

# Physics Problems for the Future of Global Ocean Modeling

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Many Collaborators, to be mentioned within...

# Ancient Worries of Ocean Modeling

- **Accurate and Stable Numerics**--many known fixes
- **Flux Adjustments**--no longer needed (ca 1995)
- **Veronis Effect**--fixed with isoneutral schemes:  
Redi & Gent-McWilliams
- **Inertial runaway**--not really a problem with vorticity sinks (Fox-Kemper & Pedlosky, 04)
- **Visualization**--graphics, movies!

# Recent Worries of Ocean Modeling

- Mesoscale eddies and boundary currents-- resolving the deformation radius?
- Tropical biases--upwelling, double ITCZ, poor ENSO, etc.
- Boundary conditions for ocean-only runs-- Flux or restoring?
- Depth, isopycnal, or sigma coordinates-- the right vertical discretization?
- Sparse data for comparison--especially subsurface and repeat observations (also double-filtering problems)

# Mesoscale eddies and boundary currents-- resolving the deformation radius?

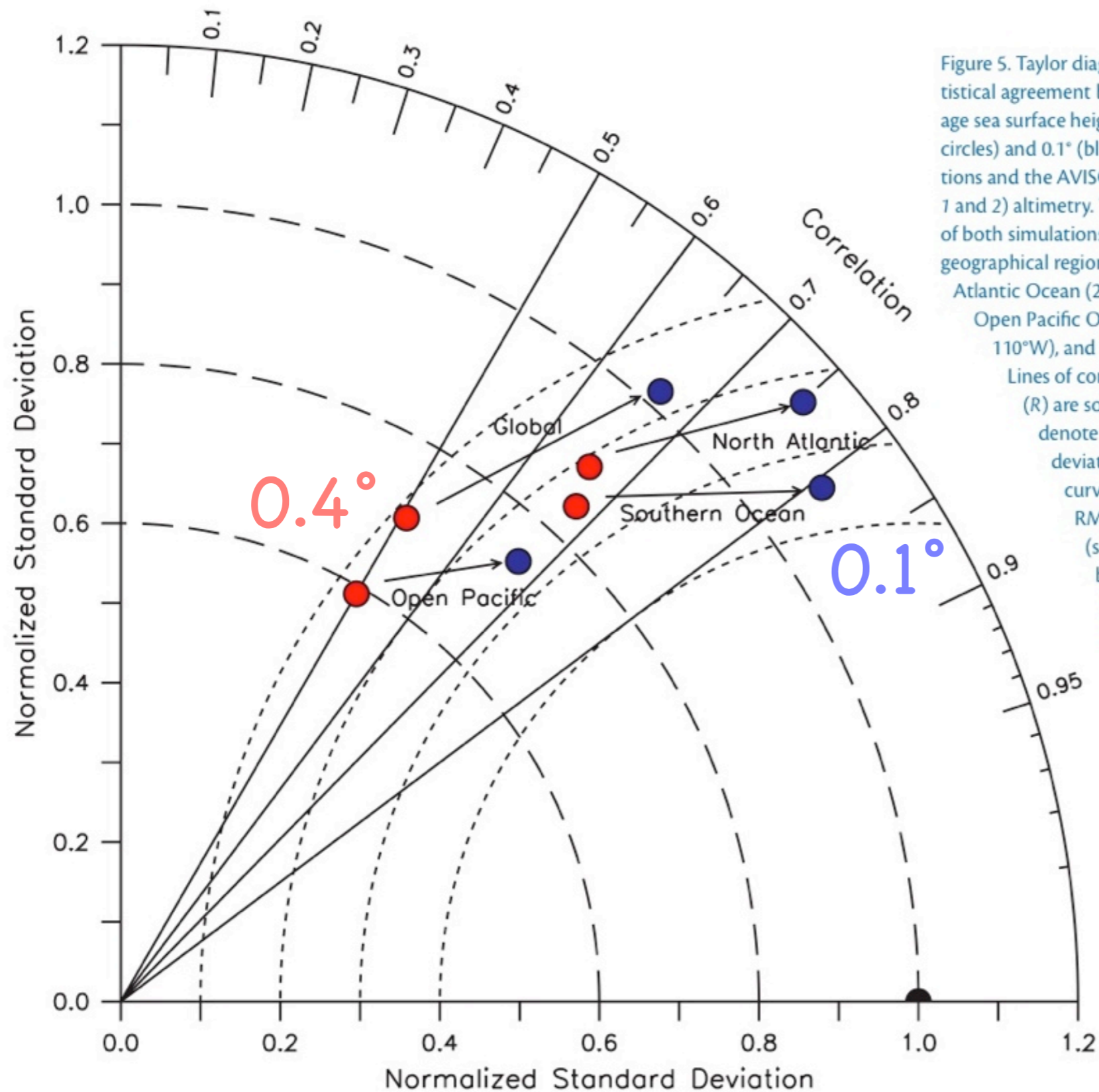
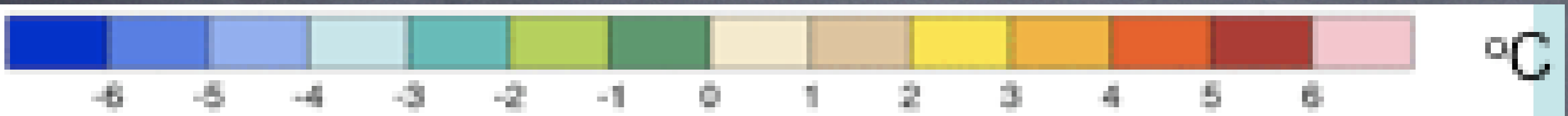


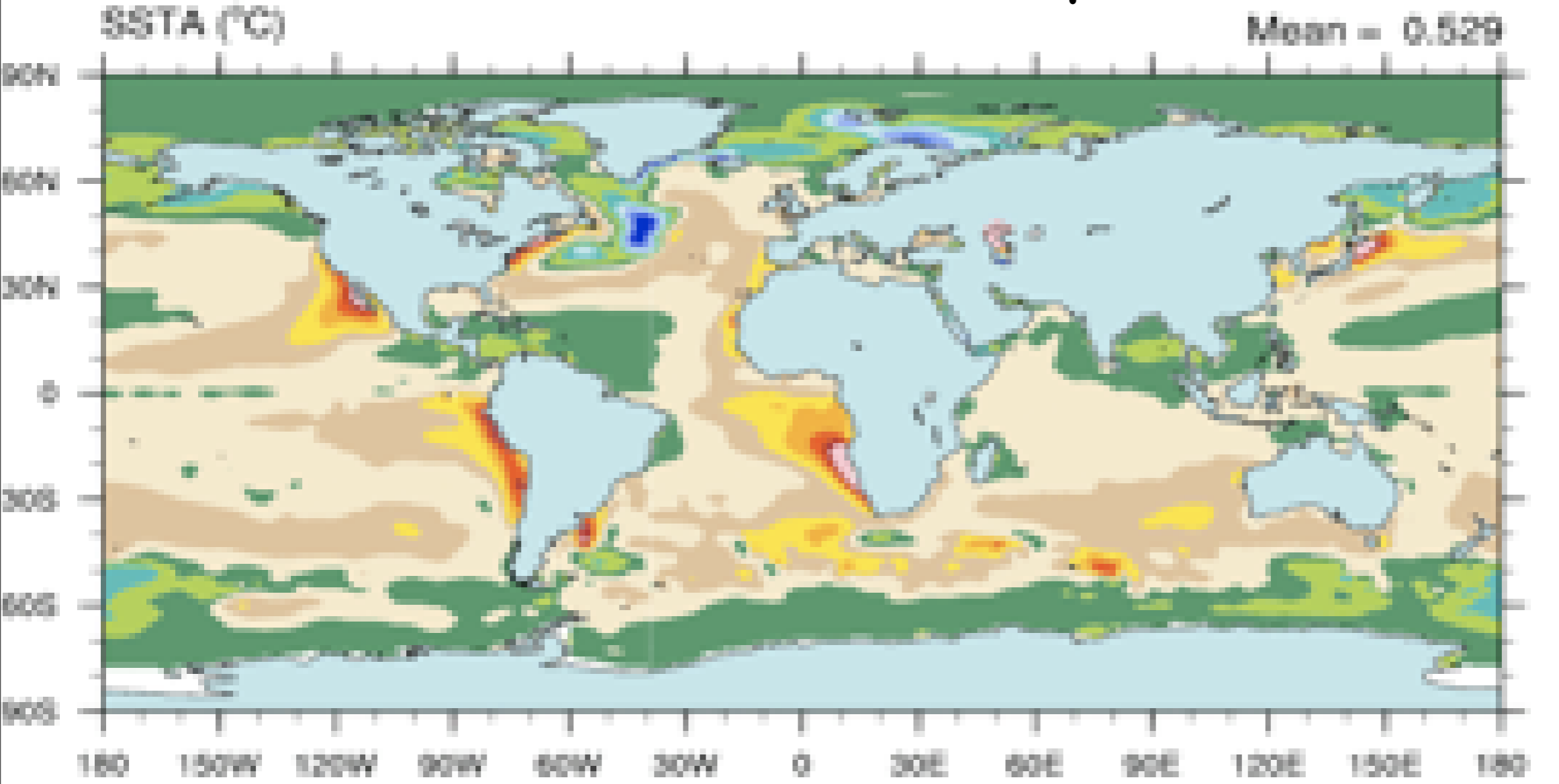
Figure 5. Taylor diagram showing the level of statistical agreement between the 1994–2001 average sea surface height anomaly from the 0.4° (red circles) and 0.1° (blue circles) global POP simulations and the AVISO (TOPEX/POSEIDON and ERS 1 and 2) altimetry. The arrows connect the results of both simulations evaluated over the following geographical regions: Global (70°S–70°N), North Atlantic Ocean (20°N–55°N, 100°W–20°W), Open Pacific Ocean (30°S–30°N, 150°E–110°W), and Southern Ocean (65°S–40°S). Lines of constant correlation coefficient ( $R$ ) are solid; the long dashed curves denote lines of constant standard deviation ratio ( $\sigma$ ); the short dashed curves denote lines of constant RMS difference, varying from 0.6 (small radius) to 0.9 by 0.1. The black semicircle represents the location of perfect agreement between the simulation and the comparison data set.

Credit:  
McClean  
et al. 06

• Tropical biases--upwelling, double ITCZ, poor ENSO, etc.



## 2° Atmosphere



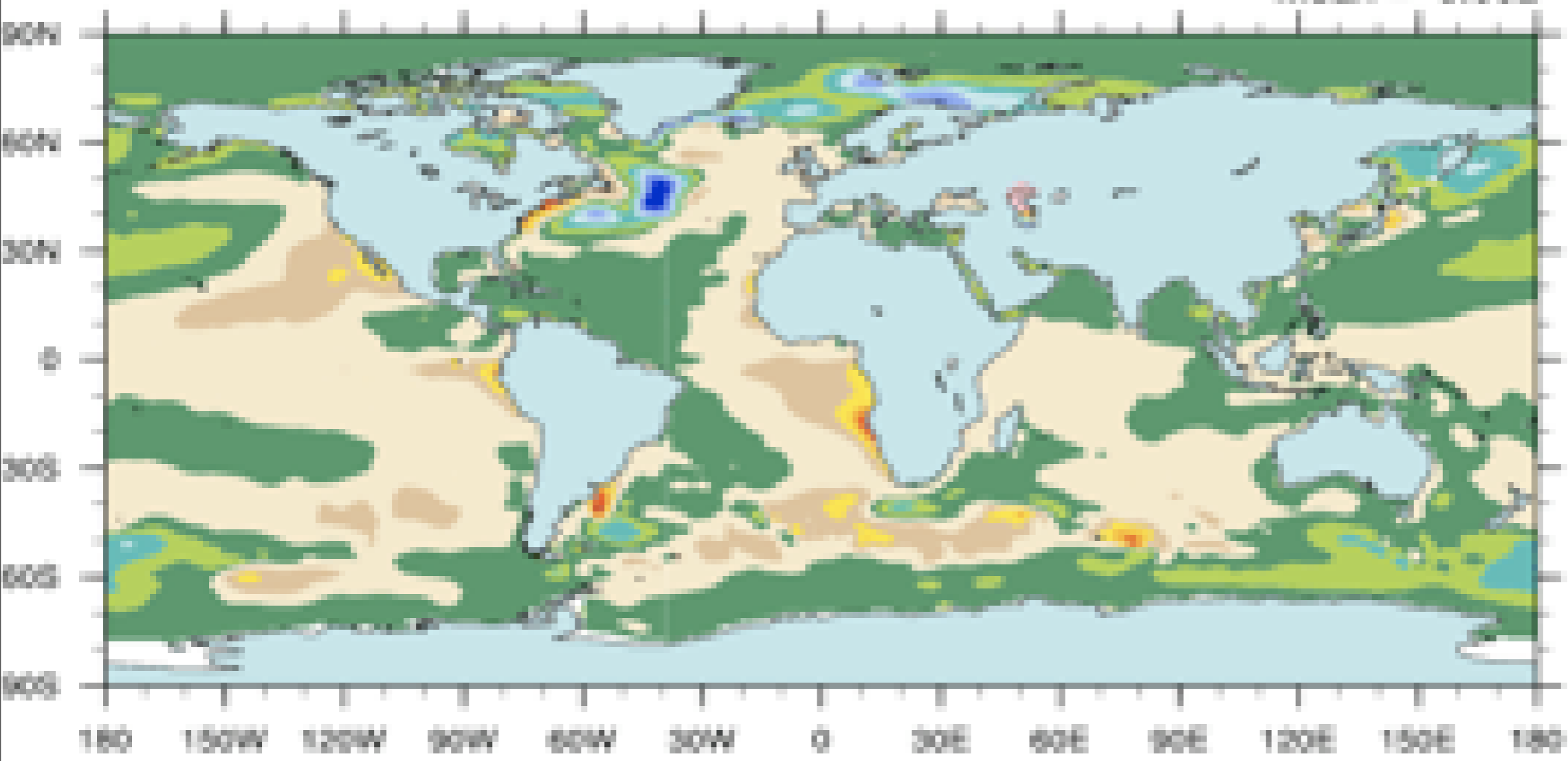
Credit:  
P. Gent

- Tropical biases--upwelling, double ITCZ, poor ENSO, etc.



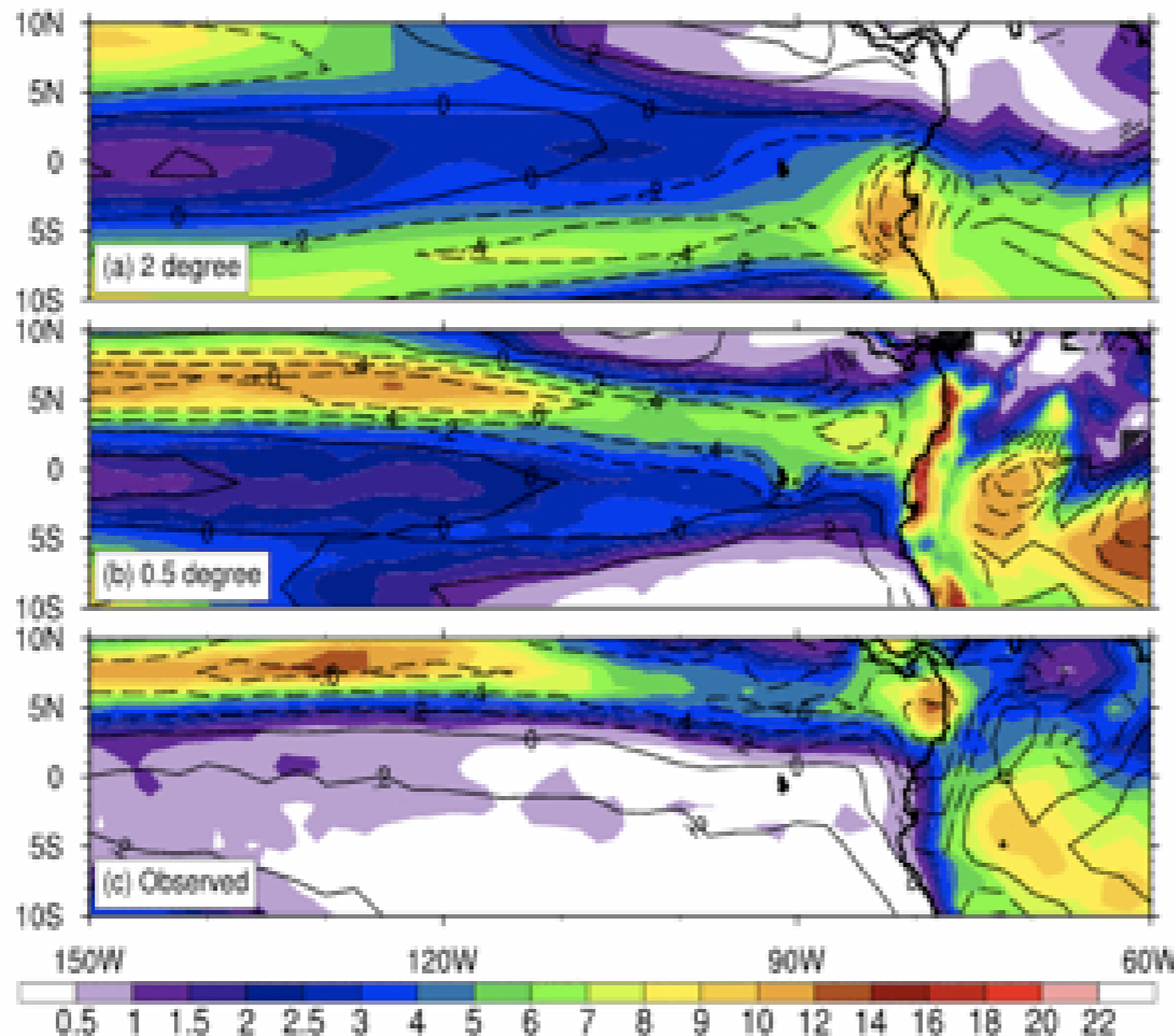
### 0.5° Atmosphere

Mean = -0.002



Credit:  
P. Gent

- Tropical biases--upwelling, double ITCZ, poor ENSO, etc.



2 Deg.  
Atm.

0.5 Deg.  
Atm.

Obs.

East Pacific DJF ITCZ precipitation (mm/day colors) and 925-hPa convergence (contours) in 2° and 0.5° CCSM 3.5

Credit:  
P. Gent

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# My Future Worries for Ocean Modeling

This talk

- Still missing physics: Multiscale interactions
- Focus on upper ocean: links to Climate and Biogeochemically active

Another day

- Energetic and Conservation consistency (enstrophy, too?)
- Internal waves and mixing
- Data assimilation and forecasting/hindcasting

# Future Opportunities in Ocean Modeling

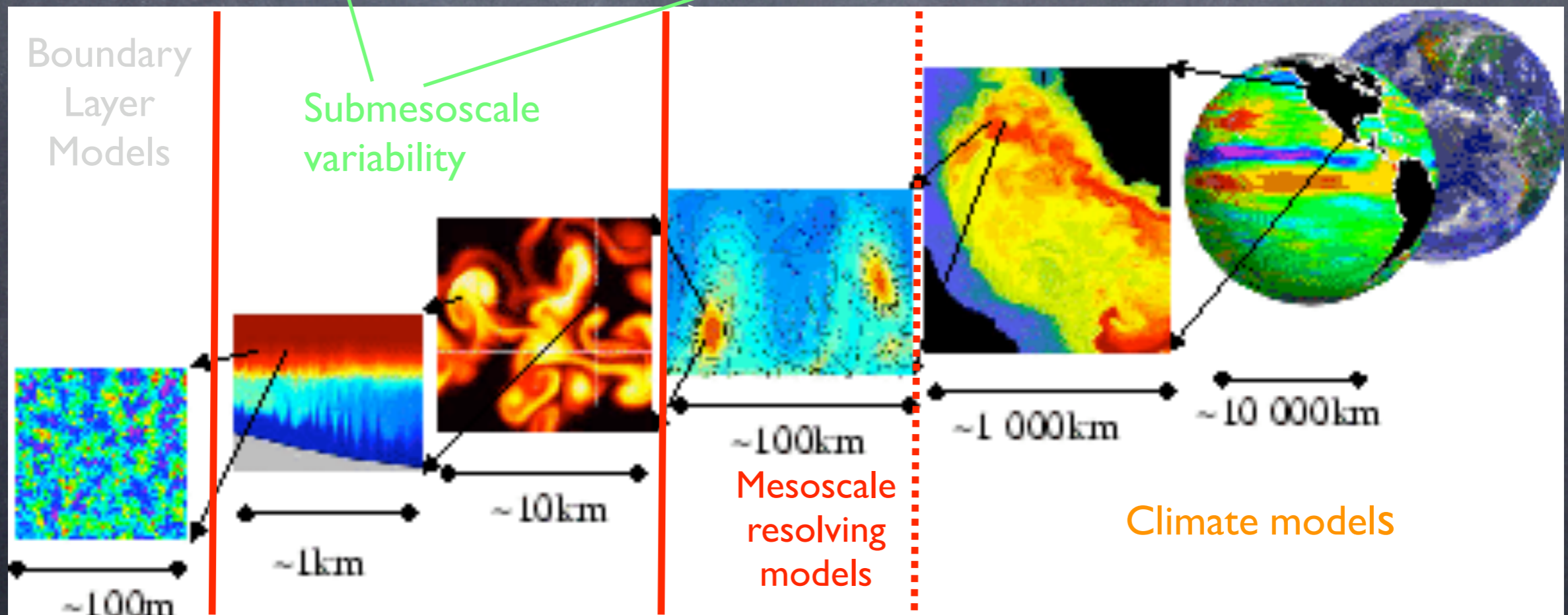
- Unfamiliar communities (at least to global modelers) of observationalists and theorists have knowledge and skills needed
- 'Climate sensitivity' of new physics--unknown
- Results will outlast a few generations of computer advances (mesoscale rich IPCC-class, perhaps 5yr away, submesoscale rich IPCC-class more than 50yr away)
- Will be fun!

# Two Examples

- **Submesoscale Eddies**--with Ferrari, Hallberg, Boccaletti, Flierl, CPT team.
- **Langmuir Mixing**--with **Adrean Webb**, Large, Peacock, Chini, Julien, Knobloch

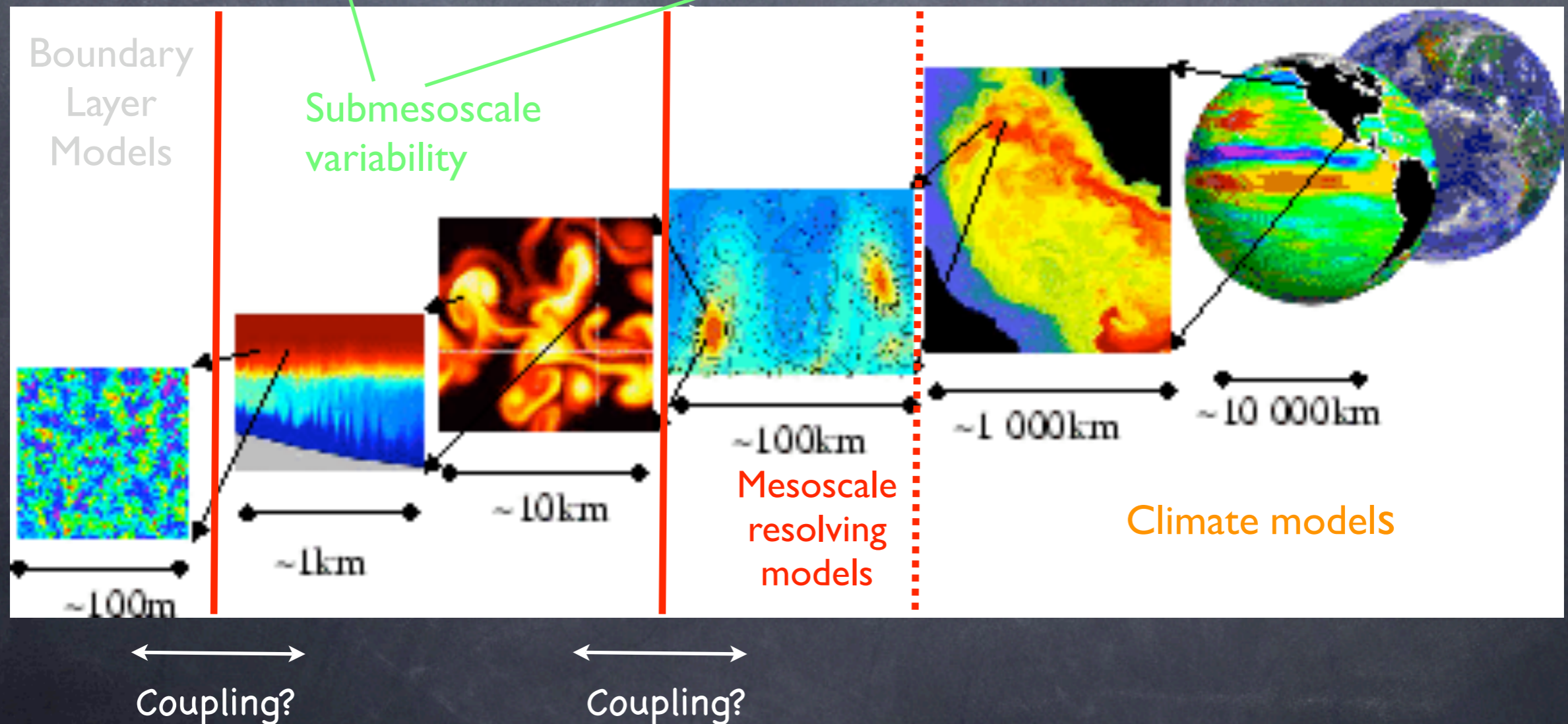
# Upper Ocean in Climate Models

- Large-scale ocean circulation (100 - 10,000 km) => resolved
- Mesoscale variability (10 - 100 km) => resolved or parameterized
- Submesoscale variability (100 m - 10 km) => ignored
- Turbulent mixing (10 cm - 100 m) => parameterized



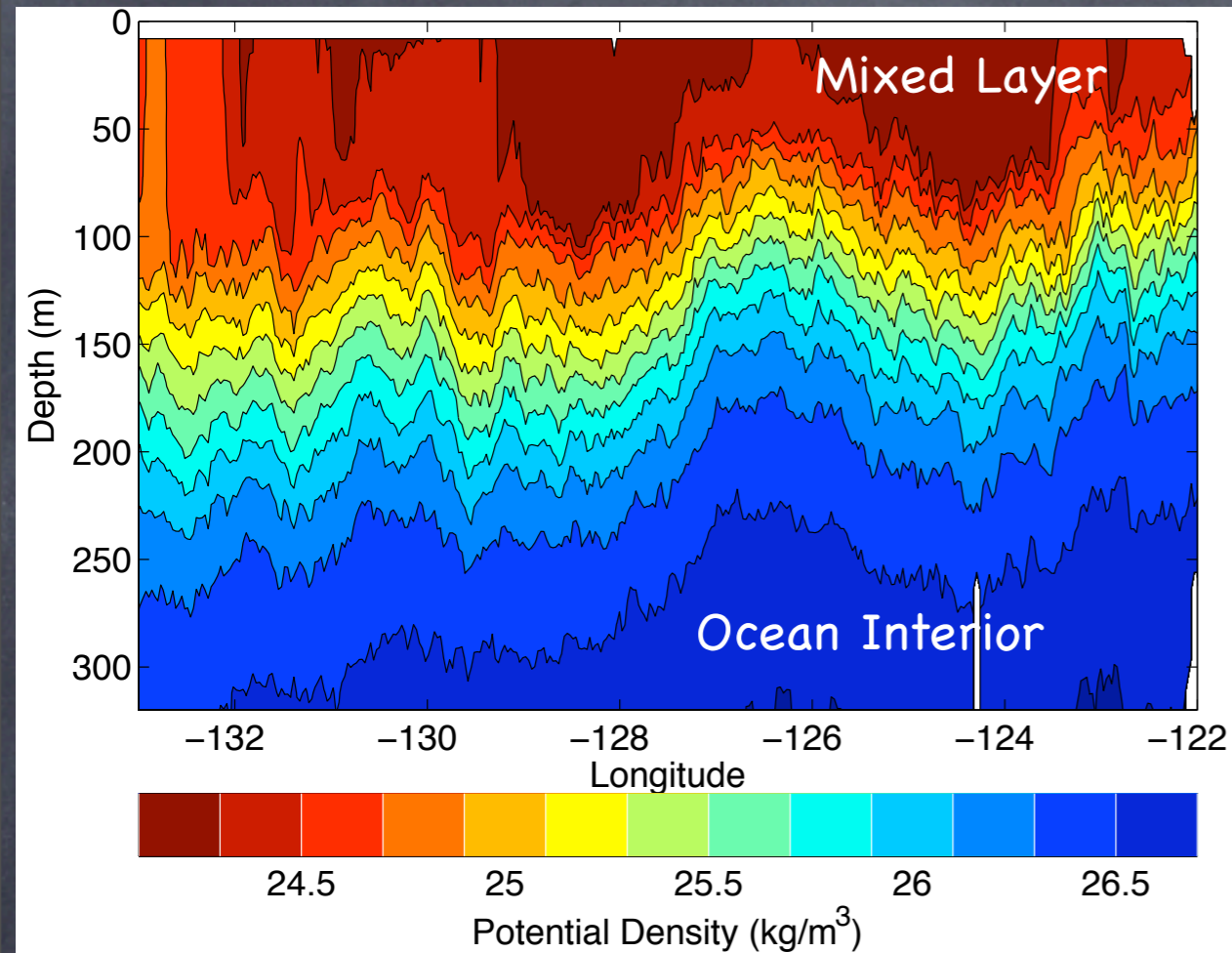
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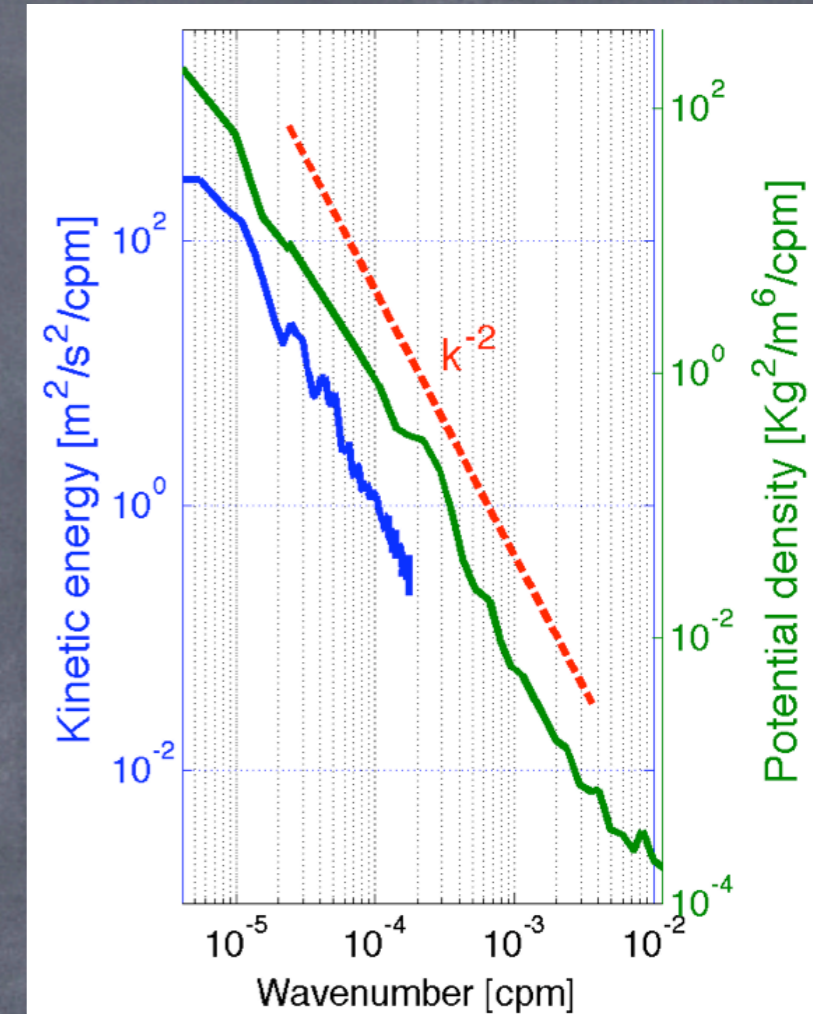


# Upper Ocean: Mixed Layer



Pot'l Density measured by a Seasoar along a straight section from (32.5N, 122W) to (35N, 132W) between the CA current and the subtropical gyre.

(as in Ferrari & Rudnick, 2000)



The mixed layer is not **TOTALLY** mixed.  
Fronts are common.

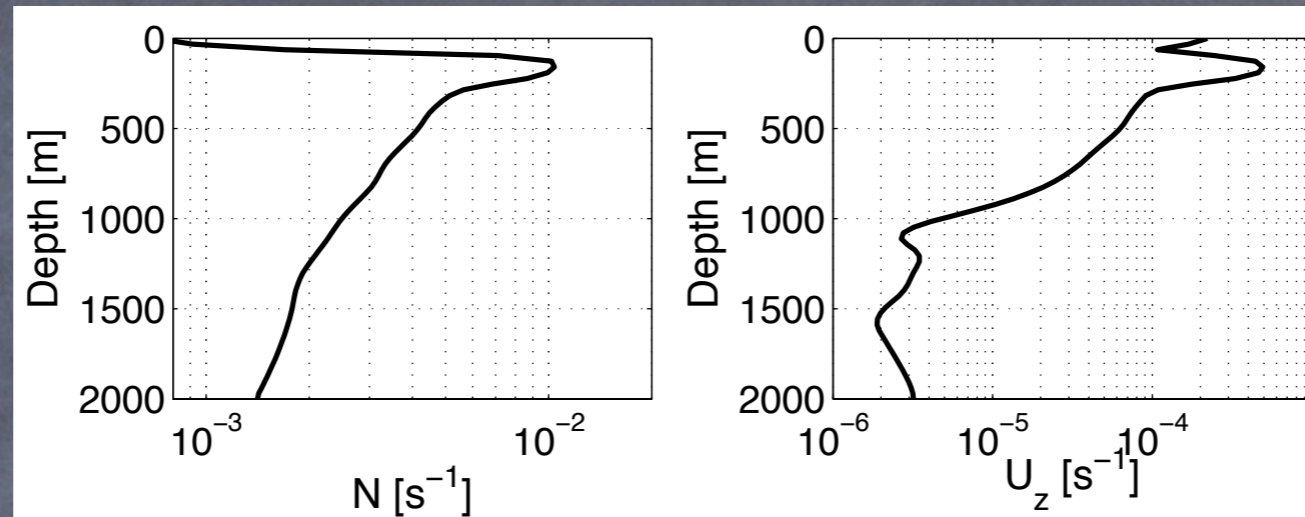
This weakly-stratified, fairly rapidly mixed region is active at the submesoscale...

# Submesoscale Features

- $Ro=O(1)$ ,  $Ri=O(1)$  (Post-geostrophic adjustment of fronts). **Multiscale-multiphysics.**
- **Frontogenesis:** McWilliams et al., Klein et al.
- **Wind, Front Effects** (Nonlin. Ekman, Wind-driven Overturn): Thomas et al.
- **Eddies and Instabilities?** Fox-Kemper et al., Molemaker et al.
- **Wave Effects?** McWilliams, Sullivan, Fox-Kemper
- **Climate Significance:** The Ocean and Atmosphere 'Talk' through the Mixed Layer, and Phytoplankton live there

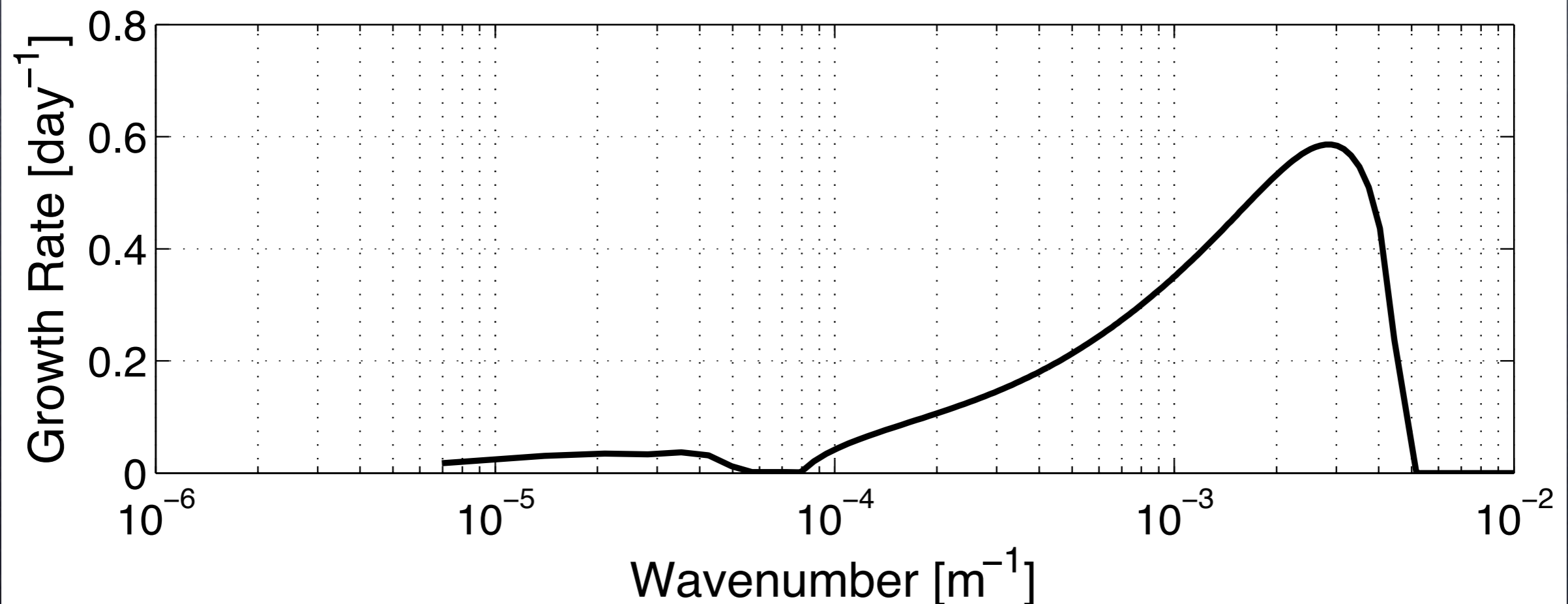
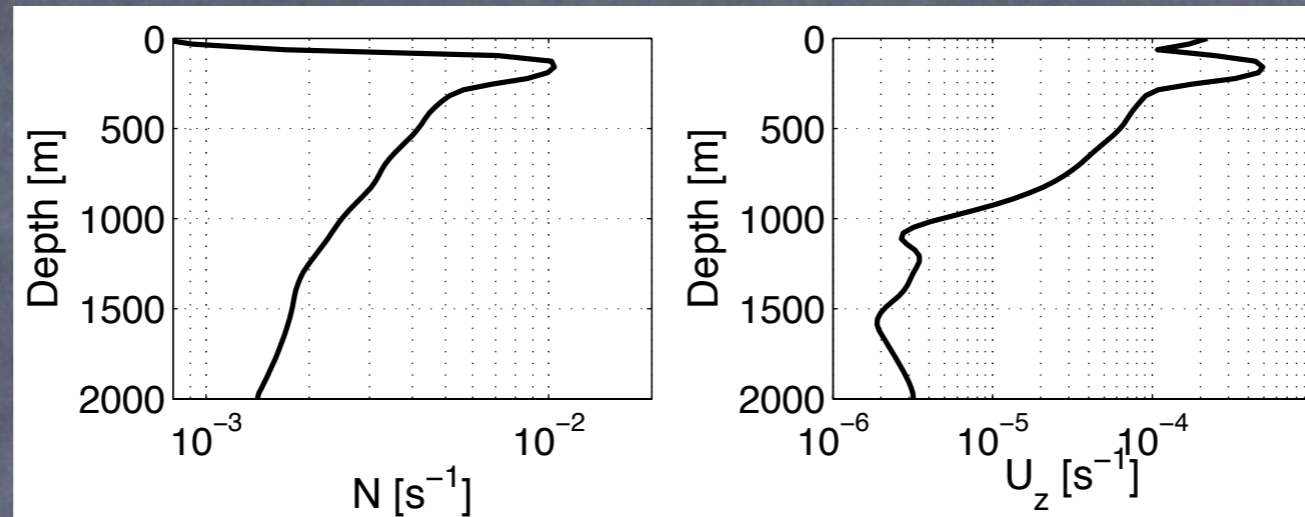
# Typical Stratification Permits Two Types of Baroclinic Instability:

