Fall 2019 GEOL0350–Homework 10 (Last One!) Based on Chapter 13 of the Notes (Stats)

1 Shop till you drop

Problem 15.1.10 of Boas (2006). A shopping mall has four entrances, one on the North, one on the South, and two on the East. If you enter at random, shop and then exit at random, what is the probability that you enter and exit on the same side of the mall?

There are 4 entrances N, S, Ea, Eb. We assume equal probability of leaving or exiting by each (uniform distribution). Thus, the possible equal likelihood, mutually exclusive combinations are:

	N	S	Ea	Eb
N	*NN*	NS	NEa	NEb
S	SN	*SS*	SEa	SEb
Ea	EaN	EaS	*EaEa*	*EaEb*
Eb	EaN	EaS	*EaEa*	*EaEb*

where rows are entering and column are exiting. Italics denote the positive outcome (enter and exit on same side). Thus, the probability of entering and exiting on the same side is 6/16.

2 Prejudice and Bayes

In class, we mentioned the logical fallacy: "which is more likely: a Brown student is an engineer, or a Brown student with glasses is an engineer?" This is commonly taken to be an example of improper statistical thinking. Use the following statistics to make some guesses: at Brown the number of engineering concentrators is about 100 per year out of about 1700 undergraduate students per year. 64% of people wear glasses. Use Venn diagrams and Bayes's theorem to examine the following cases:

- a) what's $p(E)$: the likelihood that a Brown student is an engineer?
- b) What's $p(g)$, the likelihood that someone wears glasses?
- c) What's $p_g(E)$: the likelihood that a Brown student with glasses is an engineer if wearing glasses and being an Engineer are statistically independent?
- d) The fallacy exposed: What can you say about the number of engineering students who wear glasses $N(Eg)$, versus the number of engineering students $N(E)$? What does this say about the probability that any student at Brown is one or the other of these $(p(Eg), p(E)$?
- e) What is the likelihood that a Brown student with glasses is an engineer ($p(Eg)$) if the rate of wearing glasses among engineers ($p_E(g)$) is 100%?
- Finally, here's what the fallacy seems to suggest, but actually doesn't: how does $p_E(g)$ compare to $p(g)$ and $p_g(E)$ compare to $p(E)$ given $N(Eg)/N(E) > N(g)/N$?

a) $p(E) = 100/1700 \approx 0.059$. b) $p(g) = 0.64$ c) $p_g(E) = p(Eg)/p(g) = p(E)p(g)/P(g) = p(E) = 100/1700$. d) $N(Eg) \leq N(E)$, thus $p(Eg) \leq p(E)$. e) $p(Eg) = p_E(g)p(E) = 1.0p(E) = 100/1700$. f) $\frac{p_E(g)}{p(g)} = \frac{p(Eg)}{p(E)p(g)} = \frac{N(Eg)N}{N(E)N(g)} > 1$, $\frac{p_g(E)}{p(E)} = \frac{p(Eg)}{p(g)p(E)} = \frac{N(Eg)N}{N(g)N(E)} > 1$, and indeed $\frac{p_E(g)}{p(g)} = \frac{p_g(E)}{p(E)}$.

3 Radioactive

Problem 15.6.5 of Boas (2006). The probability for a radioactive particle to decay between time t and time $t + \mathrm{d}t$ is proportional to $e^{-\lambda t}$. Find the probability density function $f(t)$ and the cumulative distribution function $F(t)$. Find the expected lifetime (called the mean life) of the radioactive particle. Compare the mean life and the so-called "half life" which is defined as the value of t when$e^{-\lambda t} = 1/2$.

The probability density function for decay to be between t and $t + \mathrm{d}t$ is $\rho(x) \propto e^{-\lambda t}$. We know that if we integrate over all (positive) distances, we will find a decay sometime, so if we choose a coefficient C, such that $\rho(t) = Ce^{-\lambda t}$, then

$$\begin{aligned} 1 &= \int_0^\infty \rho(t)\,\mathrm{d}t = \int_0^\infty Ce^{-\lambda t}\,\mathrm{d}x = \int_0^\infty \frac{C}{\lambda}e^{-\lambda t}\,\mathrm{d}\lambda x = \frac{C}{\lambda}\int_0^\infty e^{-y}\,\mathrm{d}y = \frac{C}{\lambda}(1-0), \\ C &= \lambda, \\ \rho(t) &= \lambda e^{-\lambda t}. \end{aligned}$$

