

Beyond GM

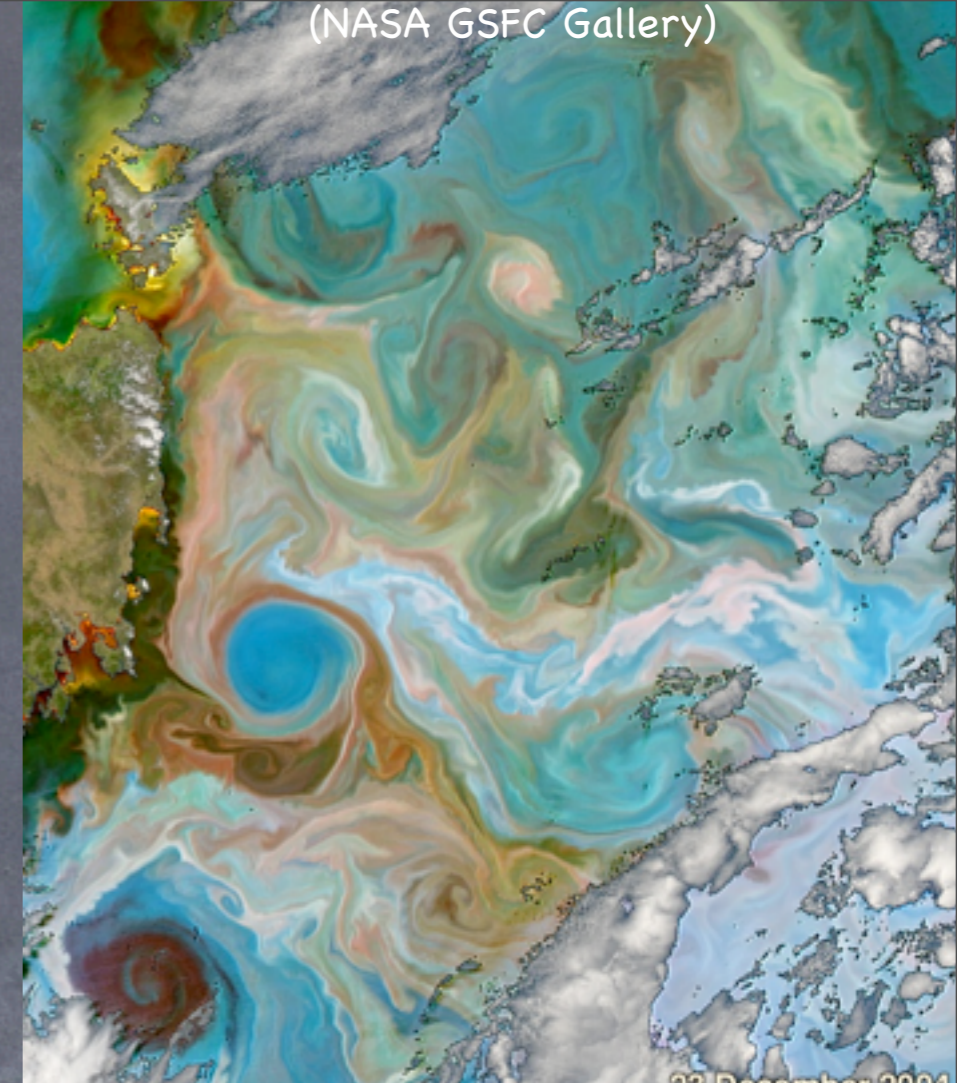
A Symposium on Oceanic Eddy Fluxes

Baylor Fox-Kemper U. Colorado-Boulder,
with Scott Bachman, Andrew Margolin (students),
Frank Bryan & John Dennis (NCAR)

NCAR CGD Symposium, Th. 3/4/2010 3-5PM

The Character of the Mesoscale

← 100 km



(Capet et al., 2008)

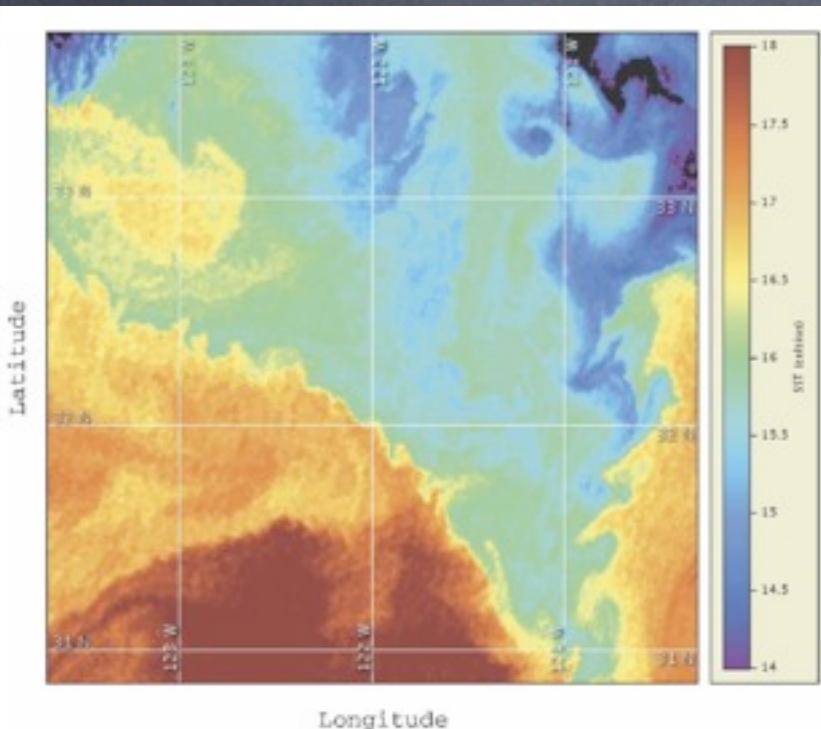
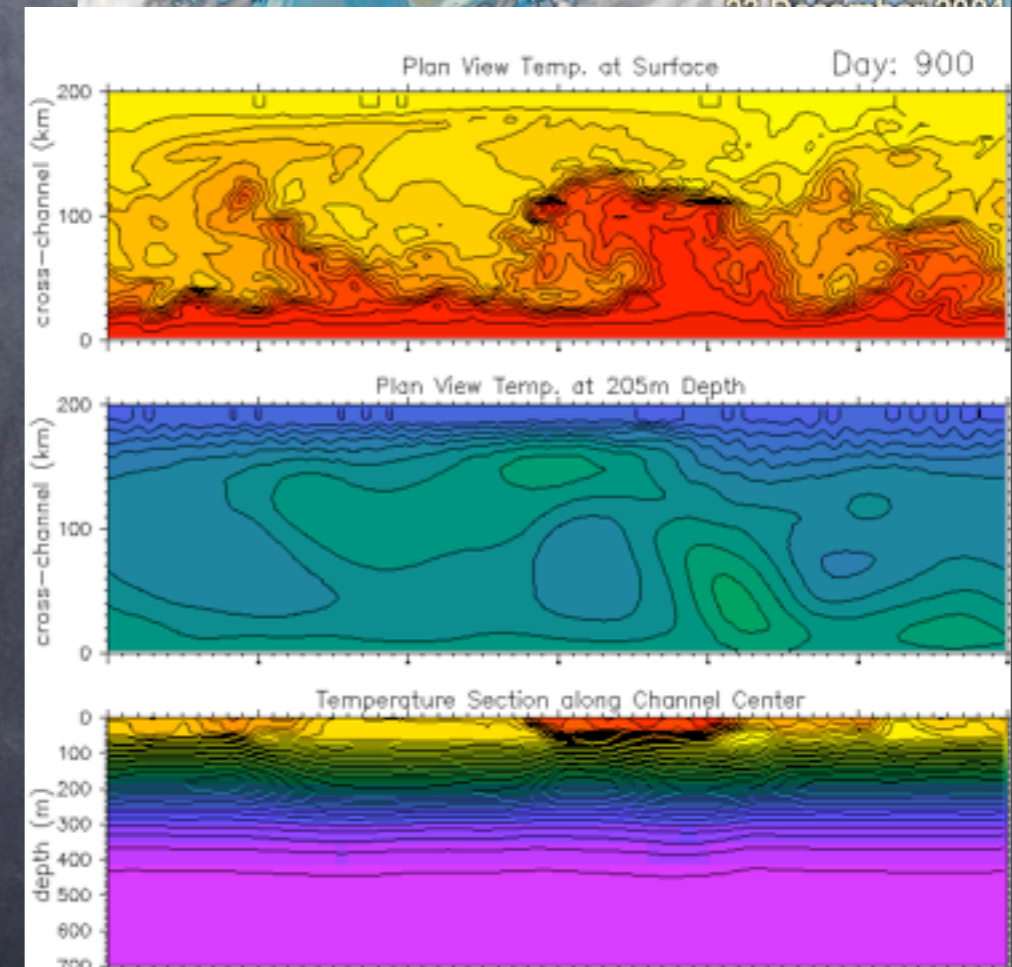


FIG. 16. Sea surface temperature measured at 1832 UTC 3 Jun 2006 off Point Conception in the California Current from CoastWatch (<http://coastwatch.pfeg.noaa.gov>). The fronts between recently upwelled water (i.e., 15°–16°C) and offshore water ($\geq 17^\circ\text{C}$) show submesoscale instabilities with wavelengths around 30 km (right front) or 15 km (left front). Images for 1 day earlier and 4 days later show persistence of the instability events.

- Boundary Currents
- Eddies
- $Ro=O(0.1)$
- $Ri=O(1000)$
- Full Depth
- Eddies strain to produce Fronts
- 100km, months

Eddy processes mainly **baroclinic & barotropic instability**. Parameterizations of baroclinic instability (GM, Visbeck...).



The Character of the Submesoscale

(Capet et al., 2008)

10 km

(NASA GSFC Gallery)

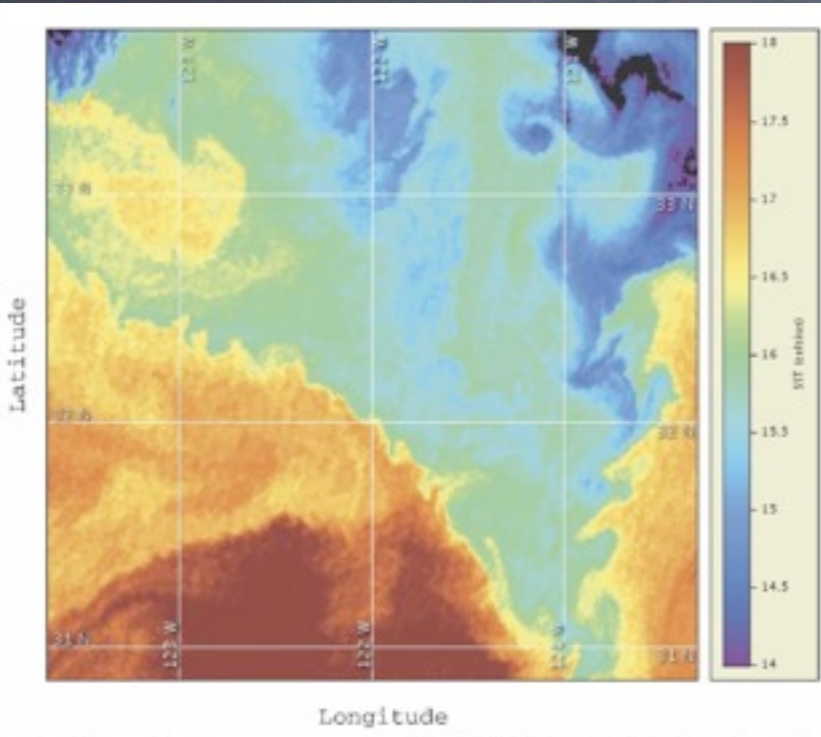
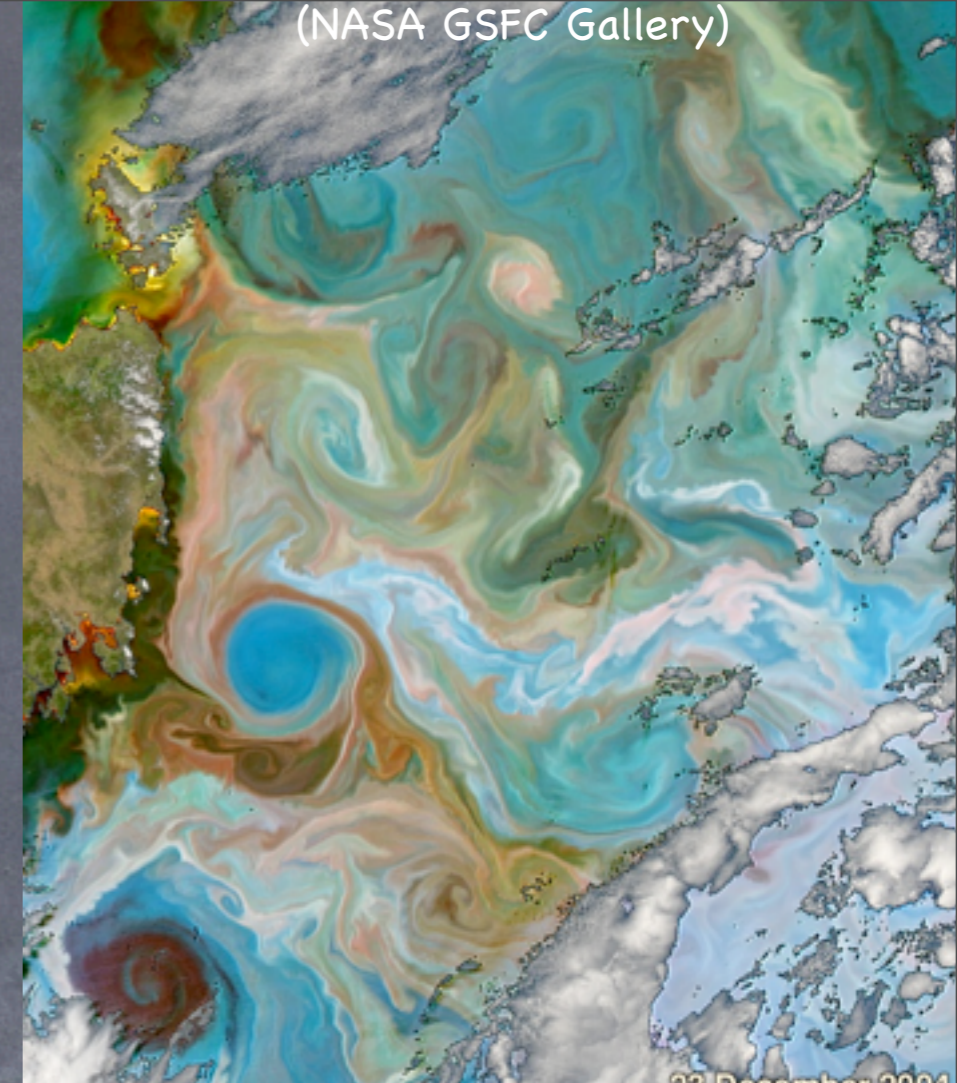
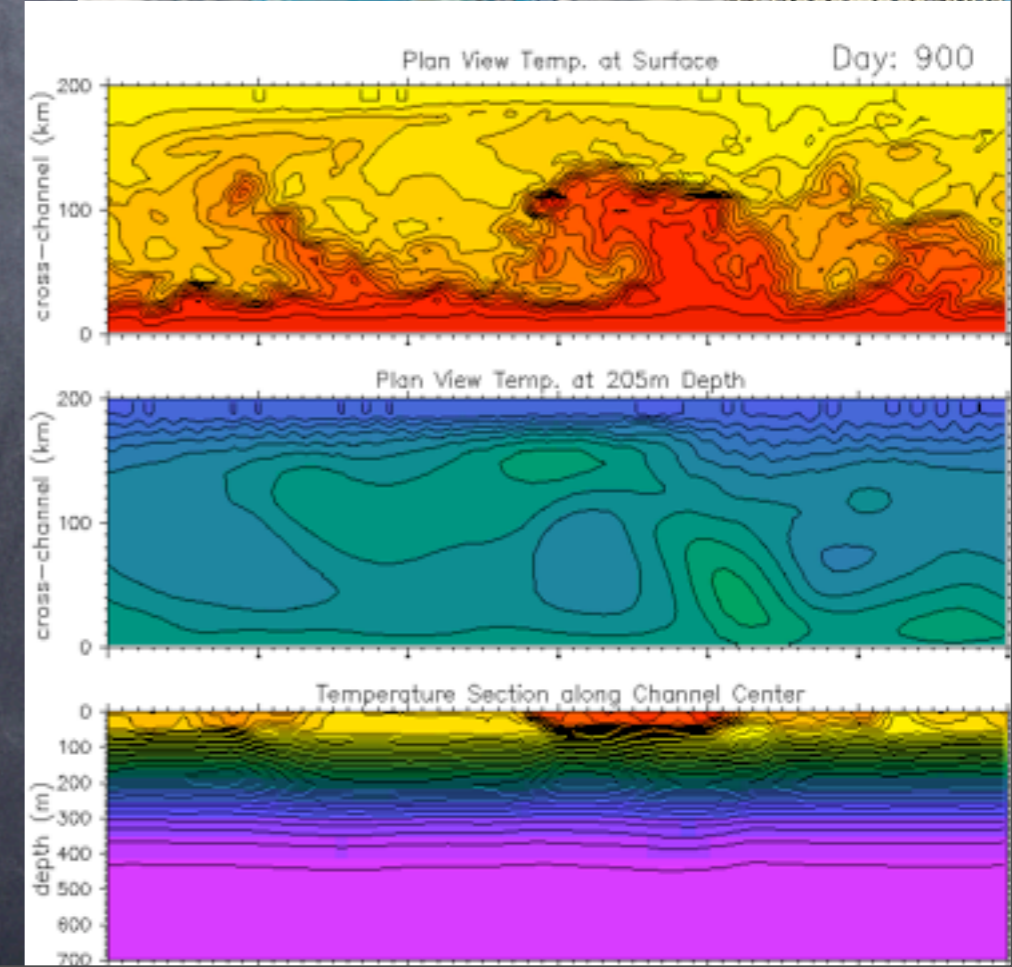
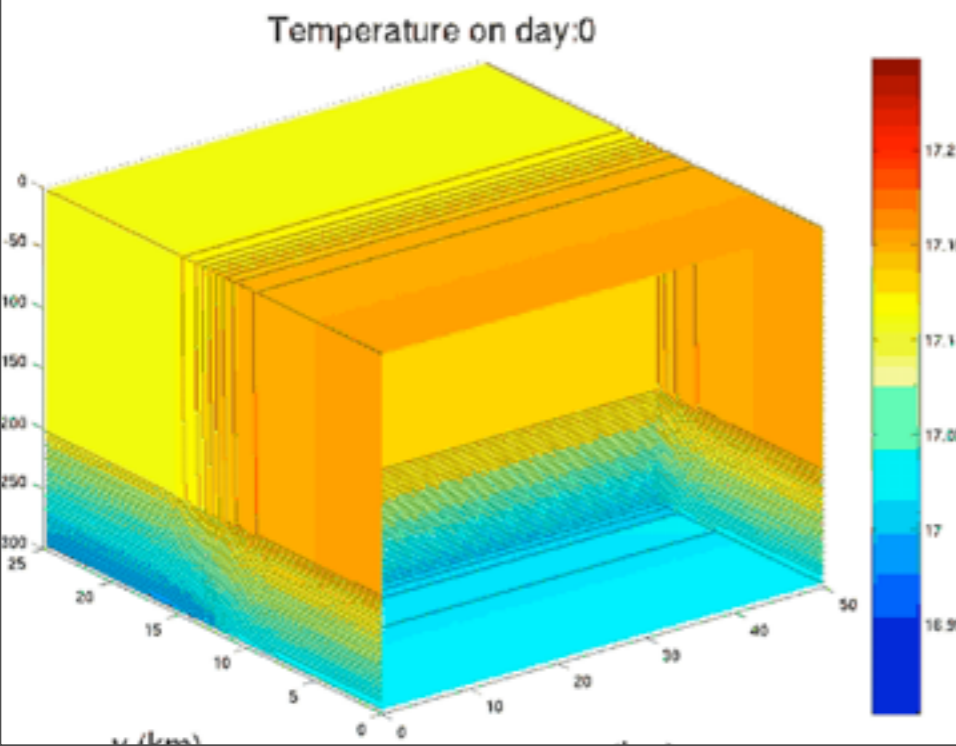


