

EDDIES, WAVES, AND FRICTION: UNDERSTANDING
THE MEAN CIRCULATION IN A BAROTROPIC OCEAN
MODEL

Baylor Fox-Kemper

Atmospheric and Oceanic Sciences Program, Princeton University

and

NOAA Climate and Global Change Program

October 28, 2003

Thanks:

Joseph Pedlosky, Paola Rizzoli (Thesis Advisors)

Glenn Flierl, Mike Spall, Vitalii Sheremet, Raffaele Ferrari
(Committee)

Steven Jayne (Computing Resources)

Geoffrey Vallis (Post-doctoral Host)

MIT, WHOI, GFDL, Princeton U., NOAA, and UCAR

0.1 Background

- **The Gulf Stream and the Wind-Driven Ocean Model**
(Sverdrup (1947), Stommel (1948), Munk (1950))
- **Inertial Domination/Vorticity Removal**
(Veronis (1966), Niiler (1966))
- **Multi-Gyre Internal Cancellation of Vorticity**
(Harrison and Holland (1981), Marshall (1984))
- **Limited Intergyre Mass Flux/Dissipative Meandering**
(Lozier and Riser (1989), Lozier and Riser (1990))

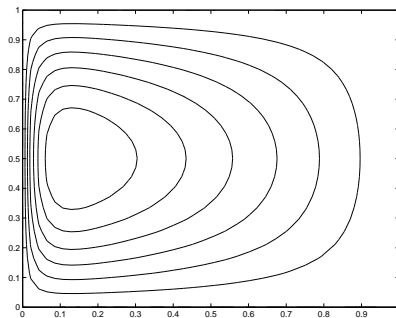
0.2 My Results

- **Control by Vorticity Flux to Enhanced Removal Region**
- **Boundary Conditions affect Intergyre Vorticity Flux**
- **Sinuious Modes affect Vorticity Flux Efficiency**
- **Resonance of Basin Mode Waves and their Nonlinear Self-Interaction**

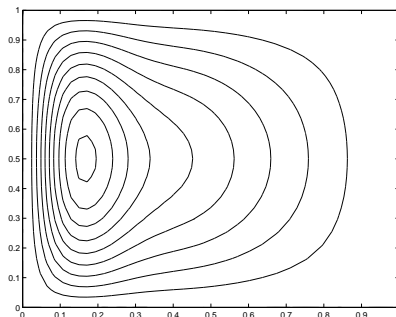
1 Sverdrup, Stommel, and Munk

(Contouring Streamfunction)

Stommel: Steady, Linear, Bottom Drag



Munk: Steady, Linear, Lateral (N-S) Friction



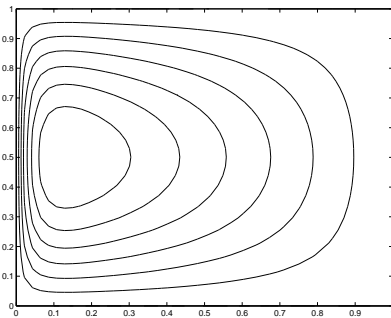
1.1 Linear Models

- Sverdrup (1947): oceanic context, linear depth-avg vort. eq. tenable
- Stommel (1948): bottom drag gives WBC as frictional return flow
- Munk (1950): asymptotic exp.: Navier-Stokes Fric. works, too.
- Require HUGE friction to produce correct BL width

2 Inertial Domination (a.k.a Runaway)

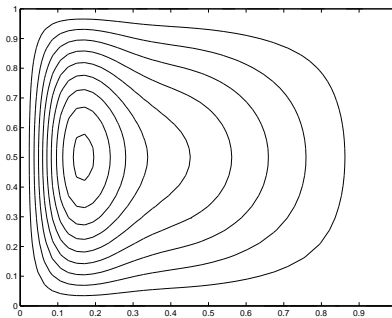
Stommel

$$\delta_I / \delta_S = 0$$

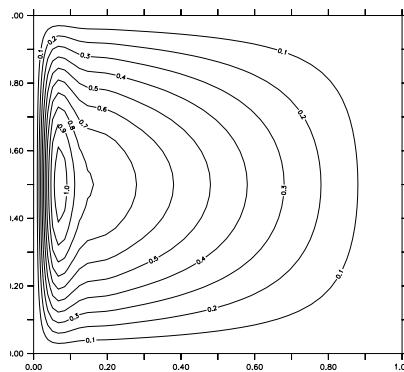


Munk

$$\delta_I^3 / \delta_M^3 \equiv \text{Re} = 0$$



Munk with Eddies: $\delta_I^3 / \delta_M^3 \equiv \text{Re} = 0$

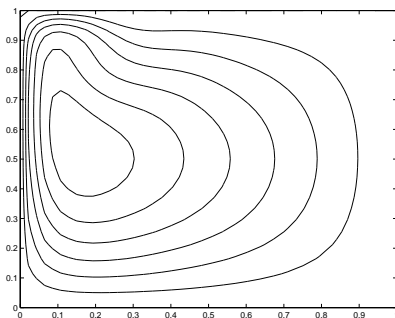


2.1 Single Gyre

- Mean flow fluxes vorticity to IWBC
- Eddies flux from IWBC to FSL
- Friction removes vorticity
- If bdy. visc. too small, inertia takes over basin
- Only eddies & fric. flux across mean streamlines

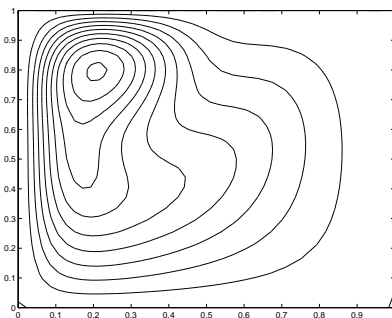
Stommel

$$\delta_I/\delta_S = 1$$

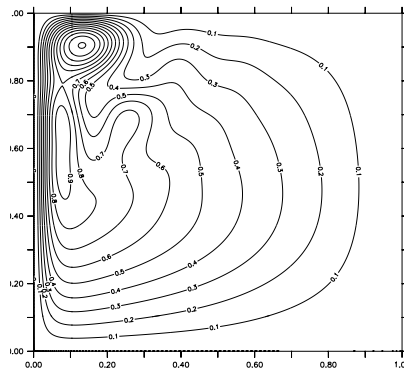


Munk

$$\delta_I^3/\delta_M^3 \equiv \text{Re} = 1$$



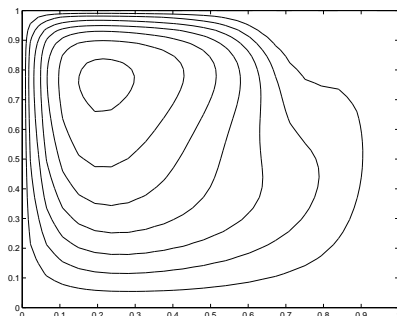
Munk with Eddies: $\delta_I^3/\delta_M^3 \equiv \text{Re} = 1$