**Mesoscale** and **SubMesoscale Eddies** (Boccaletti et al., 2006)



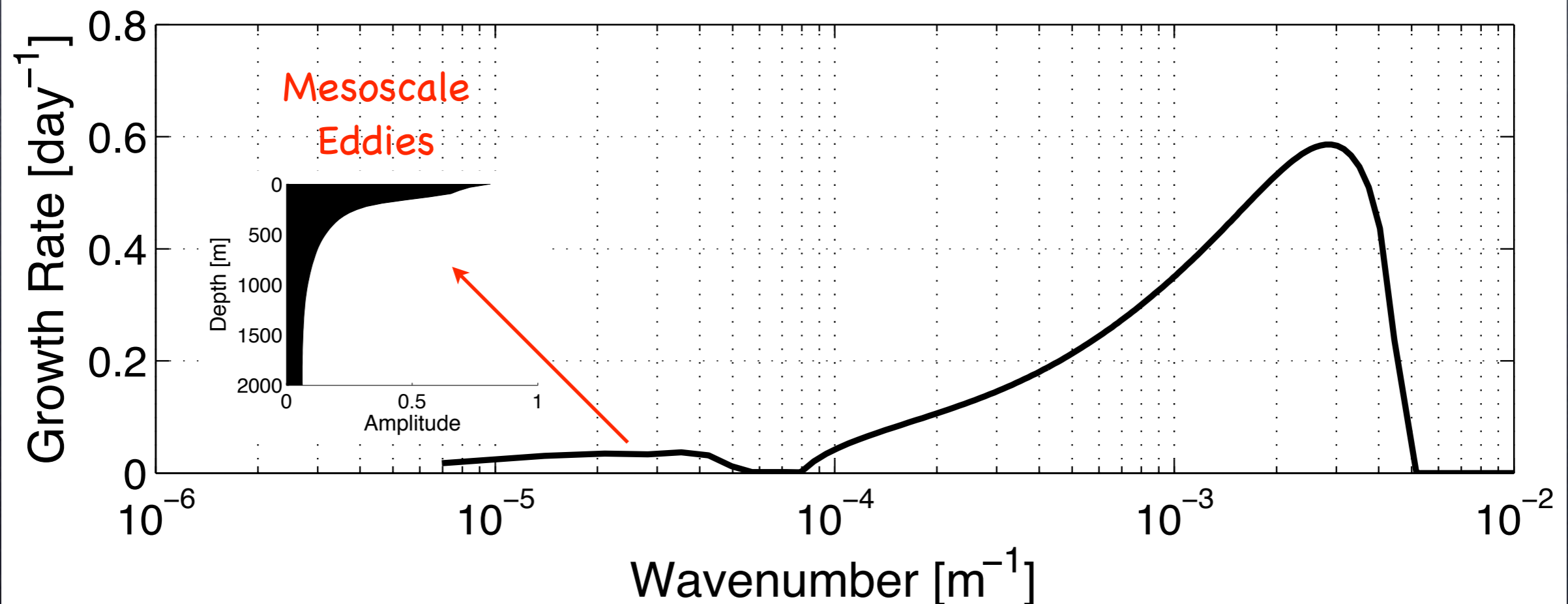
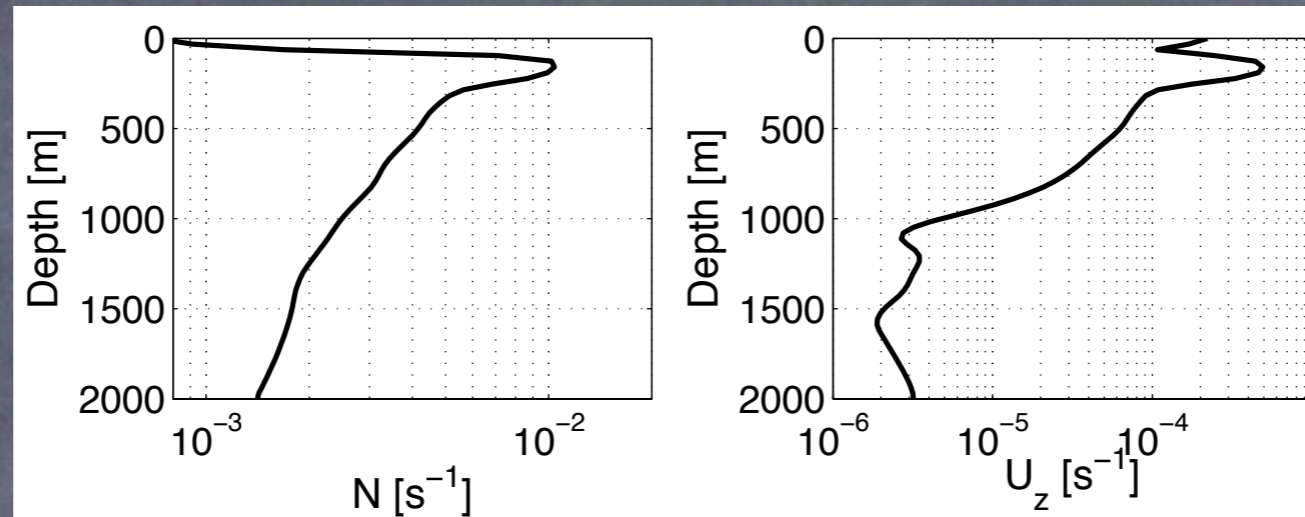
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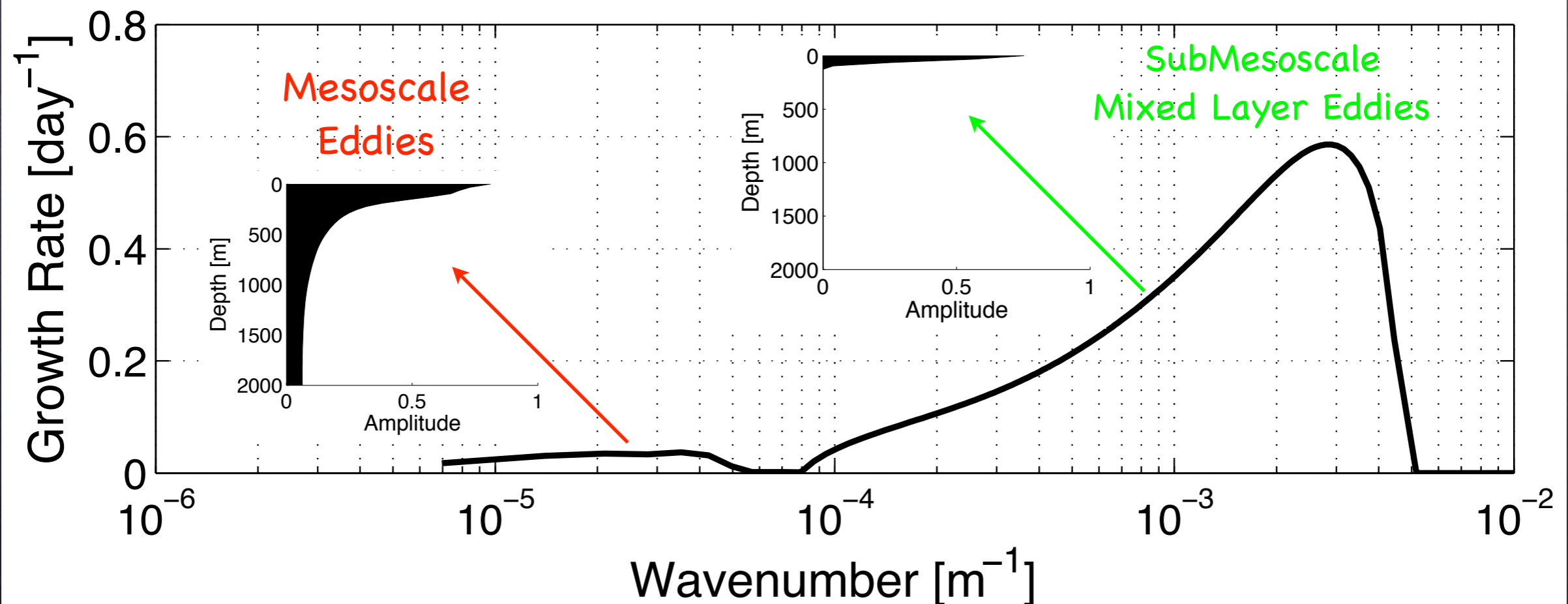
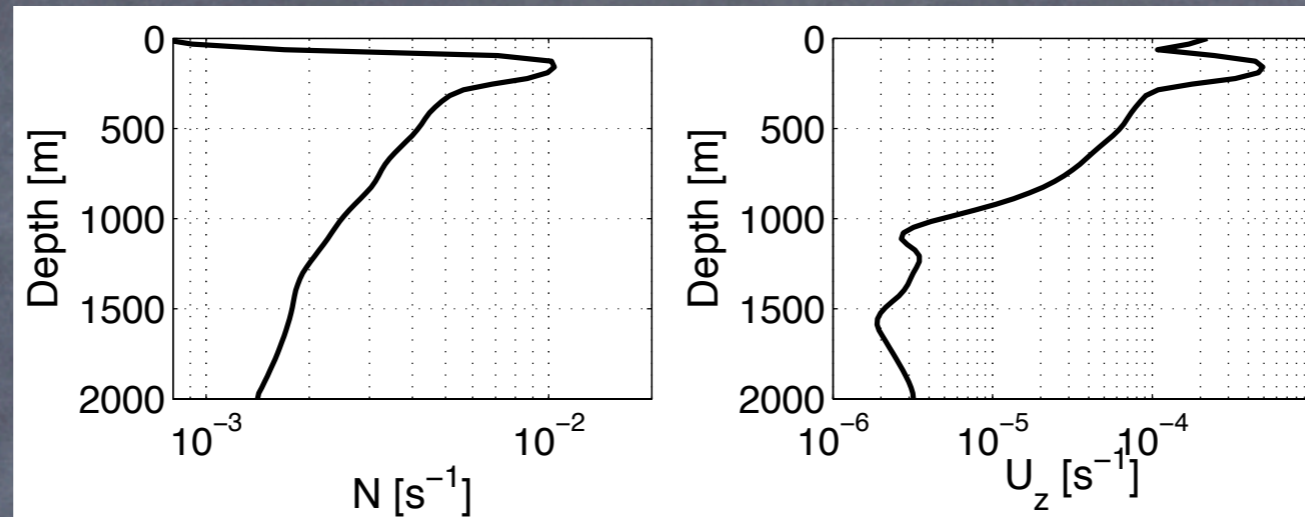
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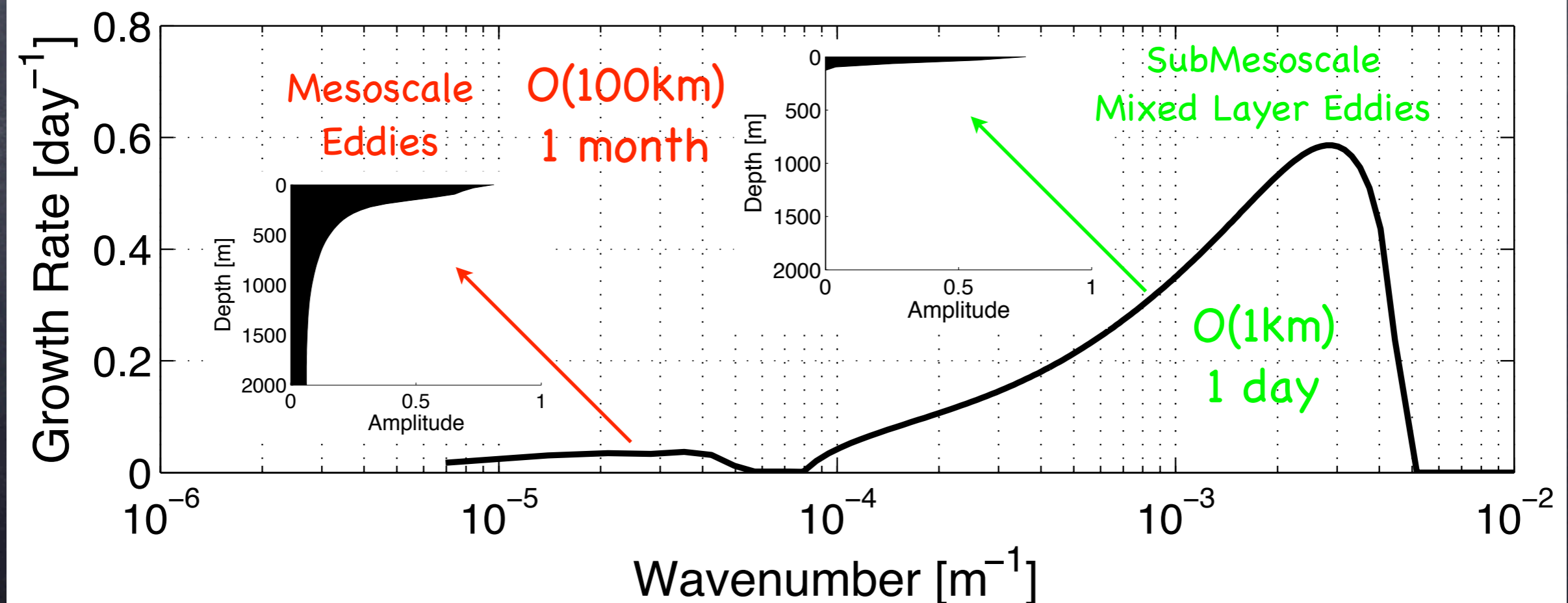
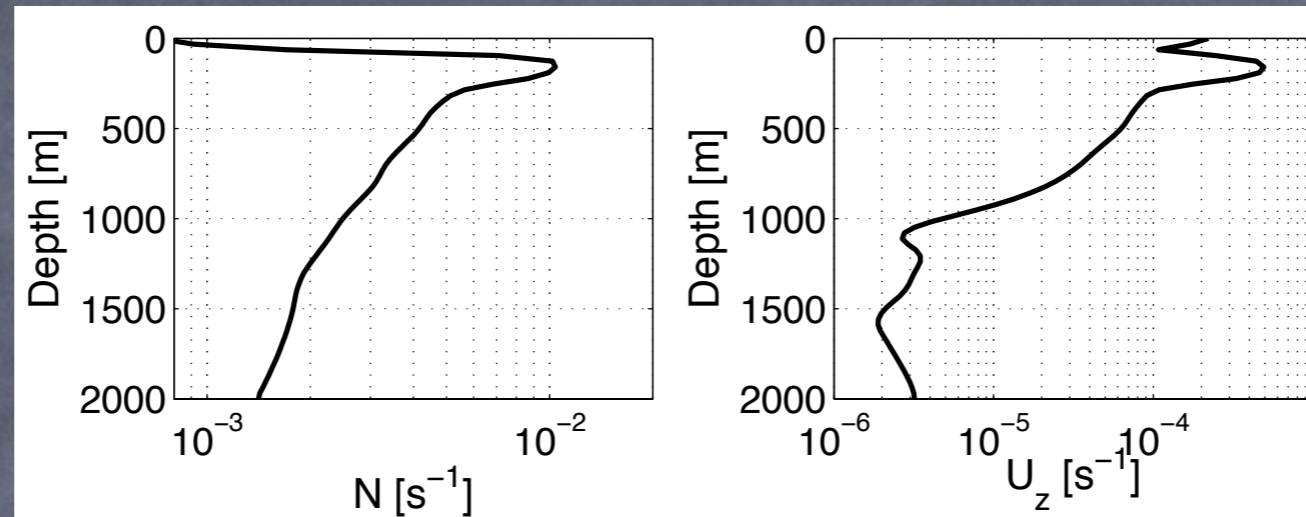
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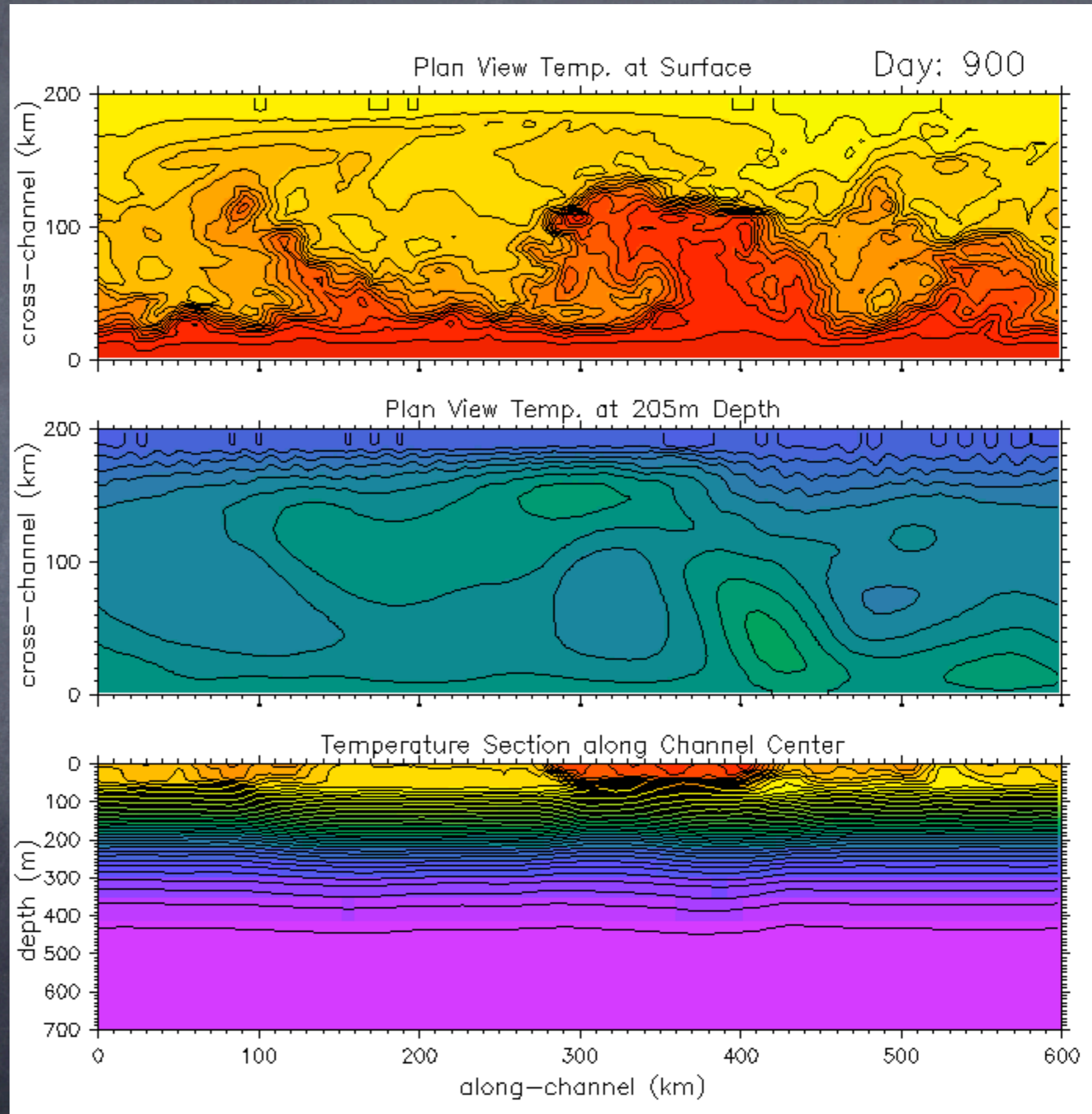
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Mesoscale and  
SubMesoscale  
are  
Coupled  
Together:

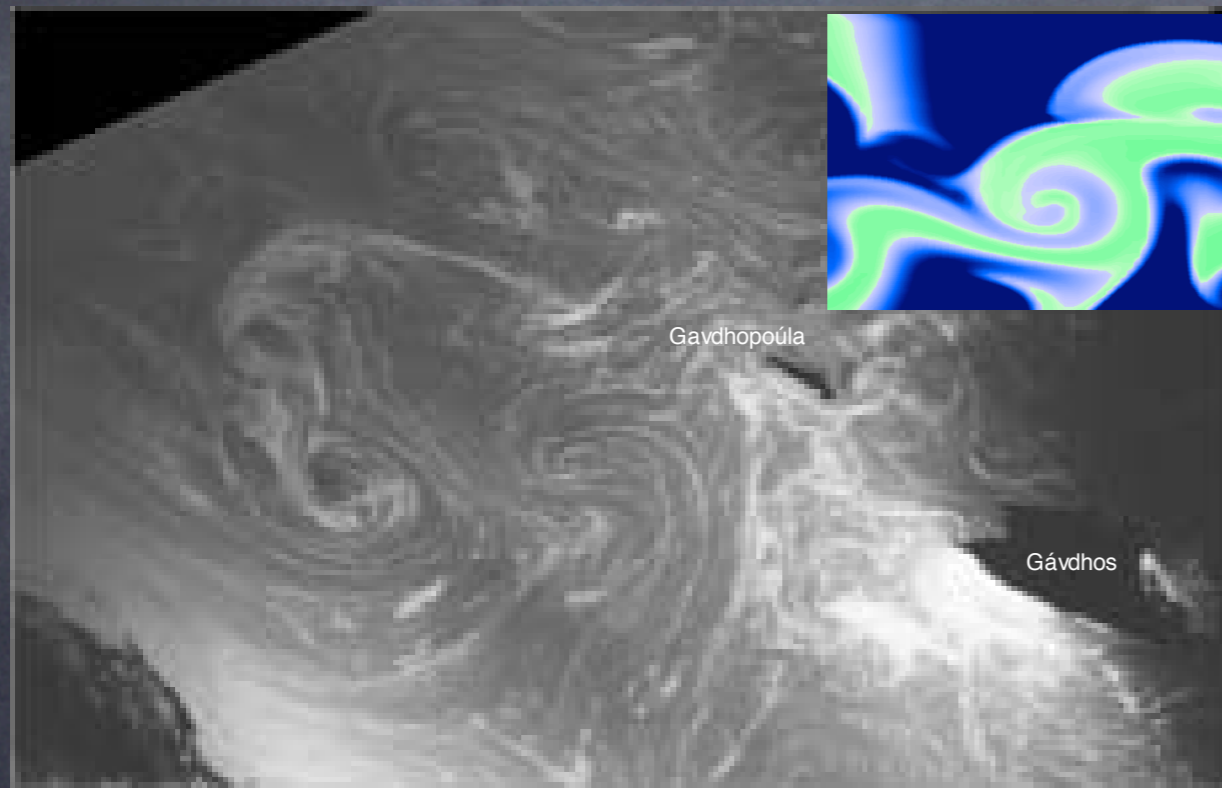
ML Fronts are  
formed by  
Mesoscale  
Straining.

Submesoscale  
eddies remove  
PE from those  
fronts.



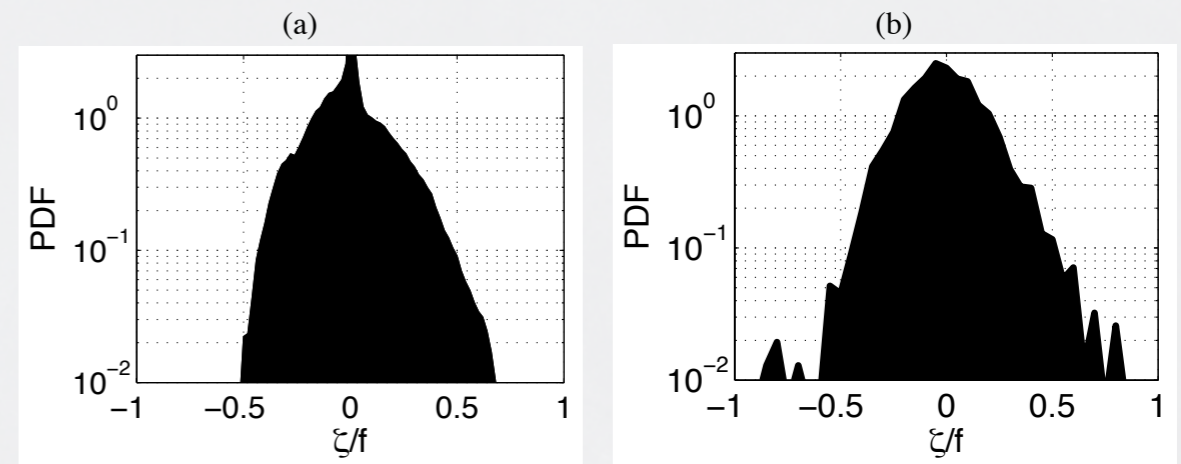


# Observed: Strongest Surface Eddies= Spirals on the Sea?



Munk, 01

**Figure 1.** A pair of interconnected spirals in the Mediterranean Sea south of Crete. This vortex pair has a clearly visible stagnation point between the two spirals, the cores of which are aligned with the preconditioning wind field. 7 October 1984.



**Figure 12:** Probability density function of relative vorticity divided by Coriolis parameter. (a) Results from the numerical simulation of a slumping horizontal density front. ( $z > 100$  only to exclude bottom Ekman layer.) The PDF is estimated using surface velocity measurements at day 25 (see also Fig. 11). A positive skewness appears as soon as the baroclinic instability enters in the nonlinear stage, and it continues to grow. Note that the peak at  $\zeta/f = 0$  is due to the model's initial resting condition; that fluid has not yet been contacted by the MLI. (b) Results from ADCP measurements in the North Pacific. The PDF is calculated in bins of width 0.02.

Vertical fluxes are Submesoscale  
and tend to restratify

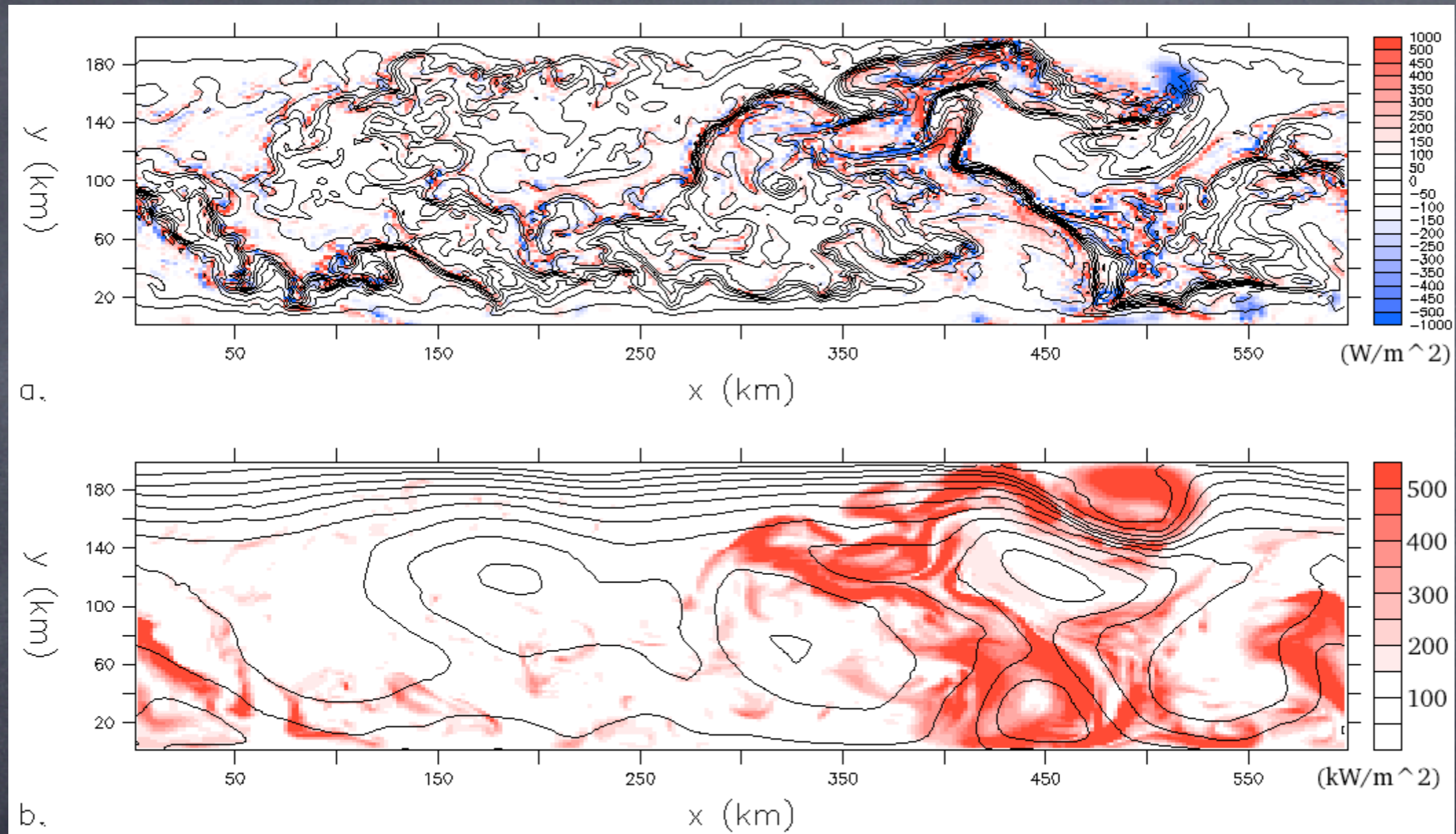
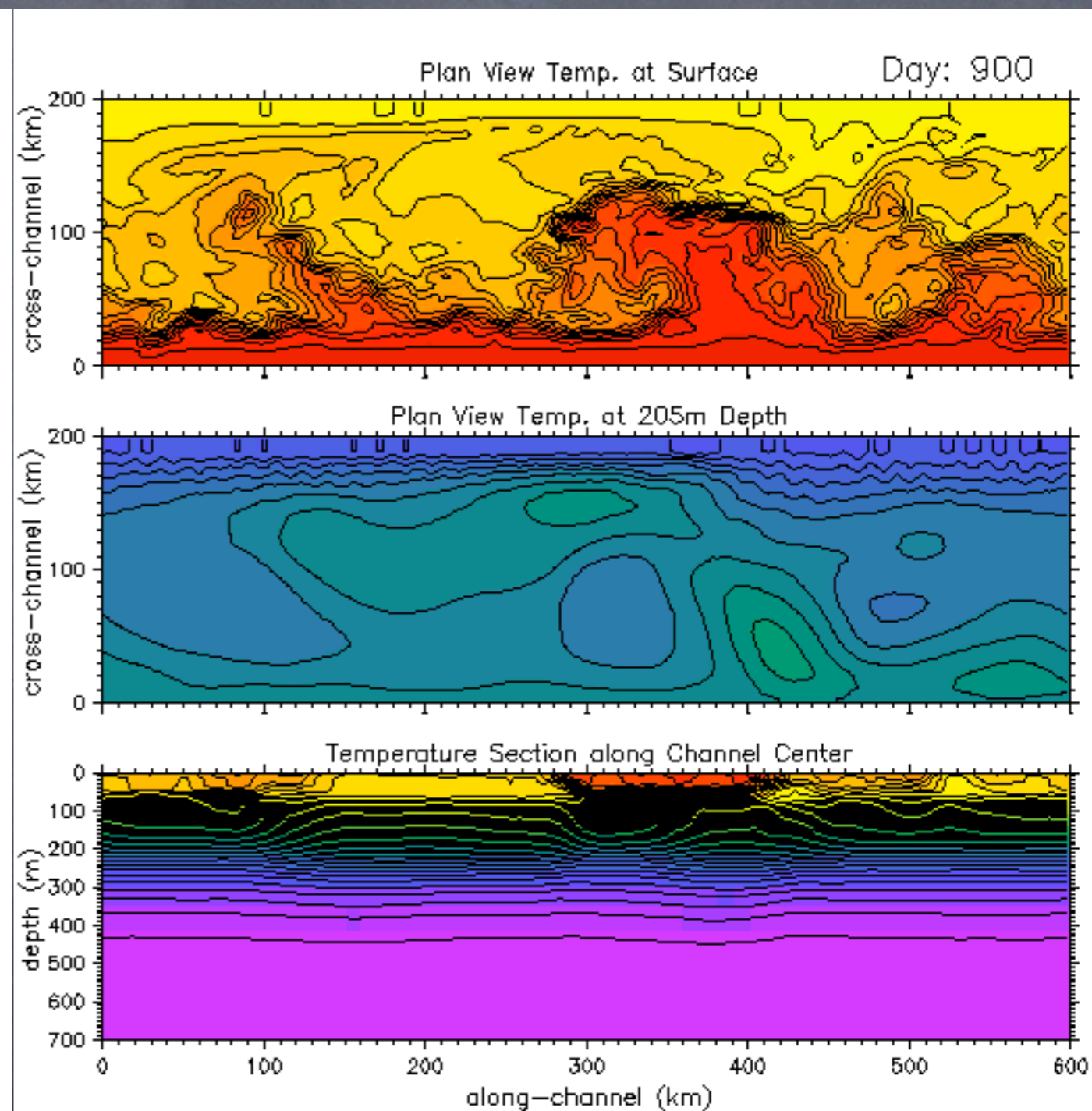
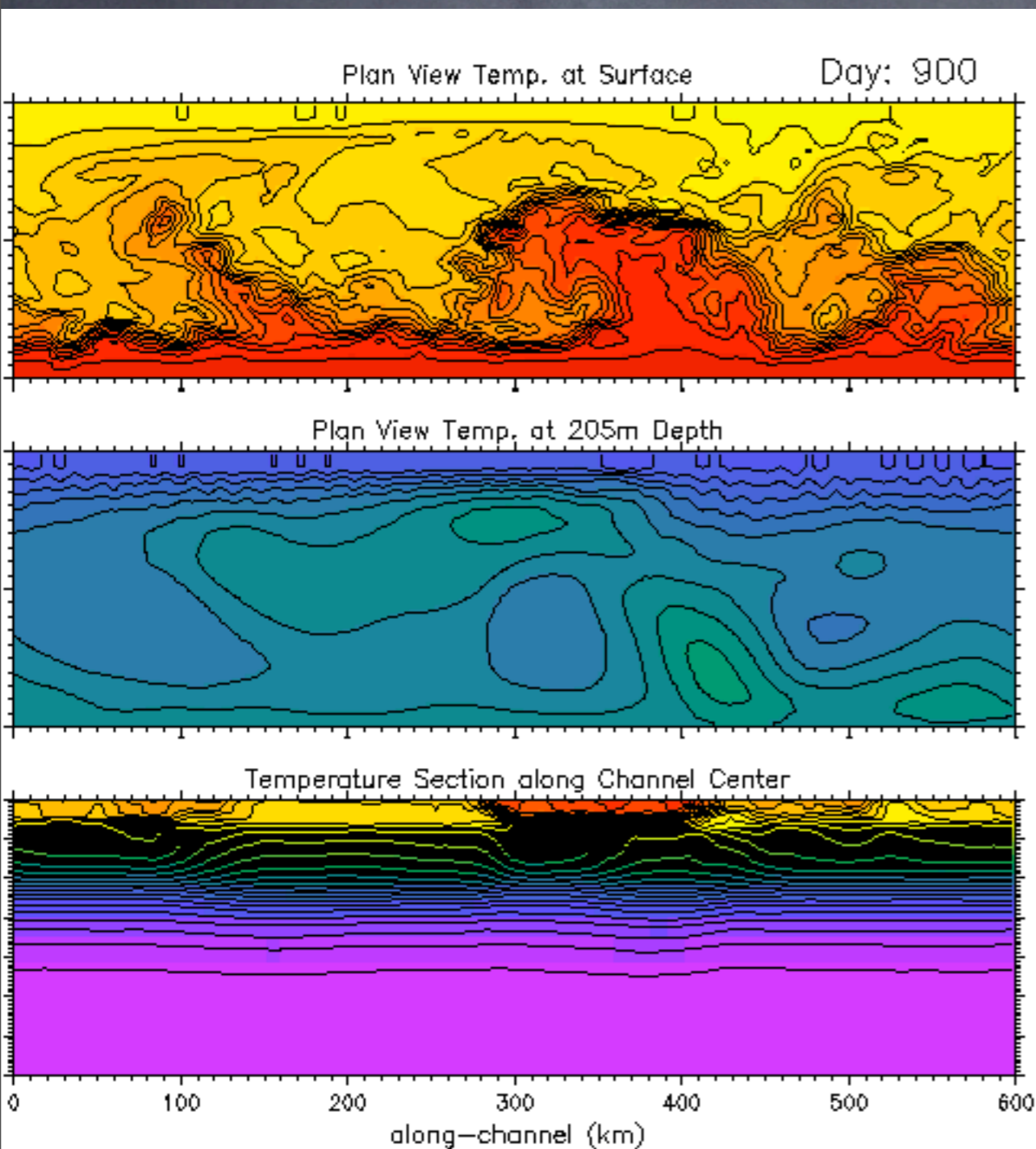


FIGURE 1: Contours of temperature at the a) surface and b) below the mixed layer base in a simulation with both mesoscale eddies and MLEs (0.2°C contour intervals). Shading indicates the value at the depth where  $\overline{w'b'}$  (upper panel) and  $|\overline{\mathbf{u}'_H b'}|$  (lower panel) take the largest magnitude.

Horizontal fluxes are Mesoscale  
and tend to stir

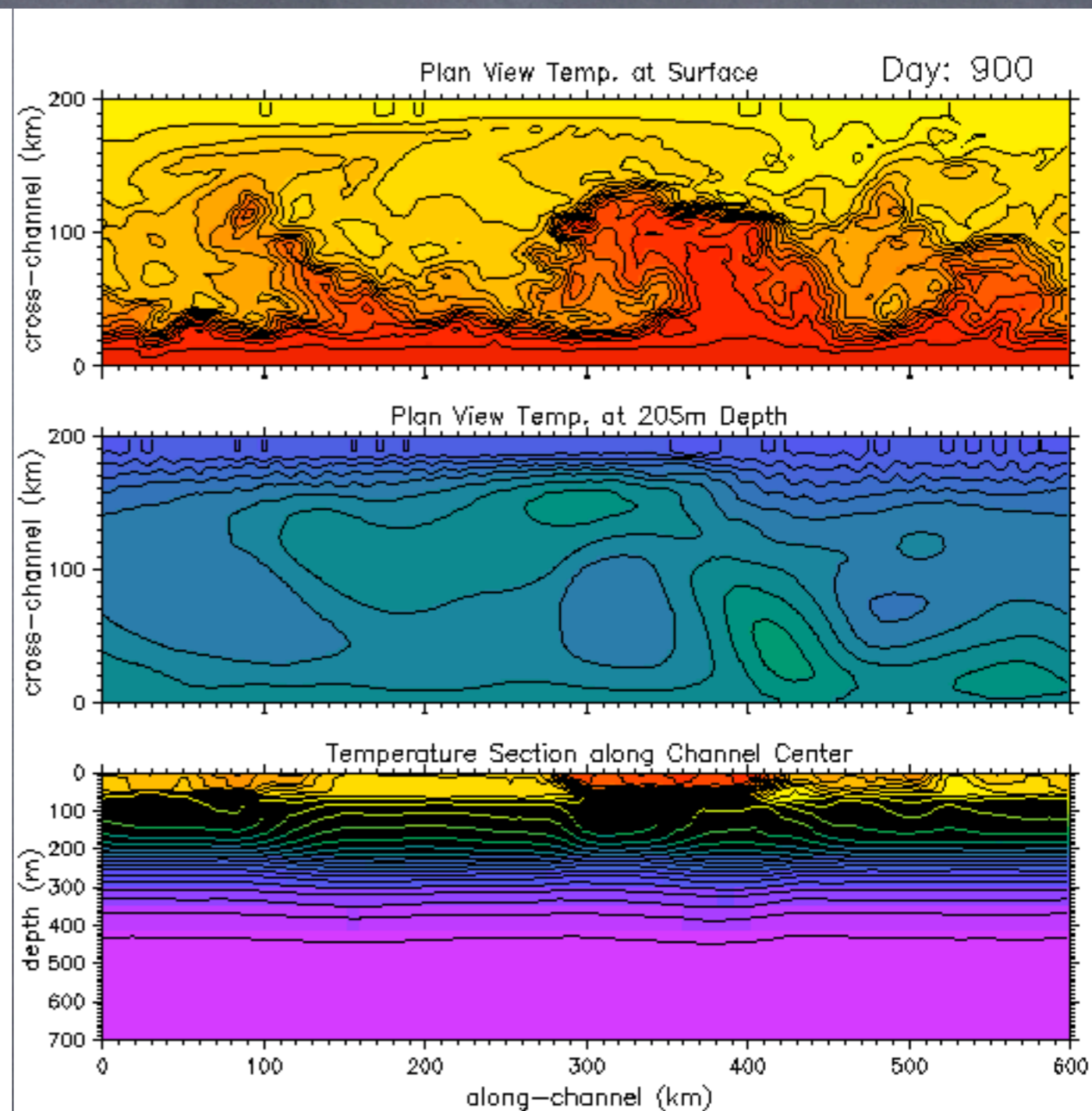
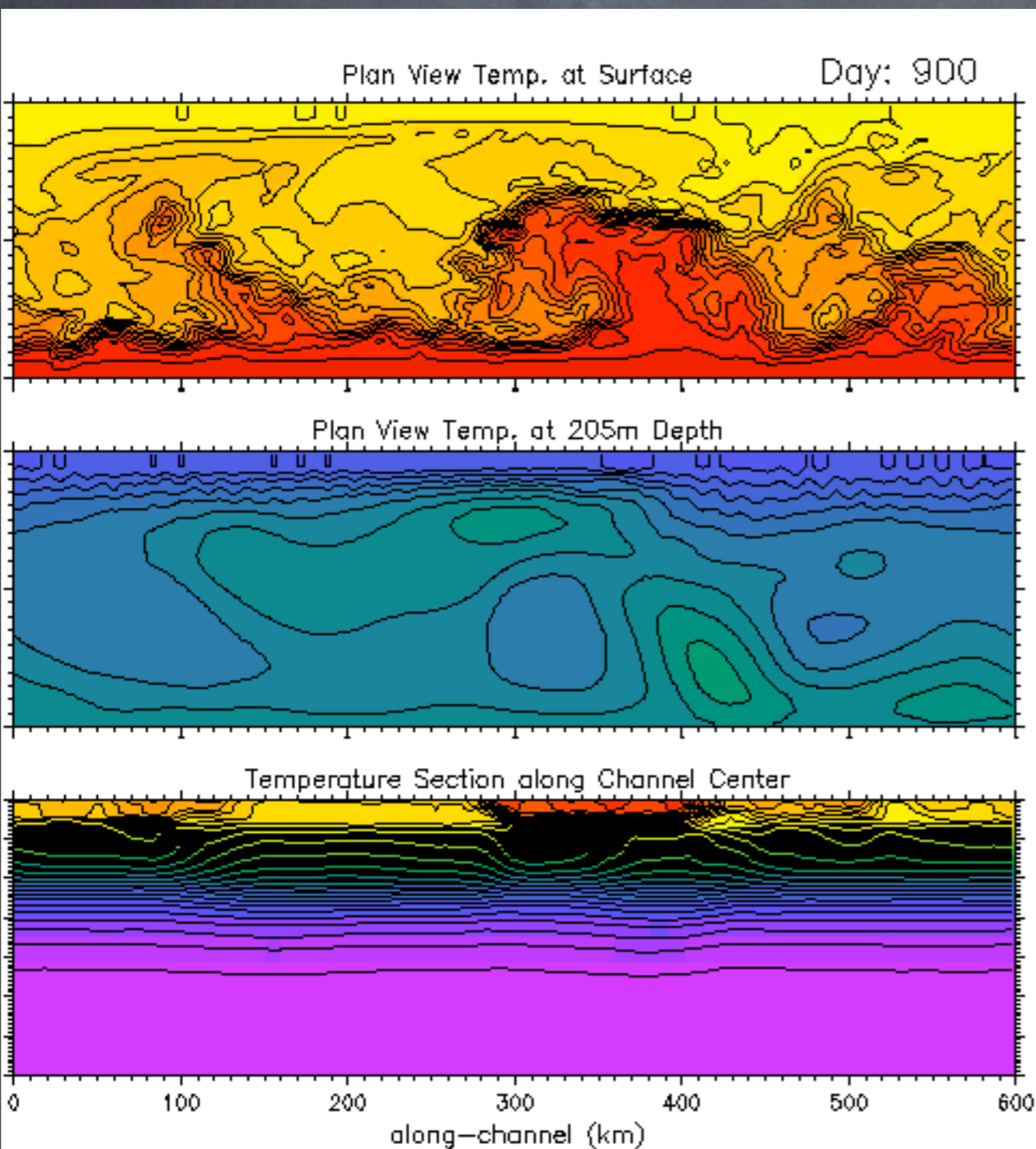
# Remixing the Mixed Layer Counts!