FIG. 16. Sea surface temperature measured at 1833 UTC 3 Jan 2006 off Point Conception in the

- Fronts
 - Eddies
 - $Ro=O(1)$
 - $Ri=O(1)$
 - near-surface
 - 1-10km, days
- Eddy processes mainly **baroclinic instability** (Boccaletti et al '07, Haine & Marshall '98). Parameterizations of baroclinic instability **apply?** (GM, Visbeck...).



Tracer Flux-Gradient Relationship

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

- Most **subgridscale eddy closures** have this form: **GM***, Redi, FFH** submesoscale
- Relates the **eddy flux** to the coarse-grain gradients **locally**
- If we knew the dependence of **M** on the coarse-resolution flow, we'd have the **optimal local eddy closure**

*Gent & McWilliams (1990)

**Fox-Kemper, Ferrari, Hallberg (2008)

A bit of theory...

- Dukowicz & Smith (97) and Smith (99) lay out the form of a **stochastic, adiabatic relocation** of particles
- The resulting **Fokker-Planck Equation** for the probability density of the particles gives a **K** and a **v**, which are closely related to the **Lagrangian mean transport** and the **diffusion of probability**.

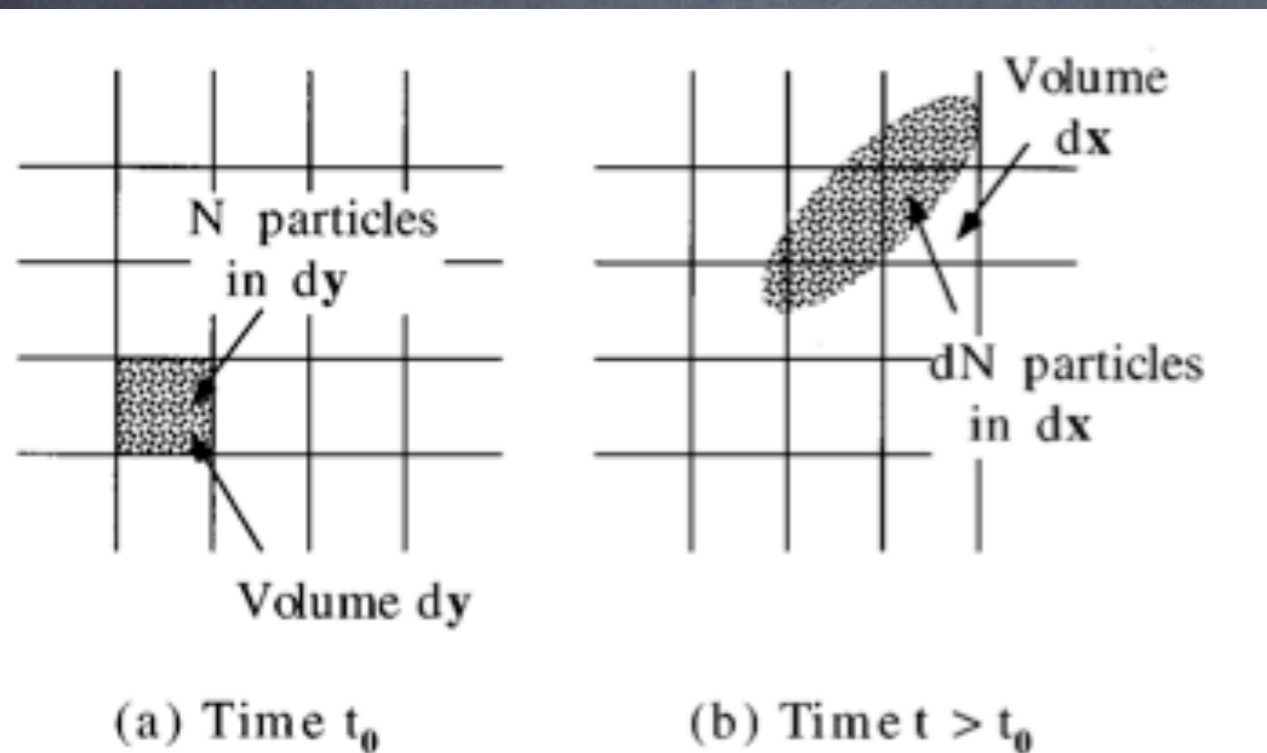


FIG. 1. A schematic illustration of the turbulent transport of a cloud of particles originating in dy at time t_0 , and the fraction dN/N which arrives in dx at a later time t .

$$\partial_t p(\mathbf{x}, t | \mathbf{y}, t_0) + \nabla \cdot \mathbf{U} p(\mathbf{x}, t | \mathbf{y}, t_0) = \nabla \cdot \mathbf{K} \cdot \nabla p(\mathbf{x}, t | \mathbf{y}, t_0) \quad (50)$$

$$\mathbf{U} = \mathbf{v} - \nabla \cdot \mathbf{K} \quad (51)$$

$$\mathbf{v}(\mathbf{x}, t) = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \int d\mathbf{x}' (\mathbf{x}' - \mathbf{x}) p(\mathbf{x}', t + \Delta t | \mathbf{x}, t) \quad (52)$$

$$\mathbf{K}(\mathbf{x}, t) = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \int d\mathbf{x}' \frac{1}{2} (\mathbf{x}' - \mathbf{x})(\mathbf{x}' - \mathbf{x}) p(\mathbf{x}', t + \Delta t | \mathbf{x}, t). \quad (53)$$

Since the pdf is positive, it is clear from (53) that \mathbf{K} is a 2×2 symmetric positive-definite tensor. We will refer to \mathbf{v} as the Lagrangian mean velocity, although this identification is not exact (see Bennett 1996, p. 7). In (52)

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\bar{\tau}$$

General Form

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

- Small slope approximation converts the horizontal stochastic reshuffling along neutral surfaces to a GM+Redi like form
- Could vary tracer by tracer, or active tracer vs. passive, etc. In practice we don't let it.
- Using the same form for all tracers amounts to 'labeling' fluid with tracer, neglecting sources, etc.

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\bar{\tau}$$

Sym Part=Anisotropic* Redi

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ K_{yx} & K_{yy} & \hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ \hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & \hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & \tilde{\nabla}_z\cdot\mathbf{K}\cdot\tilde{\nabla}_z \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

Yellow \mathbf{K} 'are' horizontal stirring & mixing

Blue factors in Redi (1982) are symmetric
and scaled to make

eddy mixing along neutral surfaces

*Anistropic form due to Smith & Gent 04

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\bar{\tau}$$

AntiSym Part=Anisotropic* GM

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ 0 & 0 & -\hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ \hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & \hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & 0 \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

Antisymmetric Elements in GM (1990)

are scaled to overturn fronts, make vertical fluxes

extract PE, and restratify the fluid
equivalent to eddy-induced advection

Q: Same horiz. mixing (\mathbf{K}) as Redi?

*Anisotropic form due to Smith & Gent 04 *Tensor Form (Griffies, 98)

An example of what a (submeso) subgrid parameterization looks like

with FLOW DEPENDENT \mathbf{M} :

Fox-Kemper, Ferrari, & Hallberg (2008) &

Fox-Kemper, Danabasoglu, Ferrari, & Hallberg (2008)

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\Psi_y \\ 0 & 0 & \Psi_x \\ \Psi_y & -\Psi_x & 0 \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

$$\Psi = \left[\frac{\Delta x}{L_f} \right] \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla \bar{b} \times \hat{\mathbf{z}}$$

$$\mu(z) = \left[1 - \left(\frac{2z}{H} + 1 \right)^2 \right] \left[1 + \frac{5}{21} \left(\frac{2z}{H} + 1 \right)^2 \right]$$

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\bar{\tau}$$

Sym Part=Anisotropic* Redi

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ K_{yx} & K_{yy} & \hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ \hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & \hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & \tilde{\nabla}_z\cdot\mathbf{K}\cdot\tilde{\nabla}_z \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

AntiSym Part=Anisotropic* GM

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ 0 & 0 & -\hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ \hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & \hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & 0 \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

Yellow \mathbf{K} 'are' horizontal stirring & mixing

Need a Natural, Mesoscale Eddy Environment to Test Out:

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\bar{\tau}$$

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

3 equations/tracer

9 unknowns (\mathbf{M} components)

BY USING 3 or MORE TRACERS, can determine \mathbf{M} !!!

(a la Plumb & Mahlman '87, Bratseth '98)

No assumptions about symmetry required.

Need a Natural, Mesoscale Eddy Environment to Test Out:

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\bar{\tau}$$

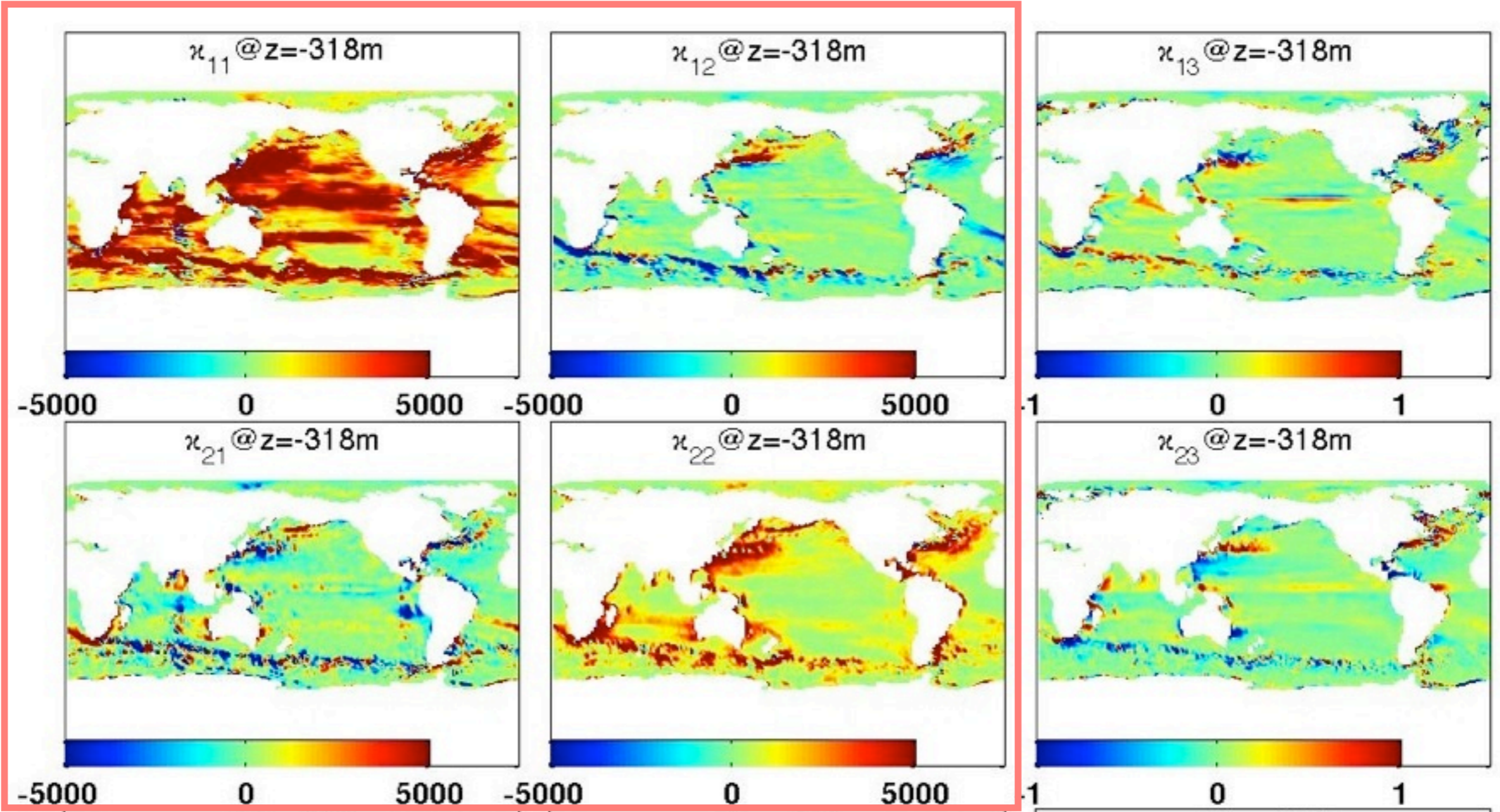
$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

With John Dennis & Frank Bryan, we took a POP0.1° Normal-Year forced model (yrs 16–20 for anal.)