- Mean flow fluxes vorticity to IWBC
- Eddies flux from IWBC to FSL
- Friction removes vorticity
- If bdy. visc. too small, inertia takes over basin
- Only eddies & fric. flux across mean streamlines

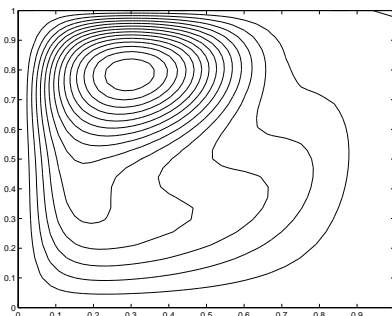
Stommel

$$\delta_I/\delta_S = 2$$

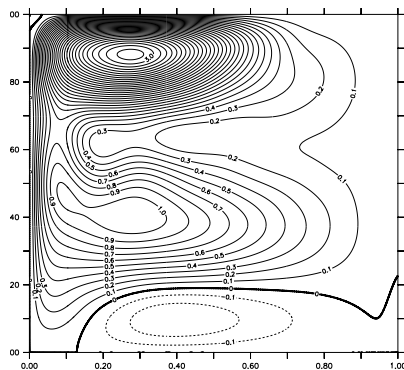


Munk

$$\delta_I^3/\delta_M^3 \equiv \text{Re} \approx 2$$



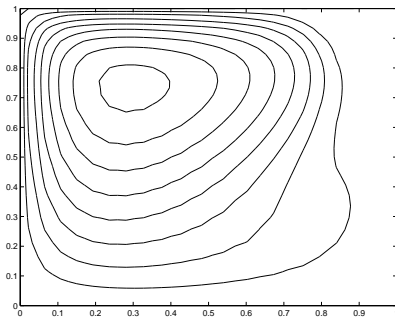
Munk with Eddies: $\delta_I^3/\delta_M^3 \equiv \text{Re} \approx 3$



- Mean flow fluxes vorticity to IWBC
- Eddies flux from IWBC to FSL
- Friction removes vorticity
- If bdy. visc. too small, inertia takes over basin
- Only eddies & fric. flux across mean streamlines

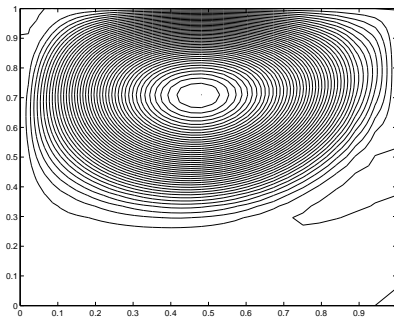
Stommel

$$\delta_I/\delta_S = 2.5$$

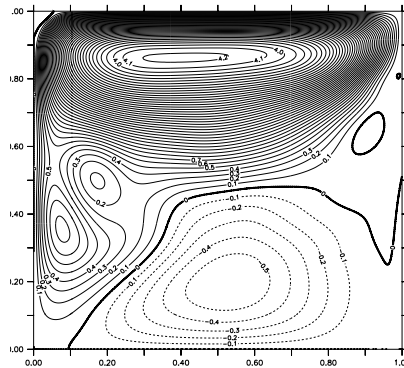


Munk

$$\delta_I^3/\delta_M^3 \equiv \text{Re} \approx 4.3$$

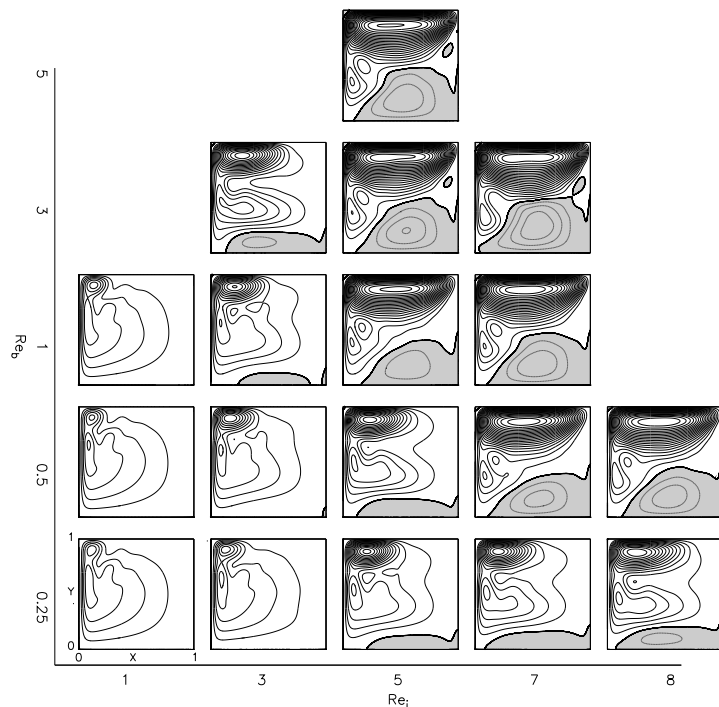


Munk with Eddies: $\delta_I^3/\delta_M^3 \equiv \text{Re} \approx 5$



- Mean flow fluxes vorticity to IWBC
- Eddies flux from IWBC to FSL
- Friction removes vorticity
- If bdy. visc. too small, inertia takes over basin
- Only eddies & fric. flux across mean streamlines

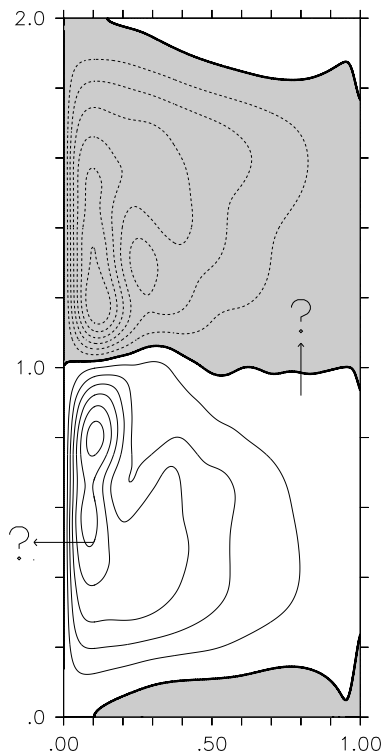
2.1.1 Circulation Control by Eddy Vorticity Flux to a Region of Enhanced Removal