The vertical buoyancy flux in the ML ( $\langle w'b' \rangle$ ) without diurnal cycle is **not less** than with cycle (ML)

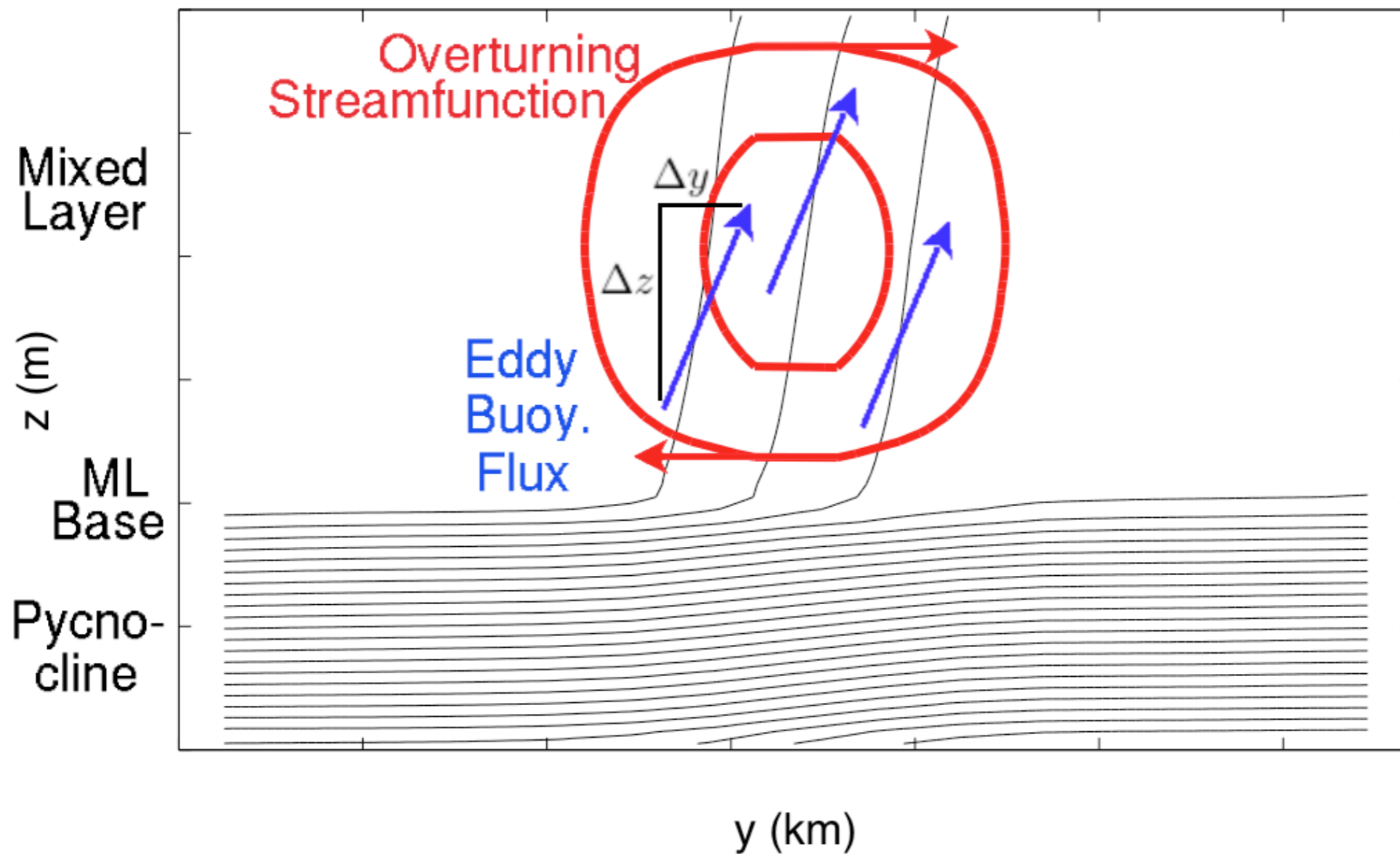


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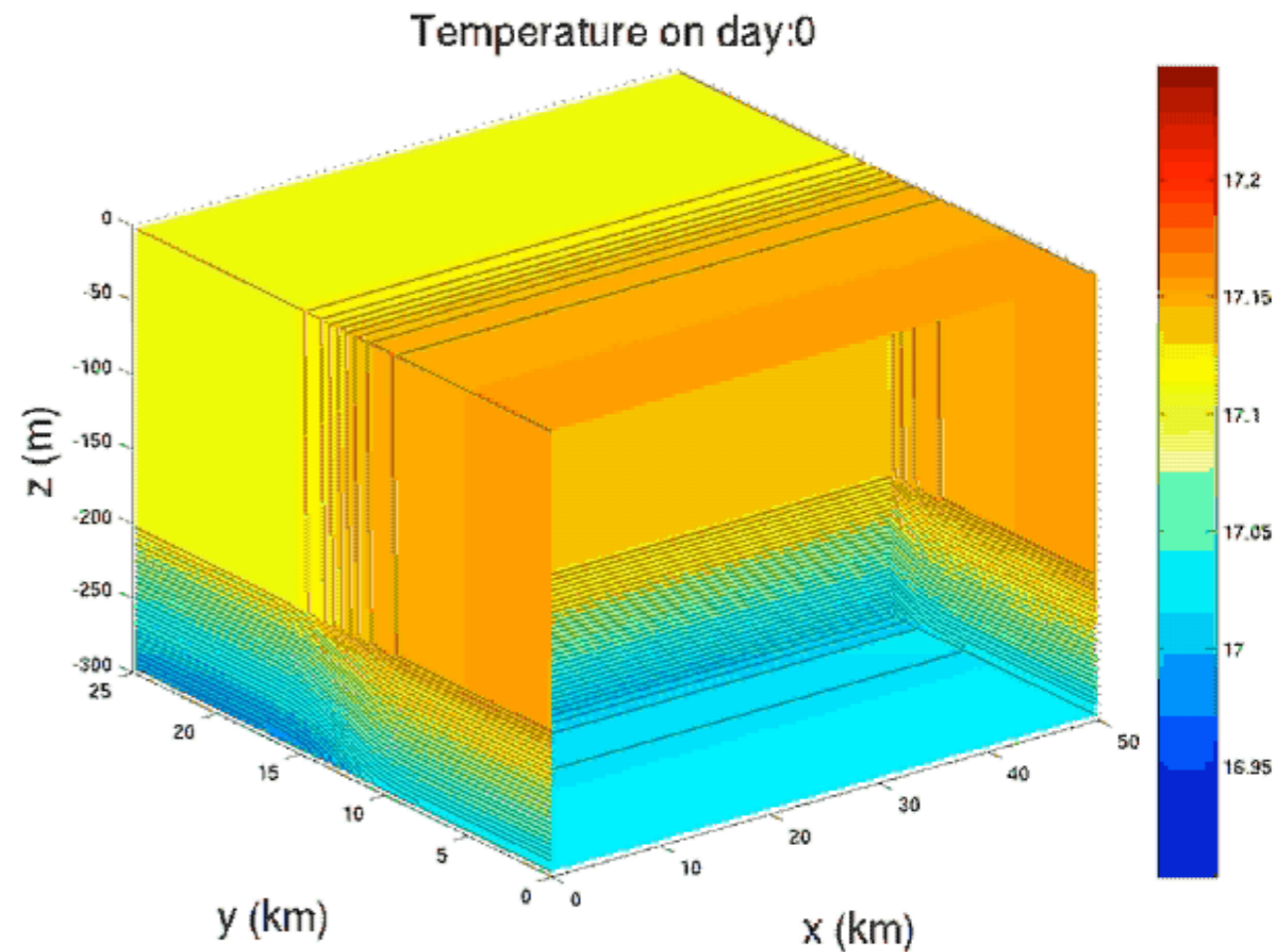
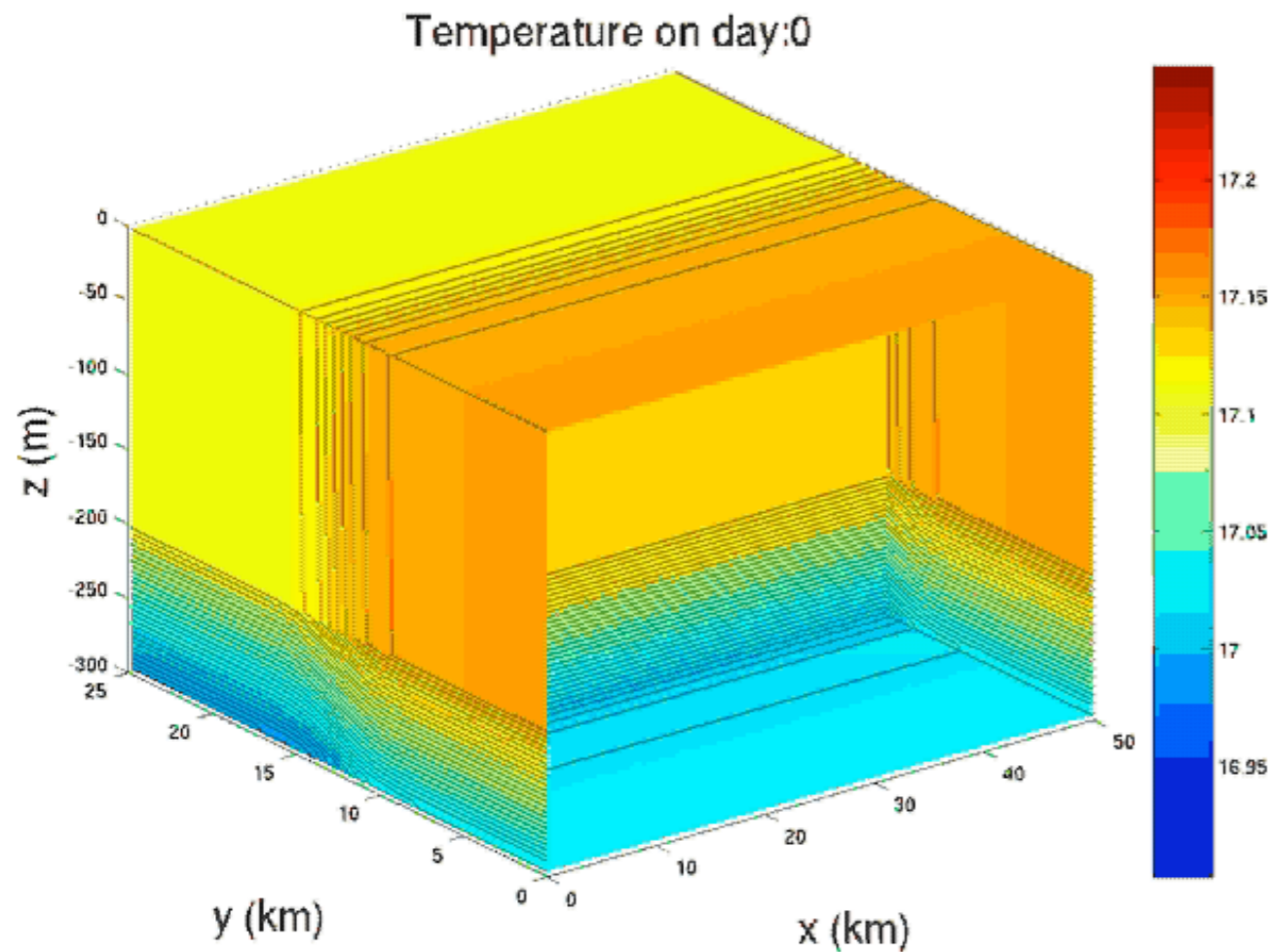
The vertical buoyancy flux in the ML ( $\langle w'b' \rangle$ ) without diurnal cycle is **4x less** than with cycle (ML)



# Overturning Schematic



# Prototype: Mixed Layer Front Adjustment

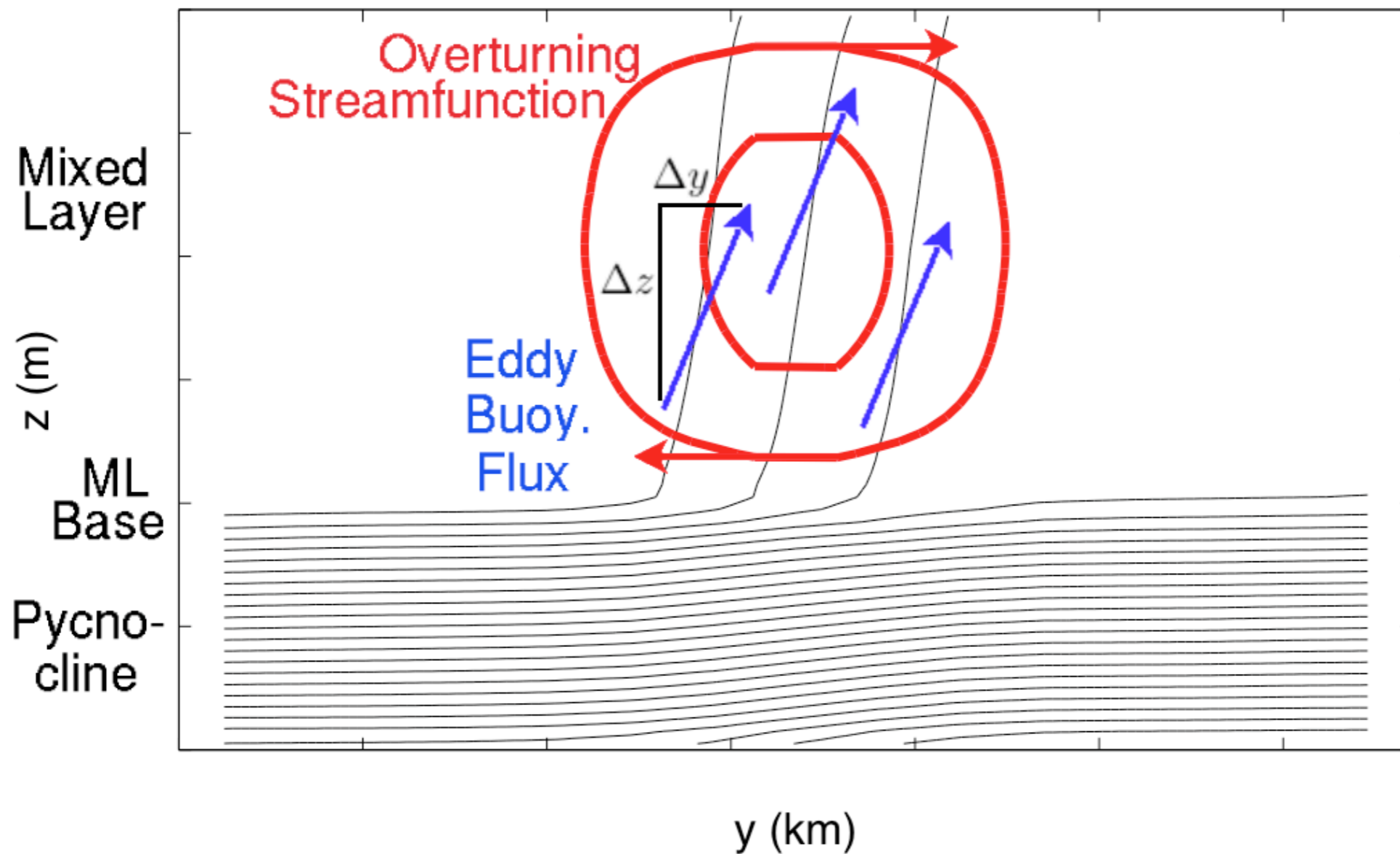


Simple Spindown

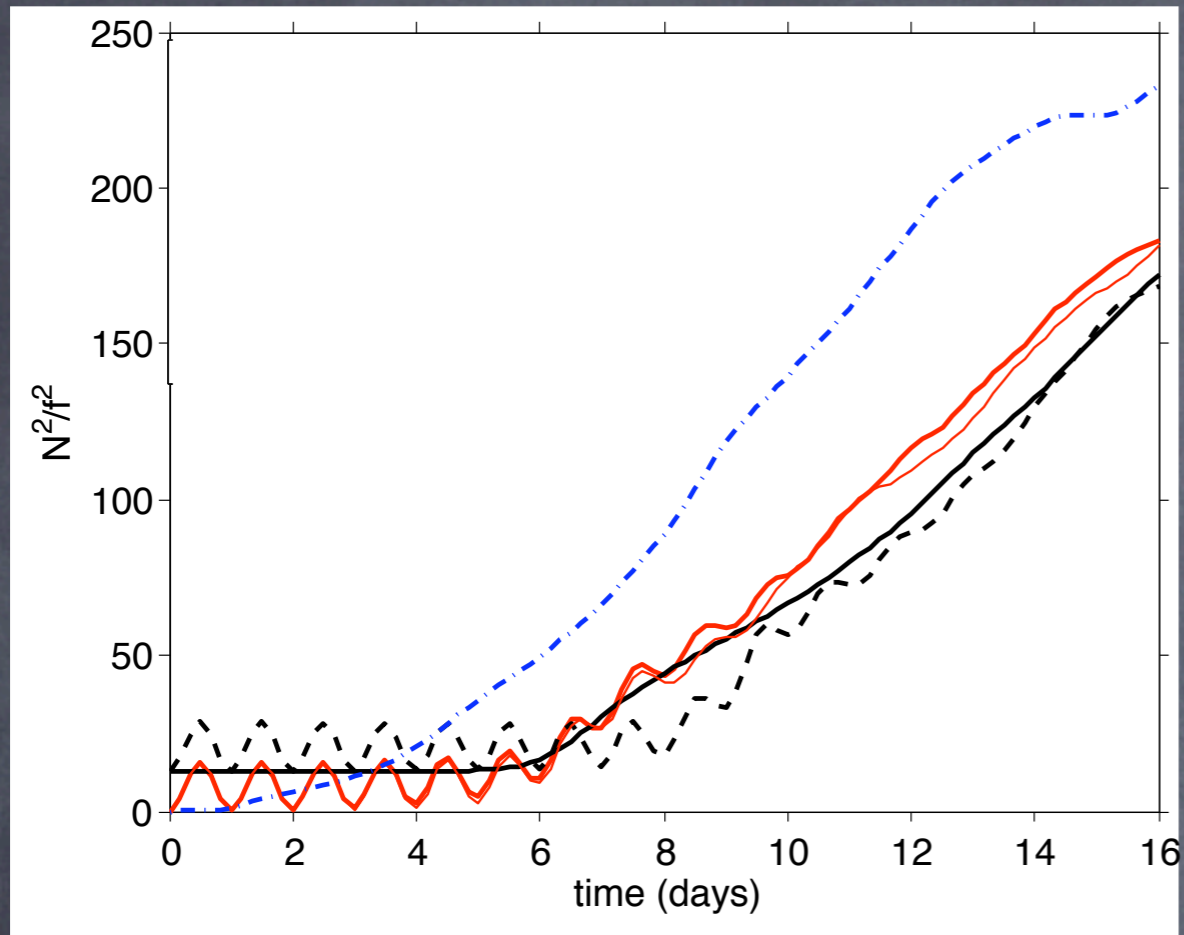
Plus, Diurnal Cycle  
and KPP

Note: initial geostrophic adjustment overwhelmed by eddy restratification

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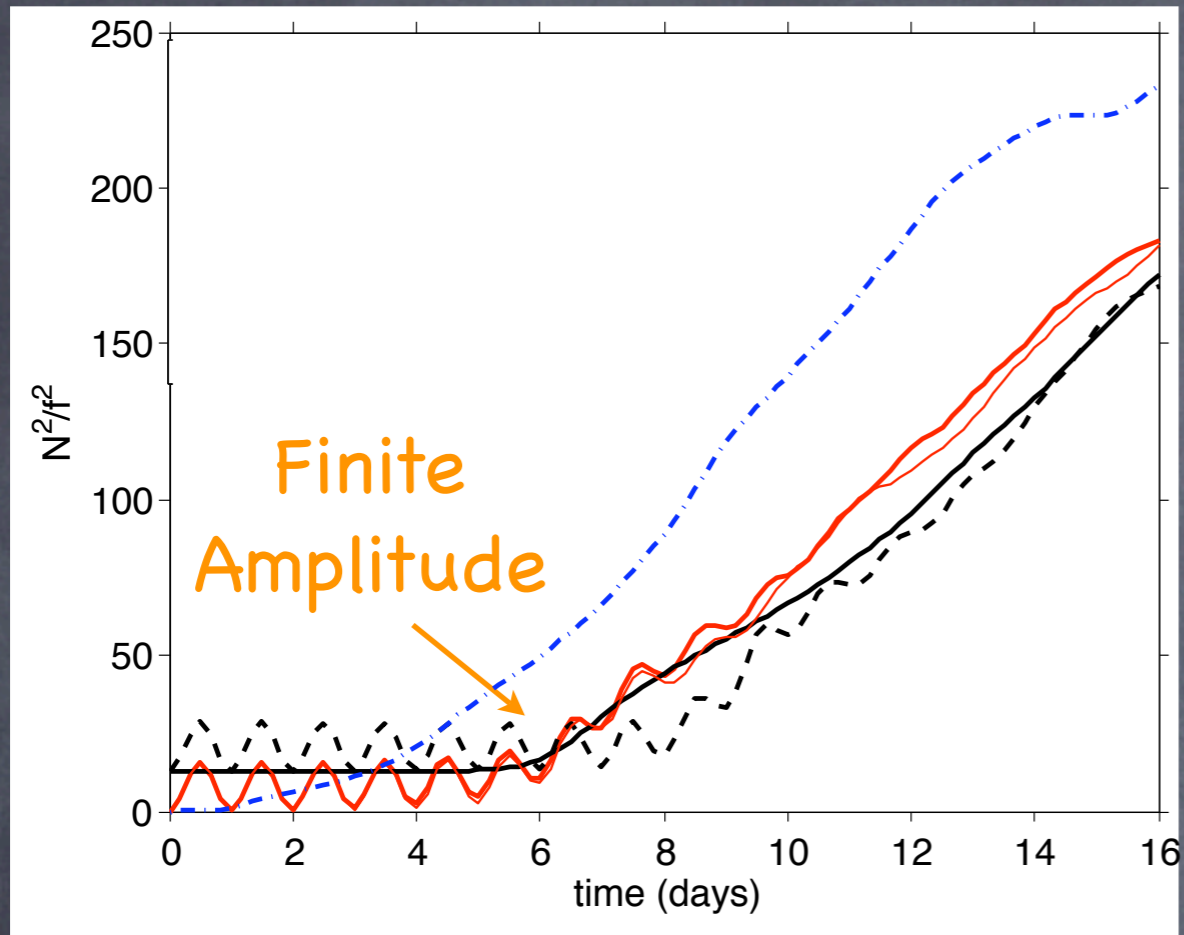


# Parameterization of Finite Amp. Eddies: Ingredients

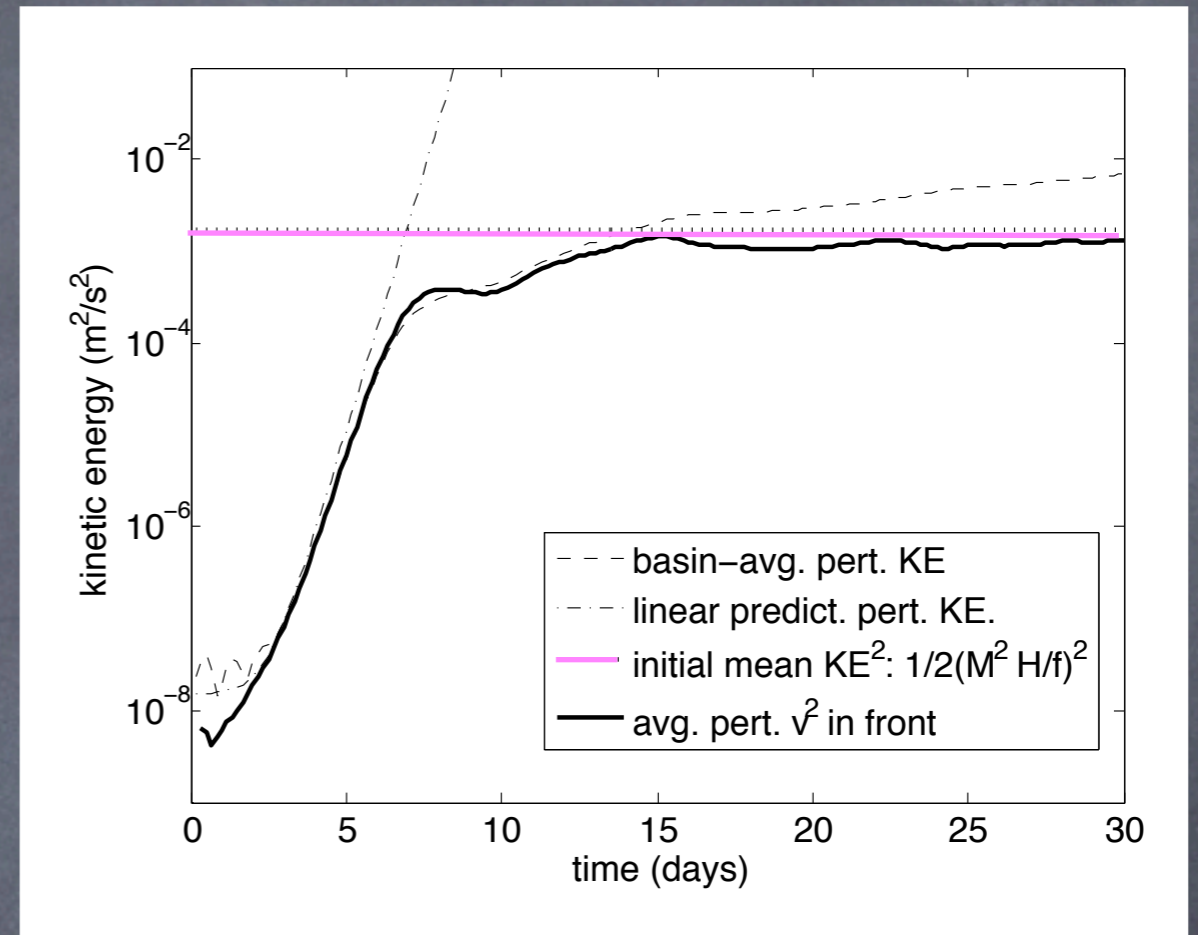
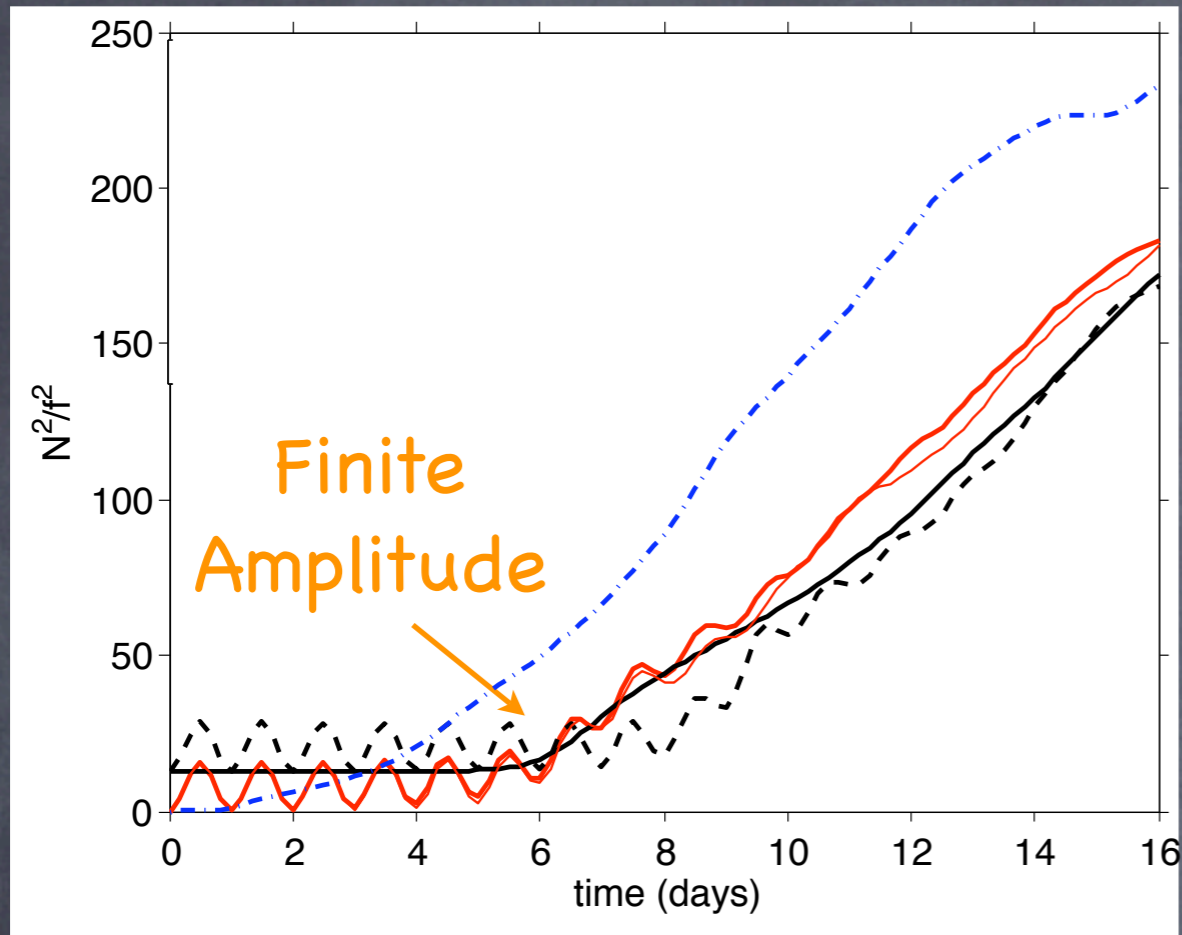