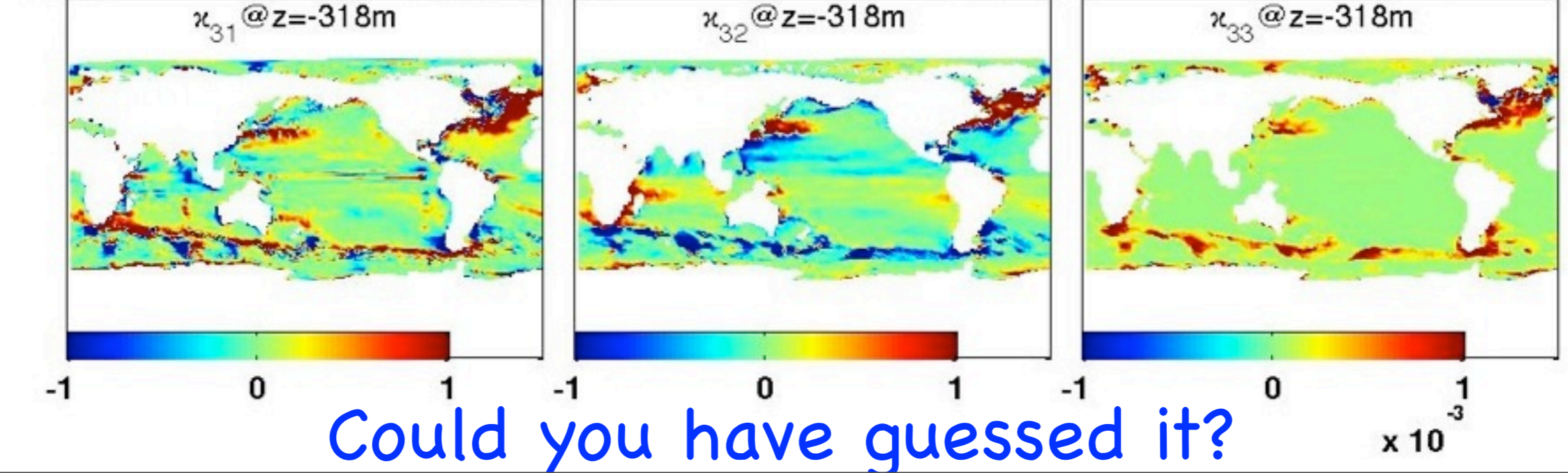
Added 9 Passive tracers--restored to x,y,z @ 3 rates
Kept all the eddy fluxes for passive & active tracers

Coarse-grained to 2°, passive tracers to find \mathbf{M}

K



M

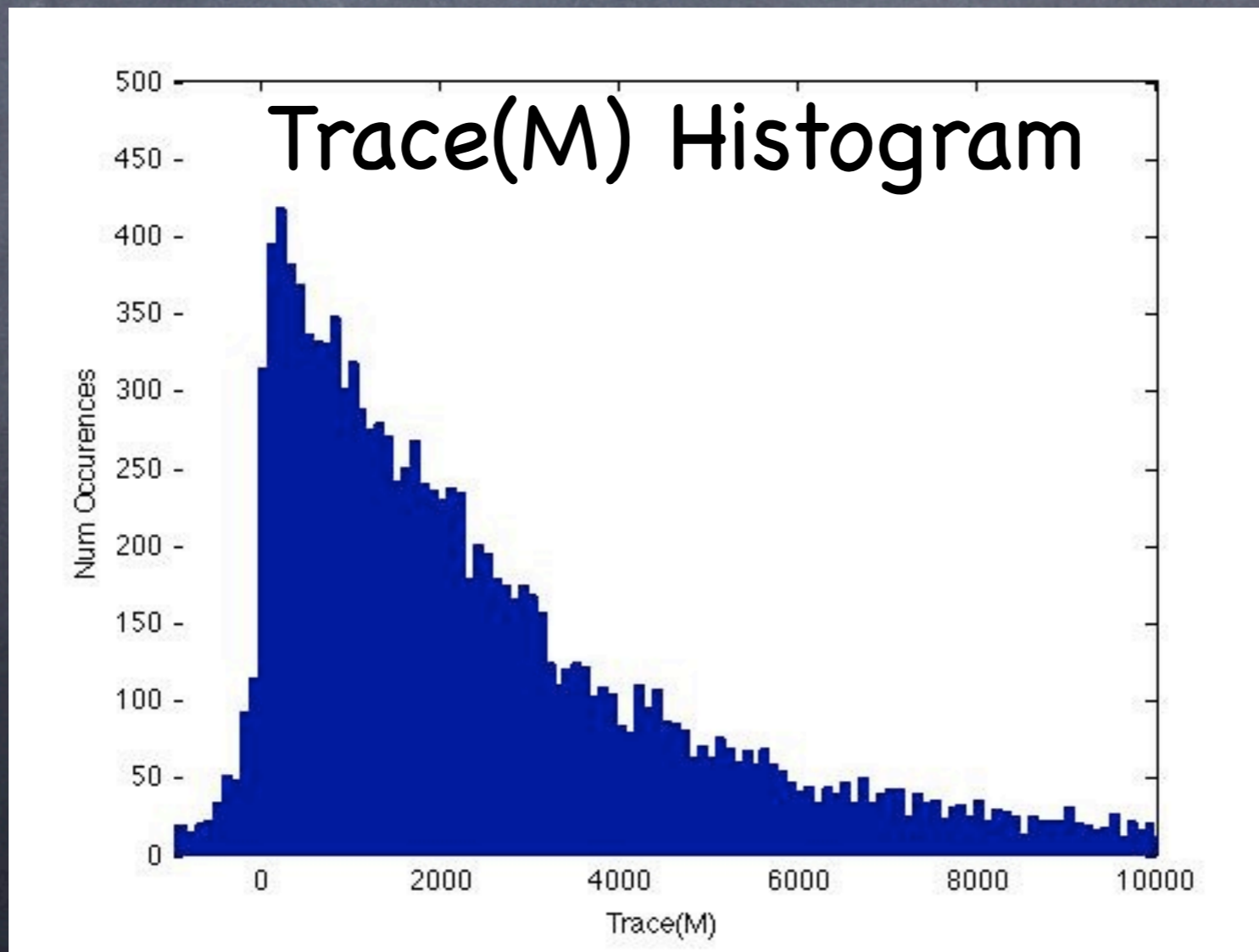


Could you have guessed it?

$\times 10^{-3}$

Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} z \\ K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} z \\ \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} z & \tilde{\nabla} z \cdot \mathbf{K} \cdot \tilde{\nabla} z \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$



Hor. Diffusivity is roughly $\text{Trace}(M)/2$

Peak of Diffusivity near $250 \text{ m}^2/\text{s}$

Median Diffusivity near $1000 \text{ m}^2/\text{s}$

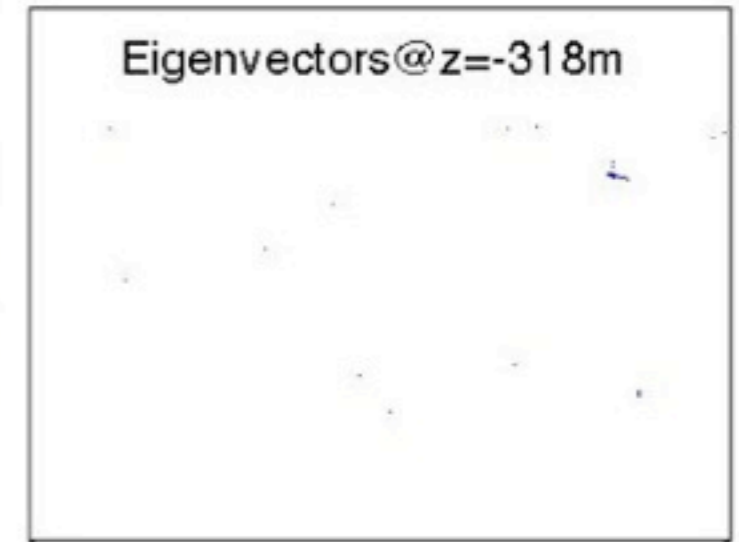
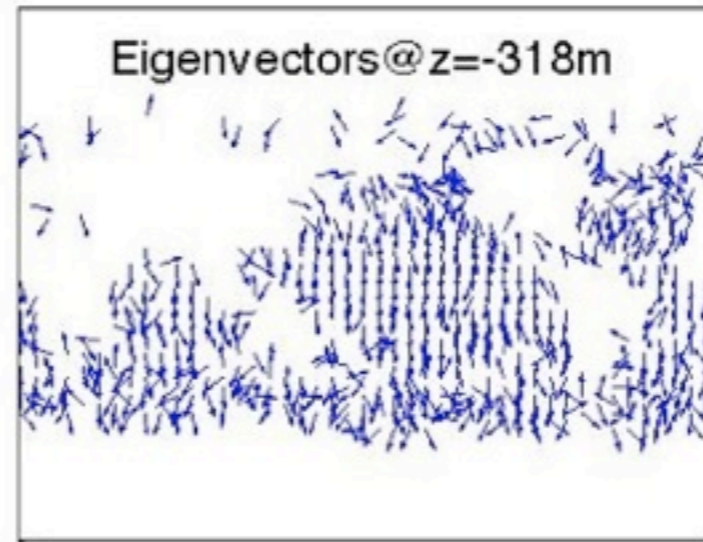
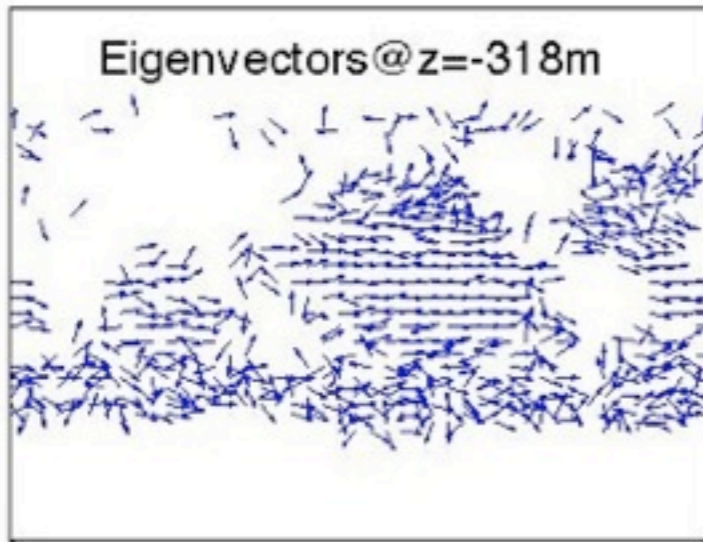
<6% negative

Interpretation?

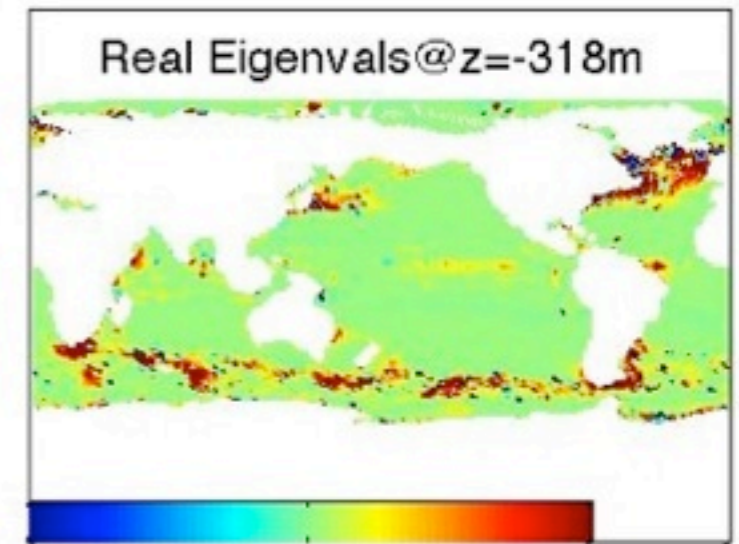
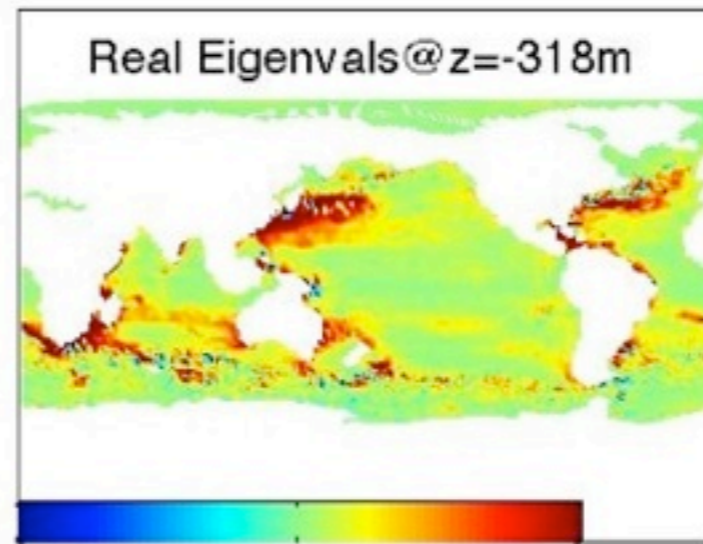
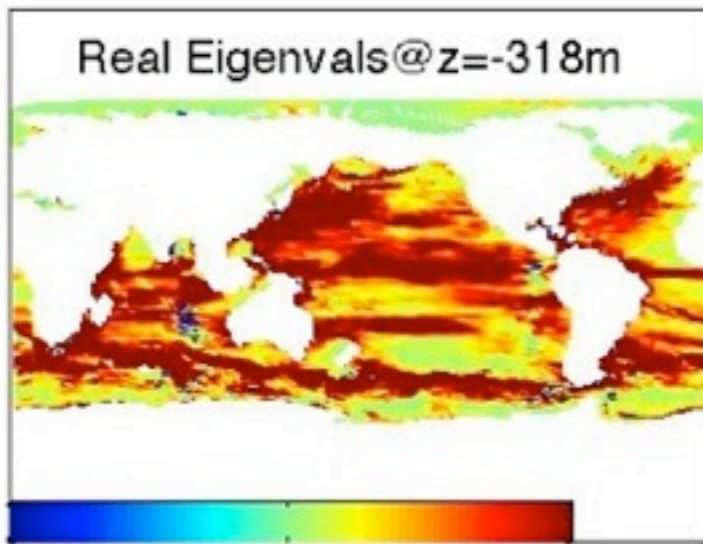
- Isonutral diffusion or 'mixing': symmetric \mathbf{K} with real, positive eigenvalues (neg \rightarrow nonlocal)
- The eigenvalues of \mathbf{M} are related, except there is one more involving the neutral to z coordinate conversion (in S&G theory, at least)
- The eigenvectors give the direction of the mixing associated with each eigenvalue
- Antisymmetric \mathbf{K} & \mathbf{M} are stirring/ overturning by an eddy-induced (quasi-stokes) streamfunction--non-orthogonal eigenvects and imaginary eigenvalues possible!

