



# Buoyancy Fluxes in the Ocean...

Friend or foe?

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Density Effects in Fluids Workshop  
Los Alamos, NM, Wed. 12/05/07, 9:00-10:00



# An Optimistic Outline

- Energy!
- Buoyancy!
- Geostrophy!
- Releasing Pent-up Energy!
- Mixing it Up!

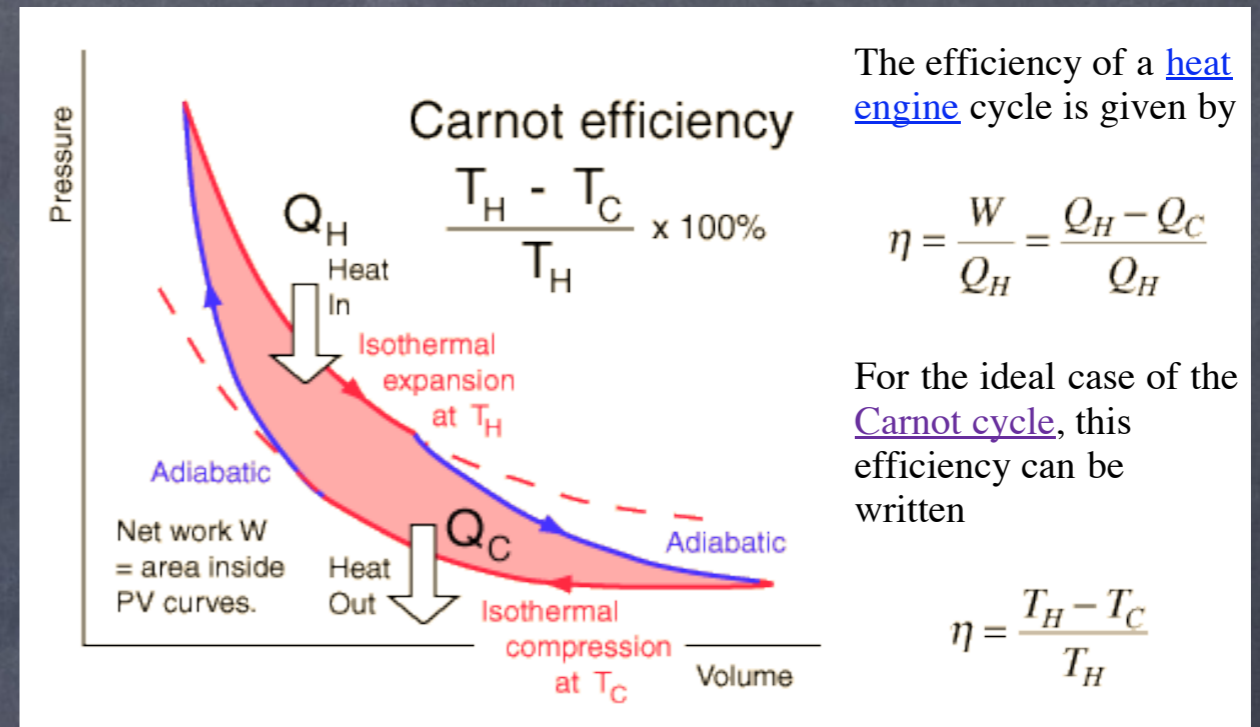


# Energy!

## The Atmosphere is a Heat Engine:

**Table 1** Global thermodynamic efficiencies versus spectral damping coefficient

Efficiency	Spectral damping coefficient (day <sup>-1</sup> )					
	0.005	0.01	0.05	0.1	0.15	0.2
$\eta$	9.12	9.21	8.74	8.17	6.69	7.29
$\eta_{rev}$	12.27	12.47	11.92	12.04	11.77	12.77
$\eta_c$	13.24	13.49	13.34	13.49	13.64	14.64



Adams & Renno 2005:

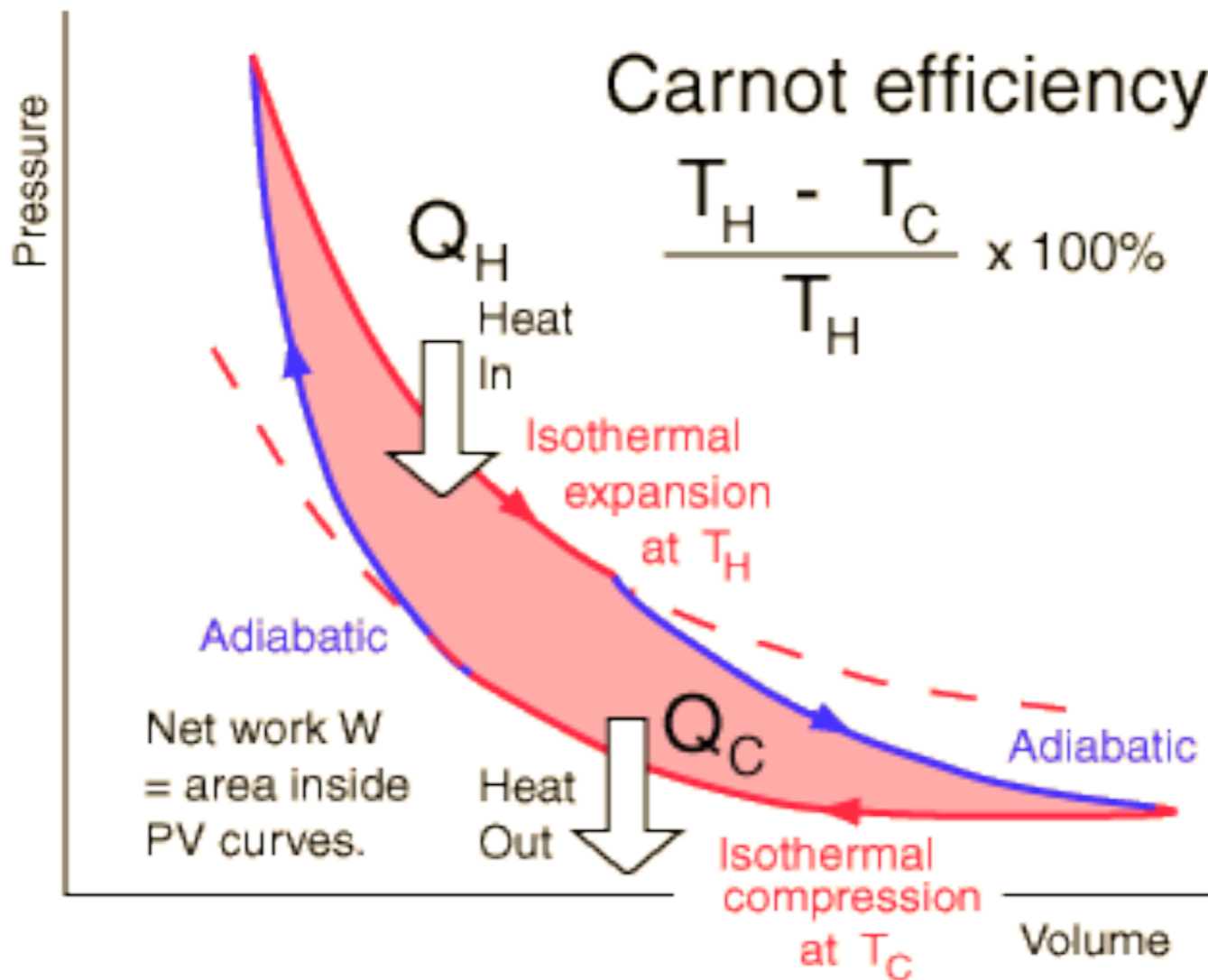
The atmosphere converts heating & cooling to kinetic energy that does work.

It's roughly 10–15% as efficient as ideal thermal engines.

Nave, 2005

# The Ocean is NOT

1 Atm



Carnot efficiency

$$\frac{T_H - T_C}{T_H} \times 100\%$$

The efficiency of a [heat engine](#) cycle is given by

$$\eta = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$$

For the ideal case of the [Carnot cycle](#), this efficiency can be written

$$\eta = \frac{T_H - T_C}{T_H}$$

Heat and Cool  
near the SURFACE:  
Almost no pressure  
difference!

Net work W  
= area inside  
PV curves.