Circulation Control

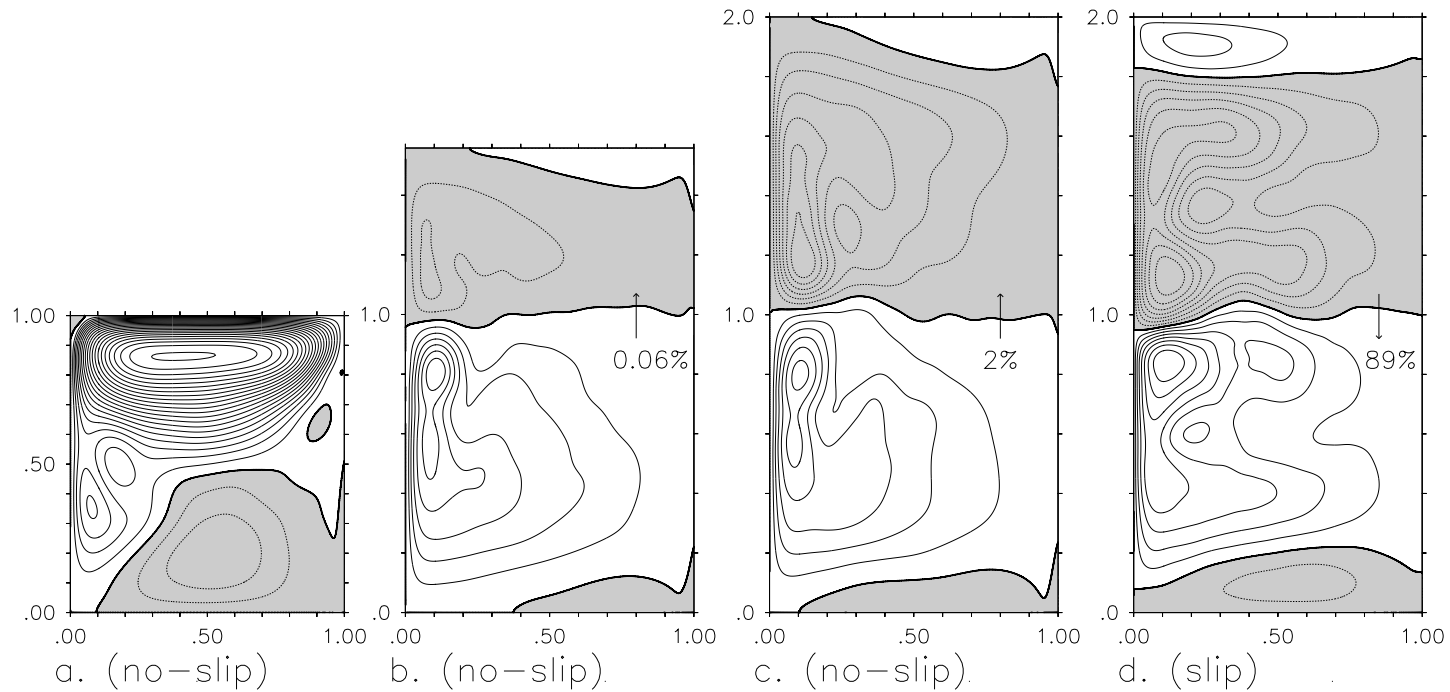
- Eddies cannot ultimately remove vorticity
- Increased viscosity in a narrow region near bdy. helps remove it
- IBL wider than FBL, even in western-intensified solns!
- Eddies *can* replace friction, but only in interior, not at bdy.

3 What about Multi-Gyres? Internal Cancellation?



- Will eddies dispose of vorticity by an intergyre eddy flux or by a flux to the frictional sublayer?
- Does internal cancellation control the circulation strength at high Re ?

3.1 Very Little intergyre Flux with No-slip

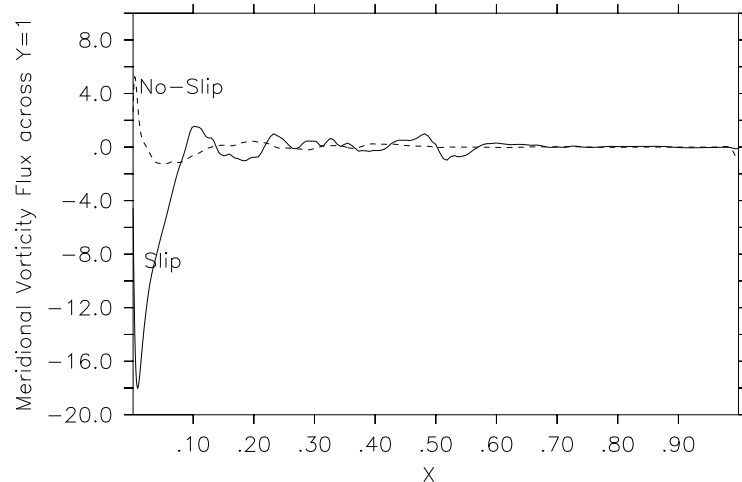


$\text{Re}(\text{bdy})=5, \text{Re}(\text{int})=5.$

Arrows=Eddy Vort. Flux as % of Subtrop. input=0.637. [Movie](#)