Result: Strong Anisotropy Along/Across Isopycnals

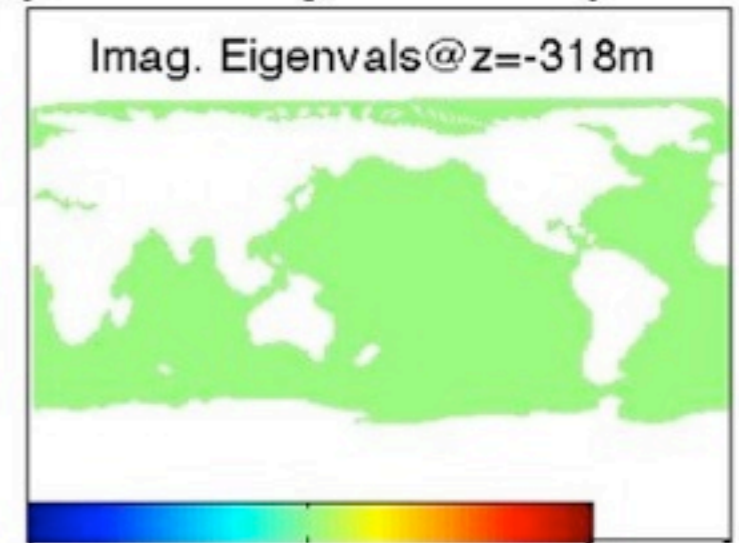
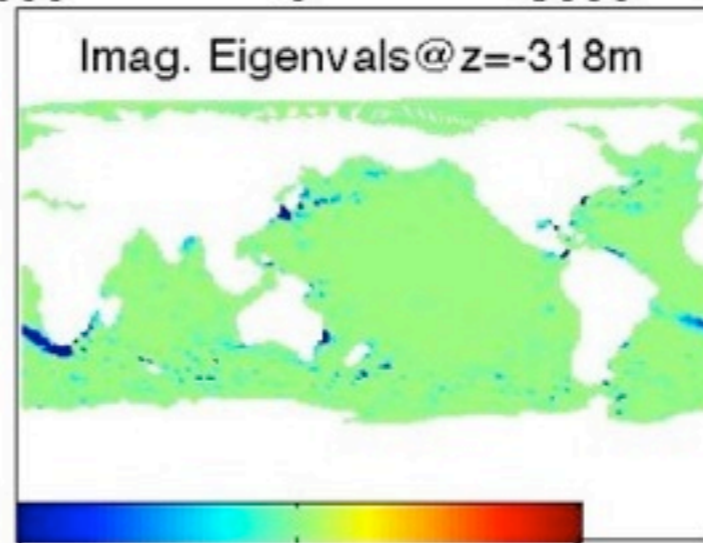
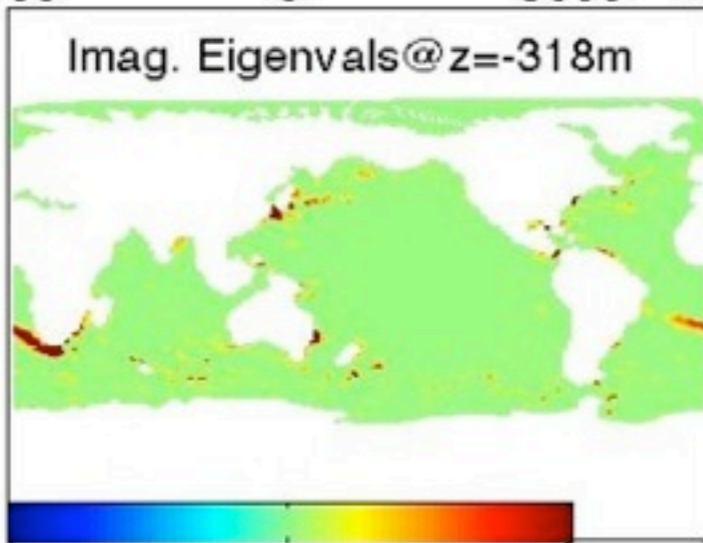
Mixing
direction



Mixing:



Stirring:



-5000 0 5000

-5000 0 5000

-1 0 1

-5000 0 5000

-5000 0 5000

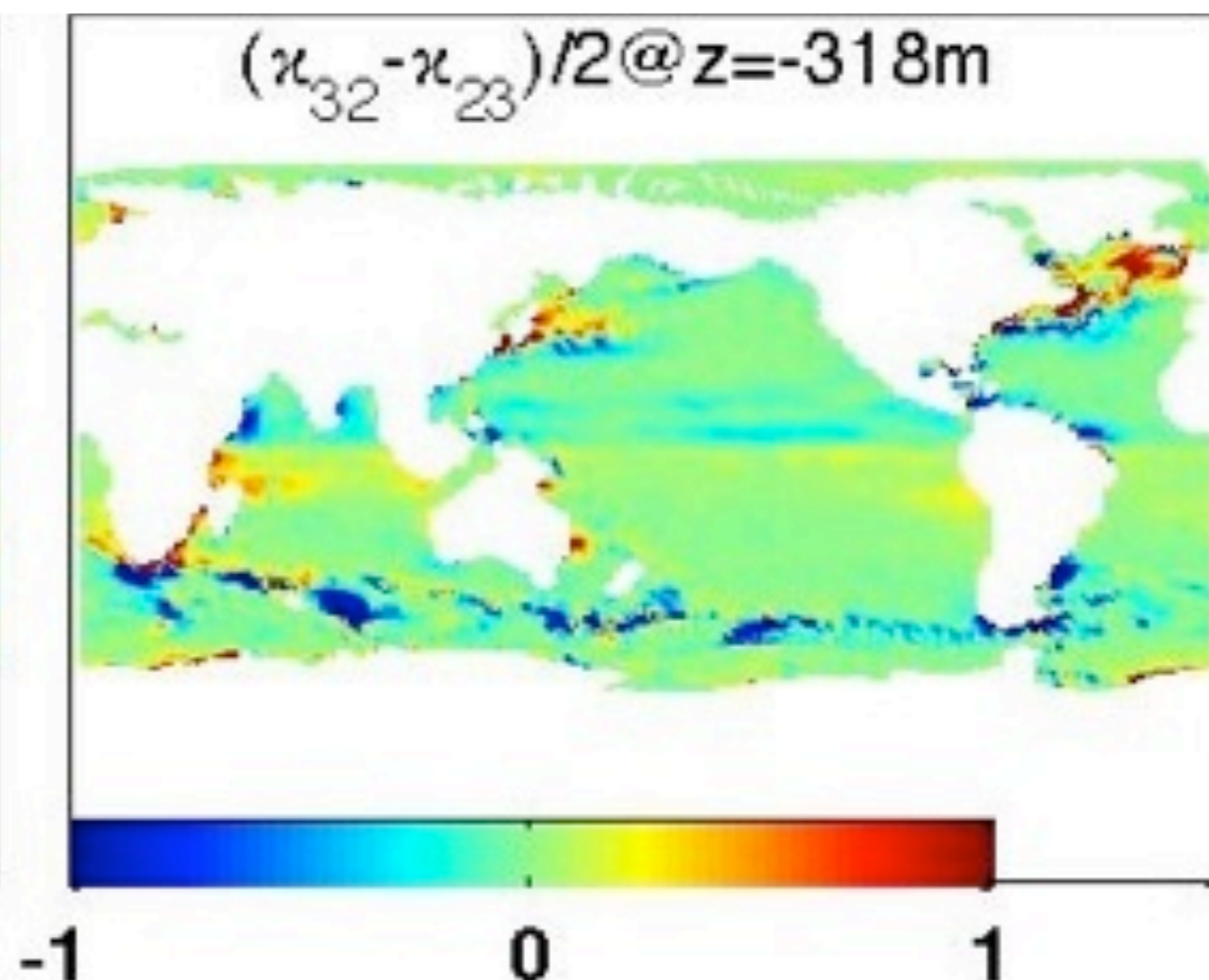
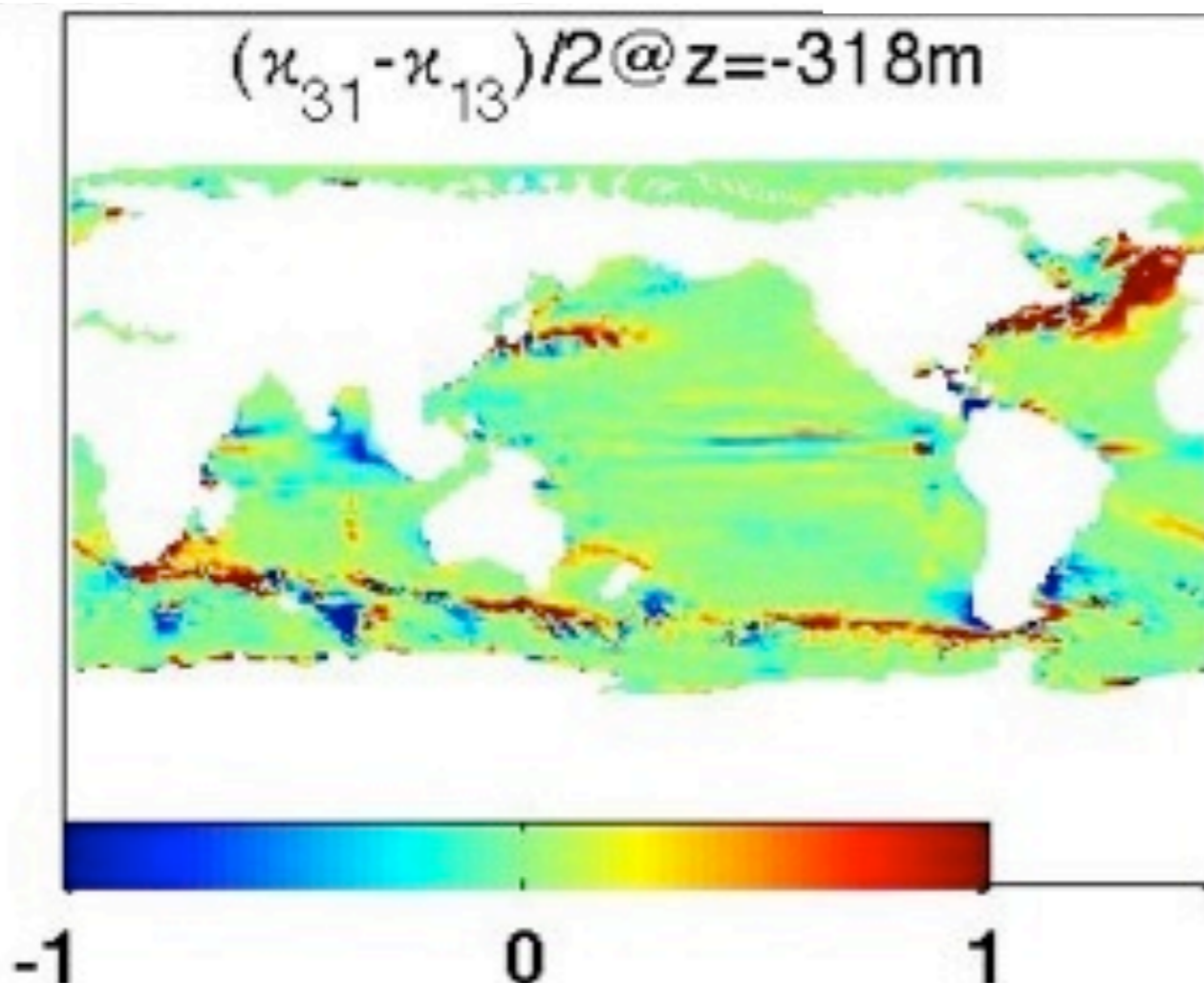
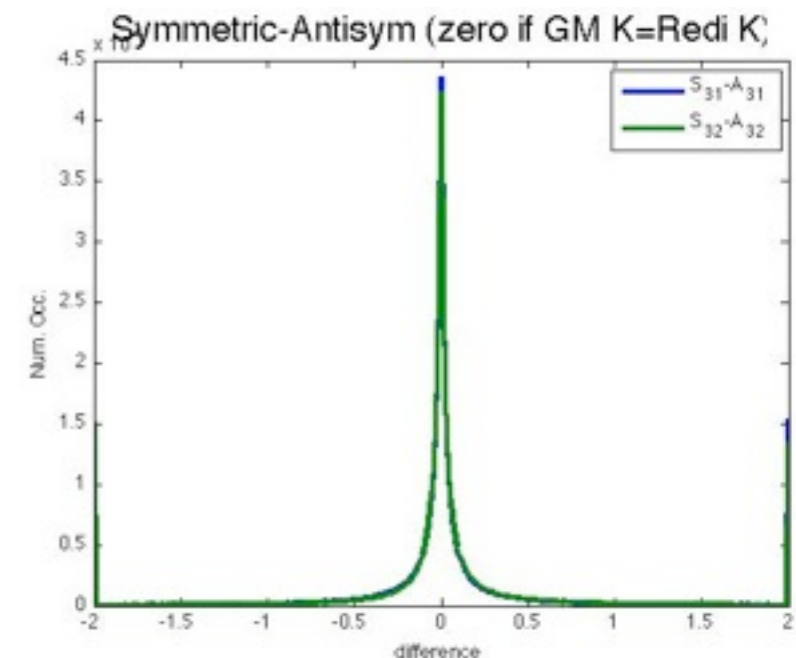
-1 0 1

$\times 10^{-3}$

Result:

Redi $K=GM$ K (mostly)