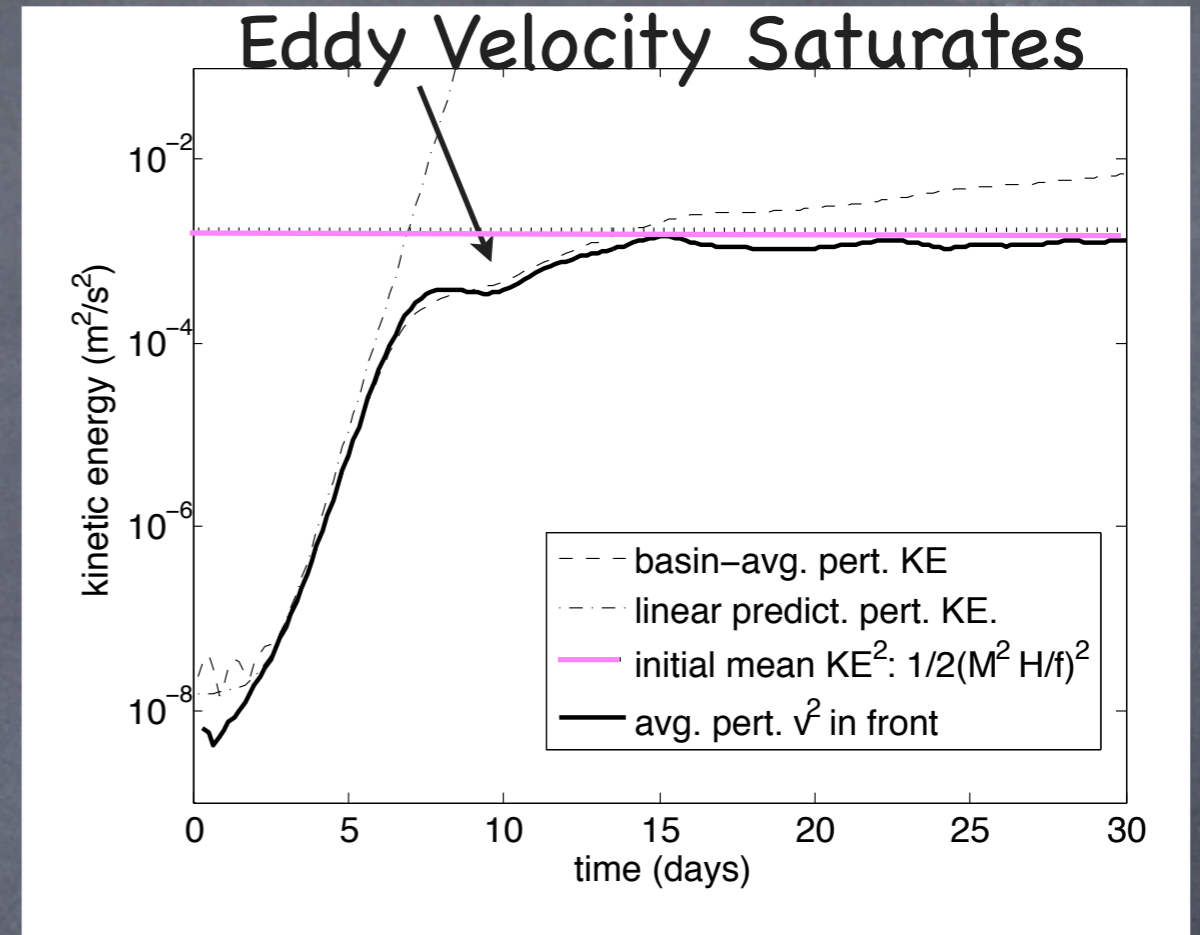
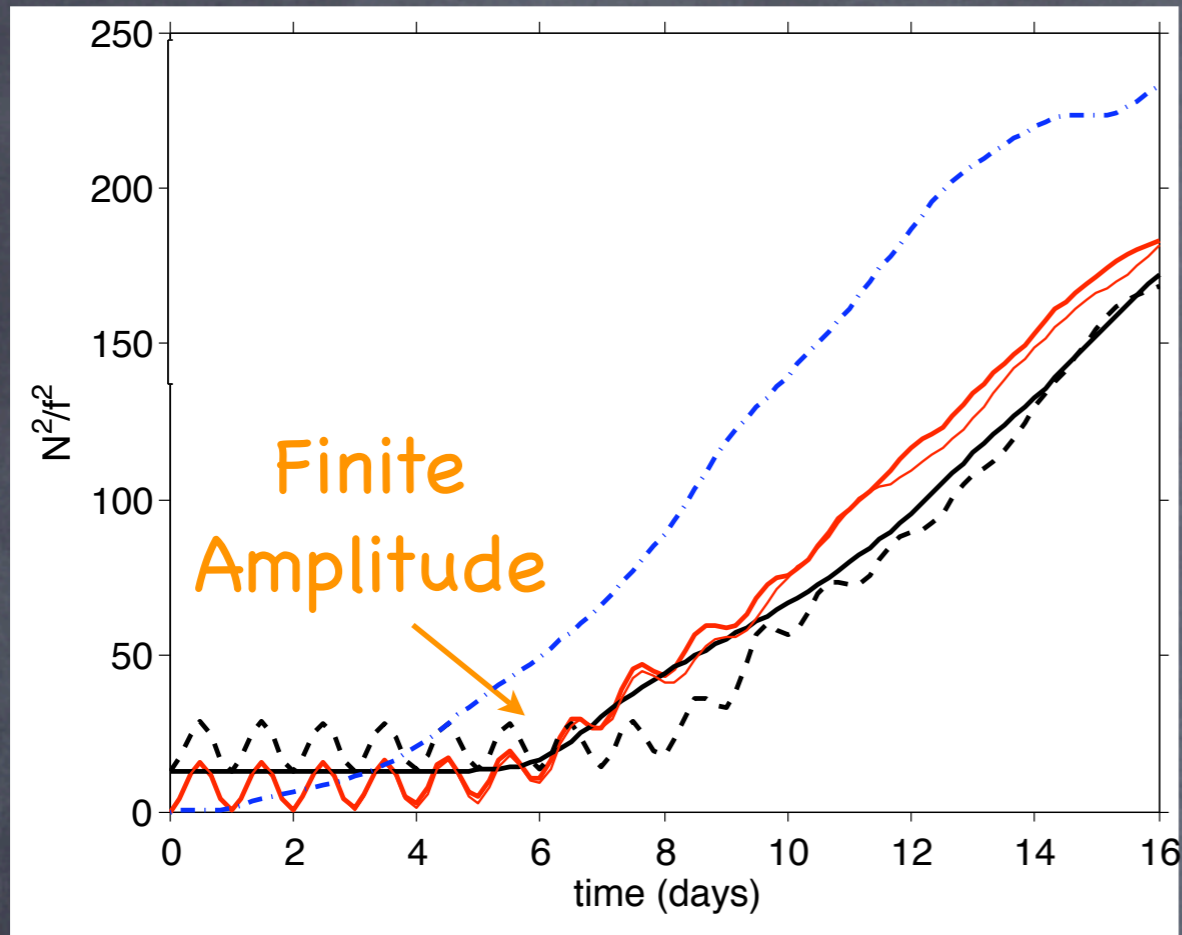
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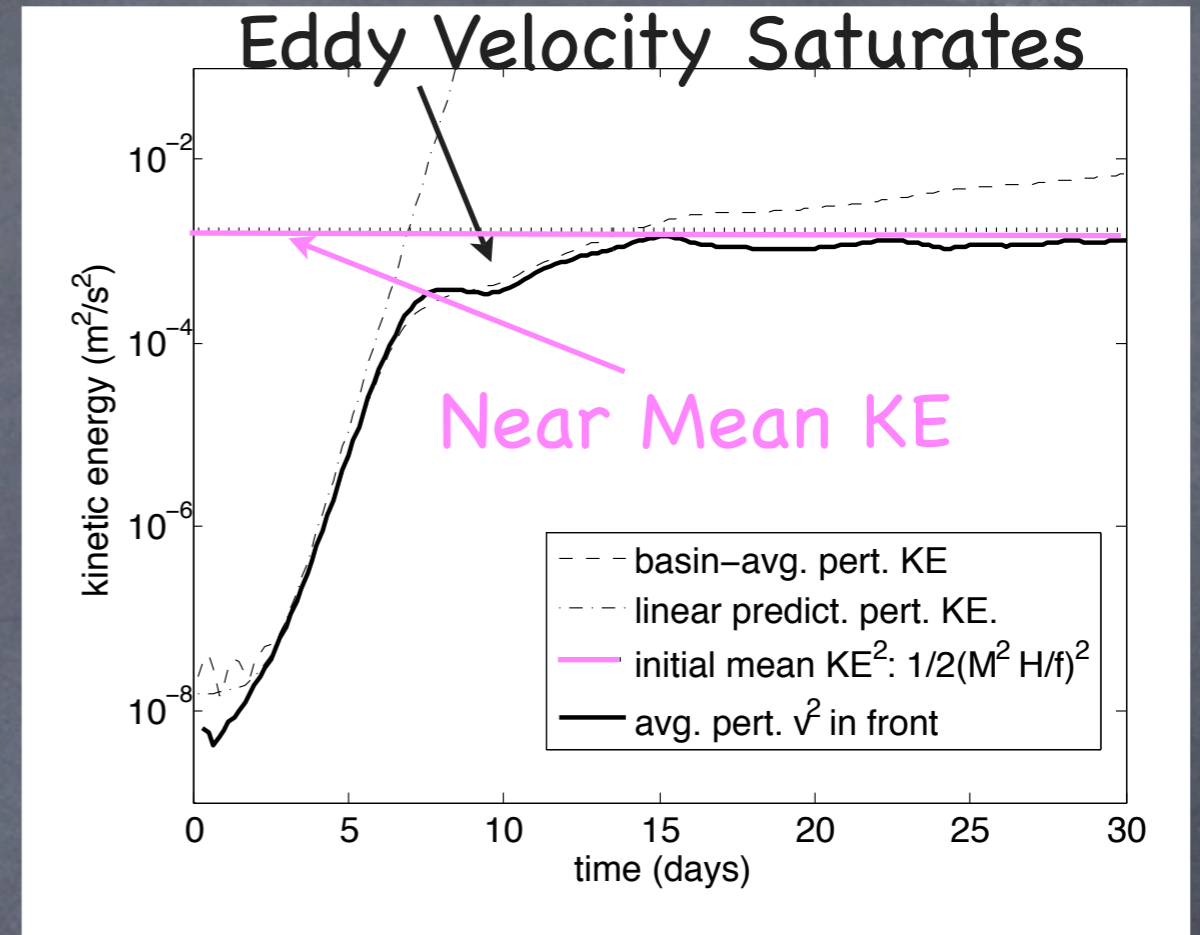
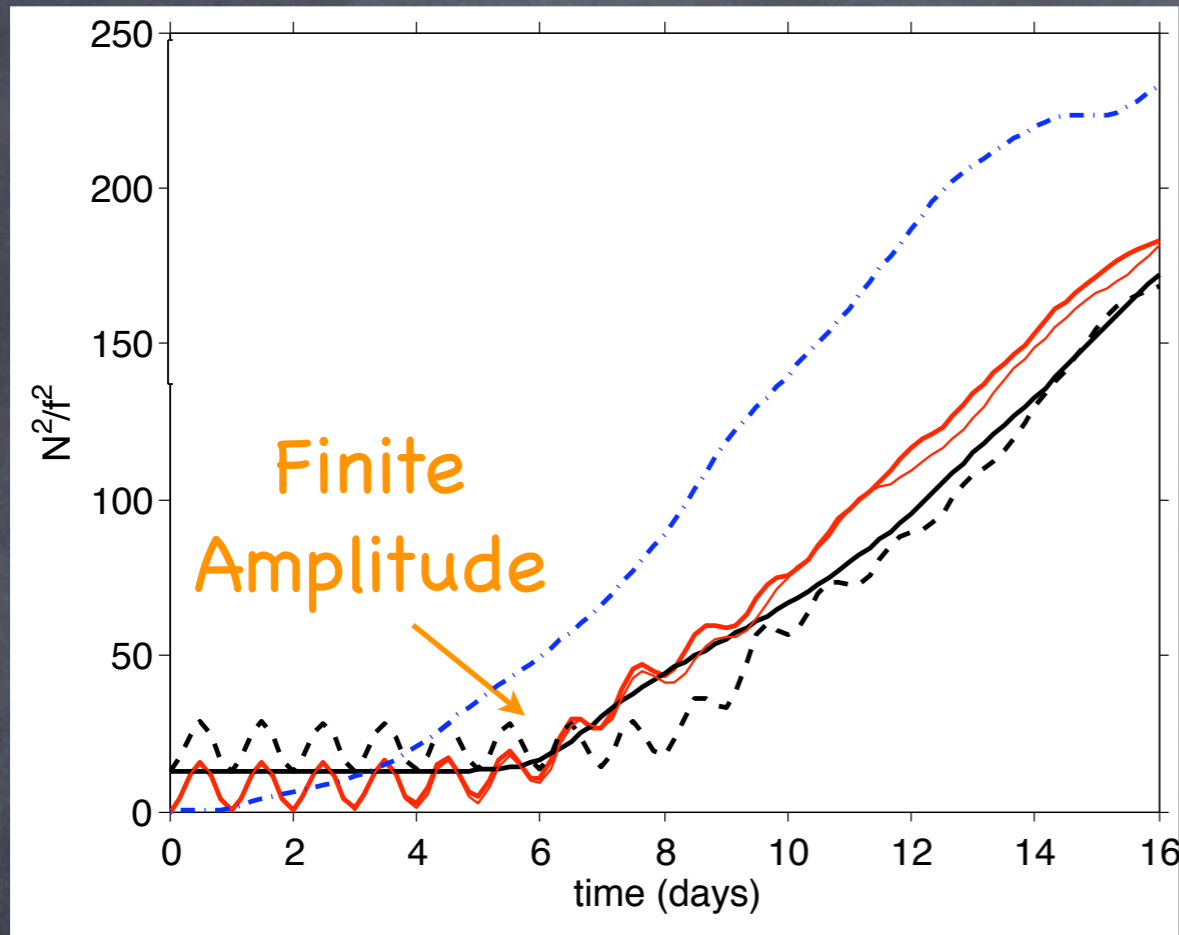
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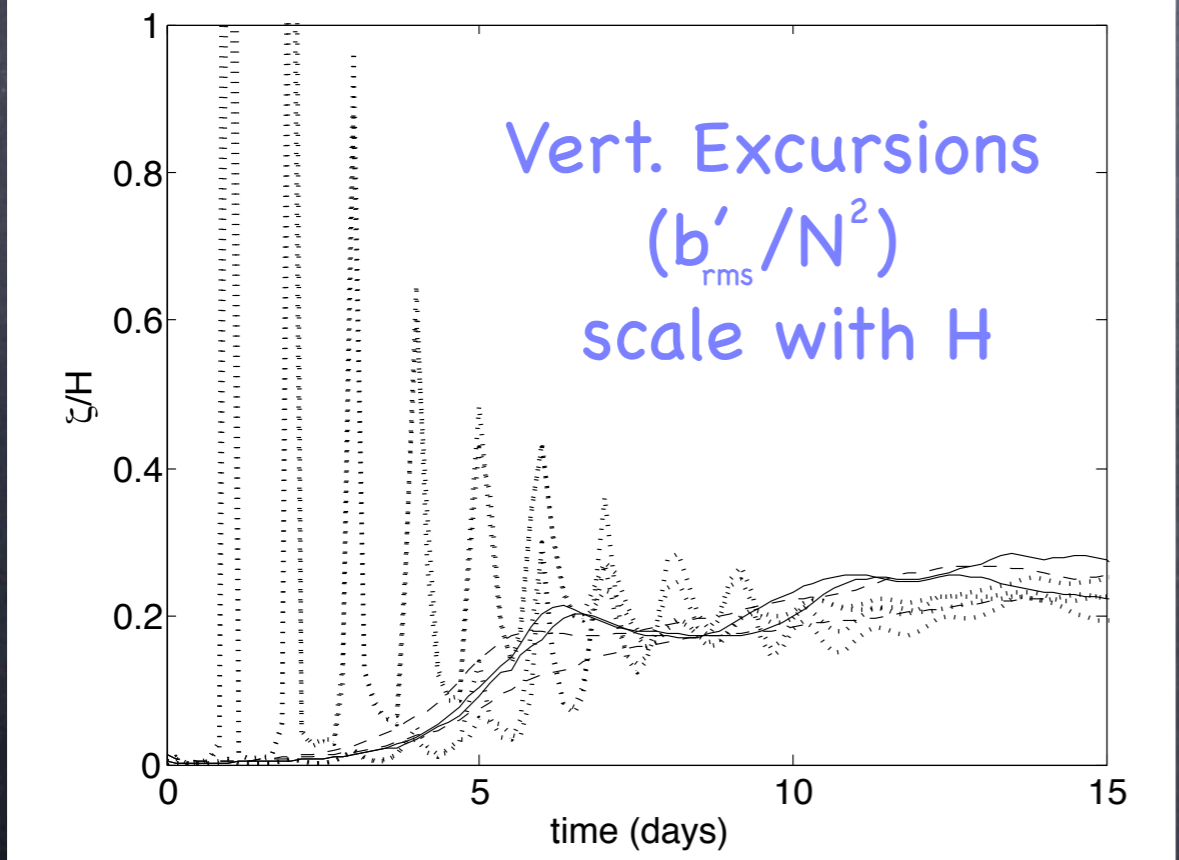
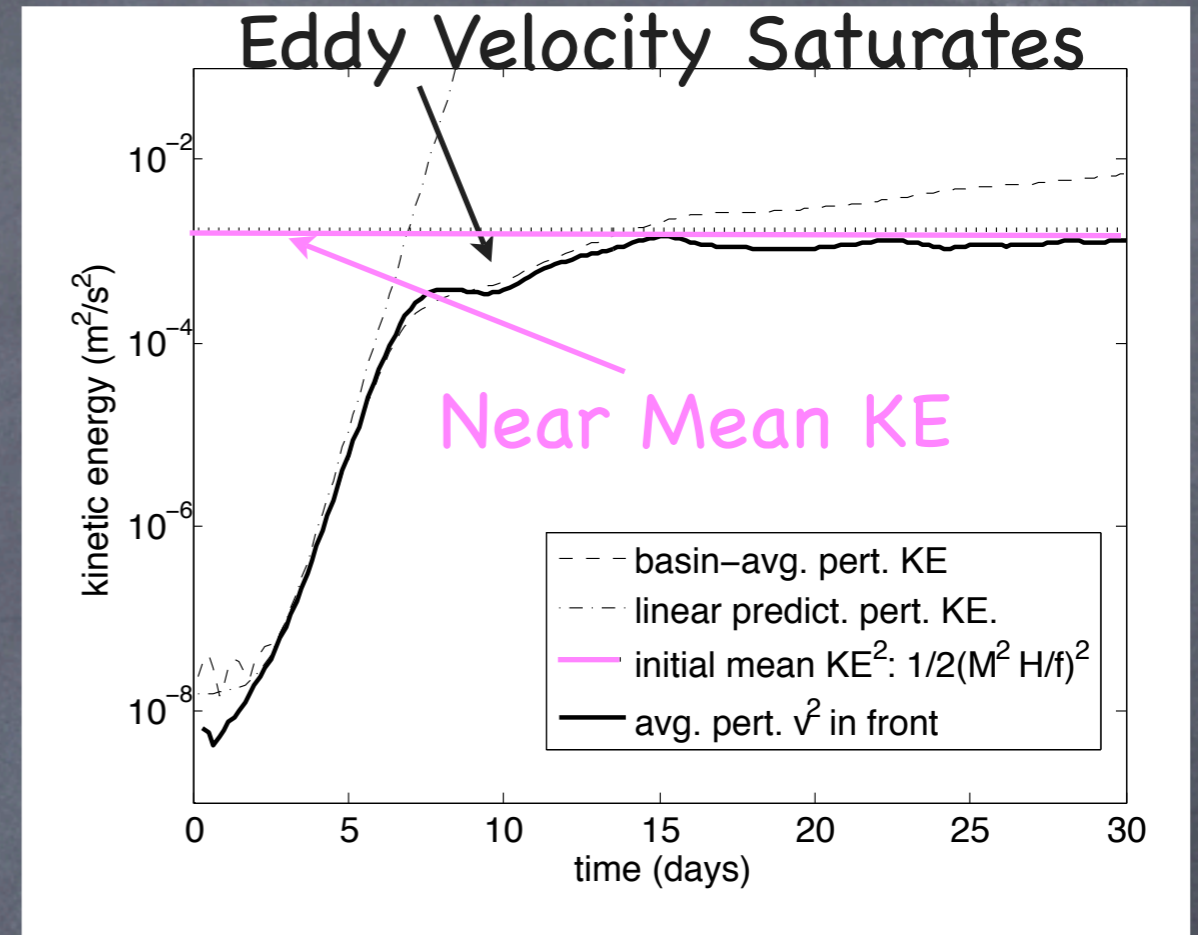
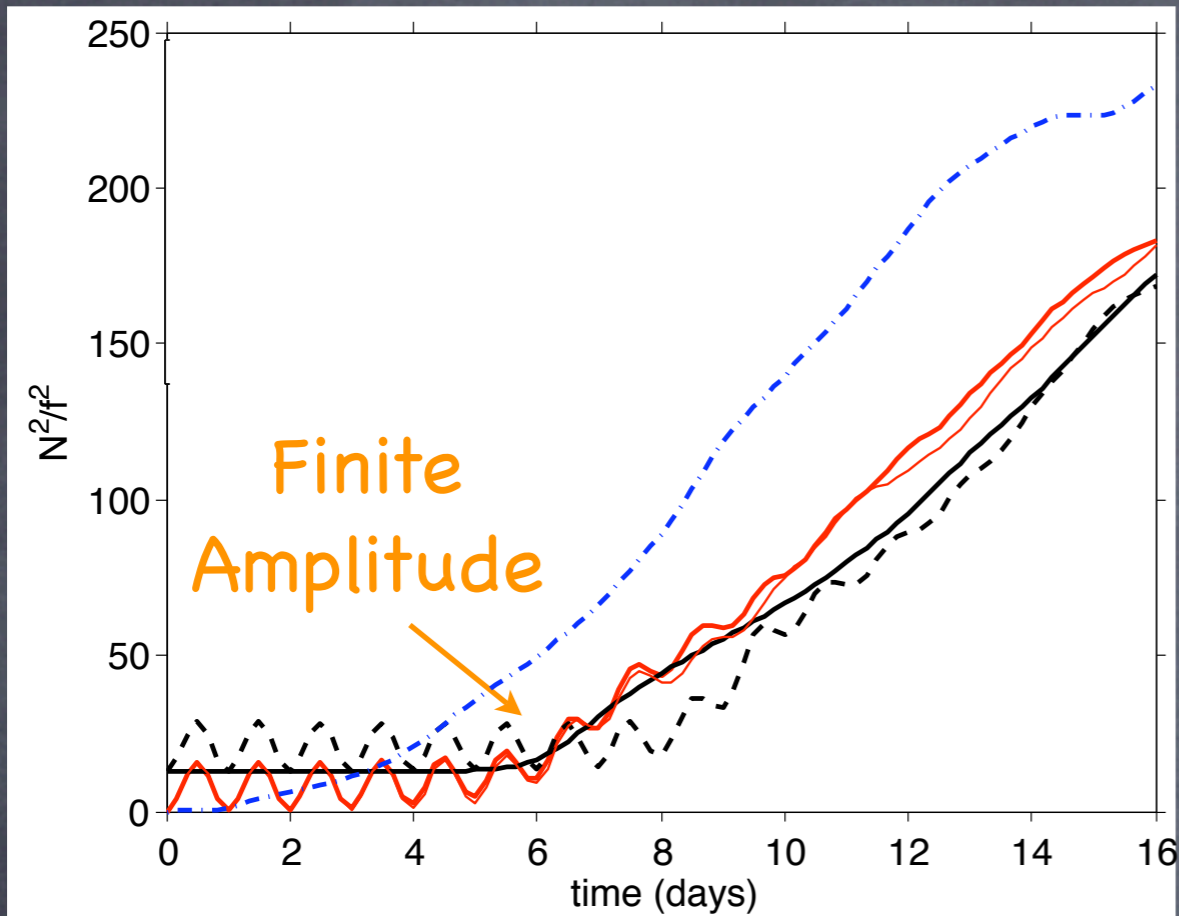
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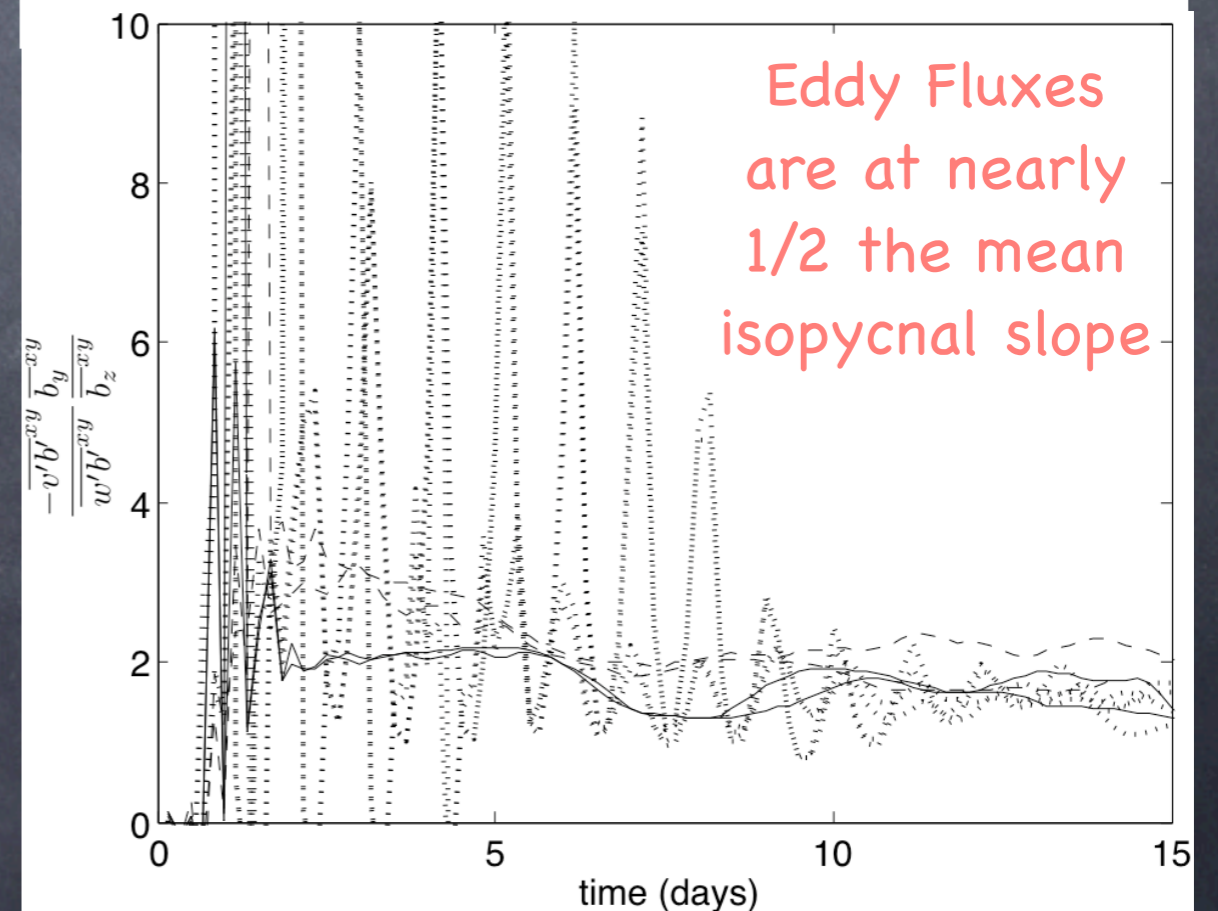
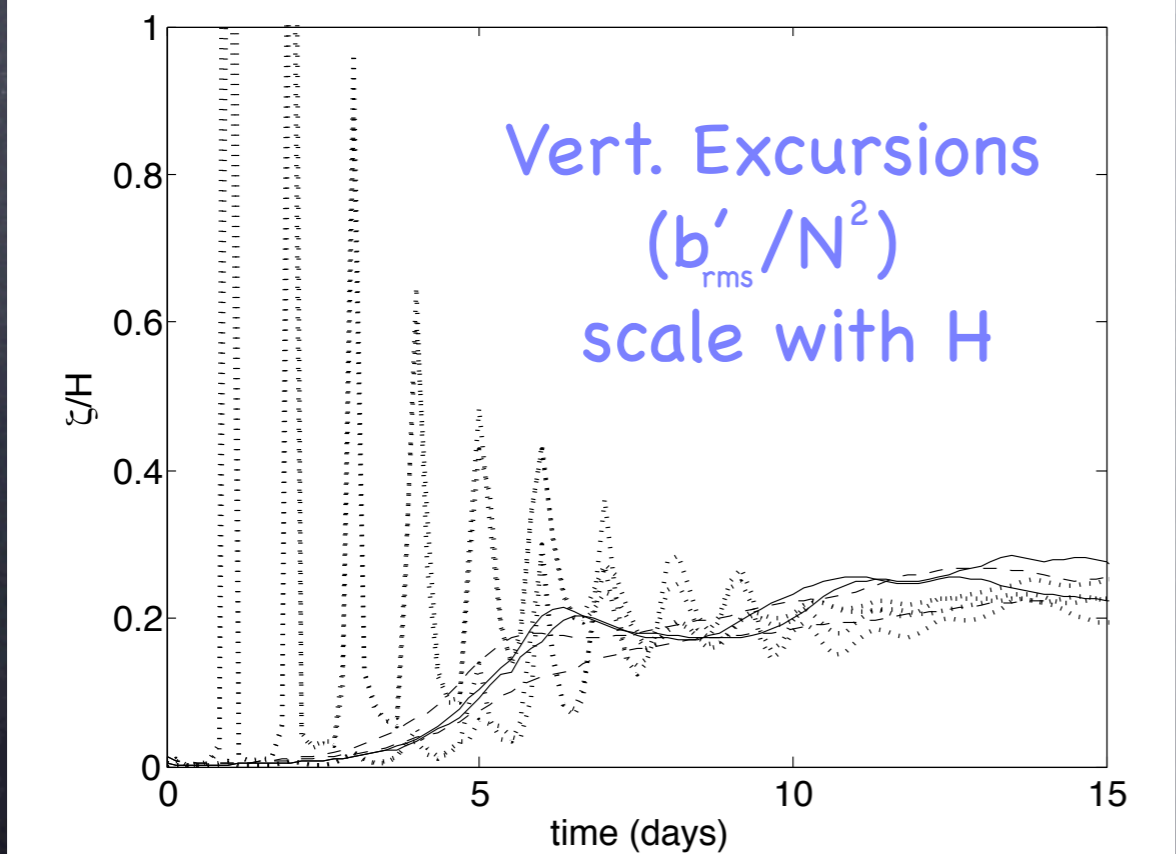
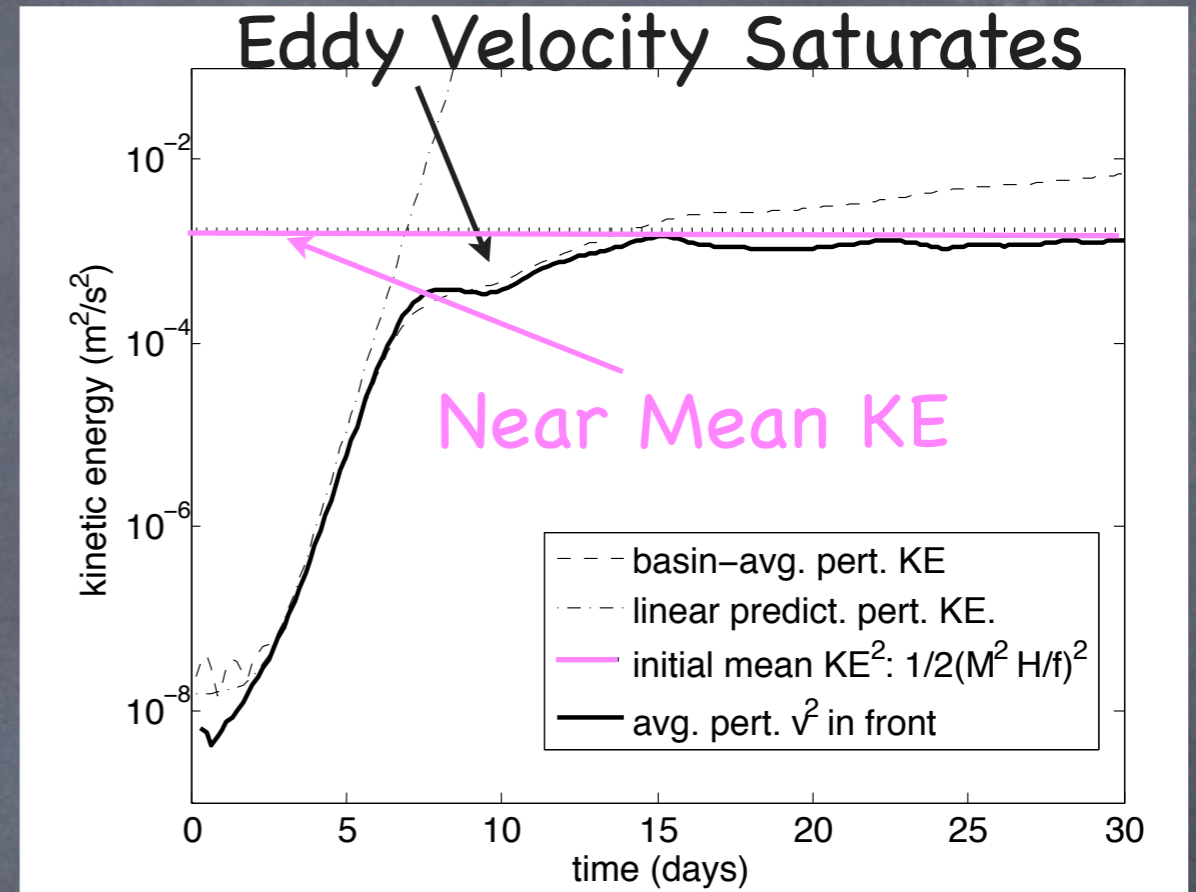
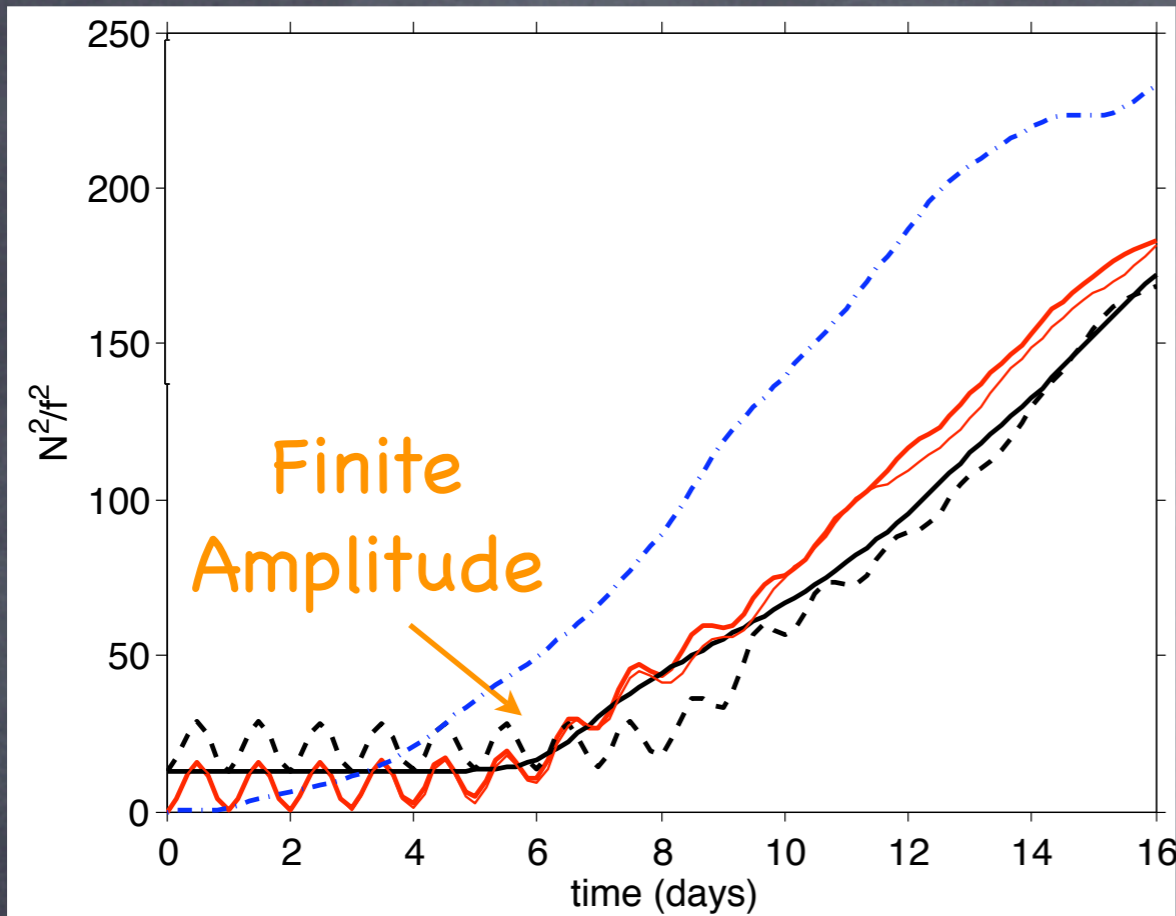
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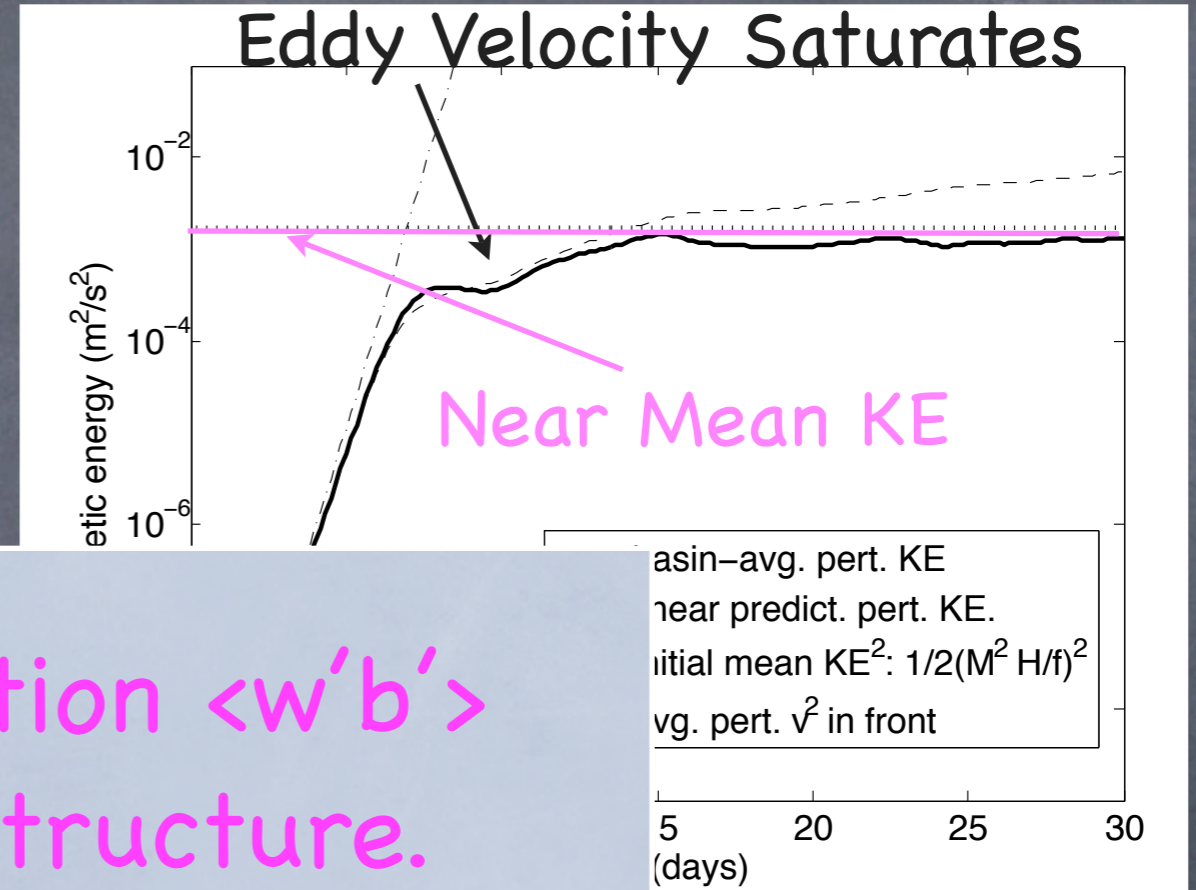
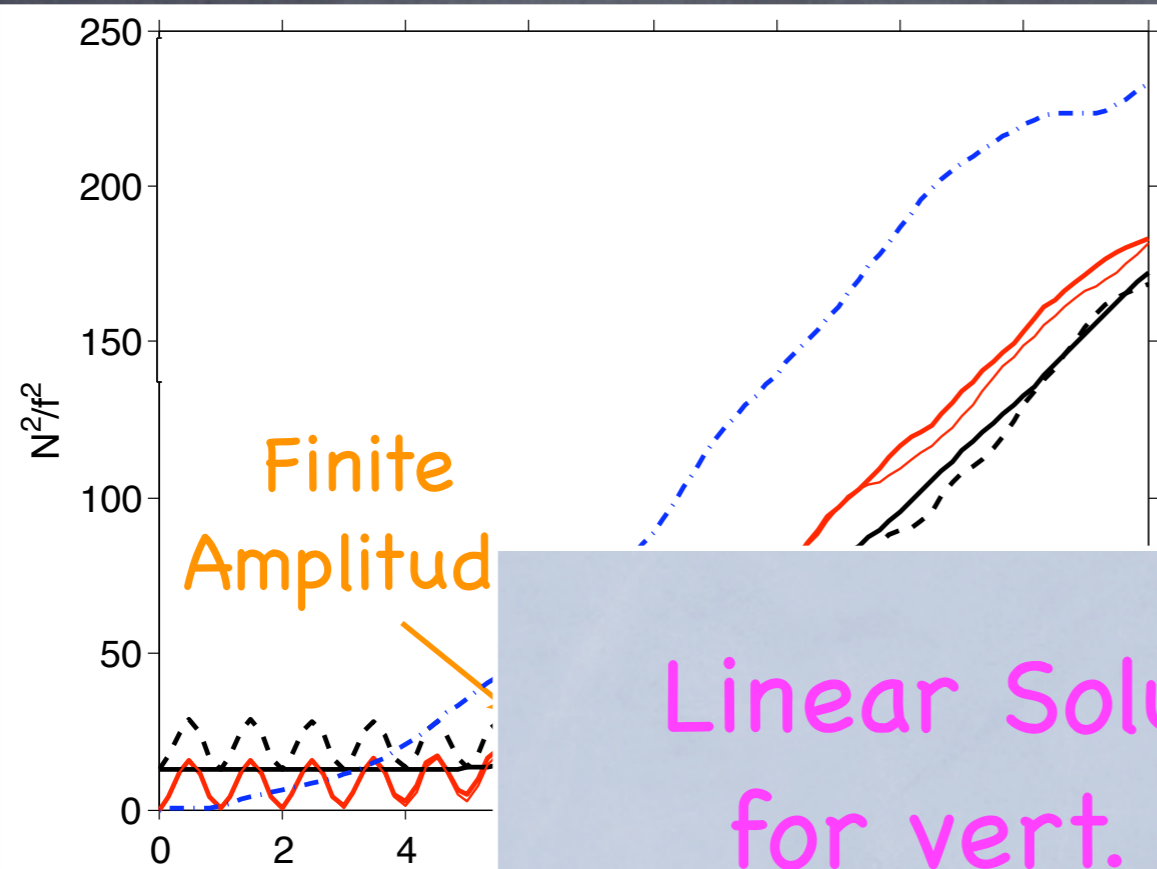
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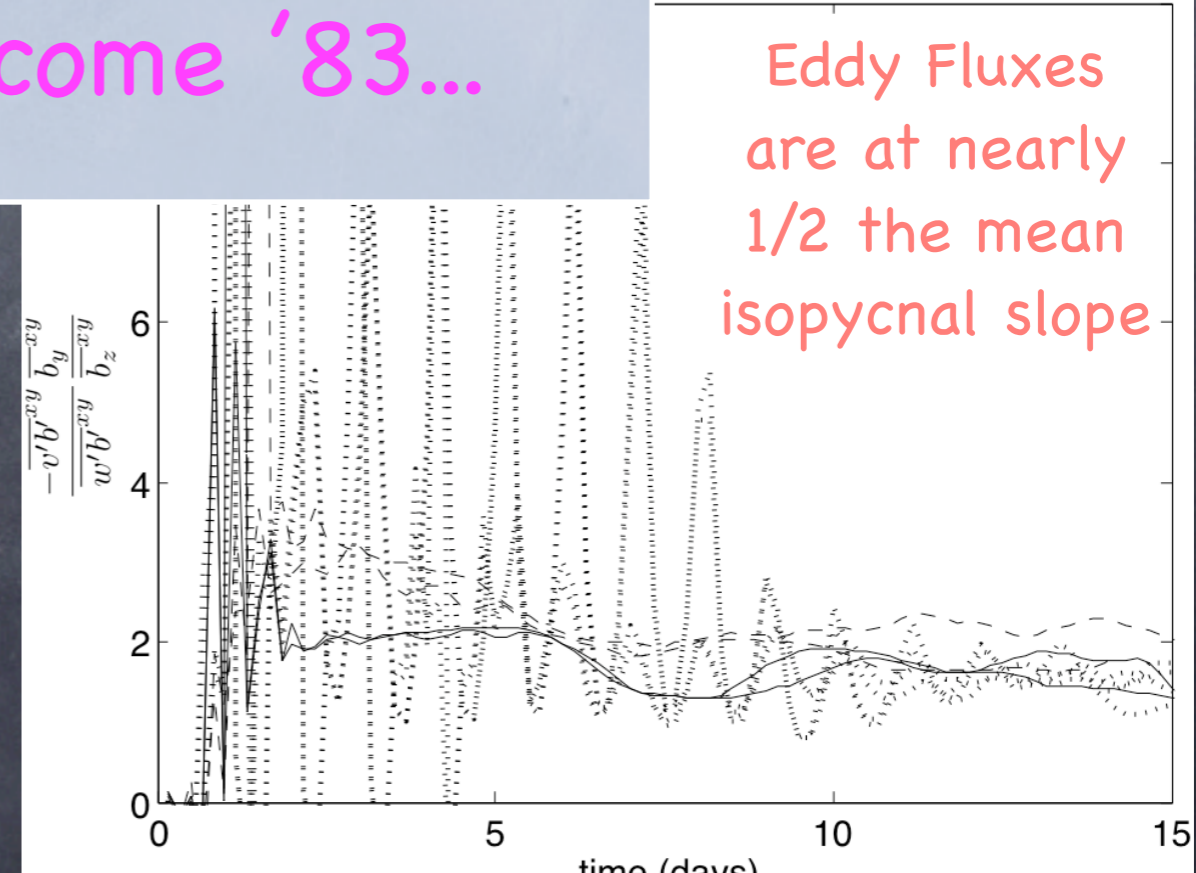
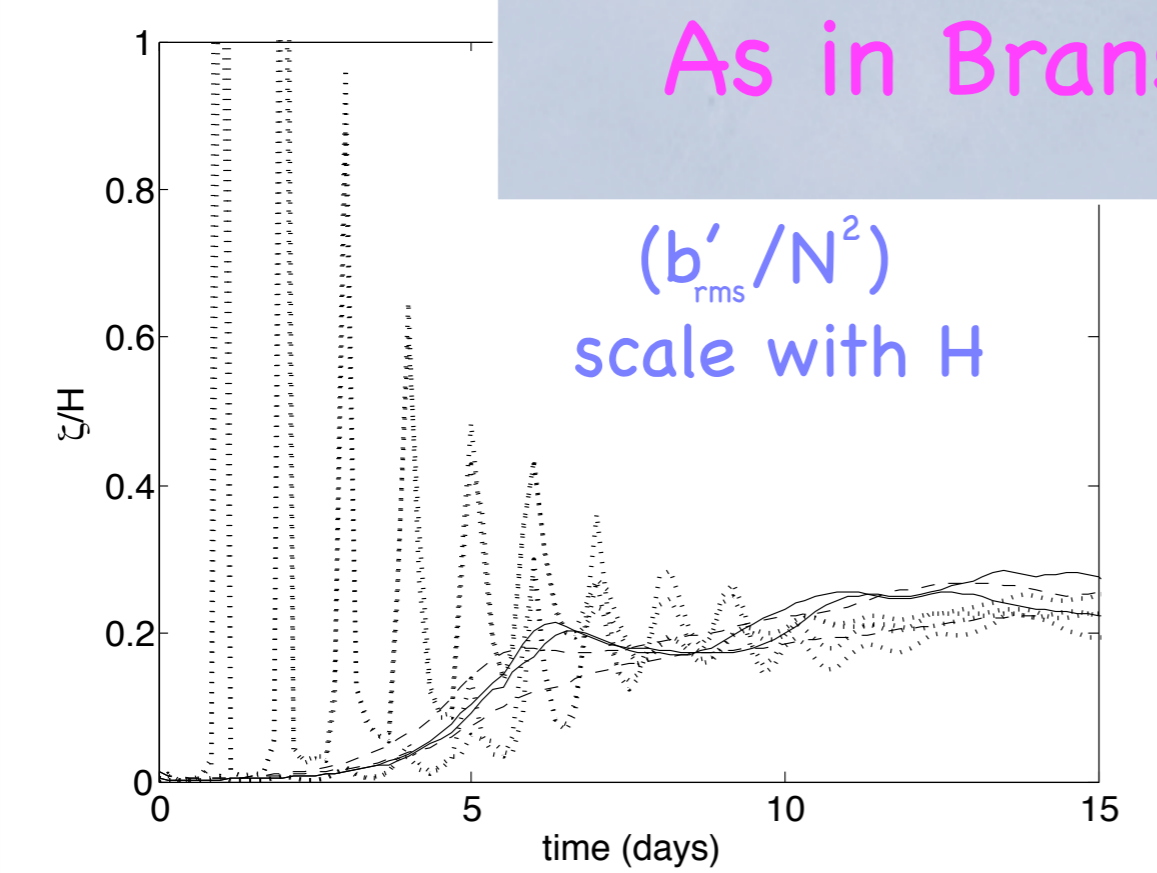


# Parameterization of Finite Amp. Eddies: Ingredients



Linear Solution  $\langle w'b' \rangle$   
for vert. structure.  
As in Branscome '83...

basin-avg. pert. KE  
near predict. pert. KE.  
initial mean  $KE^2: 1/2(M^2 H/f)^2$   
vg. pert.  $v^2$  in front



# The Parameterization:

$$\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{z}$$

$$\mu(z) = \left[ 1 - \left( \frac{2z}{H} + 1 \right)^2 \right] \left[ 1 + \frac{5}{21} \left( \frac{2z}{H} + 1 \right)^2 \right]$$

- The horizontal fluxes are downgradient:

$$\overline{\mathbf{u}'_H b'} = - \frac{C_e H^2 \mu(z) \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla_H \bar{b}$$

- Vertical fluxes always upward to restratify with correct extraction rate of potential energy:

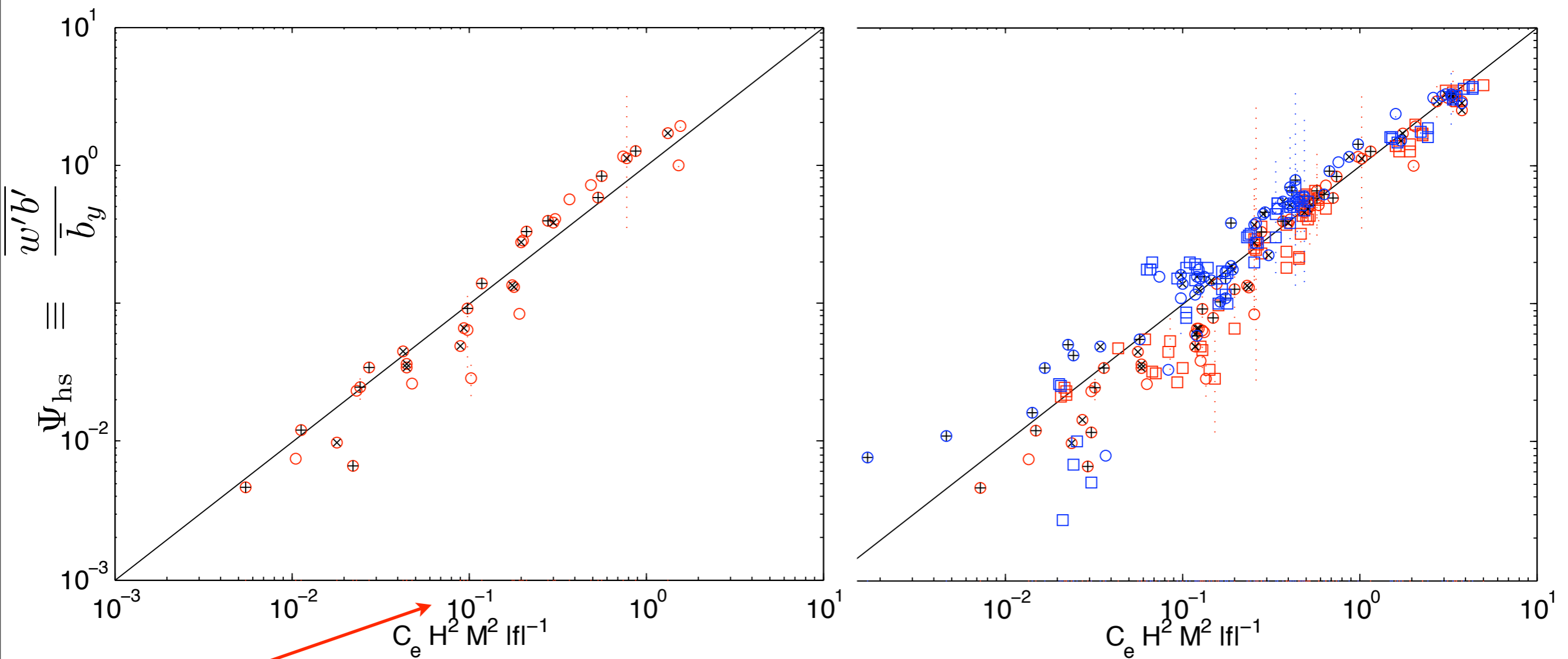
$$\overline{w' b'} = \frac{C_e H^2 \mu(z)}{|f|} |\nabla \bar{b}|^2$$



# It works for Prototype Sims:

Red: No Diurnal

Blue: With Diurnal



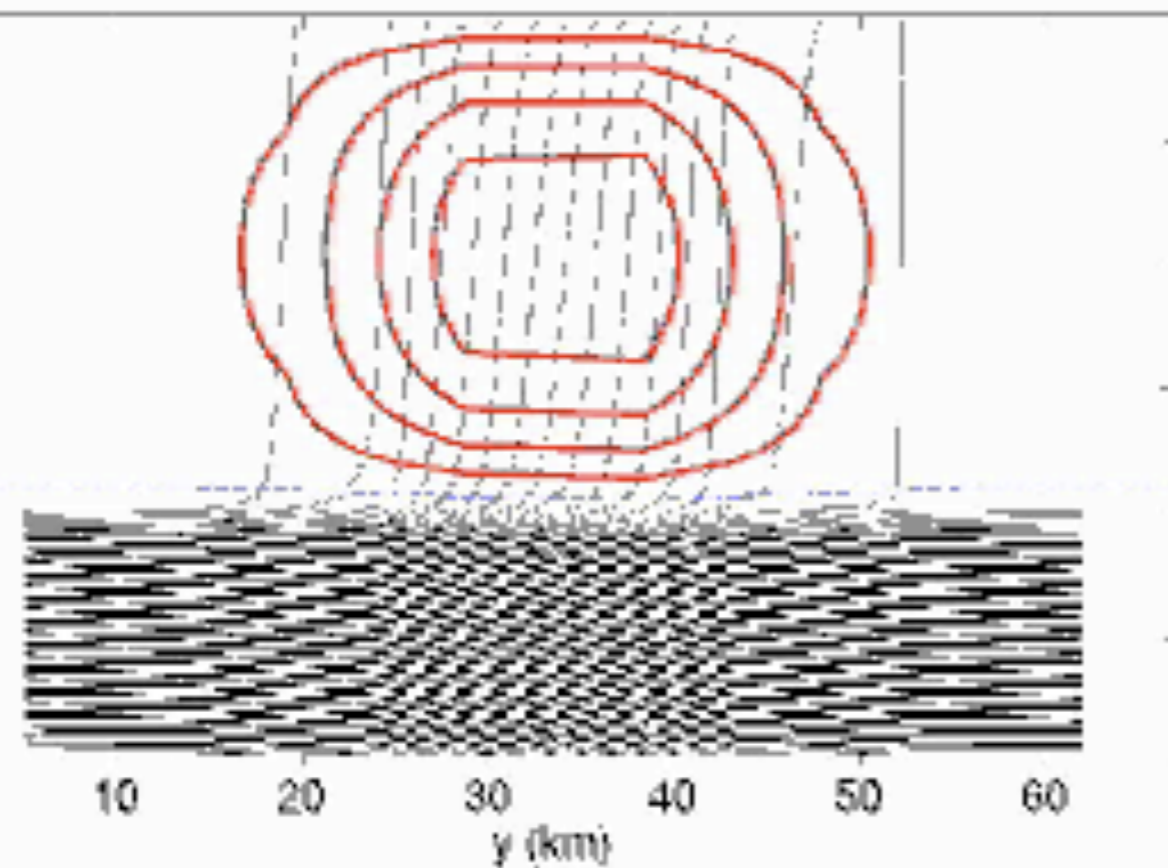
>2 orders of  
magnitude!

Circles: Balanced Initial Cond.  
Squares: Unbalanced Initial Cond.