The cumulative distribution function, mean life, and the half-life are

$$\begin{aligned} F(t) &= \int_0^t \lambda e^{-\lambda t}\,\mathrm{d}t = 1 - e^{-t\lambda}, \\ \langle t \rangle &= \int_0^\infty t\lambda e^{-\lambda t}\,\mathrm{d}t = \frac{1}{\lambda}, \\ e^{-\lambda t} &= \frac{1}{2} \to t = \frac{-1}{\lambda}\log\frac{1}{2} = \frac{\log 2}{\lambda} = \frac{0.6931\ldots}{\lambda}. \end{aligned}$$

4 Data Manipulate

Make up a small dataset of 10 or so data. a) Calculate the mean, variance, and standard error of the mean. b) Describe what statistics of the data are expected to have a distribution where the variance describes the spread and what statistics are expected to have a distribution where the standard error describes the spread of the statistic. c) Describe how jackknife estimation and bootstrap estimation can be used to produce a histogram categorizing the uncertainty in the mean. d) (optional) You may carry out the bootstrap and jackknife estimates computationally for extra credit.

dataset: 7.0605 0.3183 2.7692 0.4617 0.9713 8.2346 6.9483 3.1710 9.5022 0.3445. a) mean= 3.9782, variance= 13.0080, standard error = $\sqrt{13.0080}/\sqrt{10} = 1.1405$. b) The mean and variance of this data (generated from a random uniform distribution in matlab) describe the spread of the data points themselves, and would describe the histogram of more points drawn from the same distribution. The mean and standard error give the statistics of the *sample mean* of 10 points averaged together. This distribution is more normal (Gaussian) than the original distribution and also more narrow (smaller standard deviation by $\sqrt(10)$). c) In this case, there are 10 jackknife estimates of the mean, which are: 3.6357 4.3848 4.1125 4.3689 4.3123 3.5052 3.6481 4.0678 3.3644 4.3819. Each one is the average of 9 of the original data leaving one value out. A histogram can be made of these data, and it is clear that they cluster roughly around the mean of the original distribution with roughly the right standard error (mean of jackknife estimates is the same as sample mean, 3.9782, but in this case the variance of the jackknifes is much smaller than predicted by central limit theory 13.0080/10 0=1.30. Jackknife variance is 0.1606). To form a distribution using bootstrapping, we randomly choose sets of 10 data from the original data with replacement (i.e., repeated values are OK). Doing this, I found a mean of 3.9684, quite near the sample mean and a variances between 1.14 and 1.18. d) I did the manipulations using matlab functions jackknife and bootstrp: e.g., mean(jackknife(@mean,A)) var(jackknife(@mean,A)) mean(bootstrp(10000,@mean,A)) var(bootstrp(10000,@mean,A)).

5 Mostly False

Use Table 13.4 to consider the following scenario.

- a) Suppose an unbiased single-investigator performs a study where ten independent possible linkages between evolution of angiosperms and plate tectonics are tested. The different possibilities are mutually exclusive and estimated to be equally likely, but only one is true. This investigator works hard on the method, and she estimates only a 5% risk of a false positive and a 5% risk of a false negative, but there is only the right kind of data to test 4 of the possible linkages (each with equal power of detection). i) What are the odds that her experiment will result in a yes relationship with the correct true linkage? ii) What are the odds that her experiment will result in a no relationship with the true correct linkage? iii) What are the odds that her experiment will result in a yes relationship with a false linkage? iv) What are the odds that her experiment will result in a no relationship with a false linkage?

- b) A different investigator works on finding relationships performs a study where ten independent possible linkages between water in the mantle and melting are tested. The different possibilities are mutually exclusive and estimated to be equally likely, but only one is true. This investigator works hard on the method, and he estimates only a 5% risk of a false positive and a 5% risk of a false negative, but there is only the right kind of data to test 6 of the possible linkages (each with equal power of detection). Unlike the investigator in a), this investigator is going up for tenure, and so really wants to publish a significant result. Thus, he *does not report all 6 possible linkages tested, instead only reports 4*, and so when writing up the paper is drawn toward reporting primarily the linkages that were detected as "true" in his study. This "file-drawering" of negative results and "over-hyping" of positive results can be modeled with a $u = 0.5$ bias, meaning that he is 50% more likely to report a positive result than a negative one. i) What are the odds that his experiment will result in a yes relationship with the correct true linkage? ii) What are the odds that his experiment will result in a no relationship with the true correct linkage? iii) What are the odds that his experiment will result in a yes relationship with a false linkage? iv) What are the odds that his experiment will result in a no relationship with a false linkage?