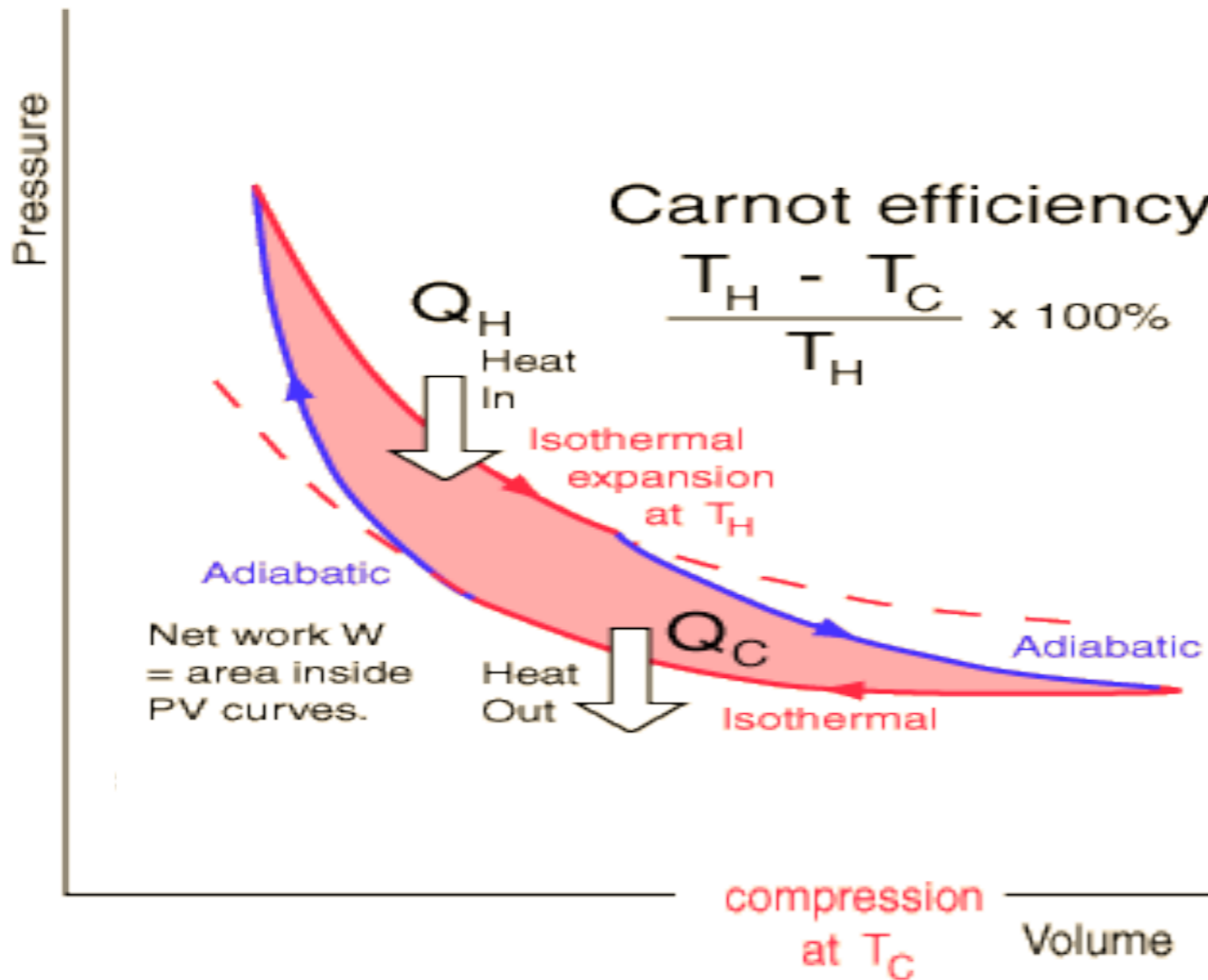
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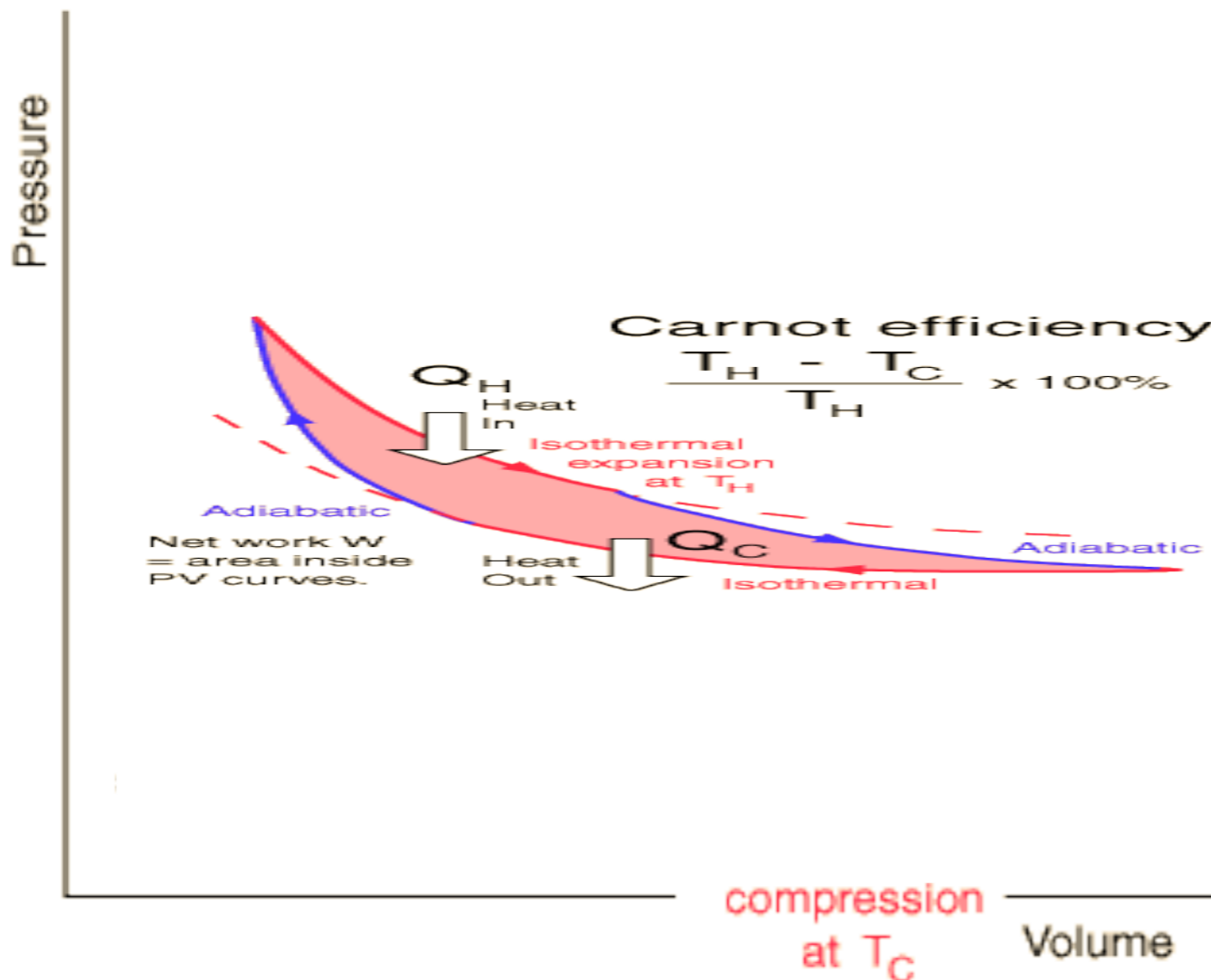
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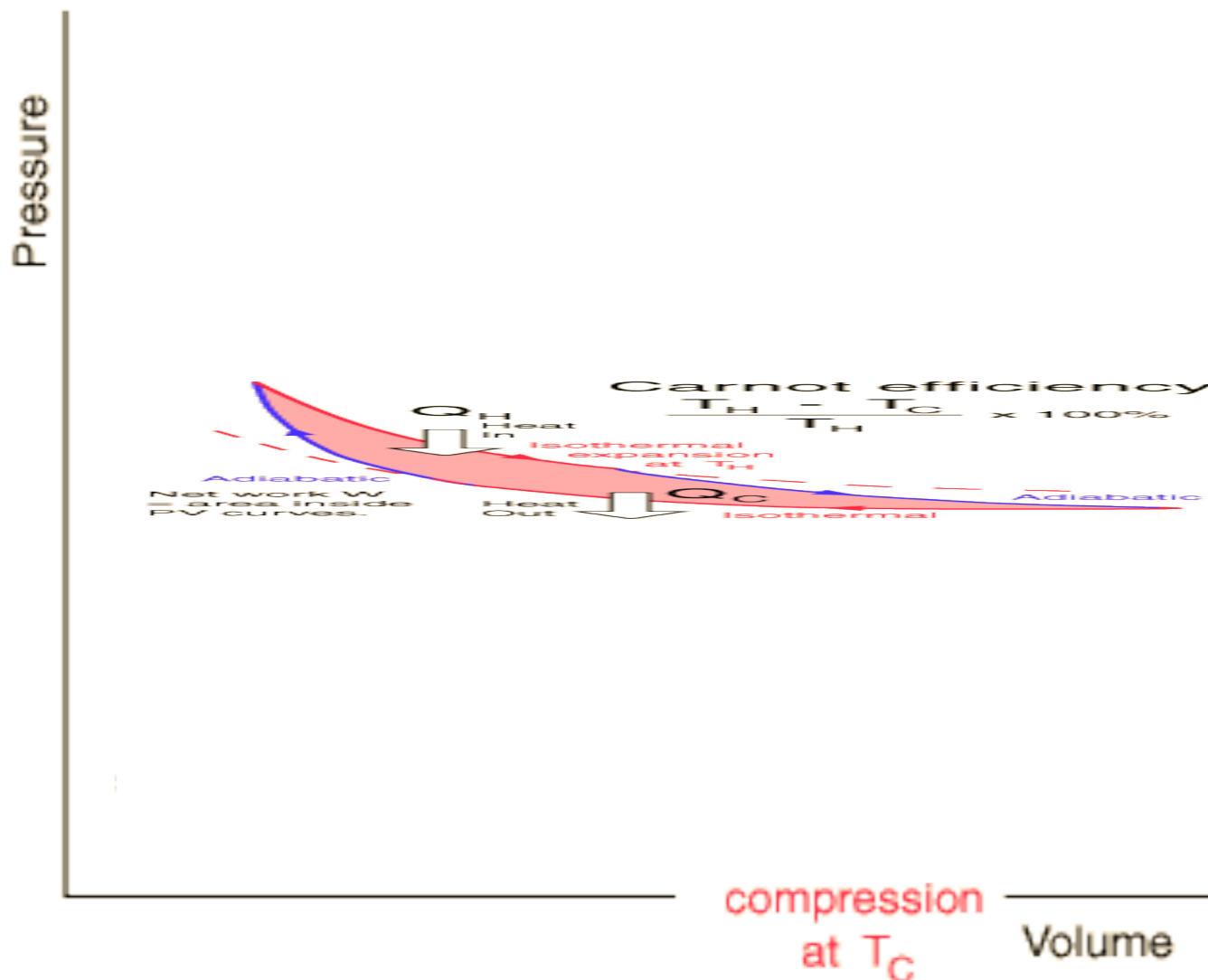
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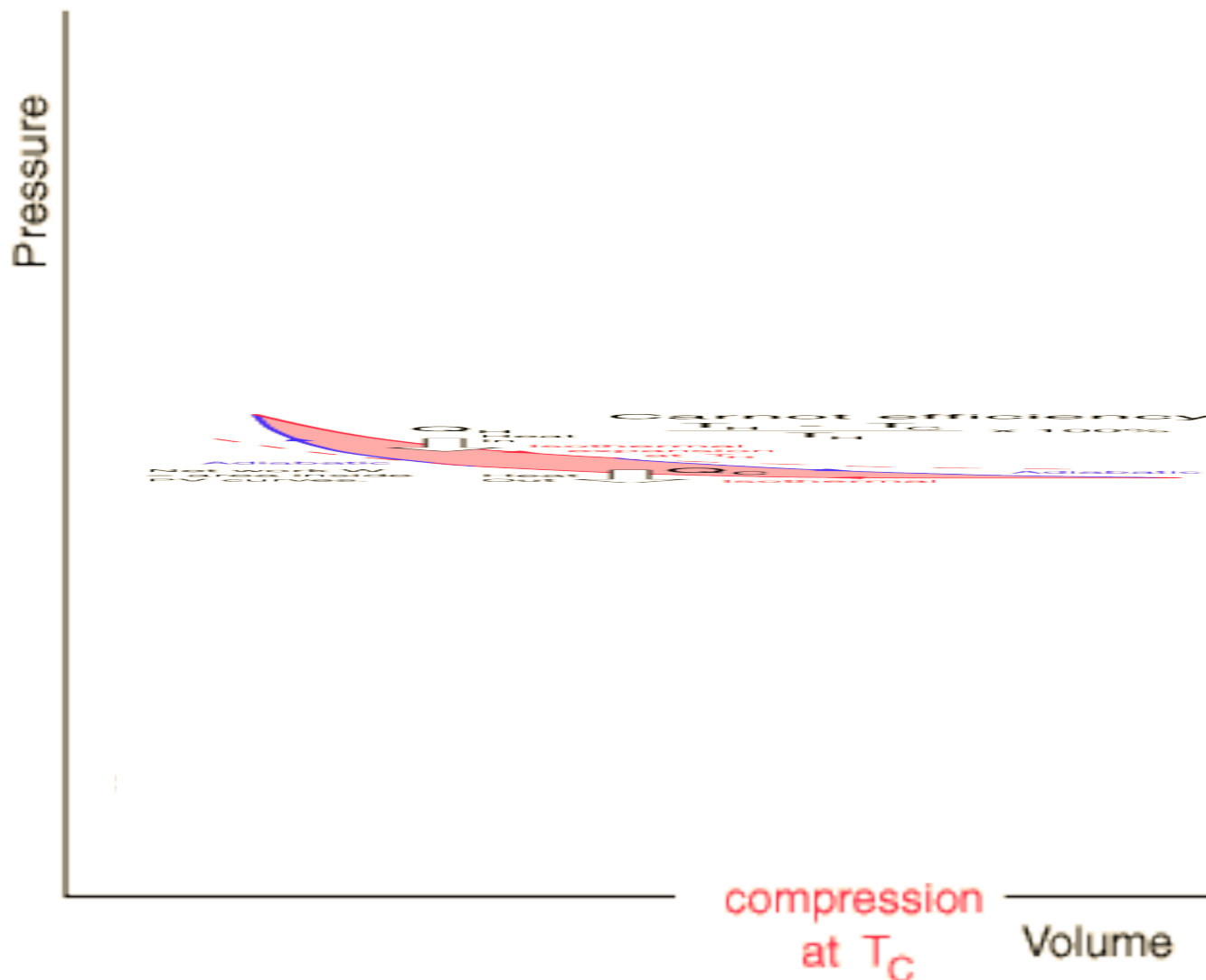
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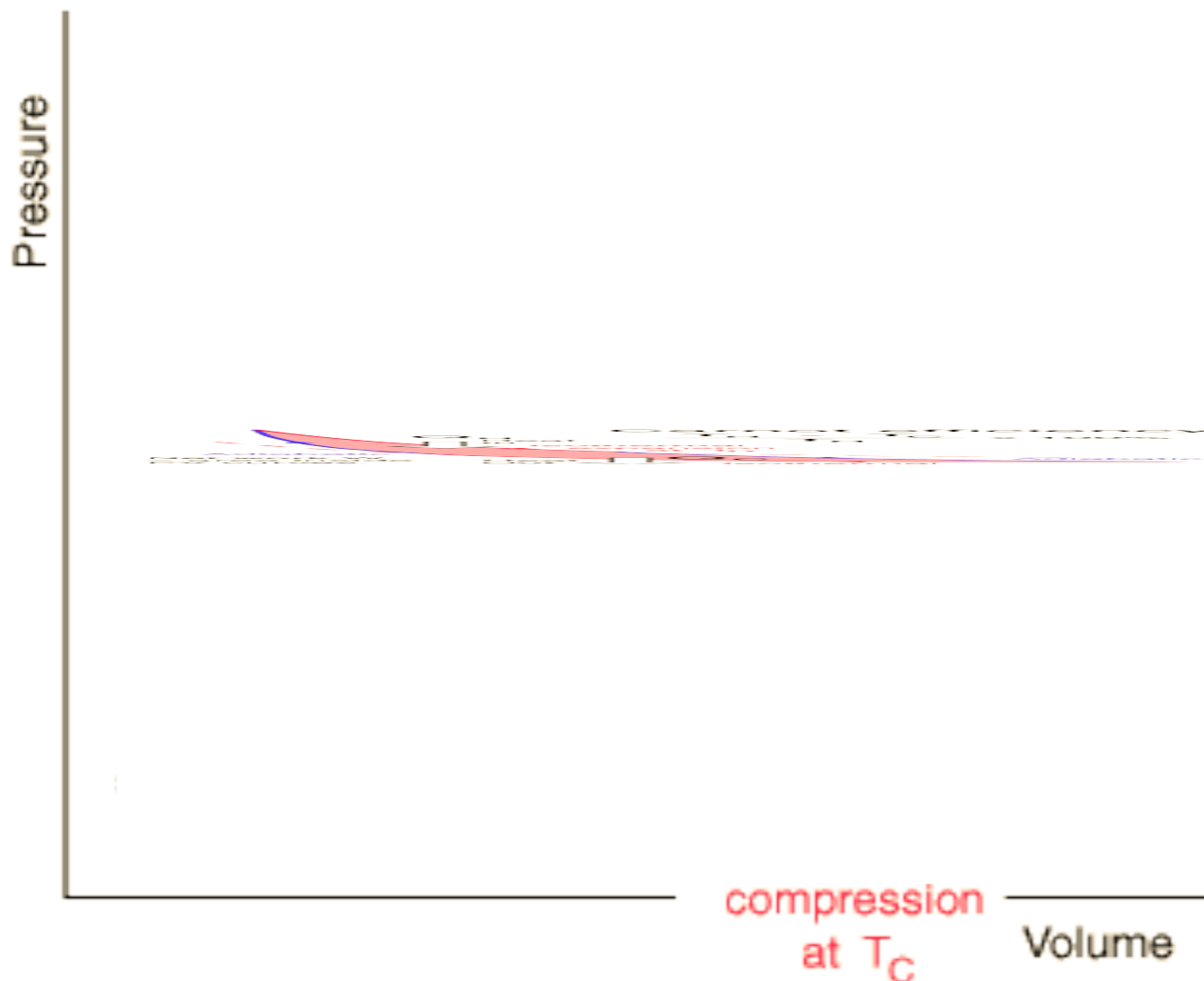
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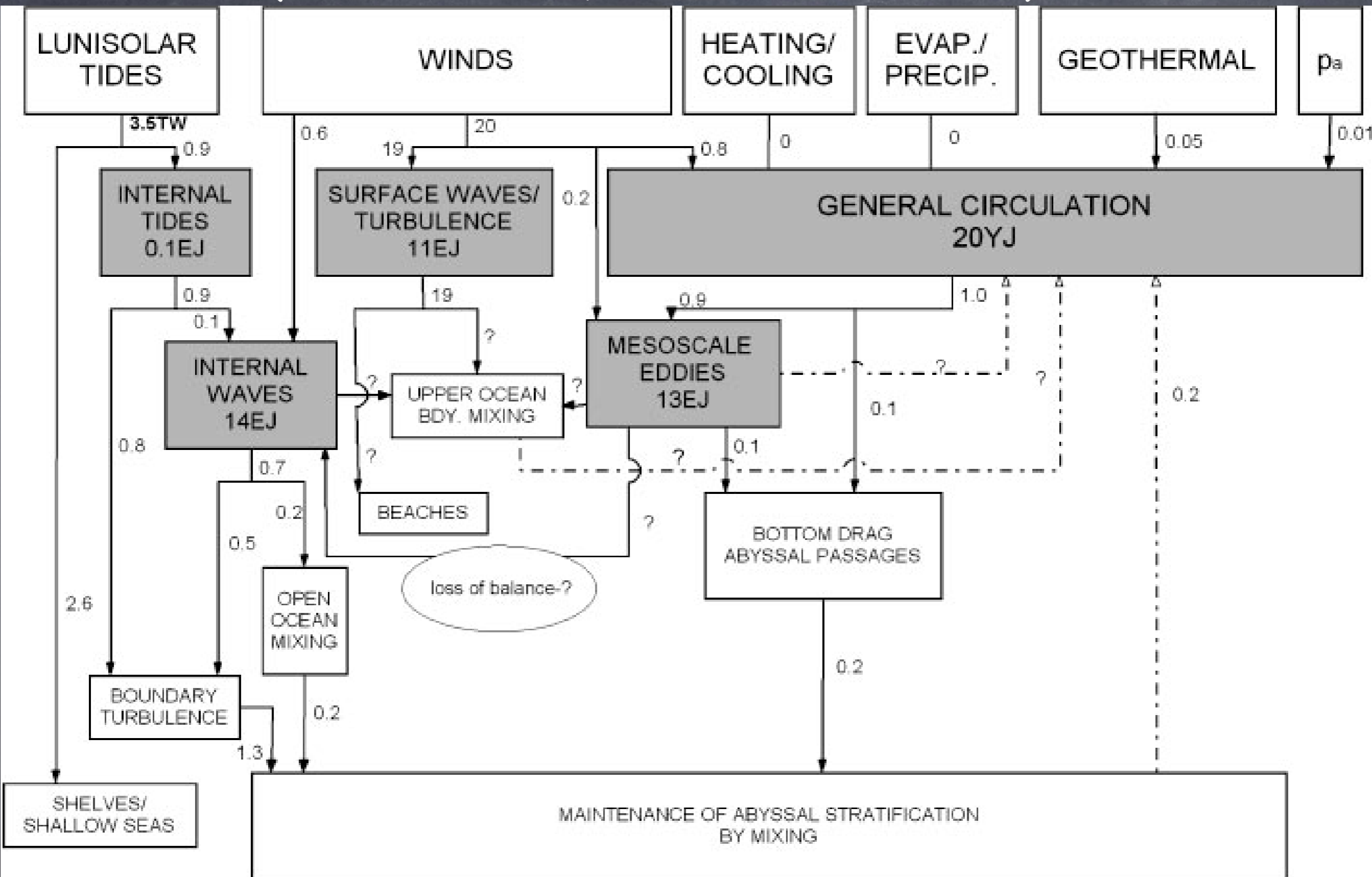
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Net work  $W$   
= area inside  
PV curves.

