Mesoscale and Submesoscale Parameterizations:

Current Practice, New Developments, and Future Plans

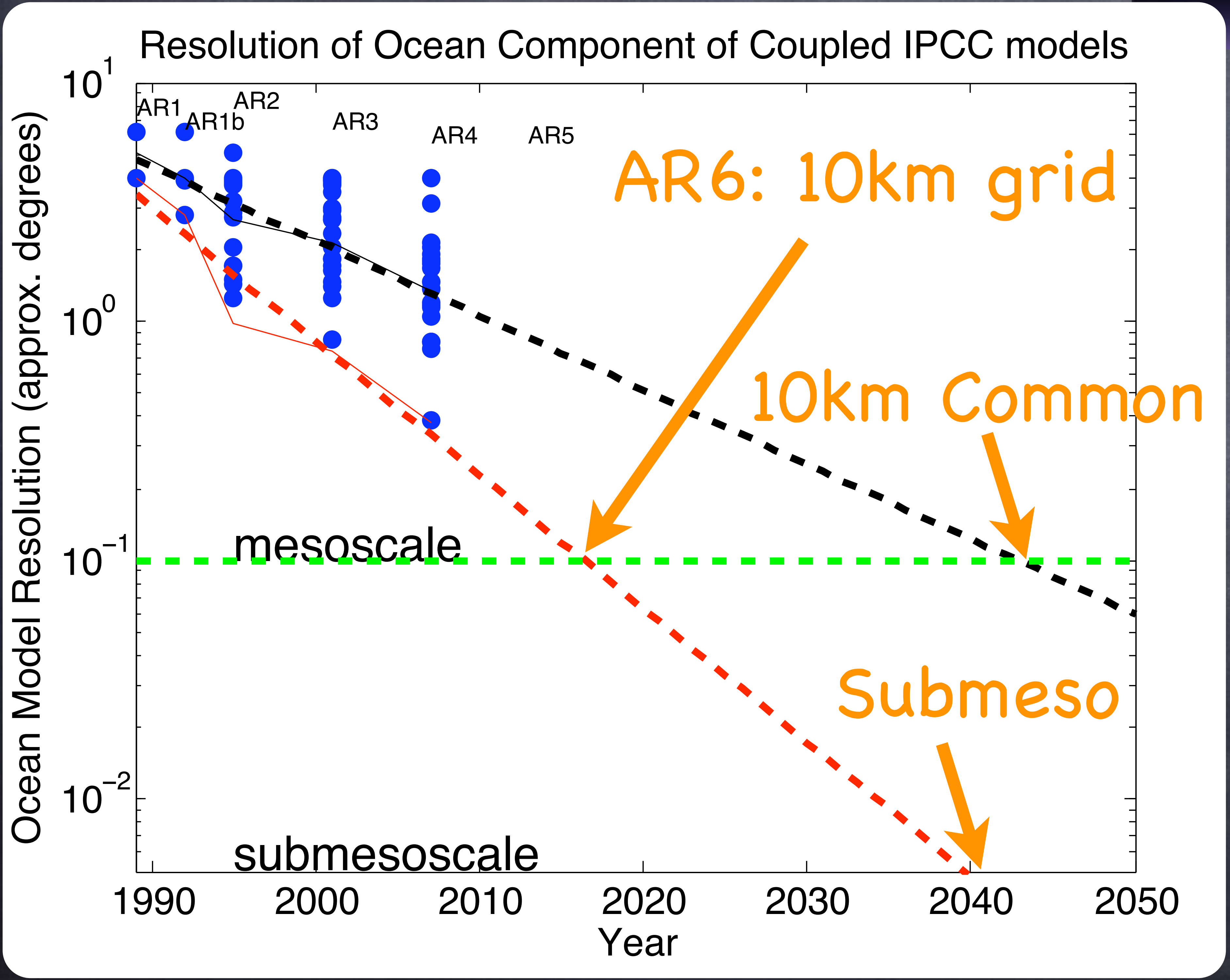
Baylor Fox-Kemper, University of Colorado

Lots of other input as mentioned in slides

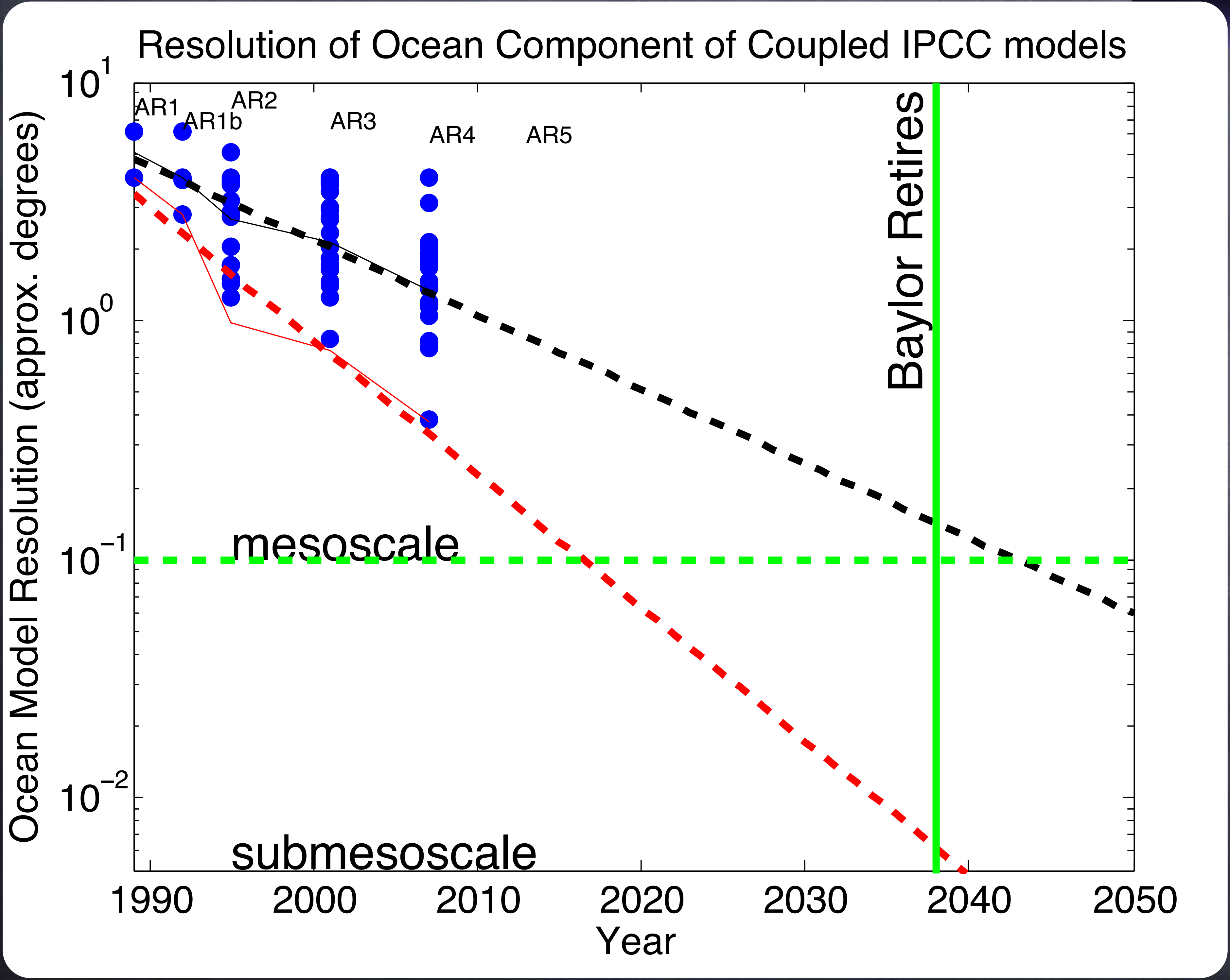
GFDL Ocean Climate Model Development Meeting

Th 29 Oct.

The Future of Resolution



The Future of Resolution



Outline

- Phenomenology of the Subgridscale--Mesoscale through Finescale
- Subgridscale Closure--in principle
- Subgridscale Closure--in practice
- Subgridscale Closure--in development

The Character of the Mesoscale

(Capet et al., 2008)

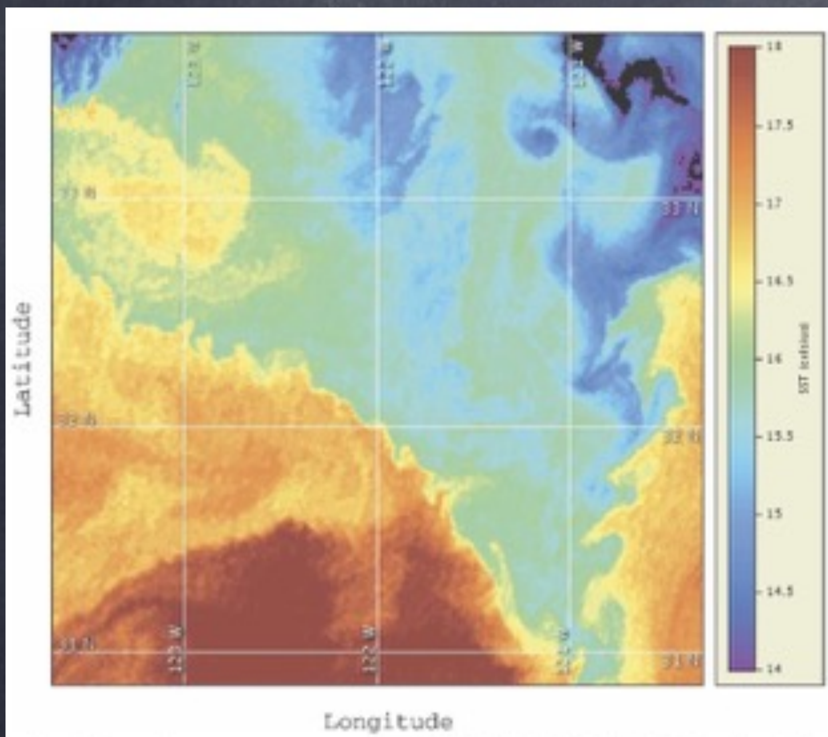
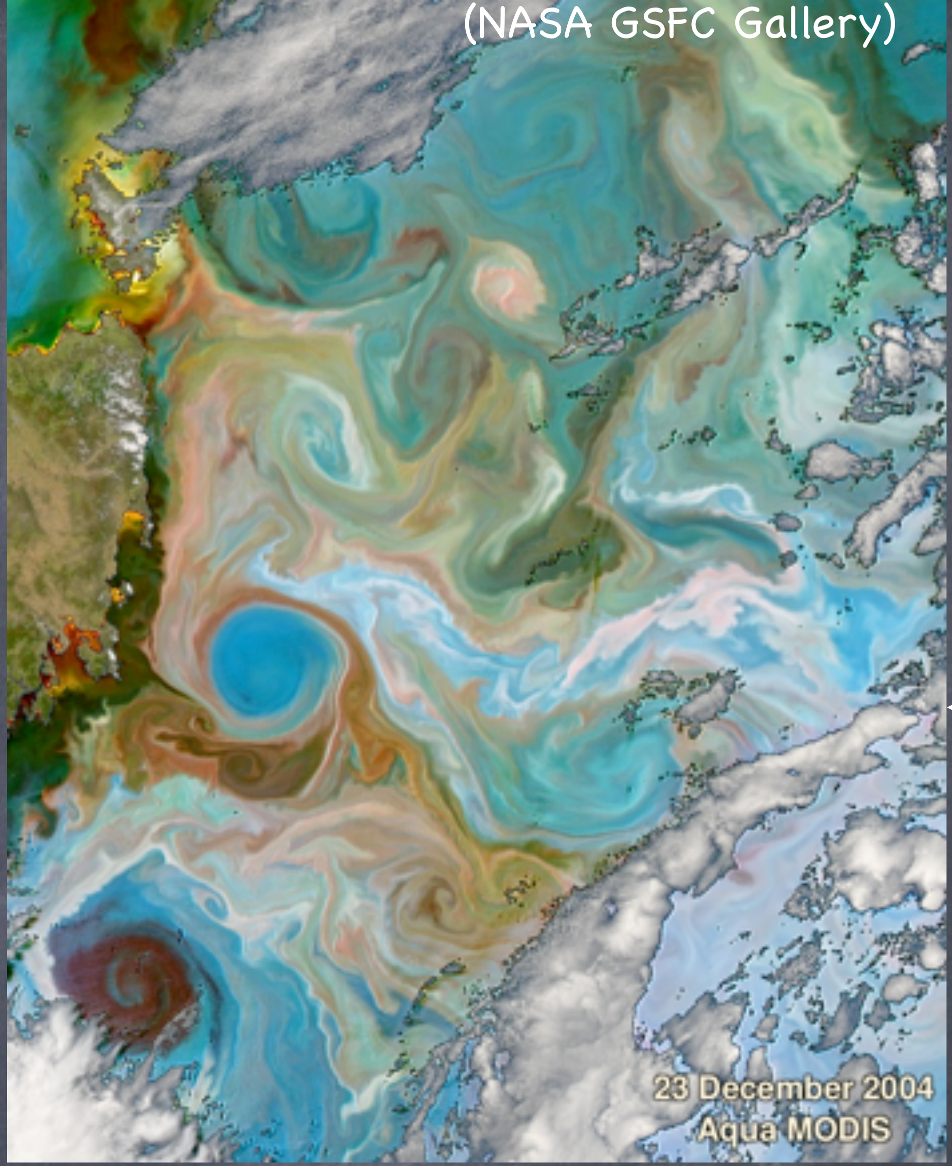


FIG. 16. Sea surface temperature measured at 1832 UTC 3 Jan 2006 off Point Conception in the California Current from CoastWatch (http://coastwatch.pfeg.noaa.gov). The fronts between recently upwelled water (i.e., 15°–16°C) and offshore water (≥17°C) show submesoscale instabilities with wavelengths around 30 km (right front) or 15 km (left front). Images for 1 day earlier and 4 days later show persistence of the instability events.

- Boundary Currents
- Eddies
- Ro=O(0.1)
- Ri=O(1000)
- Full Depth
- Projects on Fronts
- 100km, months

(NASA GSFC Gallery)

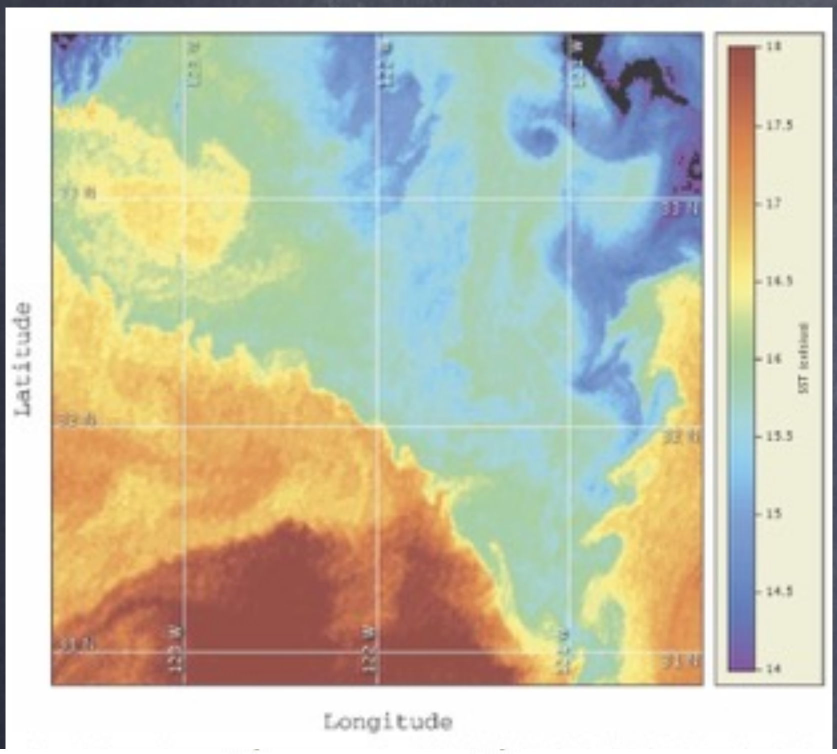


100 km

Eddy processes mainly baroclinic & barotropic instability.
Parameterizations of baroclinic instability (GM, Visbeck...).

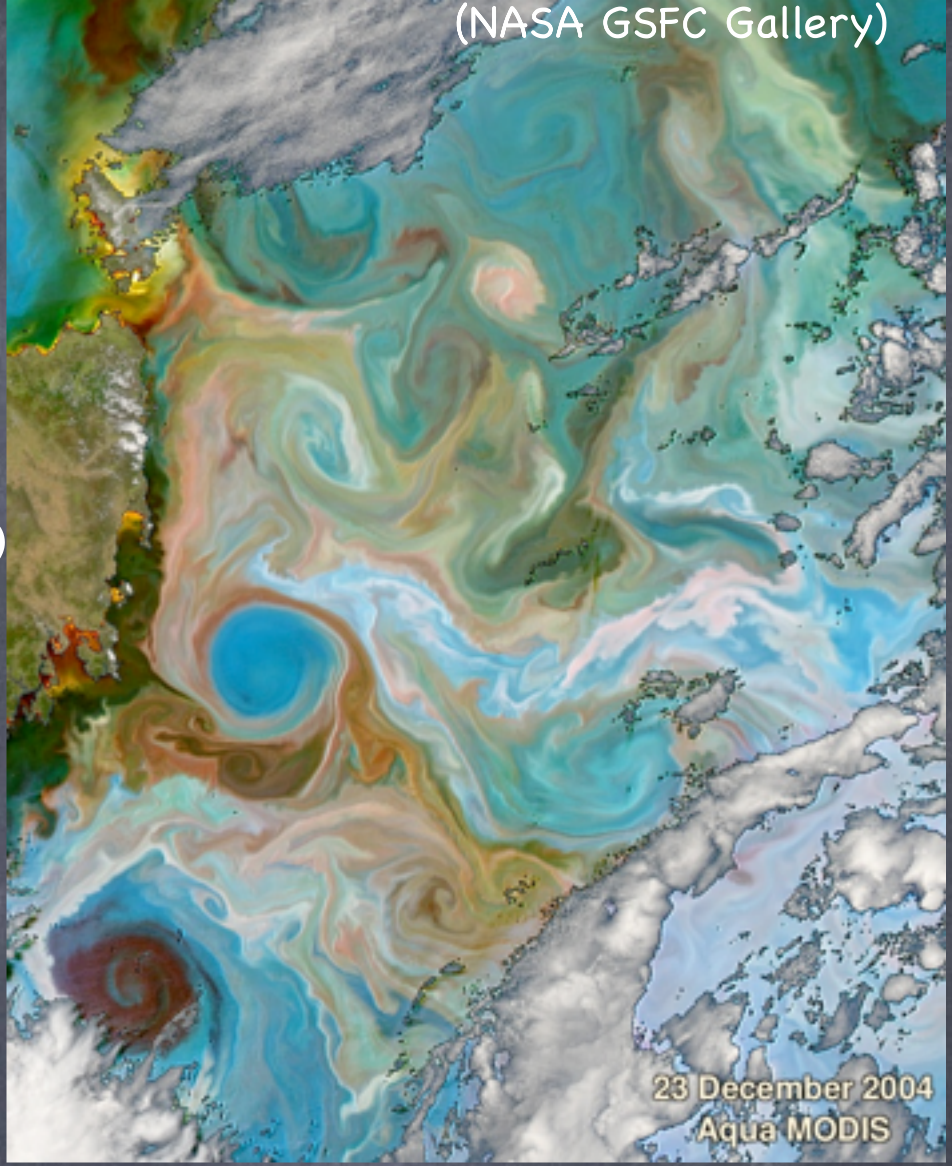
The Character of the Submesoscale

(Capet et al., 2008)

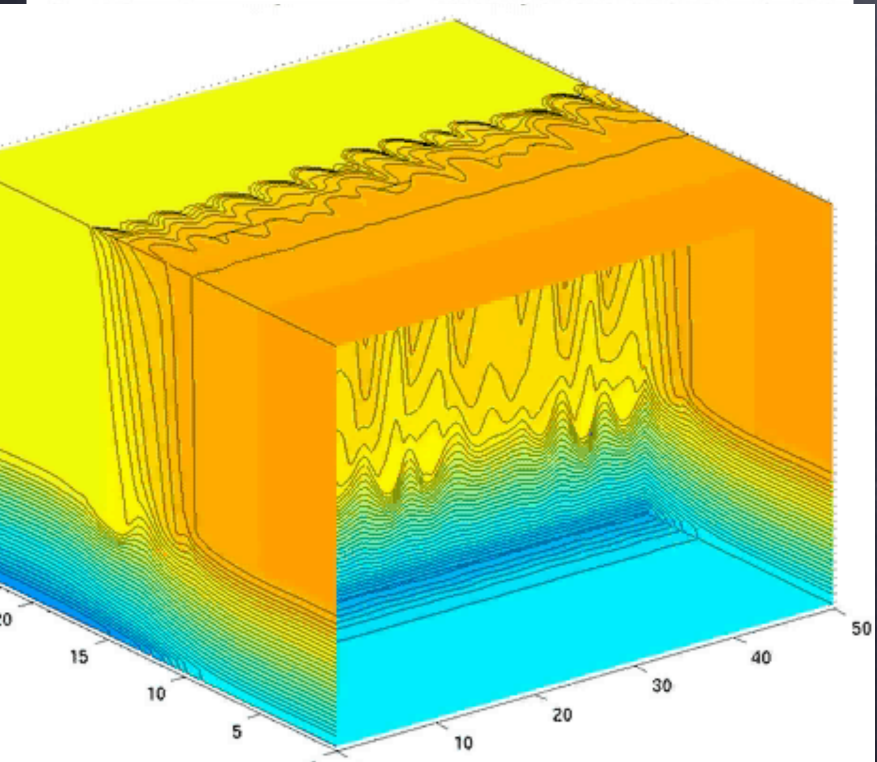


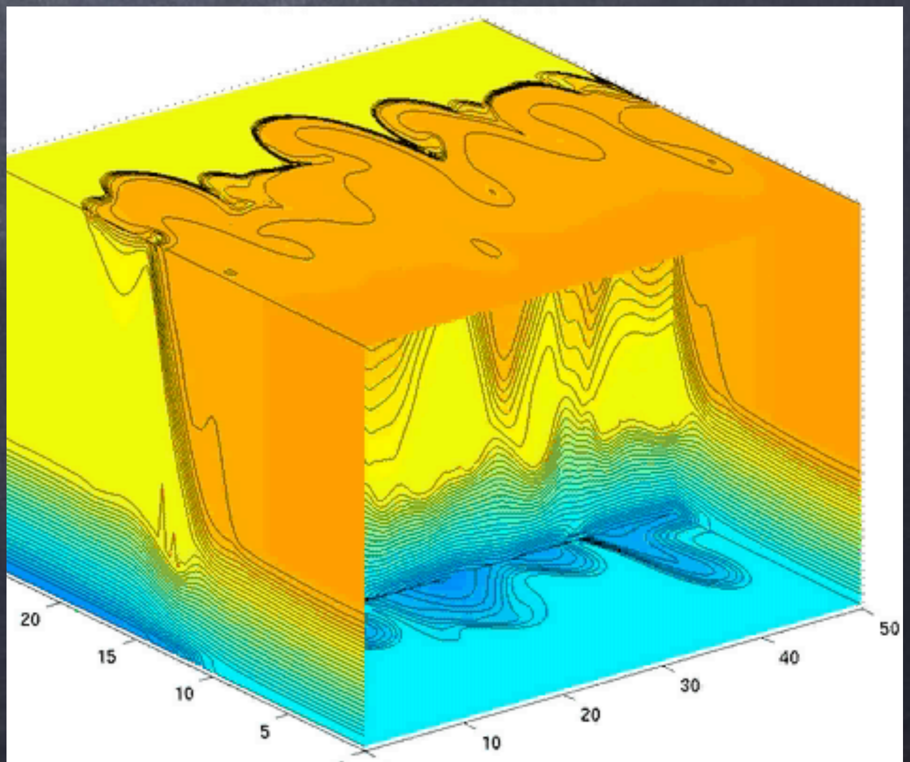
- Fronts & ageo
  wind
- Eddies
- Ro=O(1)
- Ri=O(1)
- near-surface
- 10km, days
- Parameterizations of eddies (FFH)

(NASA GSFC Gallery)



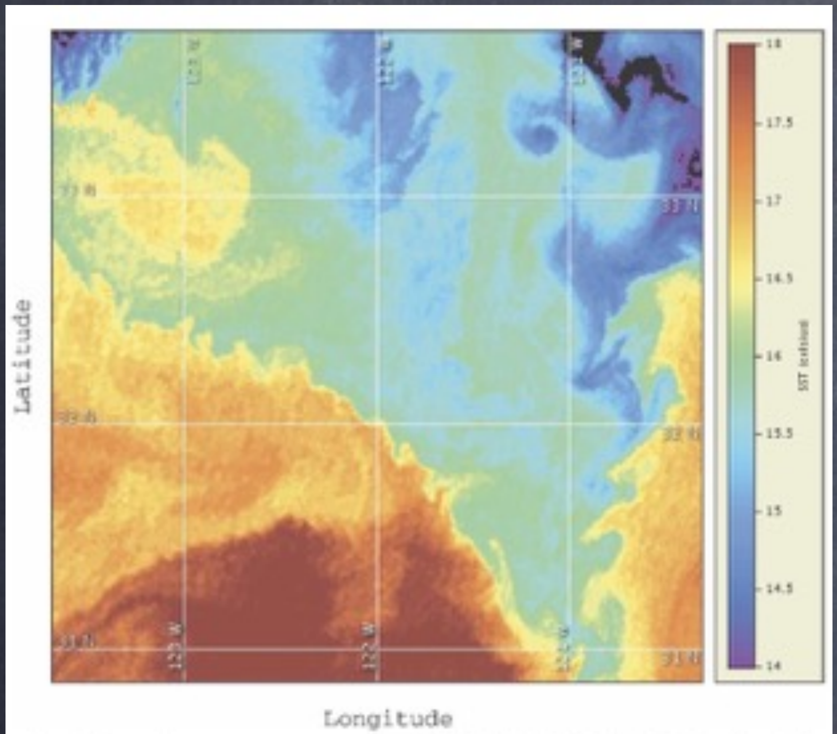
10 km





The Character of the Finescale

(Capet et al., 2008)



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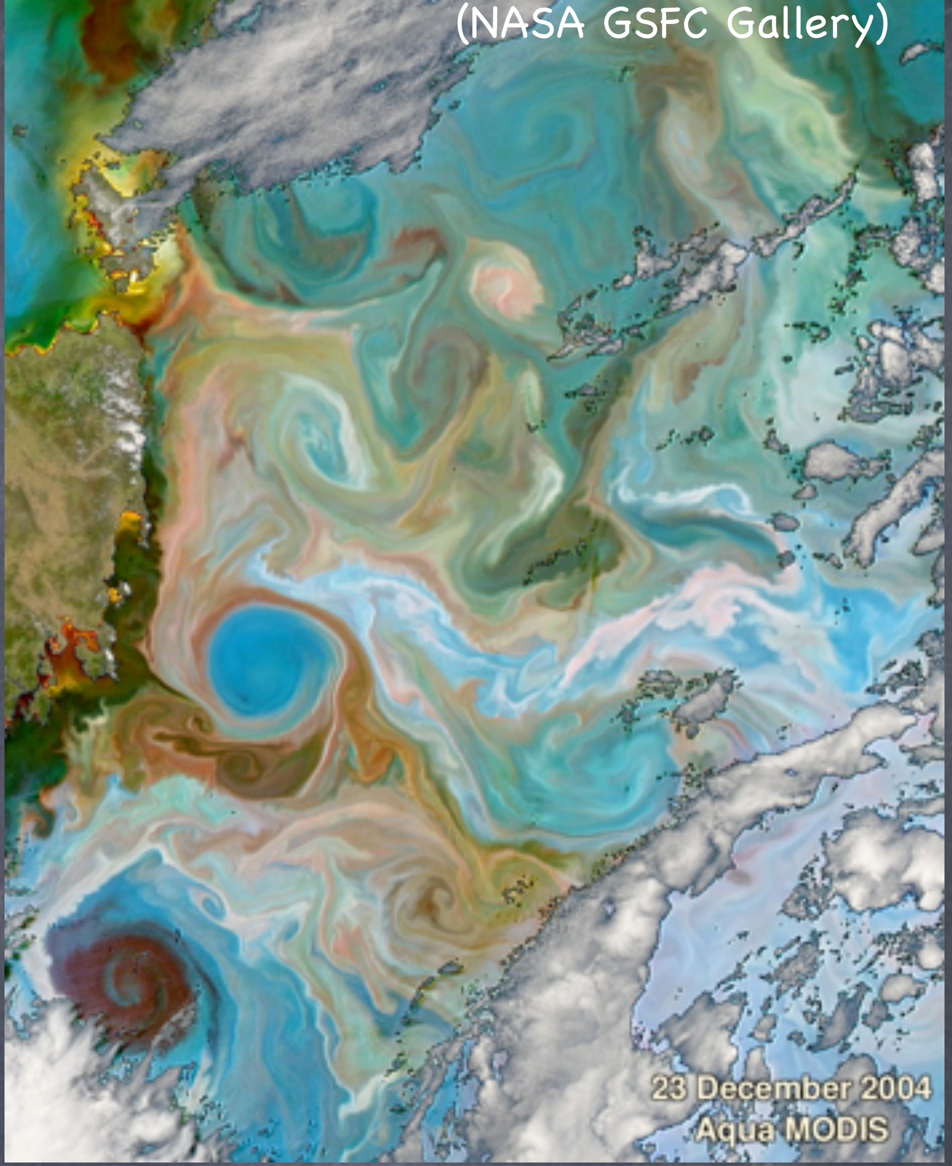
- 3d
- turbulent
- Ro>>1
- Ri<1 to <<1
- near-surface, bottom
- surface wave (Langmuir, breaking)
- internal waves/loss of balance/nonhydrostatic
- <100m, minutes-hrs.

(NASA GSFC Gallery)



100 m

Subgridscale Closure--In Principle

- Divvy the world up into spatially resolved and unresolved motions--Filter/avg. unresolved for resolved
- All nonlinear terms couple: u'v', u'b', u't', EKE, etc. Use & simulate fluctuation eqtns as guide. Beware thermodynamic constraints, e.g., nearly adiabatic flow.
  - I: MORANS->Gridscale in scale separation
  - II: MOLES->Gridscale as spectral truncation
- Grid/Filter scale sets physics: MO=Mesoscale Ocean, SO=Submesoscale Ocean, FO=Finescale Ocean, etc.
- RANS=Reynolds Avg Navier-Stokes,LES=Large Eddy

Subgridscale Closure--In Practice

- Divvy the world up into resolved and unresolved motions--Filters usually temporal/ensemble, not spatial!

- Some nonlinear terms couple: u'b', u't'. Use hodge-podge of obs., intuition, heuristics, scaling to motivate. Use neutral physics as constraint, but ignore distinctions between diabatic and dia-(coarsegrain neutral surf.)

    - Blend ideas from RANS/LES, e.g., Smag. + GM?

- Grid/Filter scale ignored, or scaled for on computational not physical reasoning (e.g., the model blows up if...)

MORANS, Objectives

- IPCC now, and with paleo/biogeochem for a long time, we will be coarser than def. radius.
- This means we are usually MORANS
- If the eddies doing the mixing are larger* than def. radius, no eddy momentum fluxes
*(how do we know how big the unresolved eddies are?)
- So, first order of MORANS business: need buoyancy & tracer flux closures

Tracer Flux-Gradient Relationship

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

- Virtually all extant subgridscale eddy closures may be written as above, e.g.: GM, Redi, FFH
- Relates the eddy flux to the coarse-grain gradients
- May have a flow/property dependent $\mathbf{M}$: (FFH, Visbeck, Green, Held & Larichev, Stone, Canuto & Dubovikov, Griffies et al '05)
- May consider gridscale (FFH, Hallberg & Adcroft)
- Isopycnal & lagrangian coordinate versions possible/known

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

General Form

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

- Diagnostically: 9 elements requires at least 3 similar-transport tracers to specify uniquely
- Could vary tracer by tracer, or active tracer vs. passive, etc. In practice we don't do this.

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

Anistropic* Redi Form

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ K_{yx} & K_{yy} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \tilde{\nabla}z\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Yellow Elements are horizontal stirring

Blue Elements in Redi (1982) are symmetric
and scaled to make
eddy mixing along neutral surfaces

*Anistropic form due to Smith & Gent 04

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

Anisotropic* Gent-McWilliams

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ 0 & 0 & -\hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Antisymmetric Elements in GM (1990)
are scaled to overturn fronts, make vertical fluxes
extract PE, and restratify the fluid
equivalent to eddy-induced advection

Q: Same K as Redi?

*Anistropic form due to Smith & Gent 04 *Tensor Form (Griffies, 98)

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

Fox-Kemper, Ferrari, & Hallberg (2008) form (a mixed layer (submeso) eddy param.):

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\boldsymbol{\Psi}_\mathbf{y} \\ 0 & 0 & \boldsymbol{\Psi}_\mathbf{x} \\ \boldsymbol{\Psi}_\mathbf{y} & -\boldsymbol{\Psi}_\mathbf{x} & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Antisymmetric Elements in Fox-Kemper, Ferrari, & Hallberg (2008) are scaled to overturn fronts, make vertical fluxes extract PE, and restratify the fluid, At a rate validated against eddying simulations!