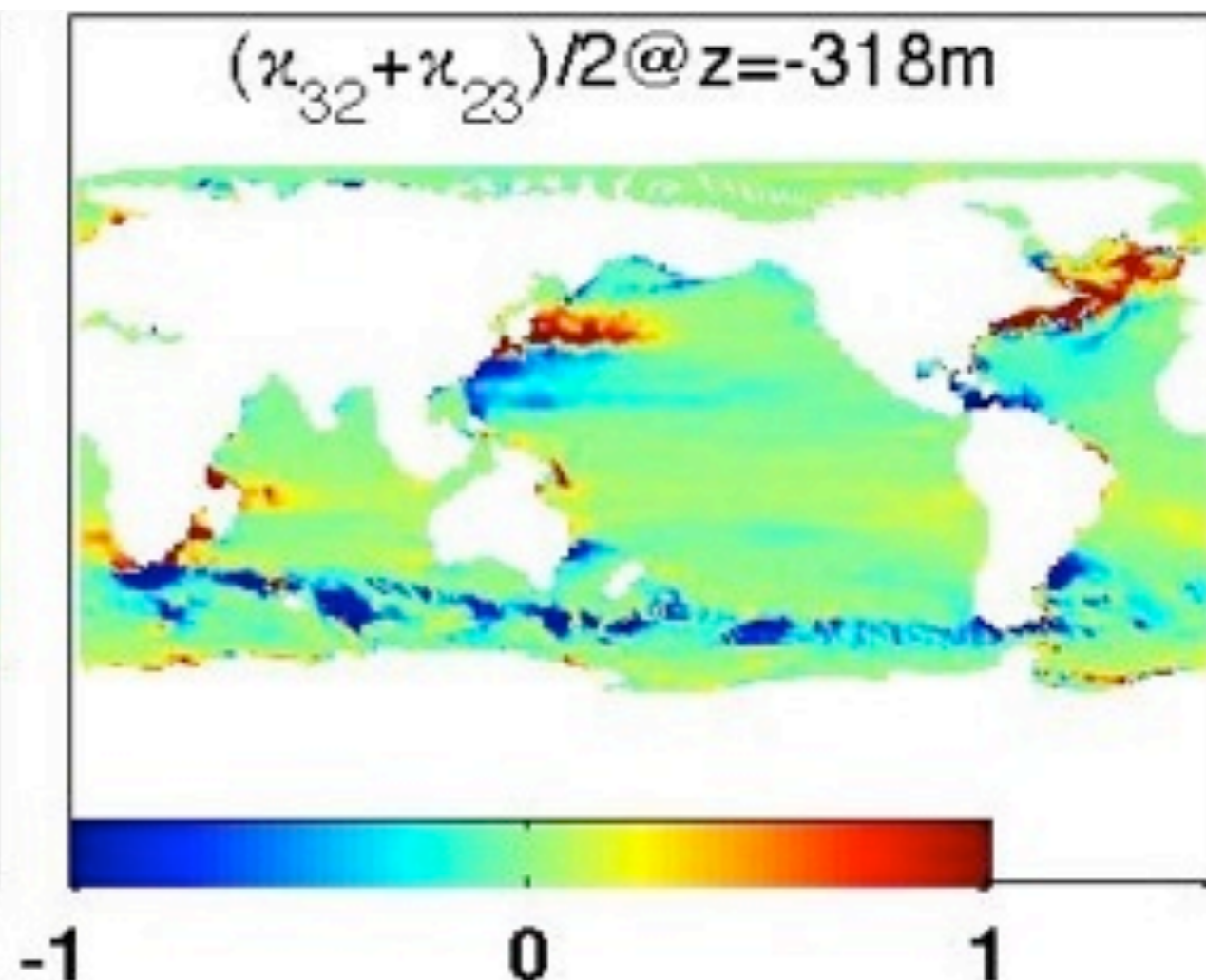
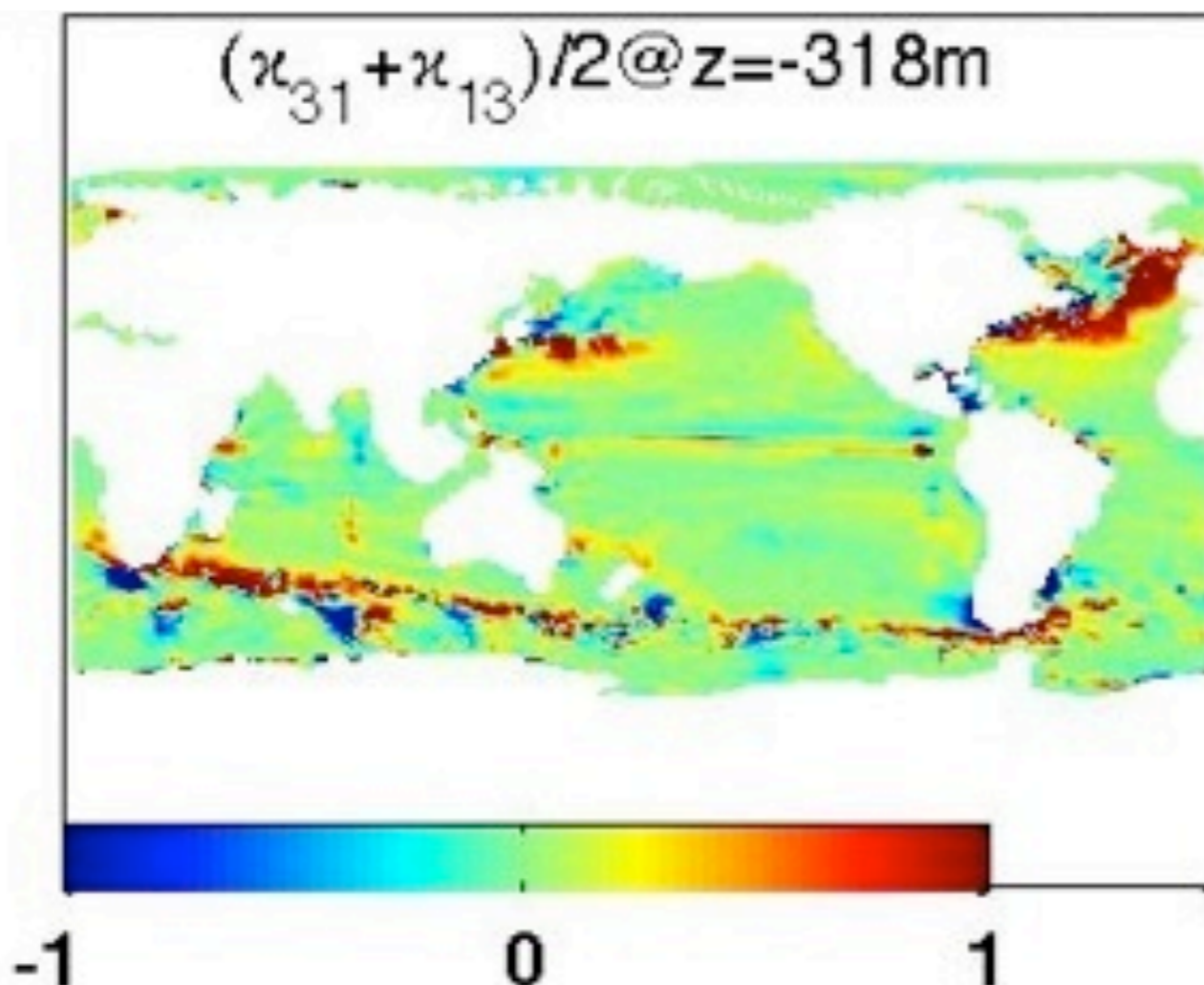
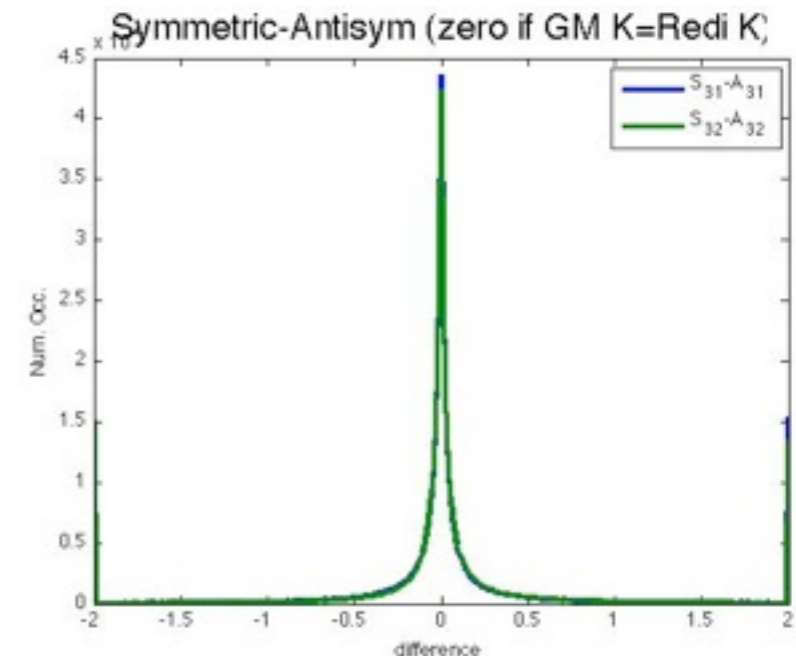
If so these 2 components should match in Sym & Antisym M



Result:

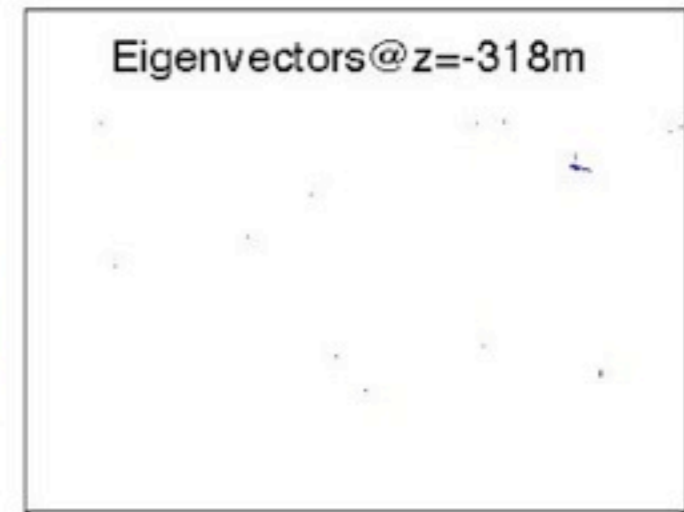
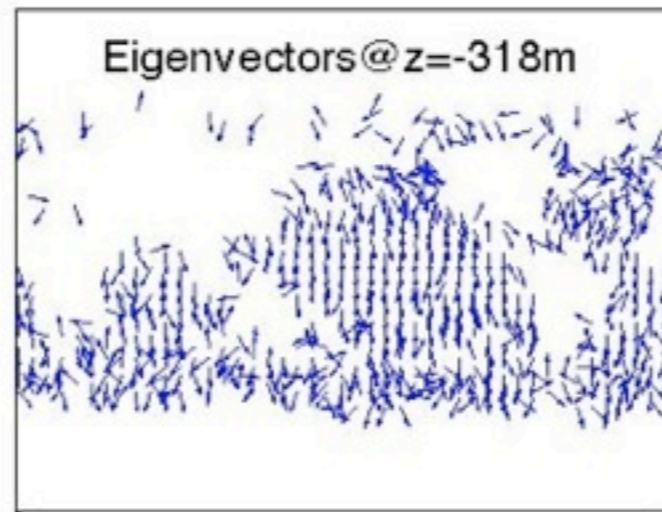
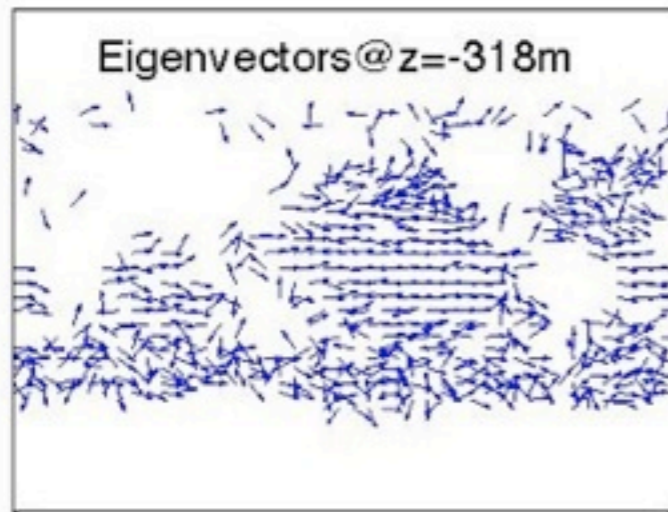
Redi $K=GM K$ (mostly)

If so these 2 components should match in **Sym** & Antisym M

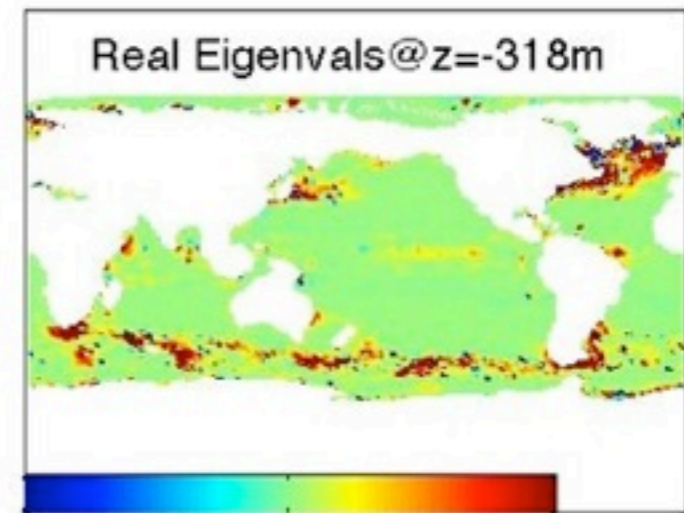
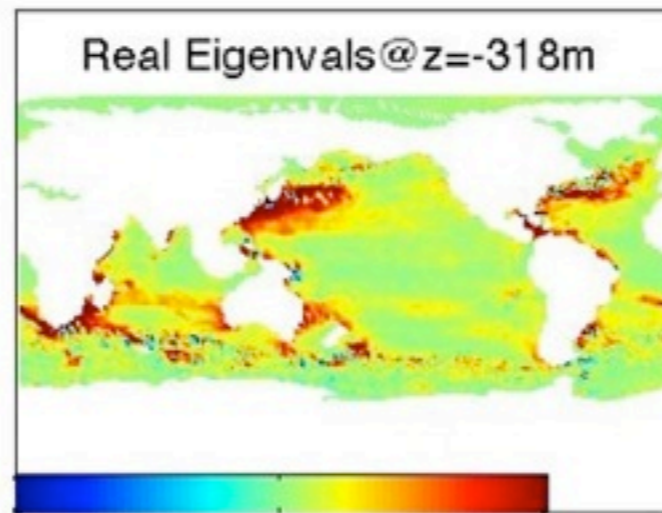
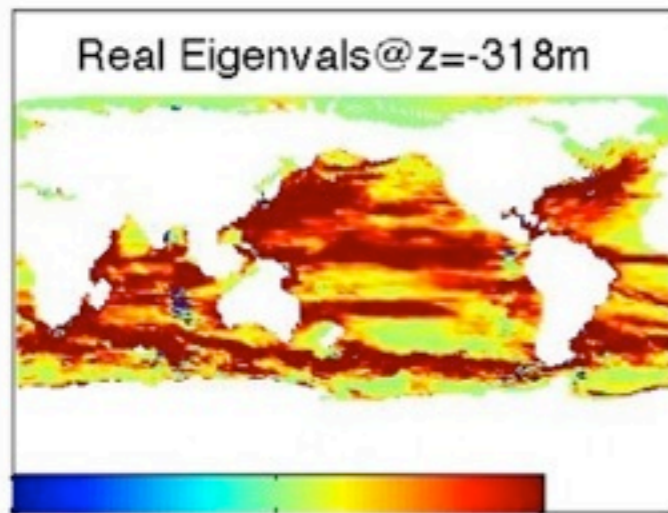


Result: Strong Anisotropy Along/Across PV Grads.