# Elaborate Schemes: Ocean Transports Heat, But winds drive the system.. plus tides and geothermal.. (Sandstrom 1916, Wunsch & Ferrari 04)





# But Does it matter?

- A man falls to his death after being pushed from the Empire State Building Observatory.

Who is responsible?

- Clearly, he was killed by kinetic energy!  
So, the source of KE is the culprit!

The push was horizontal,  
thus it supplies no PE and precious little KE.

The pusher is innocent!

Most of the KE was converted from PE,

Thus, if he took the stairs, it was suicide

Or, if he took the elevator, then the power company is to blame...

BUT, the company used oil from the Middle East...



You see how we get in  
trouble...







# Leads us to thinking about PE:

- Potential Energy is just  $\langle -zb \rangle$ , where  $z$  is height and  $b$  is buoyancy  $\equiv \frac{-g(\rho - \rho_0)}{\rho_0}$
- So increase PE by cooling at surface, decrease PE by heating at surface.
- In order to move dense water up, you have to input energy (e.g., by mixing with wind or tides)
- Instabilities rely on extracting energy. Baroclinic instabilities extract PE by net vertical transport of light water up, cold



# Leads us to thinking about Geostrophy:

In stratified, rotating flow:  
 PE can be \*stored\* in horiz.  
 buoyancy grads via geostrophic  
 balance with flow

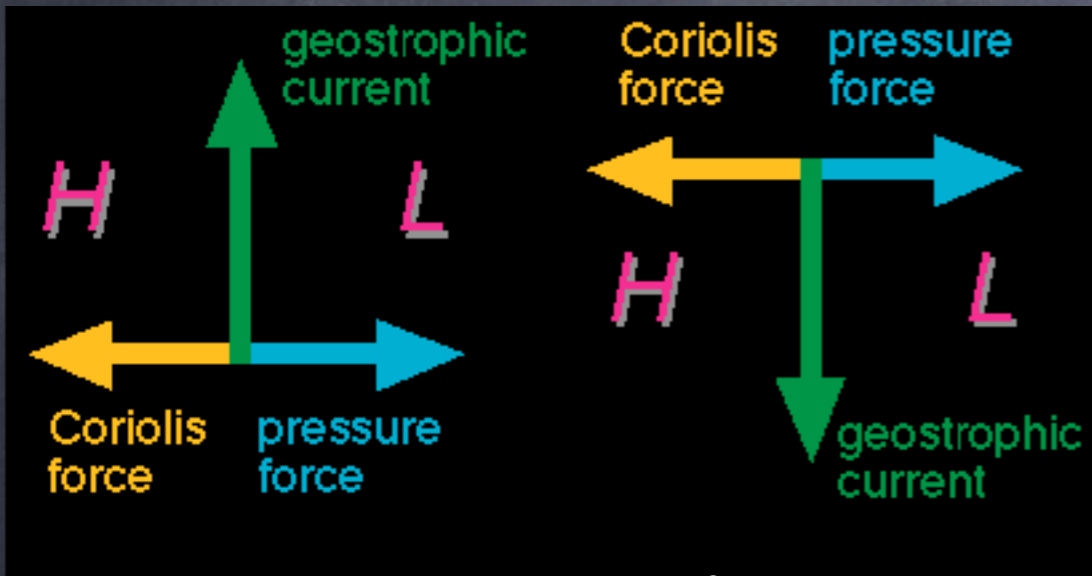
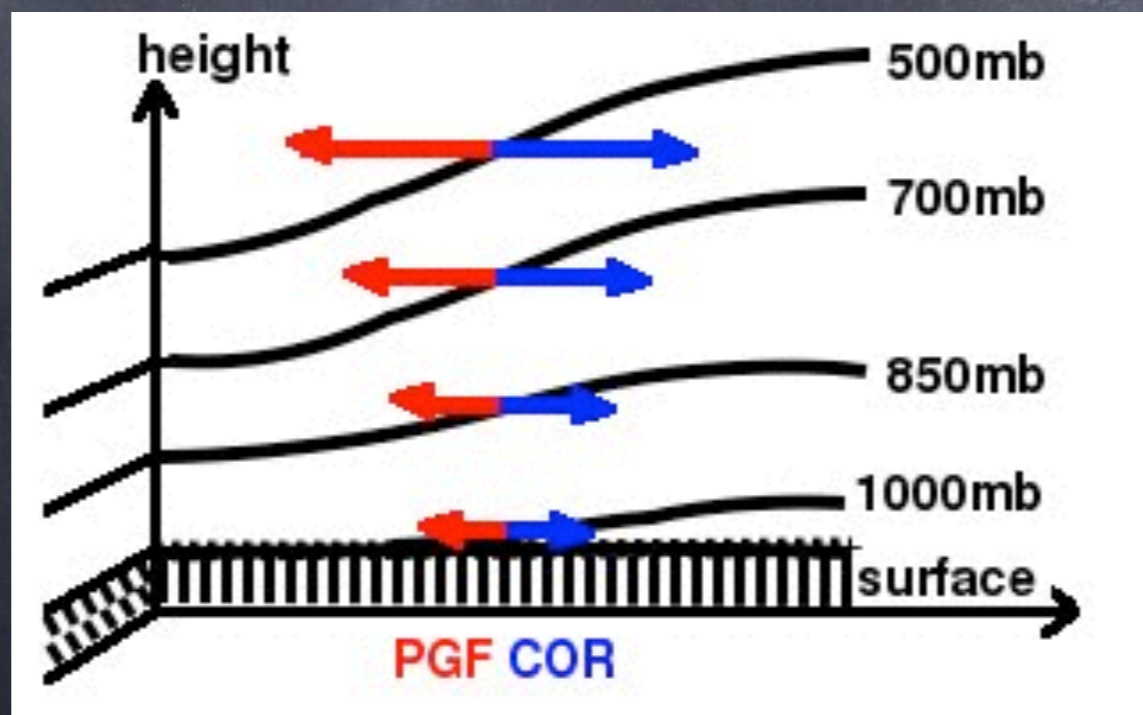


figure: M. Tomczak

Rapid  
 Pressure Change:  
 Dense Fluid



Slow  
 Pressure Change:  
 Light Fluid

figure: A. A. Lopez



# Leads us to thinking about Geostrophic Scales

In stratified, rotating flow:

Stored PE=KE at the scale of the

Deformation Radius, or Rossby Radius

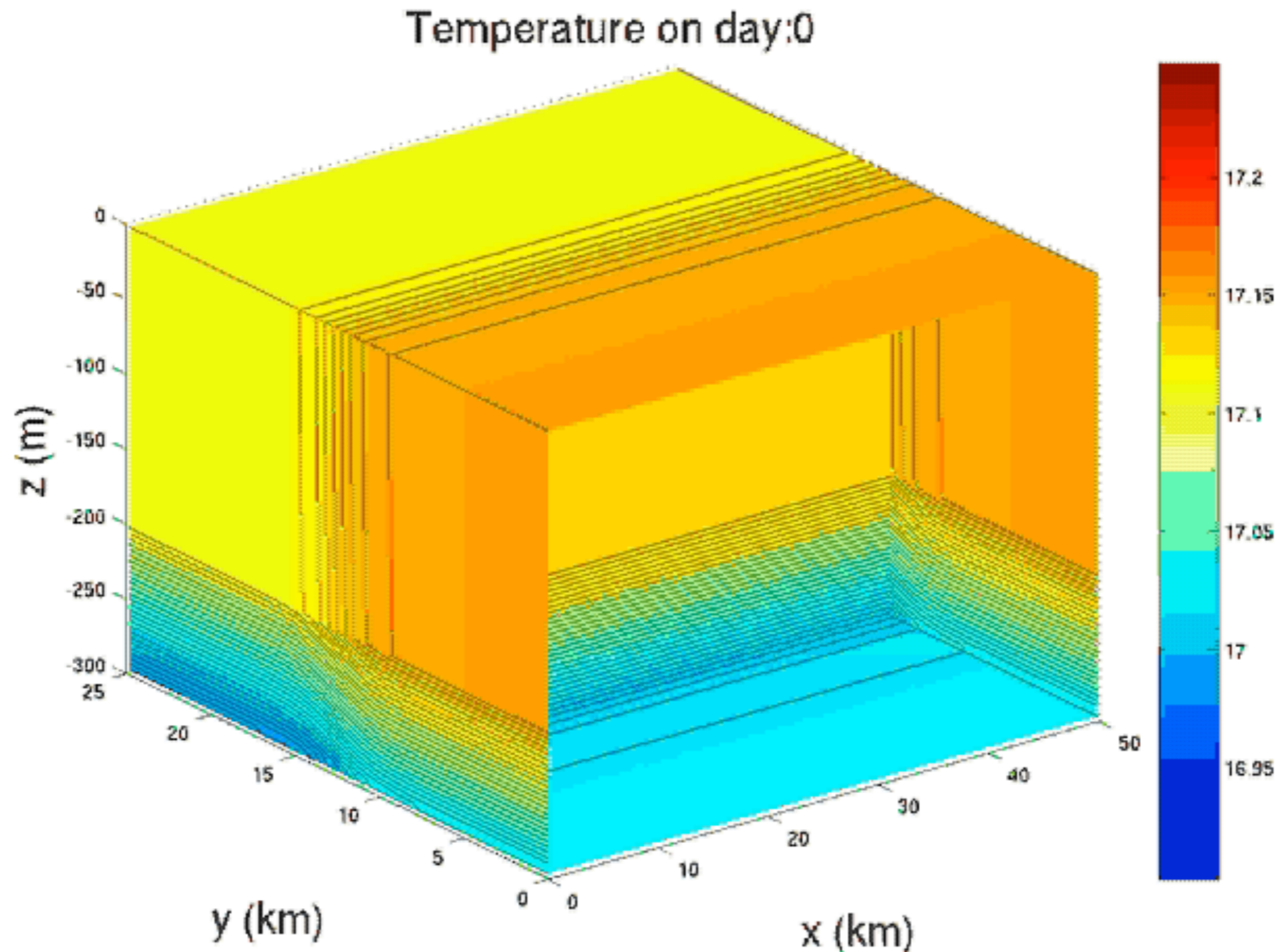
$$H N/f$$

This scale is also the width over which an  
adjusting front slumps,

And sets the typical scale of baroclinic  
instability eigenvectors



# Geostrophic Adjustment & Baroclinic Instability





# Leads us to thinking about Quasi-Geostrophy & Release

In stratified, rotating flow:  
 PE can be \*stored\* in horiz.  
 buoyancy grads via geostrophic  
 balance with flow:

Baroclinic Instability Releases It!

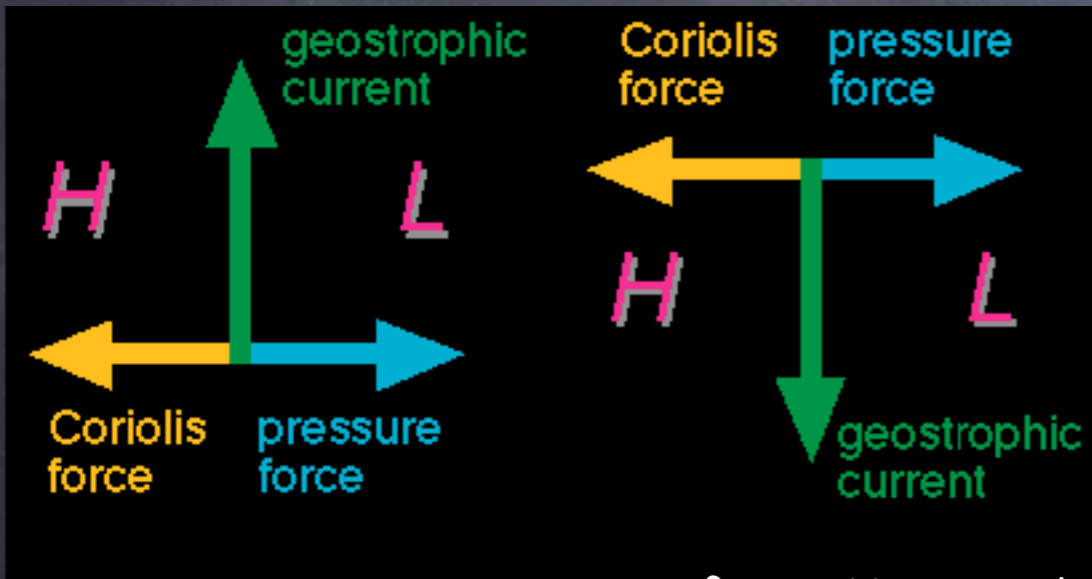
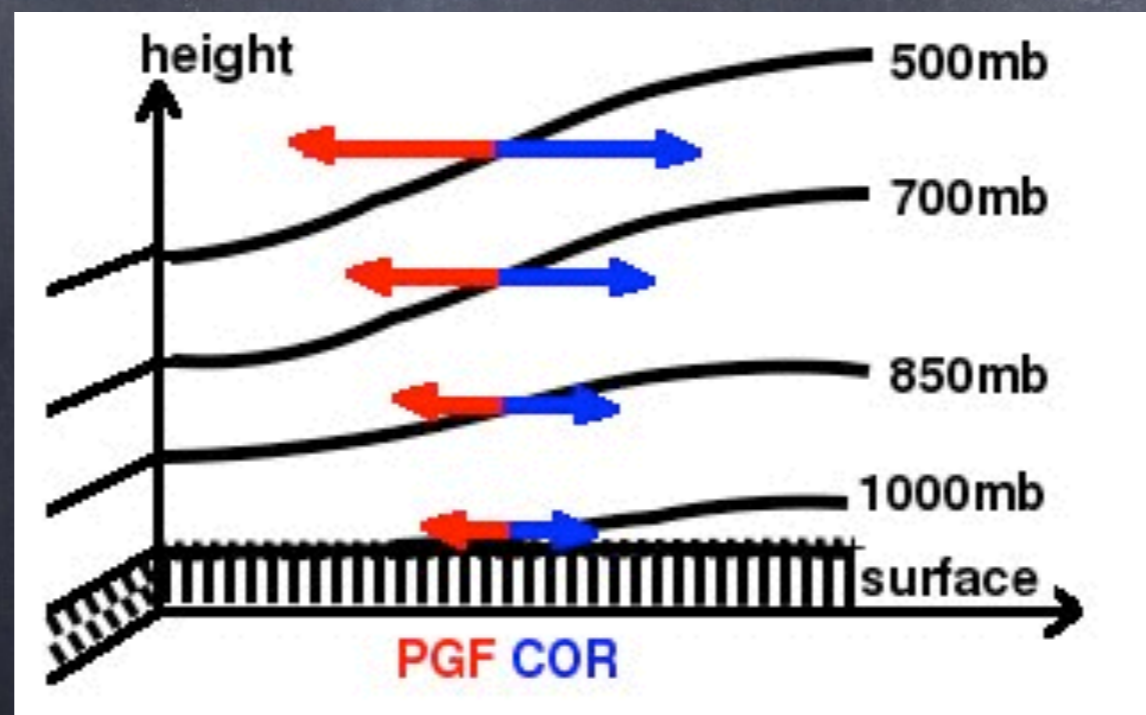