CM/MOM AR5 Practice

- Antisymmetric Part:
  - Vert. Variation: Ferrari, Griffies, Nurser, Vallis (2009)
  - Horiz. Variation: Griffies et al (2005) spatial dependent diffusivity (depends on vertically averaged baroclinicity; as in CM2.1). Max diffusivity is 800mks, min is 100mks. Implemented as skew diffusion.
  - Submeso param: FFH (in mixed layer only)
- Symmetric Part:
  - Neutral diffusion: Griffies et al (1998) with constant diffusivity of 600m^2/s (as in CM2.1)
  - Thanks to Griffies for this list

CCSM/POP AR5 Practice

- Antisymmetric Part:
  - Vert. Variation: The near-surface eddy flux parameterization of Ferrari et al. (2008) as implemented by Danabasoglu et al. (2008)
  - GM90 with vertically-varying coefficients 3000 m^2/s in surface diabatic layer to 300 m^2/s by a 2km depth(Danabasoglu and Marshall 2007).
  - Horiz. Variation: None.
  - Submeso param: FFH (in mixed layer only)
- Symmetric Part:
  - Neutral diffusion equal to GM coefficient. Matching Horiz. diffusivity in surface diabatic layer.
  - Thanks to Gokhan for this list

CM/GOLD AR5 Practice

- Antisymmetric Part:
  - Vert. Variation: None.
  - Horiz. Variation: GM diffusivity ala Visbeck et al 97. More from Alistair and/or Bob later.
  - Submeso param: FFH (in mixed layer only)
- Symmetric Part:
  - Neutral diffusion diffusivity is our final tuning, TBA?
  - Thanks to Griffies for this list

Topics for discussion: I

- Diagnosis: Spatial Variation of $\mathbf{M}$ (Ross, Shafer, Baylor)
  - Indeterminacy? (Baylor)
- Prognosis: Spatiotemporal & Flow-Dependent $\mathbf{M}$ (Baylor, Matt, Alistair)
- Dia-(coarse pycnal) $\mathbf{M}$? (All)
- Beyond $\mathbf{M}$:
  - Momentum Fluxes (Via PV, alpha-model, other) (Matt, Peter)
  - Stochastics, nonlocal closures, superparam, what have you got? (Any)
  - Too many scientists, not enough engineers: How can we go from theory to implementation?
  - Non-eddy subgridscale effects? Fronts, Wind, A-O Feedbacks, Bndy Currents?

Topics for discussion: II

- Scaling for MOLES (Bob)
- What can linear theory tell us? (Shafer)
- What can process models/idealized sims tell us? (Baylor)
- What can other theory tell us?
  - GLM? TEM? Stat Mech?
- Anisotropy, tracer type dependence, biogeochem tracers, etc.? (Baylor, others?)

Topics for discussion: I

- Diagnosis: Spatial Variation of $\mathbf{M}$ (Ross, Shafer, Baylor)
  - Indeterminacy? (Baylor)
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Need a Natural, Mesoscale Eddy Environment to Test Out:

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

Fox-Kemper, with
Frank Bryan, John Dennis (NCAR)
Students: S. Bachman, A. Margolin

Need a Natural, Mesoscale Eddy Environment to Test Out:

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

Does Redi Work?
Does GM Work?

What is the spatial/flow dependence?

Can we improve GM/Redi by comparison to eddying simulations a la FFH?

Need a Natural, Mesoscale Eddy Environment to Test Out:

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = -\begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

3 equations/tracer

9 unknowns ($\mathbf{M}$ components)

BY USING 3 or MORE TRACERS, can determine $\mathbf{M}$!!!

(a la Plumb & Mahlman '87, Bratseth '98)

Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

We Use:

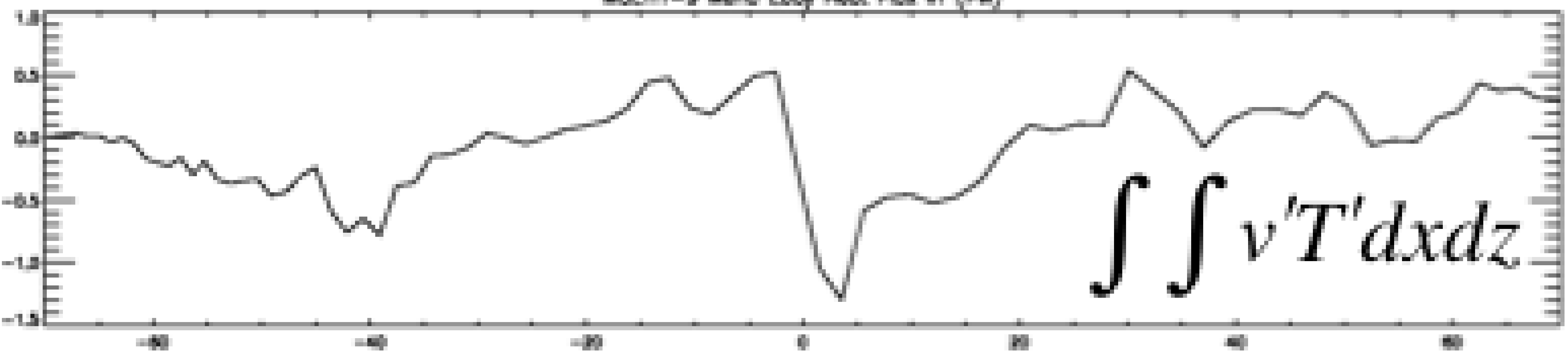
Years 16-20 of a Global 0.1 Degree Model (sim to Maltrud & McClean '06)

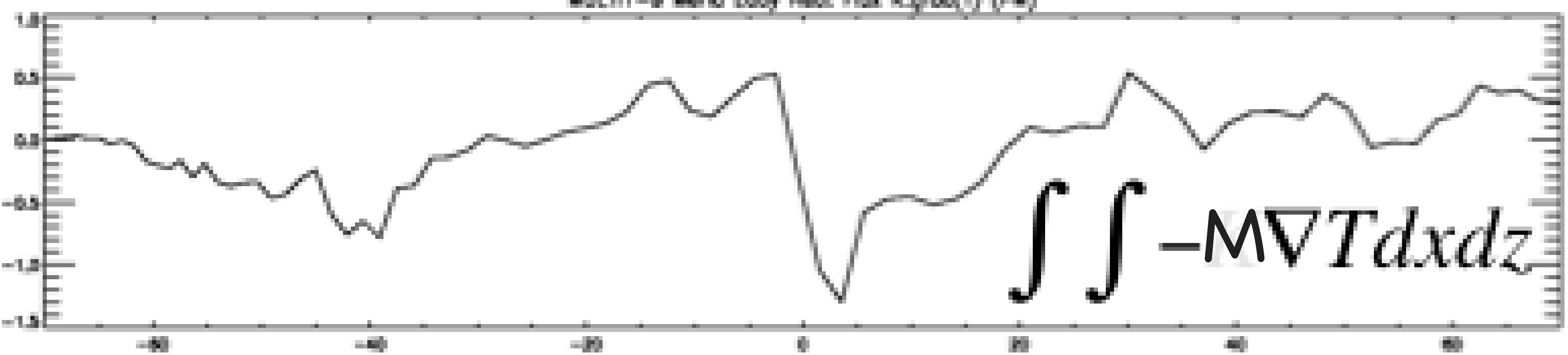
9 Passive Tracers To Overdetermine $\mathbf{M}$

Use a Natural, Mesoscale Eddy Environment to Test Out:

Testing the Diagnosis:

Note: T not used for diagnosis, active tracers are apparently transported as passive ones are!





Difference: Diffusive – Eddy

Use a Natural, Mesoscale Eddy Environment to Test Out:

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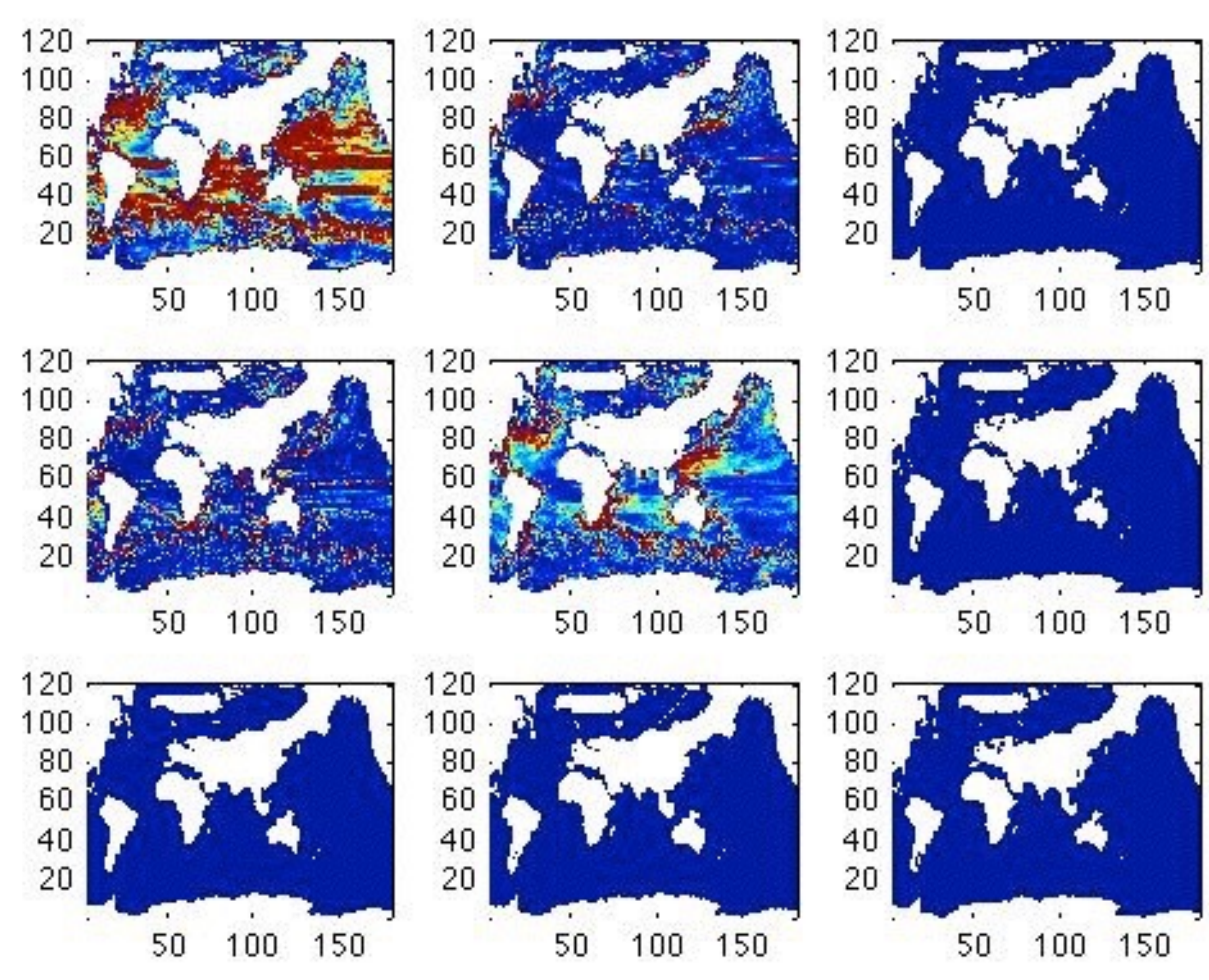
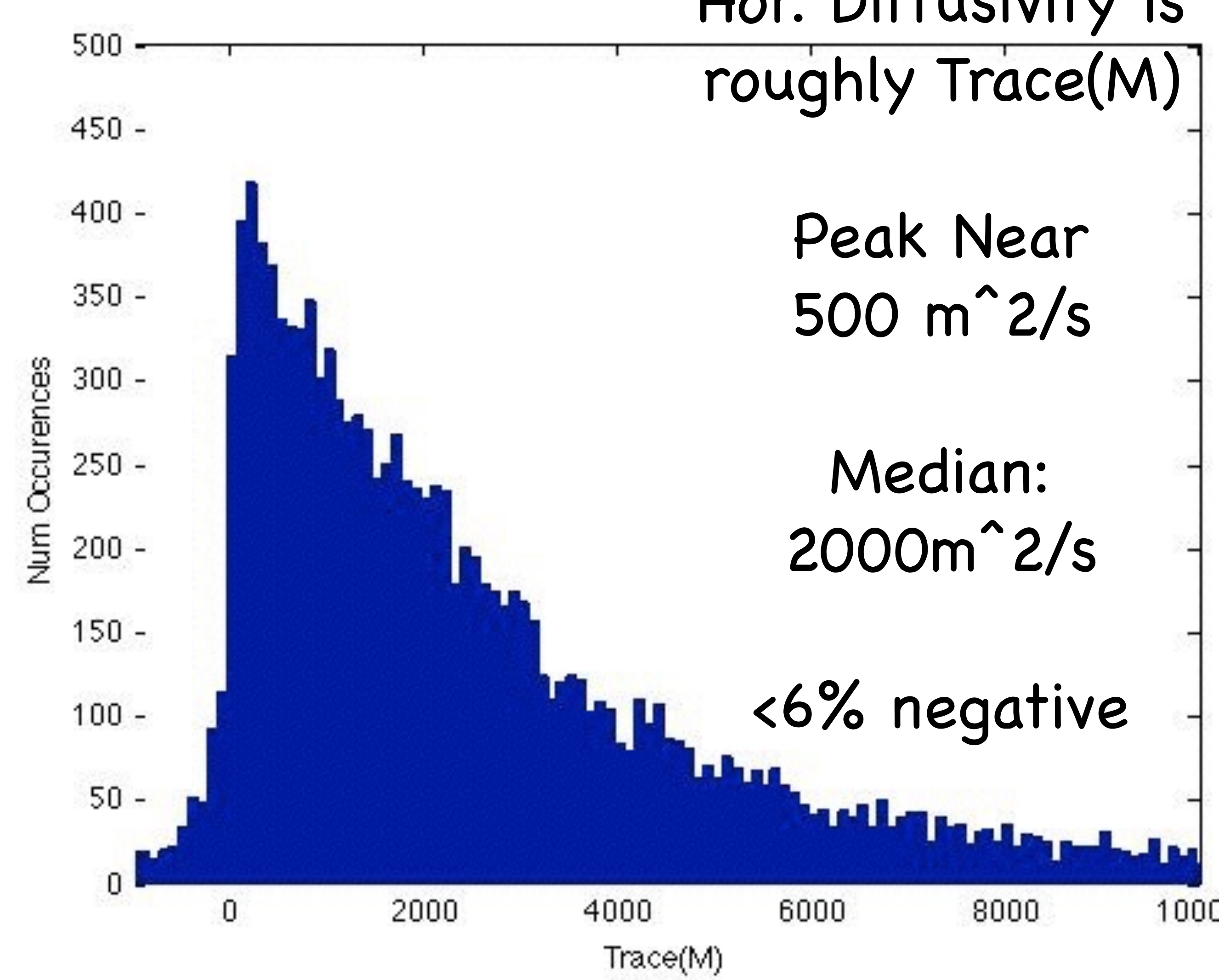
Correct shape/scale at 150m depth:





Hor. Diffusivity is roughly Trace(M)

Peak Near 500 m^2/s

Median: 2000m^2/s

<6% negative

Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ 0 & 0 & -\hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Antisymmetric (GM) Elements scale
with
corresponding Symmetric (Redi)
elements.

Thus, GM/Redi basic shape of M is
roughly correct
(some detailed validation remains)

Use a Natural, Mesoscale Eddy Environment to Test Out:

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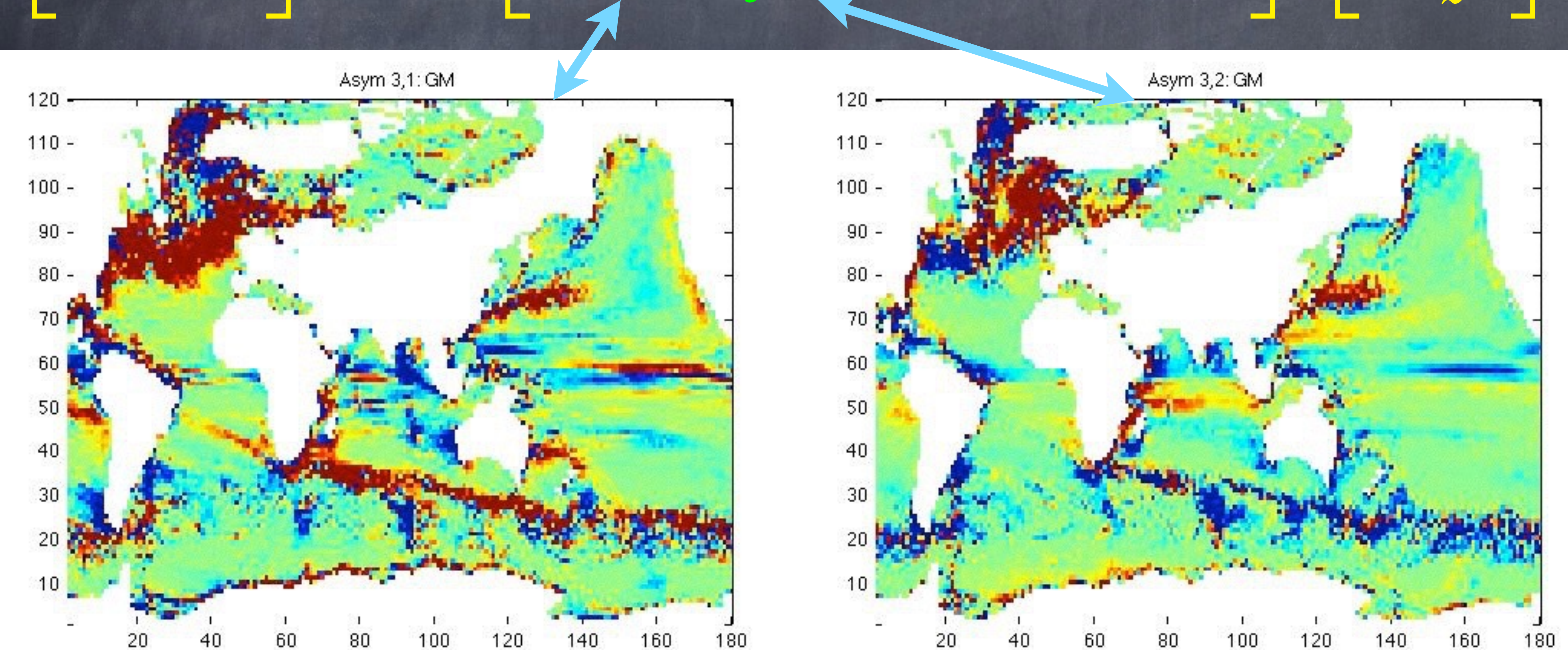

Use a Natural, Mesoscale Eddy Environment to Test Out:

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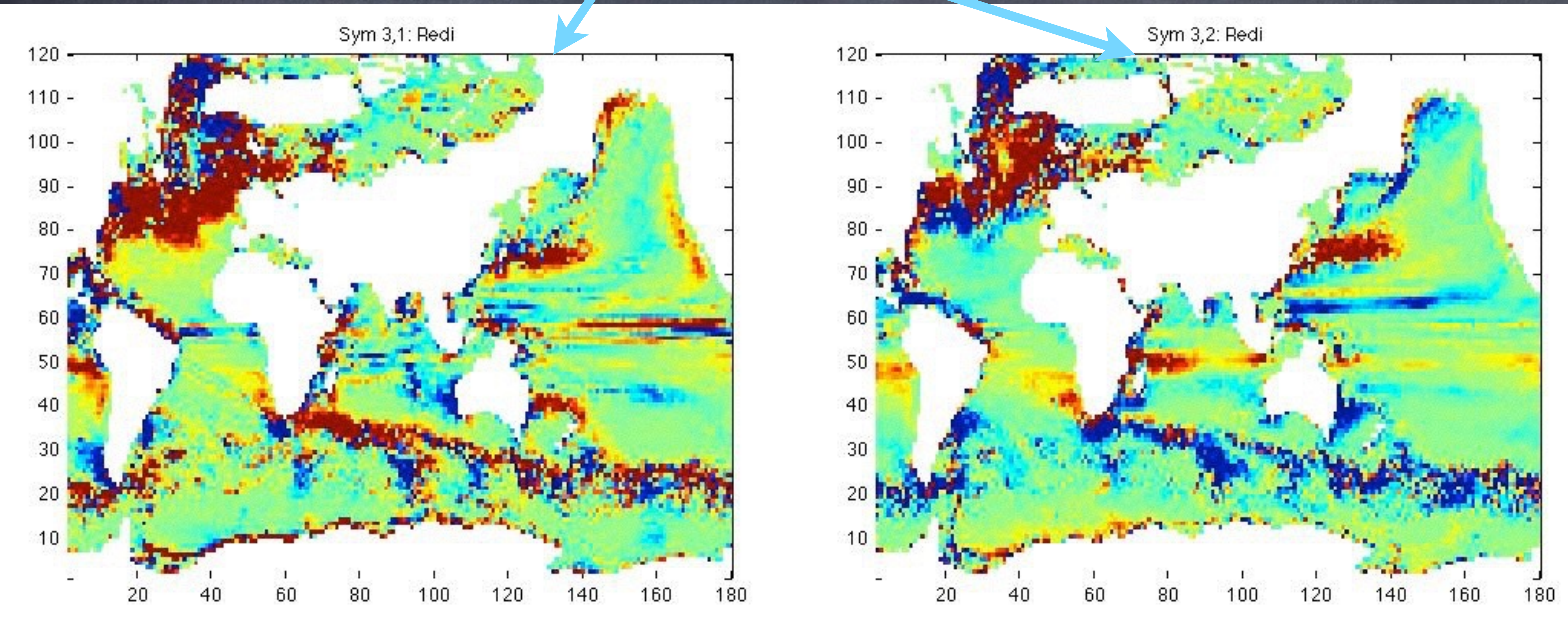

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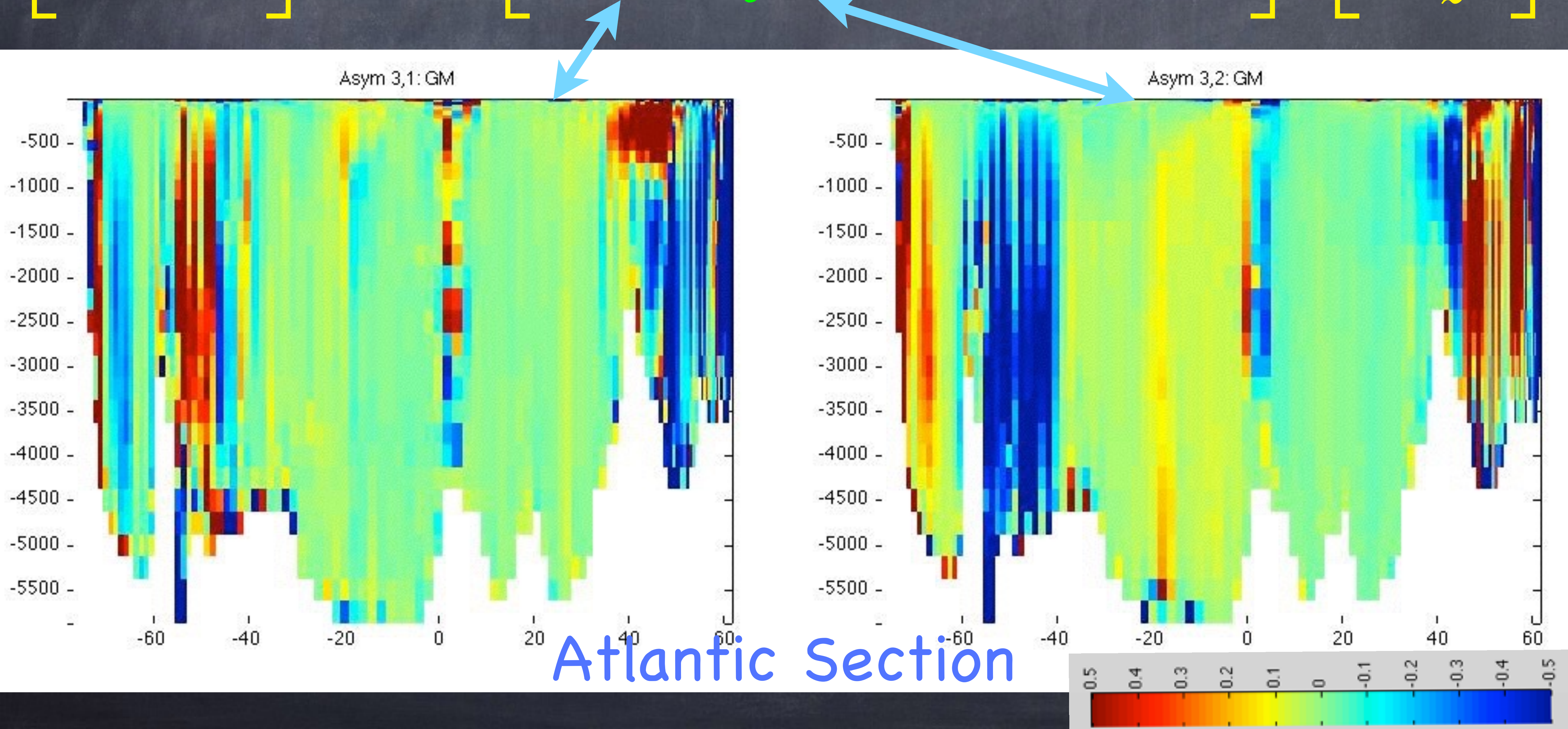

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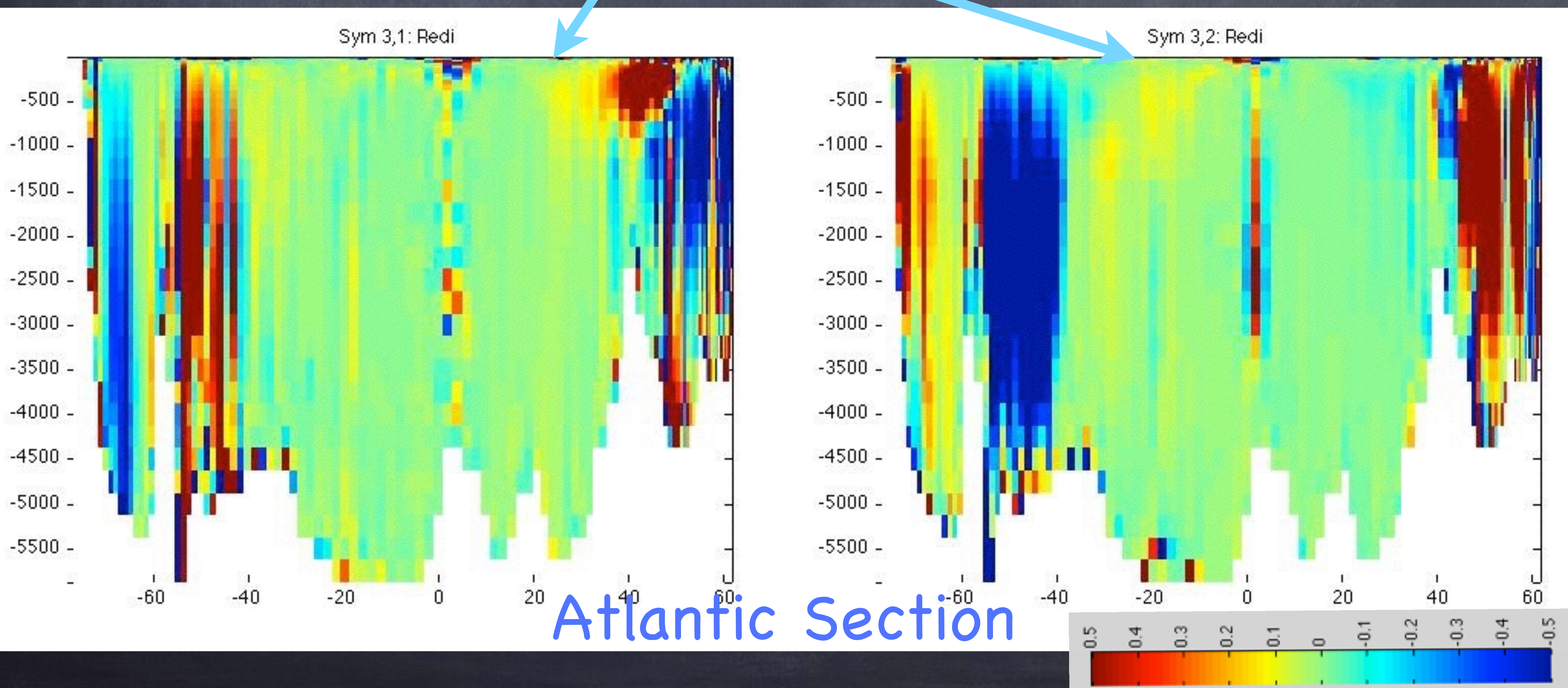

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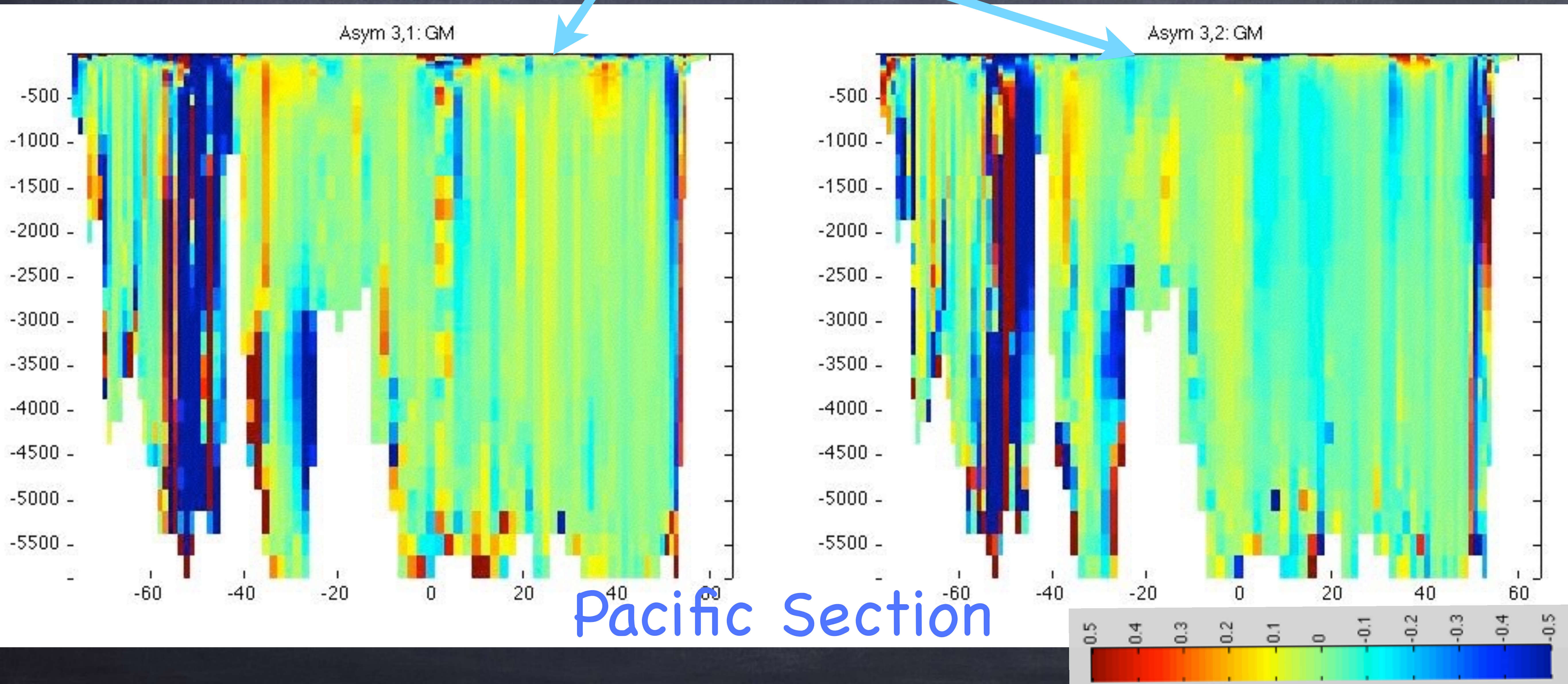