# What does it look like?

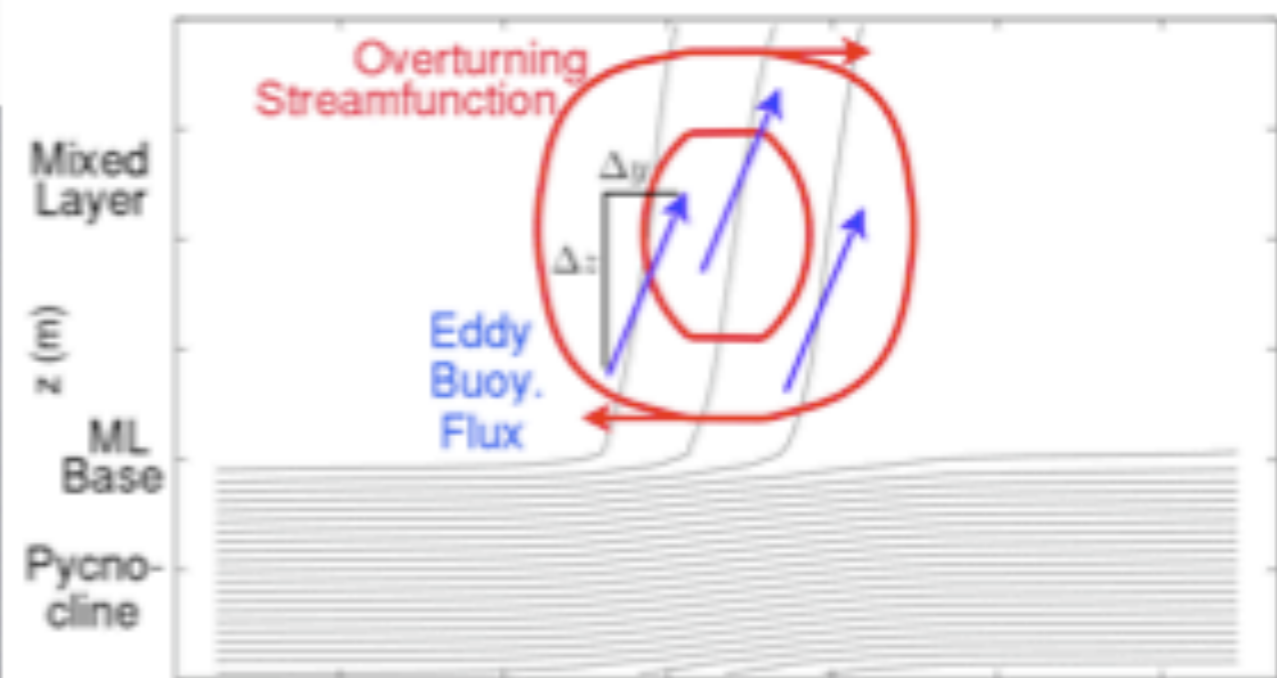
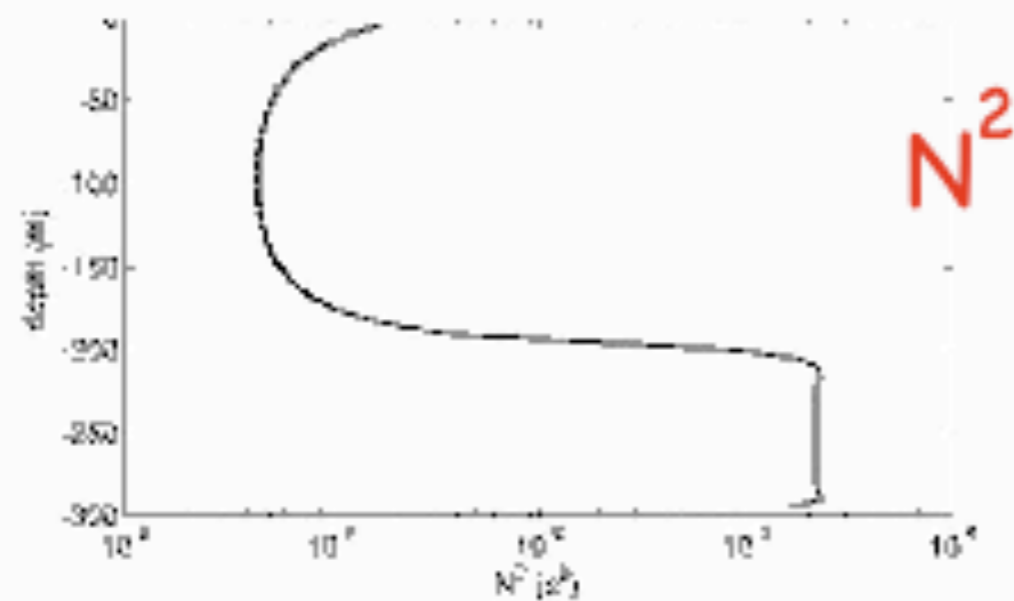
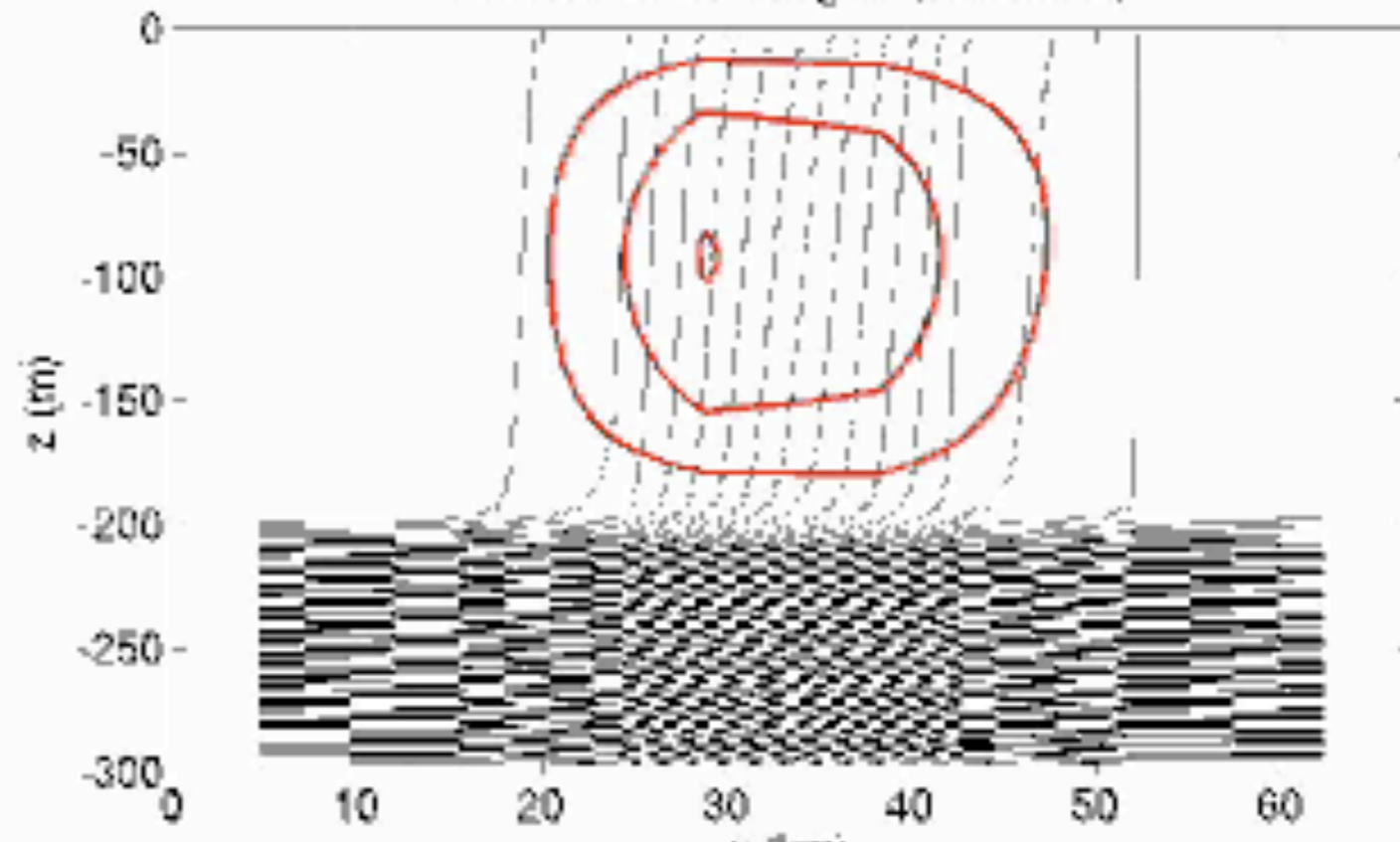
## Parameterization (2d, 10km grid)

7d01h from 2d parameterization



## Submesoscale-Resolving (3d, 500m grid)

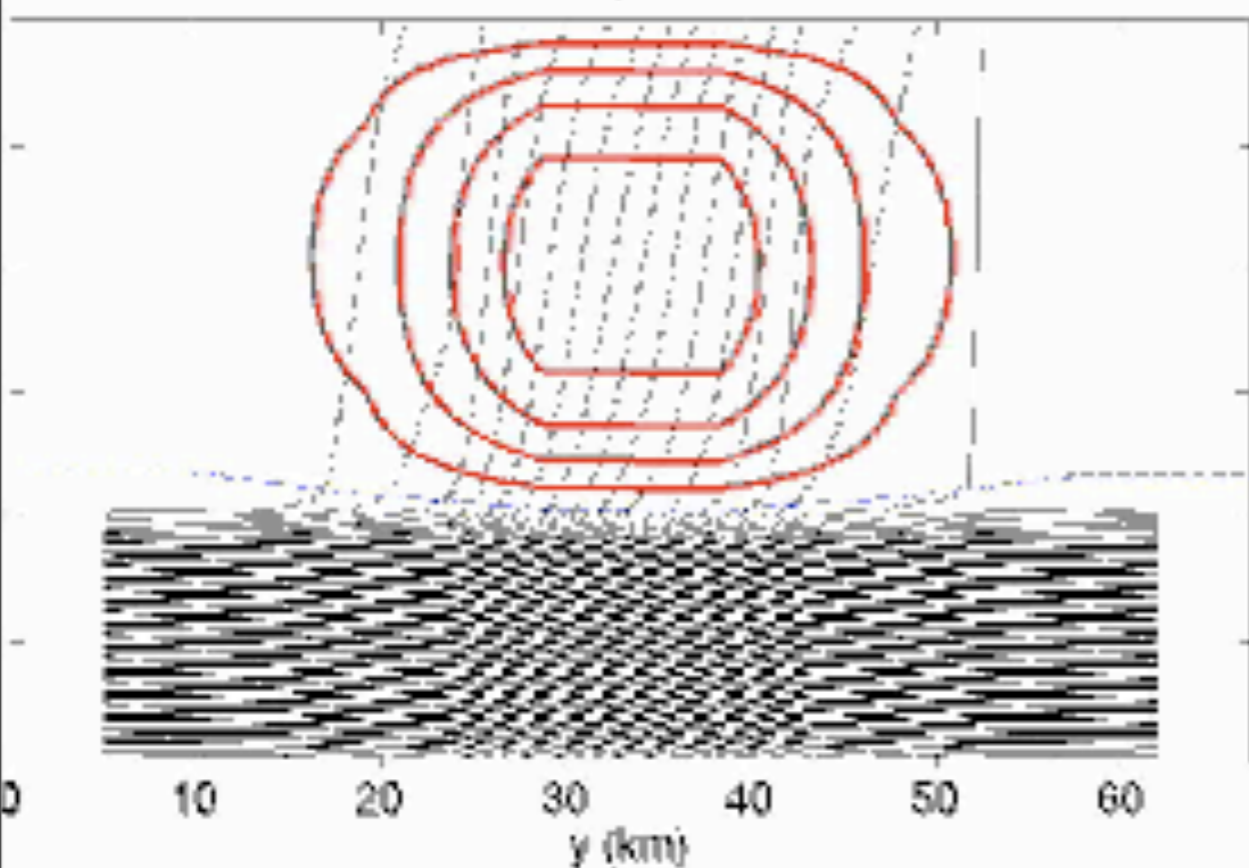
7d01h from 3d MITgcm (smoothed)



# What does it look like?

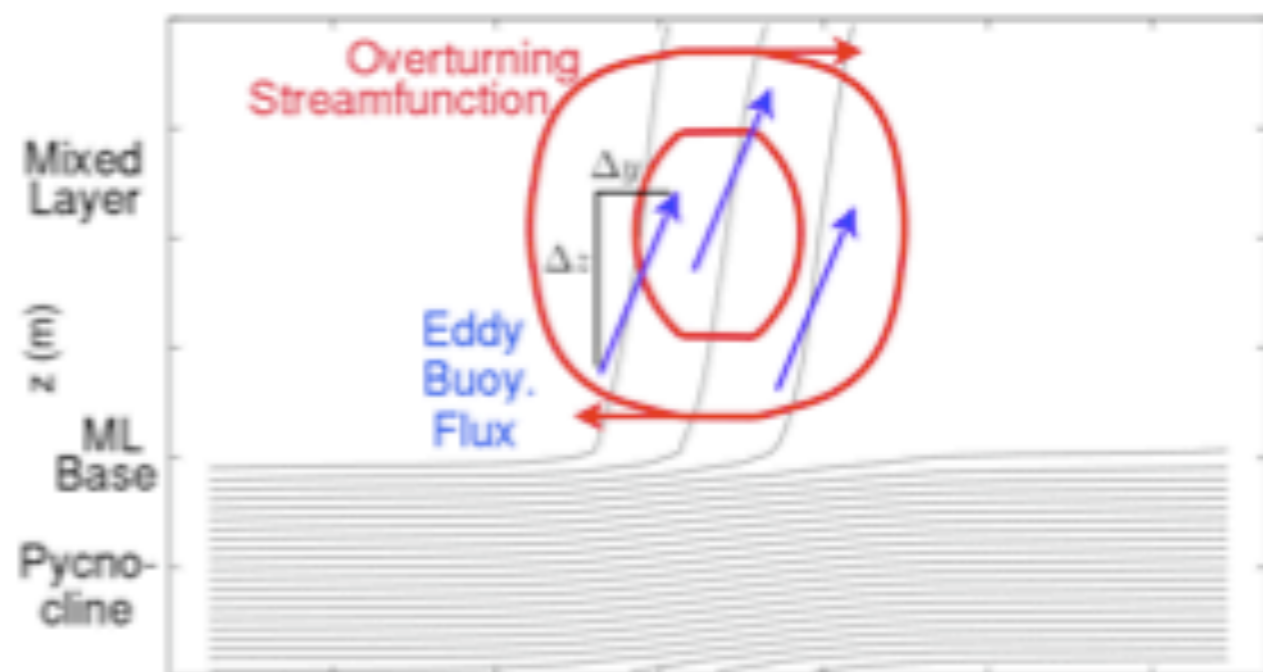
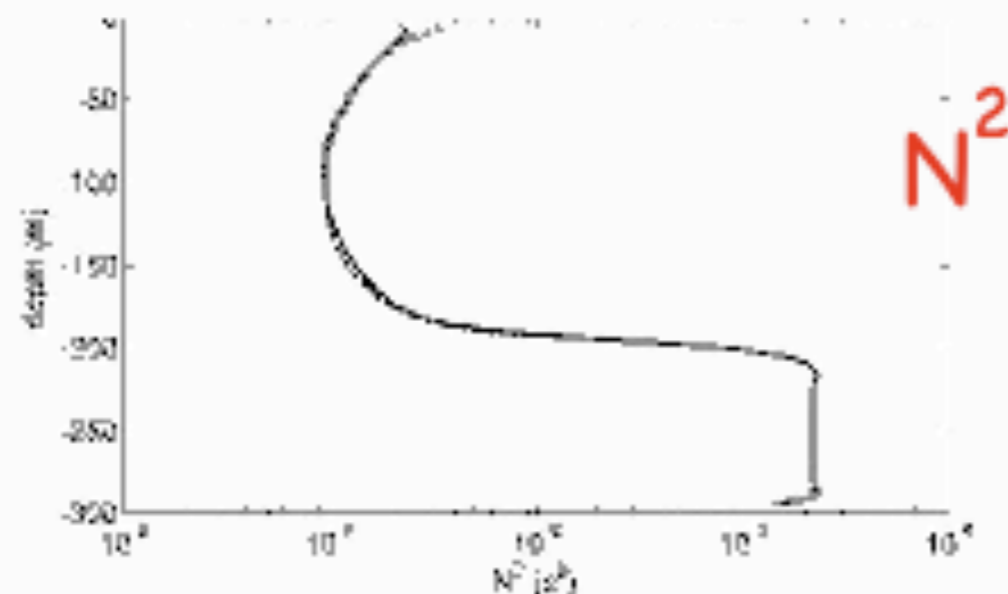
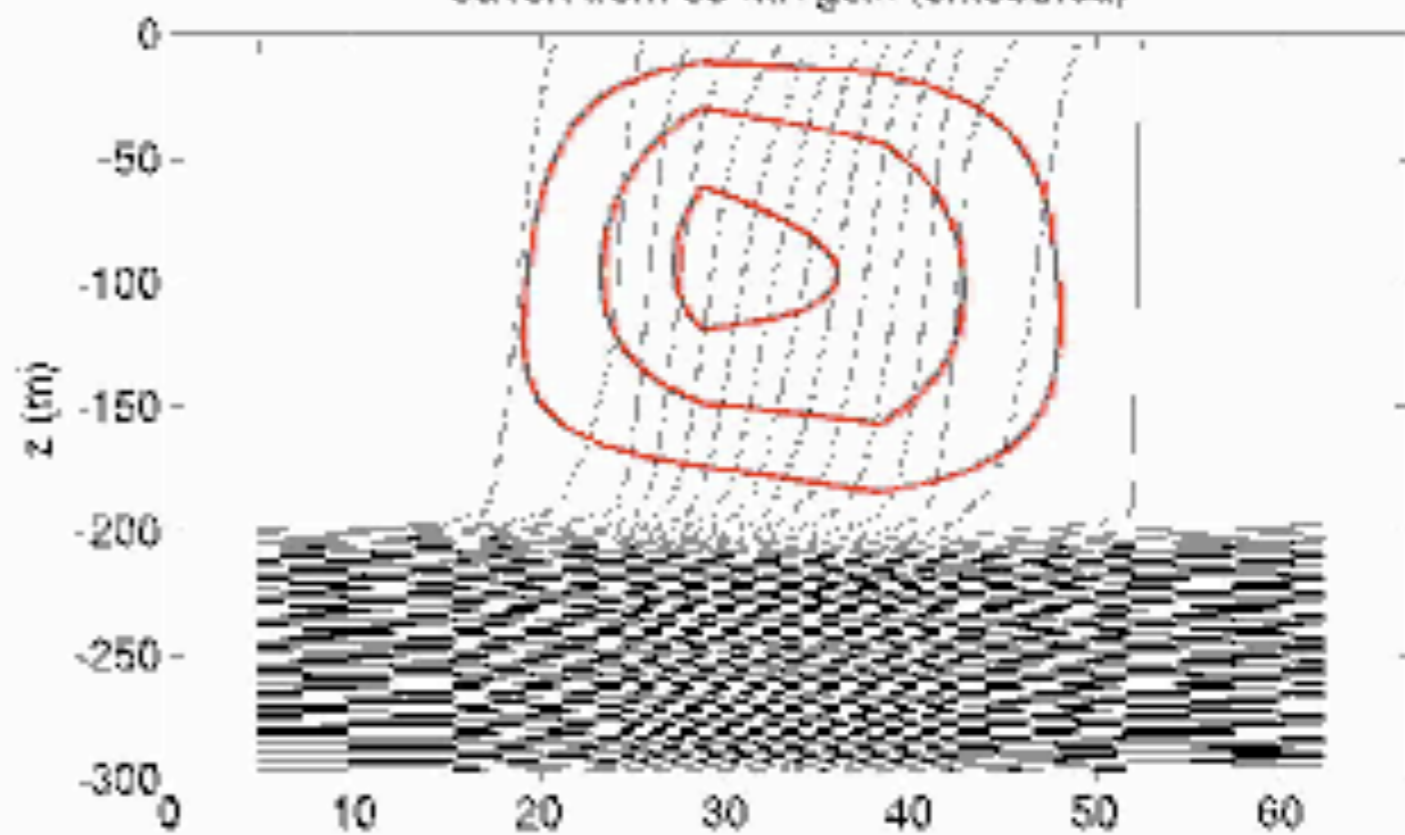
## Parameterization (2d, 10km grid)

8d16h from 2d parameterization



## Submesoscale-Resolving (3d, 500m grid)

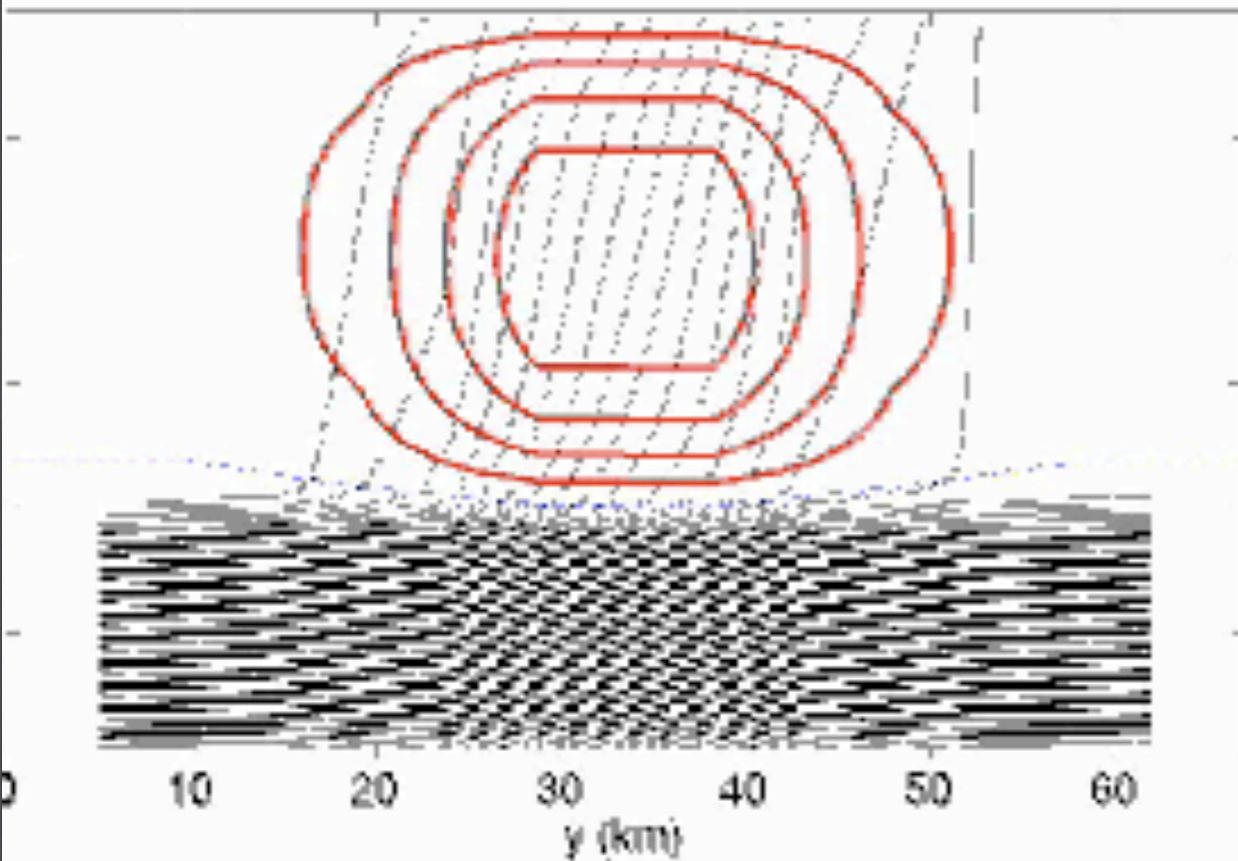
8d16h from 3d MITgcm (smoothed)



# What does it look like?

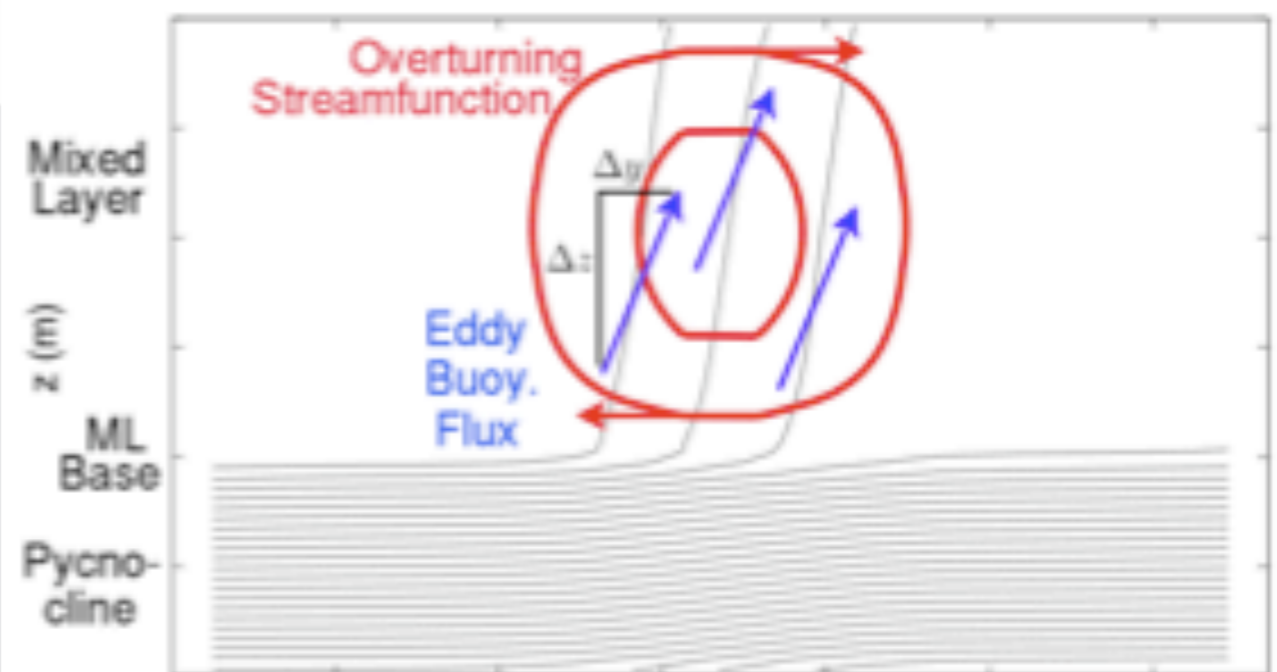
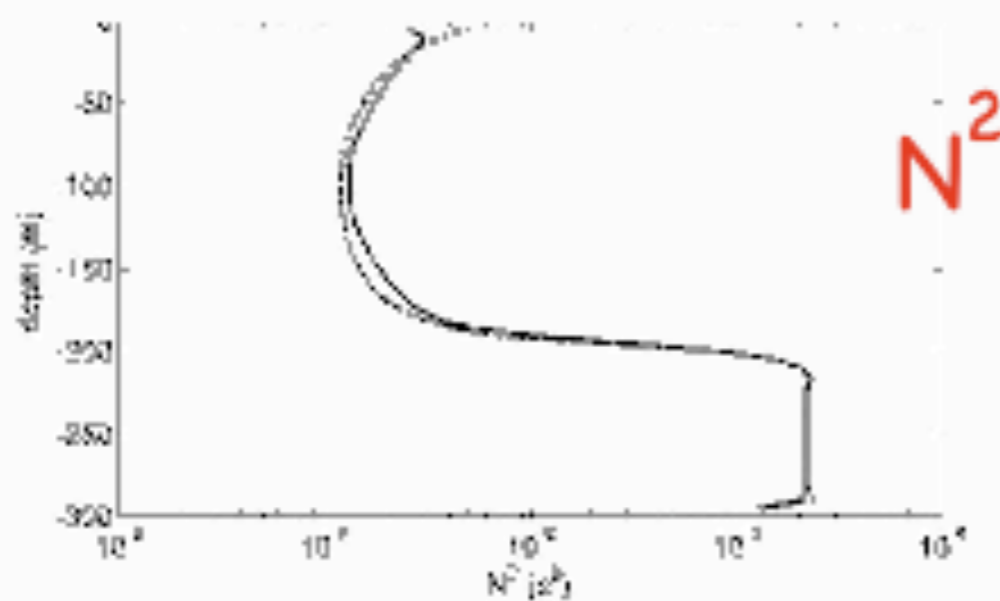
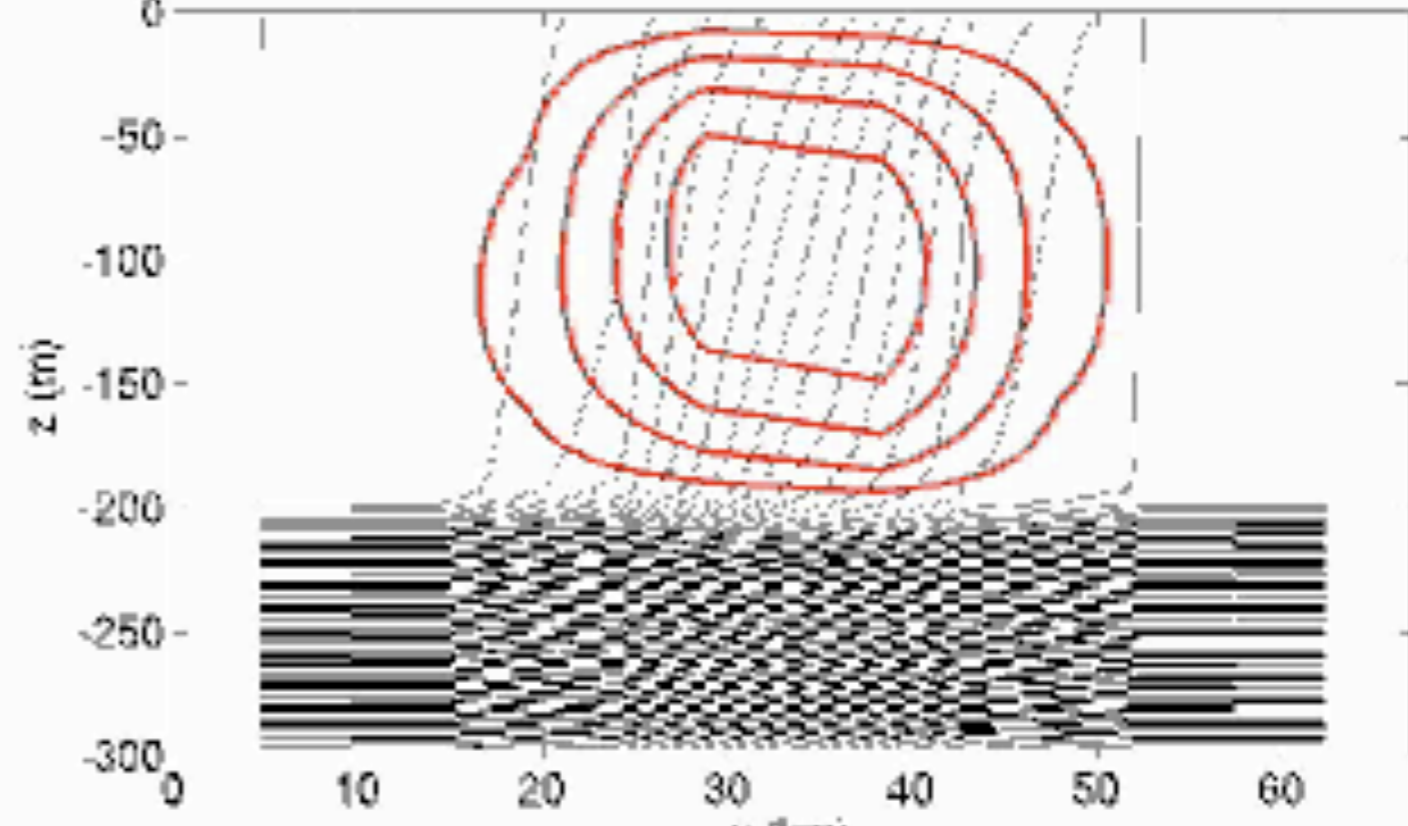
## Parameterization (2d, 10km grid)

9d16h from 2d parameterization



## Submesoscale-Resolving (3d, 500m grid)

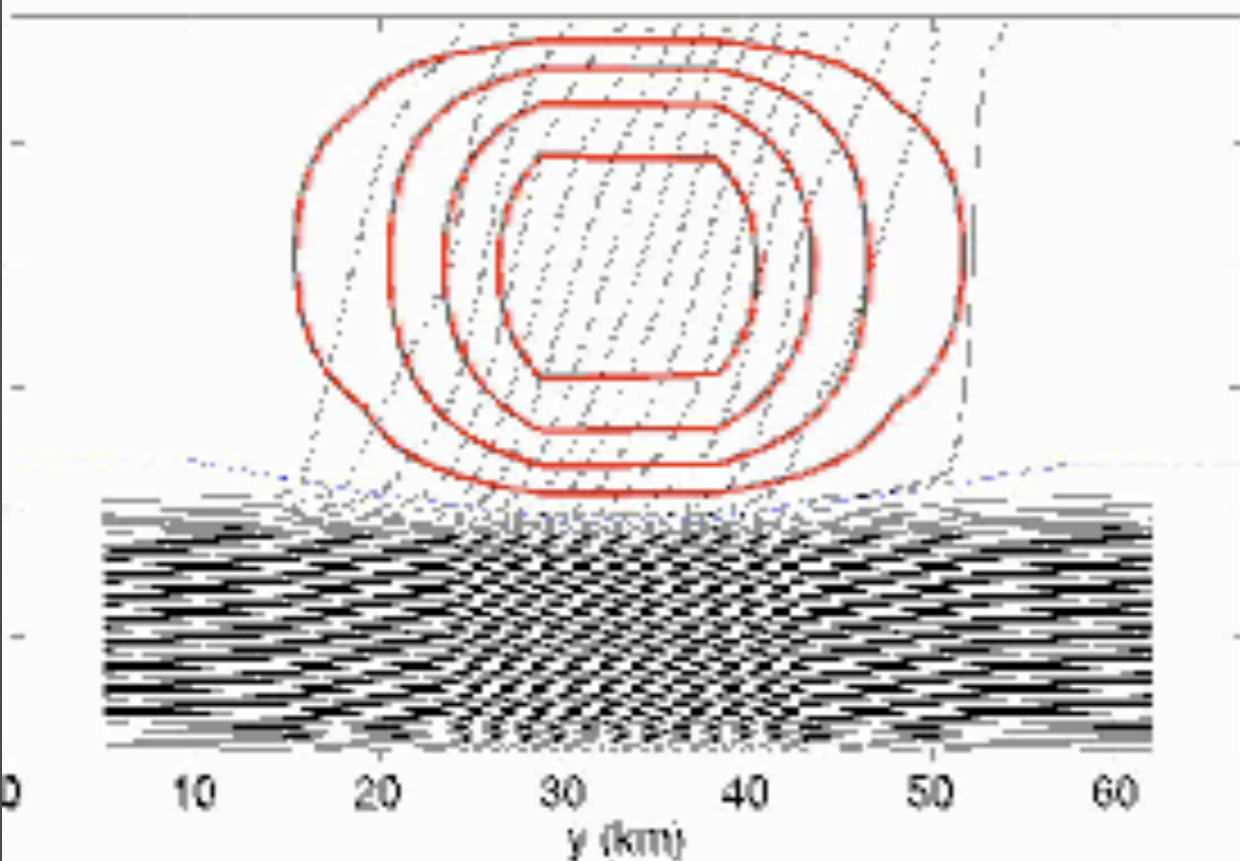
9d16h from 3d MITgcm (smoothed)



# What does it look like?

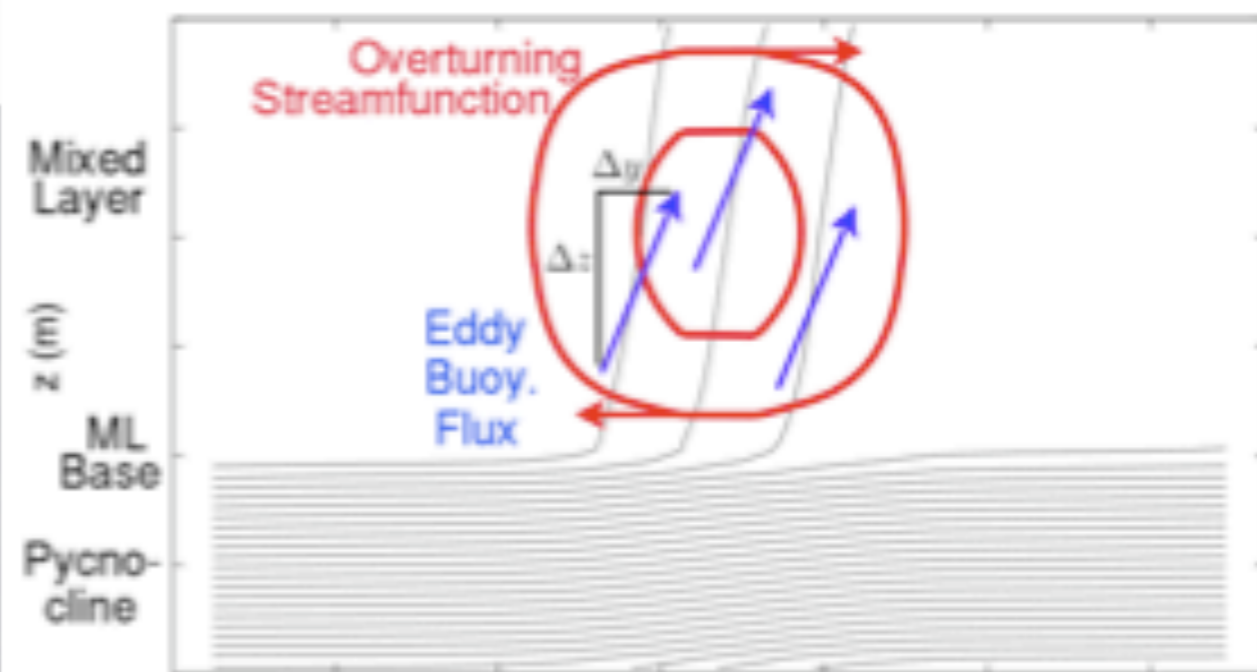
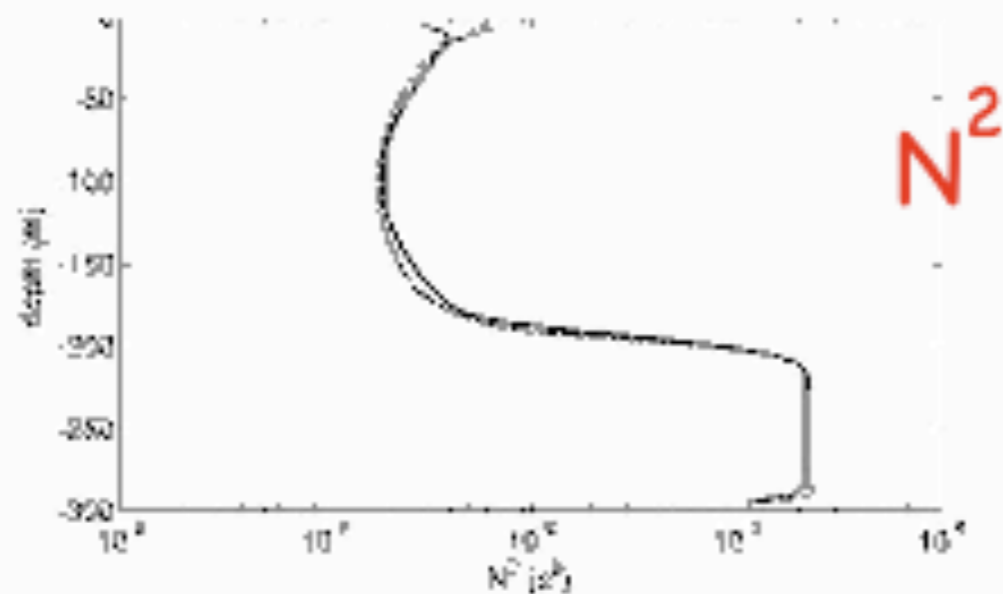
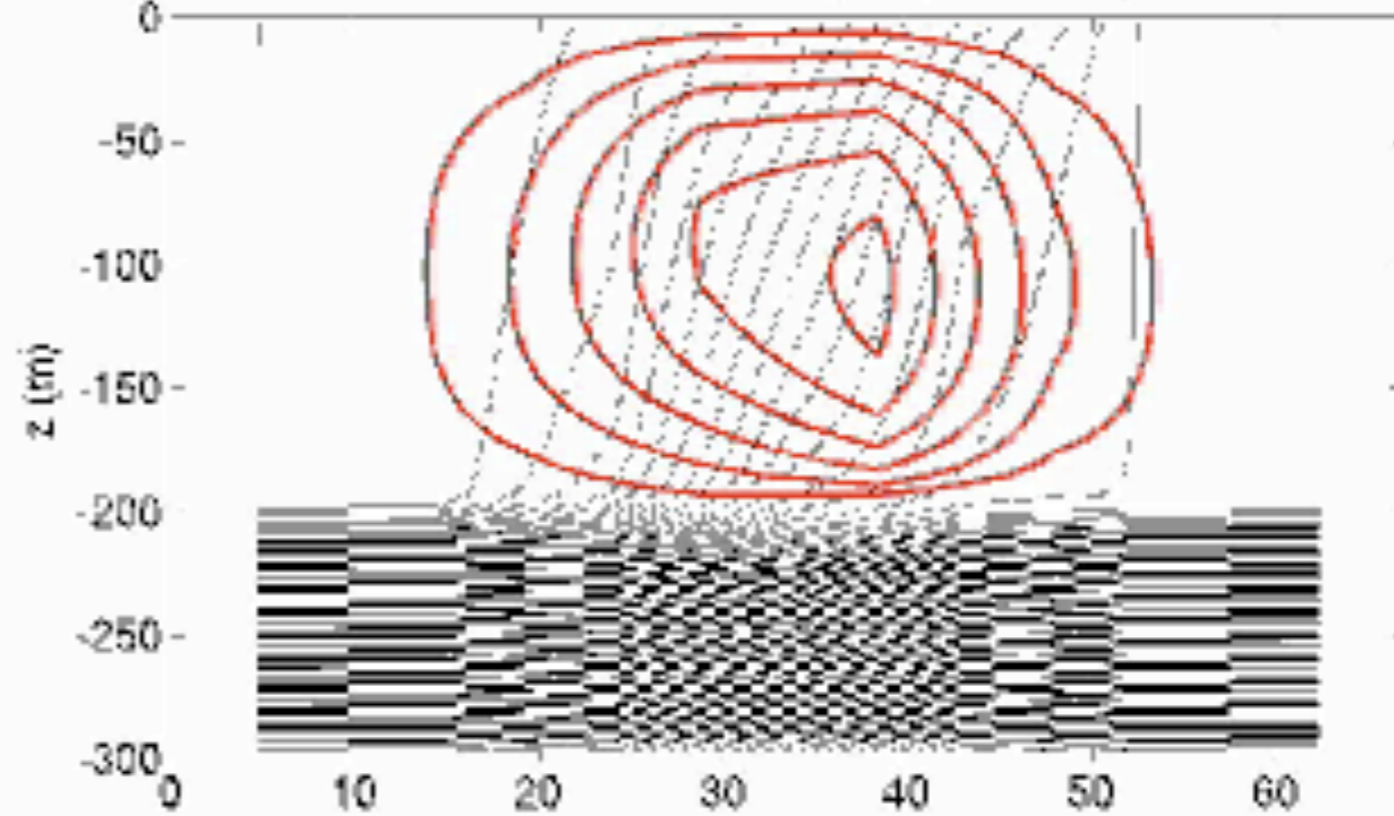
## Parameterization (2d, 10km grid)

11d16h from 2d parameterization



## Submesoscale-Resolving (3d, 500m grid)

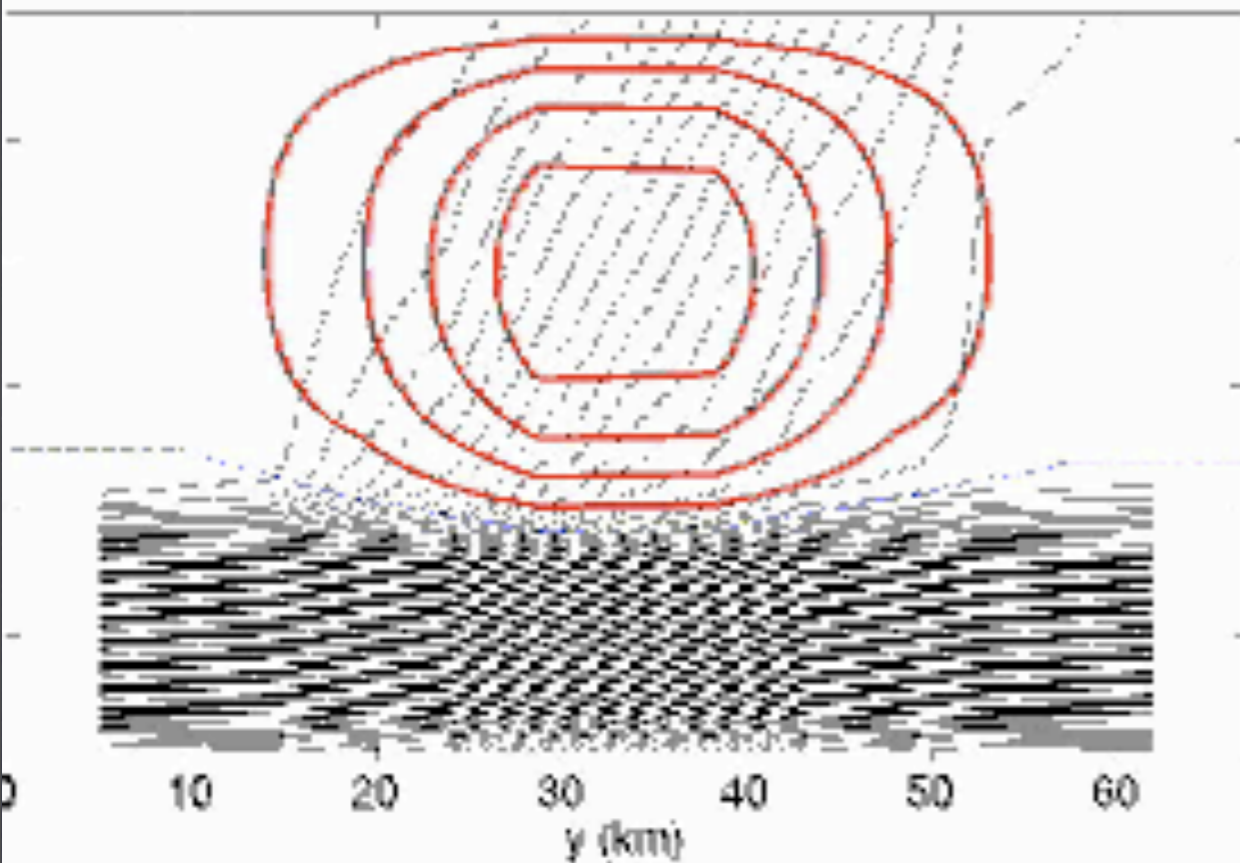
11d16h from 3d MITgcm (smoothed)