figure: M. Tomczak

Rapid  
 Pressure Change:  
 Dense Fluid



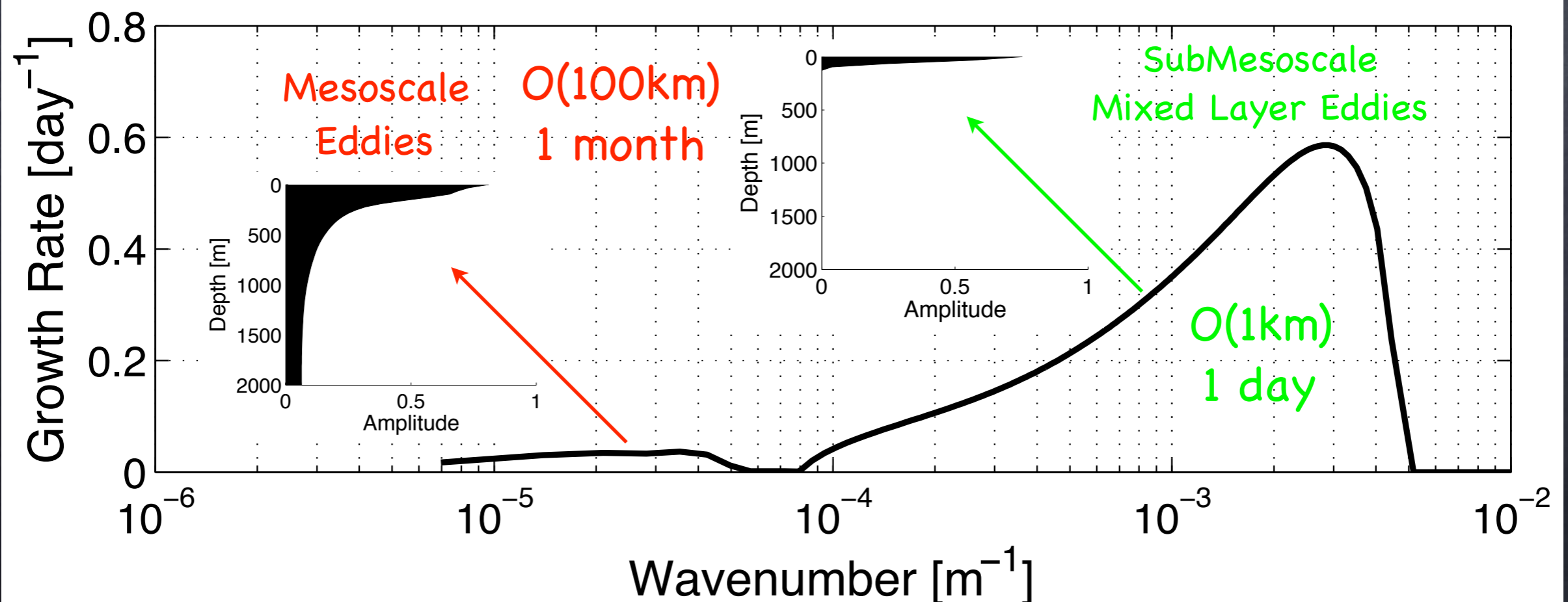
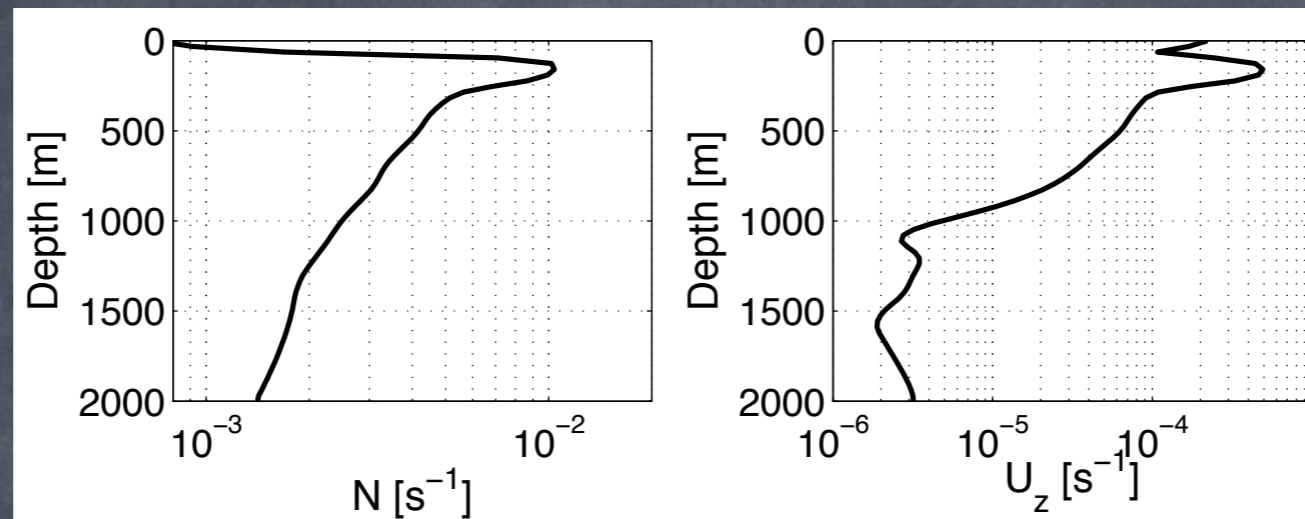
Slow  
 Pressure Change:  
 Light Fluid

figure: A. A. Lopez



# The Stratification Permits Two Types of Baroclinic Instability:

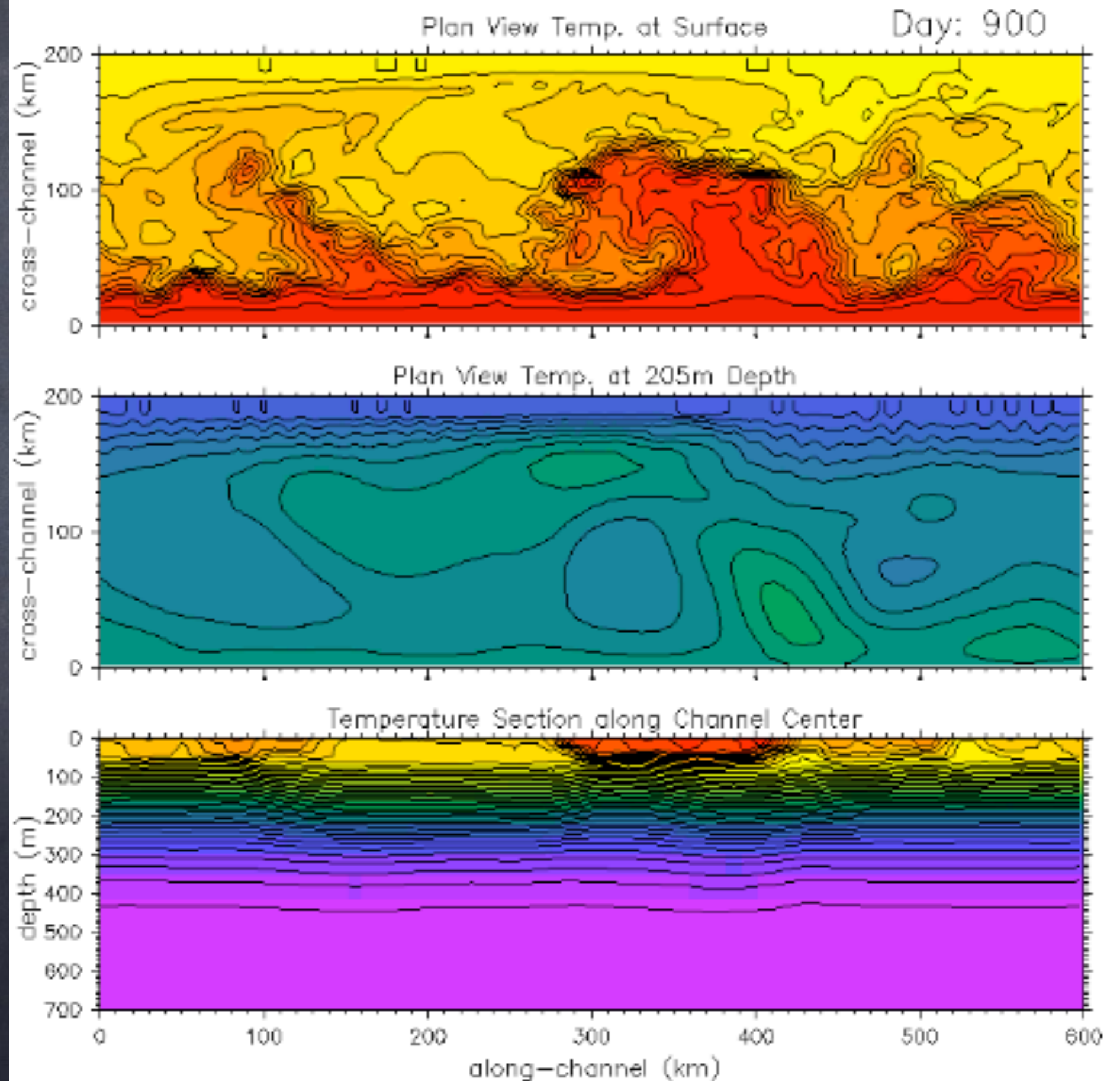
Mesoscale and SubMesoscale (Boccaletti et al., 2006)



Mesoscale and  
SubMesoscale  
are  
Coupled  
Together:

ML Fronts are  
formed by  
Mesoscale  
Straining.

Submesoscale  
eddies remove  
PE from those  
fronts.





But, Resolving both the  
Mesoscale and  
Submesoscale is  
expensive



But, Resolving both the  
Mesoscale and  
Submesoscale is  
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What we need is a  
Prototypical Problem to  
Parameterize!



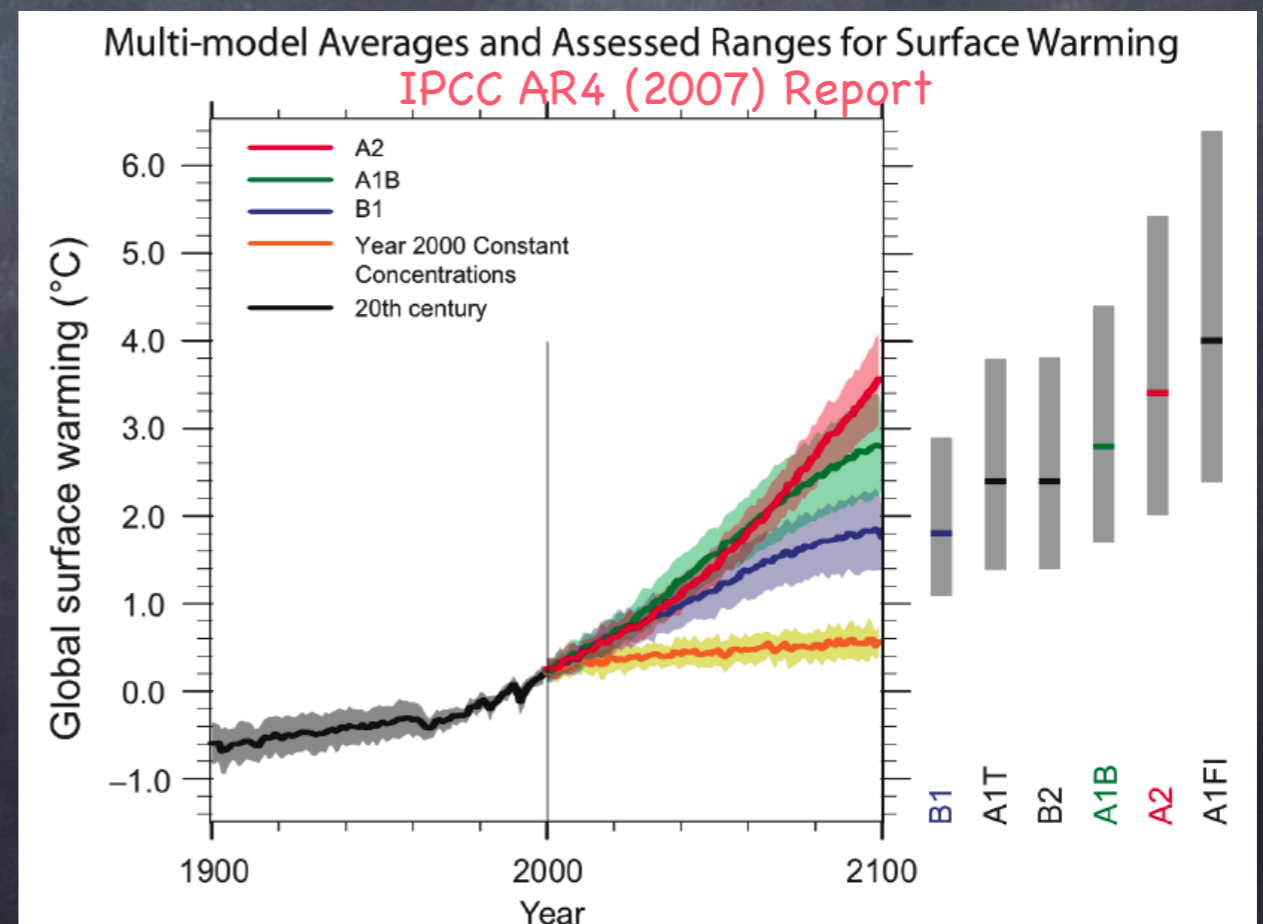
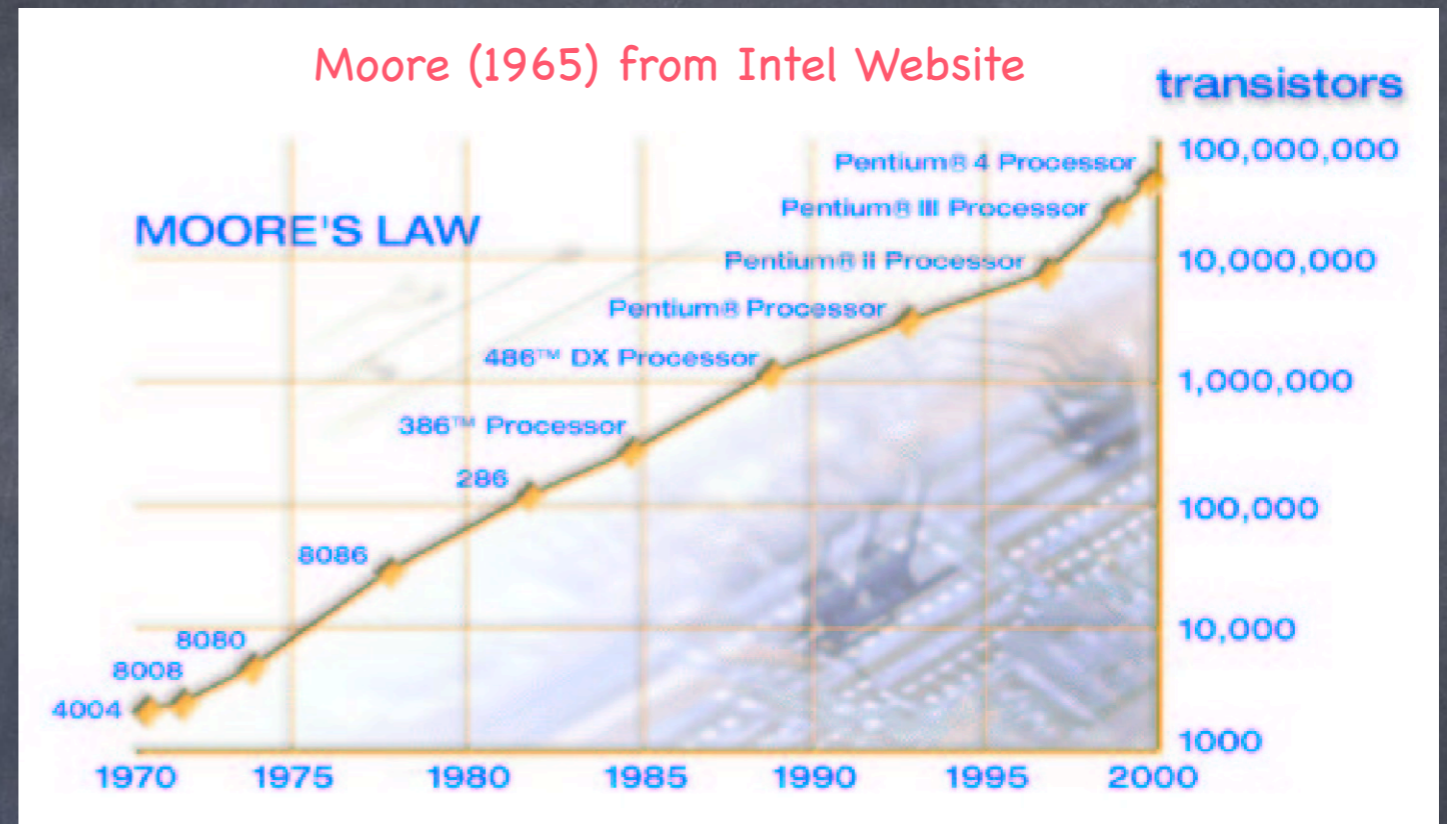
# Parameterize? Why not Resolve?