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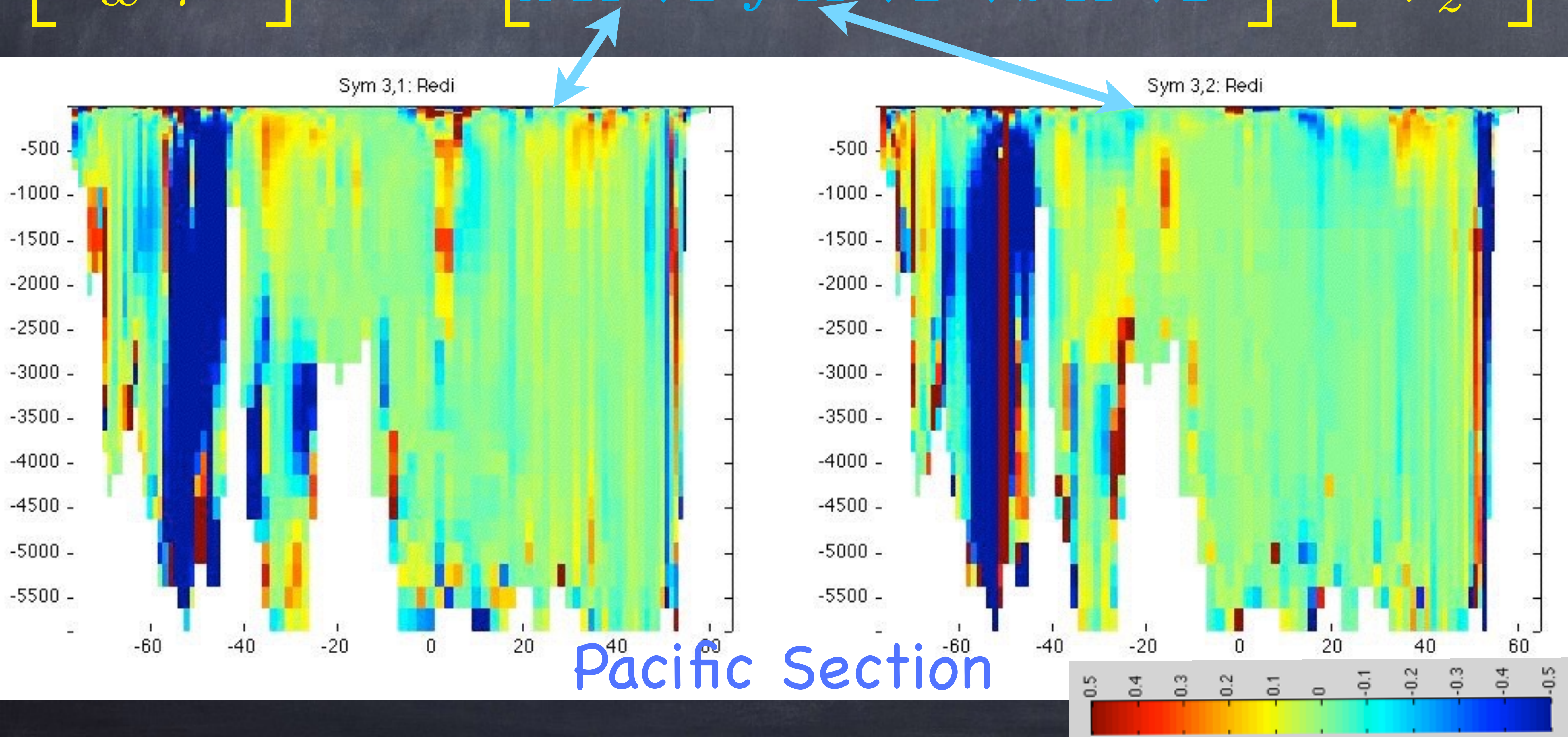
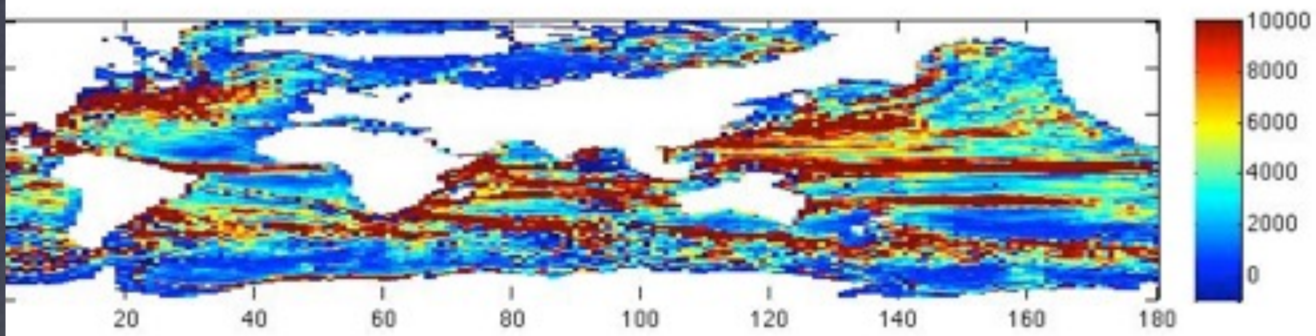
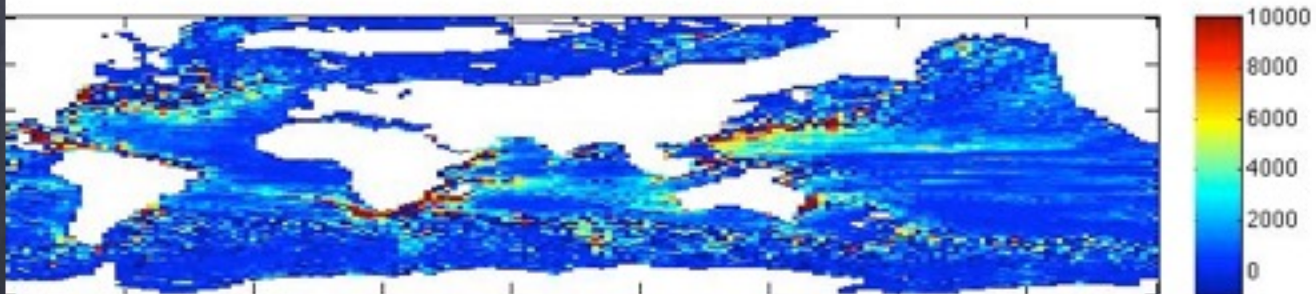
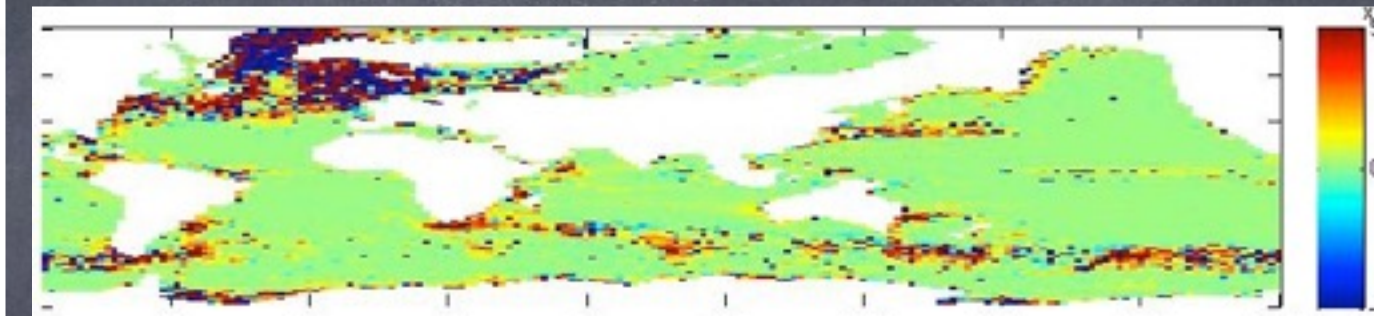




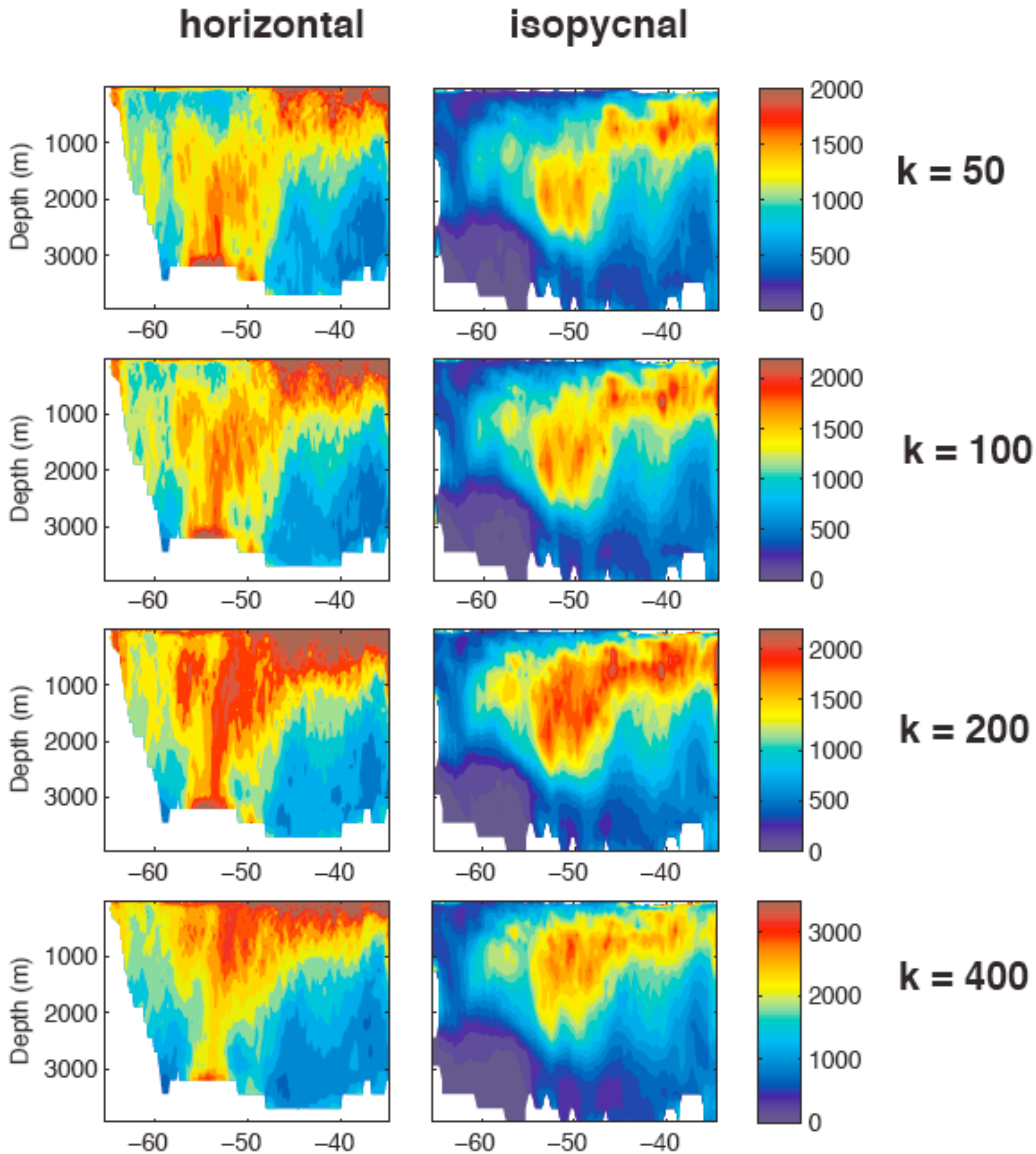
Conclusions



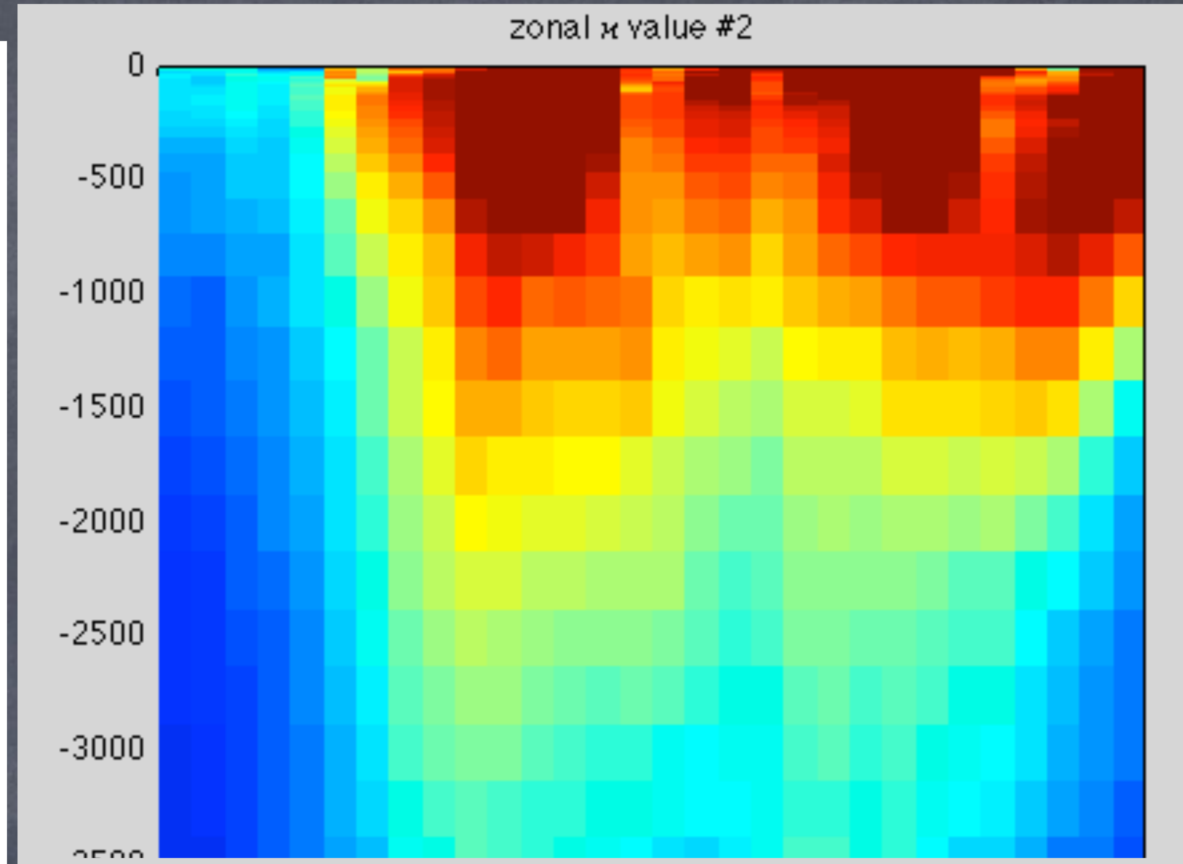


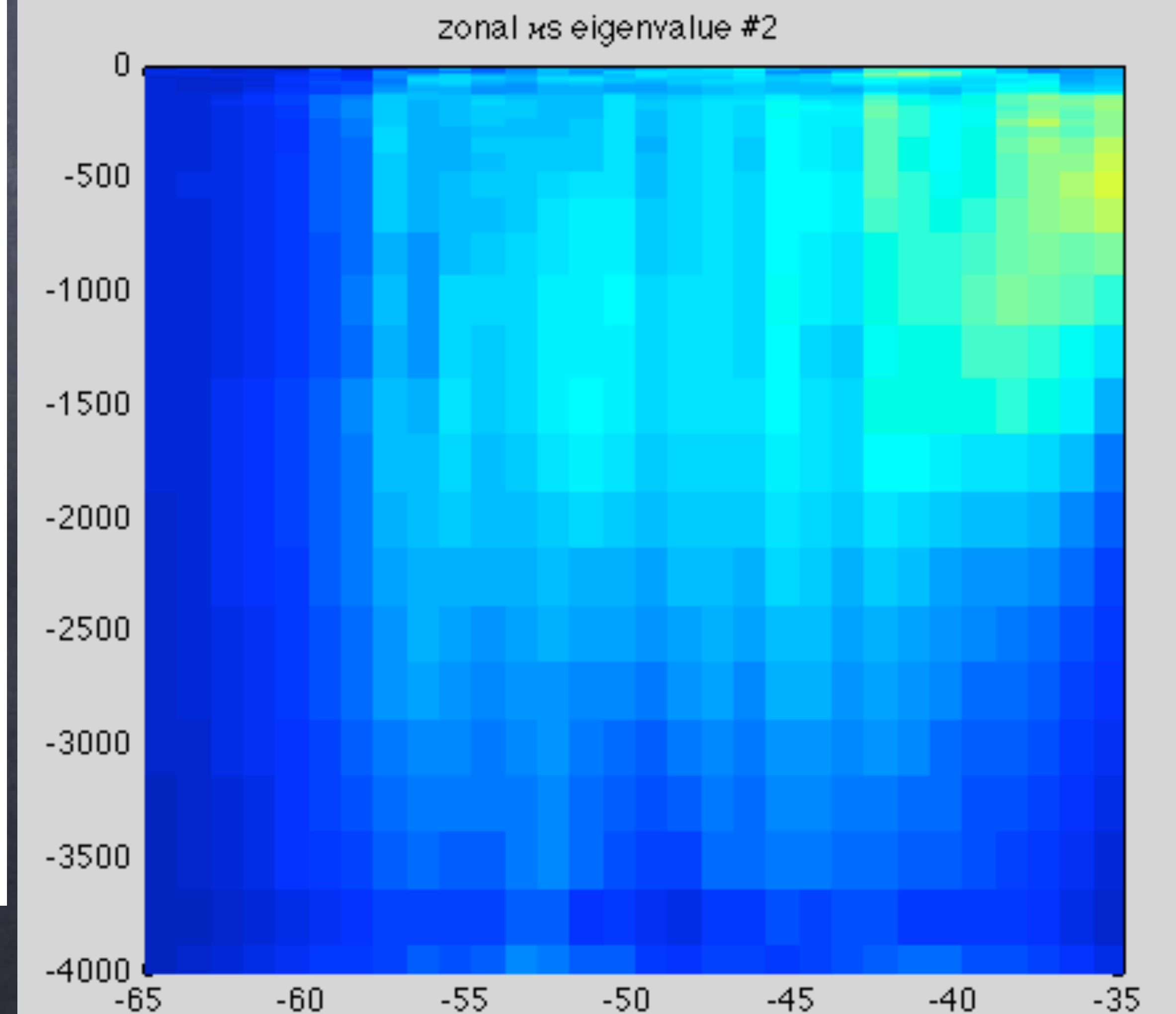
- Passive Tracers are used in a global 0.1 model to diagnose Mesoscale Flux-Gradient Relationship
- Resembles $\mathrm{GM} \approx \mathrm{Redi}$ with O(2000m^2/s) anisotropic (zonal & strong flow), Flow&Depth-dependent.
- Active vs. Passive tracers apparently not an issue
- To come: Dia-(coarse neutral) eddy fluxes? Scaling?

Comparisons with Marshall et al.



Abernathy et al 09





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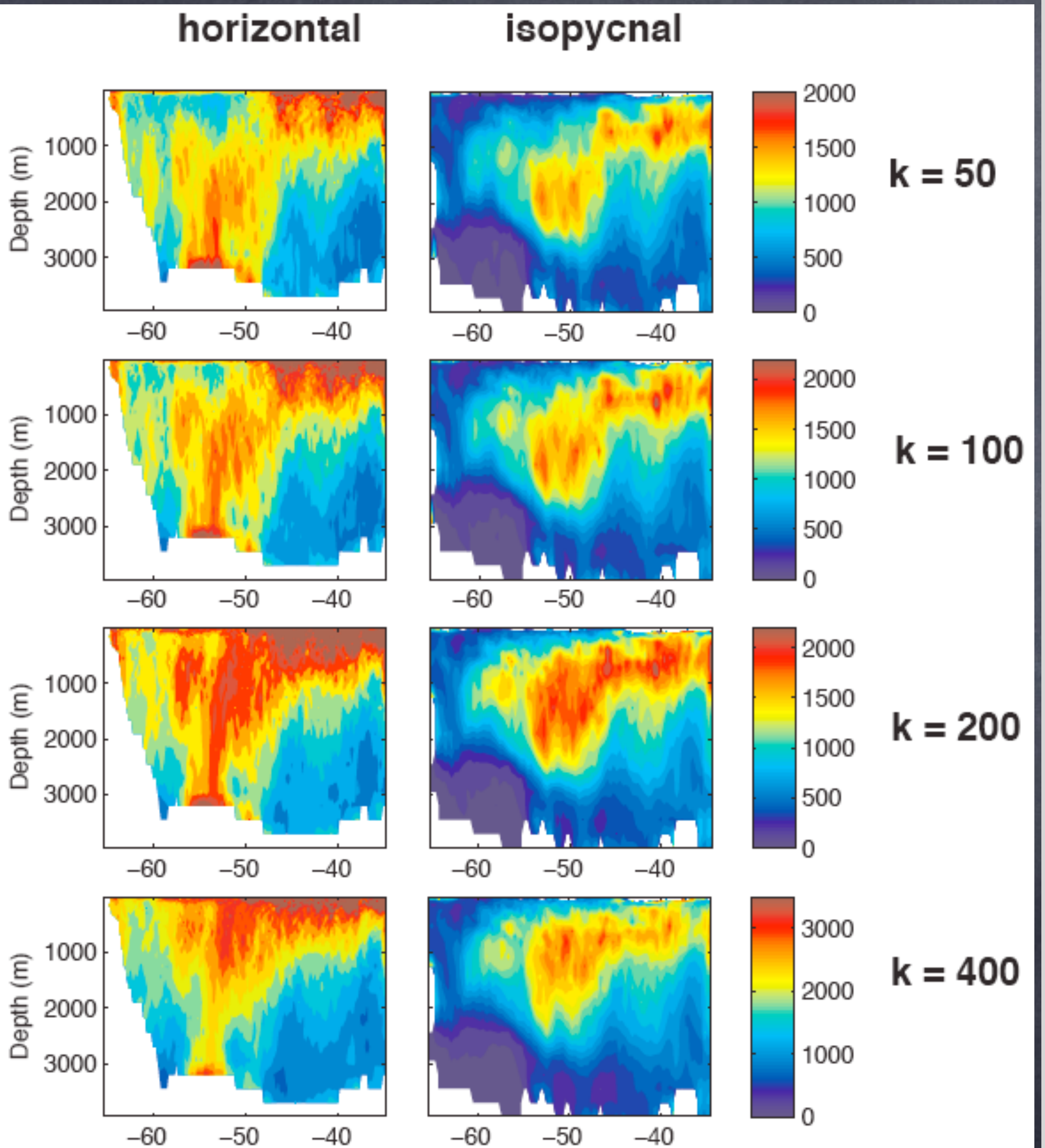

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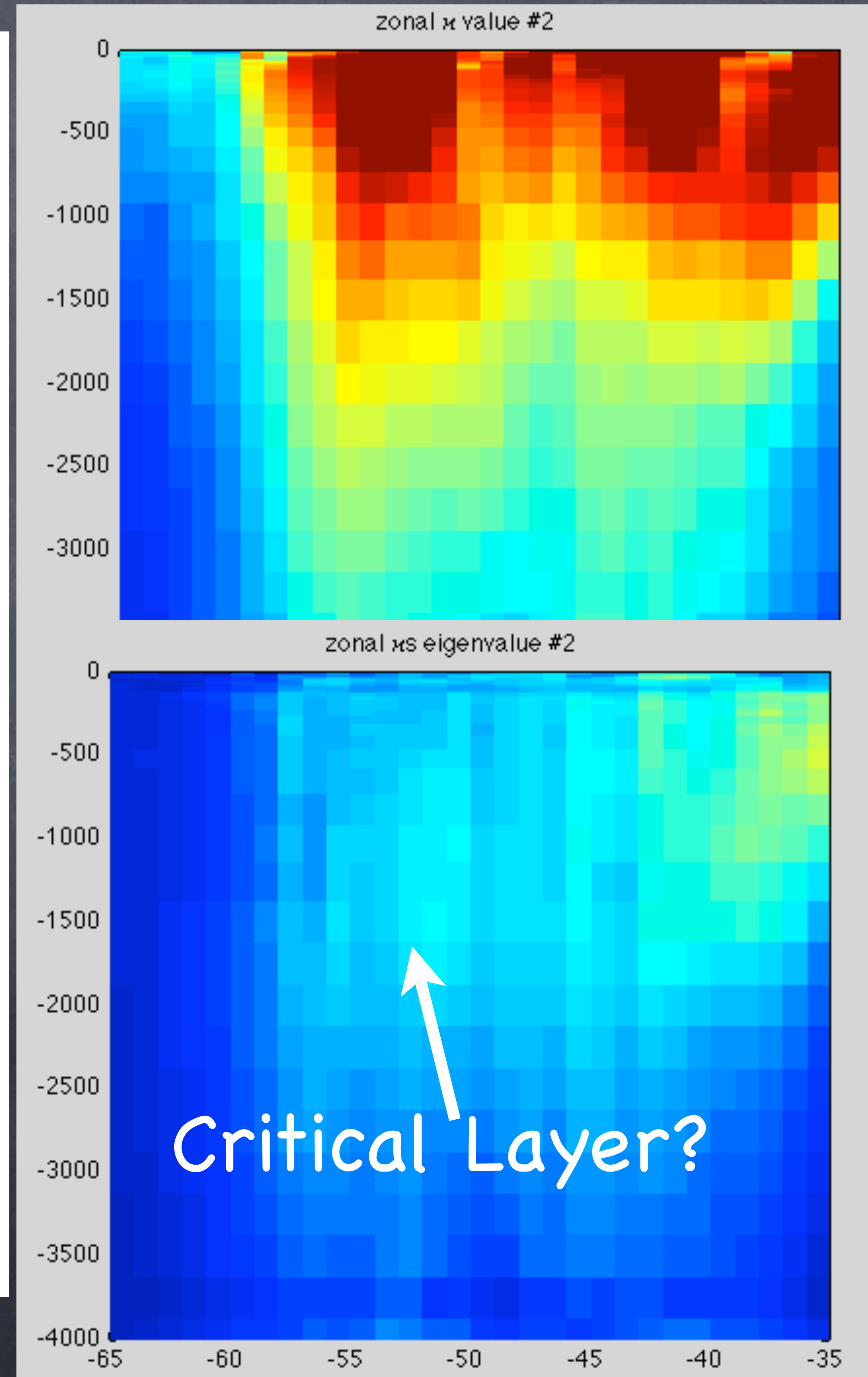

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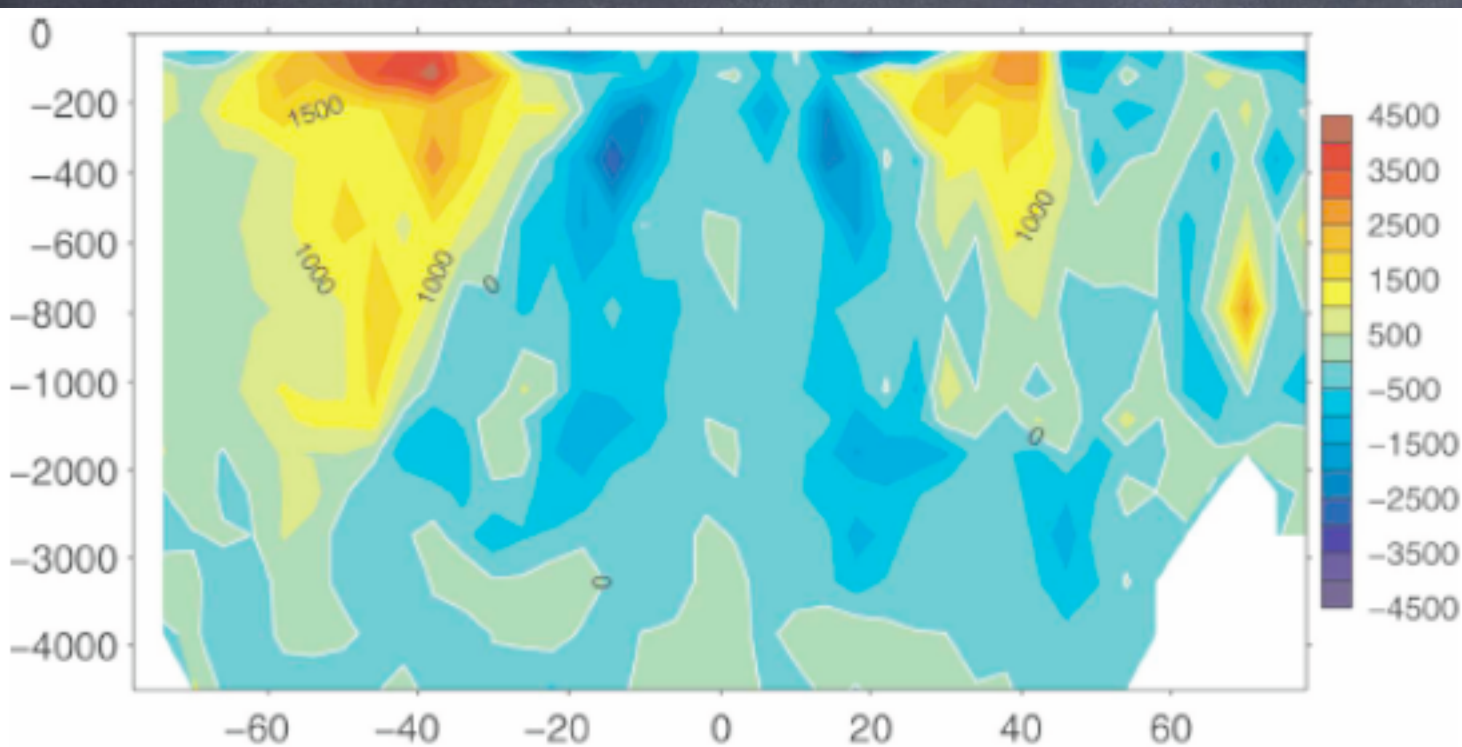
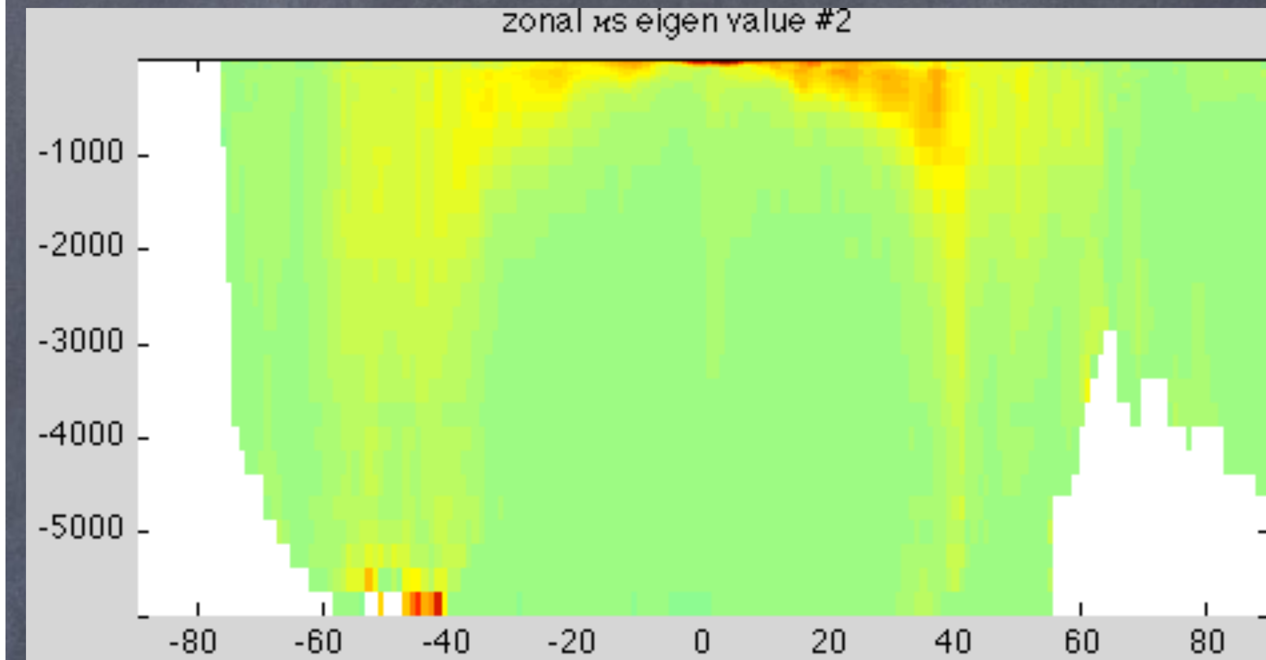
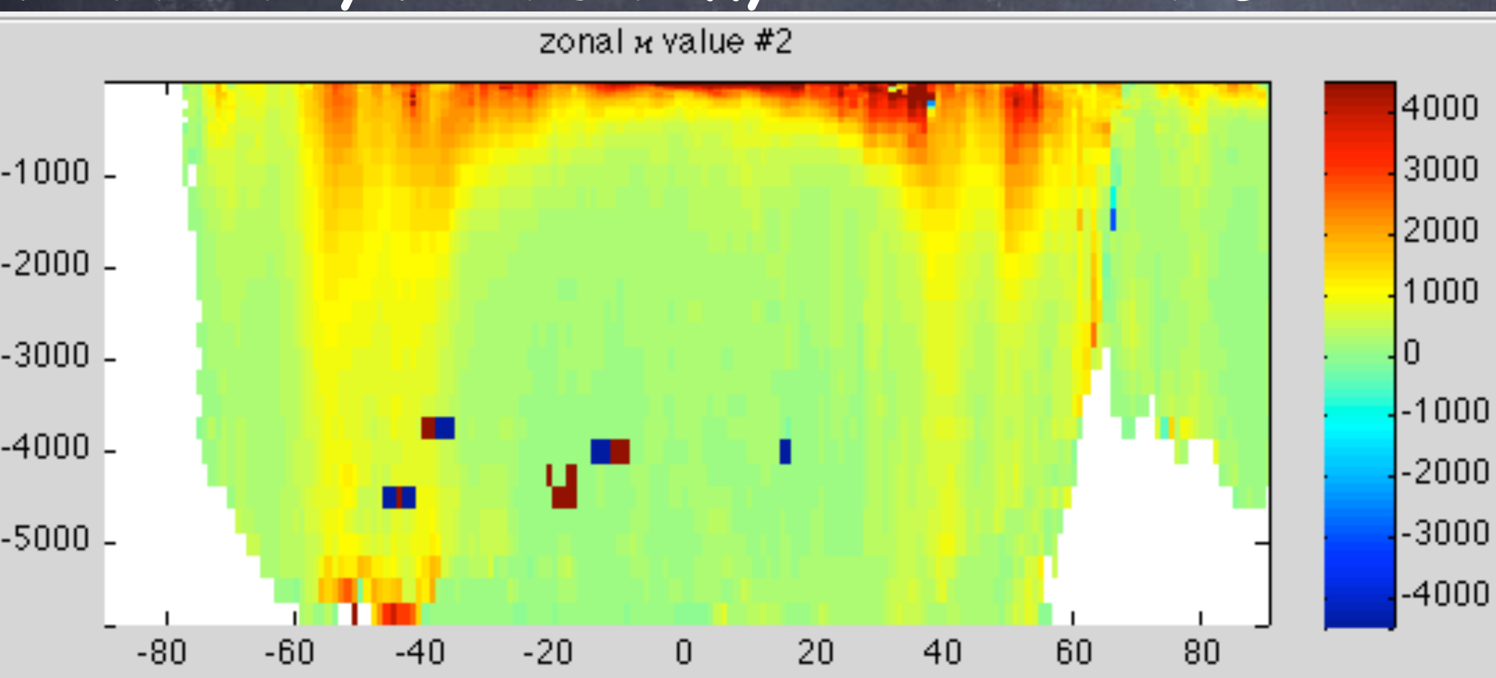
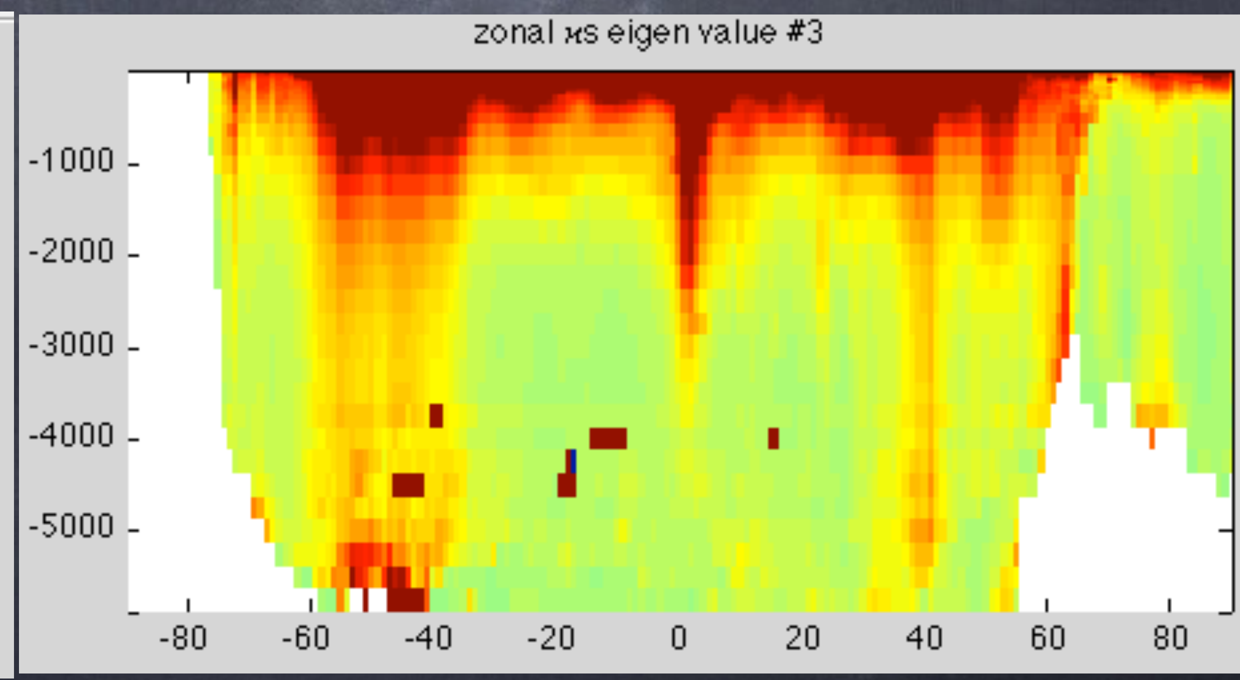