- c) Worldwide, the urgency of climate change has driven ten groups to consider possible linkages between temperature and carbon dioxide. Each of the 10 groups is able to reach the same experimental accuracies as the investigator looking into evolution and plate tectonics, and they are all unbiased (because they have built elaborate double-blind studies from fear of climate-gate like investigations by skeptics!) and independent from one another (i.e., they don't share data or methods until after the experiments and analysis are done). i) What are the odds that one group will report a yes relationship when testing the correct true mechanism? ii) What are the odds that one group will report a no relationship when testing the true correct mechanism? iii) What are the odds that one group experiment will report a yes relationship when testing a false mechanism? iv) What are the odds that one group will report in a no relationship when testing a false mechanism?

- d) Do your results make you concerned about the state of accuracy in the scientific literature?

a) $R = 1/9, \alpha = \beta = 0.05, c = 4$. i) research yes for true yes, $c(1-\beta)R/(R+1)/c = 0.095$. ii) research no for true yes, $c\beta R/(R+1)/c = 0.005$. iii) research yes for true no, $c\alpha/(R+1)/c = 0.045$. iv) research no for true no, $c(1-\alpha)/(R+1)/c = 0.855$. b) $R = 1/9, \alpha = \beta = 0.05, c = 4, u = 0.5$. i) research yes for true yes, $c((1-\beta)+u\beta)R/(R+1)/c = 0.0975$. ii) research no for true yes, $c(1-u)\beta R/(R+1)/c = 0.0025$. iii) research yes for true no, $c(\alpha + u(1-\alpha))R/(R+1)/c = 0.4725$. iv) research no for true no, $c(1-\alpha)(1-u)/(R+1)/c = 0.4275$. This researcher is thus about 50 times more likely to report a false positive than a true positive! c) $R = 1/9, \alpha = \beta = 0.05, c = 4, u = 0.0, n = 10$. i) research yes for true yes, $c((1-\beta^n))R/(R+1)/c = 0.1$. ii) research no for true yes, $c\beta^n R/(R+1)/c = 1 \cdot 10^{-14}$. iii) research yes for true no, $c(1-(1-\alpha)^n)/(R+1)/c = 0.361$. iv) research no for true no, $c(1-\alpha)^n/(R+1)/c = 0.539$. Positive results are 3.6 times more likely to be false positives than true positives! d) Absolutely!

6 Uniform Stats

Find the zeroth, first, and second moments (not normalized or centralized) of the continuous uniform distribution. Use these to derive the mean and variance of the continuous uniform distribution given in Table 13.2.

The continuous uniform probability density function is constant over the interval of possible values (here taken to be $a \le x \le b$) as in Table **??**. Thus,

$$\begin{aligned}
\rho(x; a, b) &= \frac{1}{b-a}, \\
\langle x^0 \rangle &= \int_a^b x^0 \frac{1}{b-a}\,\mathrm{d}x = \int_a^b \frac{1}{b-a}\,\mathrm{d}x = 1, \\
\langle x^1 \rangle &= \int_a^b x^1 \frac{1}{b-a}\,\mathrm{d}x = \int_a^b \frac{x}{b-a}\,\mathrm{d}x = \frac{b^2-a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \frac{b+a}{2}, \\
\langle x^2 \rangle &= \int_a^b x^2 \frac{1}{b-a}\,\mathrm{d}x = \int_a^b \frac{x^2}{b-a}\,\mathrm{d}x = \frac{b^3-a^3}{3(b-a)},
\end{aligned}$$

Then the mean and variance are

$$\begin{aligned}
\langle x \rangle = \langle x^1 \rangle &= \frac{b+a}{2}, \\
\langle x^2 \rangle - \langle x \rangle^2 &= \frac{b^3-a^3}{3(b-a)} - \frac{(b+a)^2}{4} = \frac{a^2+ab+b^2}{3} - \frac{a^2+2ab+b^2}{4} = \frac{(a-b)^2}{12}.
\end{aligned}$$

7 Creativenn

Make a Venn diagram that describes an aspect of your life or work. Does it reflect independence or mutual exclusivity?

I'm pretty happy with the ones in Fig. 13.3. Fig. 13.3b is a nice example of mutual exclusivity.

References

Boas, M. L., 2006. Mathematical methods in the physical sciences, 3rd Edition. Wiley, Hoboken, NJ.