2007 IPCC AR4 takes ~50% of GFDL's computing in 2003-2005 (4 in the top 250: 6/03).

To make eddy-resolving IPCC forecasts with the same level of commitment and approach:

Fair Resolution of Ocean Mesoscale Eddies (global 10-20km):  $10 \times 10 \times 2 \times 5 \times (\text{flops}) = 1000 \times (\text{cpu}) \approx 18 \text{ yrs}$   
By Then  $\approx 0.5\text{K}$  surface warm

Fair Resolution of Mixed Layer and Eddies (global 100m):  $1000 \times 1000 \times 10 \times 100 \times (\text{flops}) = 1 \text{ billion} \times (\text{cpu}) \approx 54 \text{ yrs}$   
By Then  $\approx 2\text{K}$  surface warm





So, for submesoscale in a  
climate model:

Not DNS...

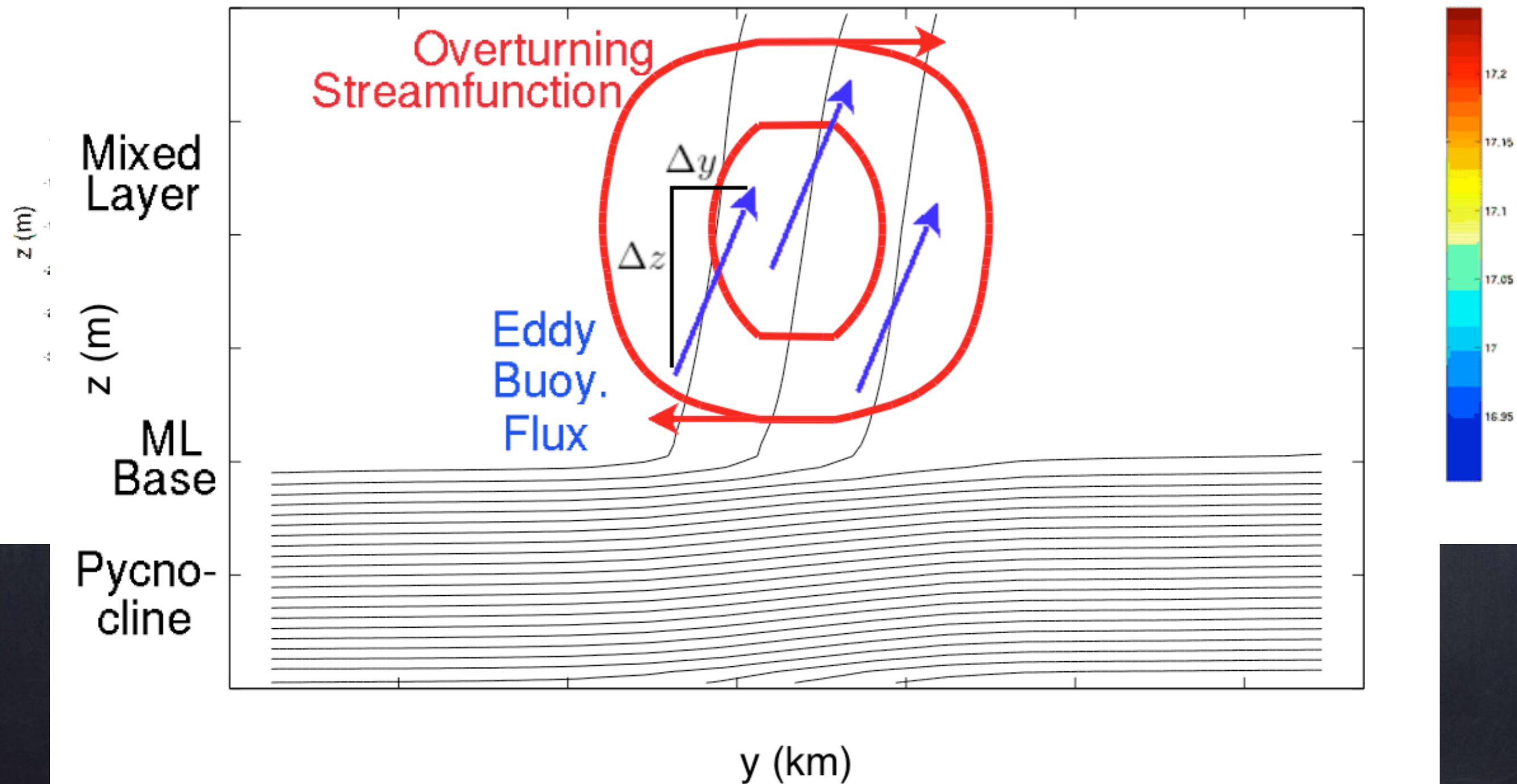
Not LES...

NES:

No Eddy Simulation!

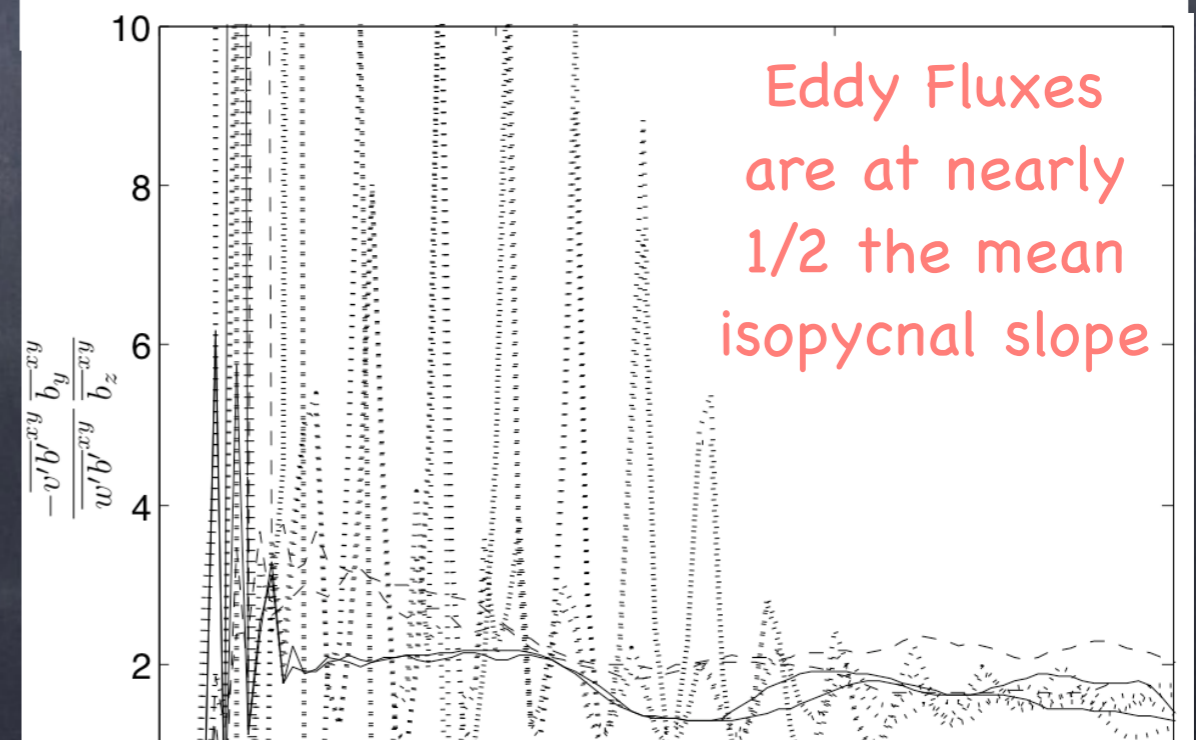
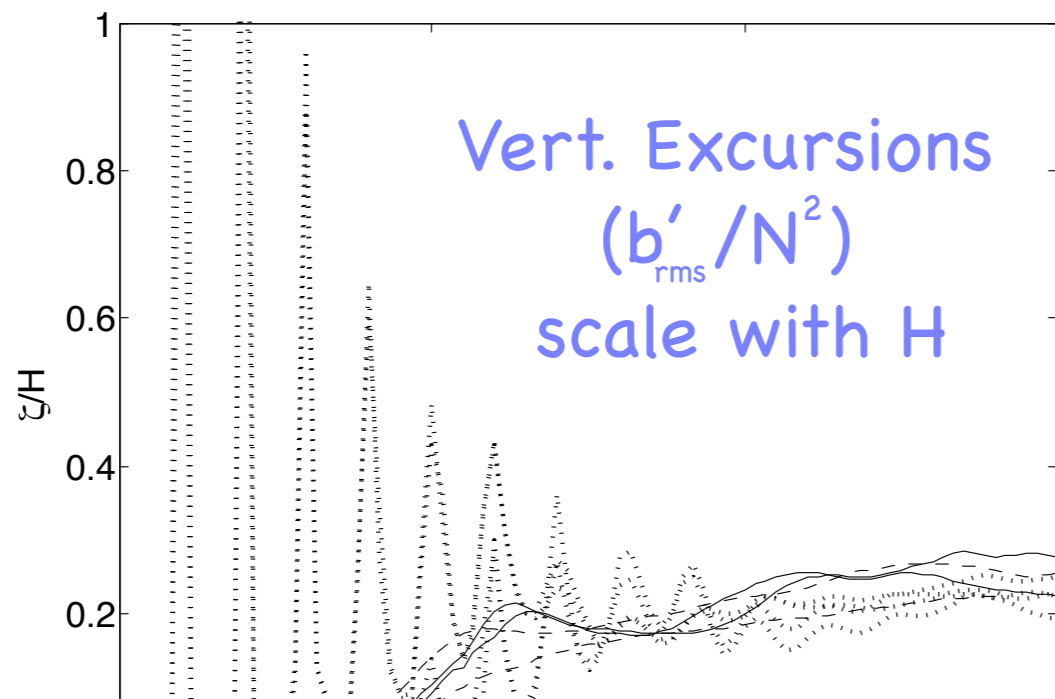
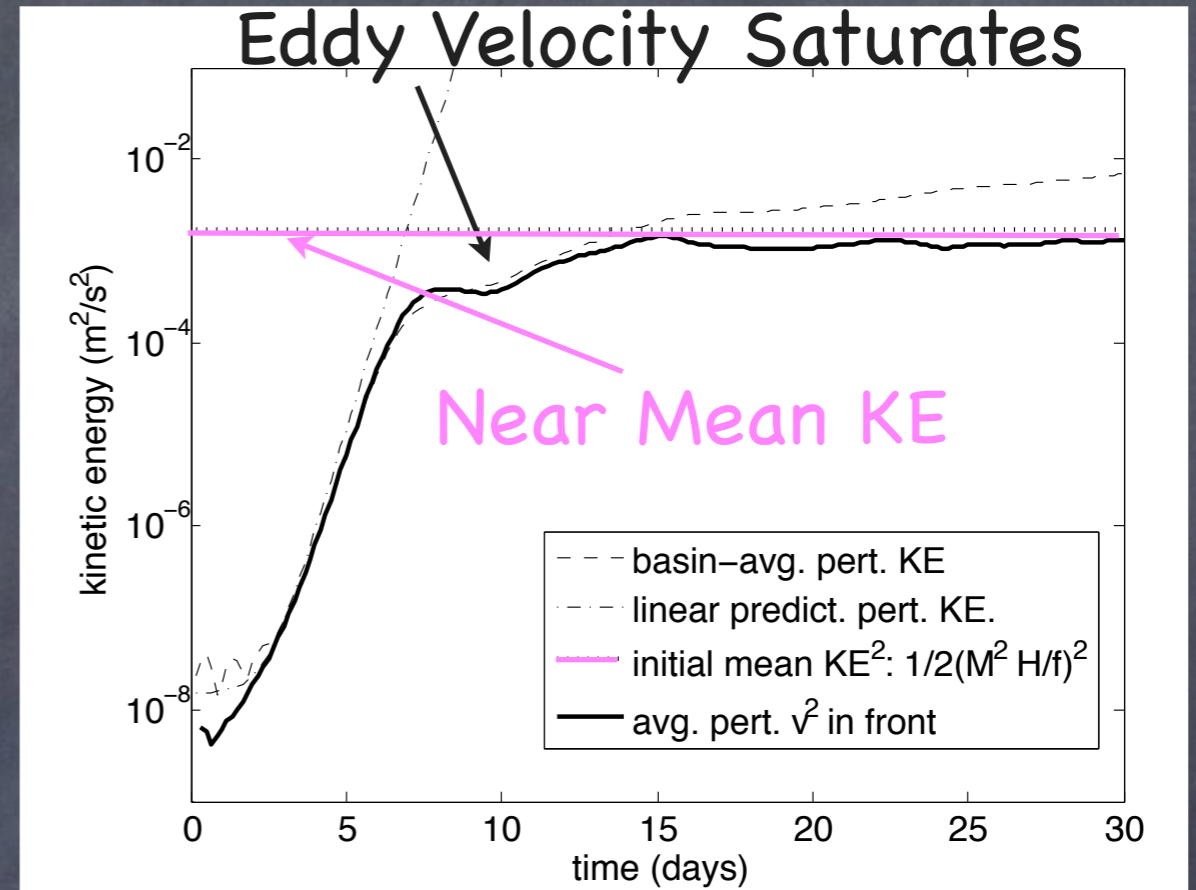
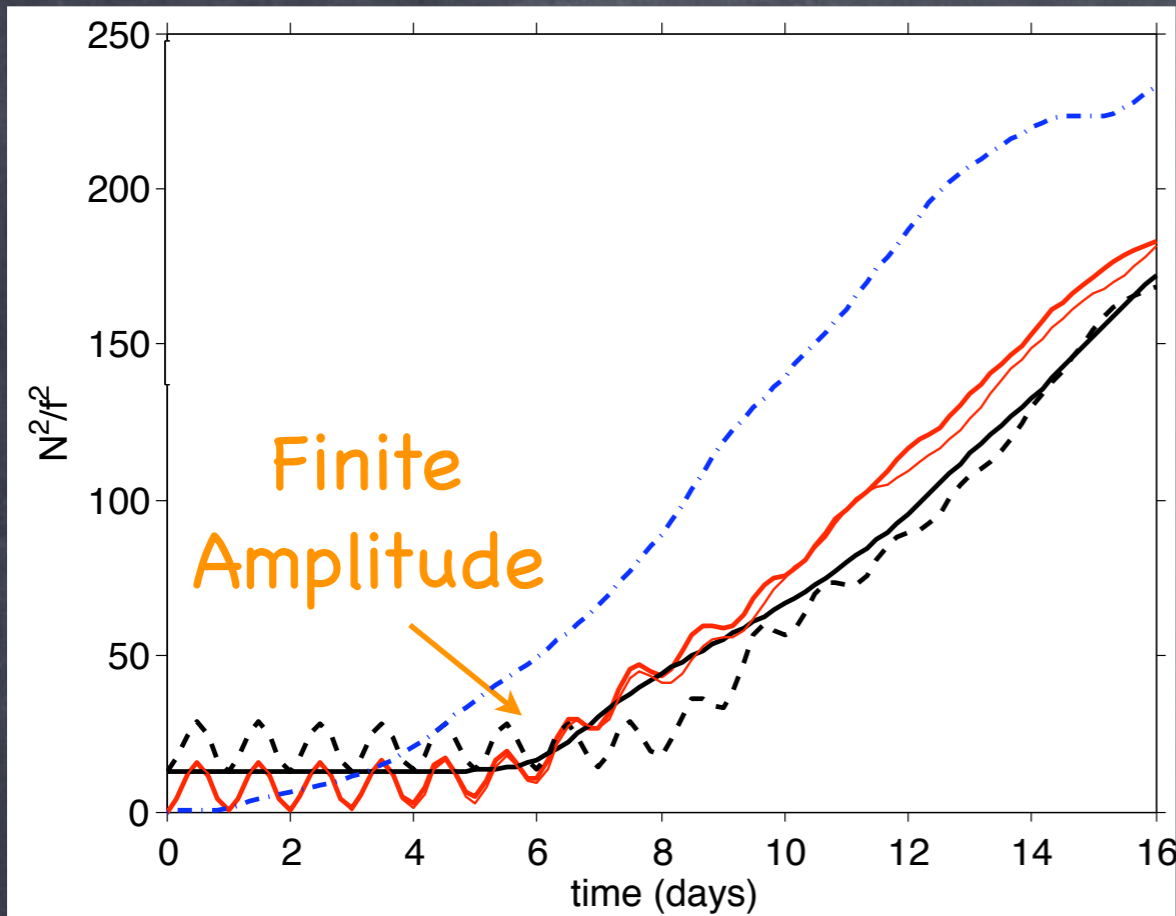