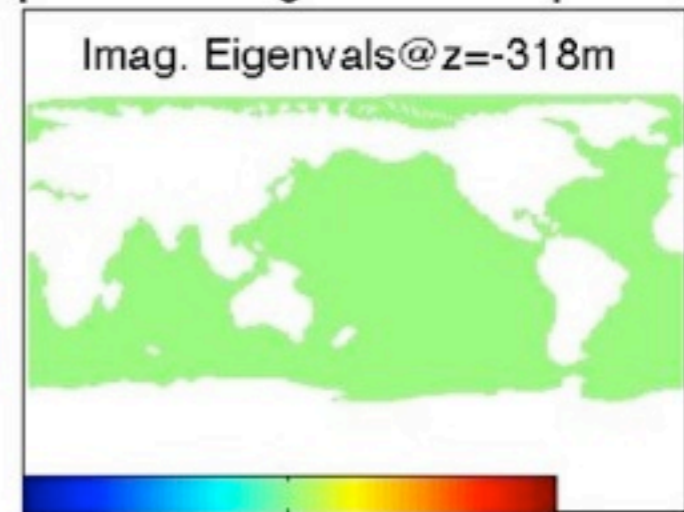
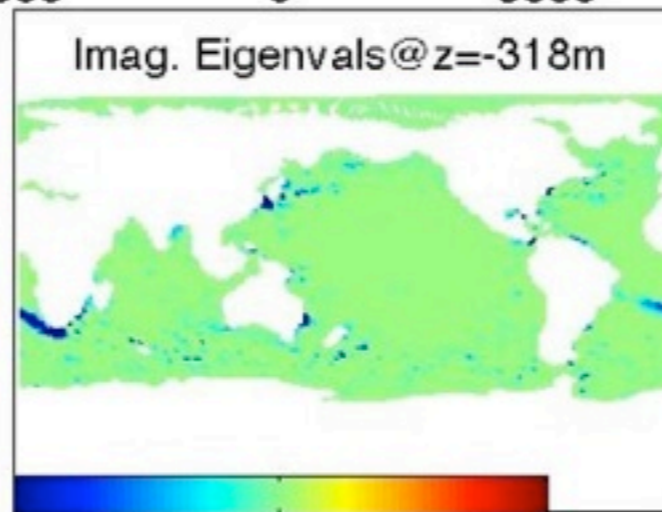
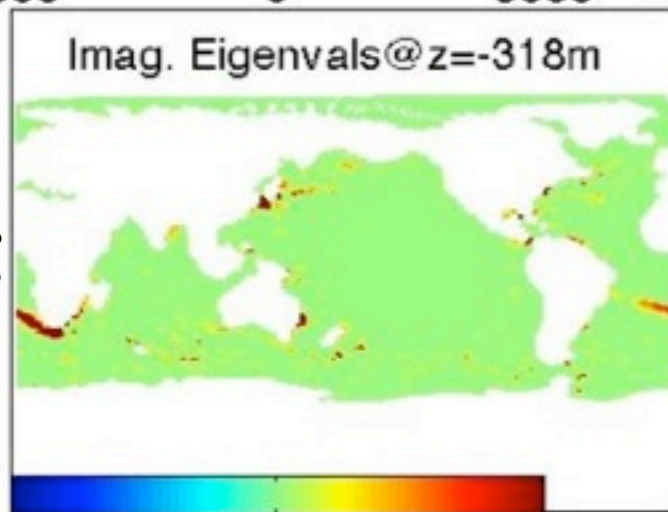
Mixing direction



Mixing:



Stirring:



$\times 10^{-3}$

Ferreira, Marshall, Heimbach 05

Comparisons with
Marshall et al.

Realistic negative
eigs. vs. spurious?

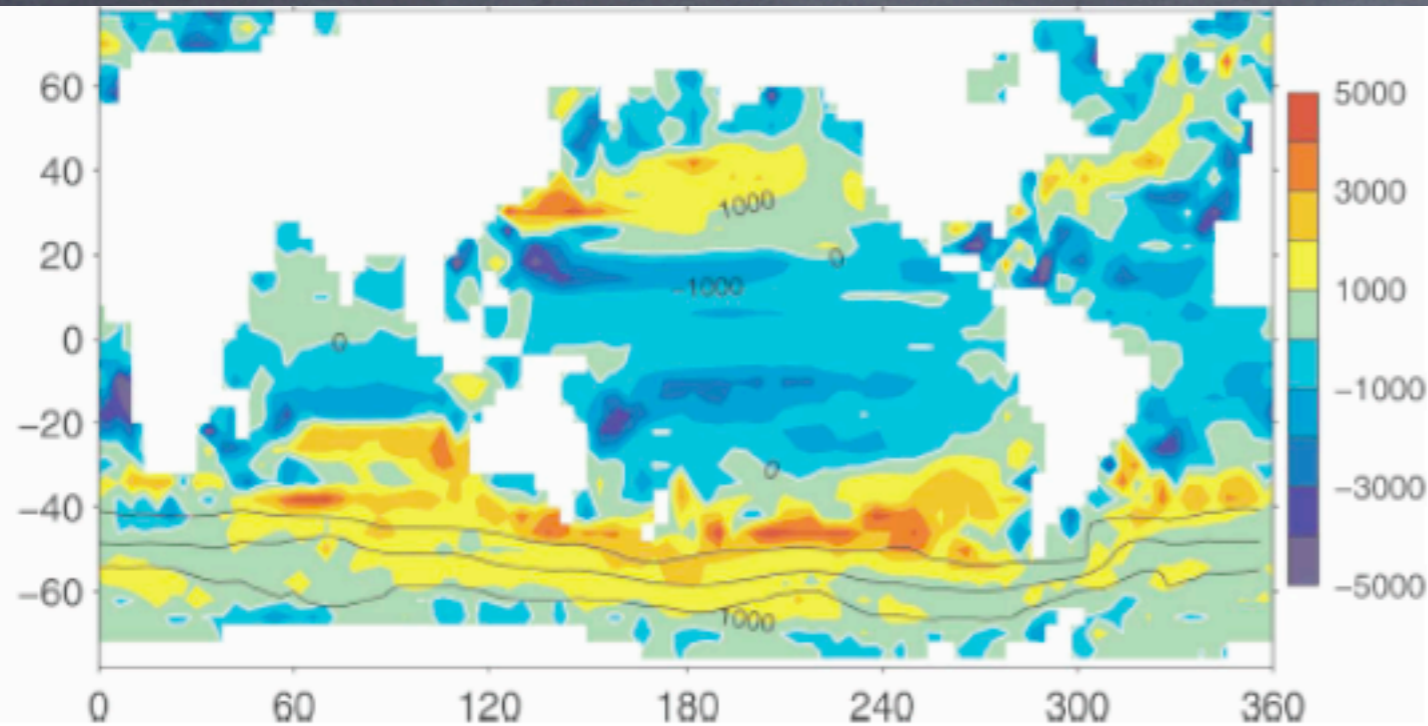
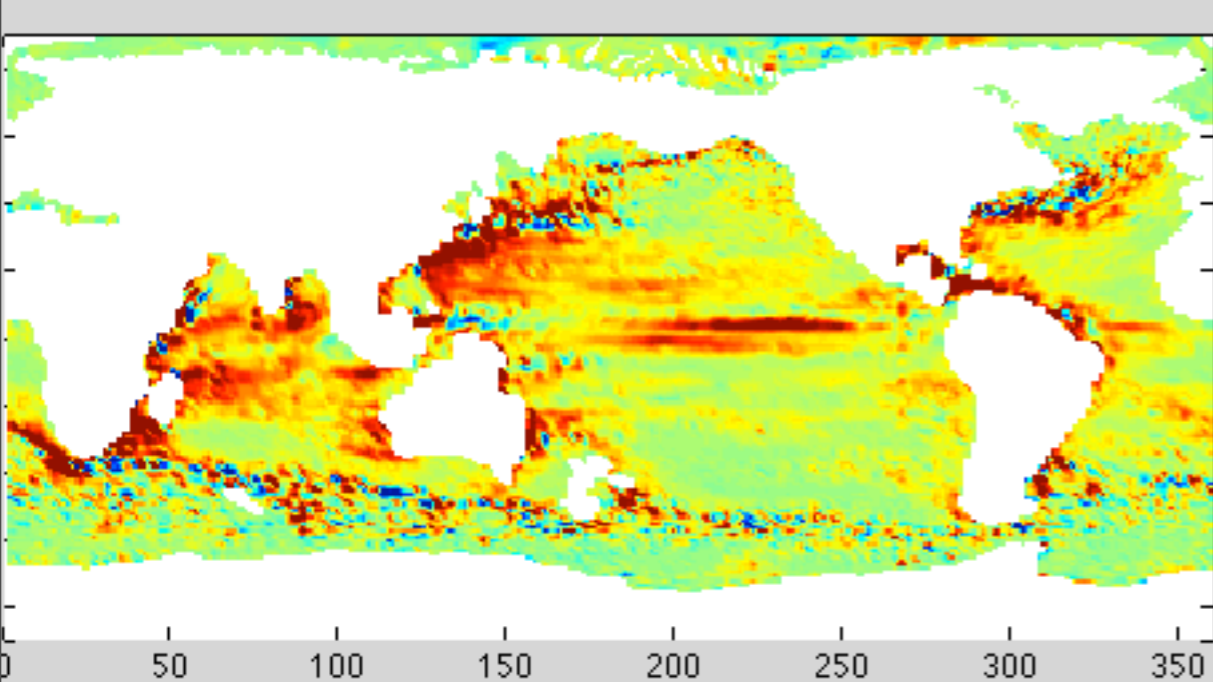
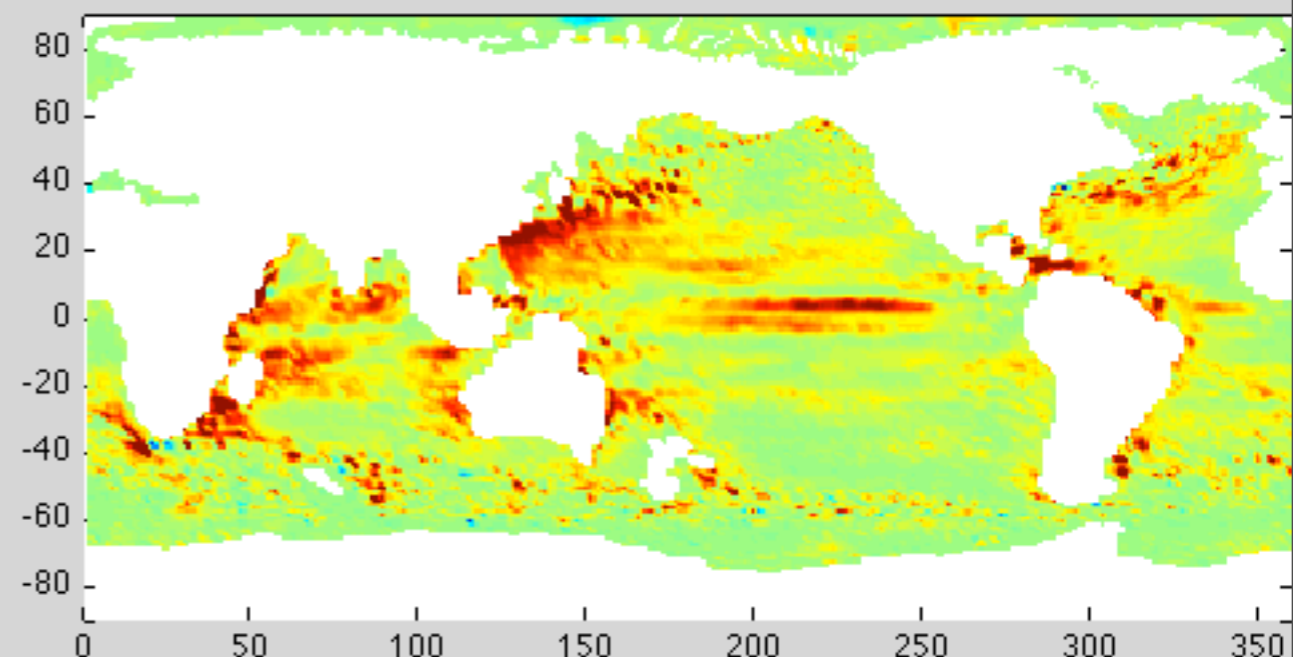
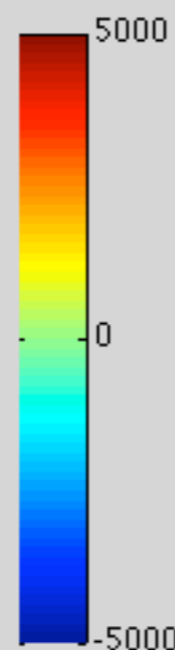


FIG. 12. Inferred horizontal eddy diffusivity κ ($\text{m}^2 \text{s}^{-1}$): (top) zonal mean and (bottom) vertical mean over the thermocline (0–1200 m). The contour intervals are (top) 500 and (bottom) 1000 $\text{m}^2 \text{s}^{-1}$. The thick line indicates the zero contour. Also indicated in the bottom panel are the 10-, 70-, and 130-Sv contours of the barotropic streamfunction.

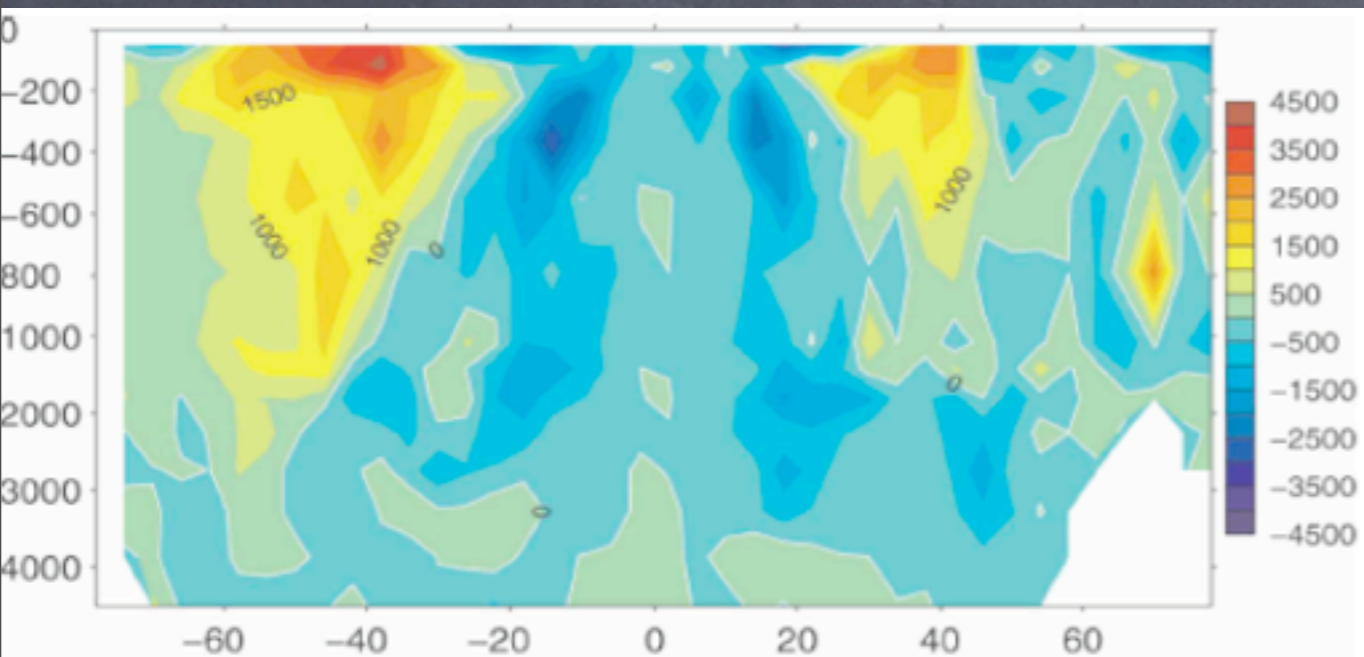


Re(2nd eigenvalue)



(2nd eigenvalue of symmetric M)

Comparisons with Marshall et al.