Ferreira, Marshall, Heimbach 05





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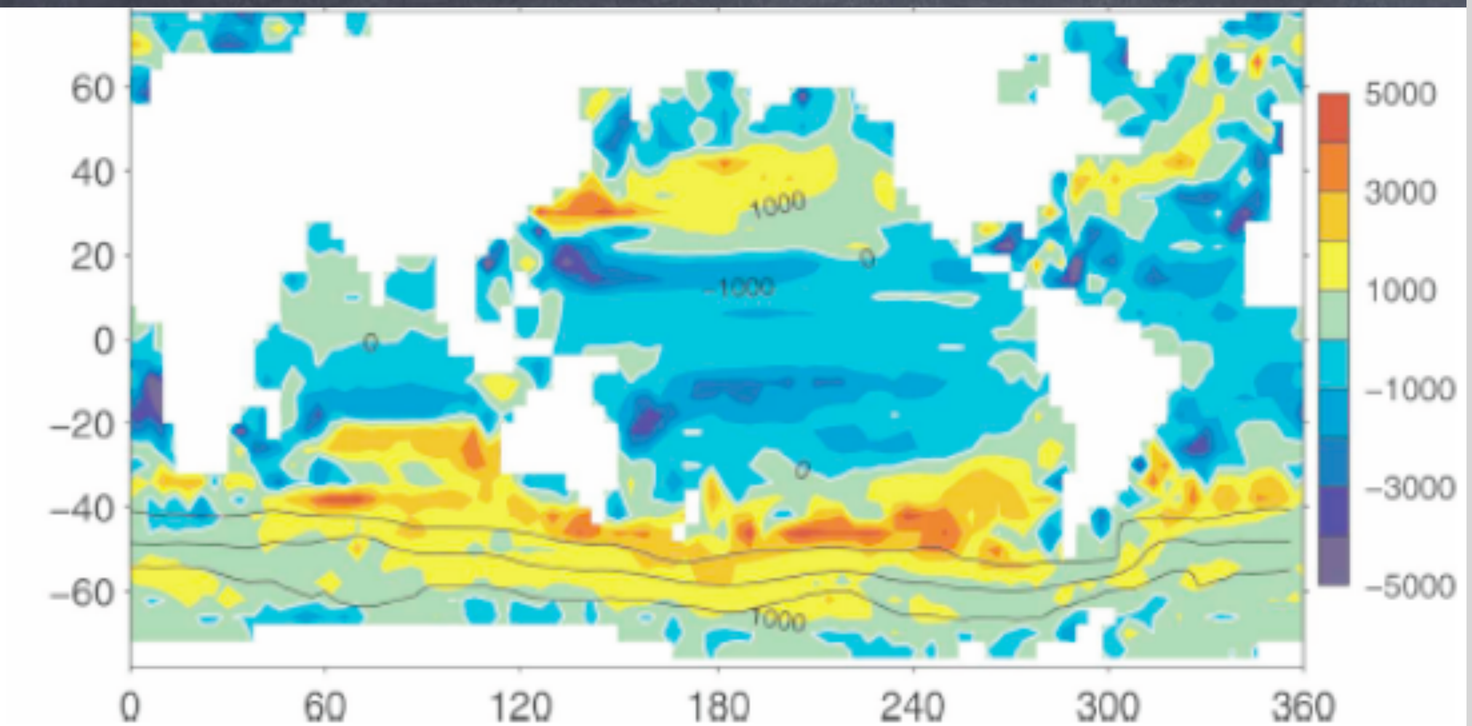
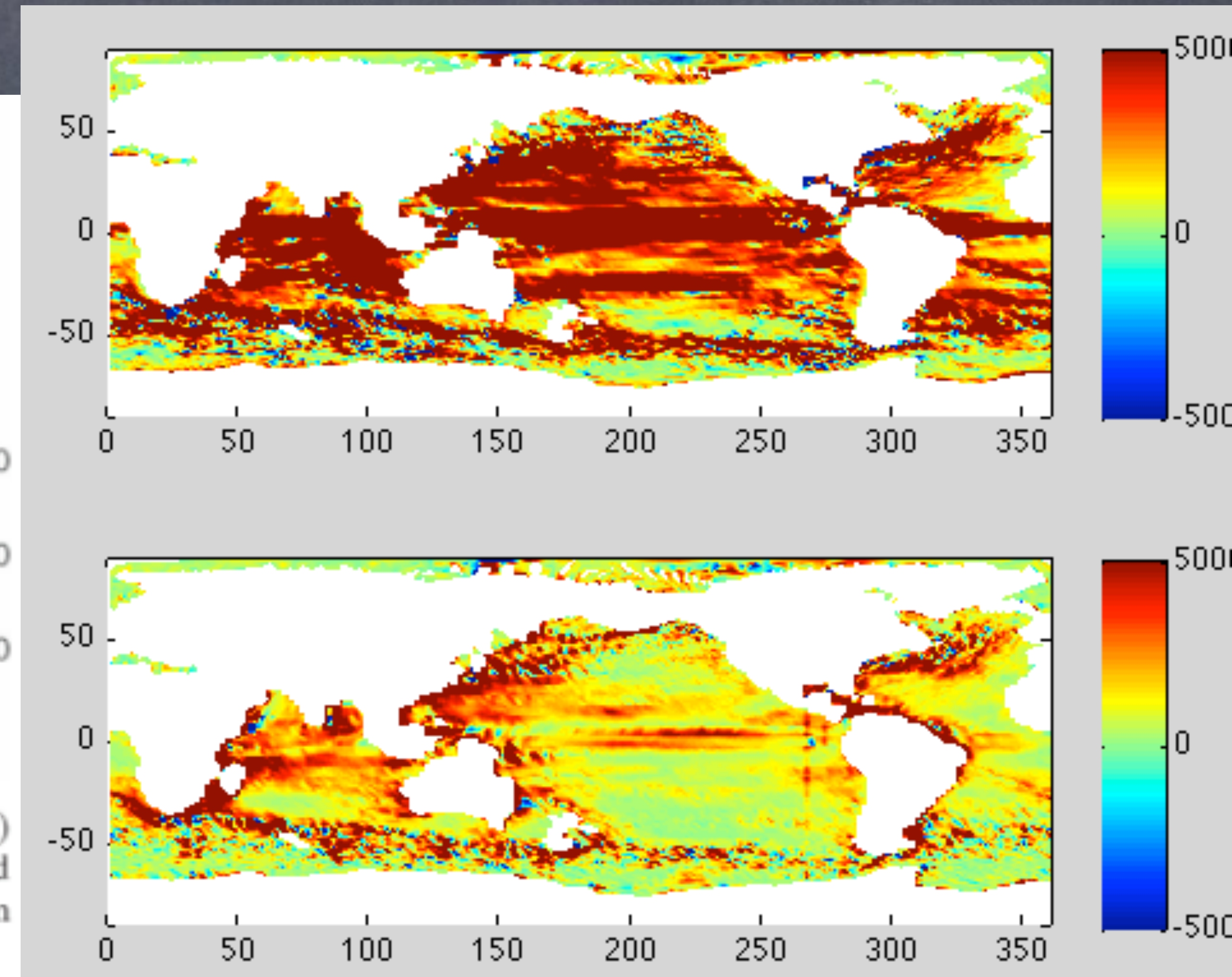
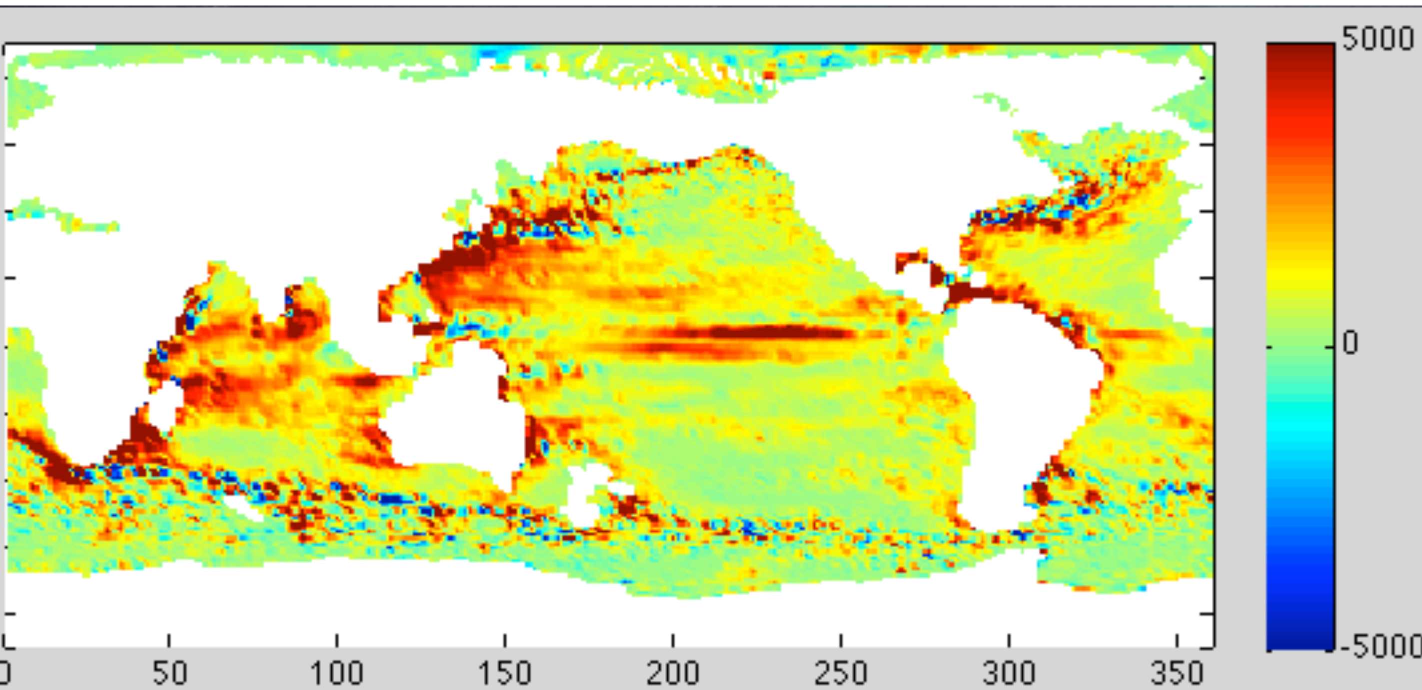
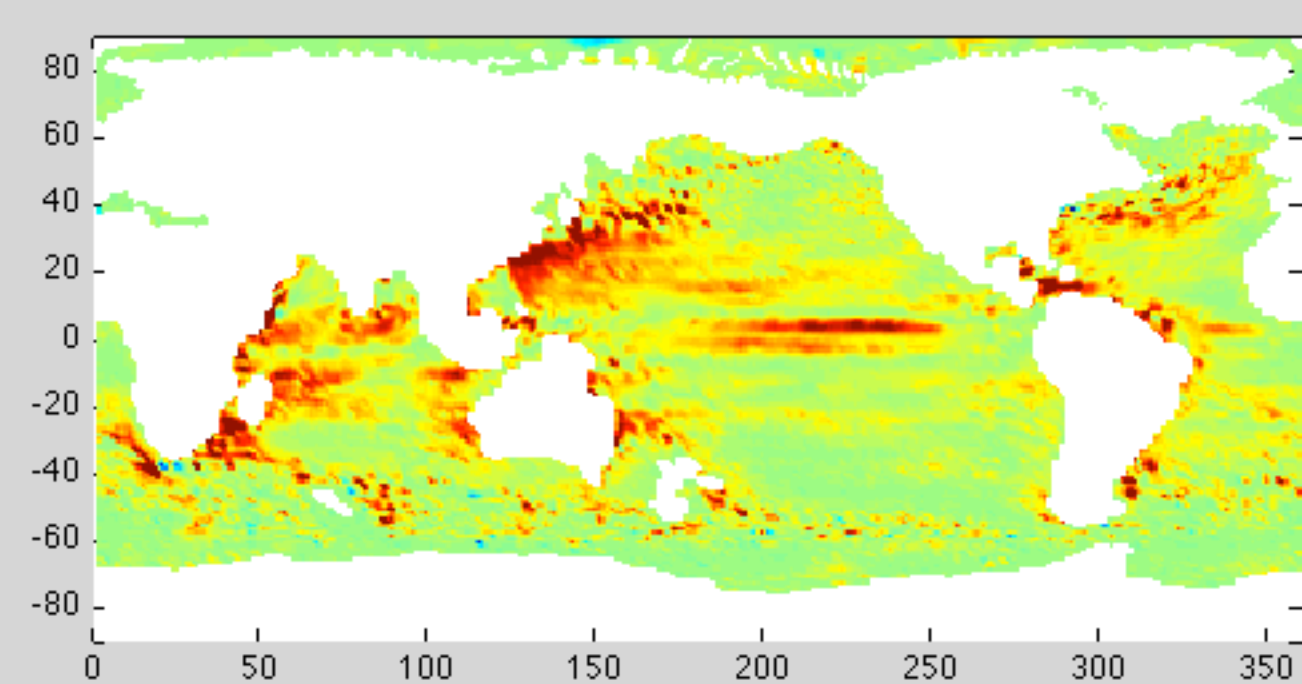


FIG. 12. Inferred horizontal eddy diffusivity κ ($m^2 s^{-1}$): (top) zonal mean and (bottom) vertical mean over the thermocline (0–1200 m). The contour intervals are (top) 500 and (bottom) 1000 $m^2 s^{-1}$. The thick line indicates the zero contour. Also indicated in the bottom panel are the 10-, 70-, and 130-Sv contours of the barotropic streamfunction.







Re(2nd eigenvalue)

(2nd eigenvalue of symmetric M)

Topics for discussion: I

- Diagnosis: Spatial Variation of $\mathbf{M}$ (Ross, Shafer, Baylor)
  - Indeterminacy? (Baylor)
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Not: $\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$ is really:

$$\nabla \cdot \overline{\mathbf{u}'\tau'} = -\nabla \cdot \mathbf{M} \cdot \nabla\overline{\tau}$$

- Eden & Greatbatch, Ferrari & Plumb, and others exploit this indeterminacy heavily.
- In practice, it is diagnostically challenging to compare eddying models to eddy parameterizations
- Consider the following for diagnosed $\mathbf{M}$ and parameterized $\mathbf{M}'$

$$0 = -\nabla \cdot (\mathbf{M} - \mathbf{M}') \cdot \nabla\overline{\tau}$$

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$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

Fox-Kemper, Ferrari, & Hallberg (2008) form (a mixed layer (submeso) eddy param.):

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\mathbf{\Psi_y} \\ 0 & 0 & \mathbf{\Psi_x} \\ \mathbf{\Psi_y} & -\mathbf{\Psi_x} & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Antisymmetric Elements in Fox-Kemper, Ferrari, & Hallberg (2008) are scaled to overturn fronts, make vertical fluxes extract PE, and restratify the fluid, At a rate validated against eddying simulations!

A Global Parameterization of Mixed Layer Eddy Restratification with FLOW DEPENDENT $\mathbf{M}$:

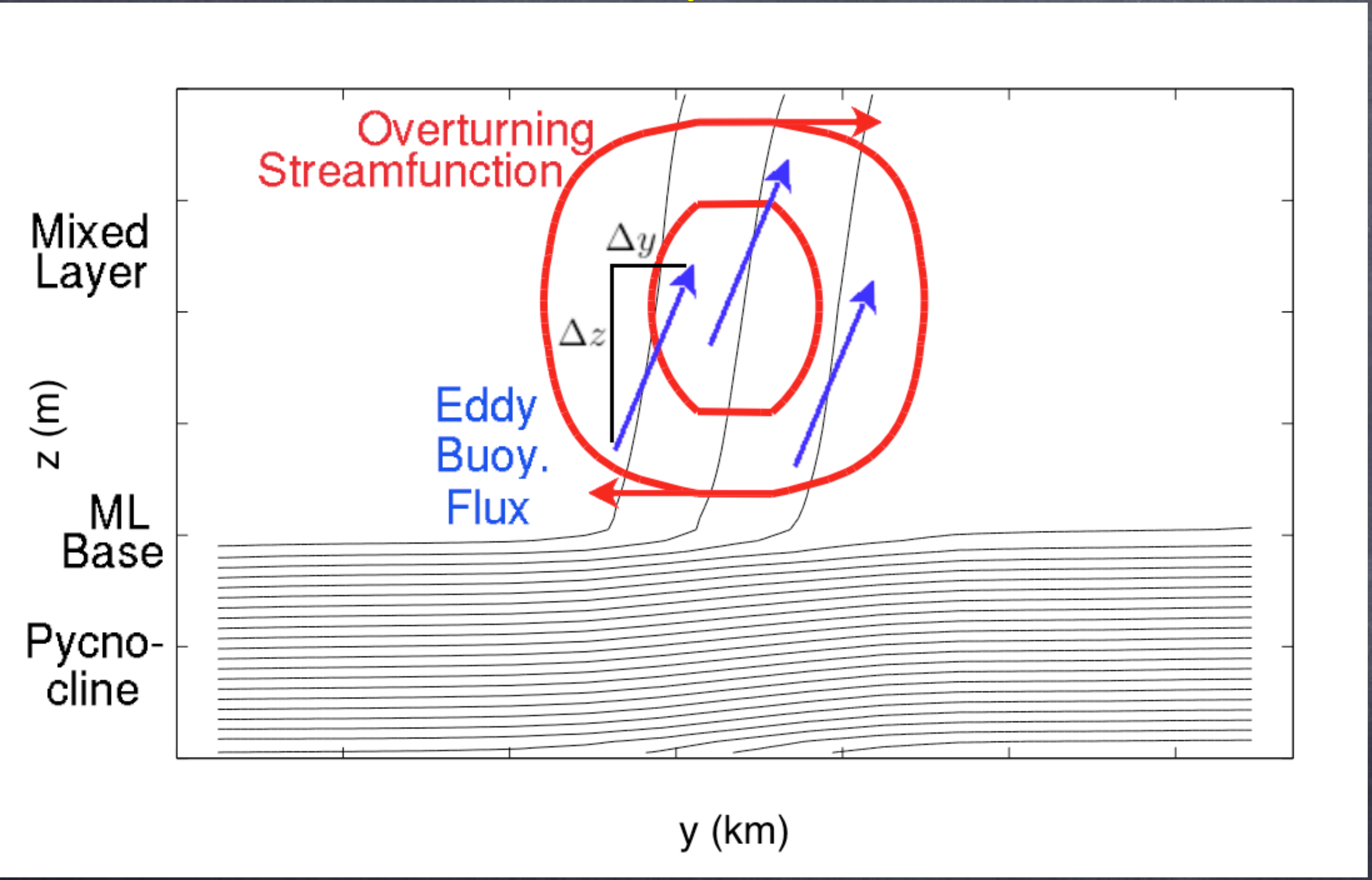
Fox-Kemper, Danabasoglu, Ferrari, & Hallberg (2008)

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\mathbf{\Psi_y} \\ 0 & 0 & \mathbf{\Psi_x} \\ \mathbf{\Psi_y} & -\mathbf{\Psi_x} & 0 \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

$$\mathbf{\Psi} = \left[\frac{\Delta x}{L_f}\right] \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla \bar{b} \times \hat{\mathbf{z}}$$

$$\mu(z) = \left[1 - \left(\frac{2z}{H} + 1\right)^2\right] \left[1 + \frac{5}{21}\left(\frac{2z}{H} + 1\right)^2\right]$$

Overturning Schematic:
An Eady-like Problem

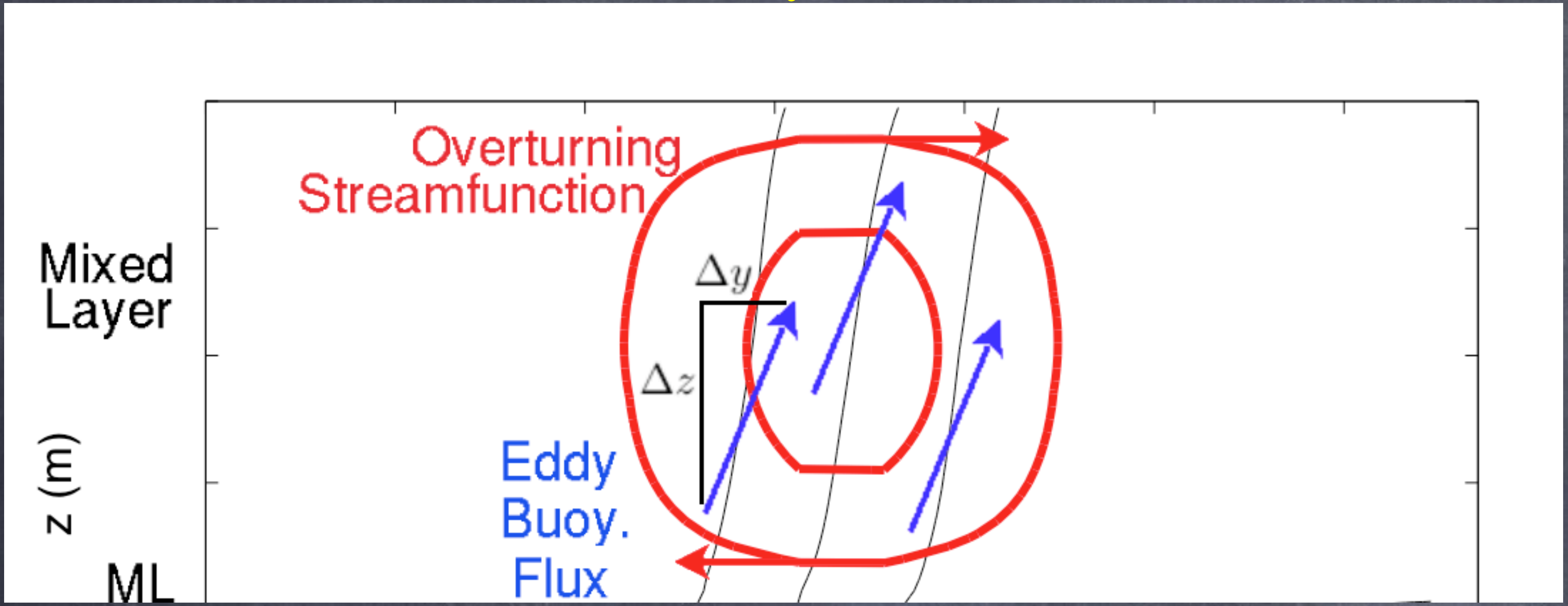


Horizontal scale of overturning = scale of front

Vertical structure of overturning = ?

Overturning Schematic:

An Eady-like Problem



$$N^2 = \frac{\partial b}{\partial z} \qquad M^2 = \frac{\partial b}{\partial y}$$

$$\mathrm{Ro} = \frac{M^2 H}{f L_f} \qquad \mathrm{Ri} = \frac{N^2 H^2}{f}$$

Fully-Developed Slumping, Big Eddy

No bkgnd strat:

$$M^2 L_f = N^2 H$$

$$\mathrm{RoRi} = 1$$

Not appropriate for mesoscale

Different Scalings, Ri>>1, Ro

FFH: finite ampl. eddies

$$\overline{v'b'} \propto -\frac{N^2H^2M^2}{|f|}$$

Stone, C&D: weak eddies

$$\overline{v'b'} \propto -\frac{N^2H^2M^2}{|f|}\frac{1}{\sqrt{\mathrm{Ri}}}$$

Green/Visbeck/L&H:

$$\overline{v'b'} \propto -\frac{N^2H^2M^2}{|f|}\frac{1}{\mathrm{Ri}^{3/2}\mathrm{Ro}^2}$$

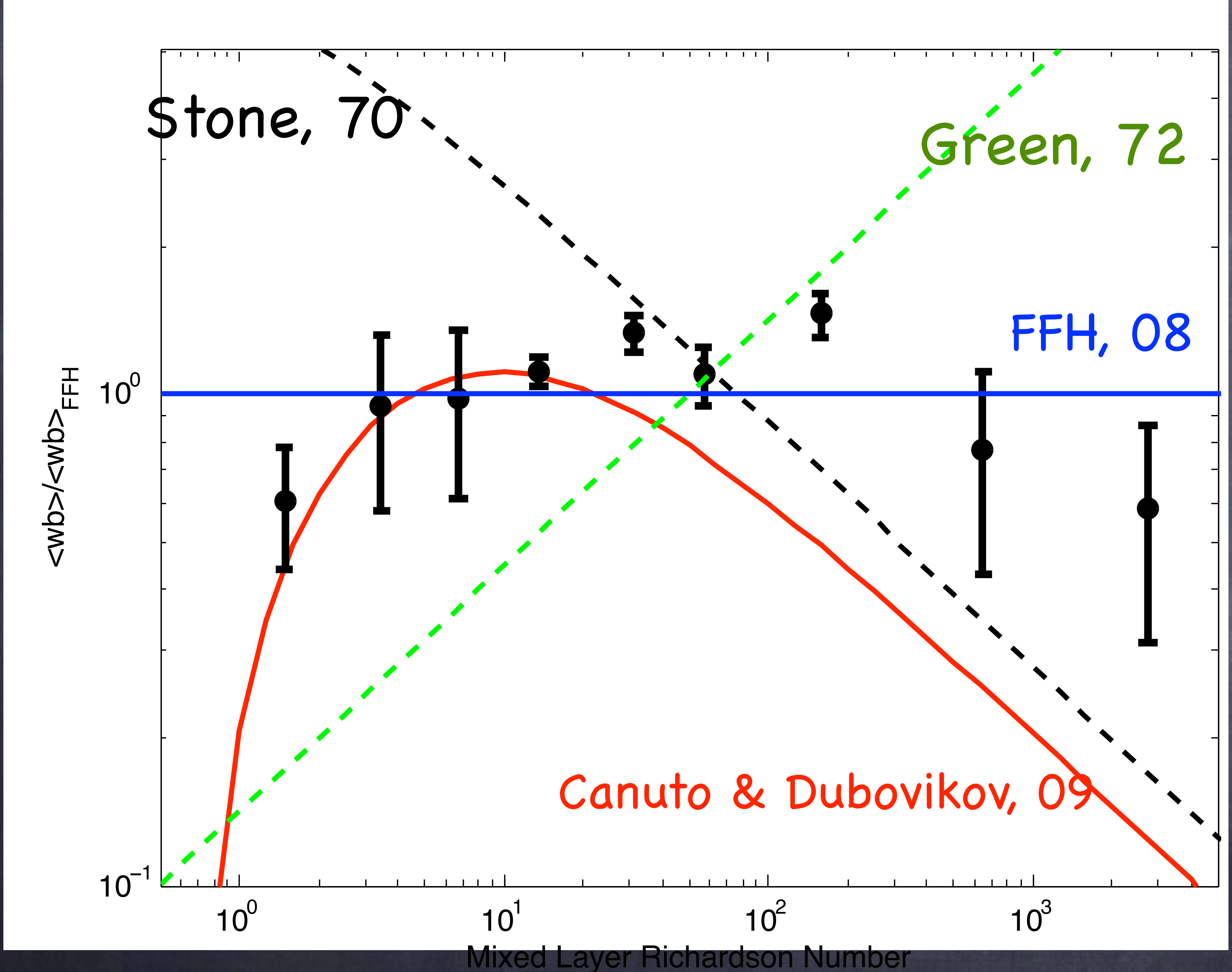
Griffies05:

$$\overline{v'b'} \propto -\frac{N^2H^2M^2}{|f|}\frac{1}{\mathrm{Ri}^{3/2}\mathrm{Ro}^2}\frac{L_0^2N}{L_f^2N_0}$$

Danabasoglu&Marshall07:

$$\overline{v'b'} \propto -\frac{N^2H^2M^2}{|f|}\frac{A_{ITDref}f}{N_{ref}^2H^2}$$

Param. Vs. Eddy-Resolving:



Green equals
Visbeck (97)
Held & Larichev (95)

Extends to Ri more mesoscale (9000) than submesoscale (1)

And Agrees with Deep Convection Studies:
Jones & Marshall (93,97), Haine & Marshall (98)

The Problem is:

The mesoscale equivalent isn't rEady

- FFH param. doesn't do interior stratification/PV gradients
- PV jumps are OK, e.g, surface & mixed layer base
- But, Mesoscale==Full Depth, so PV Varies
- Smith (07) shows interior PV gradients dominate mesoscale energy extraction

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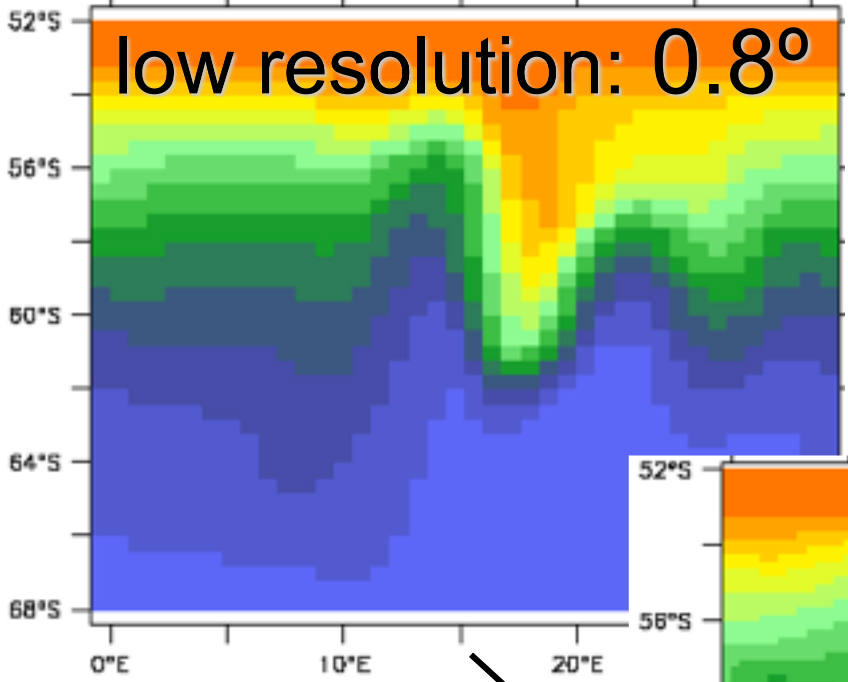
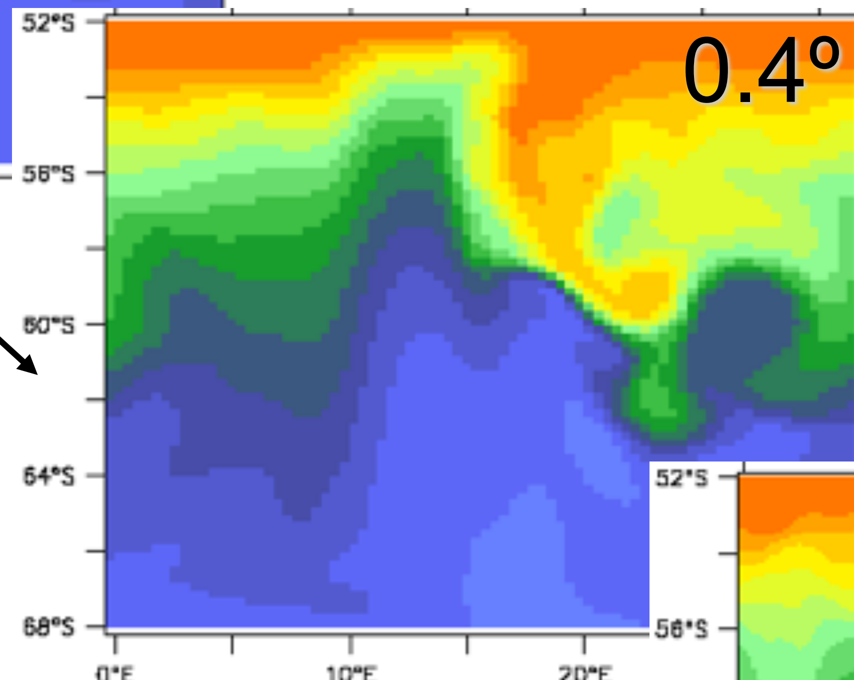
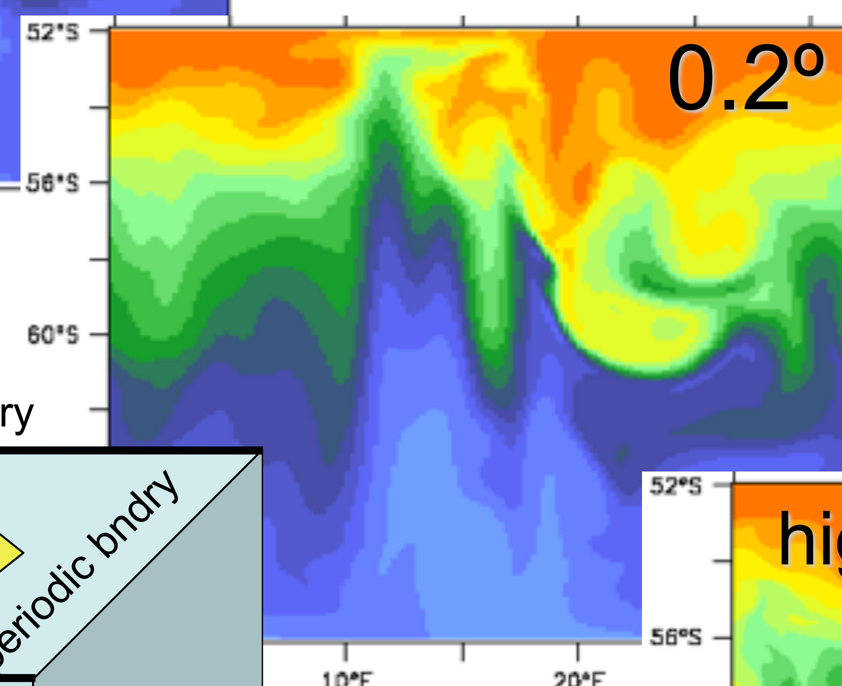
Channel model test problem for LANS-alpha turbulence parameterization in POP