# Prototype: Mixed Layer Front Overturning





# Parameterization of Finite Amp. Eddies: Ingredients



Linear Solution  $\langle w'b' \rangle$  for vert. structure.



# Magnitude Analysis: Vert. Fluxes

Extraction of potential energy by submesoscale eddies:

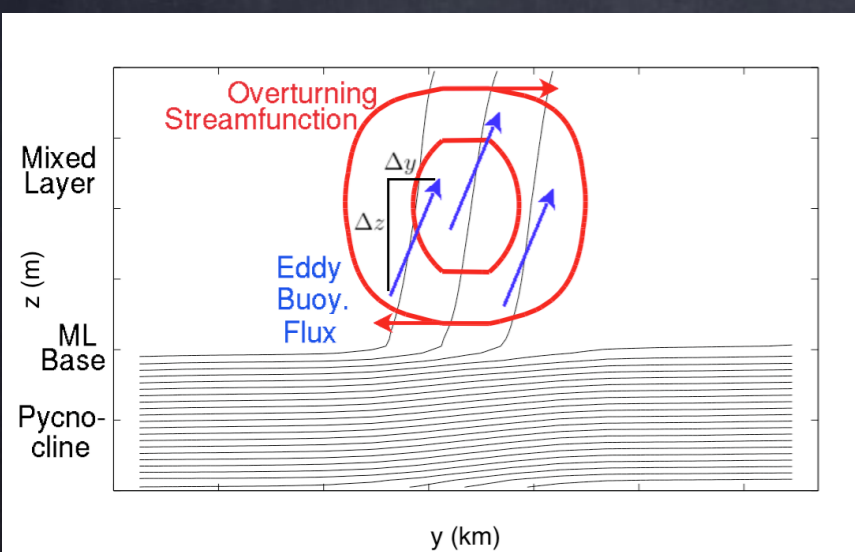
$$-\langle wb \rangle = \frac{\partial \langle PE \rangle}{\partial t} \approx \frac{\Delta PE}{\Delta t} \propto \frac{\Delta z \Delta b}{\Delta t}$$

Buoy. diff just parcel exchange of large-scale buoy.

Flux slope scales with the buoy. slope:  $\frac{\Delta y}{\Delta z} \propto \frac{-\frac{\partial \bar{b}}{\partial z}}{\frac{\partial \bar{b}}{\partial y}}$

Time scale is turnover time from mean thermal wind:

Vertical scale known:  $\Delta z \propto H$



$$\langle wb \rangle \propto \frac{\Delta z \Delta y \left( \frac{\partial \bar{b}}{\partial y} \frac{\partial \bar{b}}{\partial y} + \Delta z \frac{\partial \bar{b}}{\partial z} \right)}{|f| \Delta y \Delta t}$$

Fox-Kemper et al., 2007



# The Parameterization:

$$\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{z}$$

$$\mu(z) = \left[ 1 - \left( \frac{2z}{H} + 1 \right)^2 \right] \left[ 1 + \frac{5}{21} \left( \frac{2z}{H} + 1 \right)^2 \right]$$

$$\overline{\mathbf{u}'b'} \equiv \Psi \times \nabla \bar{b}, \quad \mathbf{u}^* = \nabla \times \Psi.$$

- The horizontal fluxes are downgradient:

$$\overline{\mathbf{u}'_H b'} = - \frac{C_e H^2 \mu(z) \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla_H \bar{b}$$

- Vertical fluxes always upward to restratify:

$$\overline{w' b'} = \frac{C_e H^2 \mu(z)}{|f|} |\nabla \bar{b}|^2$$

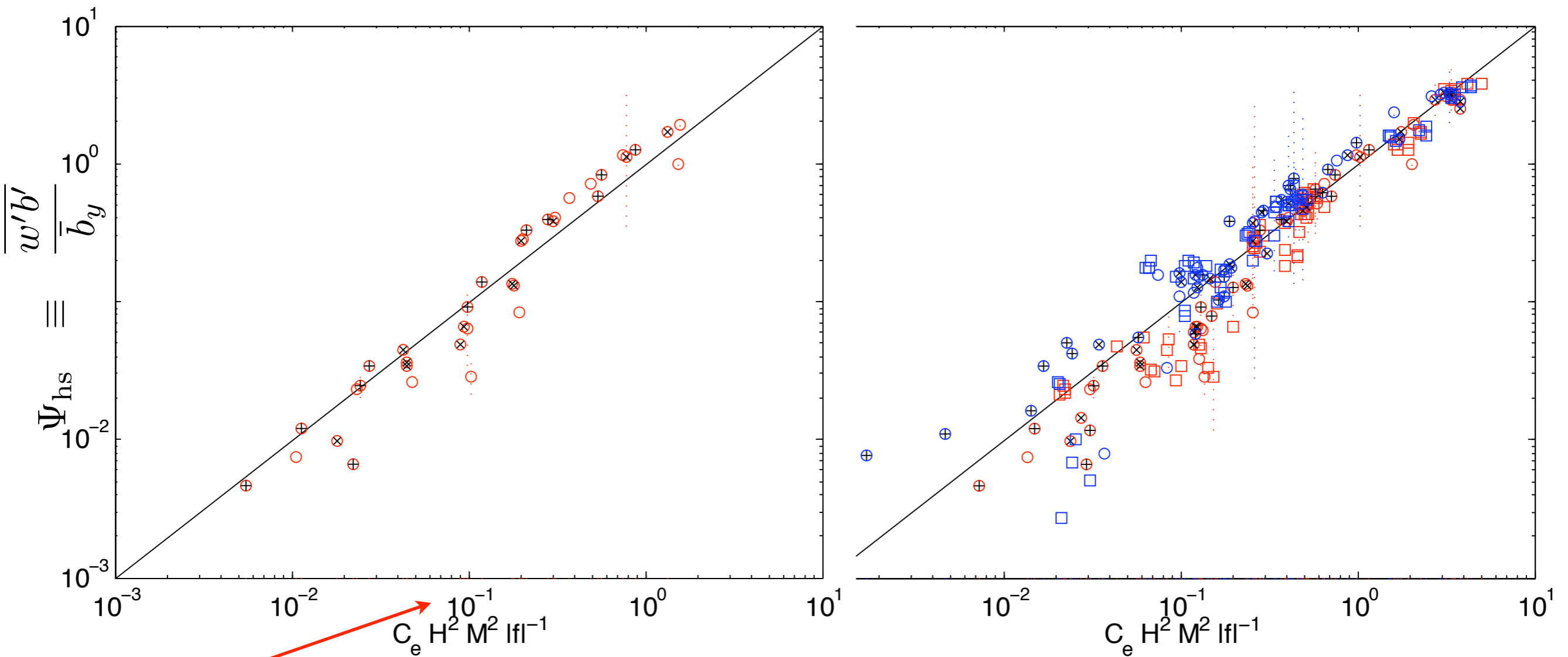
- Adjustments for coarse resolution and  $f \rightarrow 0$  are known



# It works for Prototype Sims:

Red: No Diurnal

Blue: With Diurnal



>2 orders of  
magnitude!

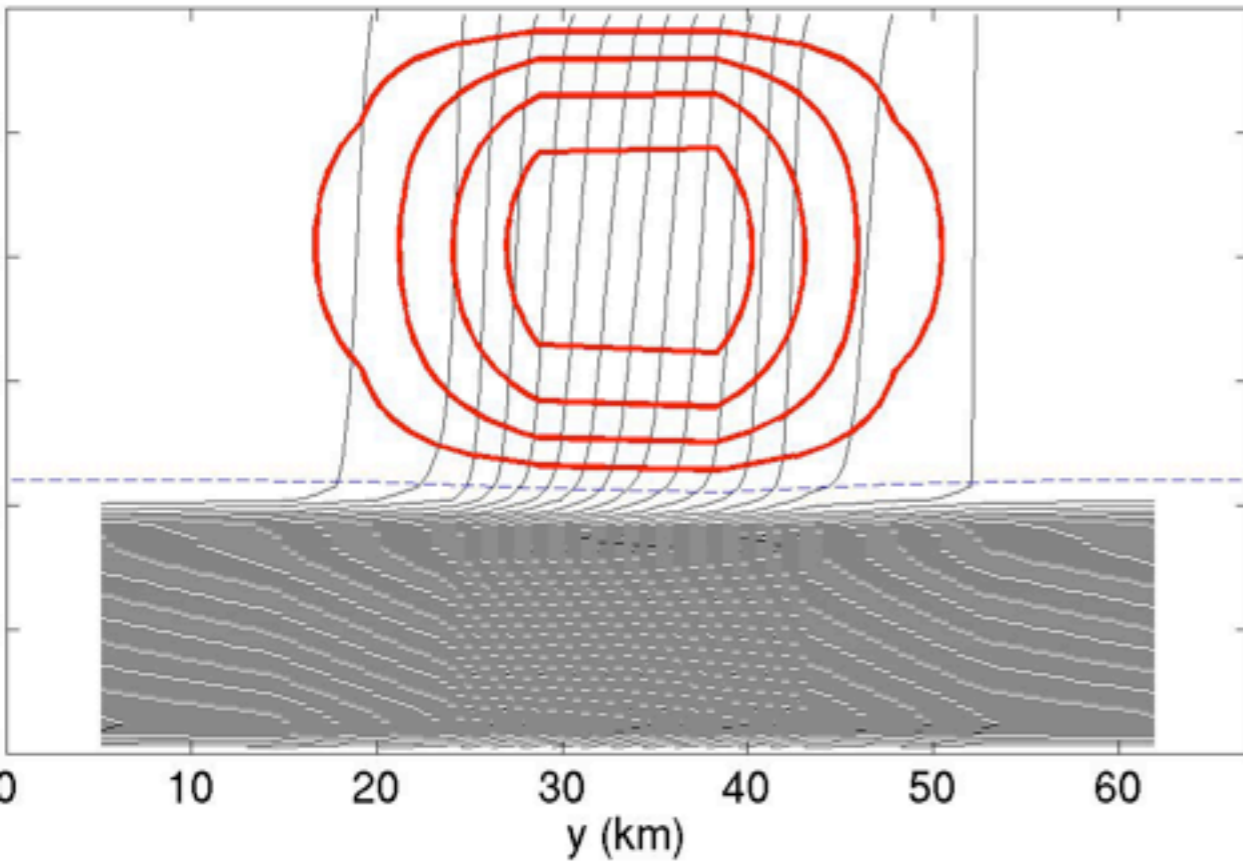
Circles: Balanced Initial Cond.  
Squares: Unbalanced Initial Cond.