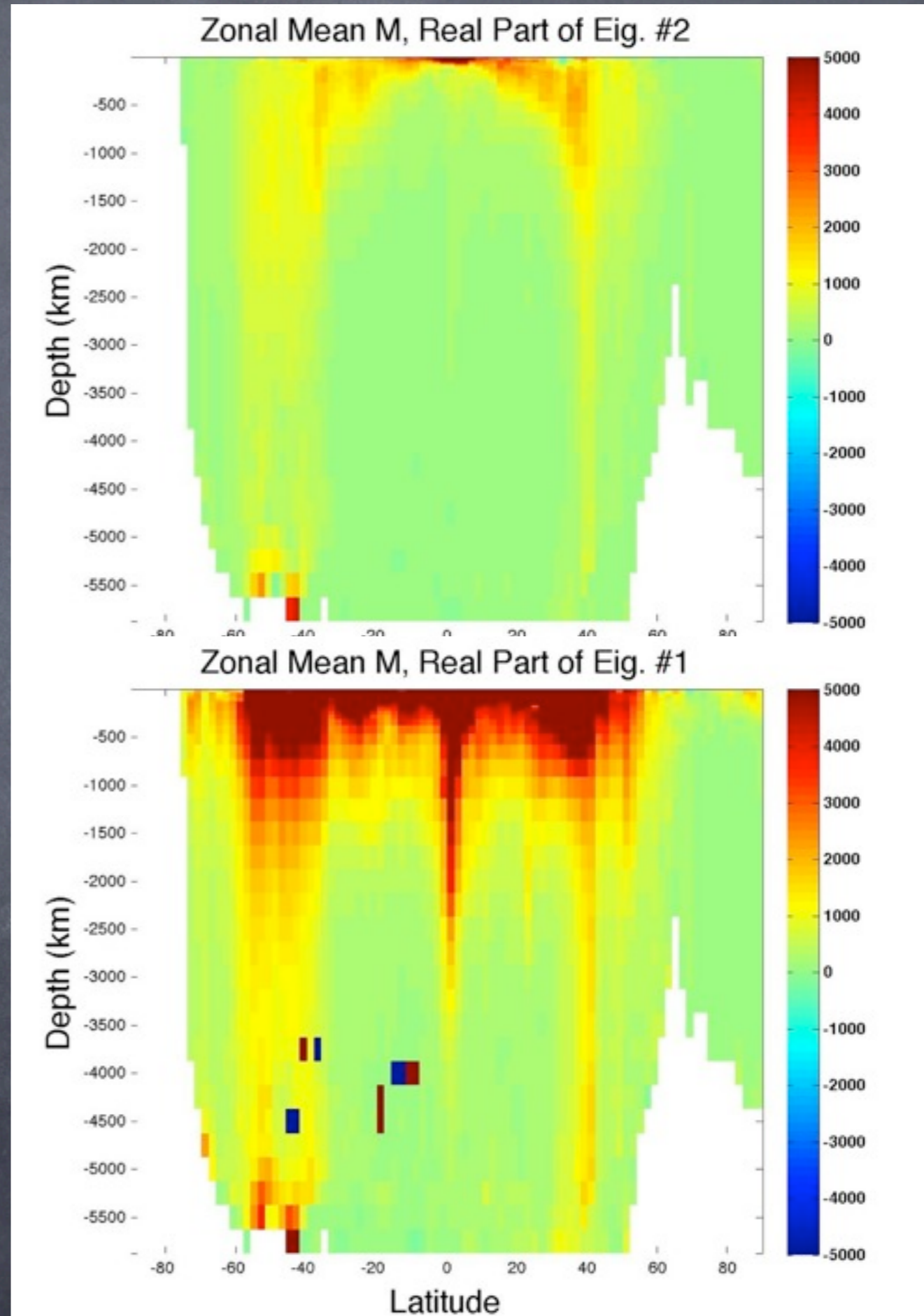


Ferreira, Marshall, Heimbach 05

Zonal mean (scalar) diffusivity
vs.

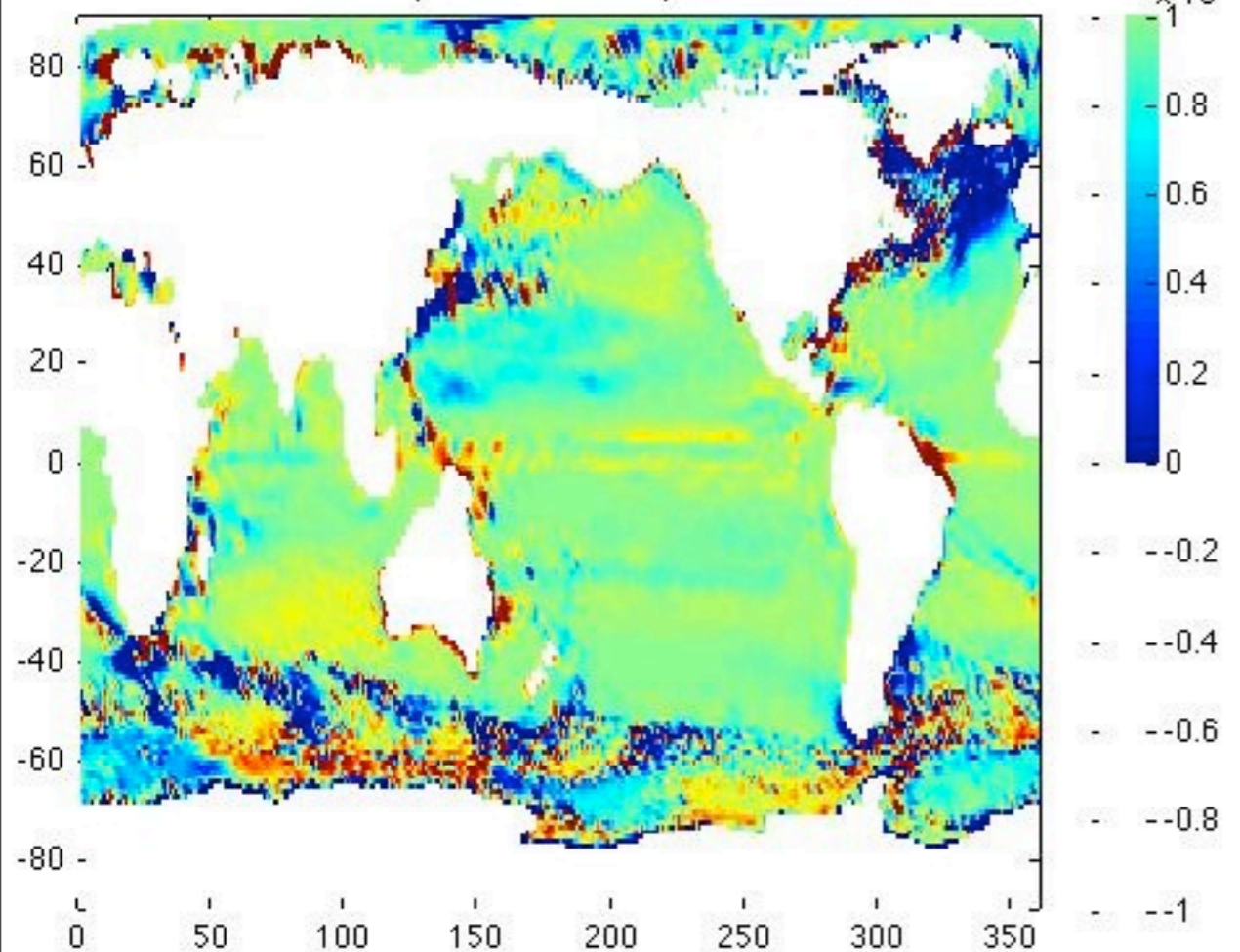
Eigenvalues of M

Same shape--few negatives!



How do we explain the
Horizontal Variations of
K?

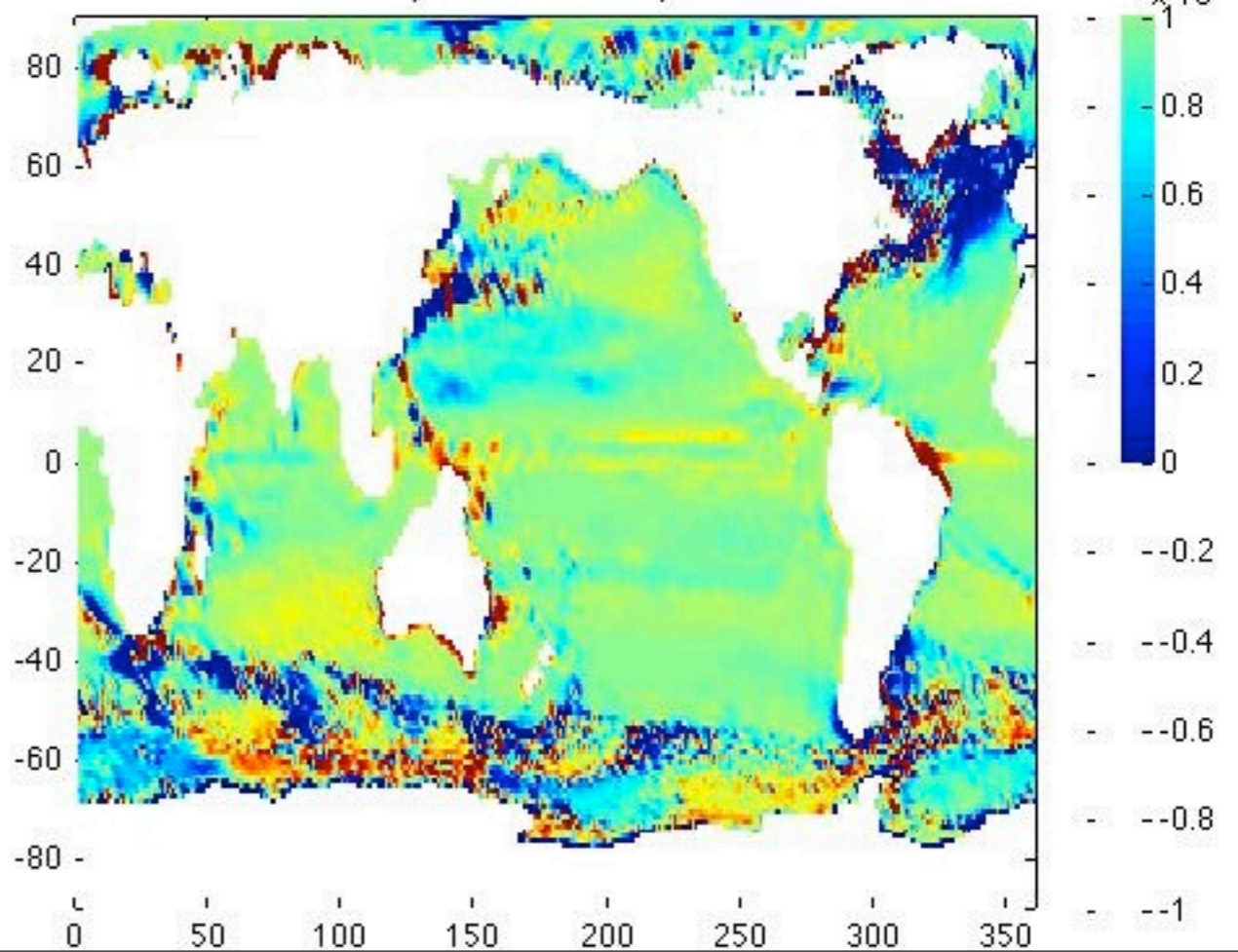
Vertical Eddy Potential Density Flux at 300m



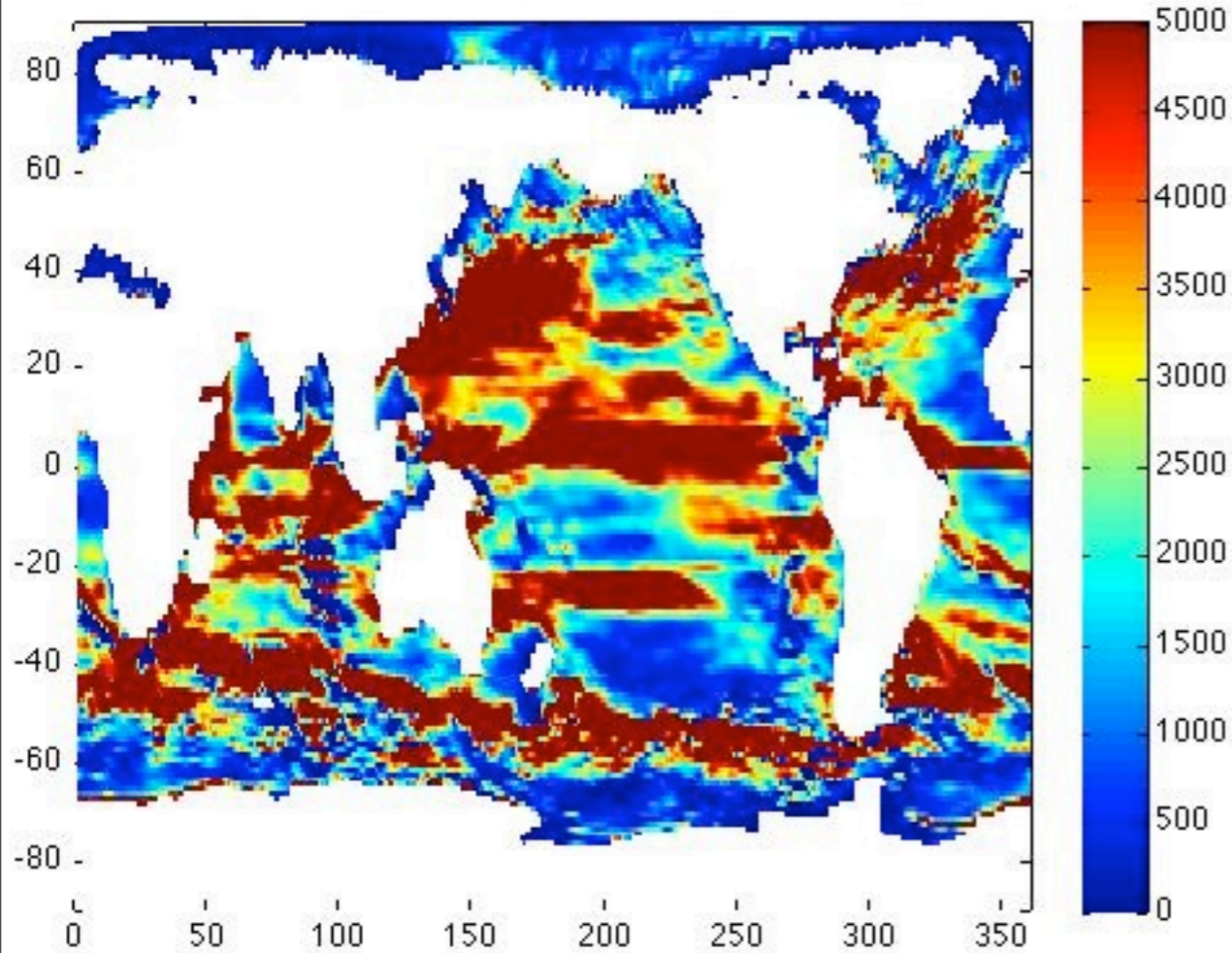
Compare to
vertical eddy
density flux
(PE Extraction)

Eden&Greatbatch (+others) propose that baroclinic instability's production of EKE from PE should guide M magnitude