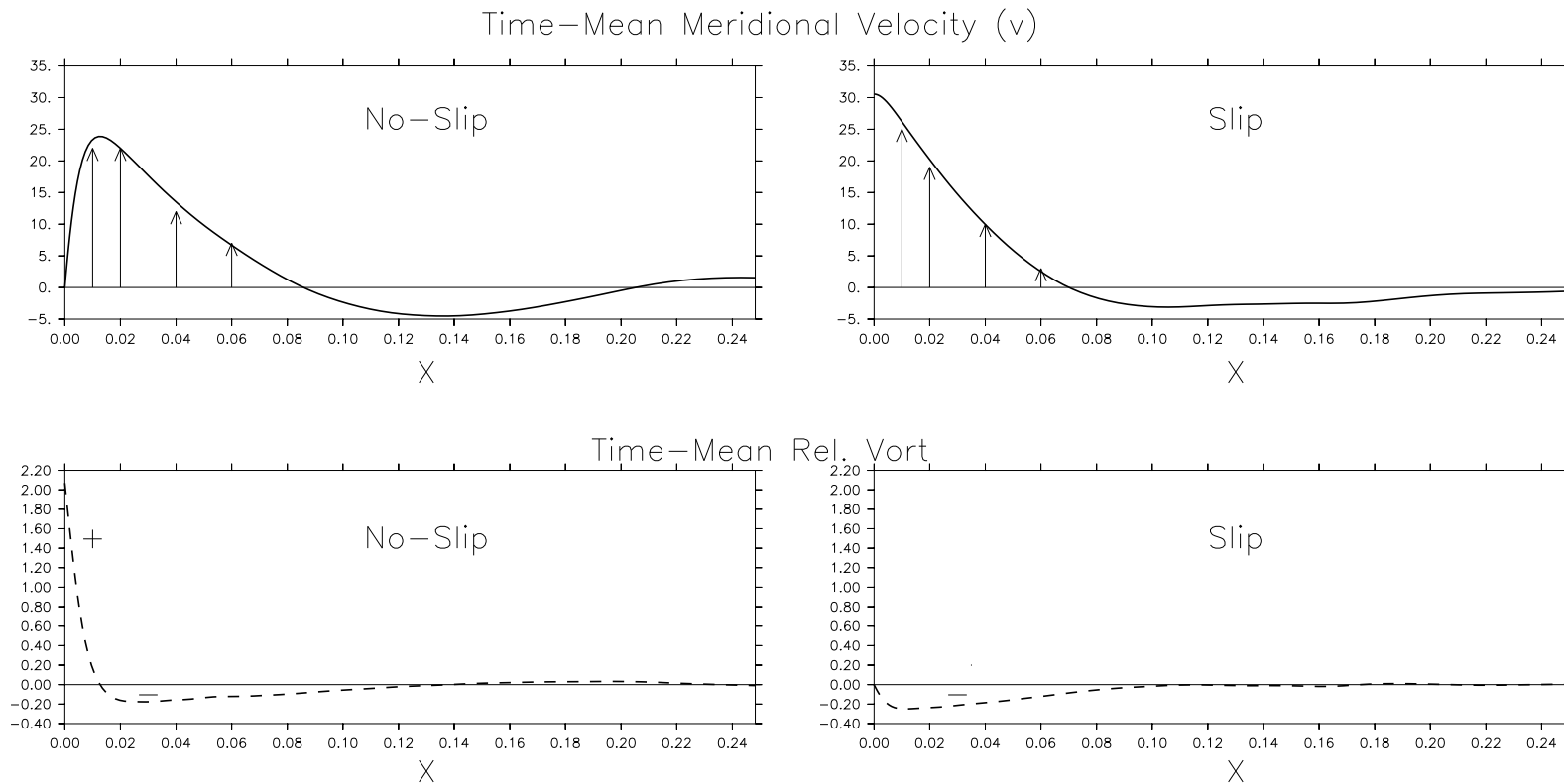
3.2 Conflict with Slippery BC results

Eddy flux across $y=1$



- Different from Harrison and Holland (1981), Marshall (1984), Lozier and Riser (1990) who use slippery bcs.
- Most intergyre eddy flux in slippery models is dissipative meandering, not by parcel exchange Lozier and Riser (1989), Berloff et al. (2002).

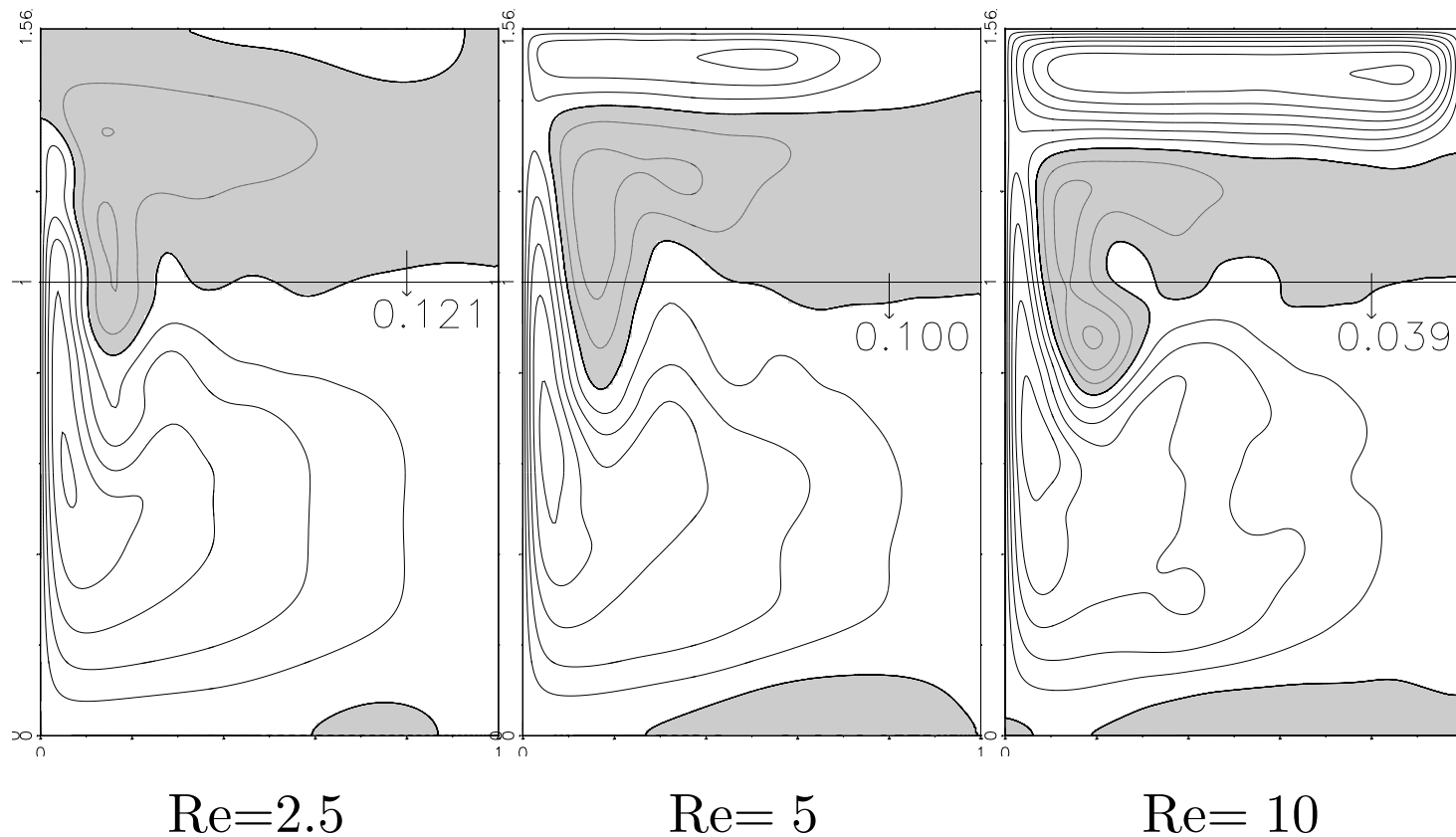
3.3 Intergyre Flux due to Rel. Vort. in BL