# What does it look like?

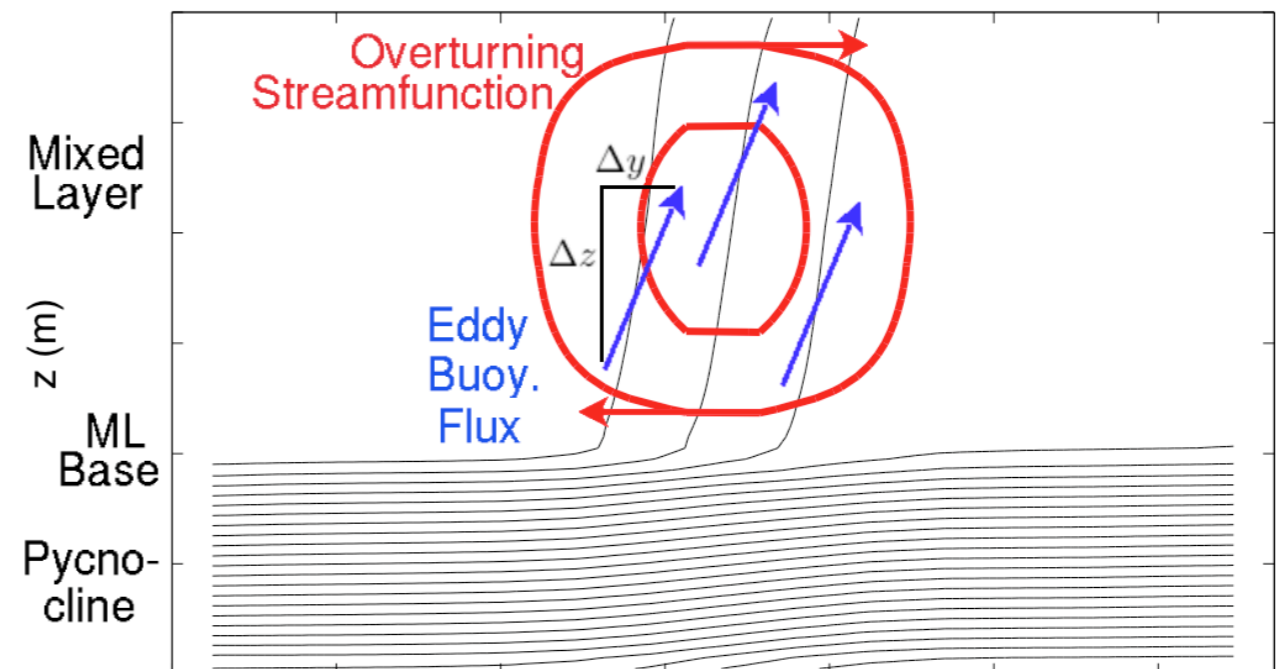
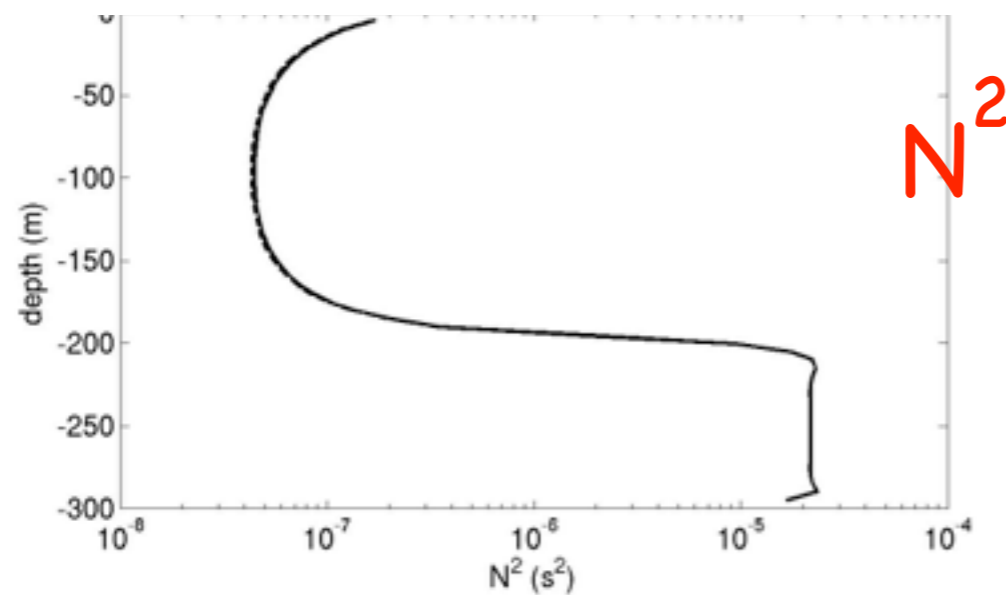
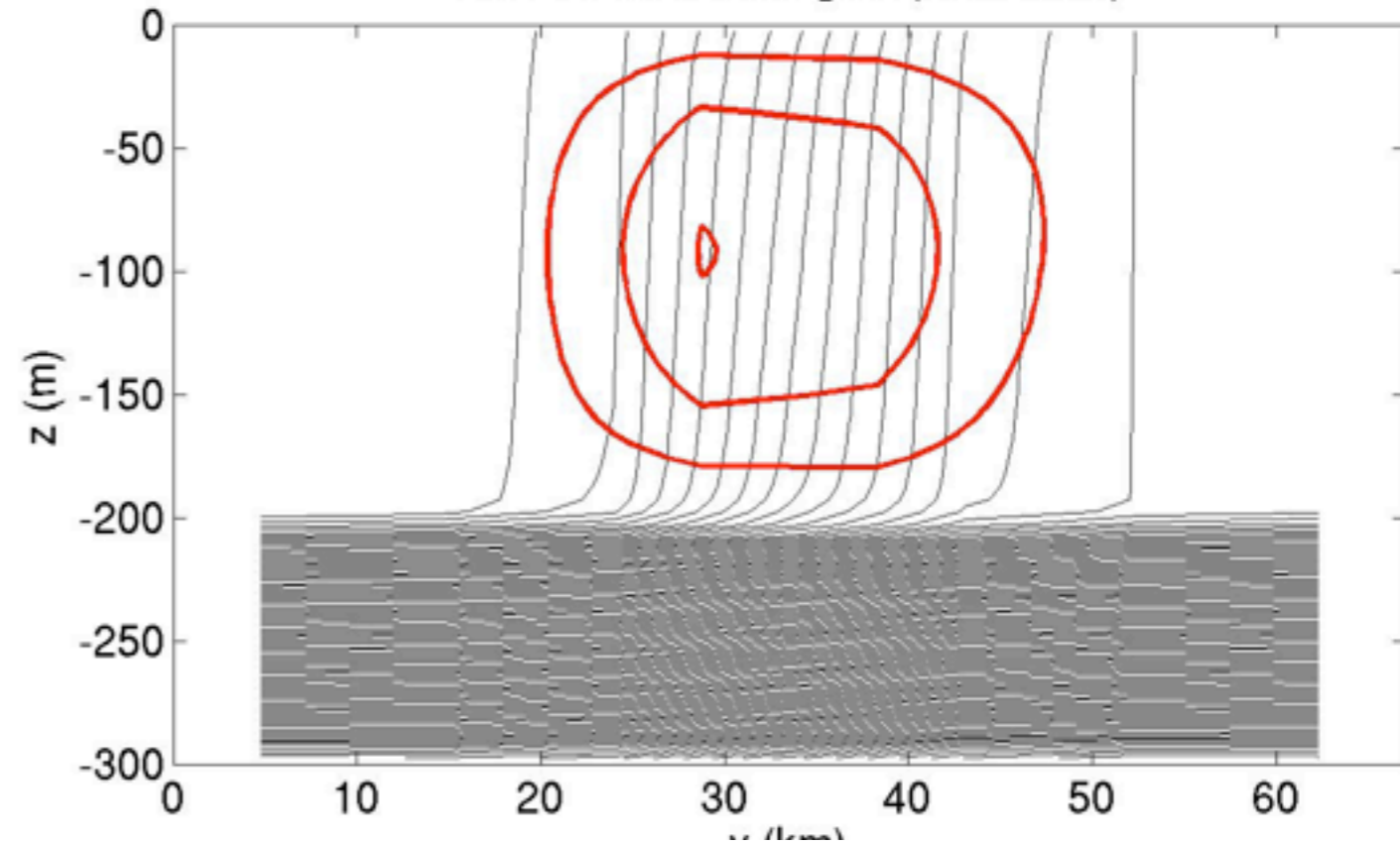
## Parameterization (2d, 10km grid)

7d01h from 2d parameterization



## Submesoscale-Resolving (3d, 500m grid)

7d01h from 3d MITgcm (smoothed)





# Implemented in GCMs

## • Hallberg Isopycnal Model (HIM/GOLD)

### • 2 Simulations in HIM/GOLD:

- MESO (Modeling Eddies in the Southern Ocean) (control & param), different resolutions of mesoscale (2 degrees to 1/6 degree)
- Global 1 degree coupled ocean-atmosphere (control & param to 20yr)

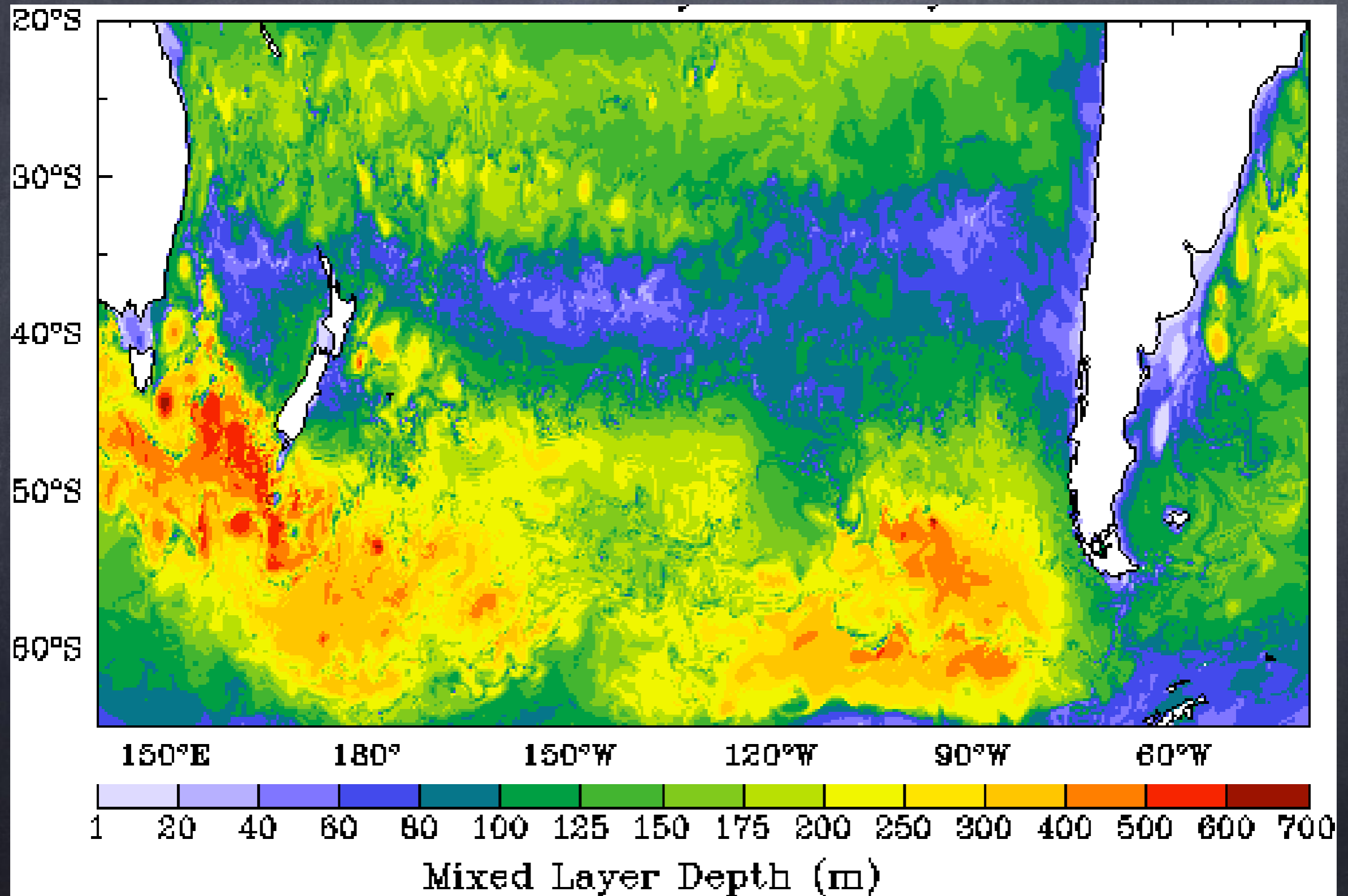
## • Community Climate System Model 3 (CCSM3)

### • 1 Simulation in CCSM so far

- Global 3 degree ocean only (control & param to 100yr)