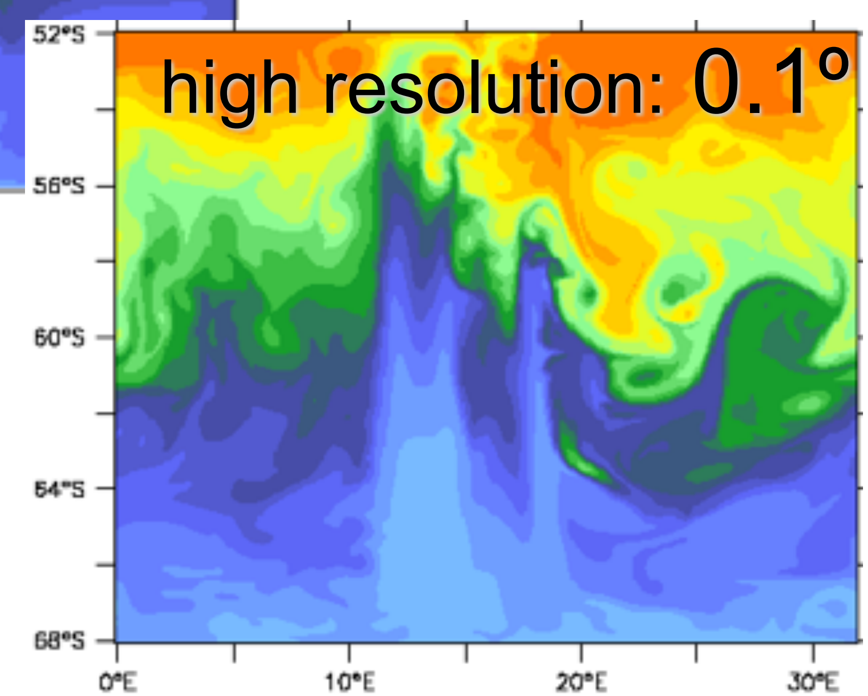
used in 2 JCP papers (2008, M. Petersen corresponding author)

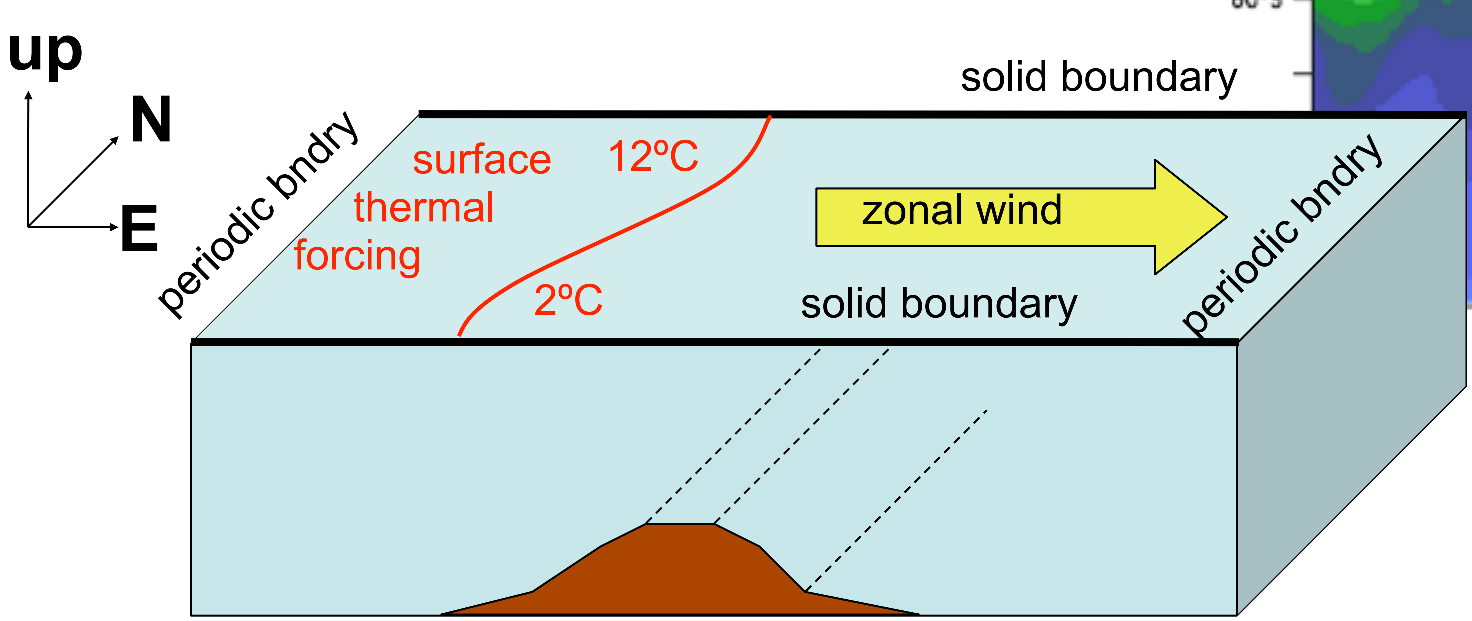


cost of doubling horizontal grid nearly factor of 10



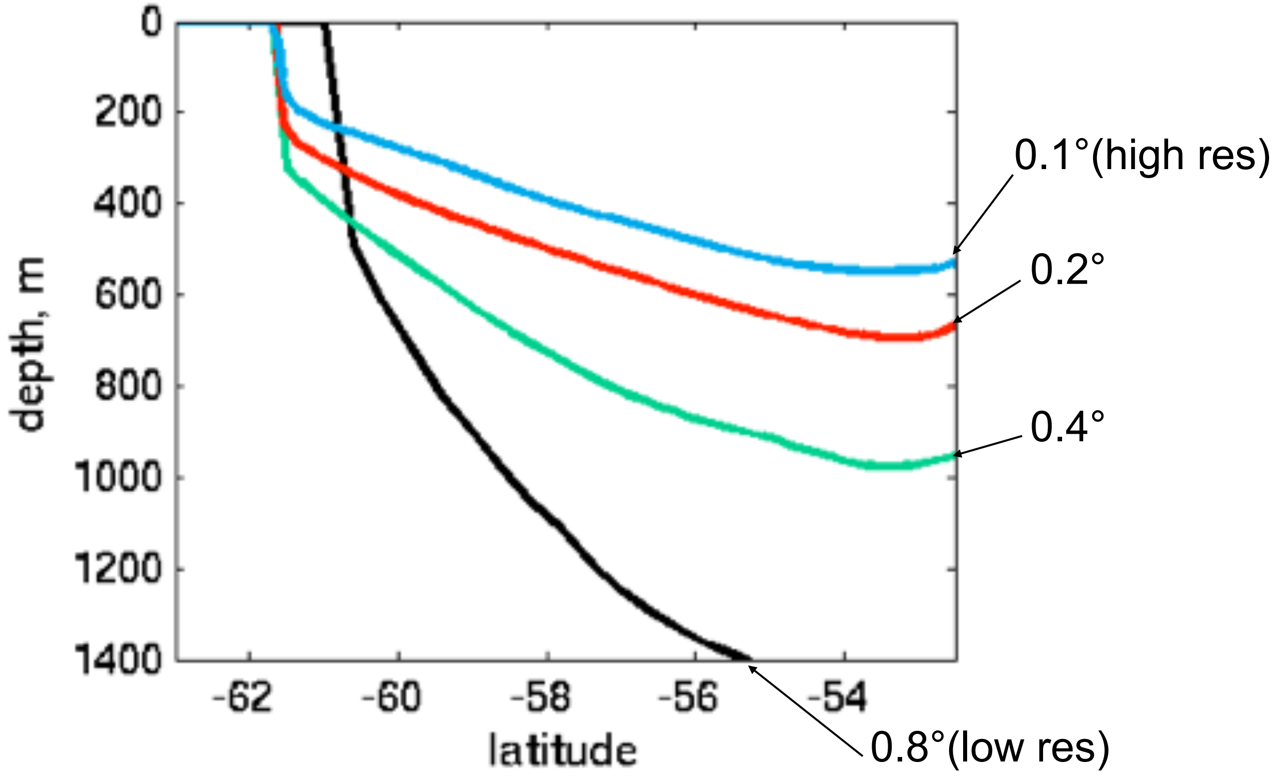




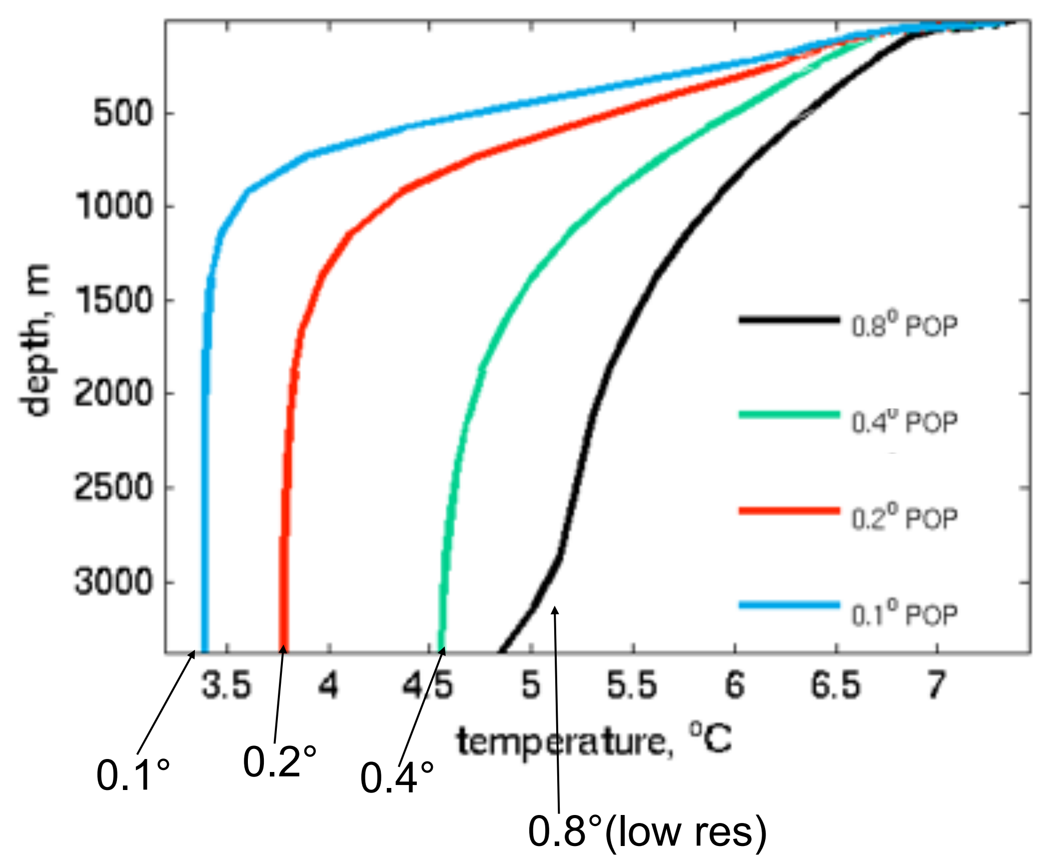


Test problem results: Dependence of vertical profiles on resolution in ordinary POP

6C isotherm

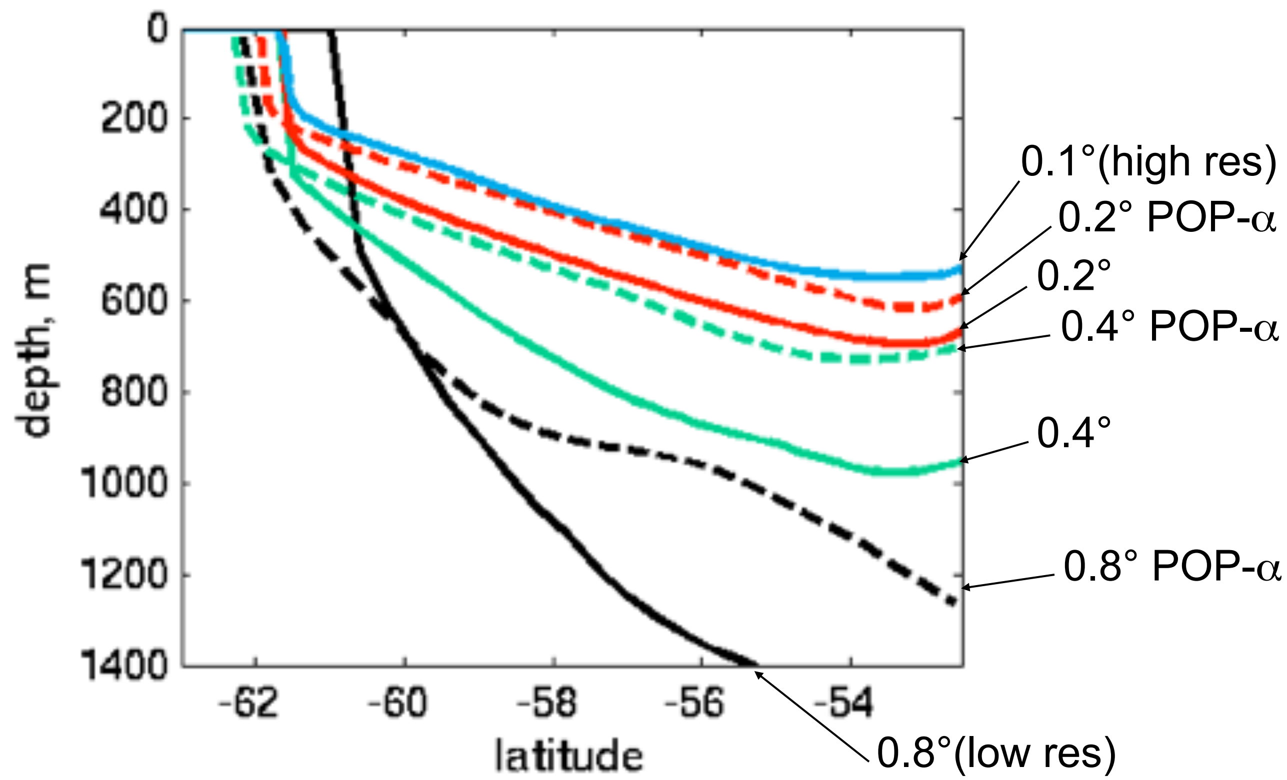


Vertical temperature profile

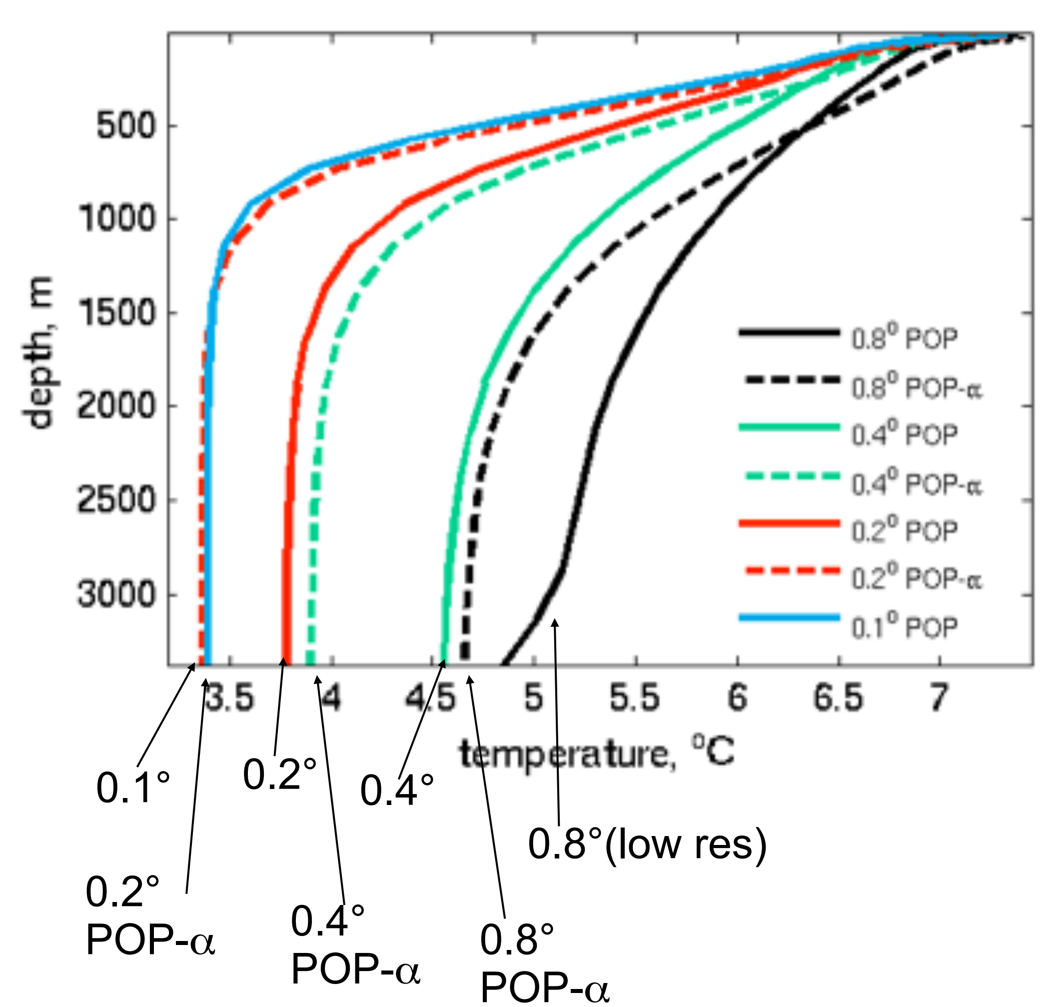


Test problem results: POP with LANS-alpha is equivalent to ordinary POP with doubled resolution, in these measures

6C isotherm

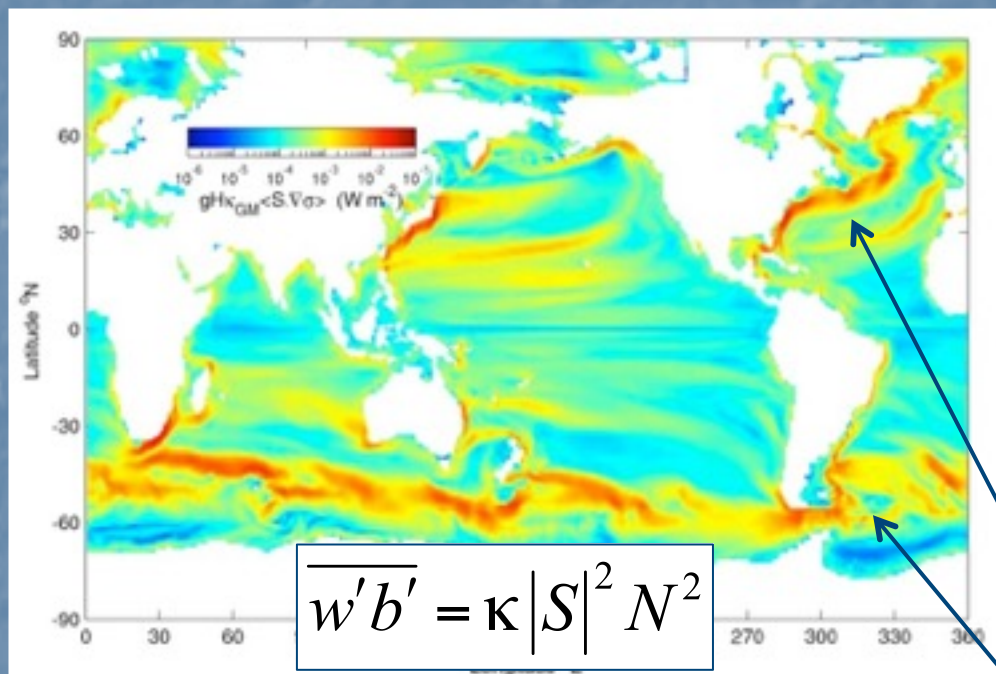


Vertical temperature profile

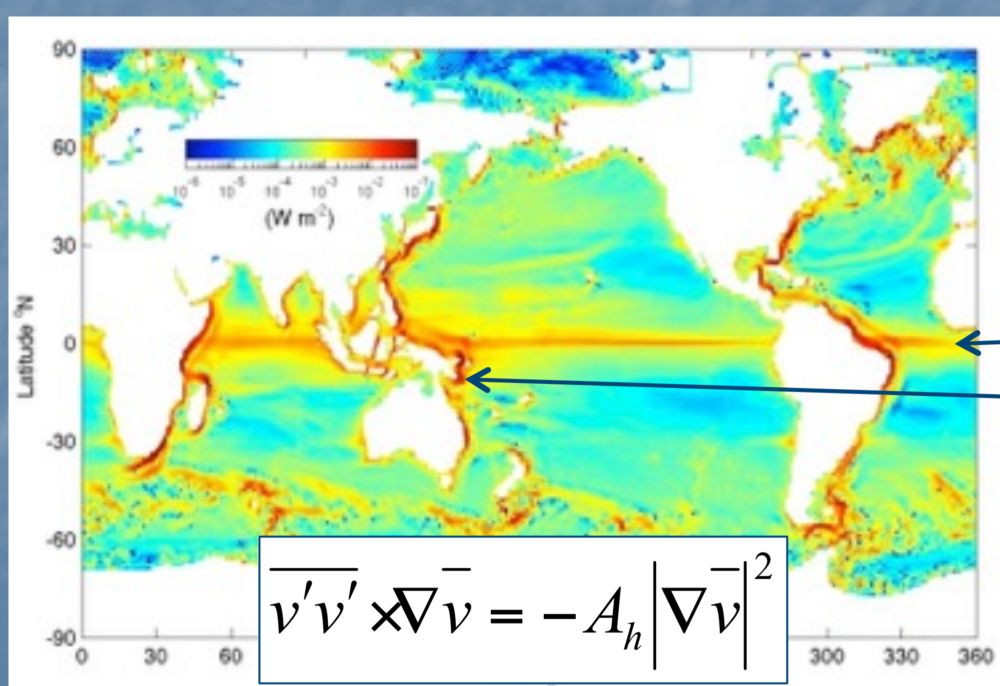


Connecting eddy activity to parameterized energy removal

- Energy removal in 1° model



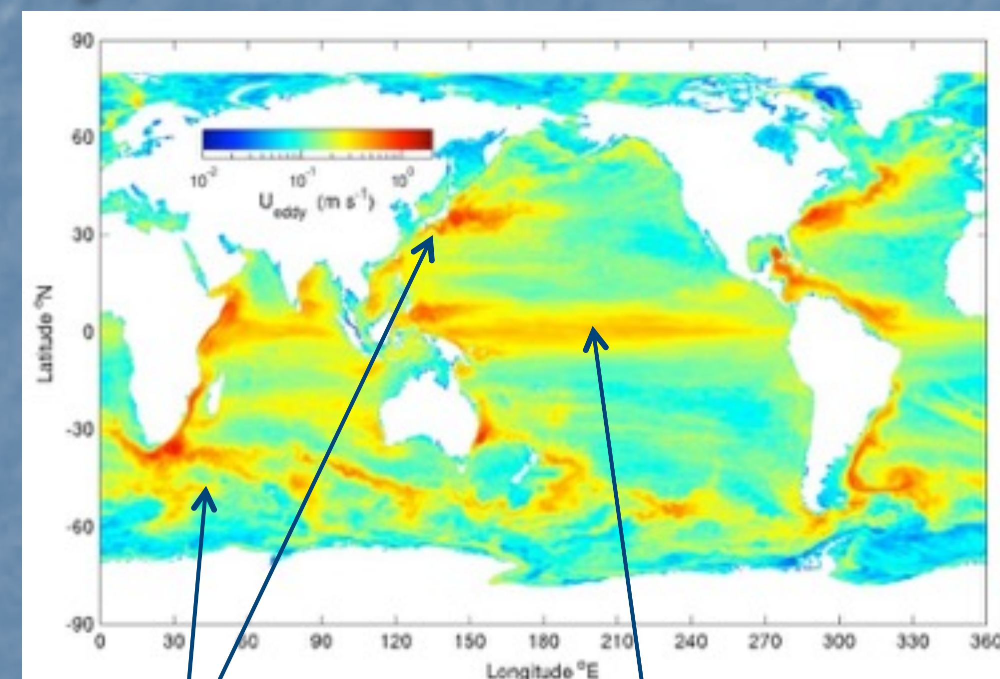
$$\overline{w'b'} = \kappa |S|^2 N^2$$



$$\overline{v'v'} \cdot \nabla \overline{v} = -A_h |\nabla \overline{v}|^2$$

  - Units: W m^{-2}

- EKE diagnosed from a 1/6° eddying model
  - Many features identifiable in 1° diagnostics



  - Barclinic zones
  - Equatorial shear zones
  - Coastal currents
  - Units: m s^{-1}

- Eden & Greatbatch, OM 2008 introduce prognostic EKE eq$^{\underline{n}}$ to calculate eddy diffusivity, K_h

Reynolds averaging II

- Mean momentum equation

$$\partial_t \bar{v} + \left(f\hat{k} + \bar{\zeta}\right) \wedge \bar{v} + \nabla \tfrac{1}{2}\left|\bar{v}\right|^2 + \tfrac{1}{\rho_o}\nabla \bar{p} - \bar{b}\hat{k} = \nabla \cdot \bar{\tau} - \overline{\zeta' \wedge v'}$$

- Perturbation momentum equation

$$\partial_t \left(\bar{v} + v'\right) + \left(f\hat{k} + \bar{\zeta} + \zeta'\right) \wedge \left(\bar{v} + v'\right) + \nabla\left(\bar{B} + B'\right) - \left(\bar{b} + b'\right)\hat{k} = \nabla \cdot \left(\bar{\tau} + \tau'\right)$$

$$\partial_t v' + \left(f\hat{k} + \bar{\zeta}\right) \wedge v' + \zeta' \wedge \bar{v} + \zeta' \wedge v' + \nabla B' - b'\hat{k} = \nabla \cdot \tau' + \overline{\zeta' \wedge v'}$$

- Eddy Kinetic Energy (EKE) equation

$$\partial_t \overline{\tfrac{1}{2}\left|v'\right|^2} - \bar{v} \cdot \overline{\left(\zeta' \wedge v'\right)} - \overline{w'b'} + \nabla \cdot \left(\overline{B'v'}\right) = \overline{v' \cdot \nabla \cdot \tau'}$$

- $\partial_t \overline{\tfrac{1}{2}|v'|^2}$: Rate of change of EKE
- $-\bar{v} \cdot \overline{(\zeta' \wedge v')}$: Conversion from Mean KE
- $-\overline{w'b'}$: Conversion from Mean PE
- $\nabla \cdot (\overline{B'v'})$: Advection of EKE + triple correlation terms
- $\overline{v' \cdot \nabla \cdot \tau'}$: Direct input to EKE from (wind) stress + Dissipation

Energetics of Reynolds Averaged Eqns

- Model (mean) equations $\nabla\cdot\bar{v}=0$

$$\partial_t\bar{v}+\left(f\hat{k}+\bar{\zeta}\right)\wedge\bar{v}+\nabla\left(\tfrac{1}{\rho_o}\bar{p}+\tfrac{1}{2}\left|\bar{v}\right|^2+\tfrac{1}{2}\overline{\left|v'\right|^2}\right)-\bar{b}\hat{k}=\nabla\cdot\bar{\tau}-\overline{\zeta'\wedge v'}$$

$$\partial_t\bar{b}+\nabla\cdot\left(\bar{b}\bar{v}\right)=\nabla\cdot\bar{Q}-\nabla\cdot\left(\overline{b'v'}\right)$$

- Resolved KE eq$^{\underline{n}}$

$$\partial_t\tfrac{1}{2}\bar{v}\cdot\bar{v}-\bar{w}\bar{b}+\nabla\cdot\left(\bar{v}B\right)=\bar{v}\cdot\nabla\cdot\bar{\tau}-\bar{v}\cdot\overline{\zeta'\wedge v'}$$

- Resolved PE eq$^{\underline{n}}$

$$\partial_t\left(-z\bar{b}\right)+\nabla\cdot\left(-z\bar{b}\bar{v}\right)+\bar{w}\bar{b}=z\nabla\cdot\bar{Q}-\nabla\cdot\left(-z\overline{b'v'}\right)-\overline{w'b'}$$

- Eddy Kinetic Energy (EKE) eq$^{\underline{n}}$

$$\partial_t\varepsilon+\nabla\cdot\left(\overline{B'v'}\right)=\overline{v'\cdot\nabla\cdot\tau'}+\bar{v}\cdot\left(\overline{\zeta'\wedge v'}\right)+\overline{w'b'}$$

$$\varepsilon\equiv\overline{\tfrac{1}{2}\left|v'\right|^2}$$

Conversion KE↔PE

Conversion KE↔EKE

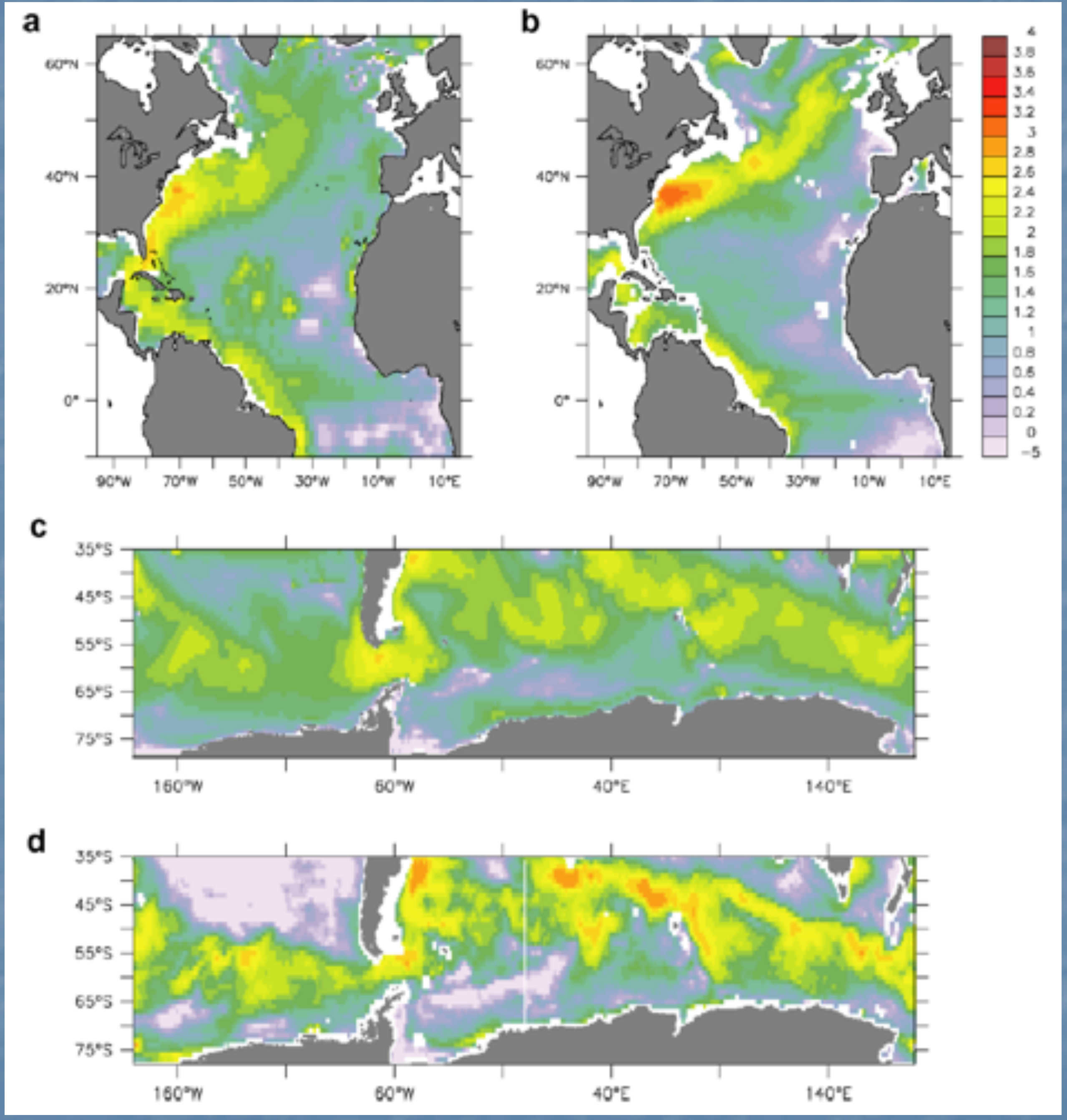
Conversion PE↔EKE

Eden & Greatbatch results

- EKE eq^{n} correctly recovers map of EKE
- Resulting K_h not very different from closed form approaches, e.g. Visbeck et al.
- EG08 approach is 3D
- Get as good results with 2D equation

  See Bob's results

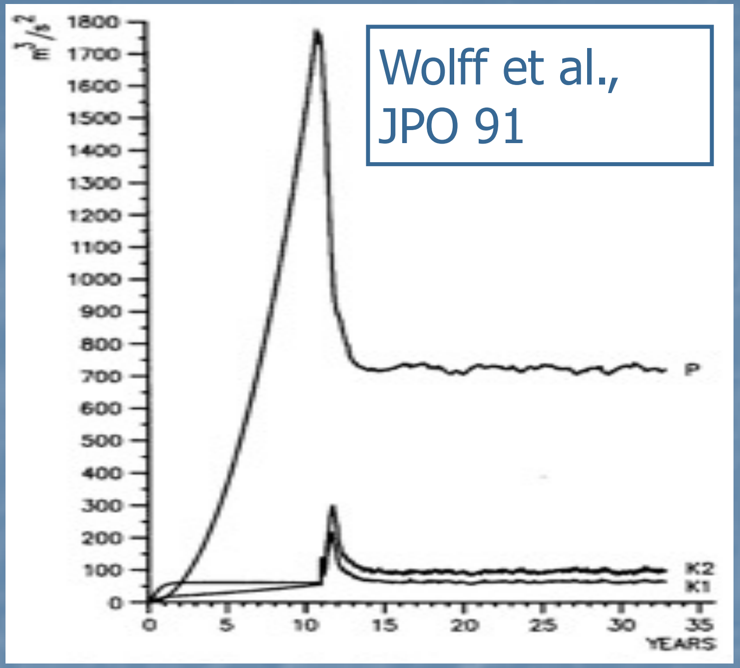
  - Avoids problems of vertical structure
  - More likely to be relevant to mesoscale eddies!



