2019) Problem 10.1

10.1 *Predictability.* The eddy turnover time of three-dimensional turbulence at a wavenumber k is given by $\tau_k = \varepsilon^{-1/3}k^{-2/3}$, and that of two-dimensional turbulence by $\tau_k = \eta^{-1/3}$ where k is the wavenumber and ε and η are constants (the energy and enstrophy cascade rates, respectively). Suppose that in weather prediction the error is confined to small scales and that the time taken for the error to contaminate the next largest scale (in a logarithmic sense) is τ_k, so that the time taken for an error at a small scale k_s to reach the large scales k_l is given by

$$T = \int_{k_s}^{k_l} \tau_k \, \mathrm{d}k. \qquad \text{(P10.1)}$$

If the inertial range extends indefinitely show that this time is infinite for classical two-dimensional turbulence and finite for three-dimensional turbulence, and discuss the implications for weather predictability and weather forecasting. If the atmosphere is two-dimensional down to a scale of 50 km, estimate a limit to weather predictability (e.g., a timescale in days), making sensible (and clearly stated) assumptions about the magnitude of the flow.

This problem had issues in the statement. I have contacted Vallis to get his take, and have updated to his response: Let's restate the problem as the predictability timescale depending on the average time, and k_s being the small scales (i.e., large wavenumbers) and k_l being the large scales (small wavenumbers). Also, the differential should go with the logarithm of k, which is because the way to think about time here is how long it takes for an error at small scales to pass to a larger scale that is a *multiple* of the original scale, e.g., wavenumber $k/2$, not a fixed additive dk larger.

$$T = \int_{k_l}^{k_s} \tau(k)\, d(\ln k) \tag{1}$$

Vallis recommended reading a few papers as well (Vallis, 1985; Lorenz, 1969b,a; Kraichnan, 1971; Lilly, 1972). Lorenz (1969b) is particularly helpful.

3 Wyngaard (2010) Problem 2.16

2.16 Before Kolmogorov published his 1941 hypotheses about the dissipative scales in turbulence, Taylor (1935) had shown that $\epsilon \sim \nu u^2/\lambda^2$, with λ a length scale defined through the behavior of the autocorrelation function at the origin (Part III). (In his honor λ is now called the Taylor microscale.) He interpreted λ as the size of the dissipative eddies. Using $\epsilon \sim \nu u^2/\lambda^2$, determine the Reynolds number $u(\lambda)\lambda/\nu$ of λ-sized eddies, $u(\lambda)$ being the velocity characteristic of λ-sized eddies. Would you say they are directly influenced by viscosity? Develop an expression for λ/η. Was Taylor correct in interpreting λ as the size of the dissipative eddies?

4 Wyngaard (2010) Problem 8.6

8.6 Show that mixing ratio is a conserved variable.

5 Wyngaard (2010) Problem 9.6

9.6 Sketch the profile of vertical temperature flux in a quasi-steady convective ABL capped by an inversion.

6 Wyngaard (2010) Problem 9.15

9.15 Write an expression for a turbulence Rossby number, the ratio of typical inertial and Coriolis forces on energy-containing eddies. Estimate its magnitude in the ABL.

7 Thorpe (2007) Problem 3.5

P3.5 (E) The Kolmogorov scale in a boundary layer. How does the Kolmogorov length scale, l_K, vary with distance from the seabed, with the friction velocity and with the stress within a law-of-the-wall constant-stress layer if $C_D = 2.5 \times 10^{-3}$? Estimate l_K at a height of 1 m from the bed in tidal flows of 0.2 and 1 m s^{-1}.

References

Kraichnan, R. H. (1971). Inertial-range transfer in two-and three-dimensional turbulence. *Journal of Fluid Mechanics*, 47(3):525–535.

Lilly, D. K. (1972). Numerical simulation studies of two-dimensional turbulence: Ii. stability and predictability studies. *Geophysical & Astrophysical Fluid Dynamics*, 4(1):1–28.

Lorenz, E. (1969a). 3 approaches to atmospheric predictability. *Bulletin of the American Meteorological Society*, 50:345–351.

Lorenz, E. N. (1969b). The predictability of a flow which possesses many scales of motion. *Tellus*, 21(3):289–307.

Thorpe, S. A. (2007). *An introduction to ocean turbulence*. Cambridge University Press, Cambridge.

Vallis, G. K. (1985). Remarks on the predictability properties of two-and three-dimensional flow. *Quarterly Journal of the Royal Meteorological Society*, 111(470):1039–1047.

Vallis, G. K. (2019). *Essentials of Atmospheric and Oceanic Dynamics*. Cambridge University Press.

Wyngaard, J. C. (2010). *Turbulence in the Atmosphere*. Cambridge University Press.