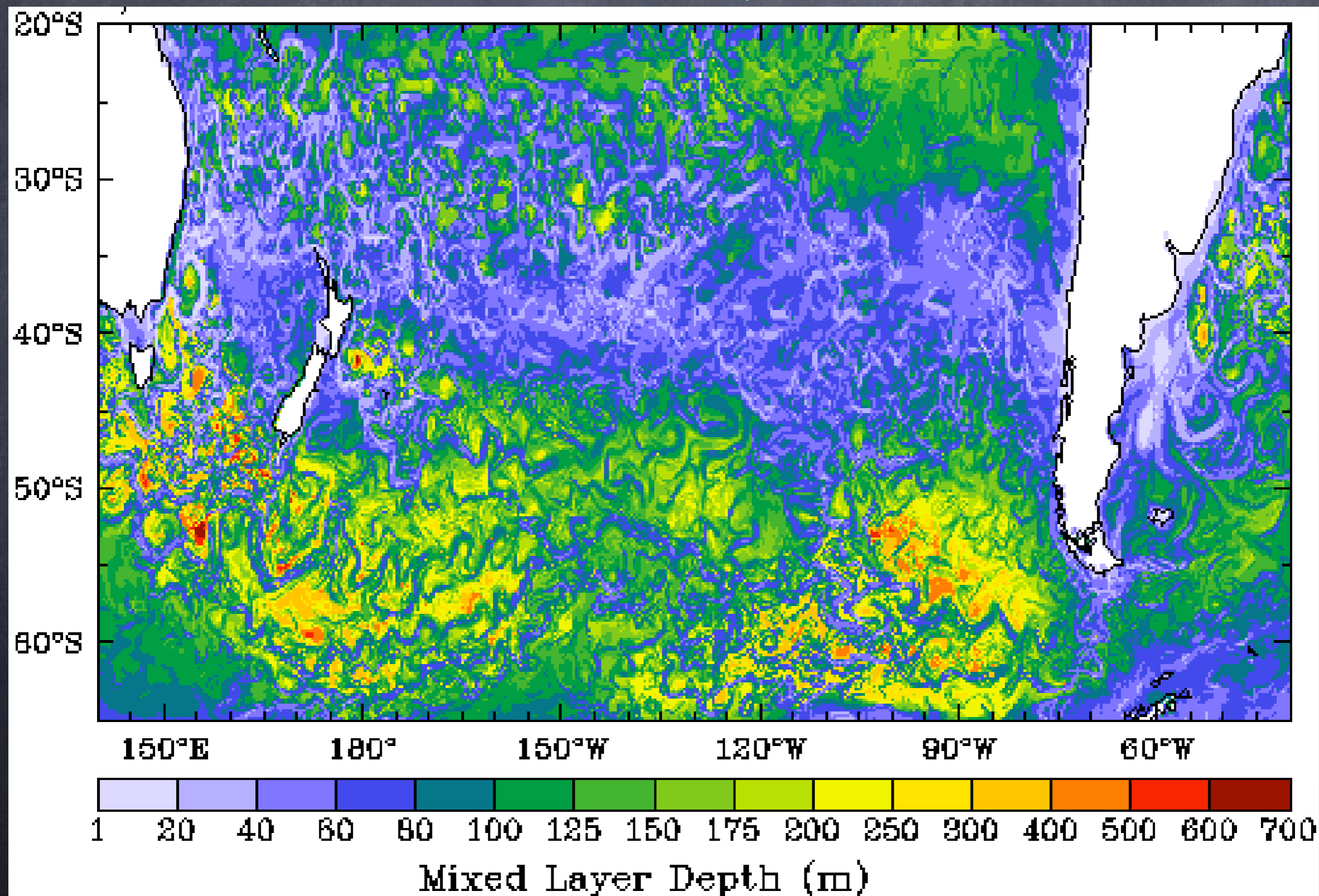


# Changes To Mixing Layer Depth in Eddy-Resolving Southern Ocean Model





# Changes To Mixing Layer Depth in Eddy-Resolving Southern Ocean Model: Where there are fronts, MLEs release PE!



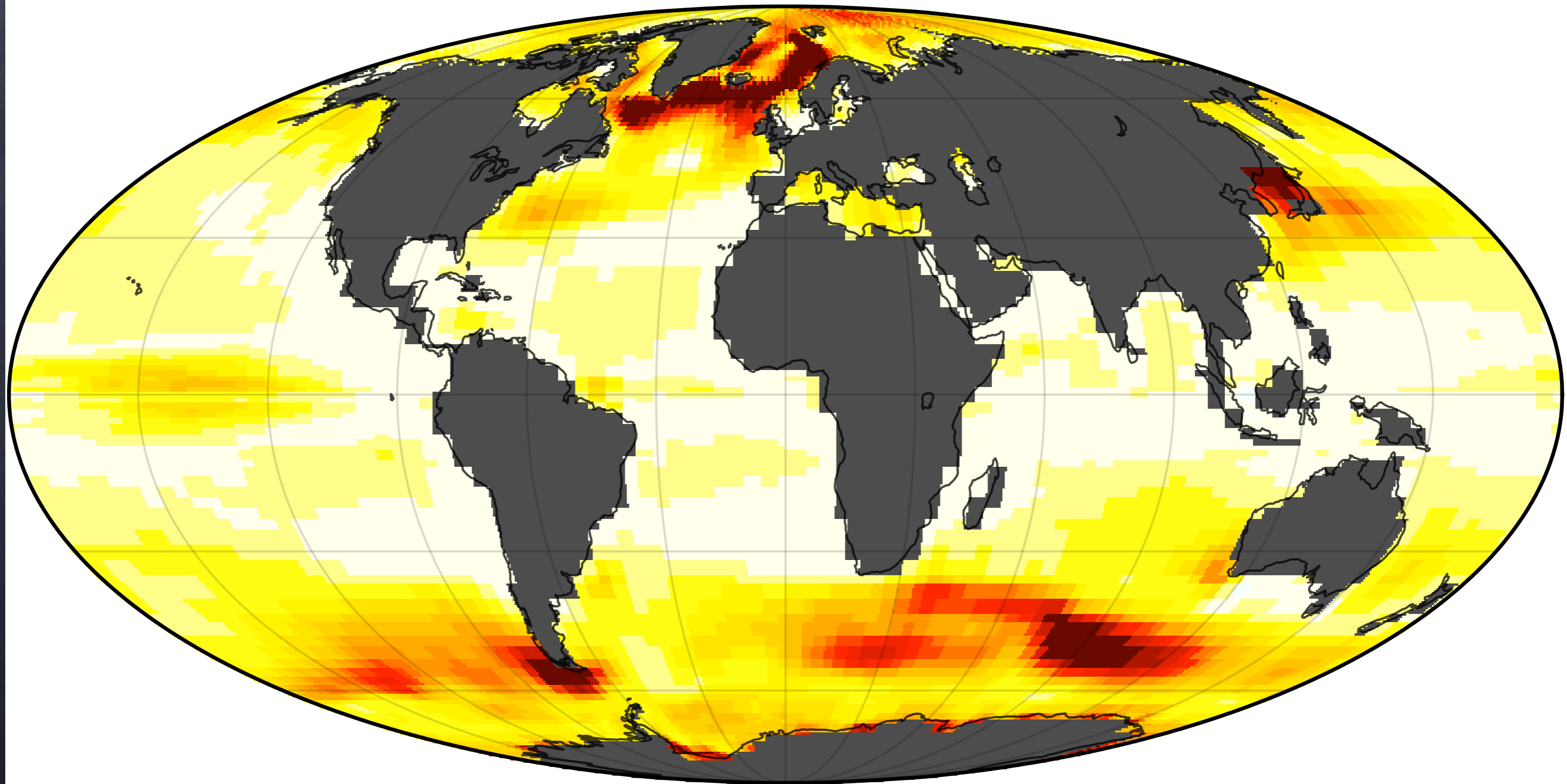


# Deep Mixed Layers Restratify Faster!

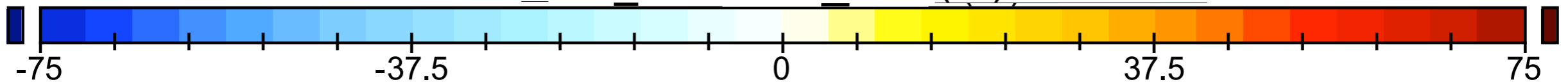
## Improves Restratification after Deep Convection

Note: param. reproduces Haine&Marshall (98) and Jones&Marshall (93,97)

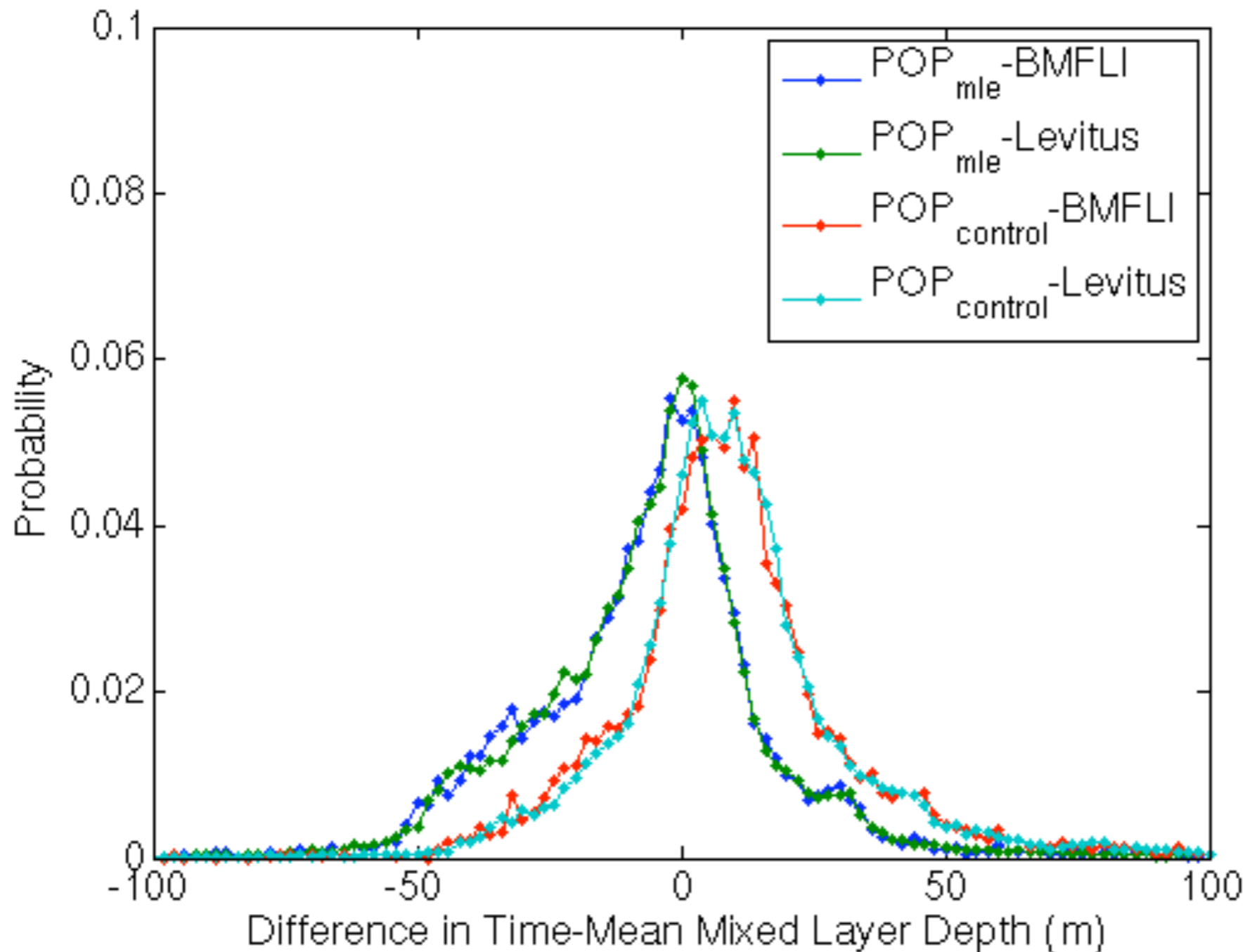
Change of Time-Mean Boundary Layer Depth in POP



BLD\_control - BLD\_mle (m)



# Bias Reduction: POP Model Mixed Layer Depth versus Observations



RMS error:  
16m  
reduced to  
8m

Skewness:  
2.4  
reduced to  
0.6



# Submesoscale Conclusion:

- Submesoscale features, and mixed layer eddies in particular, exhibit large vertical fluxes of buoyancy that are presently ignored in climate models.
- A parameterization of mixed layer eddy fluxes as an overturning streamfunction is proposed. The magnitude comes from extraction of potential energy, and the vertical structure resembles the linear Eady solution.
- Many observations are consistent, and model biases are reduced. Biogeochemical effects are likely, as vertical fluxes and mixed layer depth are changed.
- In HIM and CCSM/POP, soon to be in MITgcm & MOM.
- 3 Papers so far.. Just ask me for them.



# Mixing it up!

## Diagnosing Buoyancy Fluxes & Stirring

- Once the potential energy is extracted, the kinetic energy of the mesoscale and submesoscale eddies can be used to stir tracers.
- This is an important part of the global tracer transport, and thus of heat, freshwater, pollutants, and greenhouse-gas absorption and storage in the ocean
- However, in models with biogeochemistry, we trade high-resolution for reactions, so we have to parameterize all the eddy stirring by NES!
- Typically, the parameterizations are 'trained' on buoyancy fluxes, but...



# One version: Streamfunction

$$\overline{u'b'} = \Psi \times \nabla \bar{b}$$

$$\begin{bmatrix} \overline{u'b'} \\ \overline{v'b'} \\ \overline{w'b'} \end{bmatrix} = \begin{bmatrix} 0 & -\Psi_z & \Psi_y \\ \Psi_z & 0 & -\Psi_x \\ -\Psi_y & \Psi_x & 0 \end{bmatrix} \begin{bmatrix} \bar{b}_x \\ \bar{b}_y \\ \bar{b}_z \end{bmatrix}$$

Describes eddy buoyancy fluxes that are 'skew', i.e., along density surfaces.

These seem preferred since you don't need energy to do it. Can add along-isopycnal (Redi) diffusion of tracers, too!



# One version: Streamfunction

$$\overline{\mathbf{u}'b'} = \Psi \times \nabla \bar{b}$$

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Not general, but consider:

$$\begin{bmatrix} \overline{u'b'} \\ \overline{v'b'} \\ \overline{w'b'} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{\overline{u'b'}}{\bar{b}_z} \\ 0 & 0 & \frac{\overline{v'b'}}{\bar{b}_z} \\ -\frac{\overline{u'b'}}{\bar{b}_z} & -\frac{\overline{v'b'}}{\bar{b}_z} & \frac{\overline{w'b'}}{\bar{b}_z} + \frac{\overline{u'b'b_x + v'b'b_y}}{\bar{b}_z^2} \end{bmatrix} \begin{bmatrix} \bar{b}_x \\ \bar{b}_y \\ \bar{b}_z \end{bmatrix}$$

So, you can always do that.



Another version:

## Diffusion

$$\overline{u'b'} = -\kappa \cdot \nabla \bar{b}$$

$$\begin{bmatrix} \overline{u'b'} \\ \overline{v'b'} \\ \overline{w'b'} \end{bmatrix} = - \begin{bmatrix} \kappa_{xx} & \kappa_{xy} & \kappa_{xz} \\ \kappa_{xy} & \kappa_{yy} & \kappa_{yz} \\ \kappa_{xz} & \kappa_{yz} & \kappa_{zz} \end{bmatrix} \begin{bmatrix} \bar{b}_x \\ \bar{b}_y \\ \bar{b}_z \end{bmatrix}$$

Consider:

$$\begin{bmatrix} \overline{u'b'} \\ \overline{v'b'} \\ \overline{w'b'} \end{bmatrix} = \begin{bmatrix} \frac{\overline{u'b'}}{\bar{b}_x} & 0 & 0 \\ 0 & \frac{\overline{v'b'}}{\bar{b}_y} & 0 \\ 0 & 0 & \frac{\overline{w'b'}}{\bar{b}_z} \end{bmatrix} \begin{bmatrix} \bar{b}_x \\ \bar{b}_y \\ \bar{b}_z \end{bmatrix}$$

So, you can always do that.



# What? You can't tell diffusion from advection?

The problem is, we want to write:

$$\overline{\mathbf{u}'b'} = \mathbf{J} \cdot \nabla \bar{b}$$

As inspired by mixing lengths/scale separation arguments:

$$\overline{\mathbf{u}'b'} = \overline{\mathbf{u}'\xi'} \cdot \nabla \bar{b}$$

But, even if this form would work,  $\overline{\mathbf{u}'\xi'}$  has **9 elements** and we've only got **3 equations!**



So, we need more tracers! Getting the buoyancy fluxes right isn't enough!

$$\overline{u'\tau'_i} = \mathbf{J} \cdot \nabla \bar{\tau}_i$$

For  $i = 1, 2, 3$  distinct tracers then:

$$\begin{bmatrix} \mathbf{J}_{xx} & \mathbf{J}_{xy} & \mathbf{J}_{xz} \\ \mathbf{J}_{yx} & \mathbf{J}_{yy} & \mathbf{J}_{yz} \\ \mathbf{J}_{zx} & \mathbf{J}_{zy} & \mathbf{J}_{zz} \end{bmatrix} = - \begin{bmatrix} \overline{u'\tau'_1} & \overline{u'\tau'_2} & \overline{u'\tau'_3} \\ \overline{v'\tau'_1} & \overline{v'\tau'_2} & \overline{v'\tau'_3} \\ \overline{w'\tau'_1} & \overline{w'\tau'_2} & \overline{w'\tau'_3} \end{bmatrix} \frac{\begin{bmatrix} \left| \begin{array}{cc|cc|cc} \bar{\tau}_{2,y} & \bar{\tau}_{3,y} & \bar{\tau}_{3,x} & \bar{\tau}_{2,x} & \bar{\tau}_{2,x} & \bar{\tau}_{3,x} \\ \bar{\tau}_{2,z} & \bar{\tau}_{3,z} & \bar{\tau}_{3,z} & \bar{\tau}_{2,z} & \bar{\tau}_{2,y} & \bar{\tau}_{3,y} \end{array} \right| \\ \left| \begin{array}{cc|cc|cc} \bar{\tau}_{3,y} & \bar{\tau}_{1,y} & \bar{\tau}_{1,x} & \bar{\tau}_{3,x} & \bar{\tau}_{3,x} & \bar{\tau}_{1,x} \\ \bar{\tau}_{3,z} & \bar{\tau}_{1,z} & \bar{\tau}_{1,z} & \bar{\tau}_{3,z} & \bar{\tau}_{3,y} & \bar{\tau}_{1,y} \end{array} \right| \\ \left| \begin{array}{cc|cc|cc} \bar{\tau}_{1,y} & \bar{\tau}_{2,y} & \bar{\tau}_{2,x} & \bar{\tau}_{1,x} & \bar{\tau}_{1,x} & \bar{\tau}_{2,x} \\ \bar{\tau}_{1,z} & \bar{\tau}_{2,z} & \bar{\tau}_{2,z} & \bar{\tau}_{1,z} & \bar{\tau}_{1,y} & \bar{\tau}_{2,y} \end{array} \right| \end{bmatrix}}{|\bar{\tau}_{\pi,i}|}$$

With  $>3$  tracers, you can quantitatively assess:  
nonlocality, error, scale dependence, active vs. passive...



# Progress on J Tensor Diagnosis (ongoing)

- With John Dennis & Frank Bryan (NCAR), and help from LANL (Maltrud) and others (McClellan), we're running a 0.1 degree global ocean model with a suite > 10 tracers at BG/Watson
- We'll see what we see!
- Help in thinking about the problem would be appreciated! Difficulties in gauge invariance, etc., need to be sorted before analysis can be completed.



# Param. Applies to Other Scenarios: e.g., Hurricane Wake Recovery