No dissipative meandering with no-slip because 1) separation point doesn't meander easily and 2) rel. vorticity in BL is different as in Stewart (1964). [Movie](#)

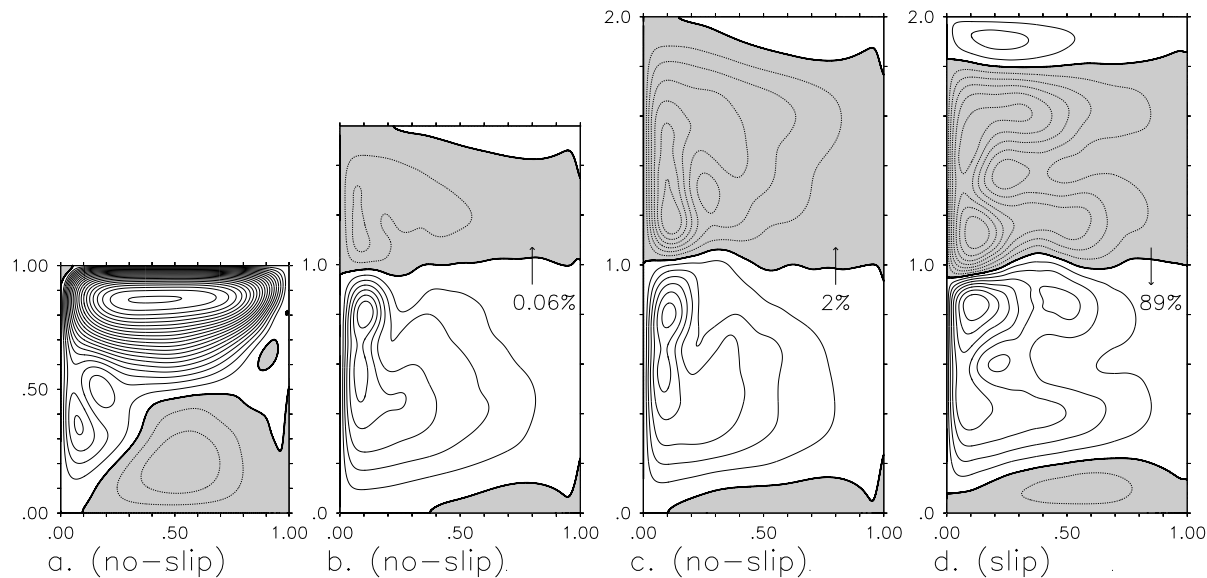
3.4 So why not Slip BCs & Eddy Fluxes?

Slip Two-Gyre: w/o *antisymmetric* wind, intergyre *eddy flux* not preferred, instead it's *mean flux*. Cessi (1991): stronger WBC no-slip/slip under/overshoots.



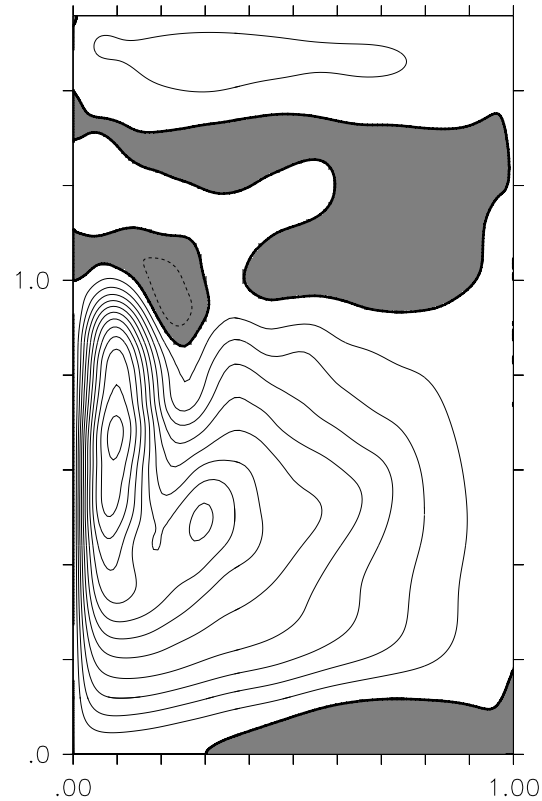
4 But why is No-Slip Multi-gyre Circ. Controlled?

Negligible intergyre eddy flux of vorticity, yet circ. is reduced with addition of a second gyre.