# What does it look like?

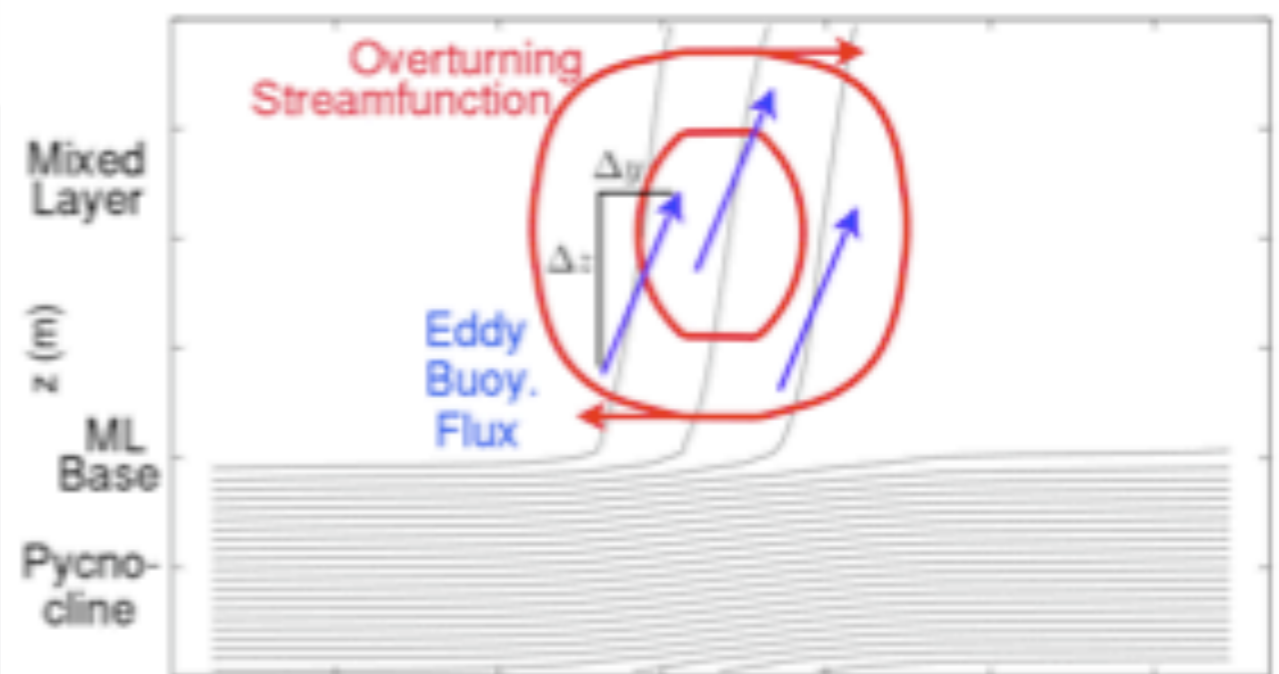
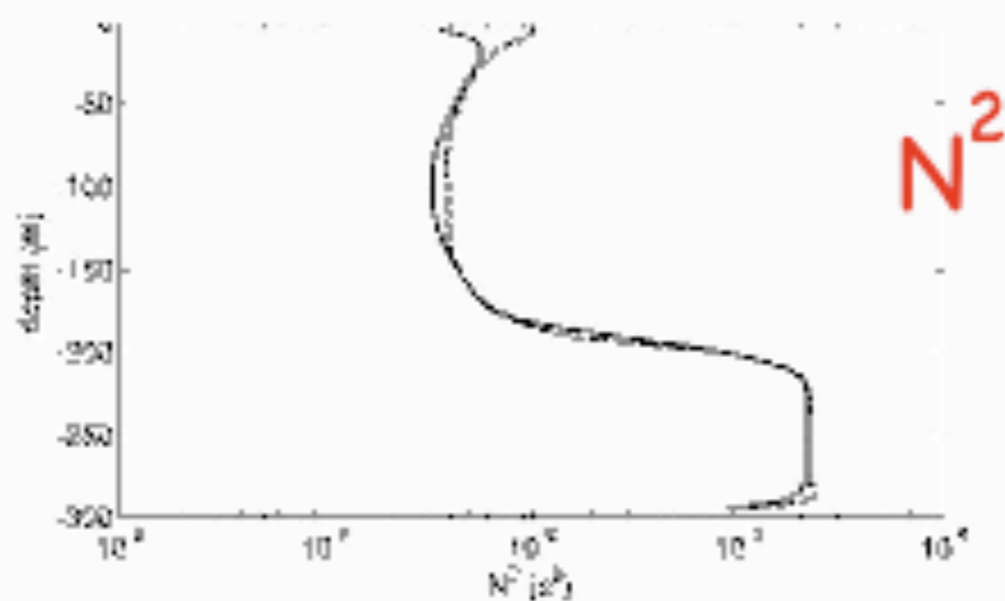
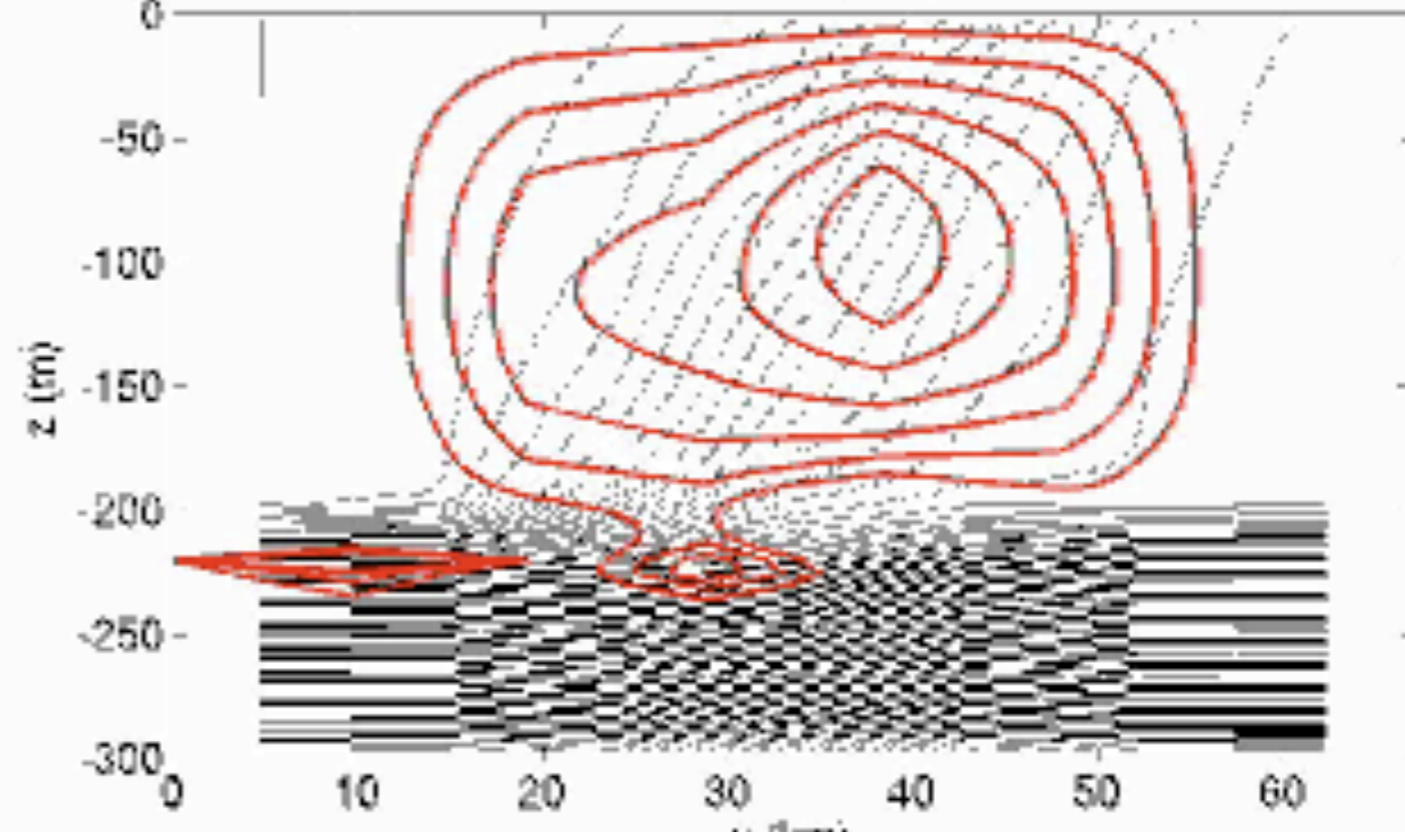
## Parameterization (2d, 10km grid)

16d00h from 2d parameterization



## Submesoscale-Resolving (3d, 500m grid)

16d00h from 3d MITgcm (smoothed)



# The Global Parameterization:

$$\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{\mathbf{z}}$$

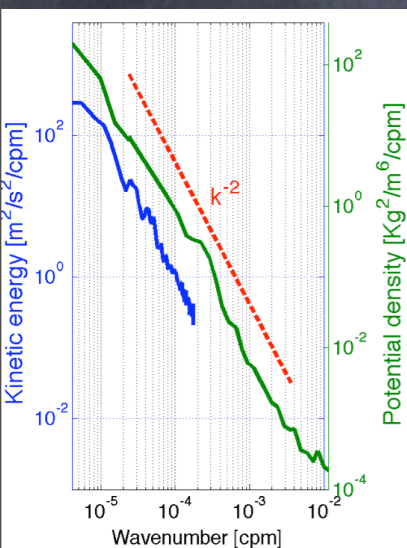
$$\mu(z) = \left[ 1 - \left( \frac{2z}{H} + 1 \right)^2 \right] \left[ 1 + \frac{5}{21} \left( \frac{2z}{H} + 1 \right)^2 \right]$$

Account for equator by going to subinertial ML approx (Young 94)

$$\Psi = \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla \bar{b} \times \hat{\mathbf{z}}$$

Account for coarse res.

$$E_b(k) \sim k^{-2} \rightarrow \Psi = \left[ \frac{\Delta x}{L_f} \right] \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla \bar{b} \times \hat{\mathbf{z}}$$

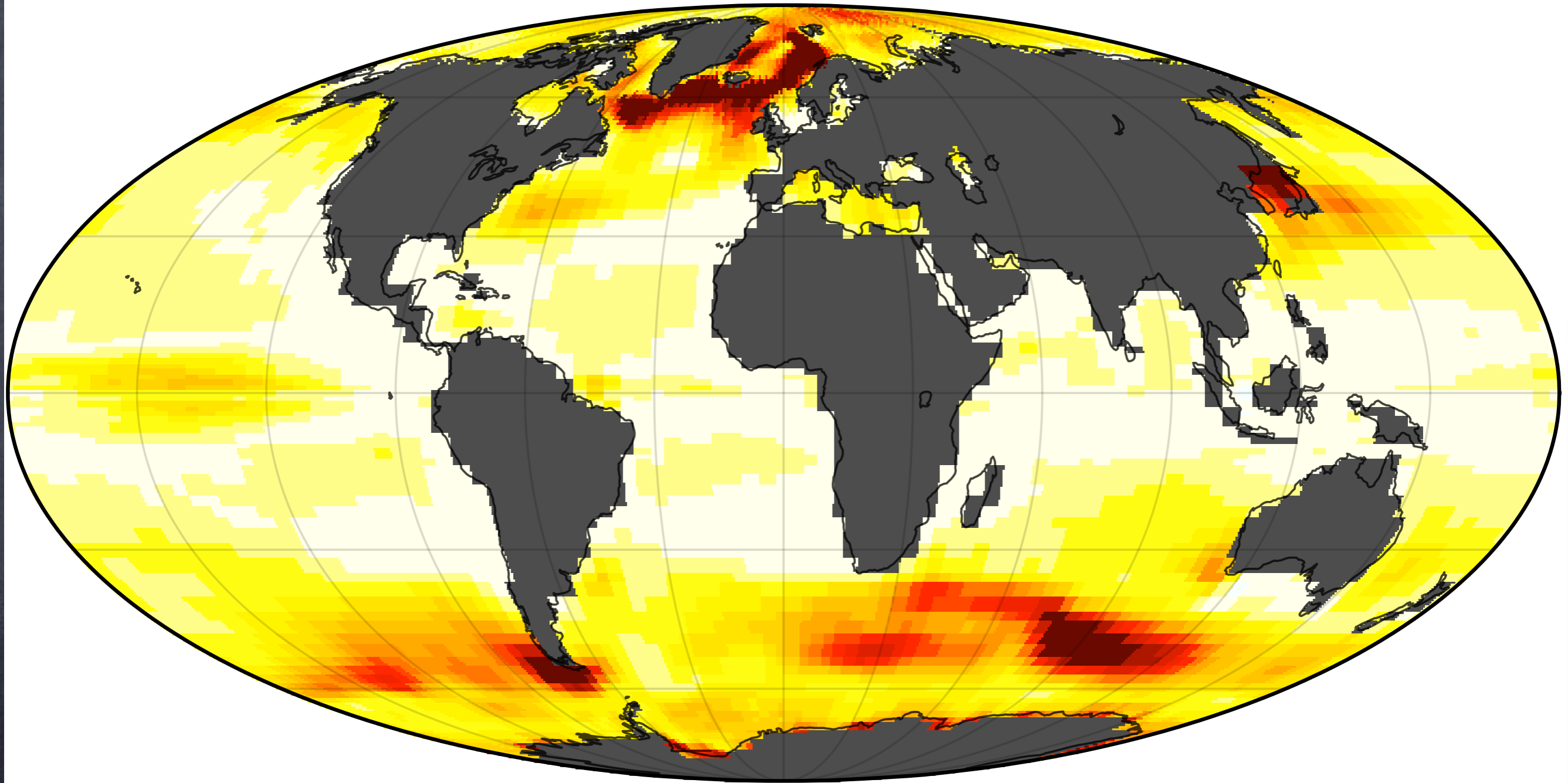


Obs. reveal (Hosegood et al., 2006):  $L_f \sim R_d$

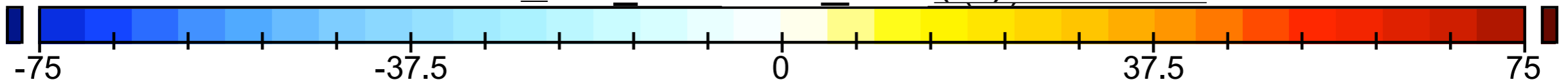
# Improves Restratification after Deep Convection

Note: param. reproduces Haine&Marshall (98) and Jones&Marshall (93,97)

## Change of Time-Mean Boundary Layer Depth in POP

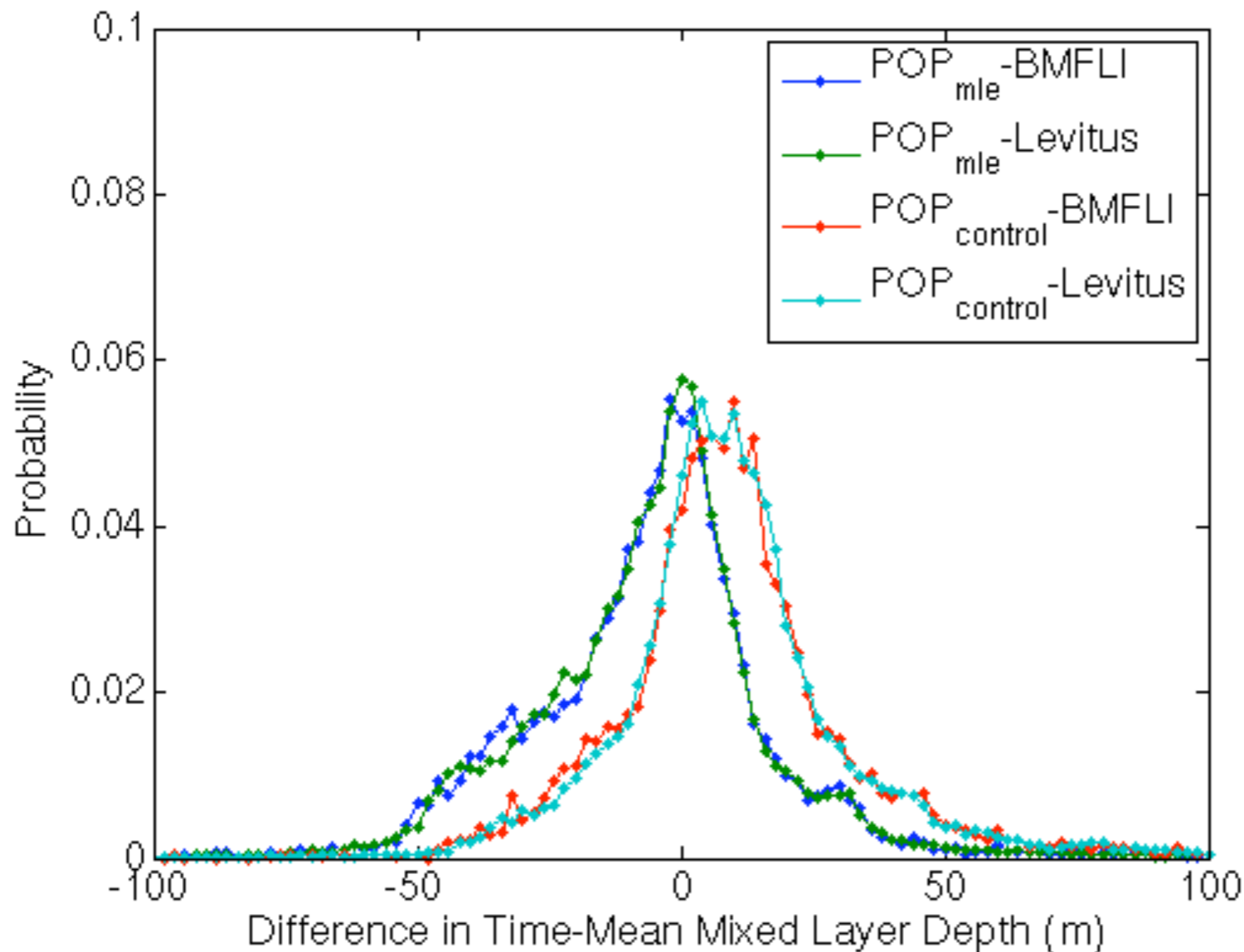


BLD\_control - BLD\_mle (m)





# Bias Reduction in POP/CCSM Mixed Layer Depth



RMS error:  
16m  
reduced to  
8m

Skewness:  
2.4  
reduced to  
0.6

# Submeso Eddy Conclusion:

- Submesoscale features, and mixed layer eddies in particular, exhibit large vertical fluxes of buoyancy and tracer that until recently were ignored in climate models.
- A parameterization of mixed layer eddy fluxes as an overturning streamfunction is proposed. The magnitude comes from extraction rate of potential energy, and the vertical structure resembles the Eady solution.
- Many observations are consistent, and model biases are reduced. Biogeochemical effects are likely, as vertical fluxes and mixed layer depth are changed.
- In HIM, CCSM, MITgcm, and MOM.
- 4 Papers so far... [fox-kemper.com/research](http://fox-kemper.com/research)

# Langmuir Circulations in CCSM

With [A. Webb](#), W. Large, S. Peacock, G. Chini, K. Julien, E. Knobloch

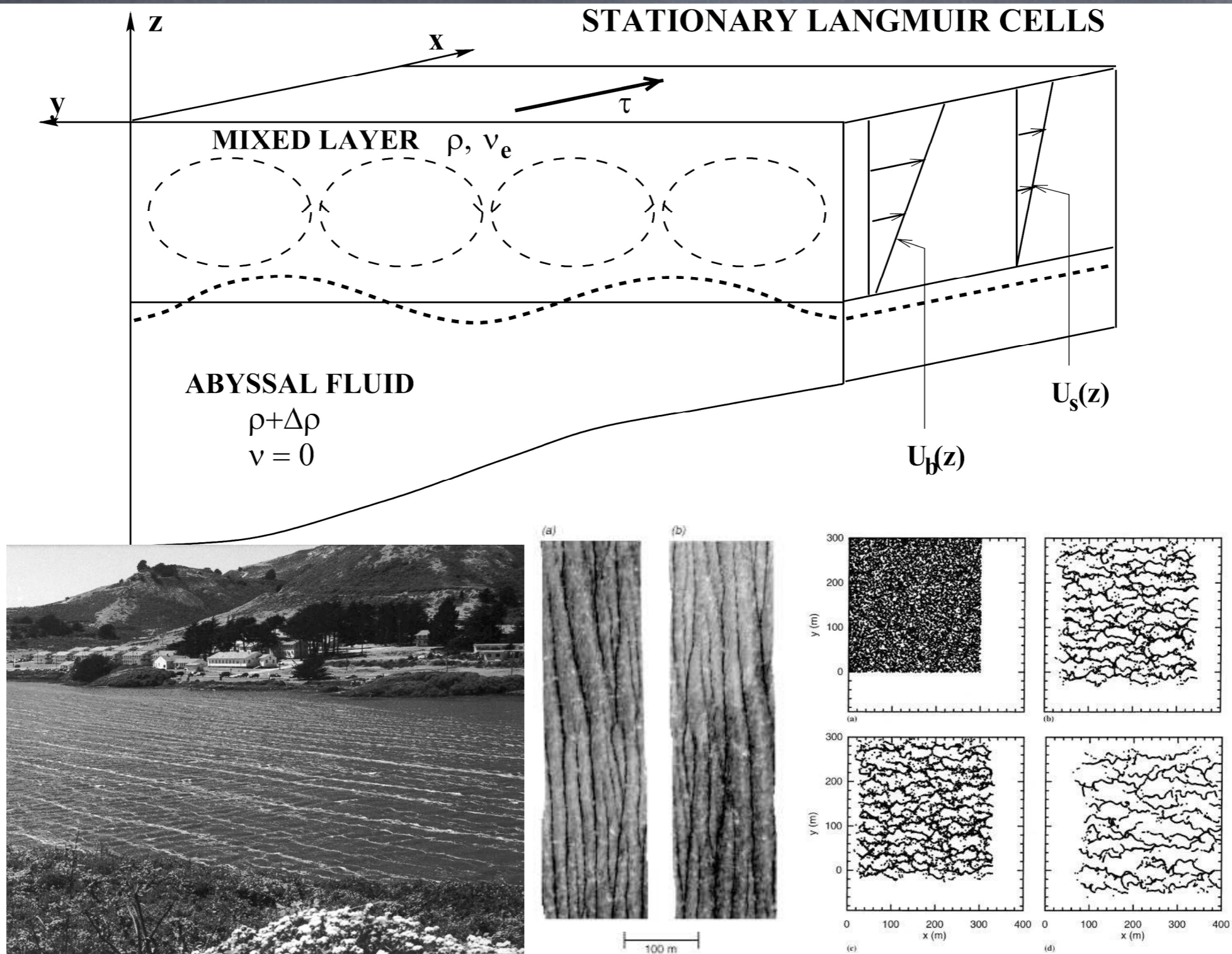


FIGURE 1: Images of Langmuir circulation windrows: (a) a photograph of Rodeo Lagoon in CA (from Szeri, 1996), (b) an infrared image of the surface of Tampa Bay (courtesy of G. Marmorino, NRL, D.C.), and (c) the evolution of surface tracers in a LES of Langmuir turbulence (McWilliams et al., 1997). Reproduced from Chini et al. (2008).

# Langmuir, do we care?

Maybe:

## **Role of Langmuir Circulation in the Deepening of the Ocean Surface Mixed Layer**

Ming Li,\* Konstantin Zahariev, Chris Garrett

Helical motions, known as Langmuir circulation, are a key physical process in the upper ocean but have not yet been incorporated into ocean models. Here, surface mixed layer deepening by Langmuir circulation was added to that due to convection or velocity shear; Langmuir circulation is more important than shear if the velocity difference across the mixed-layer base is less than about 1 percent of the wind speed. In an upper ocean data set, evidence was found for the deepening of the mixed layer by both mechanisms. Thus, Langmuir circulation influences upper ocean diurnal and seasonal changes in stratification.

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Maybe Not:

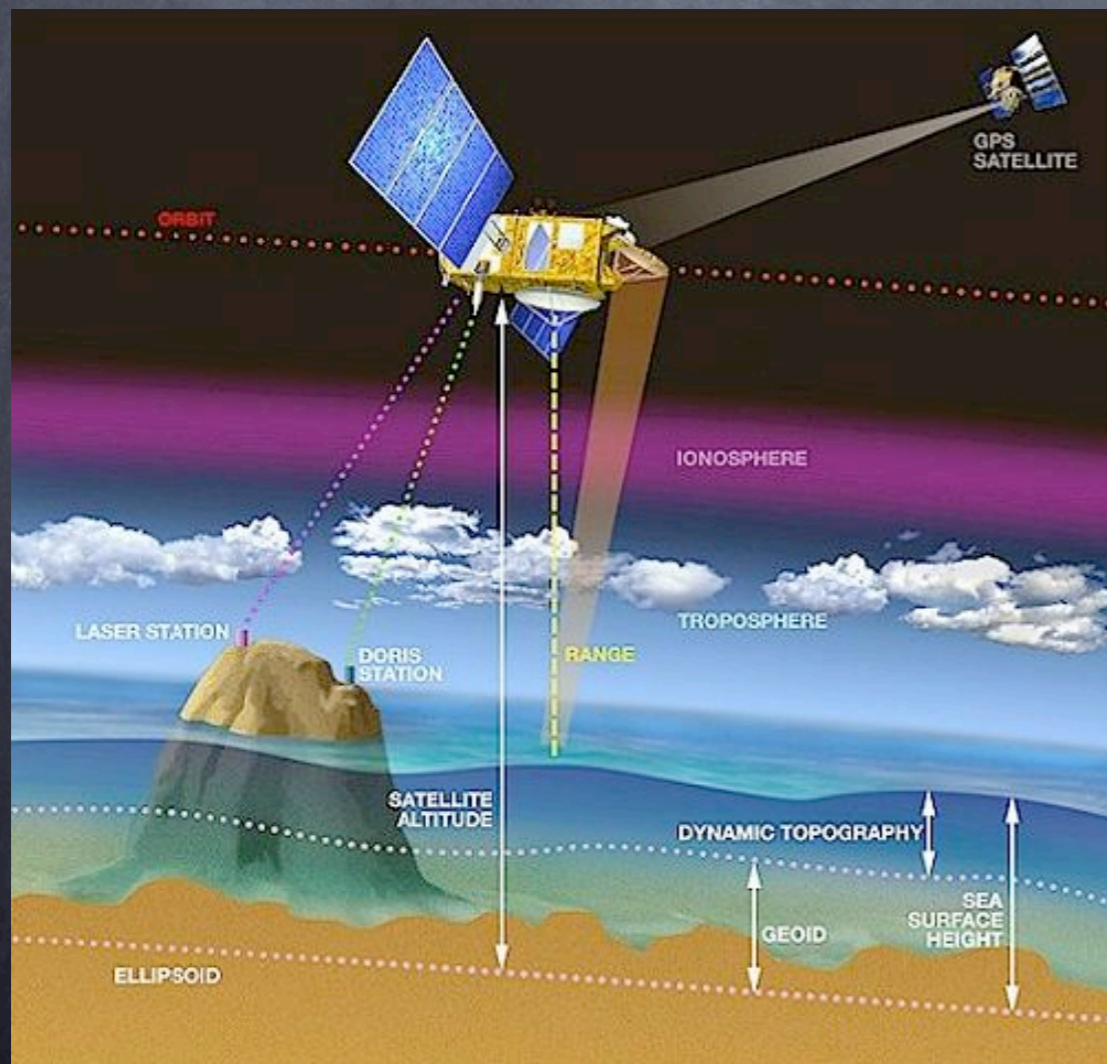
## **Langmuir circulation within the oceanic mixed layer**

ROBERT A. WELLER\* and JAMES F. PRICE\*

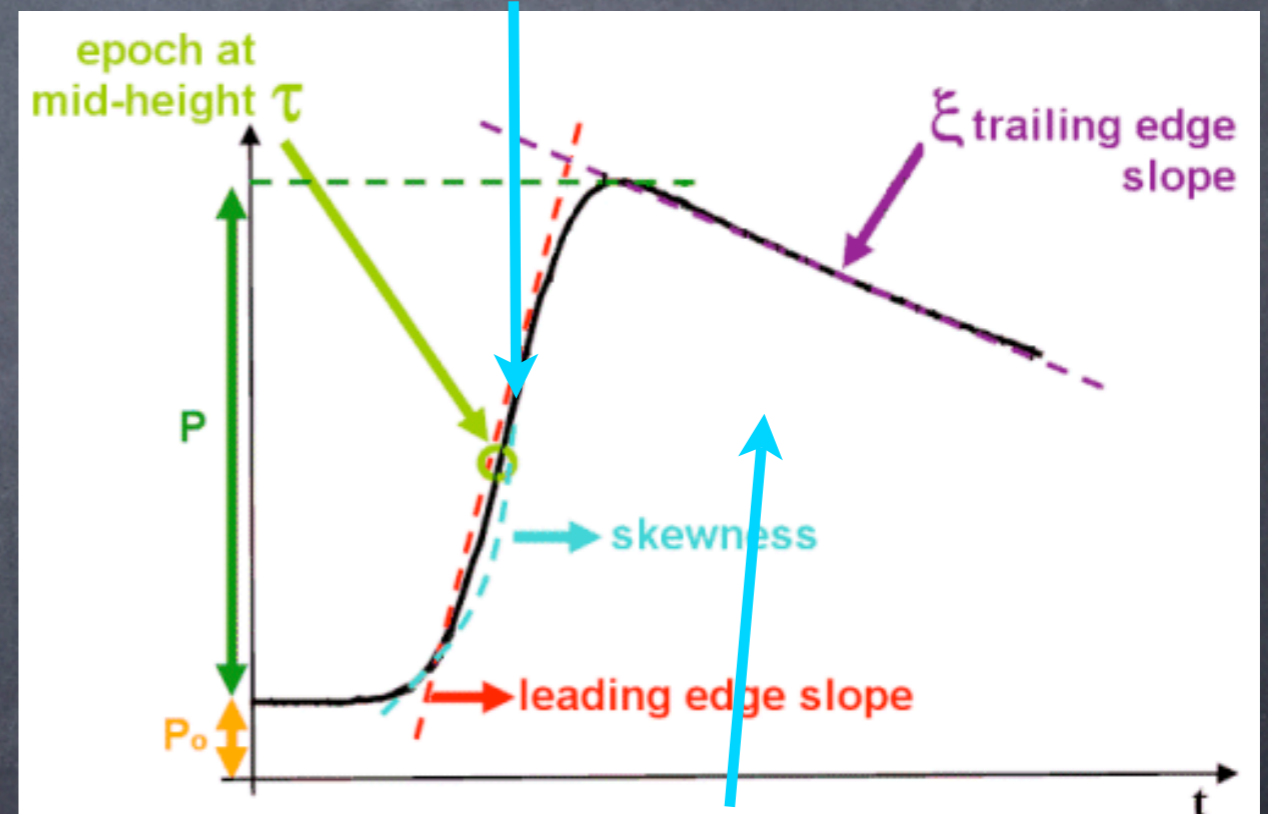
some occasions, when Langmuir cells appeared suddenly, they were able to mix the weak near-surface stratification that had formed in response to diurnal heating. They could also maintain large shears in the well-mixed fluid near the surface. They did not, however, penetrate with strength to the base of relict mixed layers observed during summer-like conditions or to the base of deeper, more isothermal, mixed layers observed during stormy conditions.

# Waves+Wind != Wind

- Langmuir is 'in' KPP, but only based on wind
- Really Langmuir depends on both  $u^*$  and  $u_s$
- Is there data? Altimeters do both simultaneously



## Leading Slope $\rightarrow$ Swell



Inverse Area  $\rightarrow$  Capillaries  
Thus, Local wind

# Waves+Wind != Wind

With wave period assumed to produce a peak at fully-developed waves (uniform period for now...)

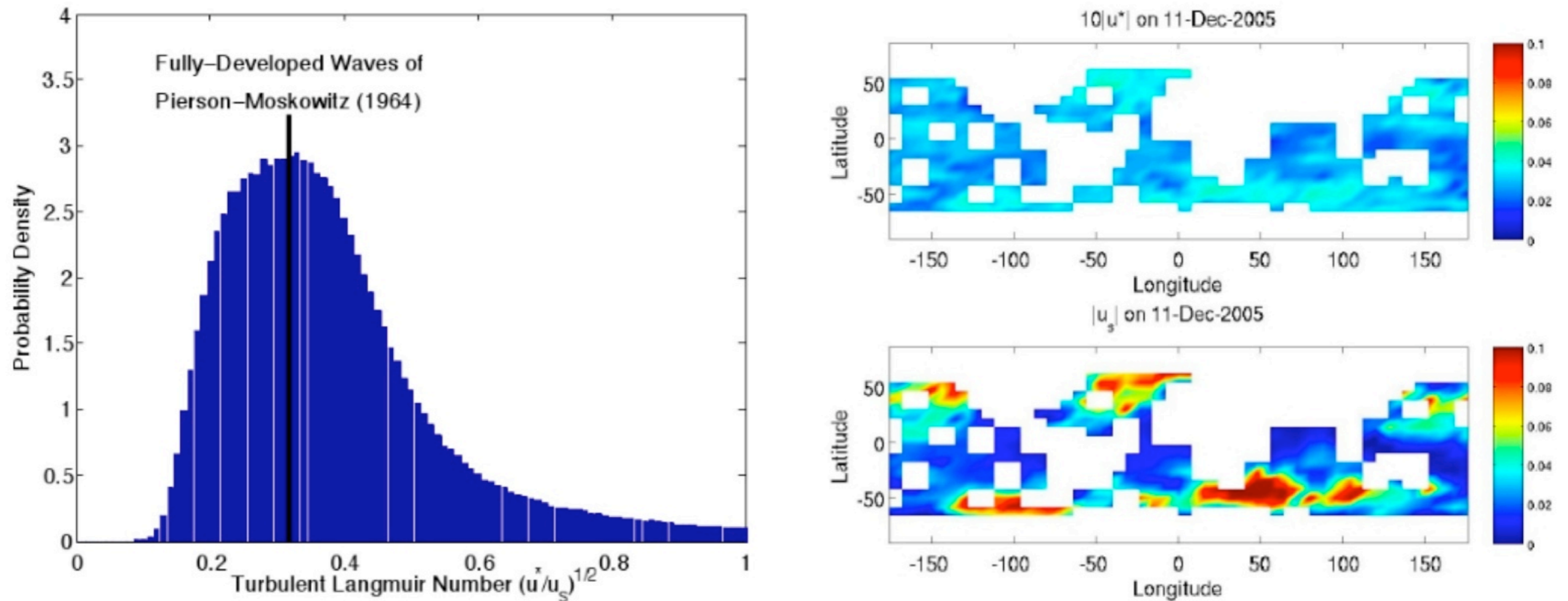


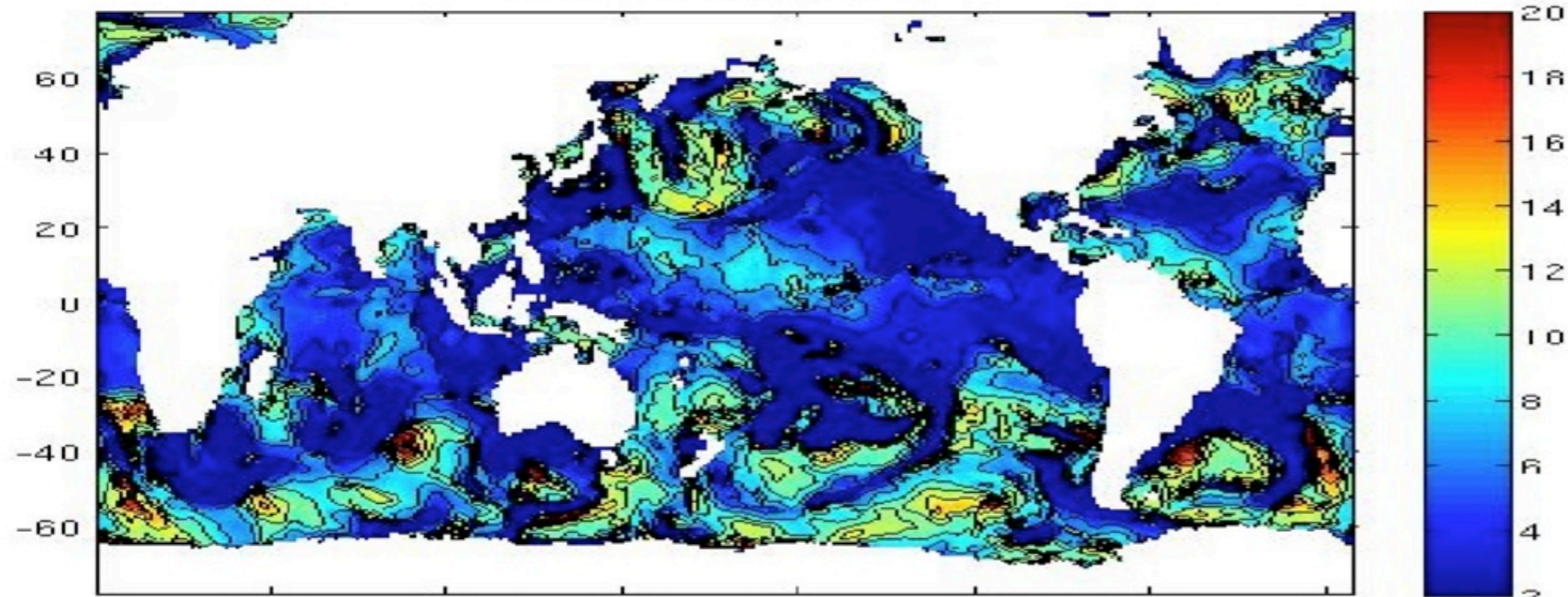
Figure 2: Aviso merged satellite dataset from 11/12/05 to 5/27/08 was used to calculate the (a) average Langmuir number and (b) compare  $10|u^*|$  to  $|u_s|$

$$u^* \equiv \sqrt{\tau/\rho} \quad u_s \approx \frac{\pi^3 H_s^2}{gT_s^3} \quad La \equiv \sqrt{u^*/u_s}$$

# Wave Model--agree with Obs, plus frequency and direction

Comparison Between  $1/La^2$  and NWW3 on 5/21/08

$1/La^2$  - Sec/Int (05/21)



NWW3 20080521 t06z hindcast

NWW3 20080521 t05z hindcast

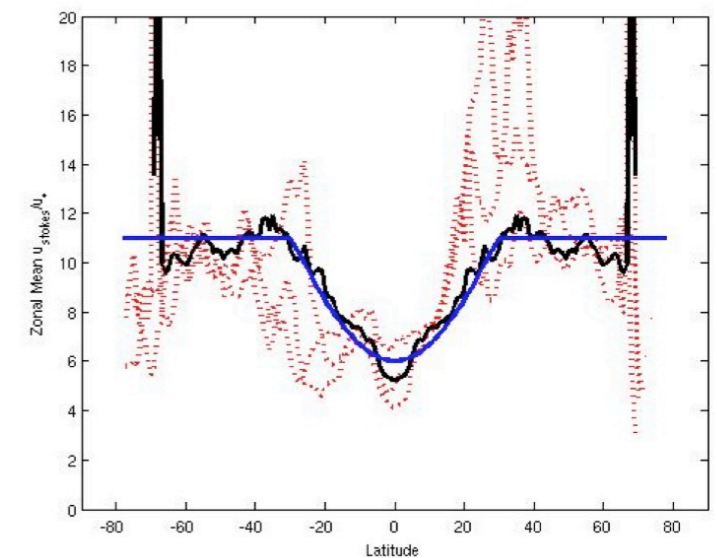
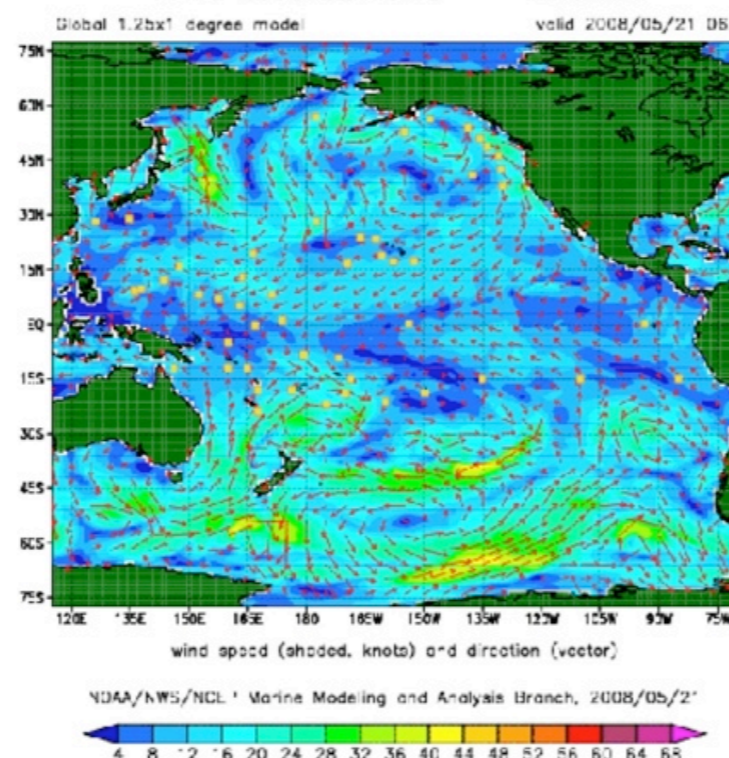
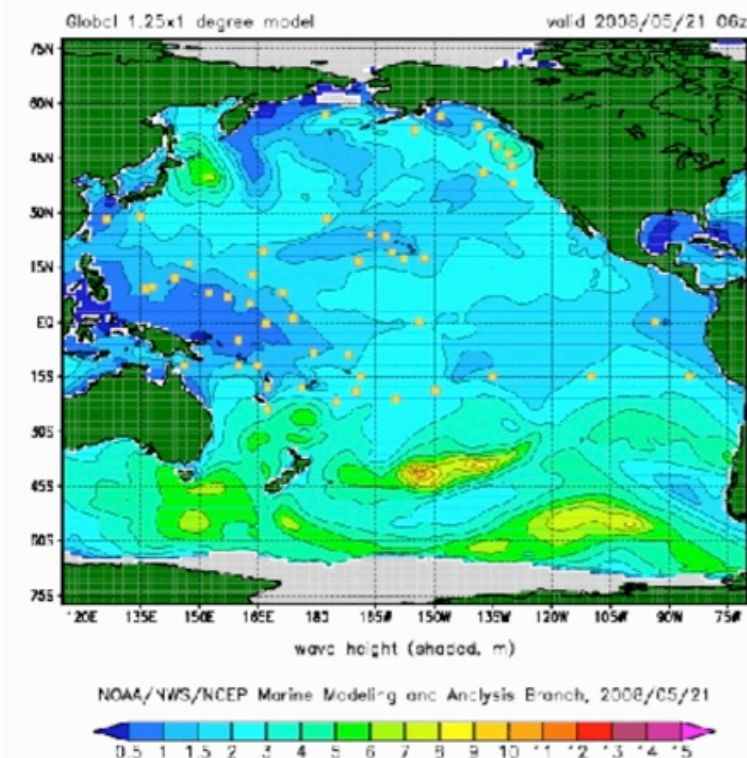


Figure 4: Climatology of  $(La^{-1})^2$  (blue) based on zonal and seasonal averages (black) with summer seasonal data (red)

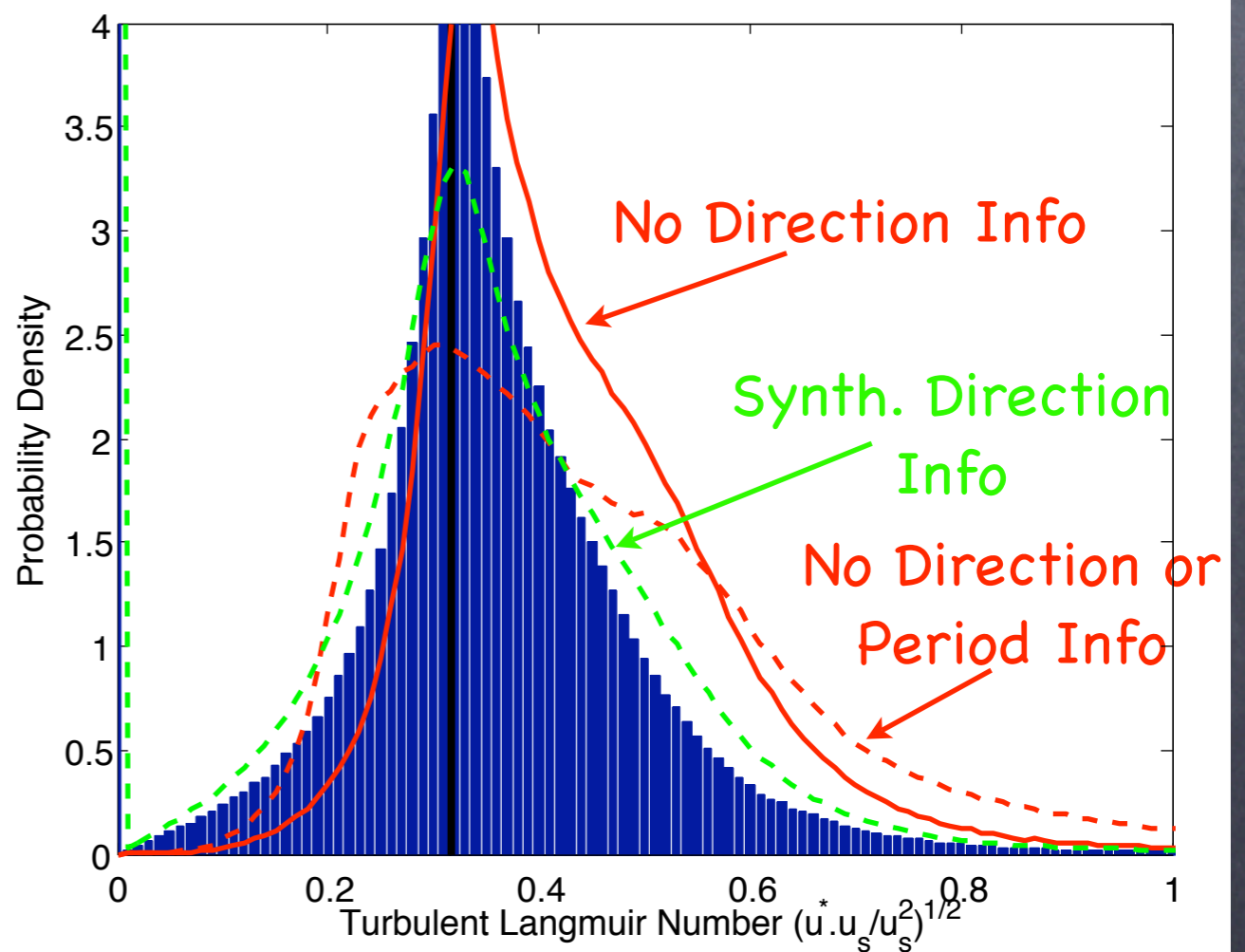
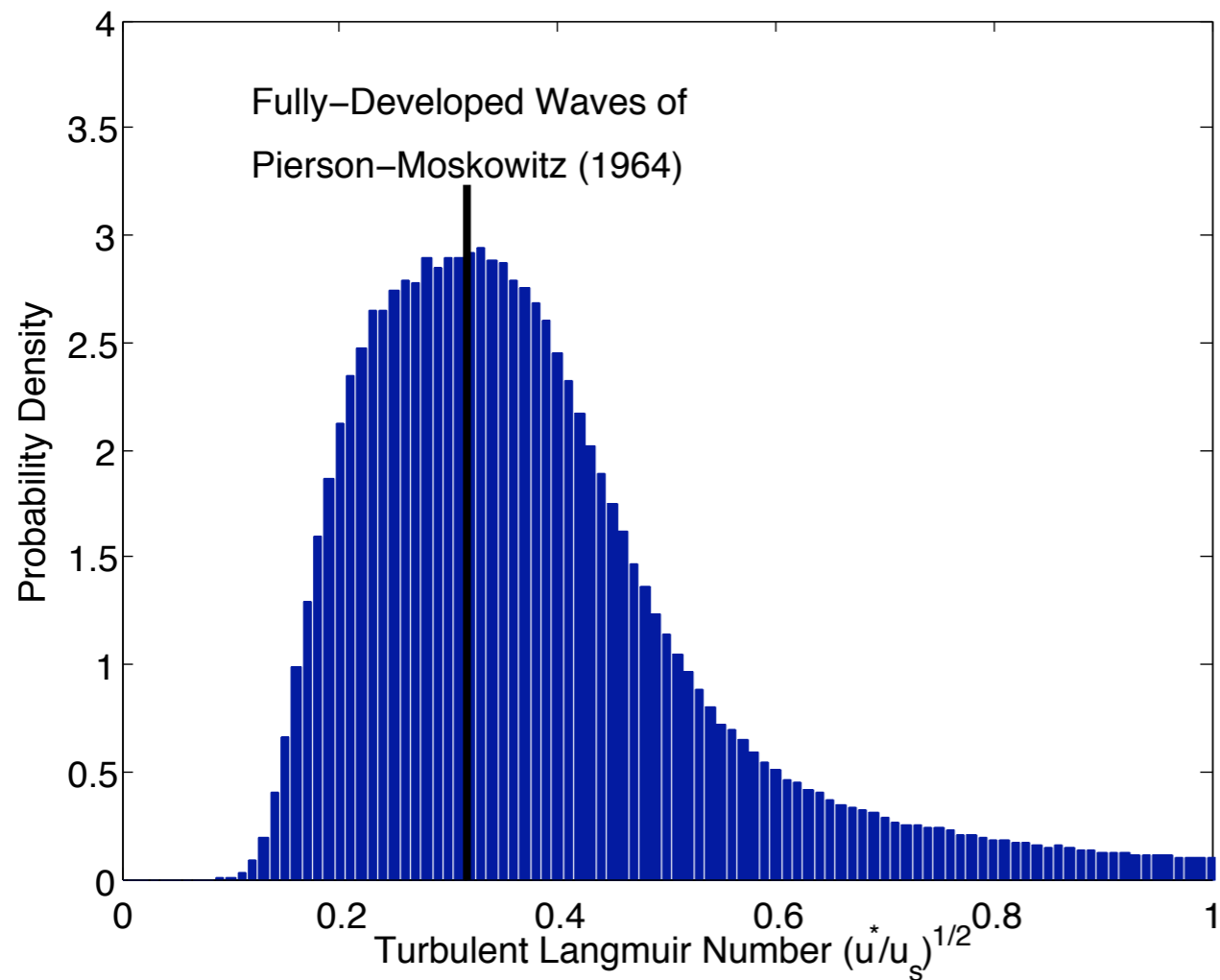
Provides wave  
period &  
direction:  
for better Stokes Drift

Figure 3: Calculation of inverse turbulent Langmuir number squared,  $(La^{-1})^2$ , (top) using NOAA WaveWatch III model global output data (bottom)

# Satellite versus WW3 Model

Stokes' Drift is *sensitive* to wave period.

$$u_s \approx \frac{\pi^3 H_s^2}{g T_w^3}$$



Aviso Merged Satellite Dataset, 11/12/05-05/27/08

WaveWatch III Data-Assimilating Wave Model  
(rescaled to find surface wind-stress)



# A Simple Scaling for Langmuir

Depth/Entrainment:

(Li & Garrett, 1997)

related to  
CAM  $u^*$  by  
WW3  
Climatology

CAM

$$Fr = \frac{\omega}{NH} \approx 0.6 \quad \omega \approx \frac{V}{1.5} \approx \frac{\sqrt{u^* u_s}}{1.5}$$

## The Algorithm

Use  $Fr$  to determine  $H$

If  $H$  is deeper than KPP Boundary Layer depth, use  $H$

Large came up with clever choices for  $N$ ,  $H$  that  
lead to a robust implementation in KPP

With these choices,  $H$  and BLD converge over time.