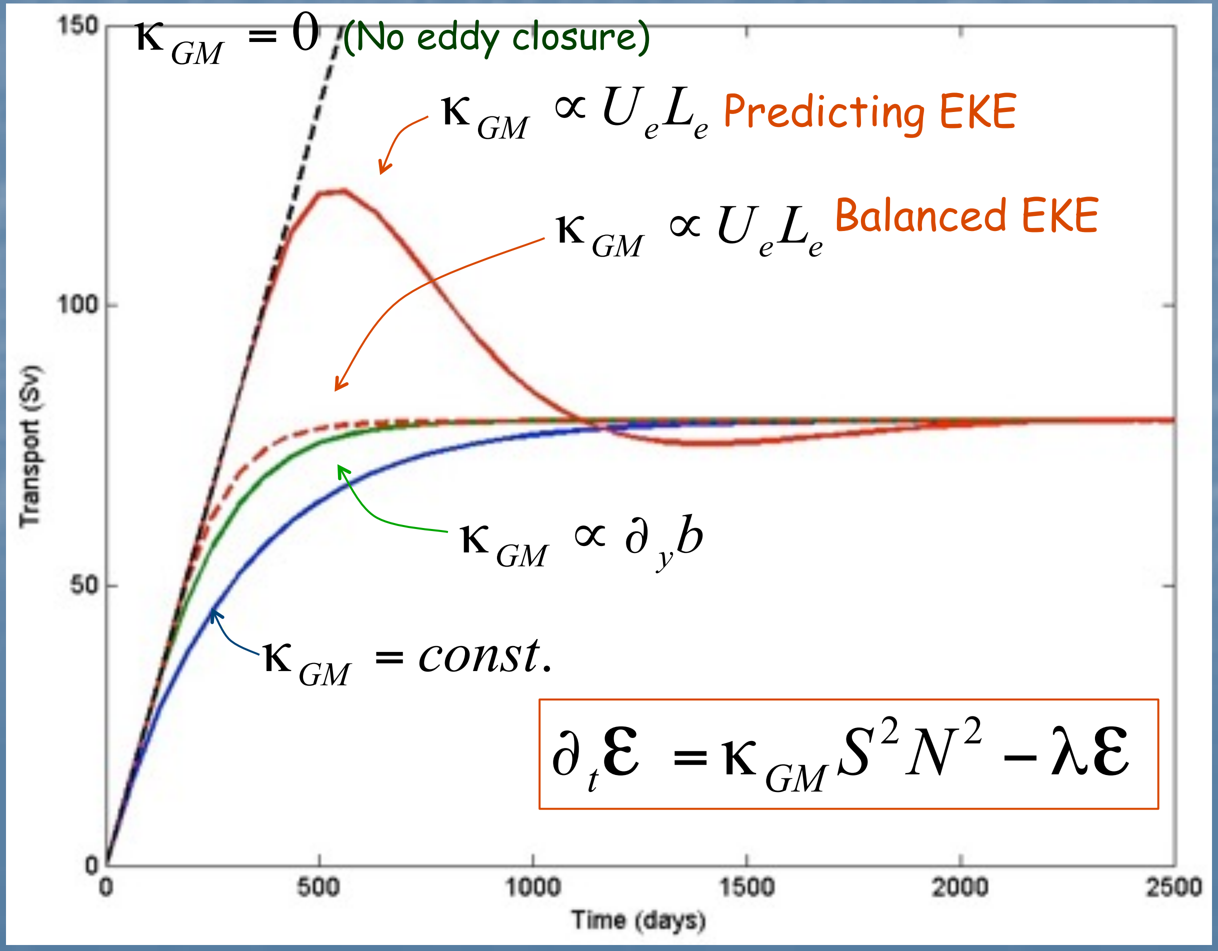
From Eden & Greatbatch, OM 2008
Log(EKE) at 300m for coarse resolution and eddy resolving models

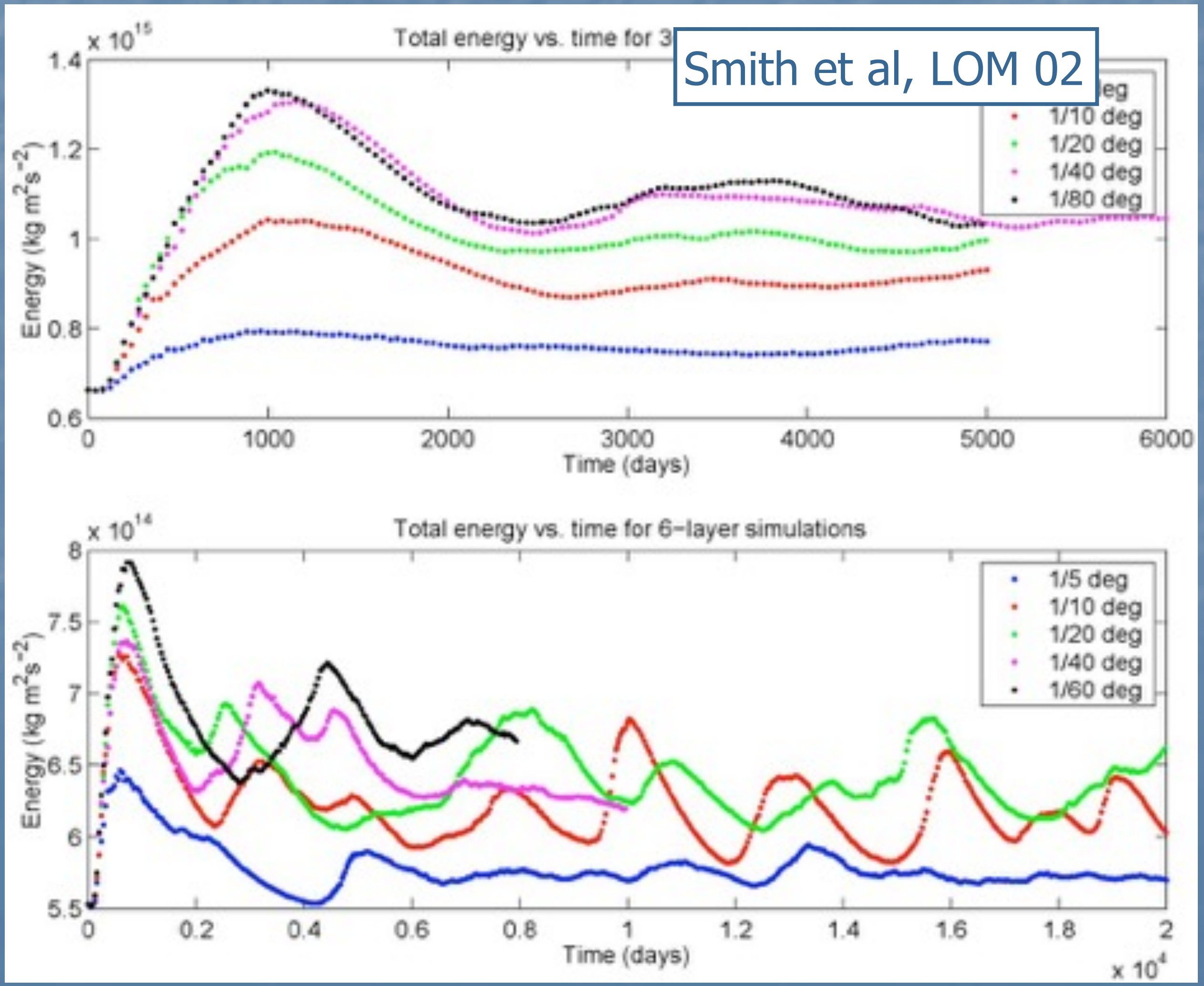
Adding extra degree of freedom

- Idealized models exhibit "delayed action" during spin up
- Possible coupled modes?
  - Delayed ocean EKE response to wind changes



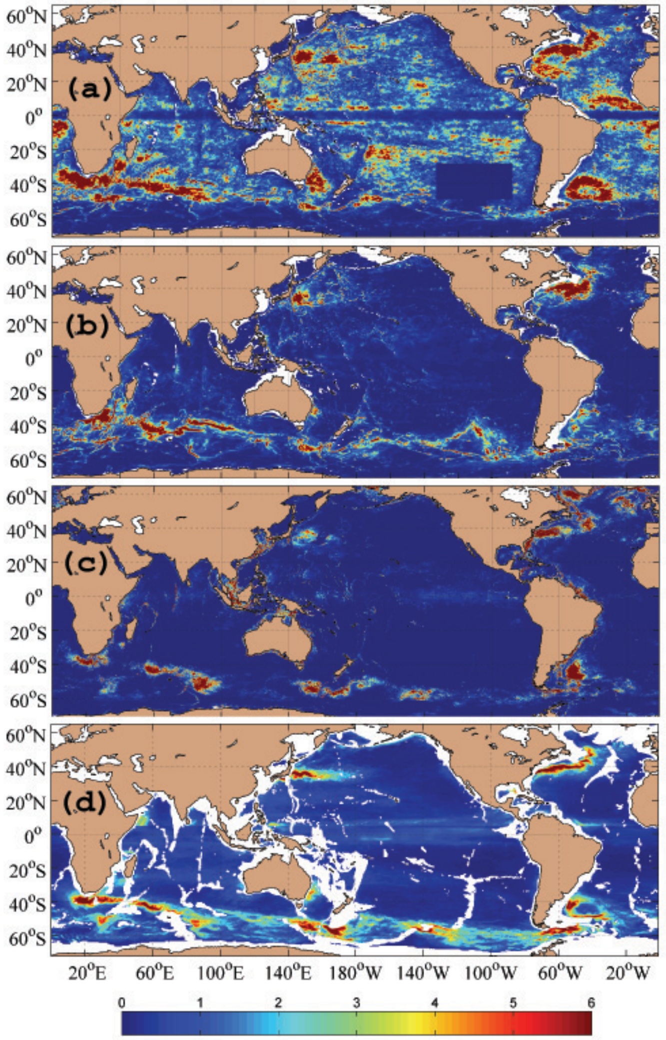
Idealized zonally averaged channel model





Maps (mW m^{-2}) and global integrals (TW) of time-averaged dissipation of eddying general circulation

(a) Data-assimilative NLOM,
(b) Non-assimilative NLOM,
(c) POP, (d) Sen et al. (2008) inference from altimetry + current meters



- $[\overline{D}] = \int \rho c_d \overline{|\mathbf{u}_b|^3}\, dA$
- Native c_d=0.003/0.002/0.001225 for DANLOM/NANLOM/POP
- Common c_d=0.0025 (used in maps to left)

Model	c_d	$[\overline{D}]$
DANLOM	Native	0.65
NANLOM	Native	0.16
POP	Native	0.14
DANLOM	Common	0.54
NANLOM	Common	0.20
POP	Common	0.29

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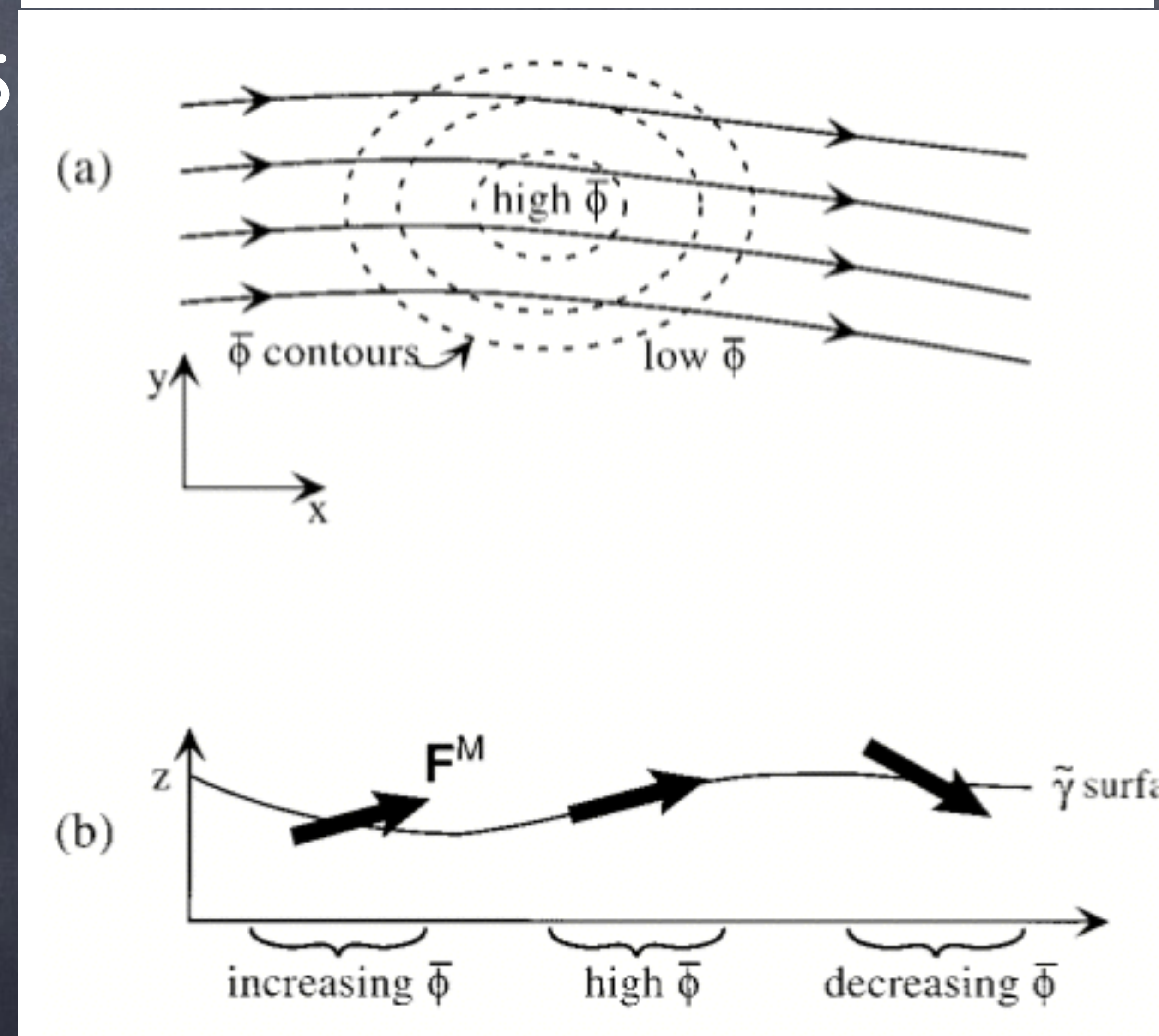
Dia-(coarse neutral) fluxes

- Featured in:
  - Eden & Greatbatch (08)
  - Canuto & Dubovikov (05
  - FFH (time-dependent)
- Different from:
  - Griffies et al (00)
  - Veronis Effect

Bachman (student)

$$\frac{\partial \overline{\tau'^2}}{\partial t} = 2\left(\overline{\tau' \cdot (\nabla \cdot \kappa \nabla)\tau'} - \overline{\tau' u'} \cdot \nabla \overline{\tau} - \bar{u} \cdot \nabla \frac{1}{2}\overline{\tau'^2}\right)$$



McDougall & McIntosh 01

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Topics for discussion: II

- Scaling for MOLES (Bob)
- What can linear theory tell us? (Shafer)
- What can other theory tell us?
  - GLM? TEM? Stat Mech?
- Anisotropy, tracer type dependence, biogeochem tracers, etc.? (Baylor, others?)

Scaling Eddy Parameterizations with Locally Eddy Resolving Models

Robert Hallberg and Alistair Adcroft
NOAA/GFDL & Princeton University

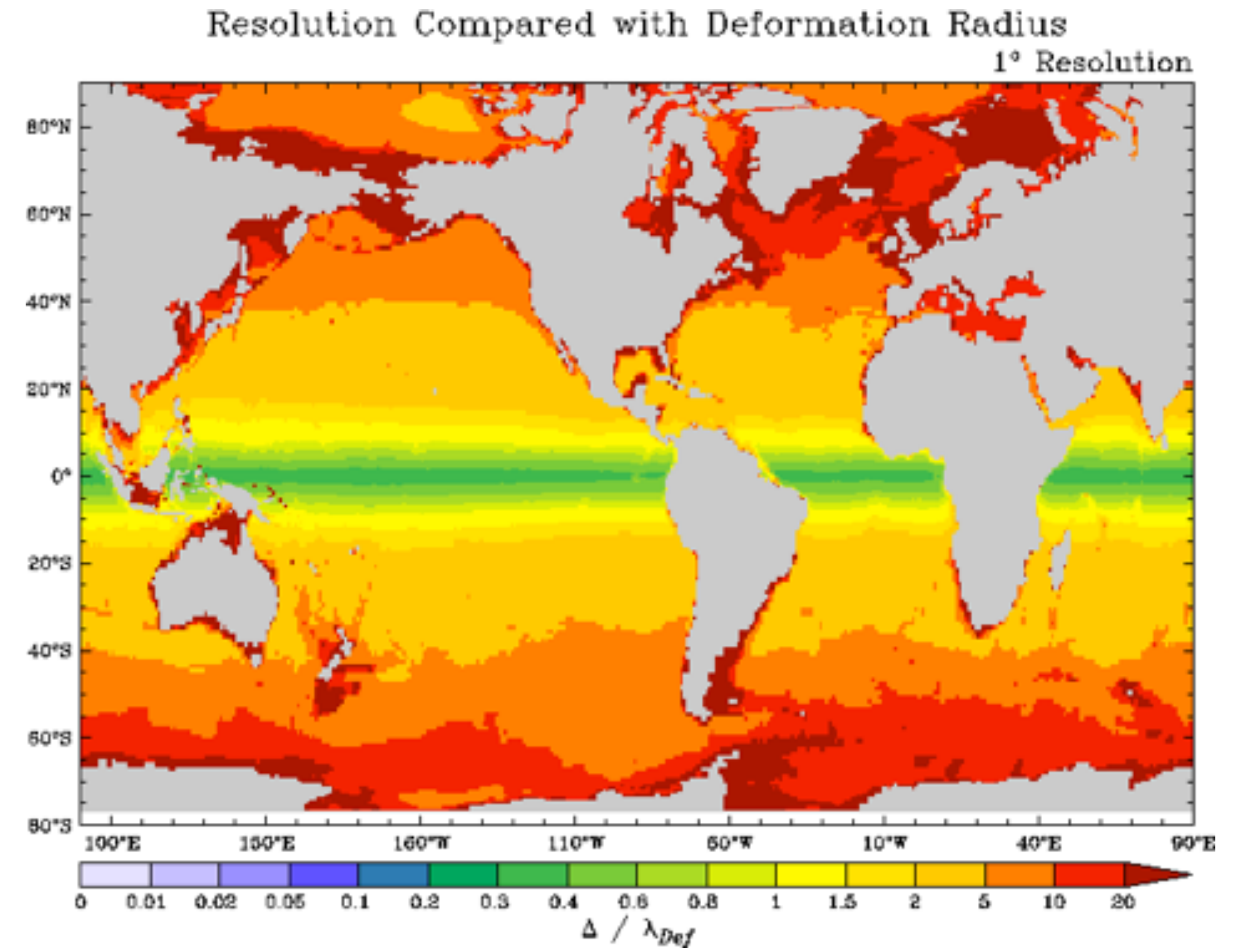
Typical (1°) ocean climate models resolve the equatorial deformation radius.

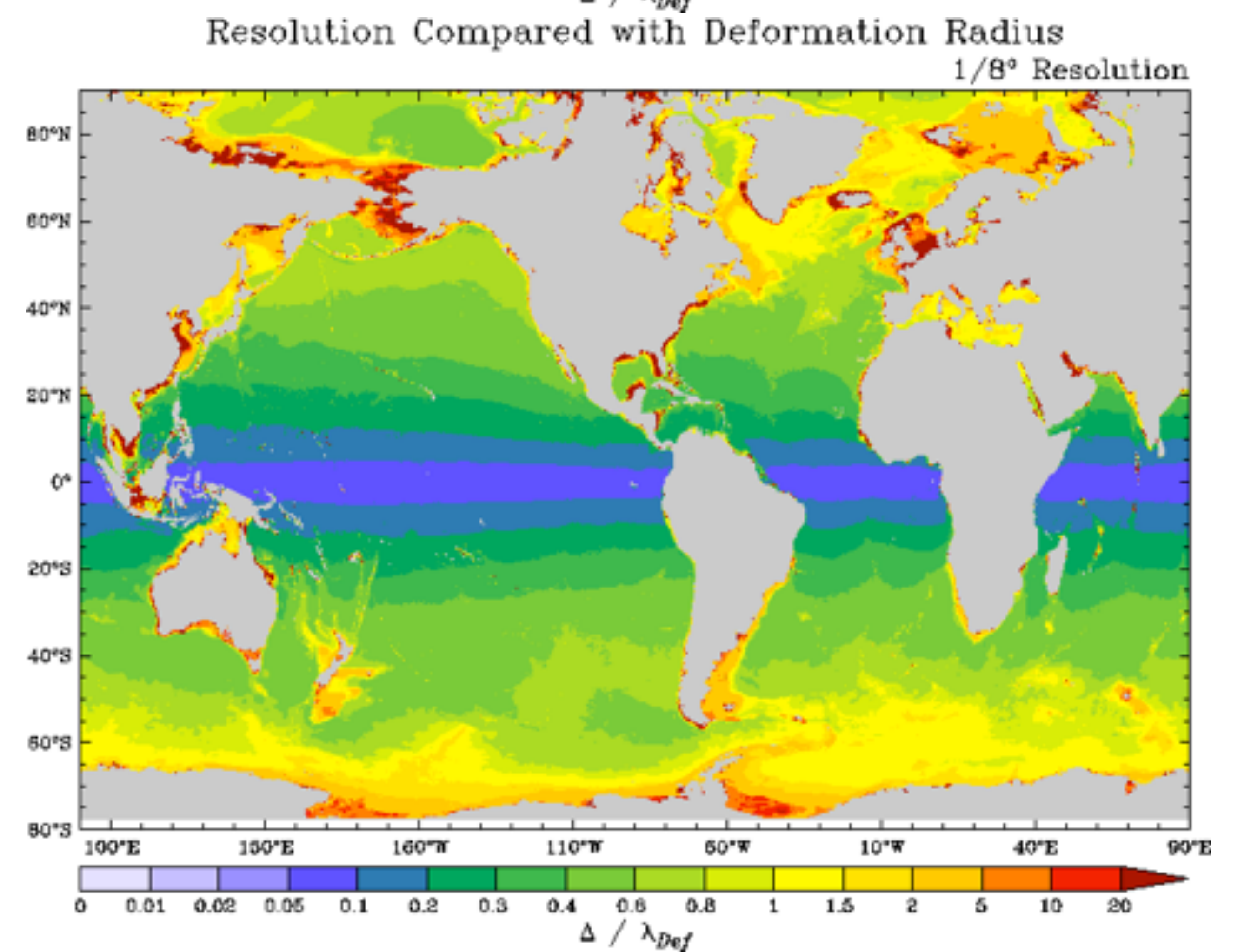
Even high resolution global models do not resolve the eddy scales in coastal regions and weakly stratified high latitude regions.

$$\tilde{\Delta} = \sqrt{\Delta x^2 + \Delta y^2}$$

$$R = \lambda_{Def} / \tilde{\Delta}$$

$$\lambda_{Def} = \sqrt{\frac{c_{g1}^{\;2}}{f^2 + \beta c_{g1}}}$$



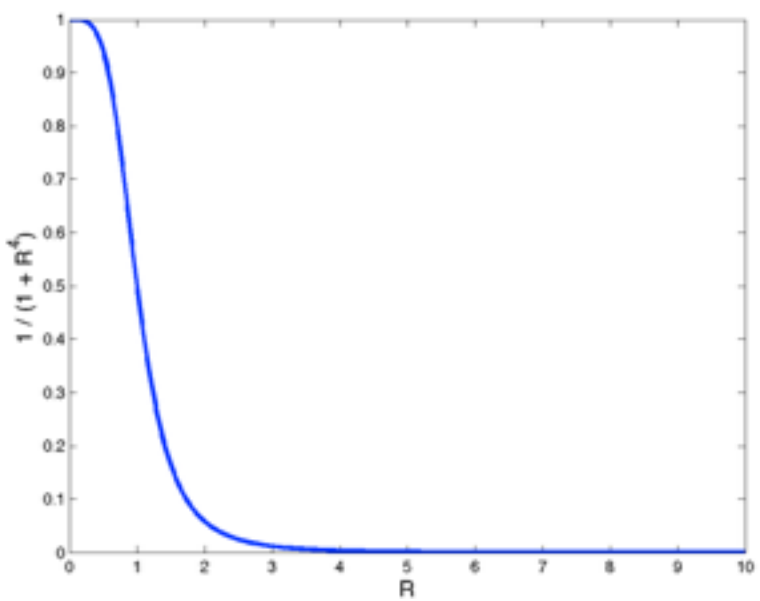


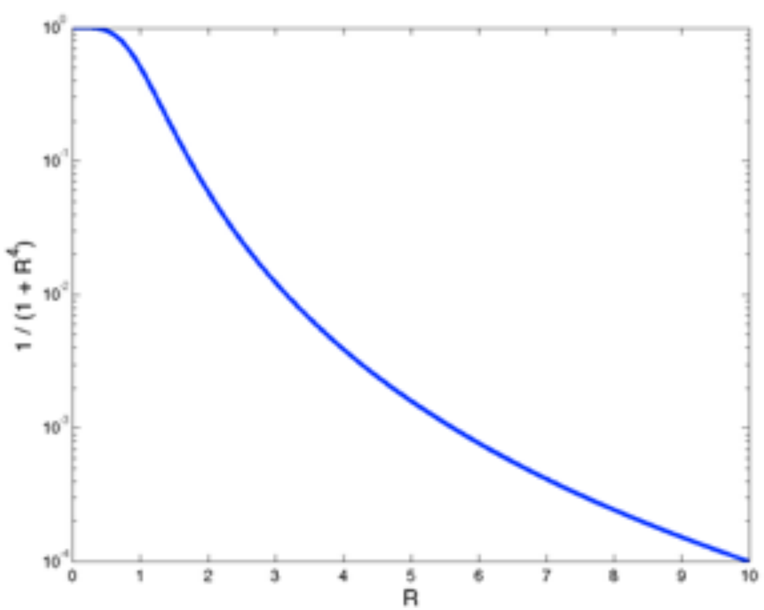
Eddy parameterizations tend to suppress resolved eddies.

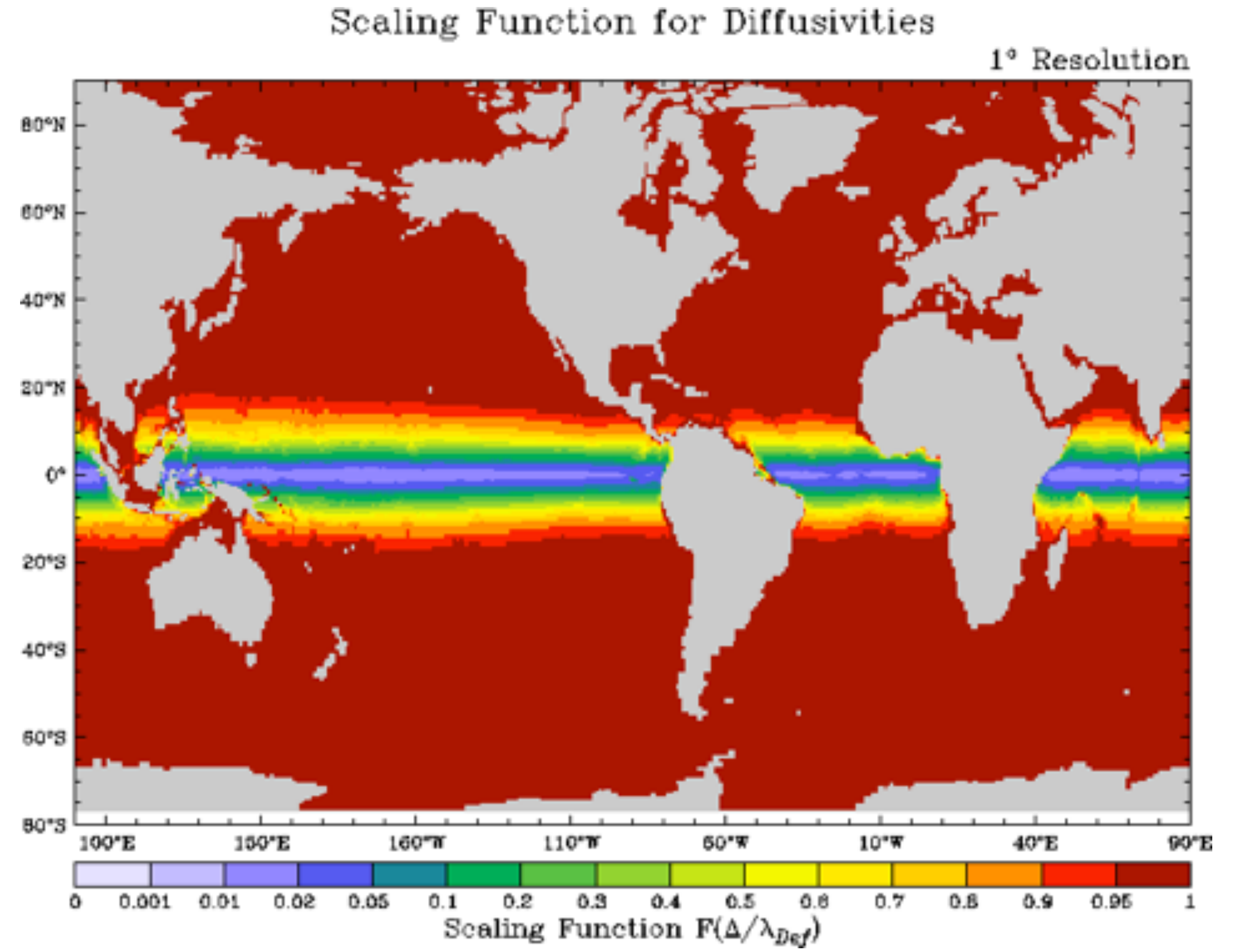
The parameterizations should be scaled away where the eddy scales (1st baroclinic deformation radius?) are well resolved. A reasonable function to do this is:

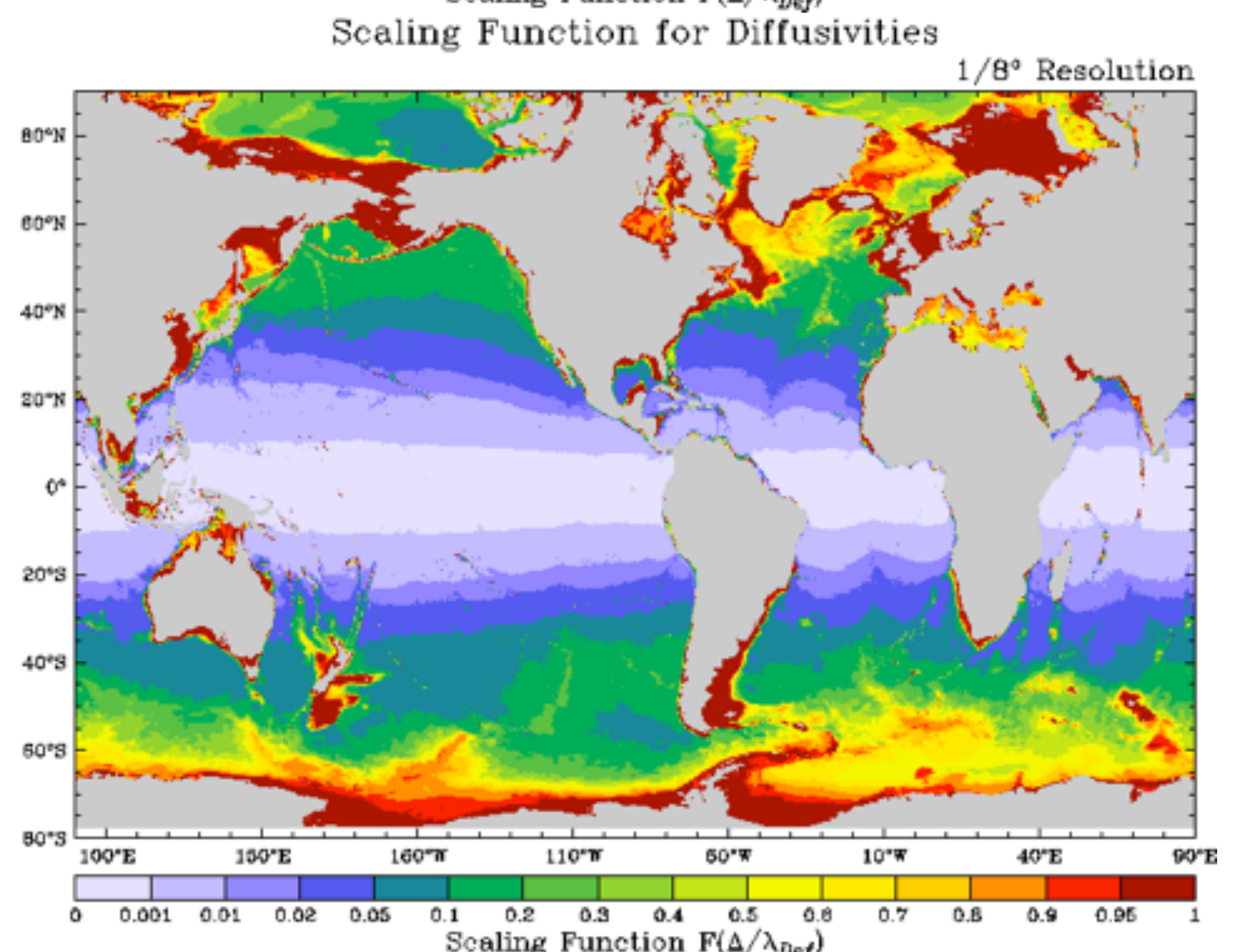
$$F(R) = \frac{1}{1+R^4}$$

$$R = \lambda_{Def} / \tilde{\Delta}$$









Scaled MEKE-Derived Diffusivity in a 1° Model

This scaling works very naturally with MEKE (see Alistair's slides), but could apply to any scheme.

