Spring 2020 GEOL1820
Homework 2, due Monday, February 24, 9AM

1 Vallis (2019) Problem 3.2

3.2 Consider a dry, hydrostatic, ideal-gas atmosphere whose lapse rate is one of constant potential temperature. What is its vertical extent? That is, at what height does the density vanish? Is this result a problem for the any of the assumptions we have made in the first three chapters?

2 Vallis (2019) Problem 3.5

3.5 Using approximate but realistic values for the observed stratification, calculate the buoyancy period for (a) the midlatitude troposphere, (b) the stratosphere, (c) the oceanic thermocline, (d) the oceanic abyss.

3 Vallis (2019) Problem 4.4

4.4 Linearize the f-plane shallow water system about a state of rest. Suppose that there is an initial disturbance given in the general form

$$\eta = \iint \widetilde{\eta}_{k,l} \mathrm{e}^{\mathrm{i}(kx+ly)} \, \mathrm{d}k \, \mathrm{d}l, \tag{P4.2}$$

where η is the deviation surface height and the Fourier coefficients $\widetilde{\eta}_{k,l}$ are given, and that the initial velocity is zero.

(a) Obtain the geopotential field at the completion of geostrophic adjustment, and show that the deformation scale is a natural length scale in the problem.

(b) Show that the change in total energy during the adjustment is always less than or equal to zero. Neglect any initial divergence.
N.B. Because the problem is linear, the Fourier modes do not interact.

4 Vallis (2019) Problem 4.5

4.5 If energy conservation is one of the most basic physical laws, how can energy be lost in geostrophic adjustment?

5 Vallis (2019) Problem 4.7

4.7 In the shallow water equations show that, if the flow is approximately geostrophically balanced, the energy at large scales is predominantly potential energy and the energy at small scales is predominantly kinetic energy. Define precisely what 'large scale' and 'small scale' mean in this context.

6 Vallis (2019) Problem 4.8

4.8 In the shallow water geostrophic adjustment problem, show that at large scales the velocity essentially adjusts to the height field, and that at small scales the height field essentially adjusts to the velocity field. Your derivation may be detailed and mathematical, but explain the result at the end in words and in physical terms.

7 Cushman-Roisin (1994) Problem 3-1

3-1. A laboratory tank consists of a cylindrical container 30 cm in diameter, filled at rest by 20 cm of fresh water and then spun at 30 rpm. After a state of solid-body rotation is achieved, what is the difference in water level between the rim and the center? How does this difference compare with the minimum depth at the center?

8 Cushman-Roisin (1994) Problem 3-3

3-3. Using the scale given in (3-34), compare the dynamic pressure induced by the Gulf Stream (speed = 1 m/s, width = 40 km, and depth = 500 m) with the main hydrostatic pressure due to the weight of the same water depth. Also, convert the dynamic-pressure scale to its equivalent height of hydrostatic pressure (head). What can you infer about the possibility of measuring oceanic dynamic pressures by a pressure gauge?

9 Cushman-Roisin and Beckers (2011) Problem 3-5

3-5. Within the Boussinesq approximation and for negligible diffusion in (3.24), show that for an ocean at rest, density can only be a function of depth: $\rho = \rho(z)$. (*Hint*: The situation at rest is characterized by the absence of movement and temporal variations.)

References

Cushman-Roisin, B. (1994). *Introduction to geophysical fluid dynamics.* Prentice Hall, Englewood Cliffs, N.J.

Cushman-Roisin, B. and Beckers, J.-M. (2011). *Introduction to geophysical fluid dynamics: physical and numerical aspects*, volume v. 101 of *International geophysics series.* Academic Press, Waltham, MA, 2nd ed edition.

Vallis, G. K. (2019). *Essentials of Atmospheric and Oceanic Dynamics.* Cambridge University Press.