$$\text{Re}(\text{bdy})=5, \text{Re}(\text{int})=5.$$

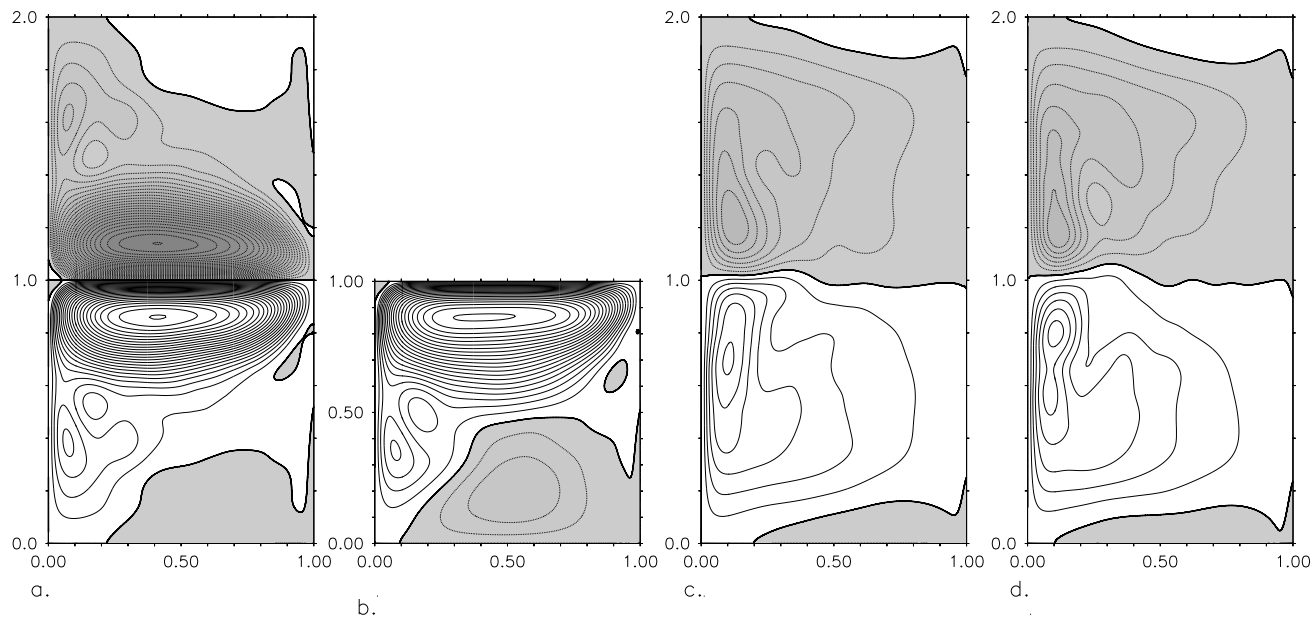
Circulation reduced even *without subpolar wind forcing!*



$\text{Re}(\text{bdy})=5$, $\text{Re}(\text{int})=5$, and no-slip boundary conditions.

5 Sinuous Modes

Removing northern boundary changes eddies that flux vorticity to the frictional sublayer. Rapidly-growing *sinuous modes* are then present: [Movie](#)



Rest initial

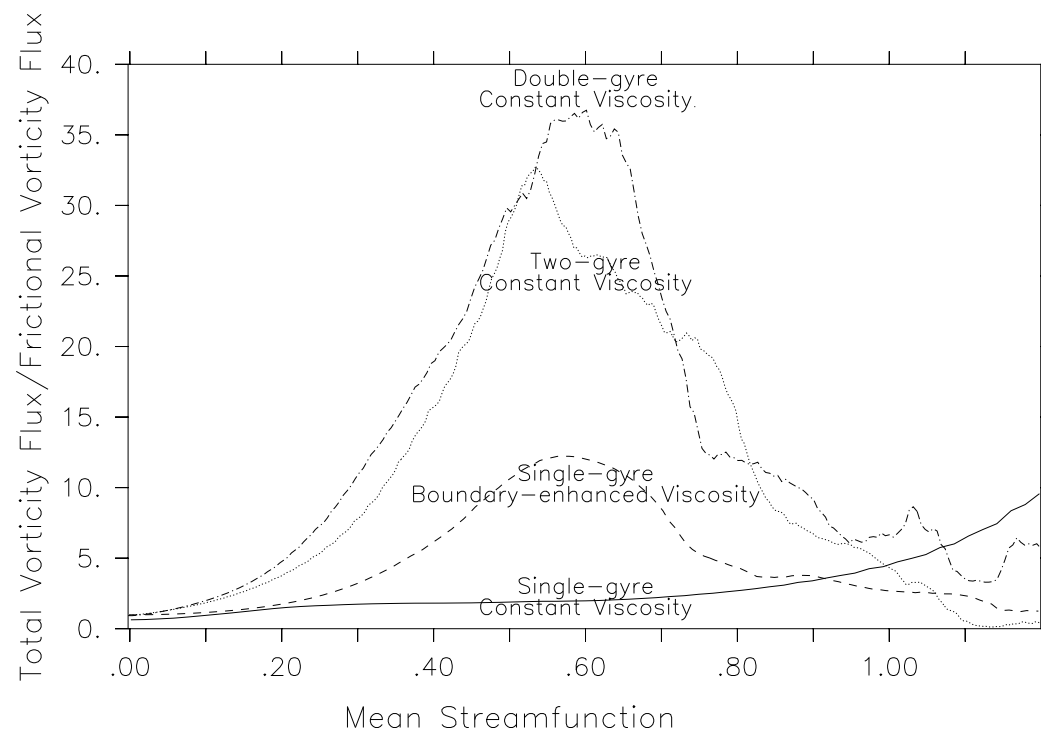
Single-gyre

Rest initial after

Asym. initial

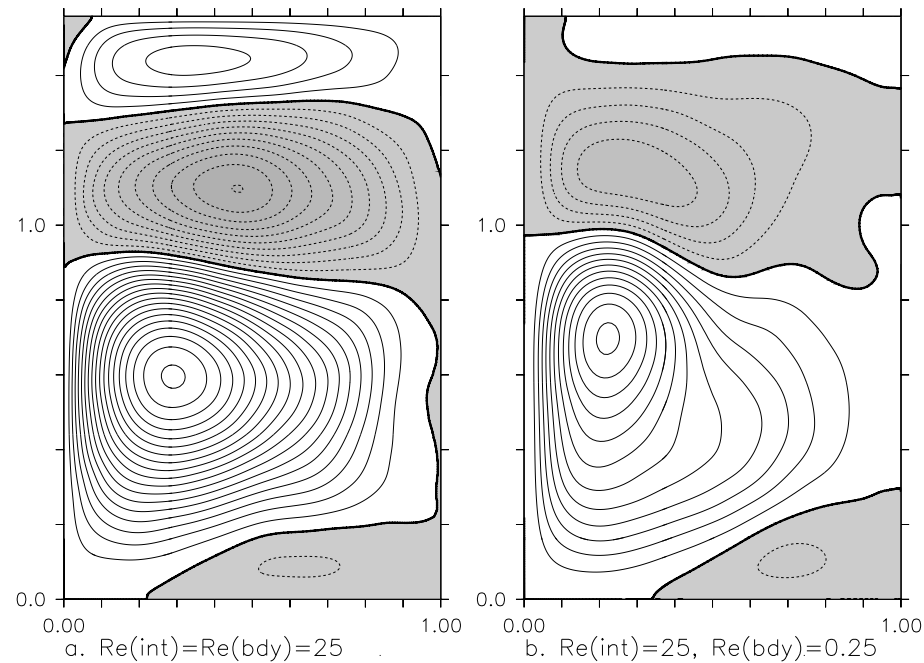
5.1 Sinuous Efficiency (Total Flux/Fric. Flux)

Sinuous modes known to mix strongly on either side of the jet: *e.g.*, Balmforth and Piccolo (2001), Rogerson et al. (1999). No intergyre flux needed!