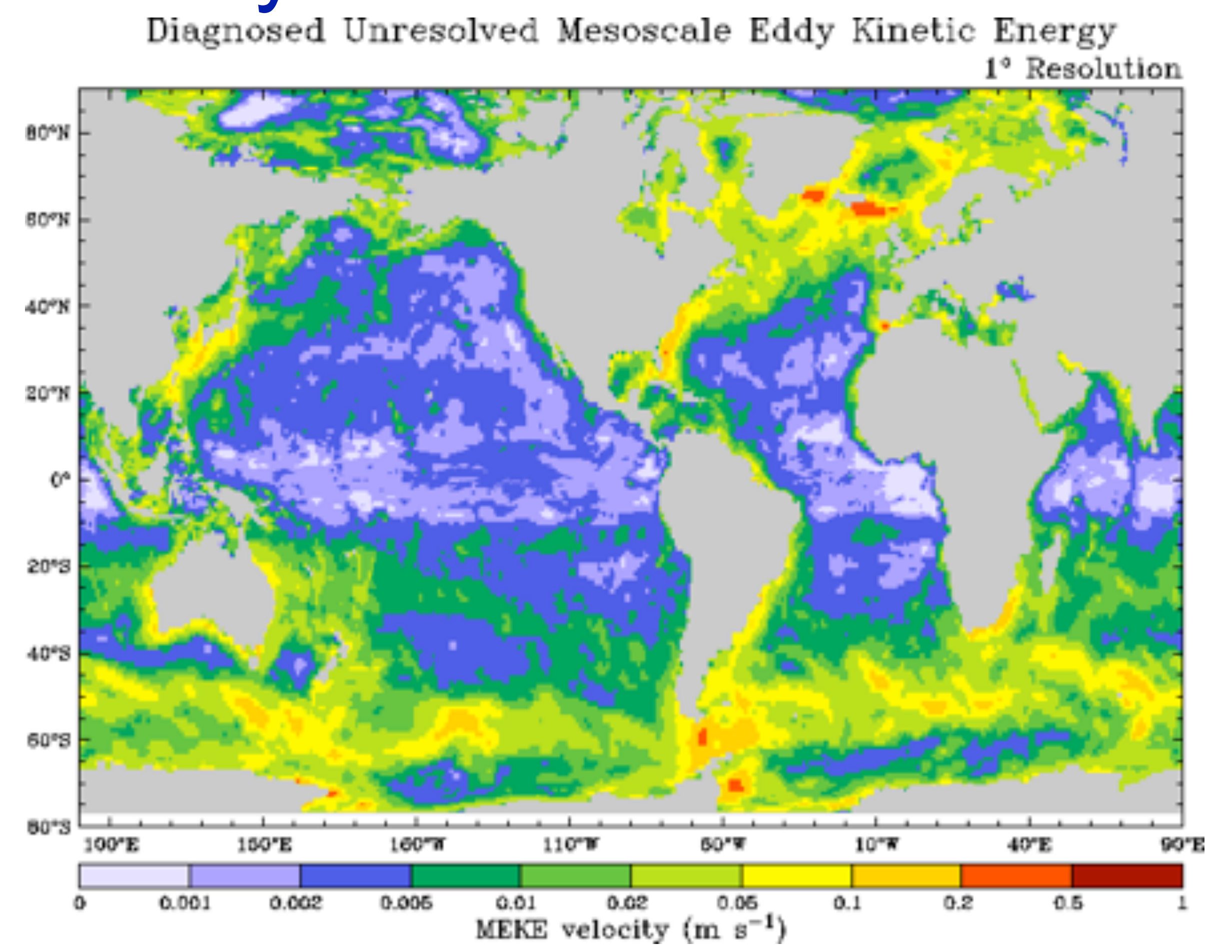
$$\frac{\partial E}{\partial t} = Src - \gamma E - \frac{c_d \|u_{bot}\|}{H} E + \frac{1}{H} \nabla \cdot \left(H \kappa_E \nabla E \right)$$

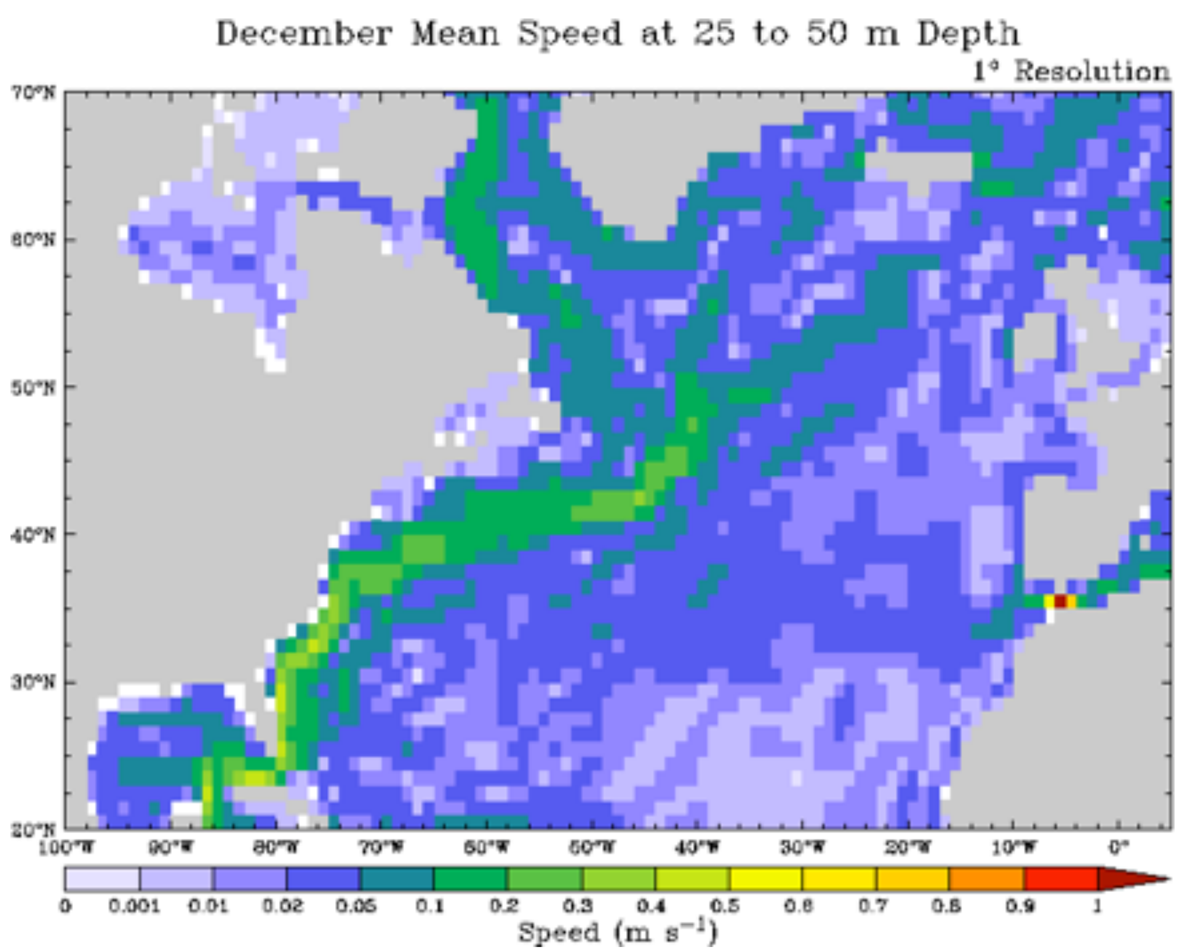
$$Src = \frac{1}{H} \sum_{k=1}^{K} g'_k \kappa_{Int} \|\nabla \eta_k\|^2 - \frac{0.001}{H} \sum_{k=1}^{K} h_k u_k \cdot \nabla \tau_{Visc}$$

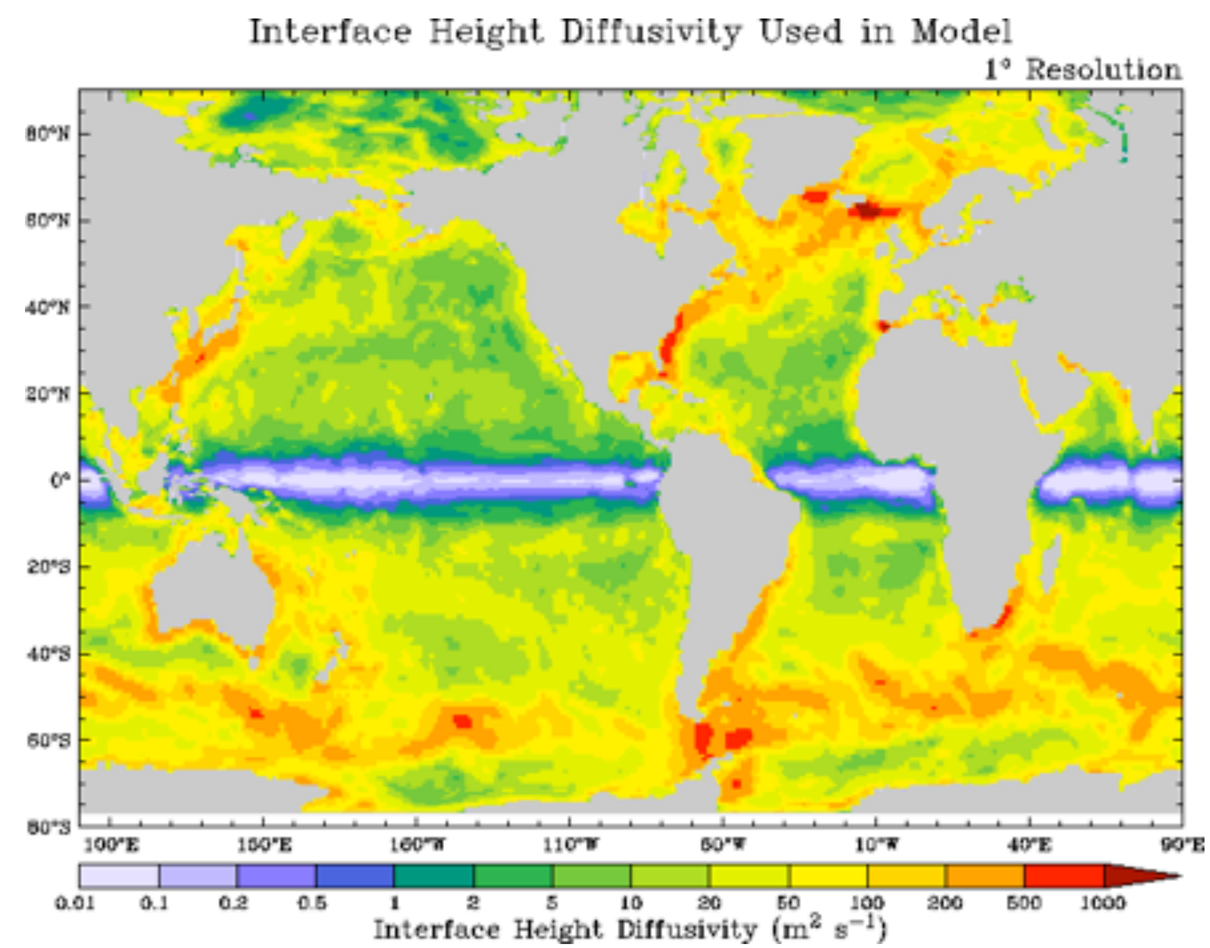
$$\kappa_{MEKE} = 0.03 \tilde{\Delta} \sqrt{2E}$$

$$\kappa_{Int} = F(R) \left(\kappa_{MEKE} + \kappa_{Background} \right)$$

At 1 ° resolution, eddy parameterizations are suppressed only in the tropics.



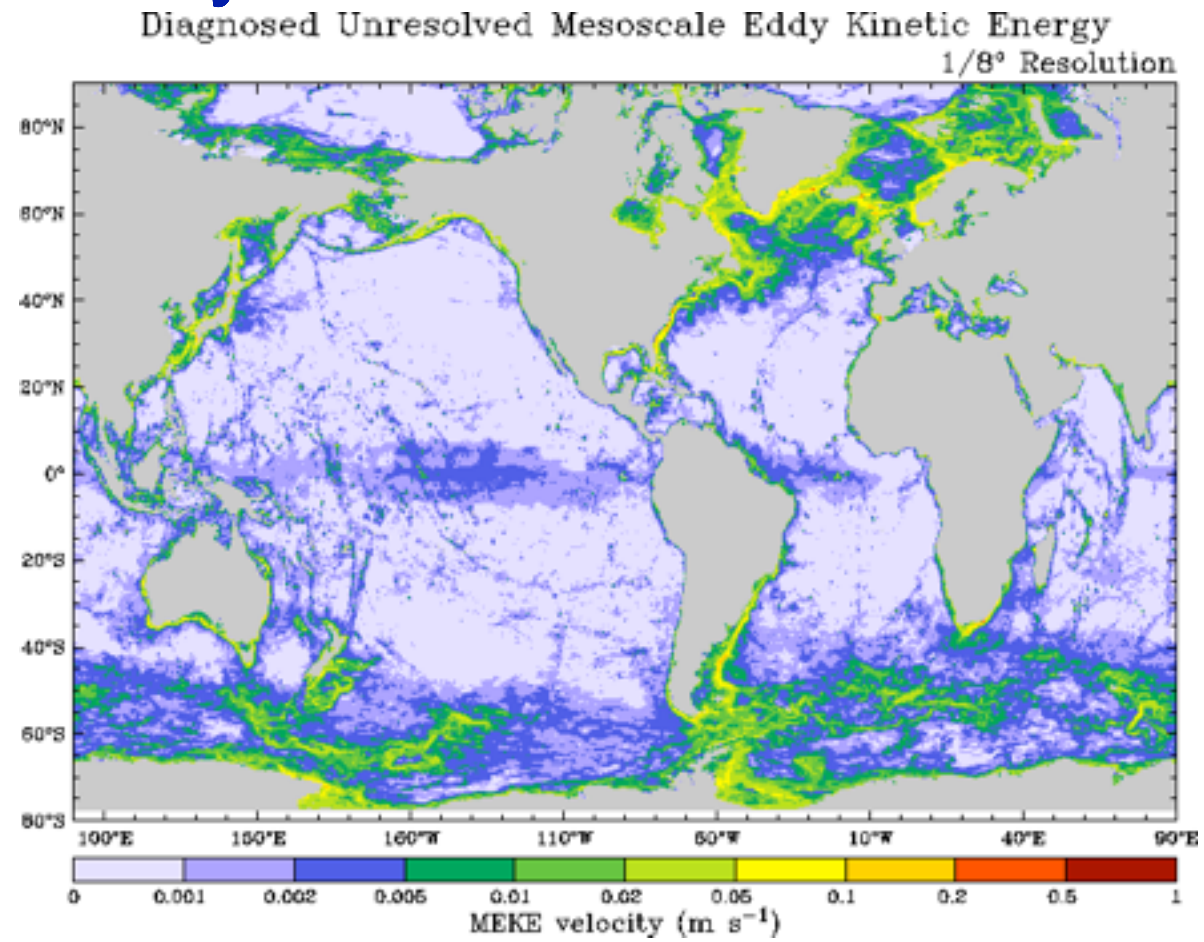


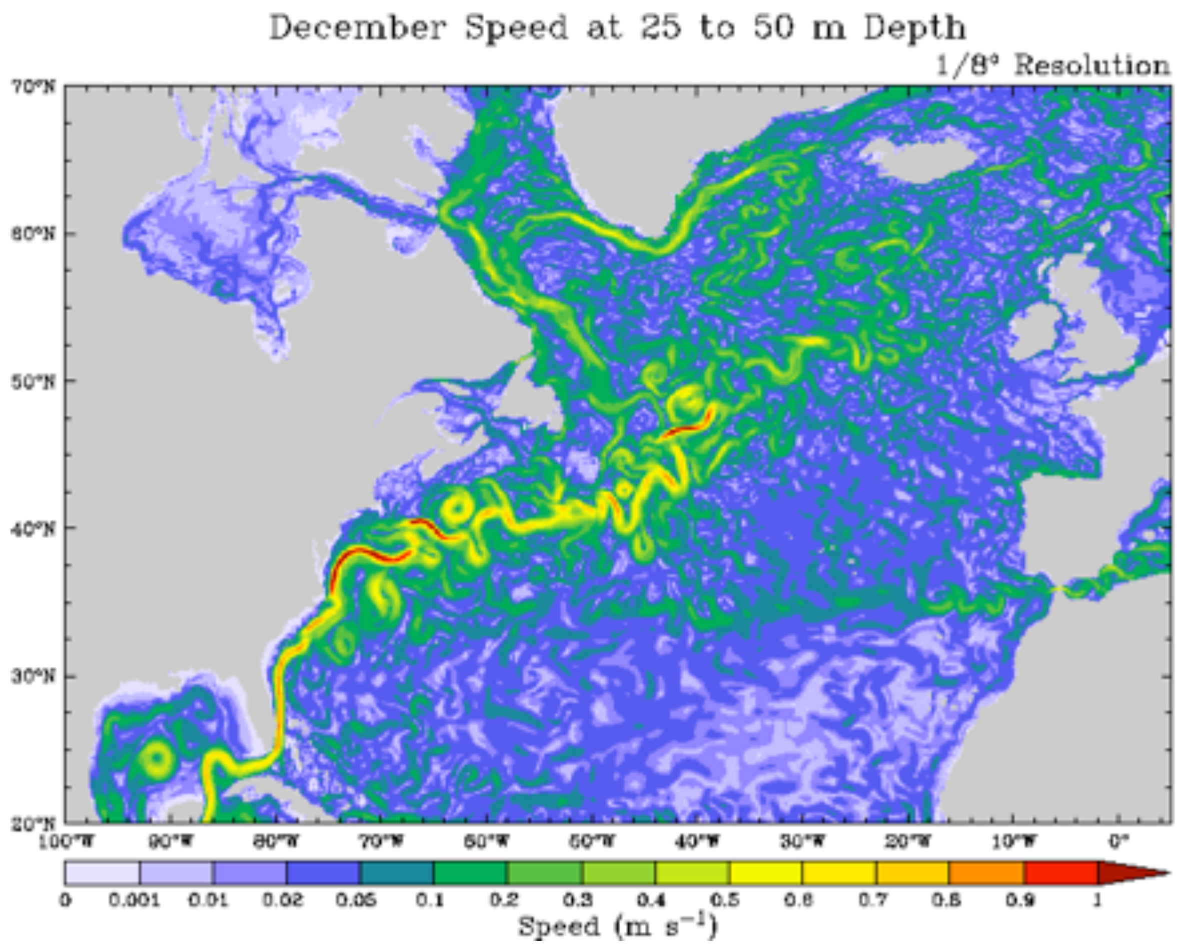


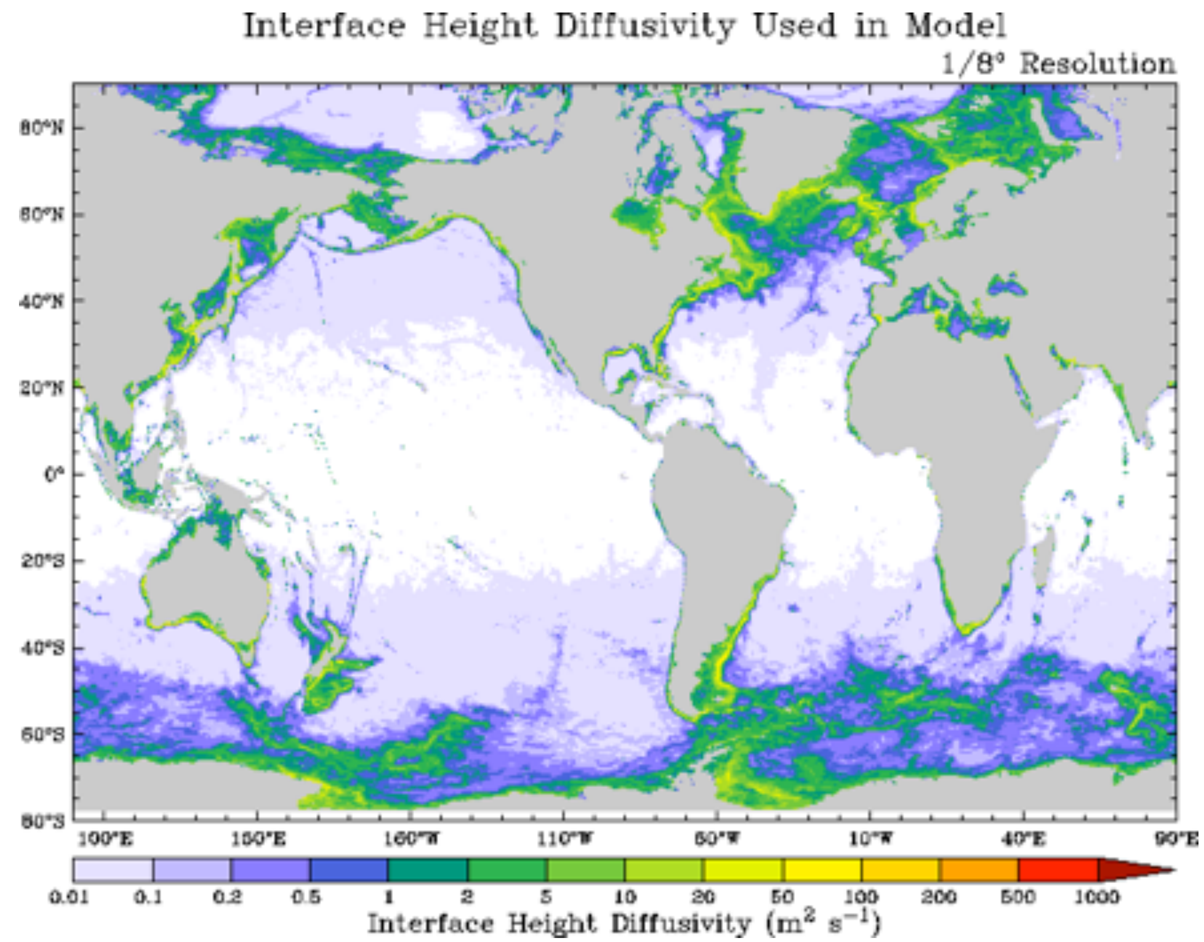
Scaled MEKE-Derived Diffusivity in a 1/8° Model

At high resolution, diffusivities are very small except in coastal regions and high-latitudes.

This 1/8° resolution model uses *identical* (nondimensional) eddy parameters as the previous 1 ° resolution model!







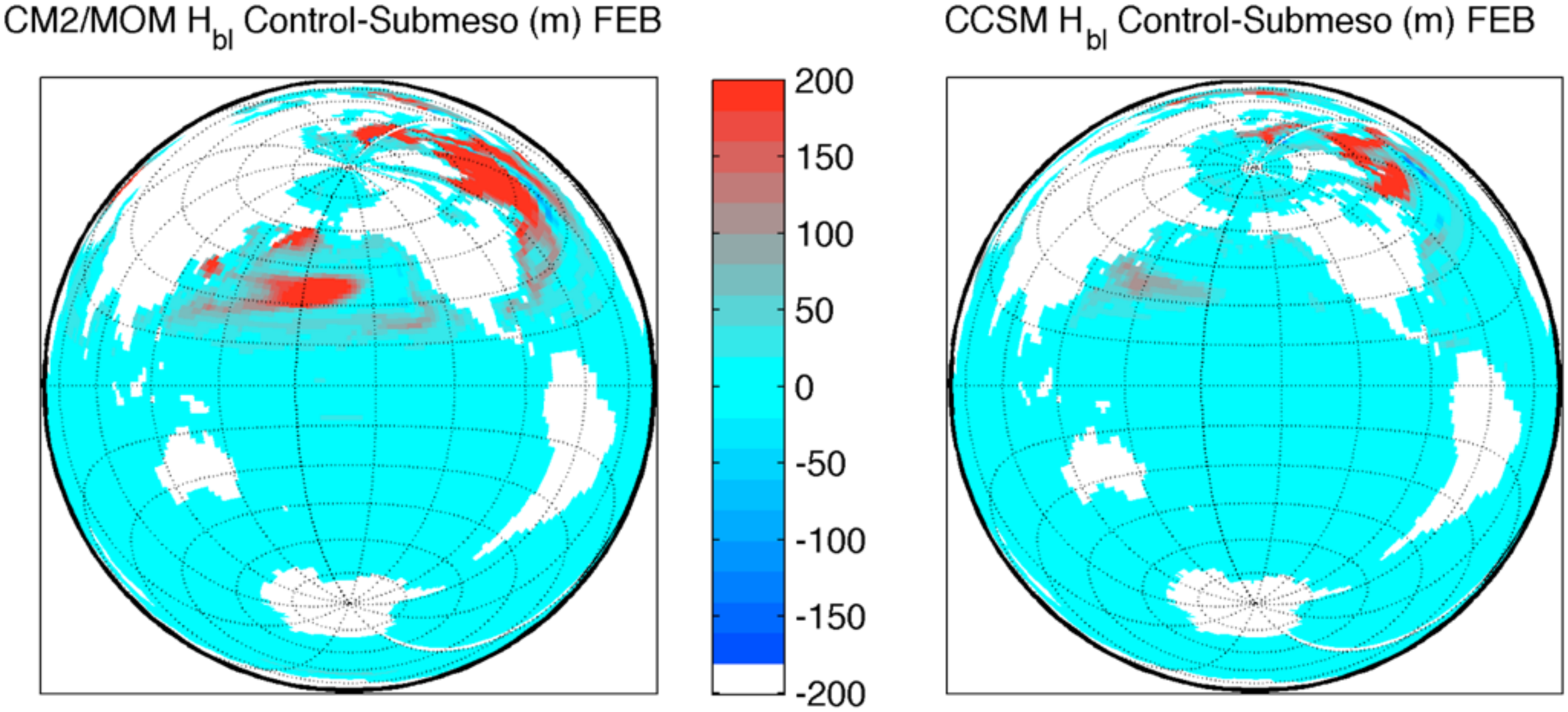
Topics for discussion: II

- Scaling for MOLES (Bob)
- What can linear theory tell us? (Shafer)
- What can other theory tell us? (Isaac?)
  - GLM? TEM? Stat Mech?
- Anisotropy, tracer type dependence, biogeochem tracers, etc.? (Baylor, others?)

Extra Slides!!

- FFH Comparison Slides
- What would AR6 Success look like (IMHO)
- Ferrari et al (2010) vs Ferrari et al (2009) vs Danabasoglu et al (2008)... Vicissitudes of boundary conditions!

FFH As Implemented in CCSM, CM/MOM, CM/GOLD: A Comparison:



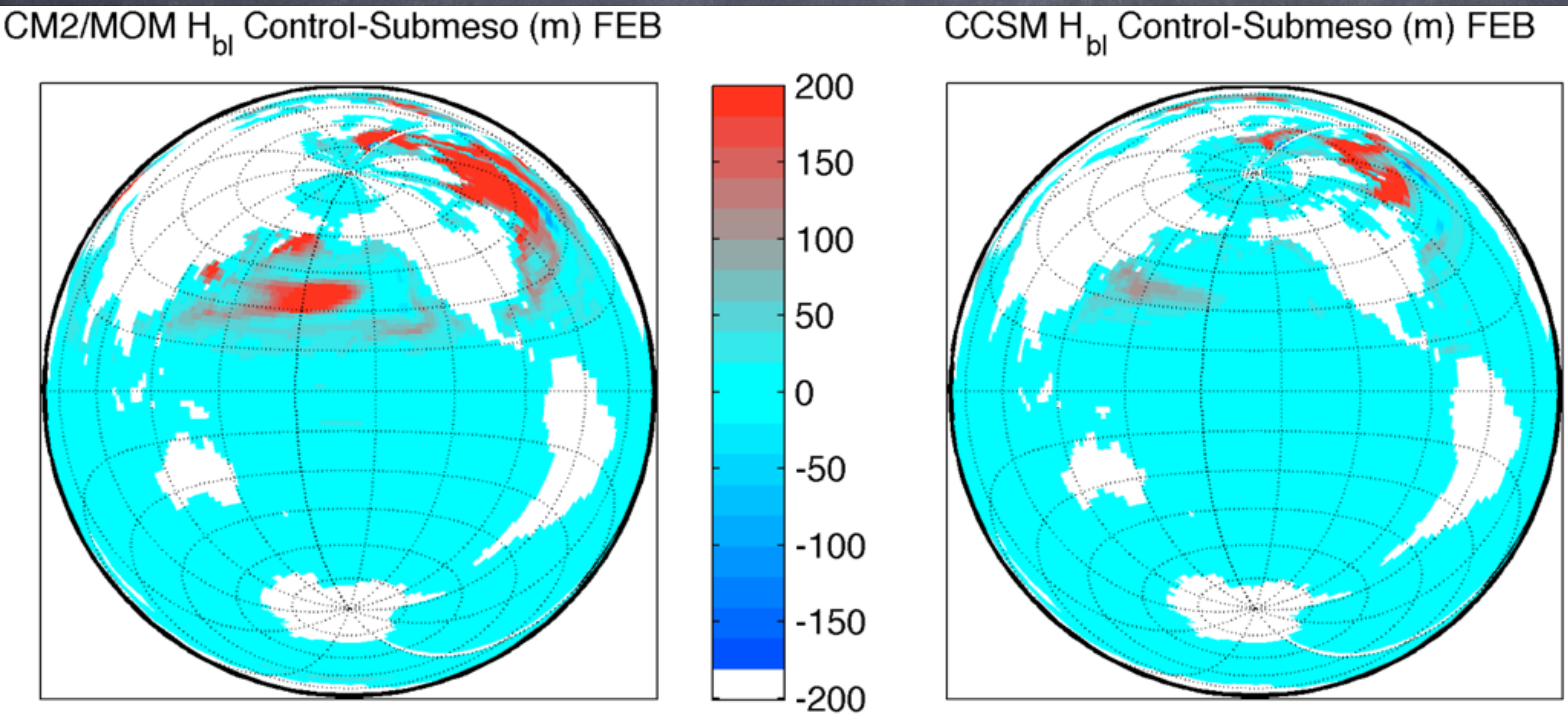
Thanks to Samuels, Griffies, Danabasoglu for these!