# Wave Model--agree with Obs, plus frequency and direction

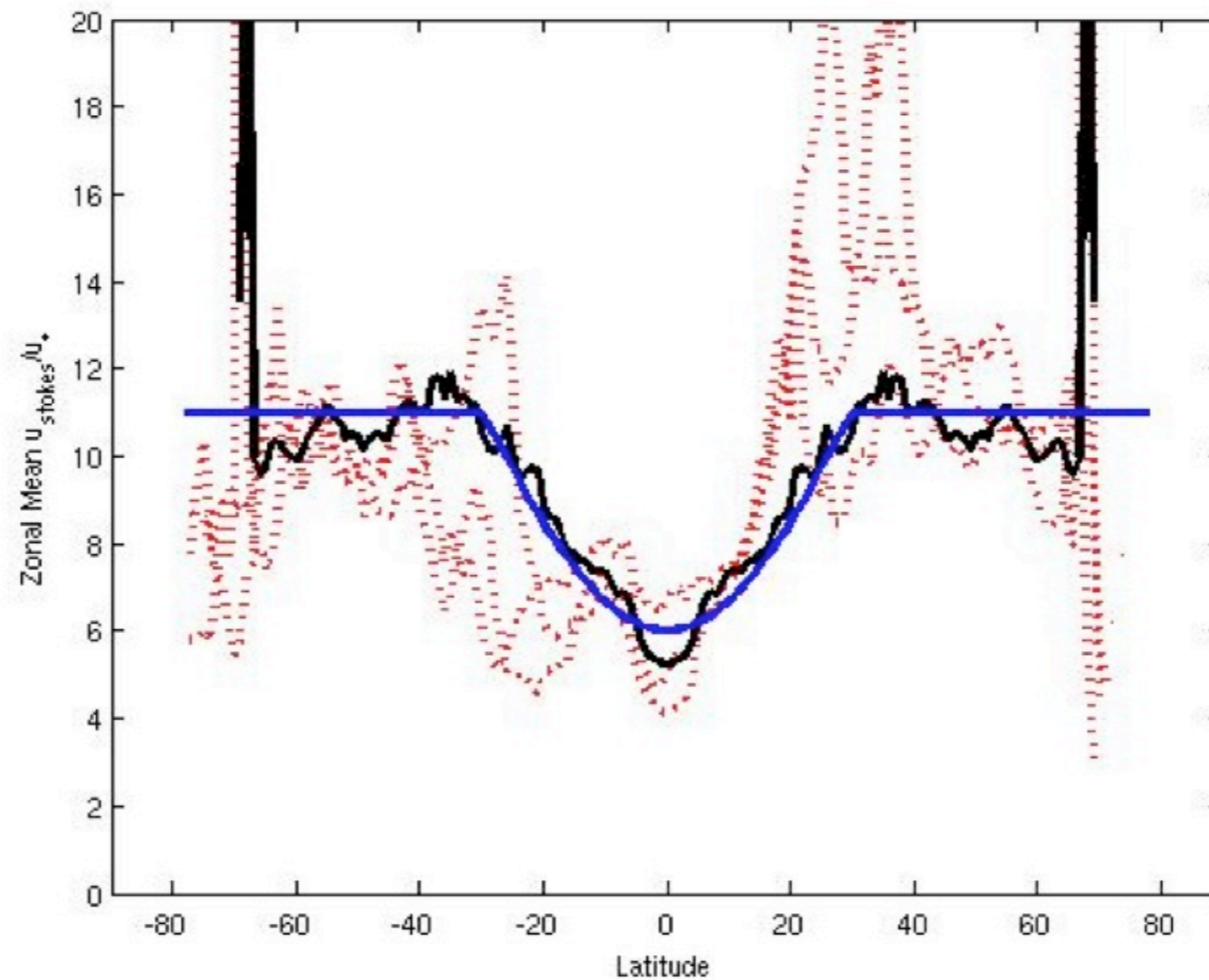


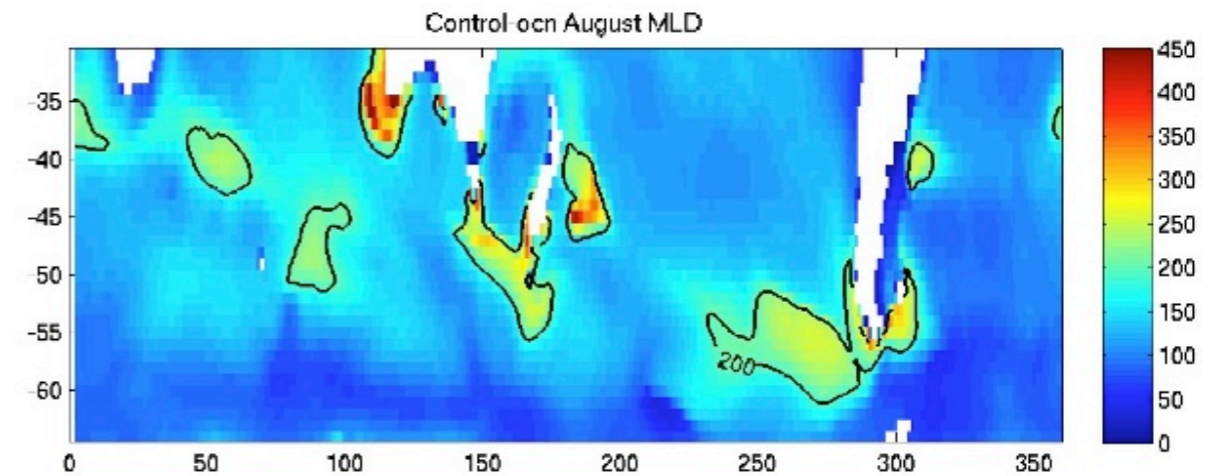
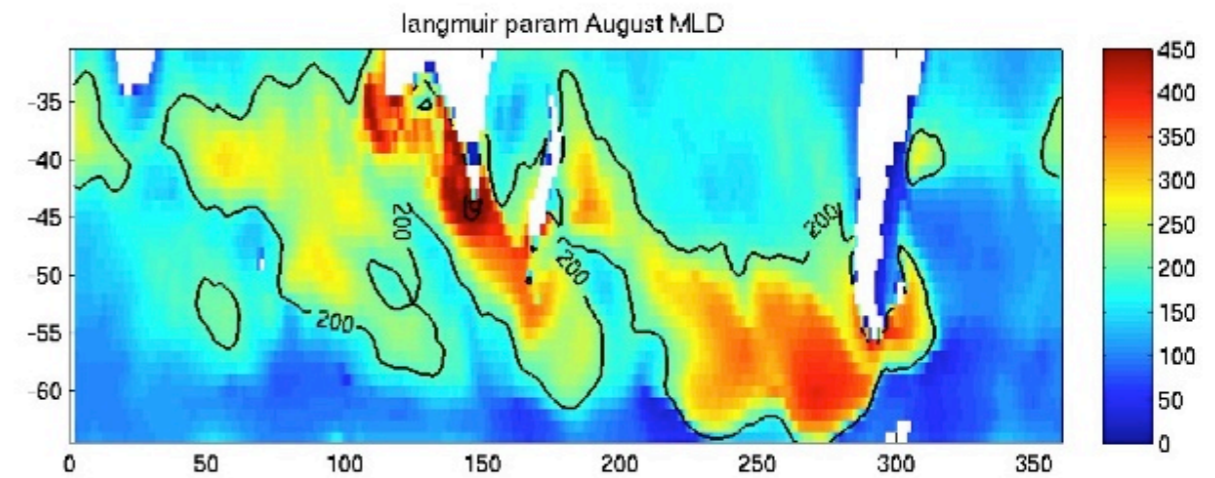
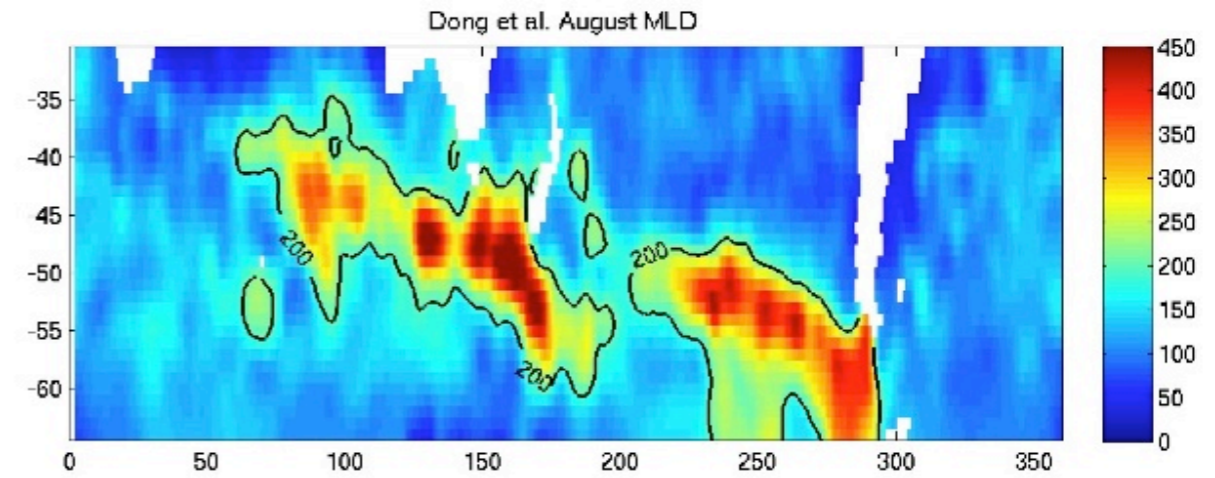
Figure 4: Climatology of  $(La^{-1})^2$  (*blue*) based on zonal and seasonal averages (*black*) with summer seasonal data (*red*)

Assuming for a moment  
that we already know  
how Langmuir  
Circulations scale, then...

Let's try it in a model!

# CCSM3.5 Impact: MLD

- With reasonable parameters, can produce deeper mixed layers
- This often reduces bias in some regions, e.g., ACC

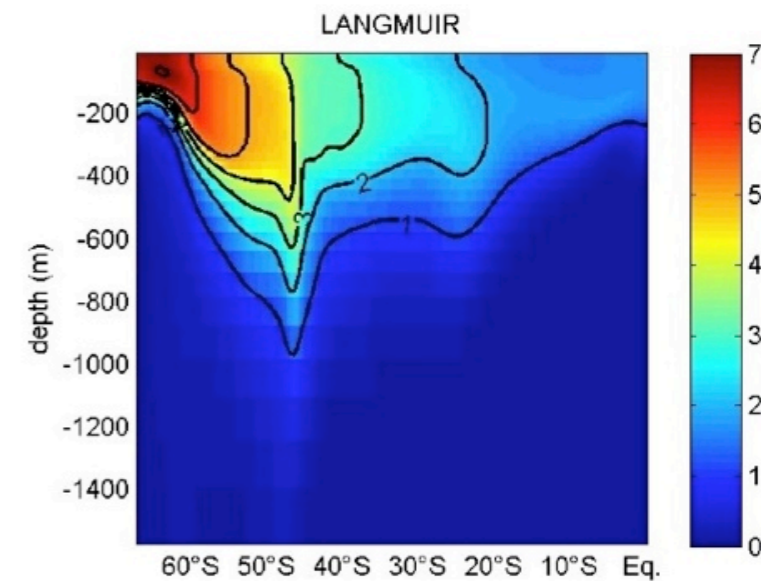
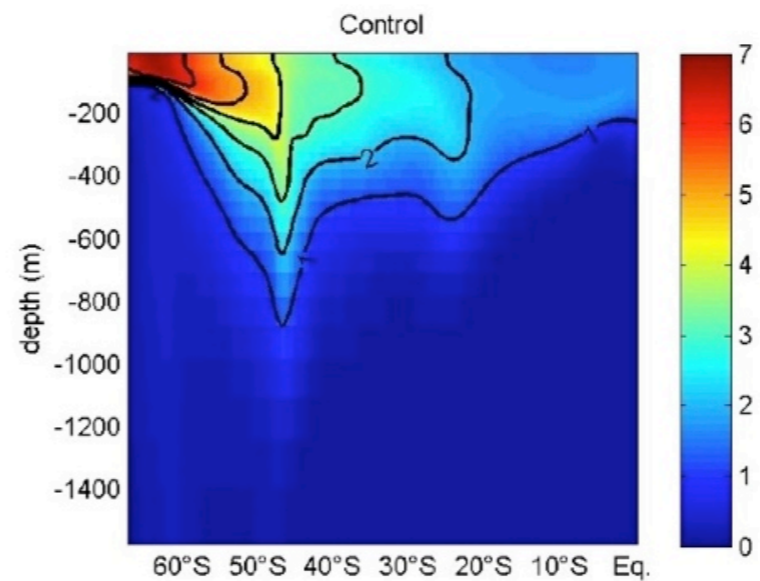
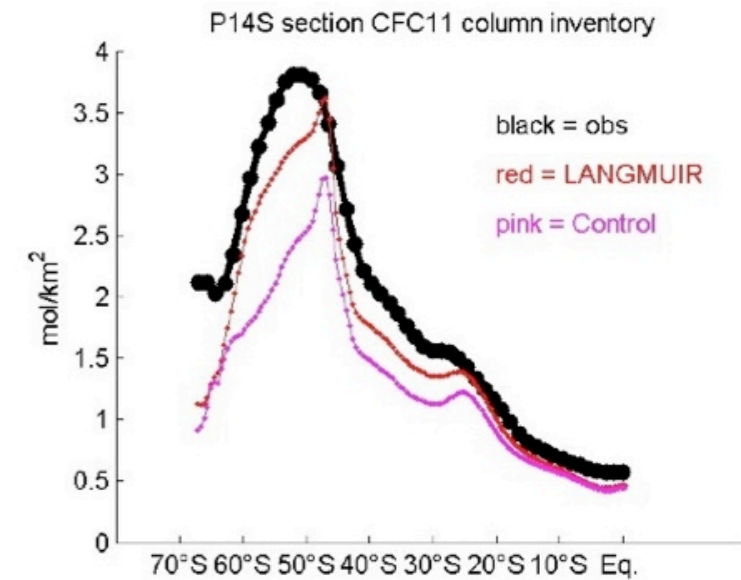
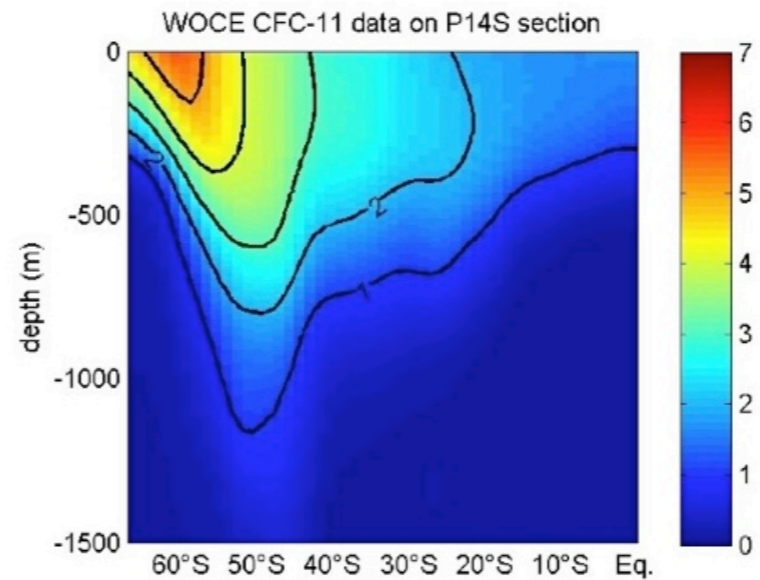


August mixed layer depths.

# CCSM3.5 Impact:

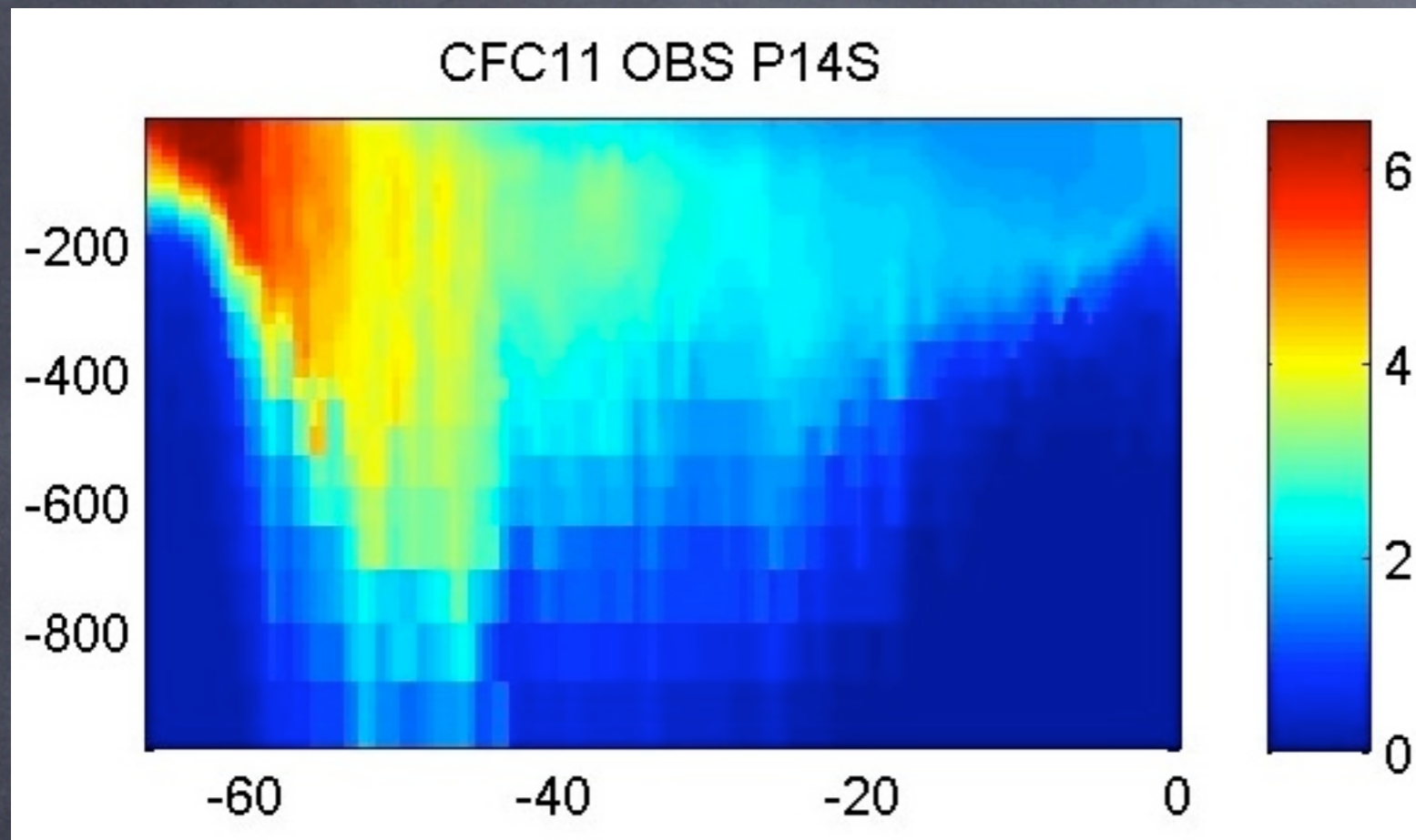
## CFCs

- With reasonable parameters, can affect CFCs
- This reduces bias in some regions, e.g., ACC versus WOCE
- Potentially Large impact, change as large as bias



CFC in CCSM & P14S WOCE observations.

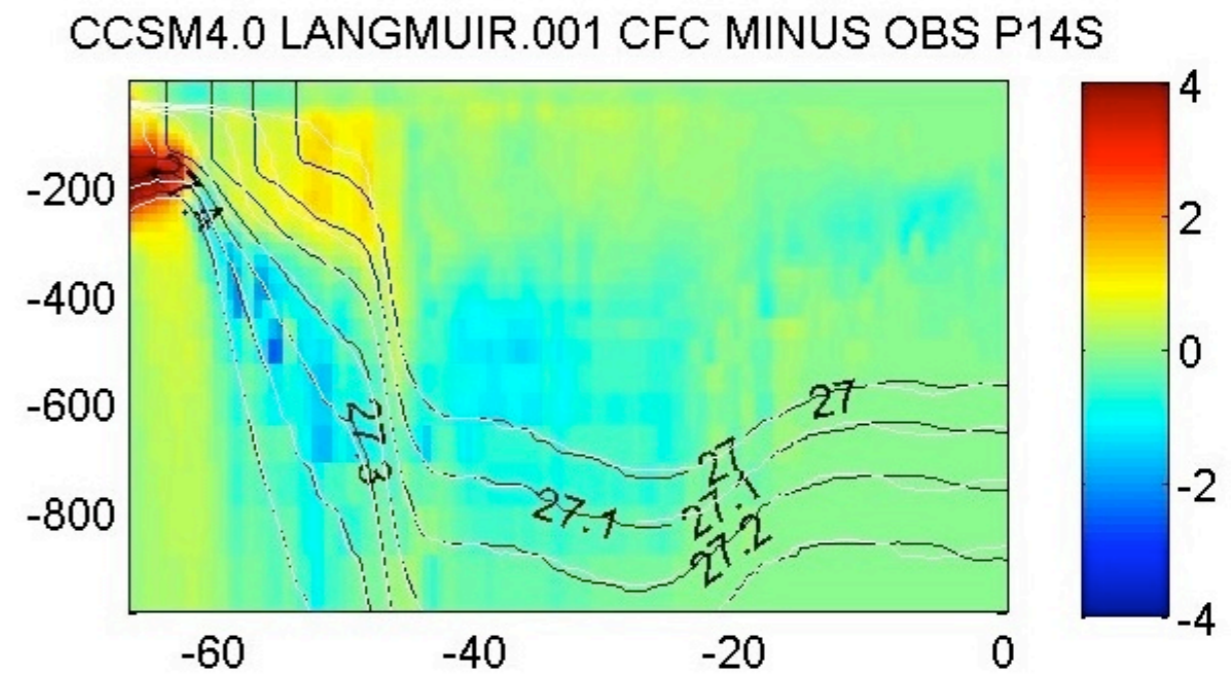
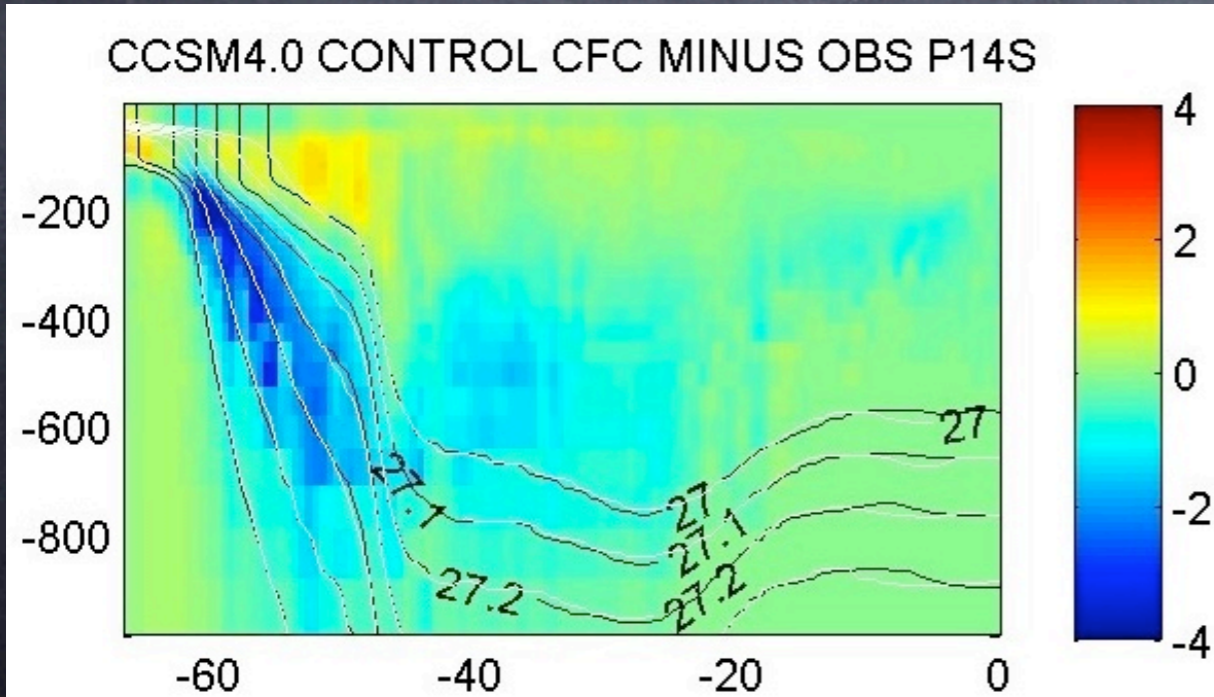
# Nuance--CCSM3.5 and CCSM4.0



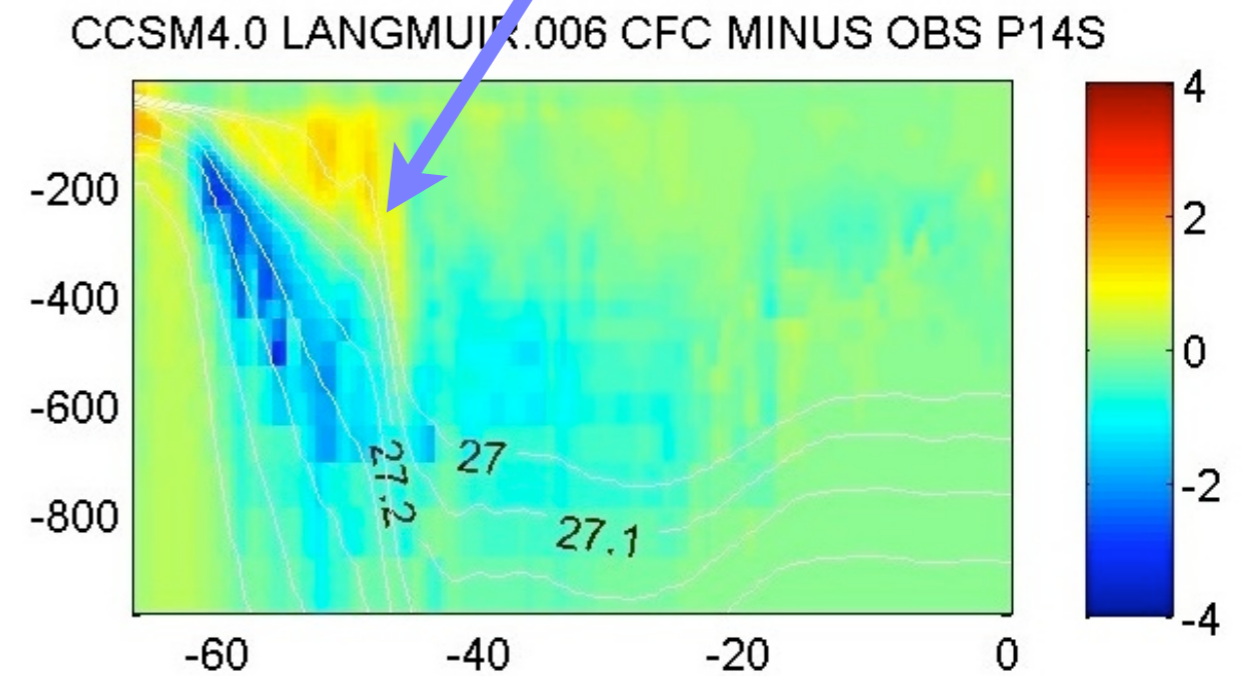
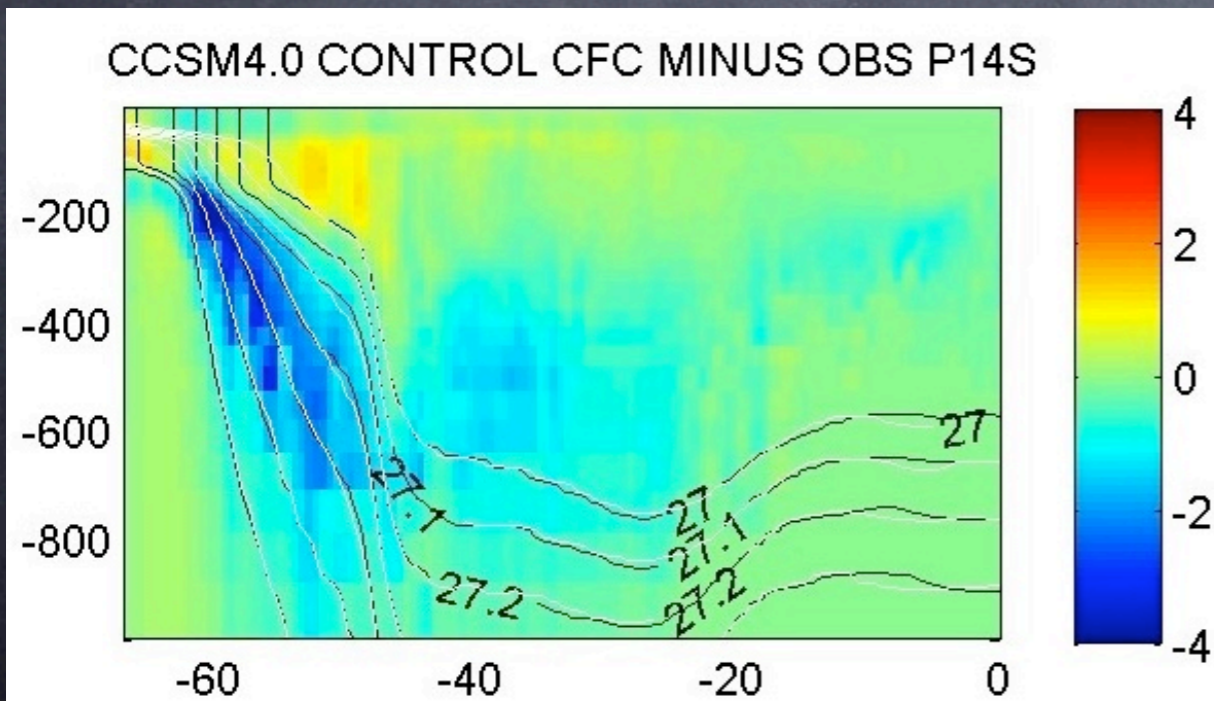
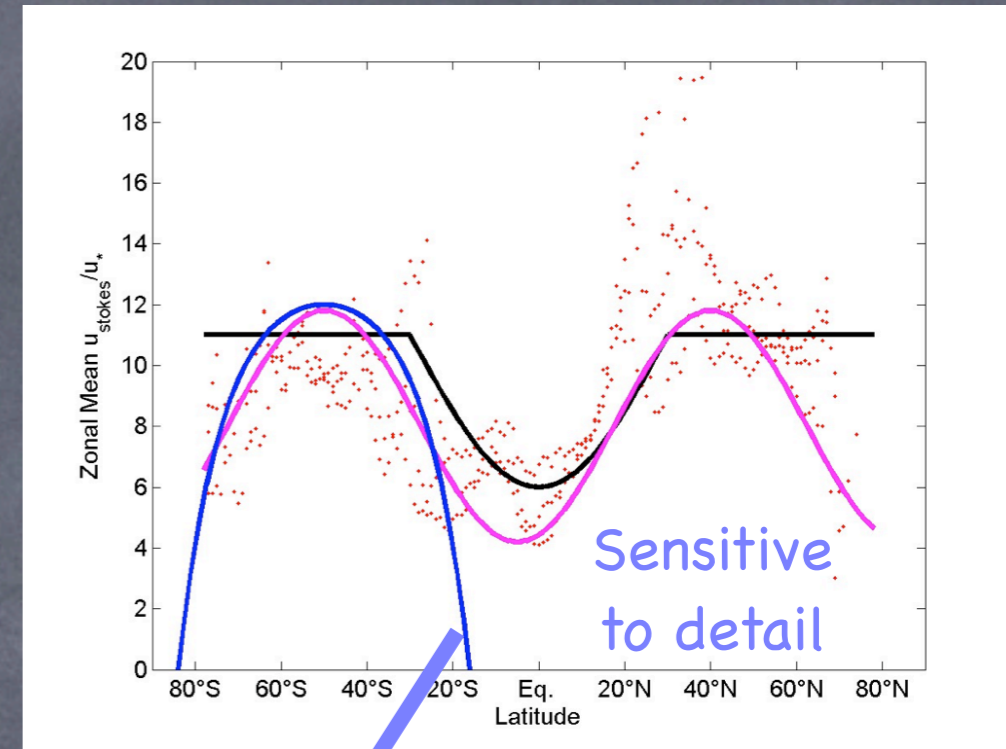
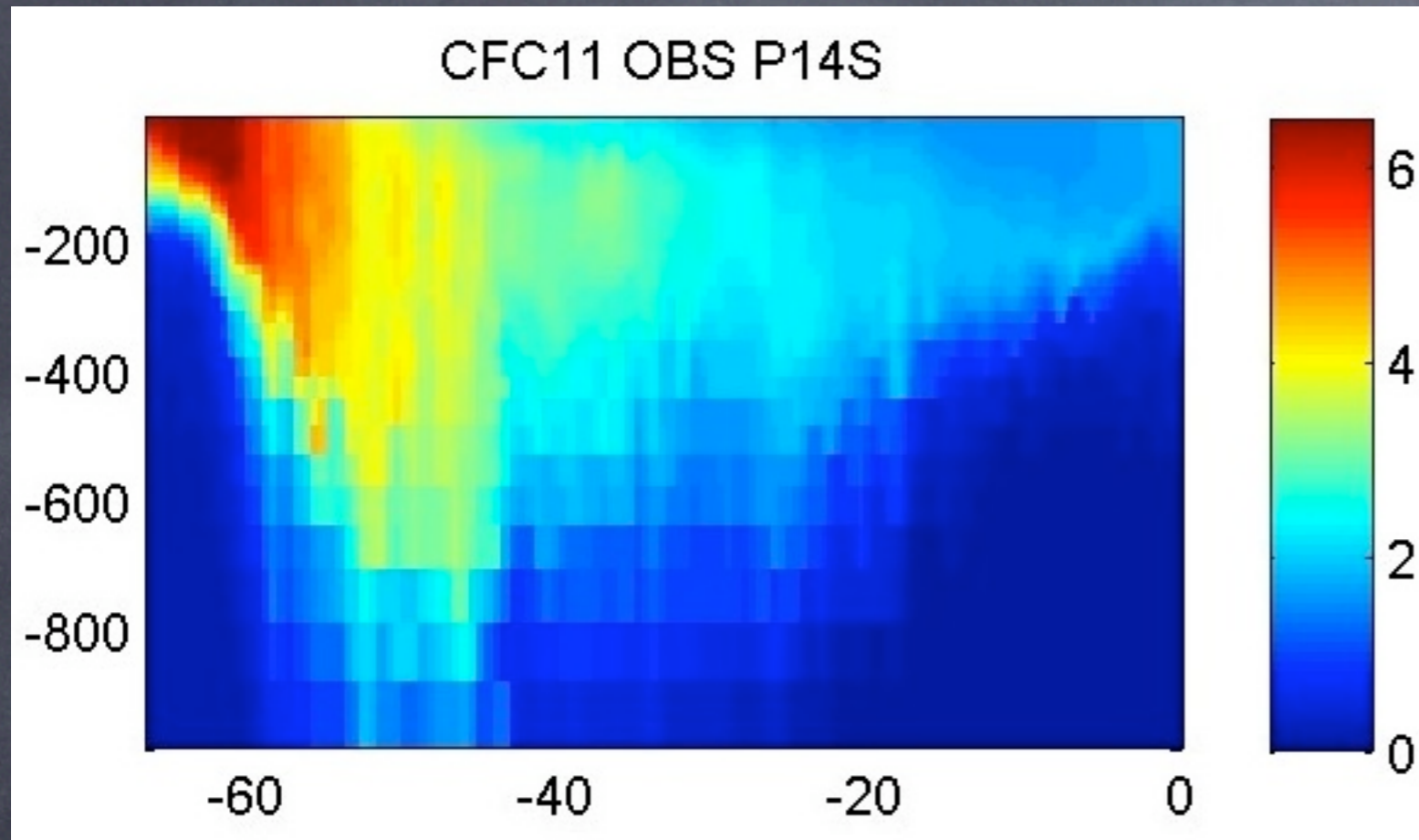
CCSM4.0 did not have the same initial improvement!

S & T particularly bad

Interactions with submeso?