5.2 Eventual Inertial Domination

At a sufficiently high Reynolds number inertial domination returns even with sinous modes,

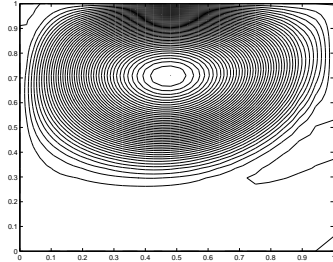


Thus, vorticity removal at high Re must still be considered.

6 Eddies \neq Friction: Non-locality, for example

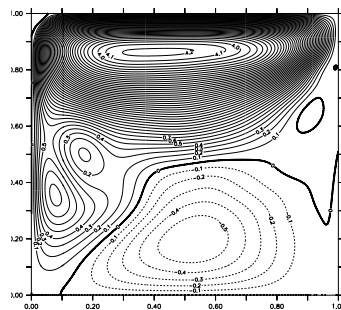
Munk Steady

$$\delta_I^3 / \delta_M^3 \equiv \text{Re} \approx 4.3$$



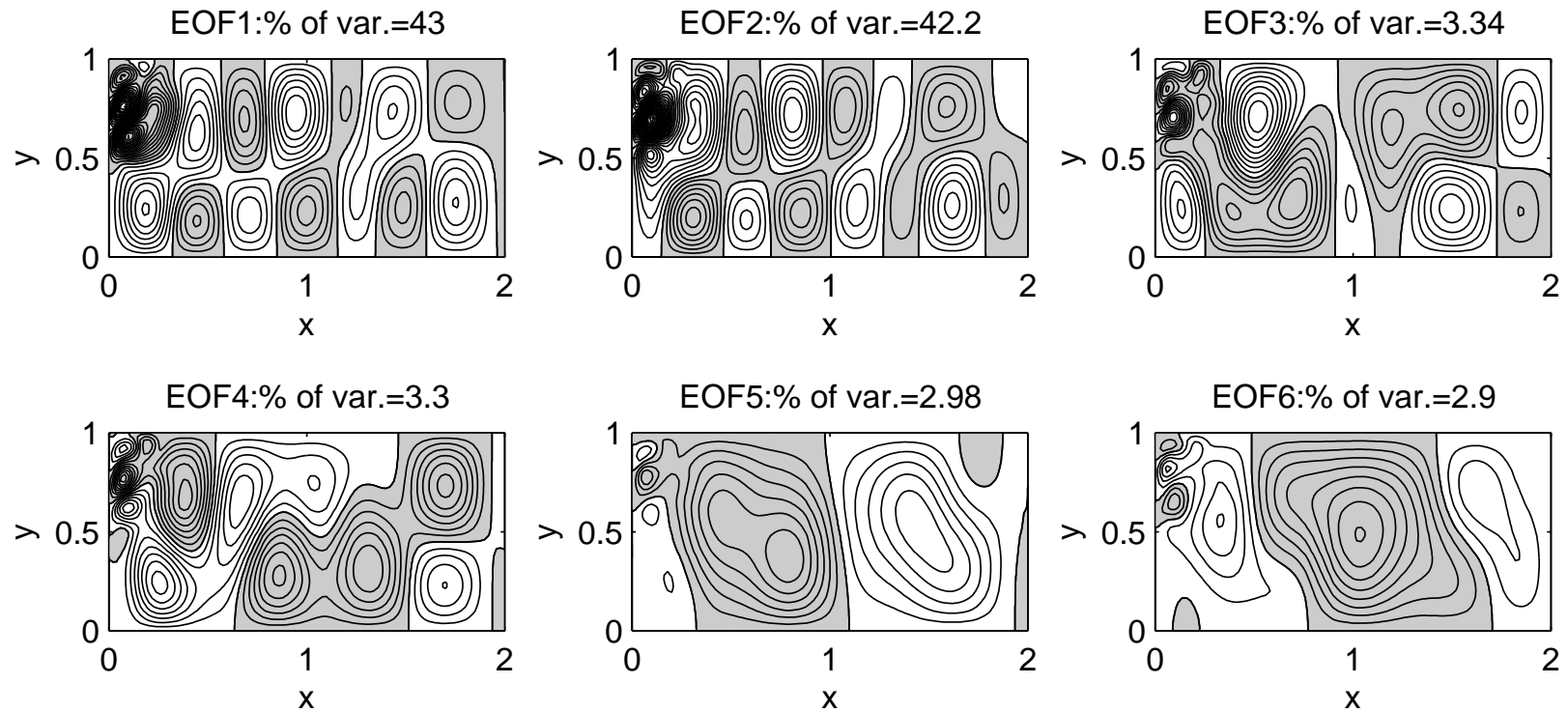
Munk with Eddies:

$$\delta_I^3 / \delta_M^3 \equiv \text{Re} \approx 5$$



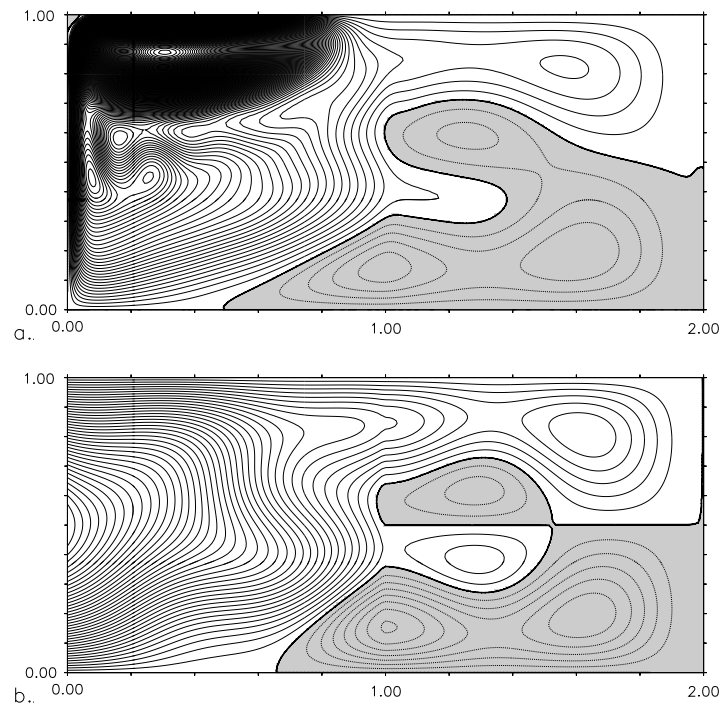
Note the *counter-rotating* regions only present in the time-dependent calculation.