MLE-Control:Climatologies at end of > 100yr simulation

Improves Restratification after Deep Convection

Note: param. reproduces Haine&Marshall (98) and Jones&Marshall (93,97)

& generally shallower boundary layers



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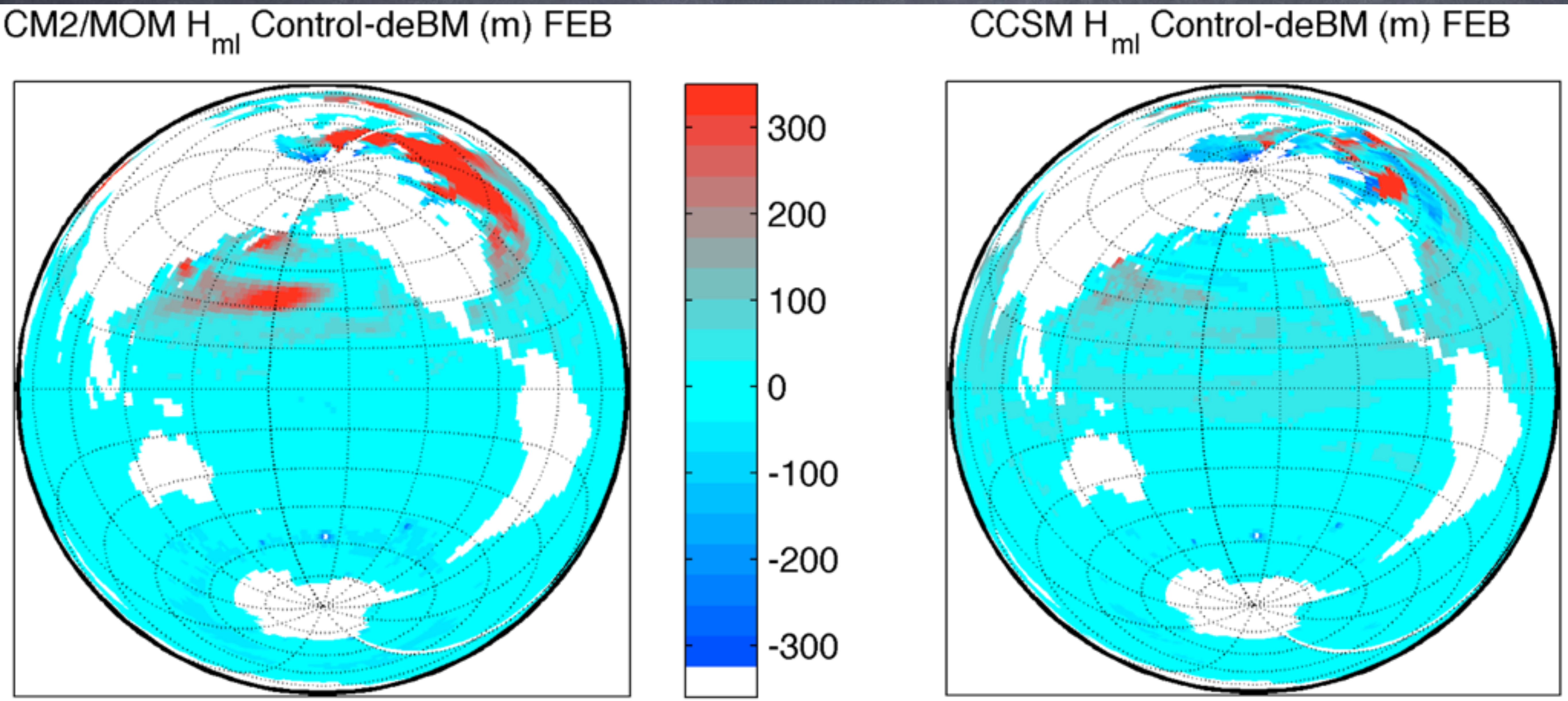

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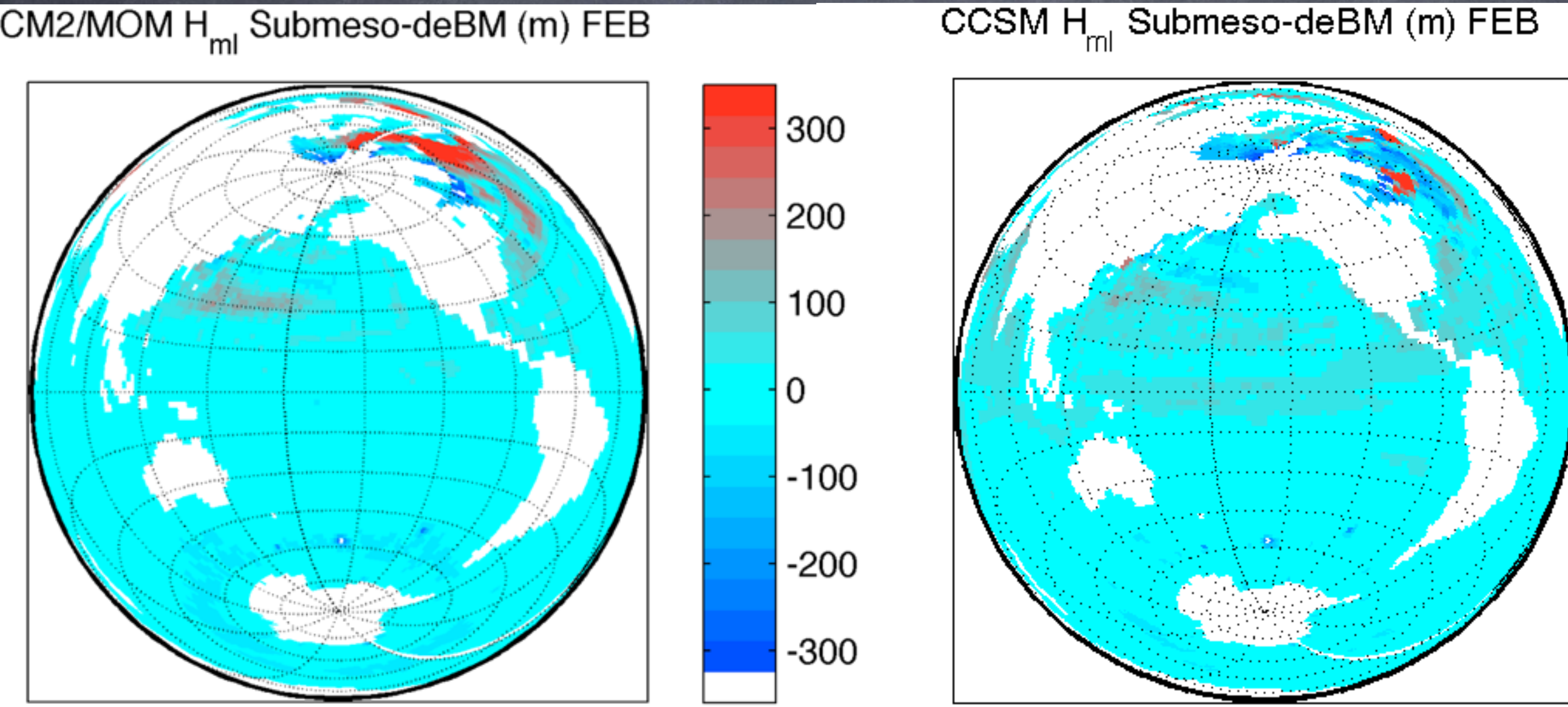

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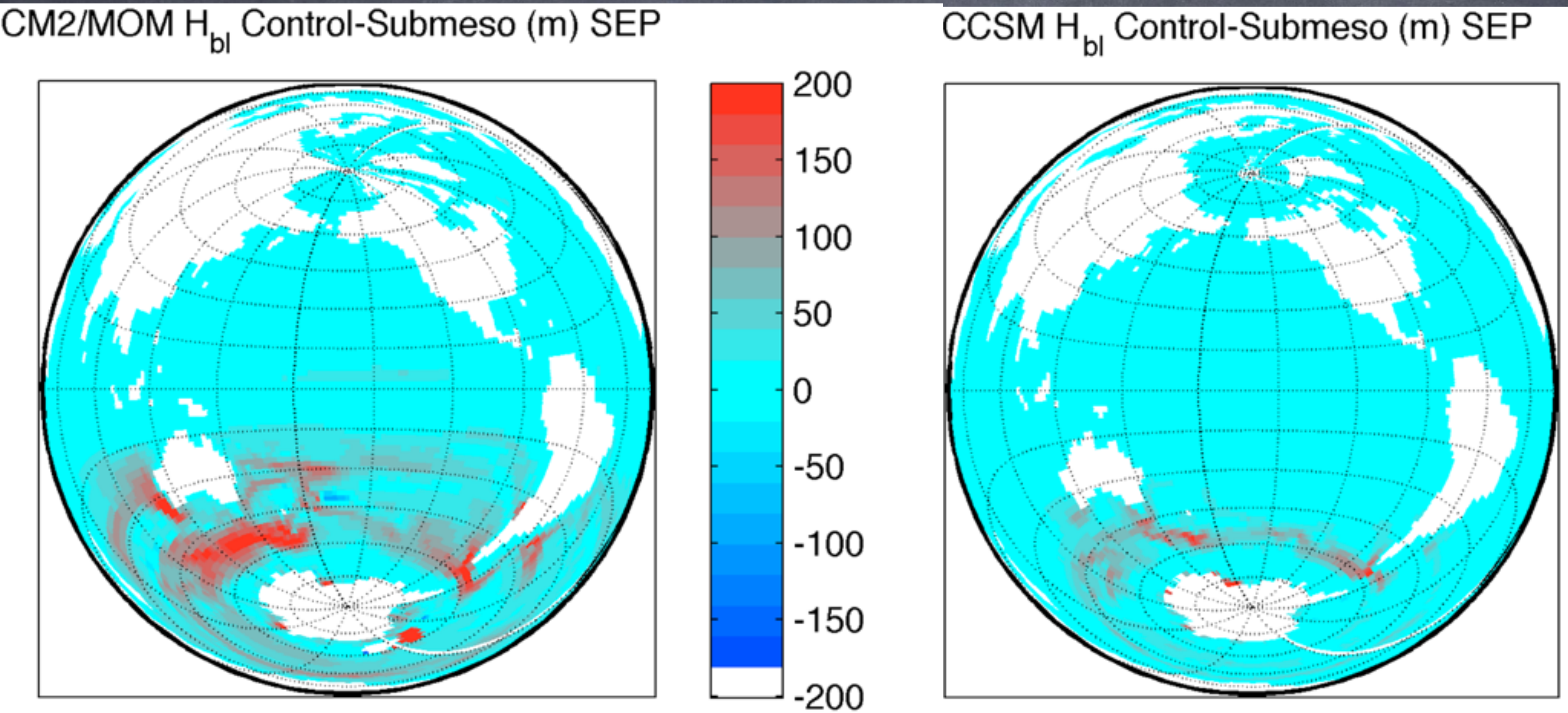

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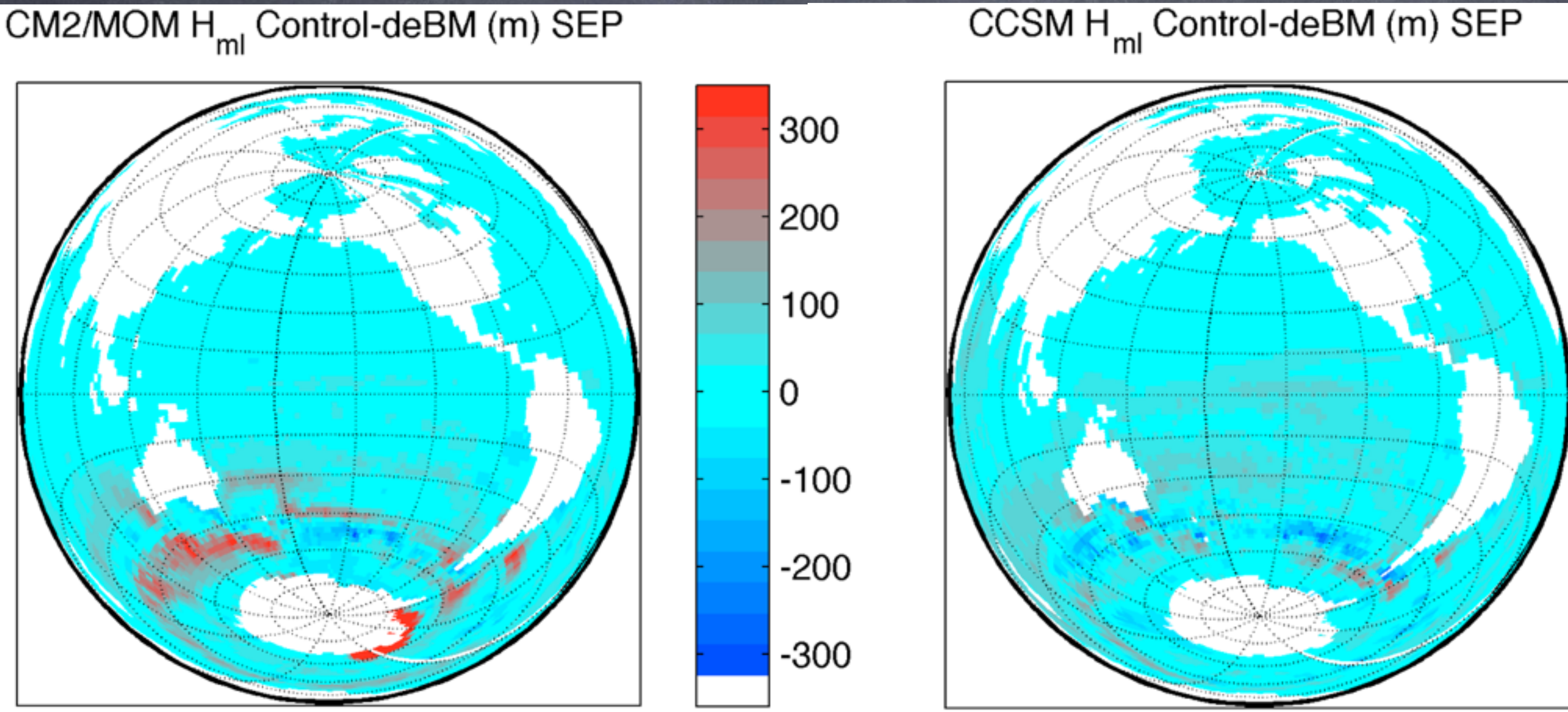

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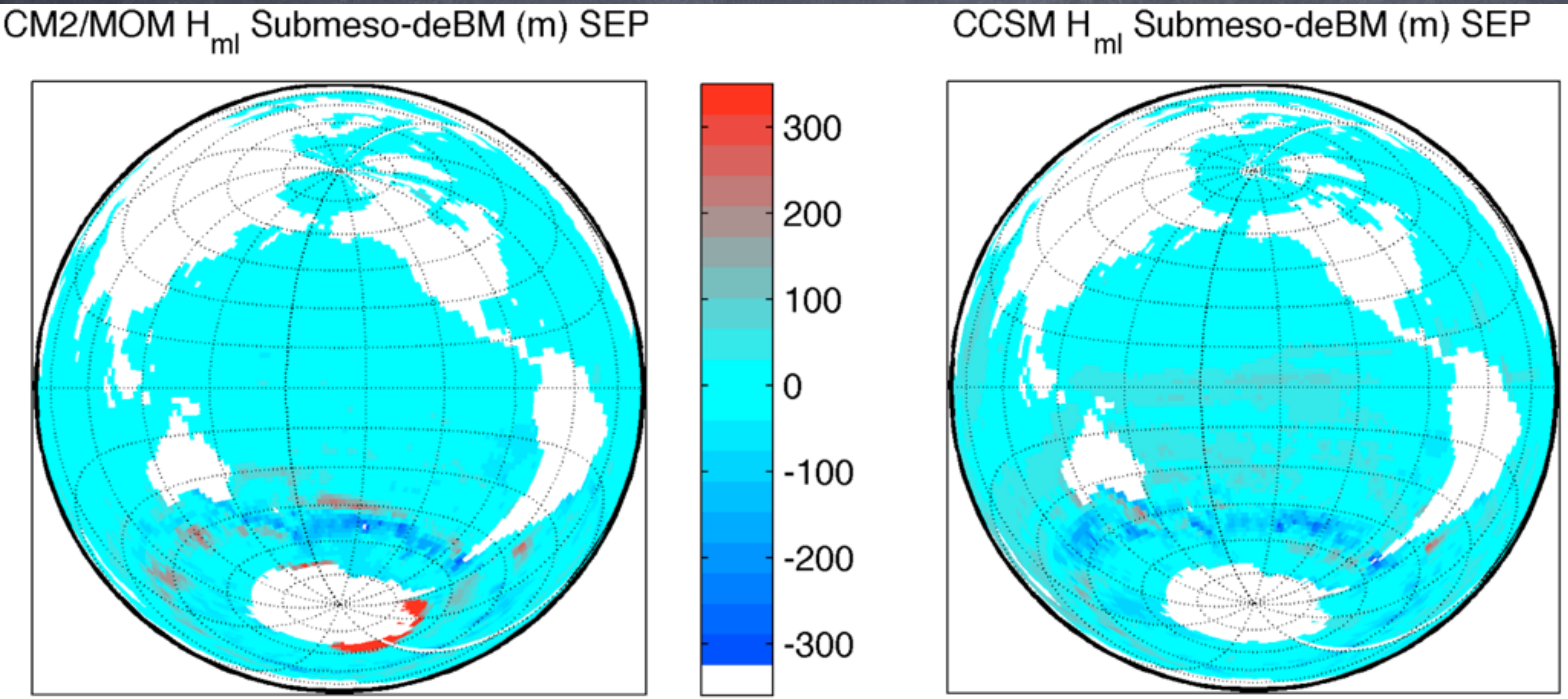

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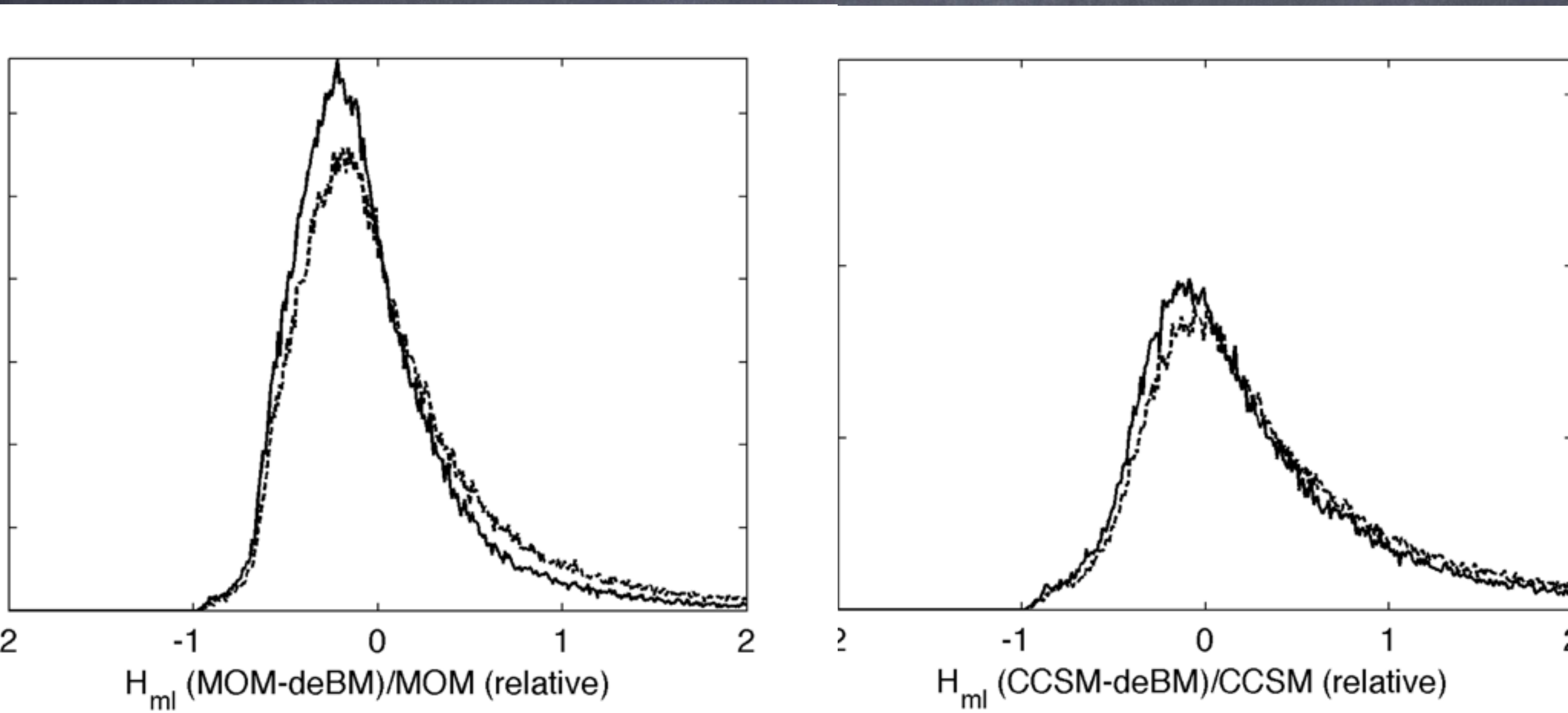
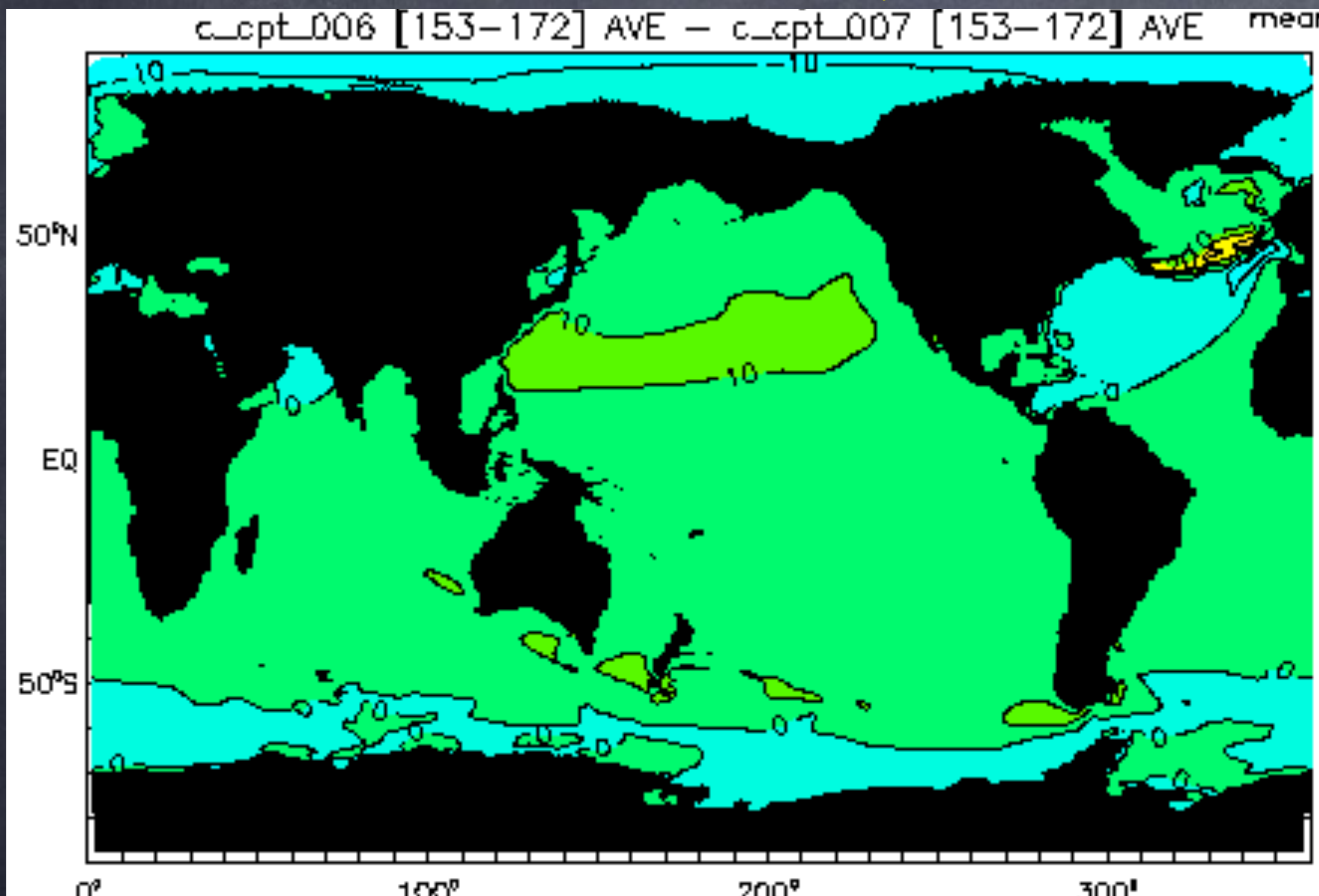


Thanks to Samuels, Griffies, Danabasoglu for these!

MLE-Control:Climatologies at end of > 100yr simulation

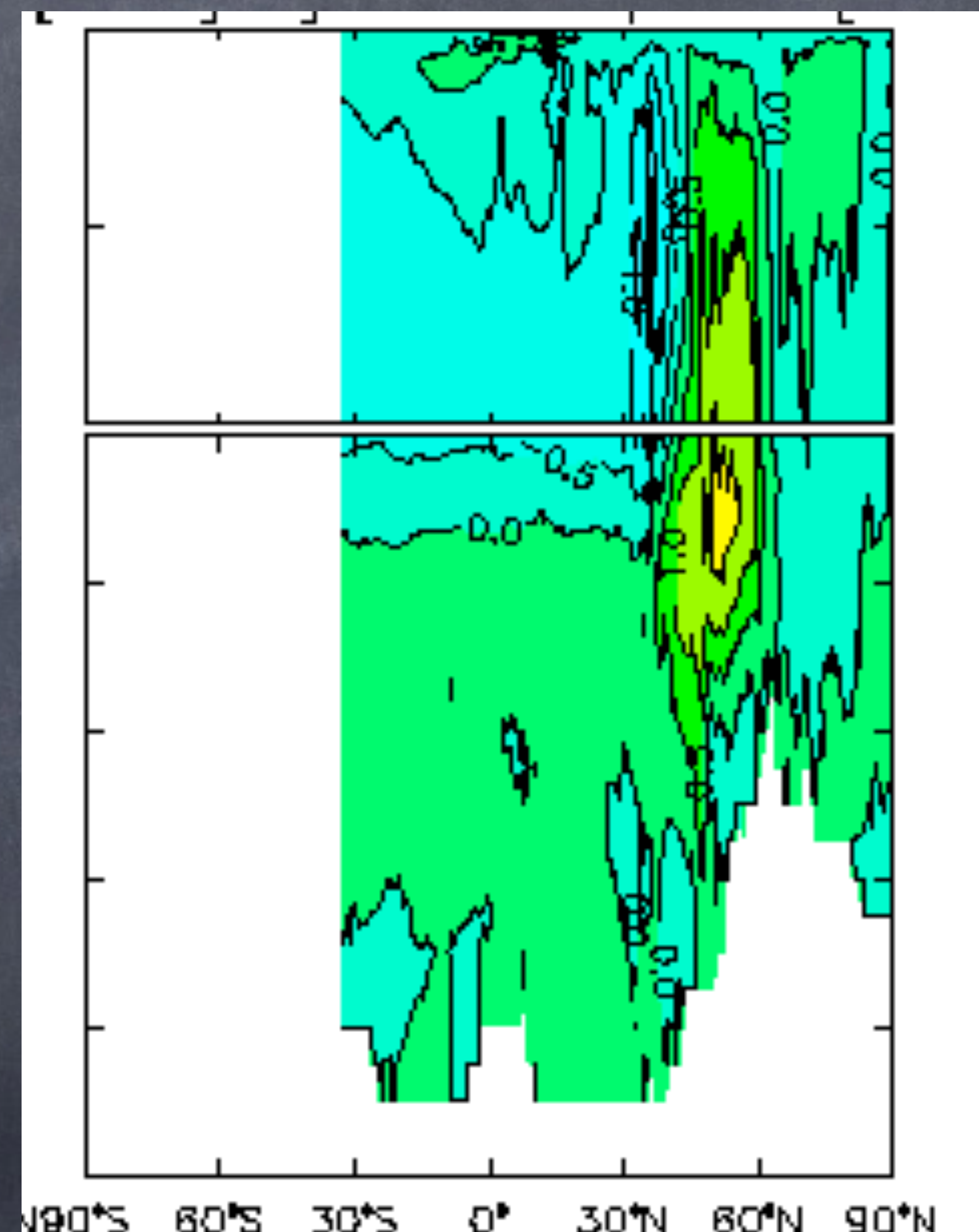
Changes other variables we care about... CCSM

Avg. Ideal Age 4 yrs older at 500m with MLE (up to 30%)



(as big as coarse vs 10km, Frank)

MOC 10% greater with MLE



Thanks to Danabasoglu for these!

MLE-Control:Climatologies at end of > 100yr simulation

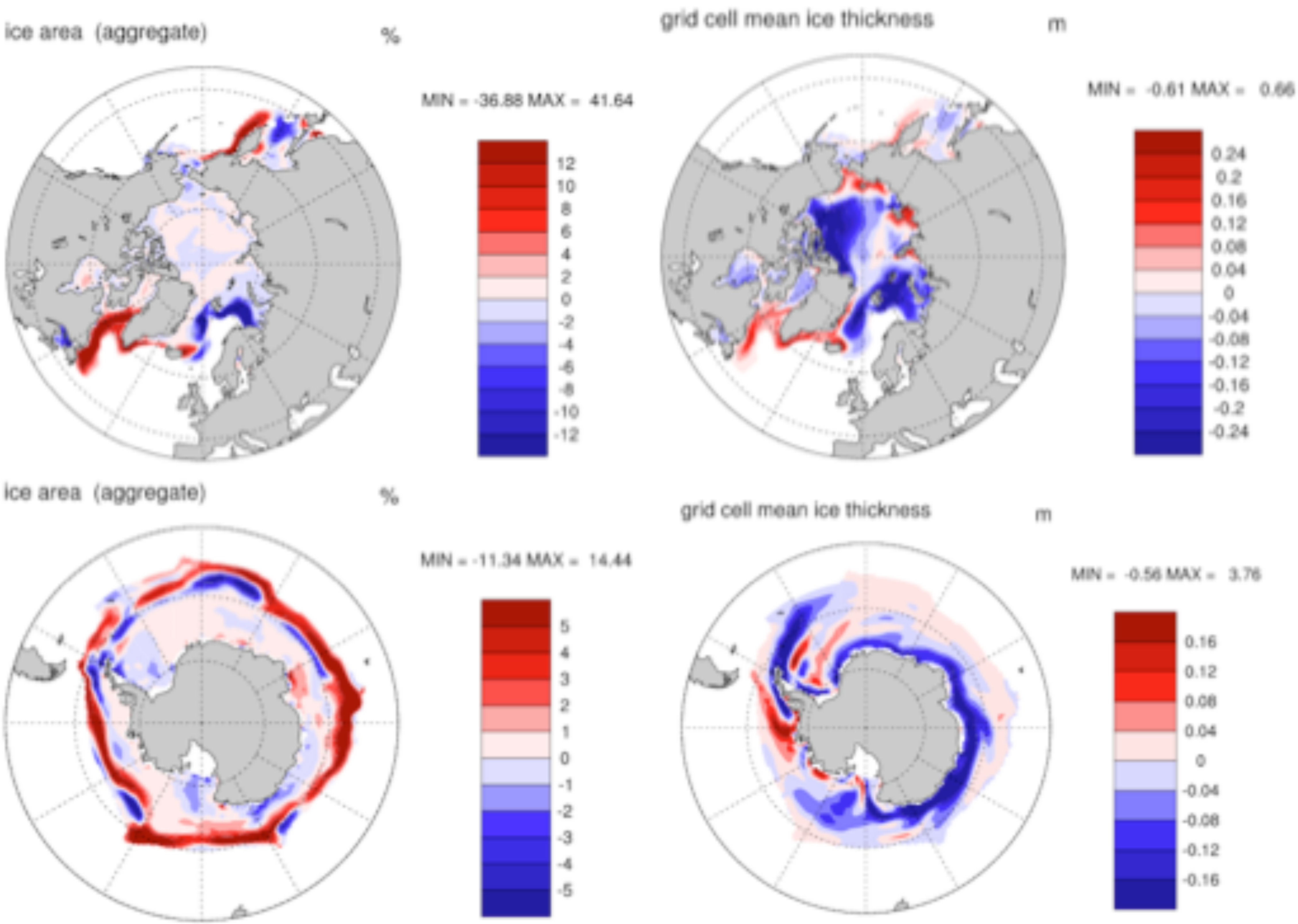
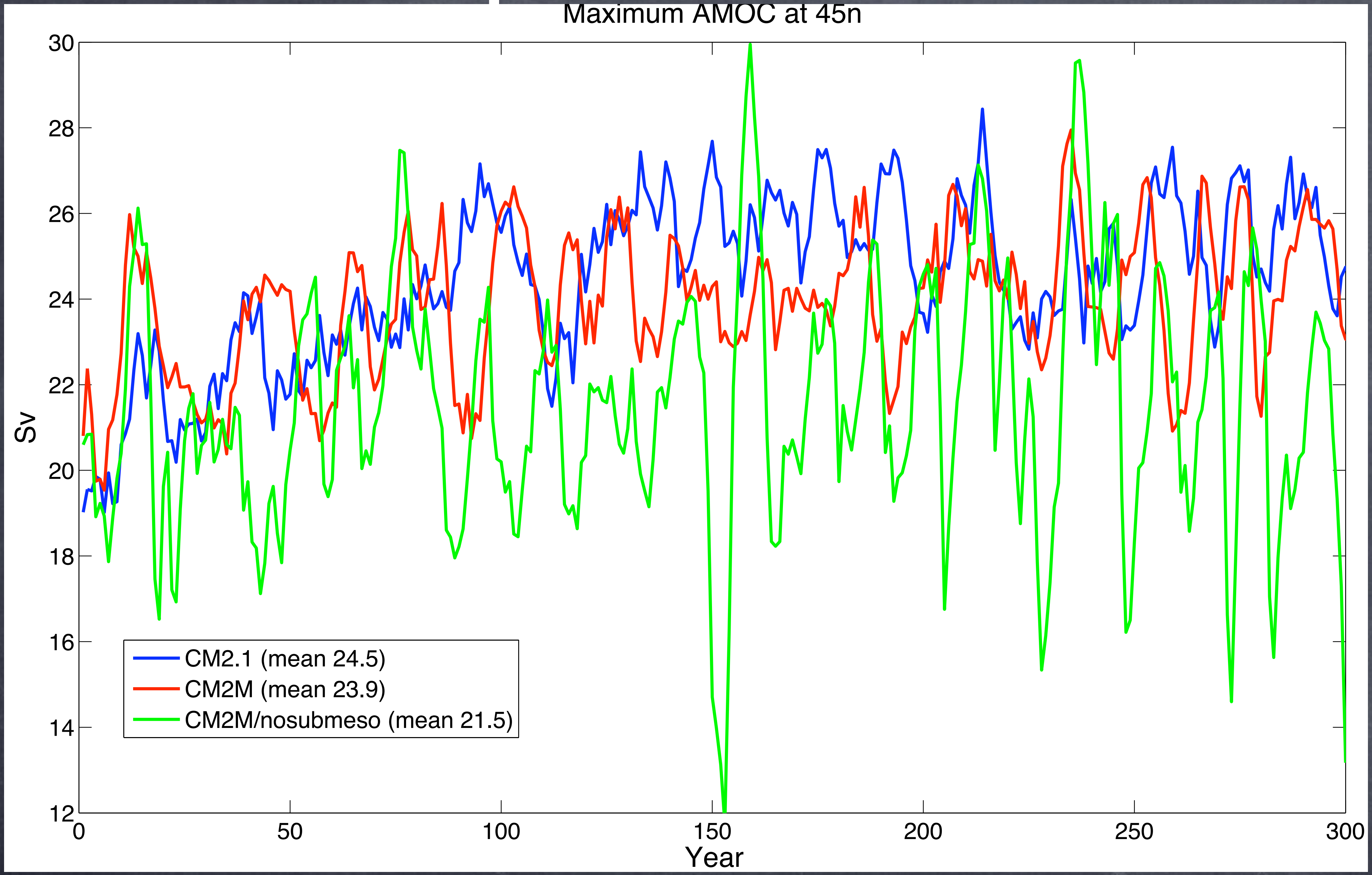


FIGURE 5: Figures demonstrating the change in wintertime sea ice from $CCSM^+$ to $CCSM^-$.

Thanks to Marika Holland for these!

Coupled MOM Shows



Submeso increases MOC stability

Recent Submeso Results:

- Capet, X., E. J. Campos, and A. M. Paiva: 2008
  - <wb> scaling OK, differs maybe due to other effects or resolution
- Mahadevan, Tandon, Ferrari (in press, JGR)
  - <wb> scaling diminished somewhat by downfront winds, but eddy-driven frontal meandering eventually reduces how down-front the winds are!

What would AR6 Success look like? (IMHO)

- Distinguish MOLES from MORANS
- Distinguish MO, SO, FO
- No Arbitrary Dimensional Constants
- Process model/Fluctuation equation basis for all balances; including energetics, PV, Pot'l enstrophy
- Tracer (active & passive) and momentum handled sensibly and respecting tensor order
- Science->Engineering transfer