# Nuance--CCSM3.5 and CCSM4.0



# Remaining Problems in Langmuir...

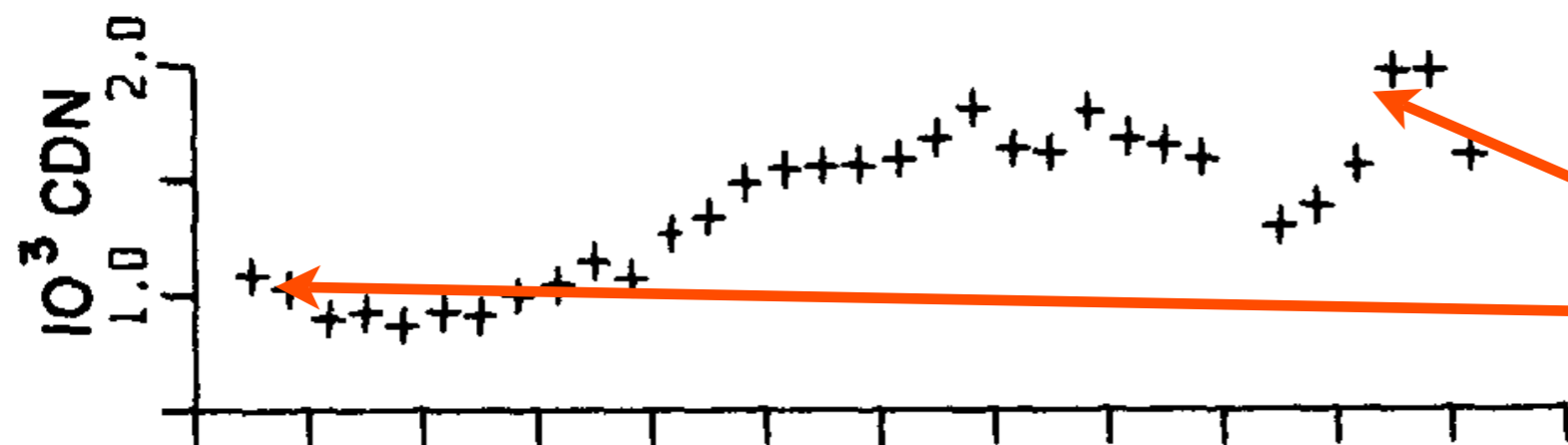
- Demonstrated potential sensitivity and impact, so accuracy needed. It will require:
  - Prognostic Wave Model coupled to CCSM
  - Better Parameterization of Langmuir Circulation mixing
    - Better understanding of regimes of Langmuir scalings



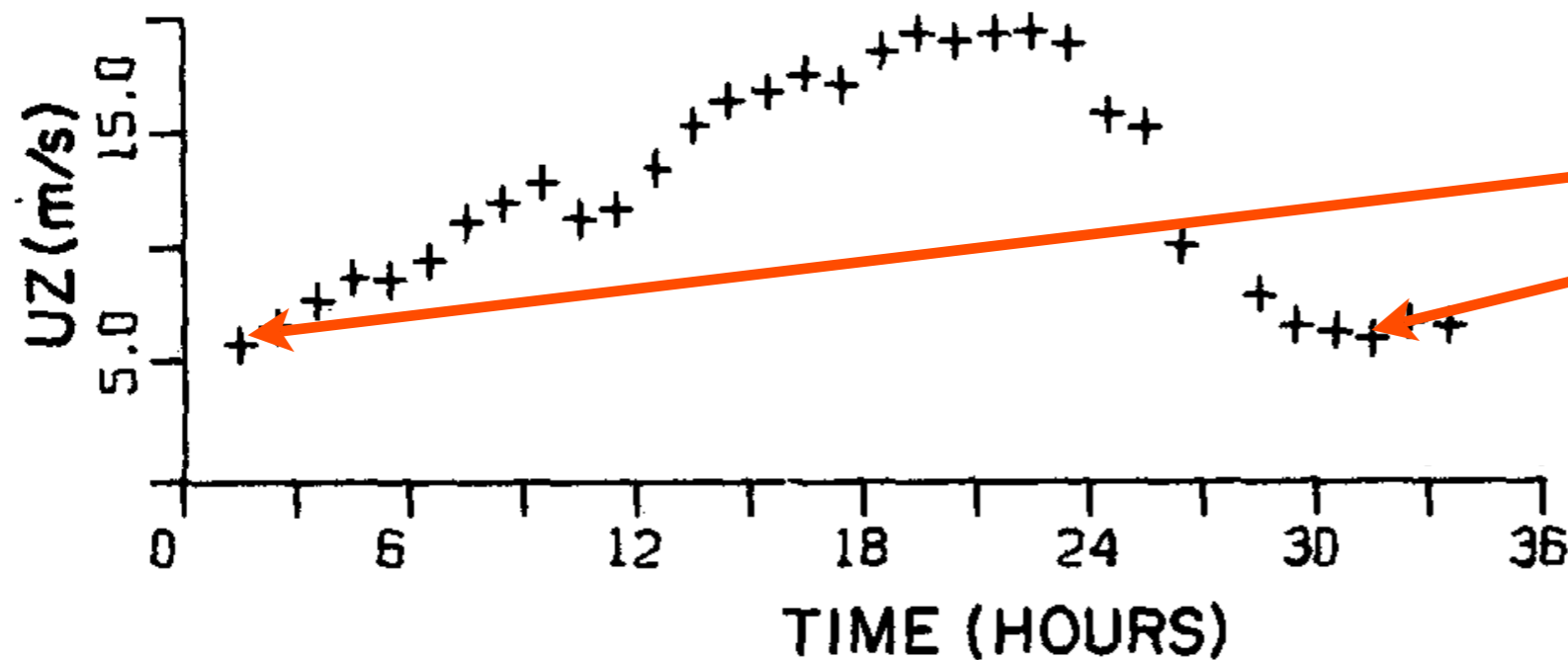
# Other Effects of Wind+Waves != Wind

MARCH 1981

W. G. LARGE AND S. POND



Different  
Drag

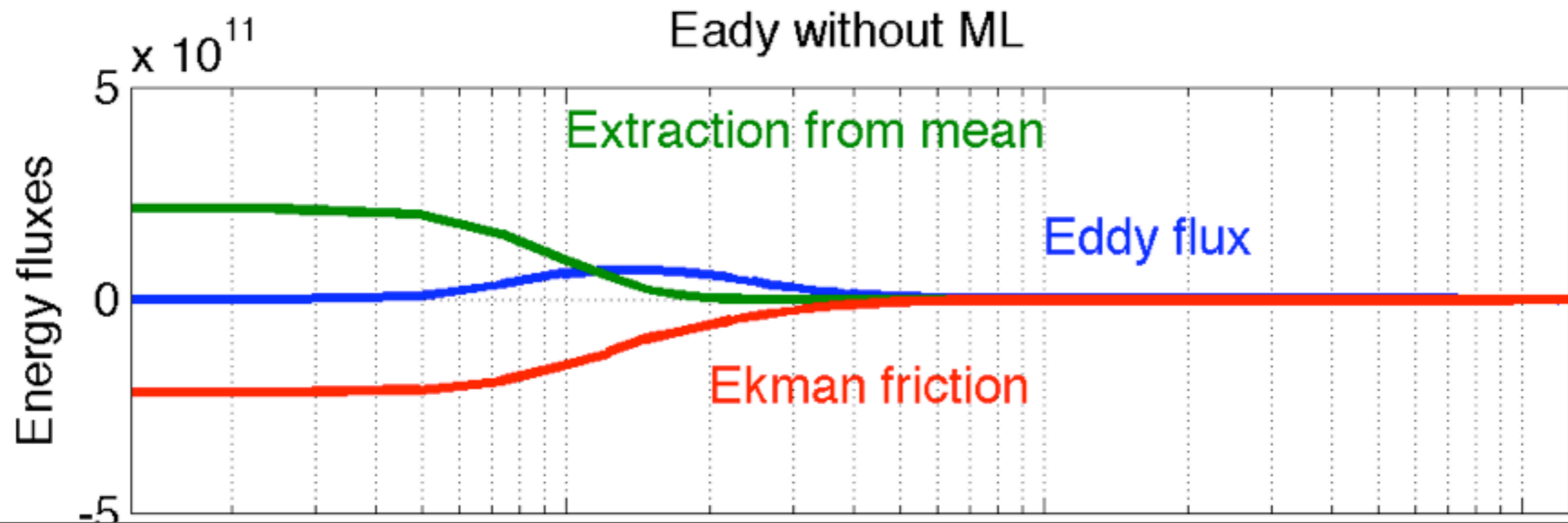
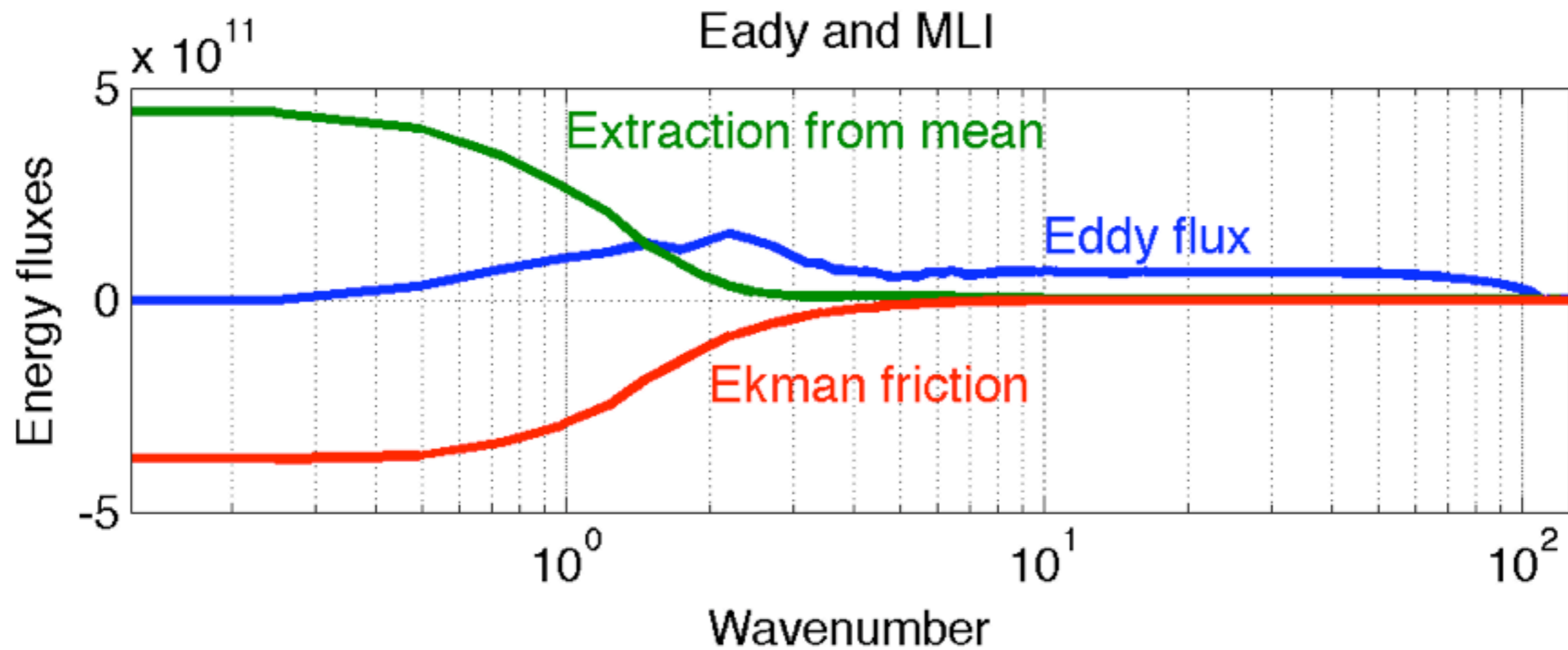


Same  
Wind

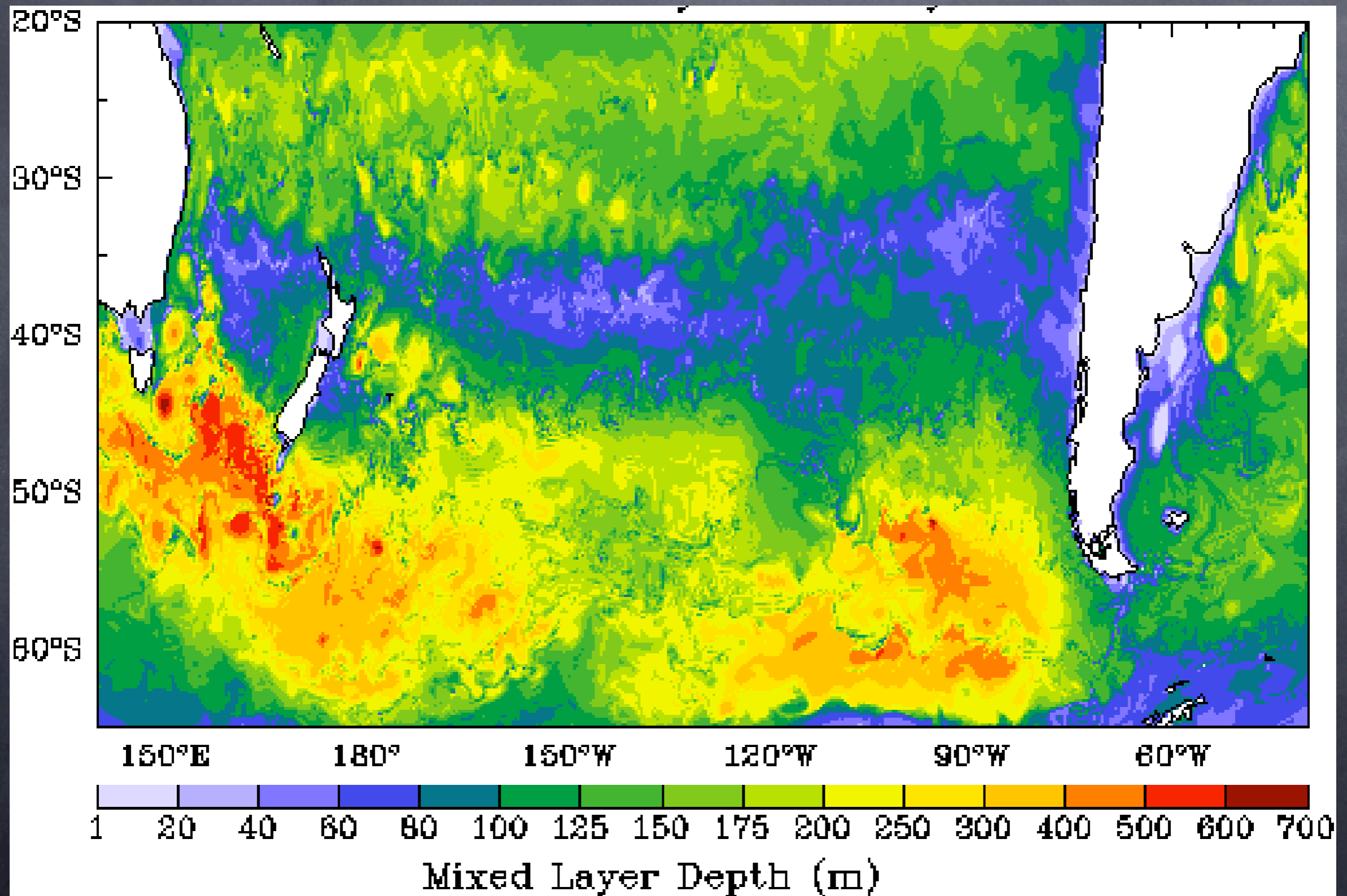
# Conclusions

- The focus of physics for ocean modeling is moving again to smaller scales, pushing ahead of resolution
- From submesoscale to finescale, new significant couplings are being found
- While these processes and couplings are poorly understood, their affect on air-sea exchanges is estimable
- Progress on parameterizations will be fun, and should involve interaction with new observationalists, so speak up folks!

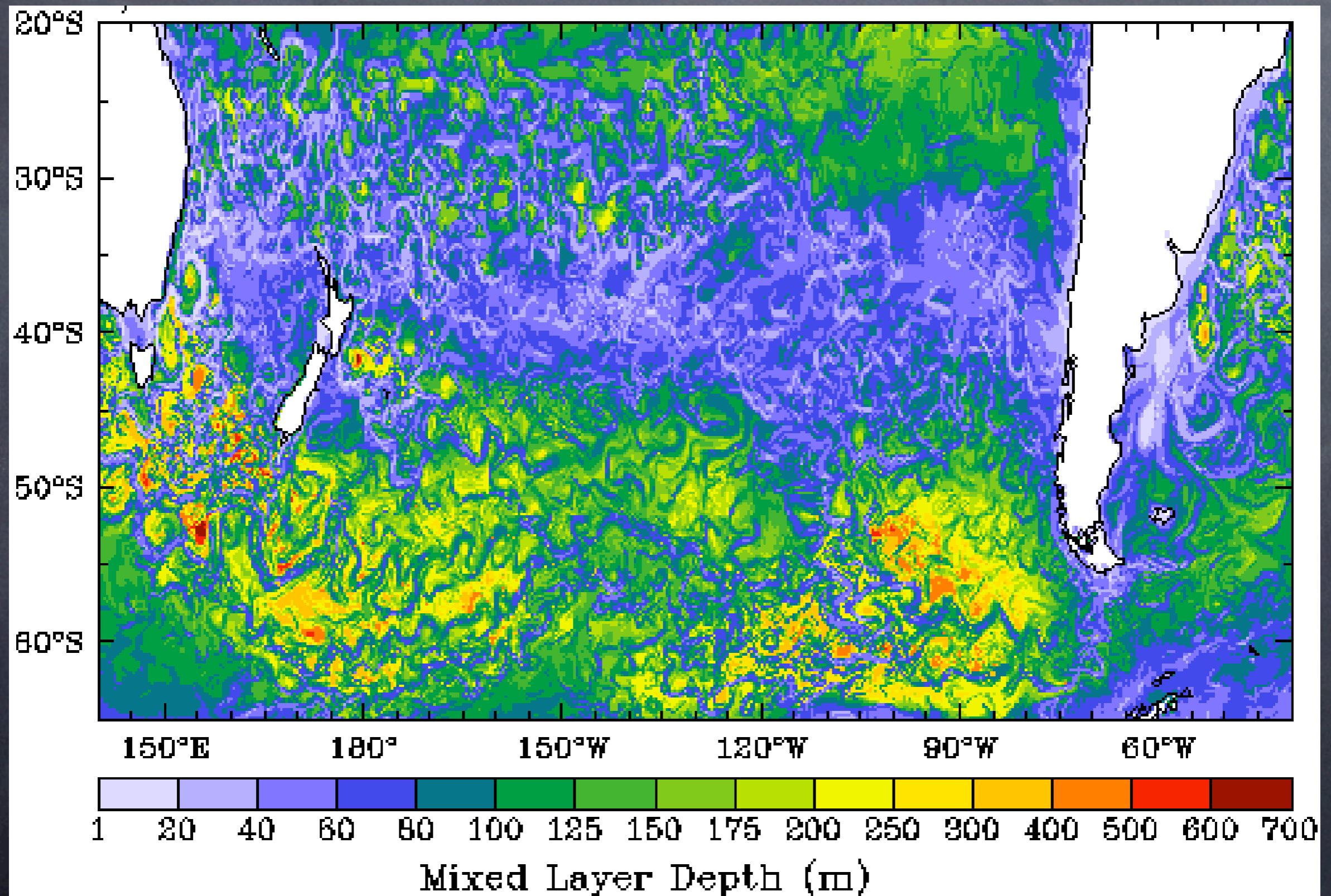
# Extensions: Forward Cascade?



# Changes To Mixing Layer Depth in Eddy-Resolving Southern Ocean Model



# Changes To Mixing Layer Depth in Eddy-Resolving Southern Ocean Model



# The Scaling of MLIs

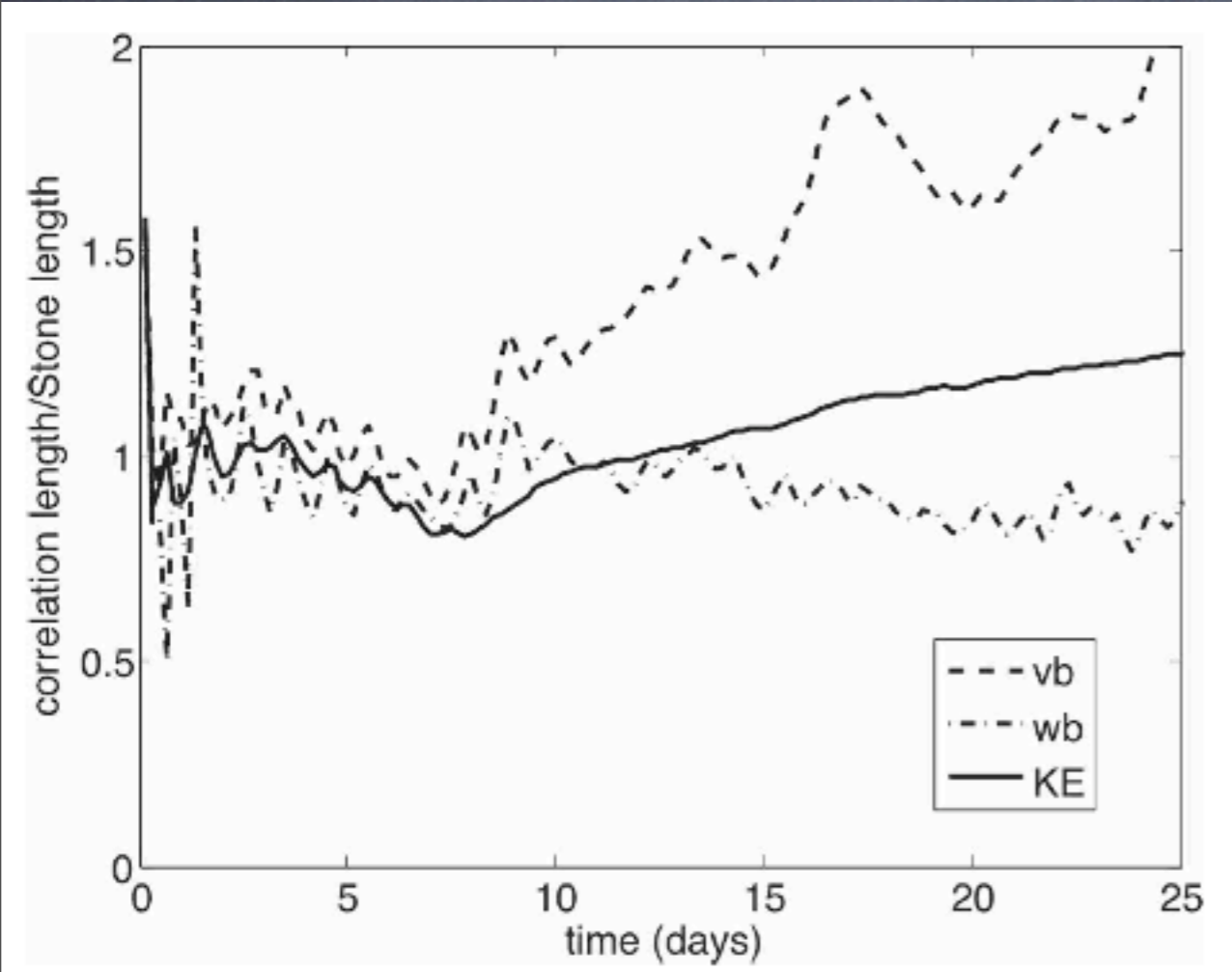
Mixed Layer Eddies (MLEs) begin as ageostrophic baroclinic instability of a front in the Mixed Layer:  
the Mixed Layer Instability (MLI)

MLI=infinitesimal  
MLE=finite amplitude

$$L_s = \frac{2\pi U}{|f|} \sqrt{\frac{1 + Ri}{5/2}} \approx 5.6 \frac{NH}{|f|}$$

$$\tau_s = \sqrt{\frac{54}{5}} \frac{\sqrt{1 + Ri}}{|f|} \approx \frac{4.6}{|f|}$$

(Fastest growing modes of Stone 66, 70, 72)



See Boccaletti et al 07,  
Fox-Kemper et al 08  
& Hosegood et al 06

# The Scaling of MLEs

MLEs form from MLIs, but scale differently due to an inverse cascade.

- (i) the relevant time scale  $\Delta t$  is *advective*: the time it takes for an eddy to traverse the decorrelation length with typical eddy velocities,  $\mathcal{V}$ , is

$$\Delta t \propto \Delta y / \mathcal{V}; \quad (7)$$

- (ii) the horizontal eddy velocity  $\mathcal{V}$  scales as the mean thermal wind  $U$  (see Fig. 5):

$$\mathcal{V} \propto U = \frac{M^2 H}{f}; \quad (8)$$

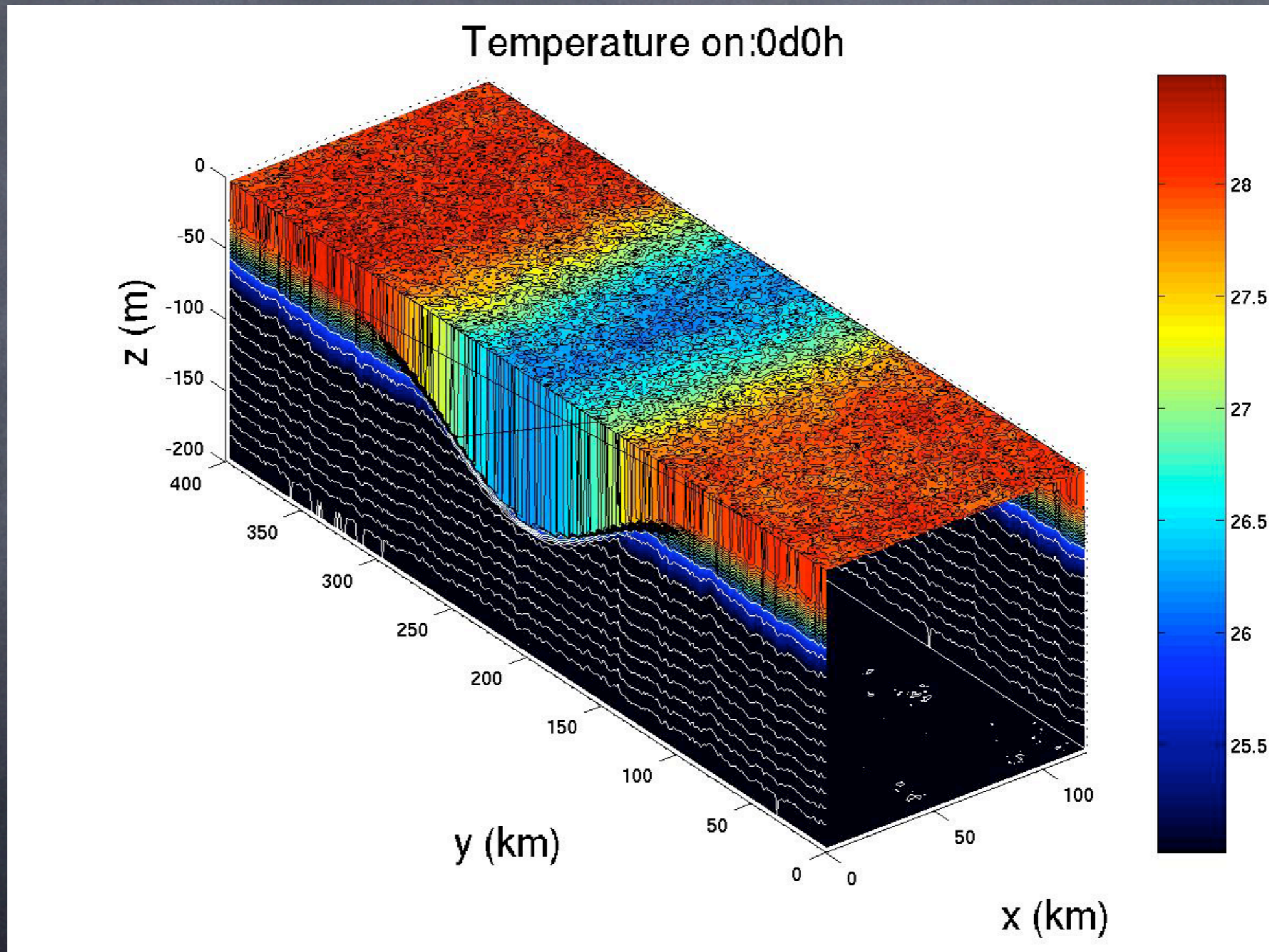
- (iii) the vertical decorrelation length scales with the ML depth (see Fig. 6):

$$\Delta z \propto H; \quad \text{and} \quad (9)$$

- (iv) fluid exchange occurs along a shallower slope (i.e., PE extracting) and proportional to the mean isopycnal slope (see Fig. 7):

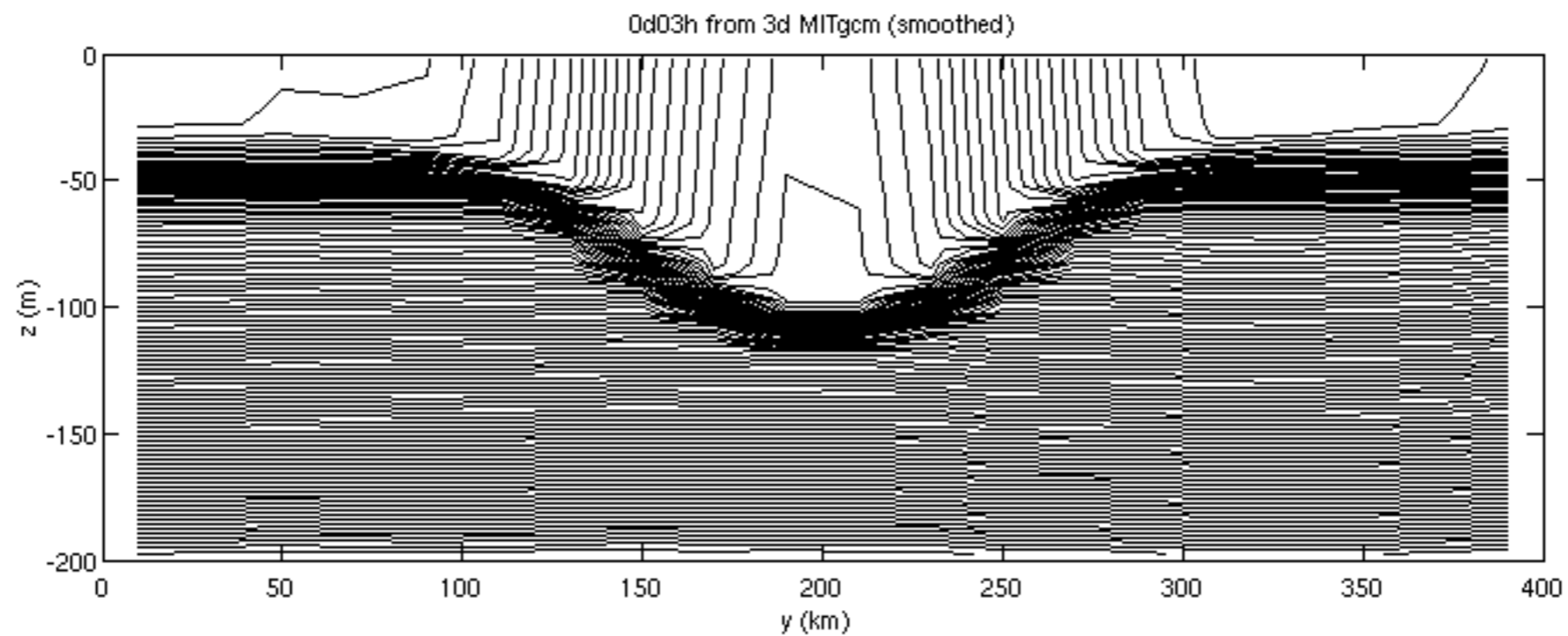
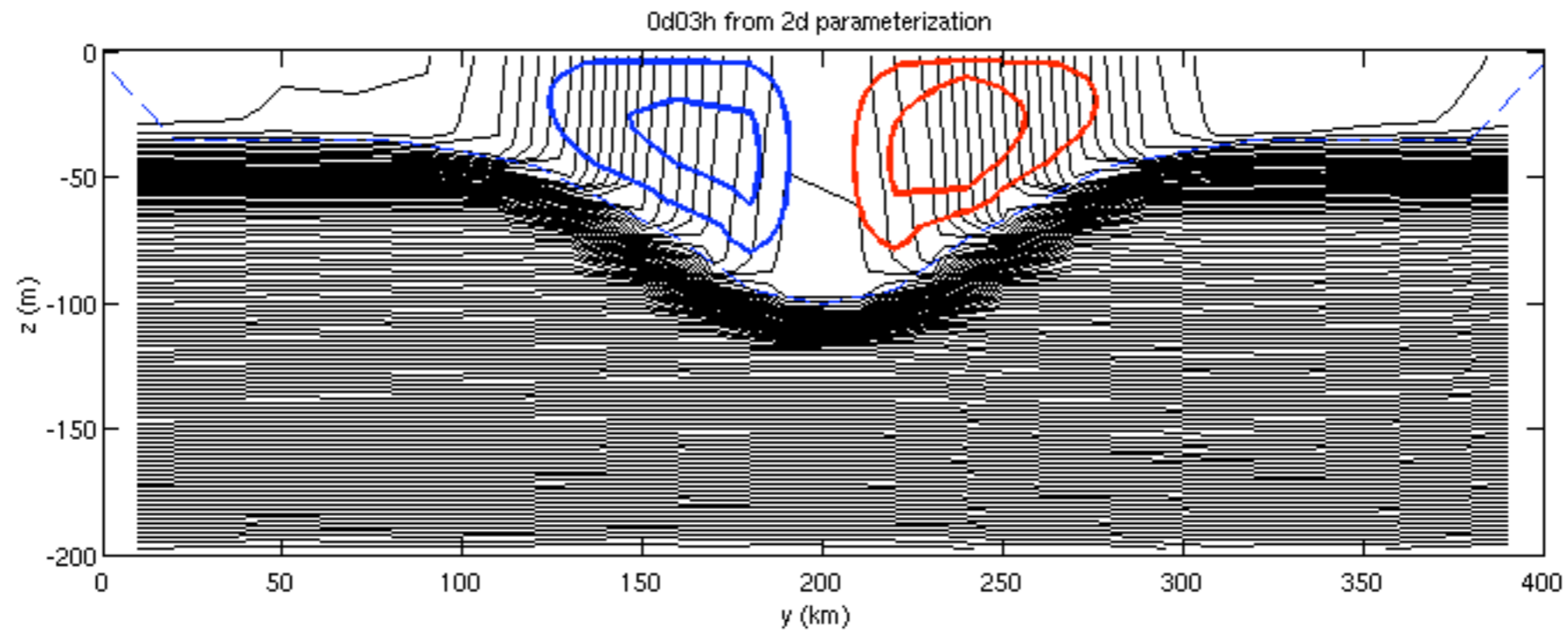
$$\frac{\Delta z}{\Delta y} = \frac{1}{C} \frac{M^2}{N^2}, \quad C > 1. \quad (10)$$

# Extensions: e.g., Hurricane Wake Recovery

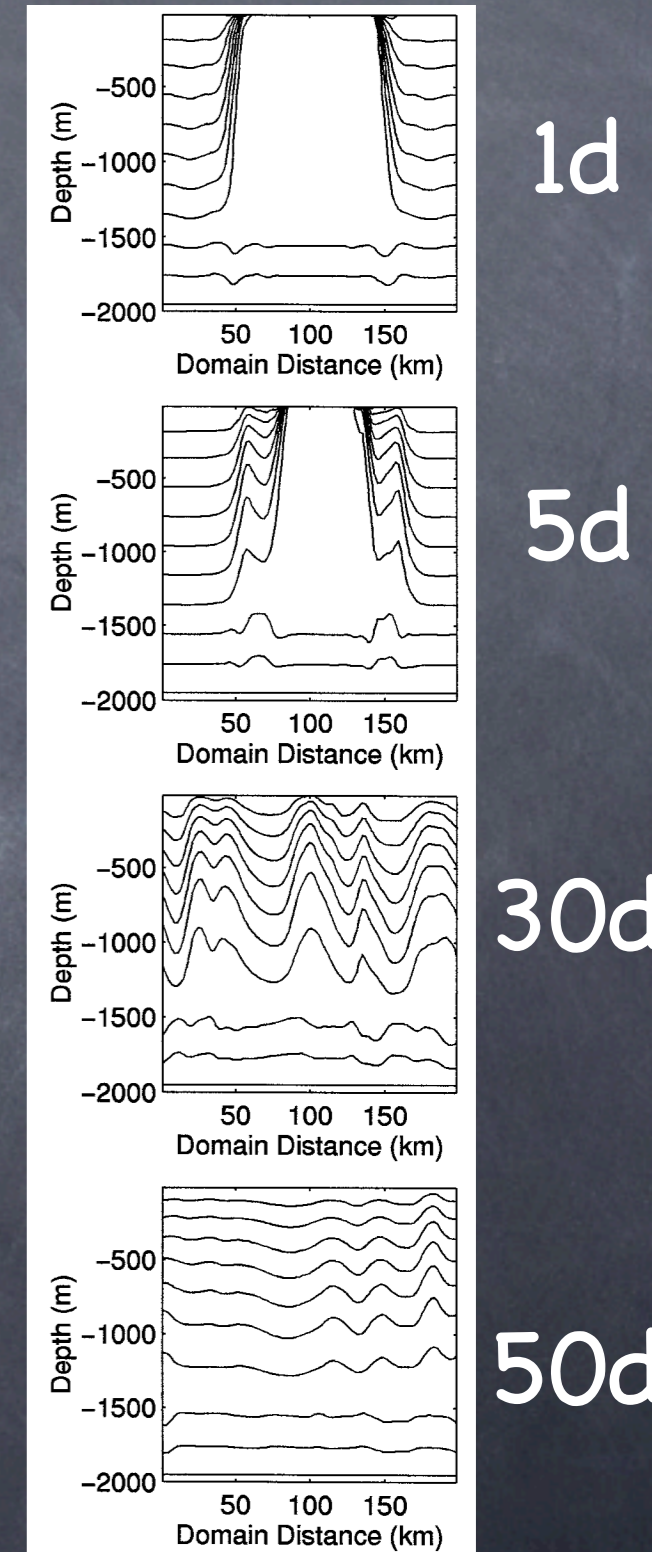
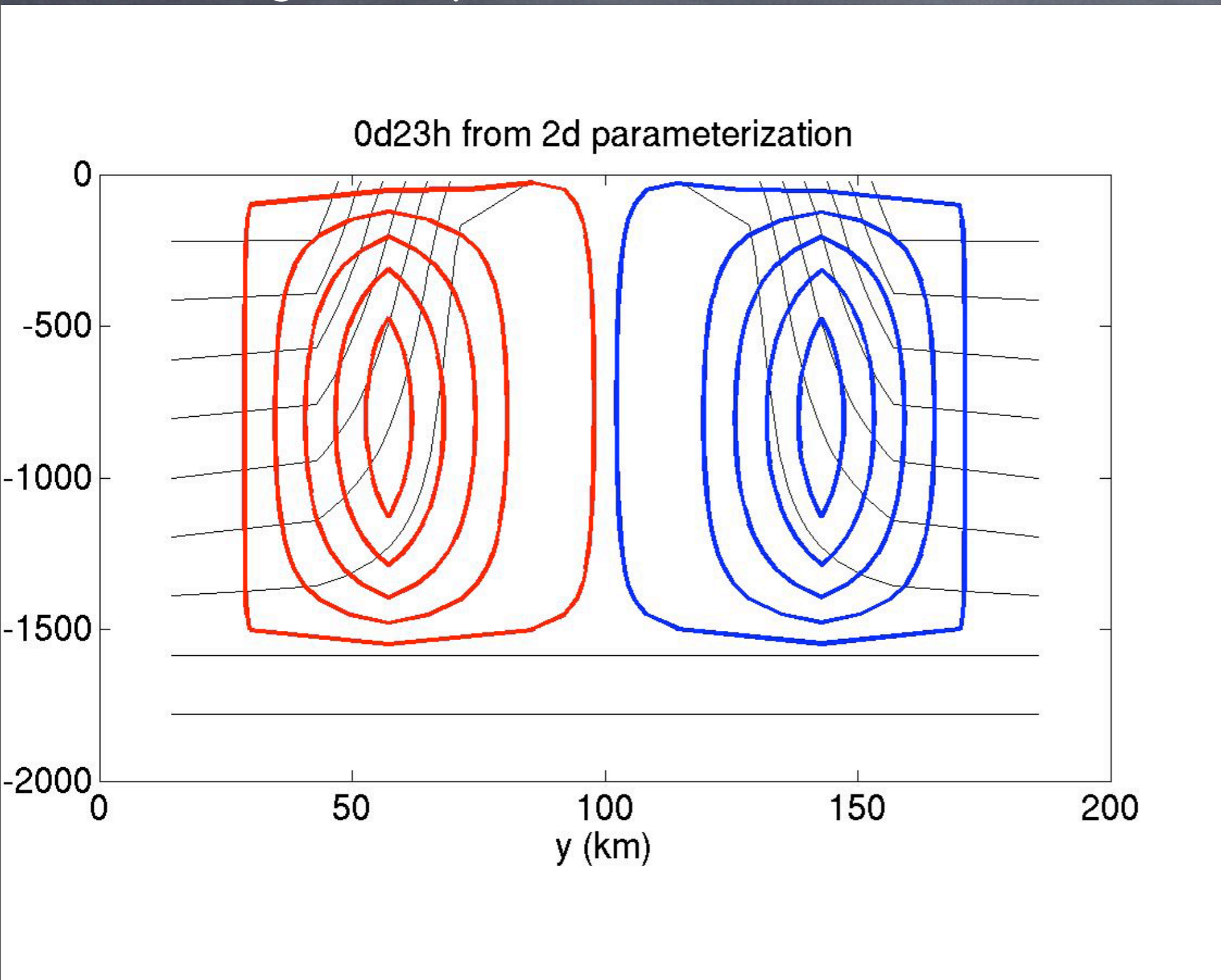




# Param vs. unforced model



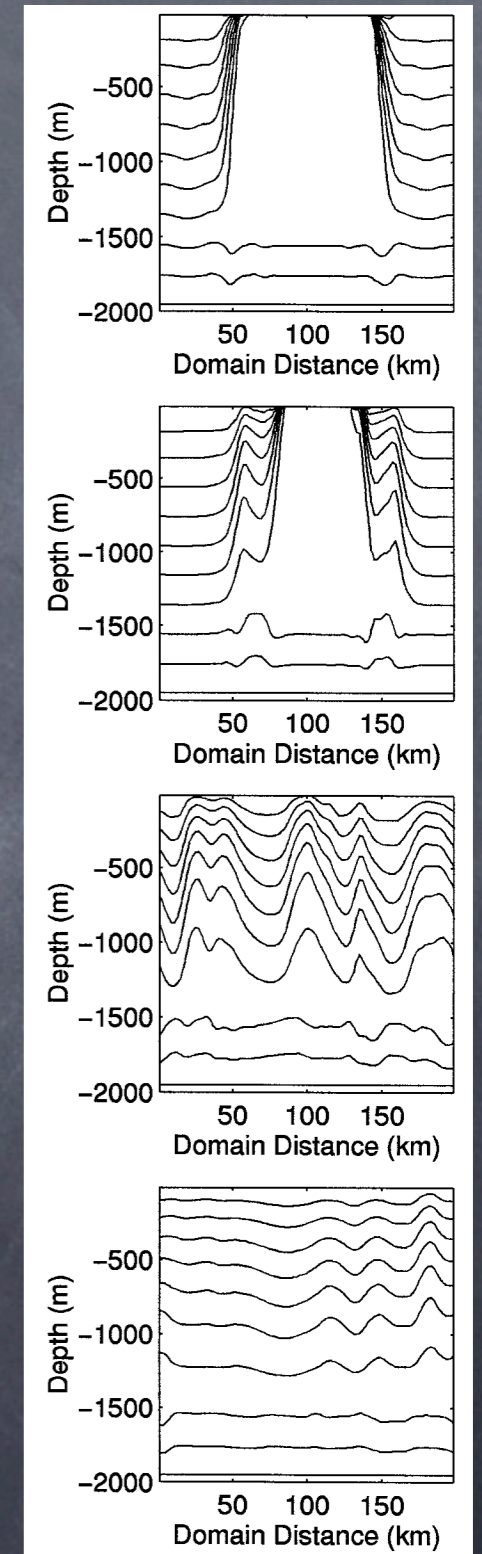
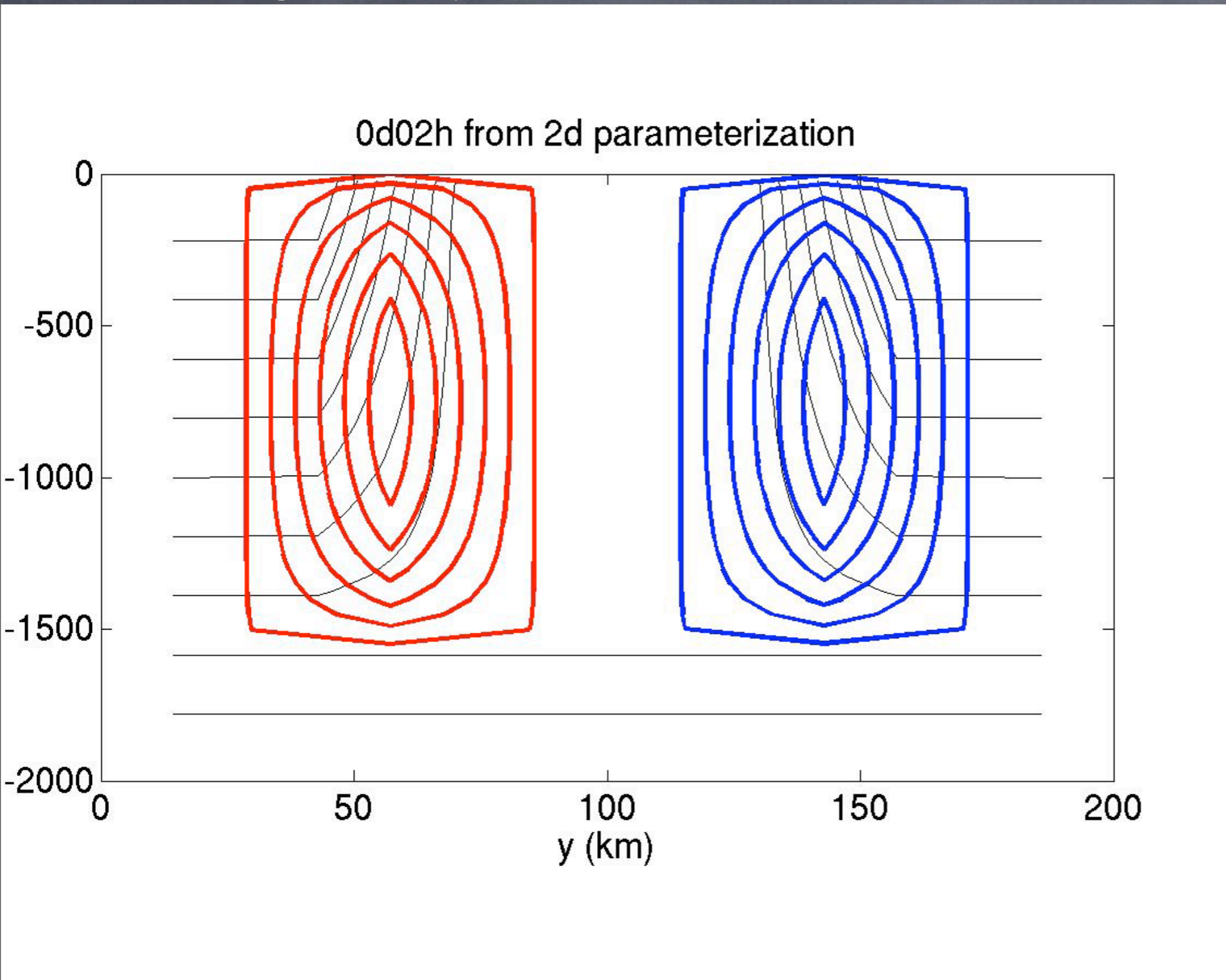
# Param. Applies to Other Scenarios: e.g., Deep Convection (versus Jones & Marshall)



Param gives same scaling, but...

Jones & Marshall 97

# Param. Applies to Other Scenarios: e.g., Deep Convection (versus Jones & Marshall)



Vertical structure is different..

Jones & Marshall 97