They rotate in a sense opposite of the wind forcing!



By examining the EOFs of the variability, it becomes clear that most of the variance away from the WBC is in basin modes.

6.1 A Non-local Theory



Assuming that these are basin mode waves generated in WBC with the same variance as their EOF, a theory for the wave-mean flow in the interior explains counter-rotating gyres.

7 Conclusions

- Friction \neq eddies: only friction removes vorticity; eddies have barriers to transport, nonlocal effects, upgradient regions...
- However, eddies can prevent inertial domination so long as vorticity removal is assured.
- Sources and sinks of vorticity are not the only important consideration, *efficiency* of eddies also an important consideration.
- Sinuous modes increase the efficiency of vort. flux to the FSL and reduce circ. w/o requiring intergyre eddy flux.

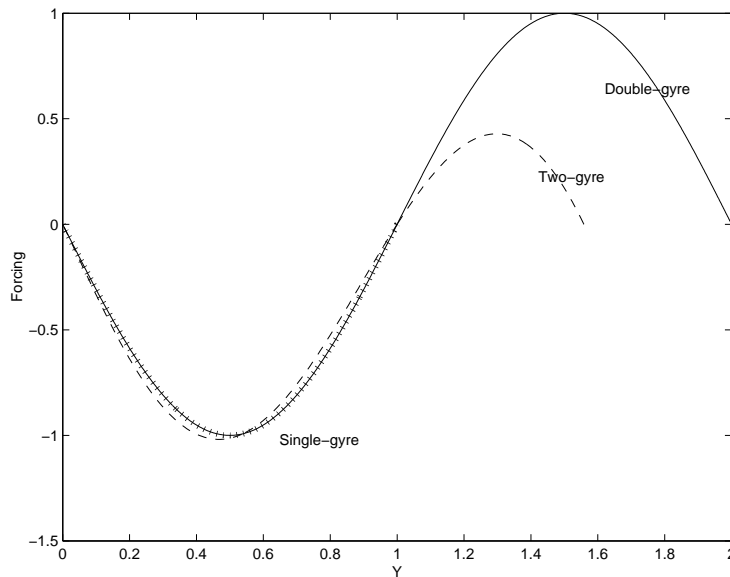
8 Implications?

- Eddy vort. flux is very important at high Re when mean streamlines are closed. True also in real ocean.
- The removal of vorticity at the boundary can be very important in determining the interior solution. *Nonlocal control*.
- If eddies are more efficient—as sinuous modes are—circulation strength can be reduced, but vorticity removal always important
- Intergyre eddy vort. flux seems to be restricted to symmetric slip double-gyre, probably not a major player in real ocean.

9 Issues?

- Baroclinicity? Thickness fluxes, outcropping, buoyancy budget.
- Precisely how does boundary remove vorticity? Perhaps bottom drag (Hughes and De Cuevas, 2001).
- What are the instabilities in the real ocean, and how efficient are they?
- Are basin modes active also in ocean, or just this model?

9.1 We Compare 3 Models: Vorticity Input



Single-gyre is in square basin.

Two-gyre is in asymmetric basin. $0 \leq y \leq 1.56$

Double-gyre is in symmetric basin. $0 \leq y \leq 2$

References

- Balmforth, N. J. and C. Piccolo: 2001, The onset of meandering in a barotropic jet. *Journal of Fluid Mechanics*, **449**, 85–114. 19
- Berloff, P. S., J. C. McWilliams, and A. Bracco: 2002, Material transport in oceanic gyres. part I: Phenomenology. *Journal of Physical Oceanography*, **32**, 764–796. 13
- Cessi, P.: 1991, Laminar separation of colliding western boundary currents. *Journal of Marine Research*, **49**, 697–717. 15
- Harrison, D. E. and W. R. Holland: 1981, Regional eddy vorticity transport and the equilibrium vorticity budgets of a numerical model ocean circulation. *Journal of Physical Oceanography*, **11**, 190–208. 3, 13
- Hughes, C. W. and B. A. De Cuevas: 2001, Why western boundary currents in realistic oceans are inviscid: A link between form stress and bottom pressure torques. *Journal of Physical Oceanography*, **31**, 2871–2885. 26

- Lozier, M. S. and S. C. Riser: 1989, Potential vorticity dynamics of boundary currents in a quasi-geostrophic ocean. *Journal of Physical Oceanography*, **19**, 1373–1396. 3, 13
- 1990, Potential vorticity sources and sinks in a quasi-geostrophic ocean: beyond western boundary currents. *Journal of Physical Oceanography*, **20**, 1608–1627. 3, 13
- Marshall, J. C.: 1984, Eddy mean flow interaction in a barotropic model. *Quarterly Journal of the Royal Meteorological Society*, **100**, 573–590. 3, 13
- Munk, W. H.: 1950, On the wind-driven ocean circulation. *Journal of Meteorology*, **7**, 79–93. 3, 5
- Niiler, P. P.: 1966, On the theory of the wind-driven ocean circulation. *Deep-Sea Research*, **13**, 597–606. 3
- Rogerson, A. M., P. D. Miller, L. J. Pratt, and C. K. R. T. Jones: 1999, Lagrangian motion and fluid exchange in a barotropic meandering jet. *Journal of Physical Oceanography*, **29**, 2635–2655. 19

Stewart, R. W.: 1964, Studies on oceanography: A collection of papers dedicated to Koji Hidaka. University of Washington Press, Seattle, Washington, chapter The Influence of Friction on Inertial Models of Oceanic Circulation, 3–9. 14

Stommel, H. M.: 1948, The westward intensification of wind-driven ocean currents. *Transactions, American Geophysical Union*, **29**, 202–206. 3, 5

Sverdrup, H. U.: 1947, Wind-driven currents in a baroclinic ocean; with application to the equatorial currents of the eastern Pacific. *Proceedings of the National Academy of Sciences*, **33**, 318–326. 3, 5

Veronis, G.: 1966, Wind-driven ocean circulation—part II. numerical solution of the nonlinear problem. *Deep-Sea Research*, **13**, 30–55. 3