Vertical Eddy Potential Density Flux at 300m



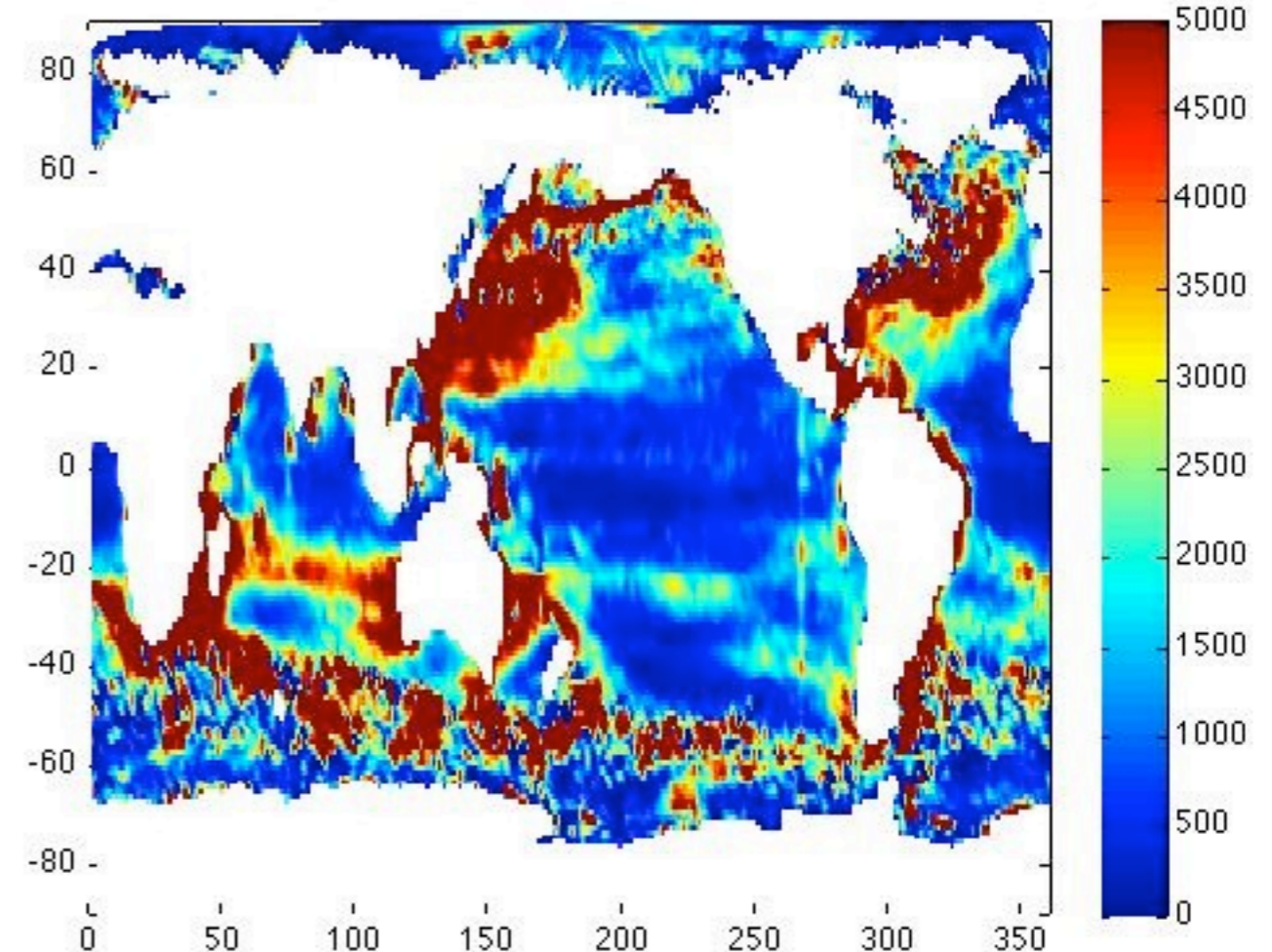
κ_{11}



Locations of
PE extraction
are

Locations of
large eigs of
K

κ_{22}



Compare with Eden, Jochum, Danabasoglu compilation of present parameterizations

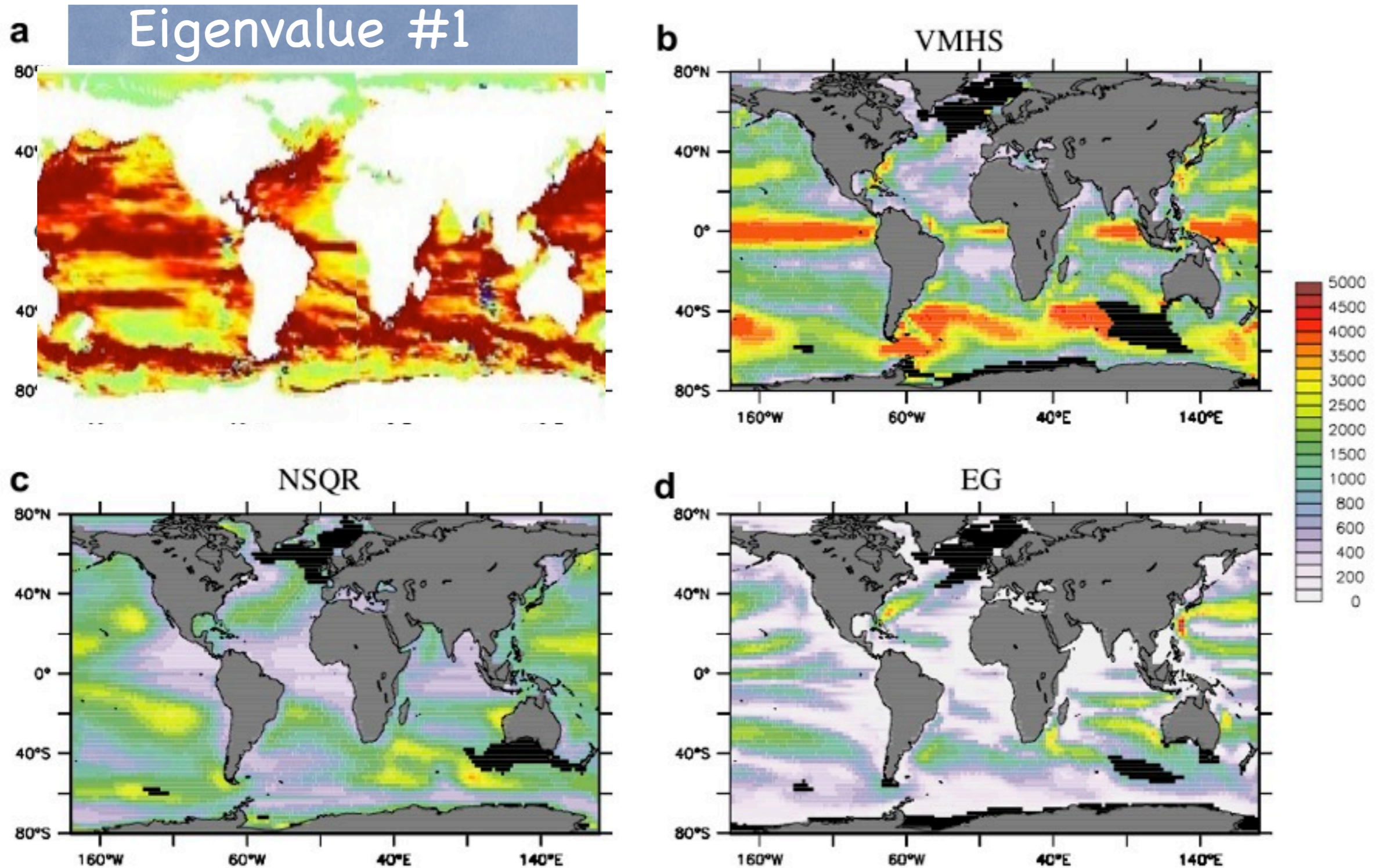


Fig. 1. Annual mean thickness diffusivity (K) in m^2/s at 300 m depth in experiment CONST (a), VMHS (b), NSQR (c) and EG (d) after 500 years integration. Values of K are shown for the interior region only, i.e. values of K in the (seasonal maximum) diabatic surface and transition layer are not shown and shaded black. Note the non-linear colour scale for the thickness diffusivity. Note also that the data have been interpolated from the model grid to a regular rectangular grid of similar resolution prior to plotting. The land mask in the figure (taken from Smith and Sandwell (1997)) differs therefore slightly from the model's land mask.

Compare with Eden, Jochum, Danabasoglu compilation of present parameterizations

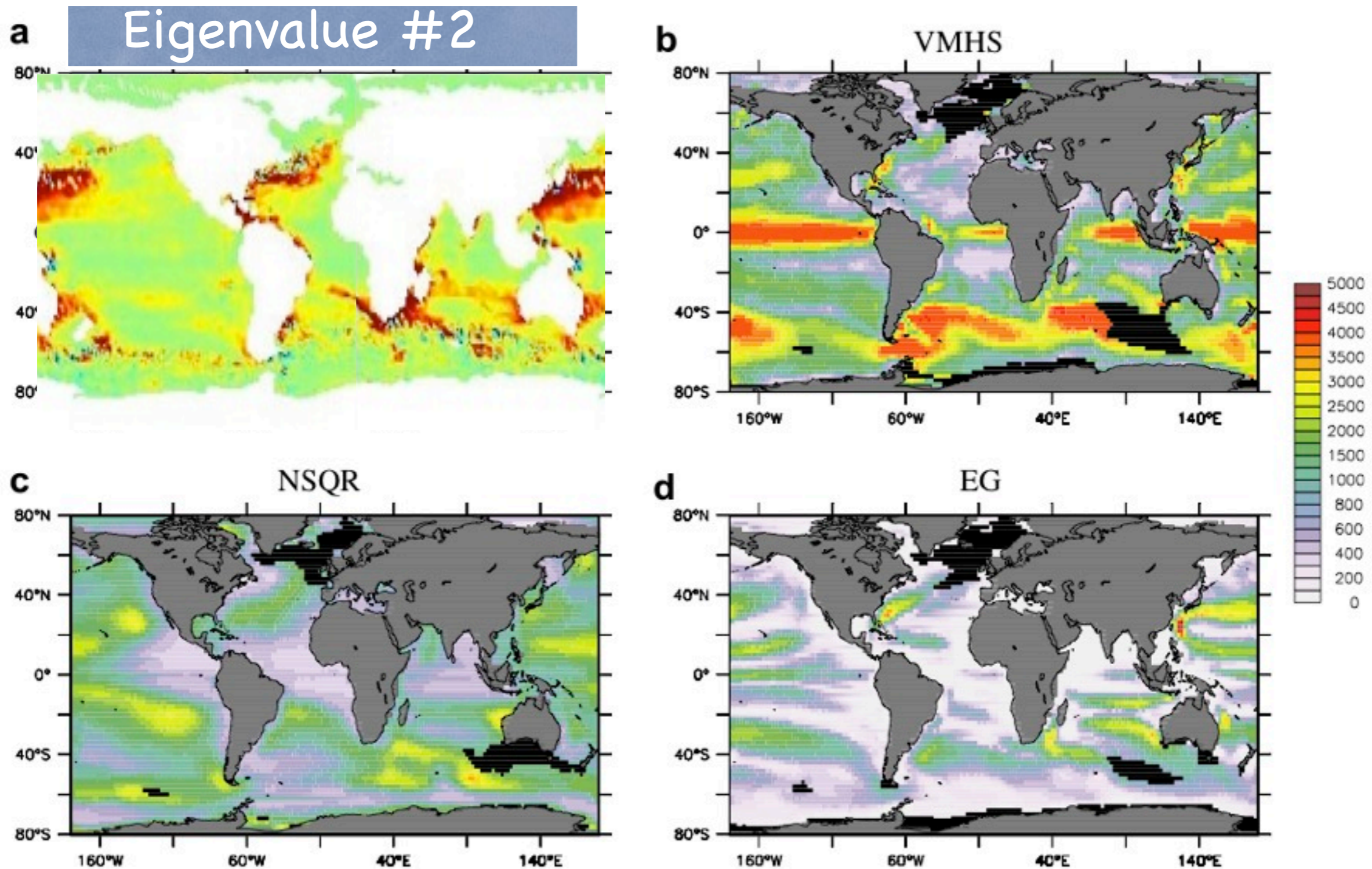


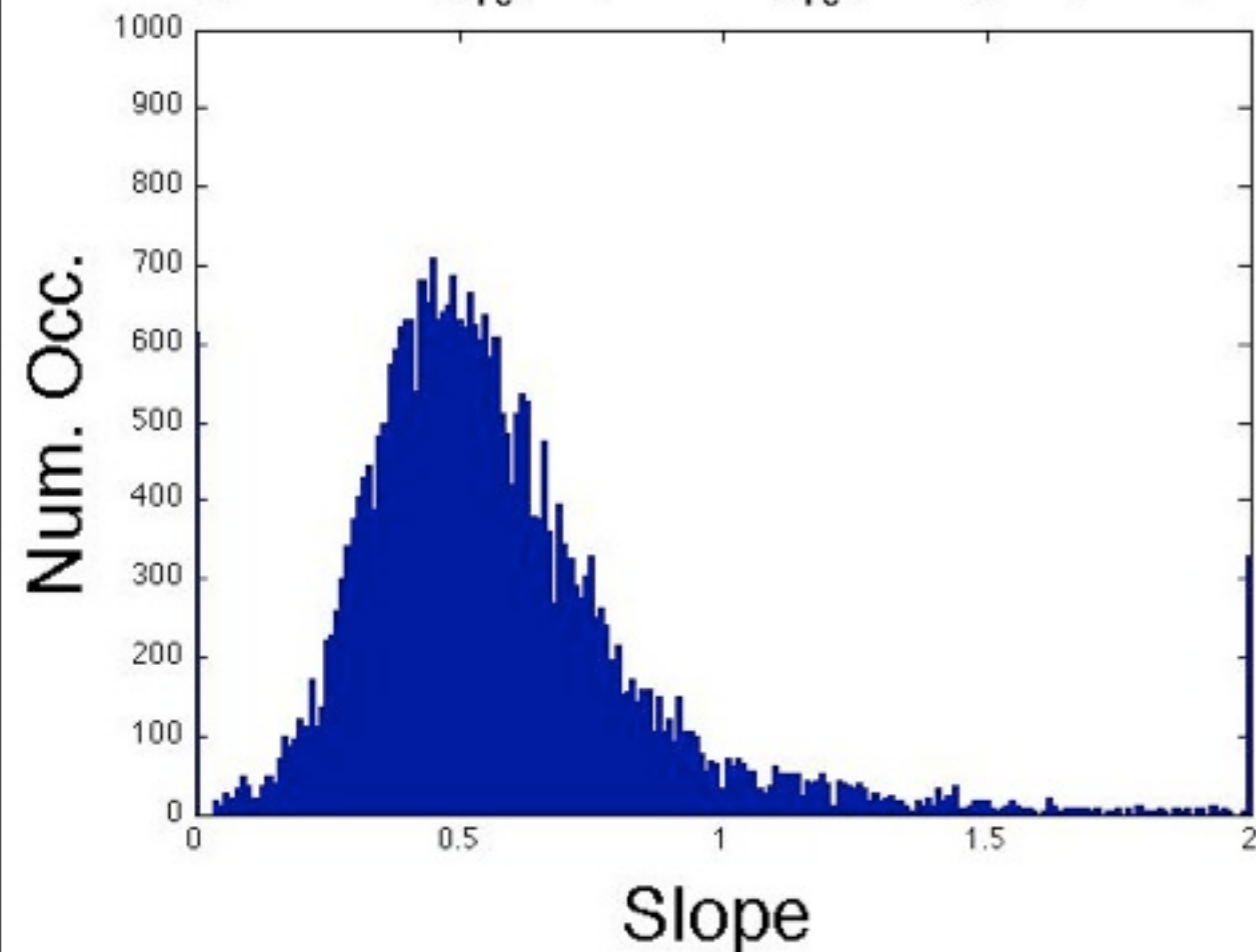
Fig. 1. Annual mean thickness diffusivity (K) in m^2/s at 300 m depth in experiment CONST (a), VMHS (b), NSQR (c) and EG (d) after 500 years integration. Values of K are shown for the interior region only, i.e. values of K in the (seasonal maximum) diabatic surface and transition layer are not shown and shaded black. Note the non-linear colour scale for the thickness diffusivity. Note also that the data have been interpolated from the model grid to a regular rectangular grid of similar resolution prior to plotting. The land mask in the figure (taken from Smith and Sandwell (1997)) differs therefore slightly from the model's land mask.

How do we explain the
Vertical Variations of K ?

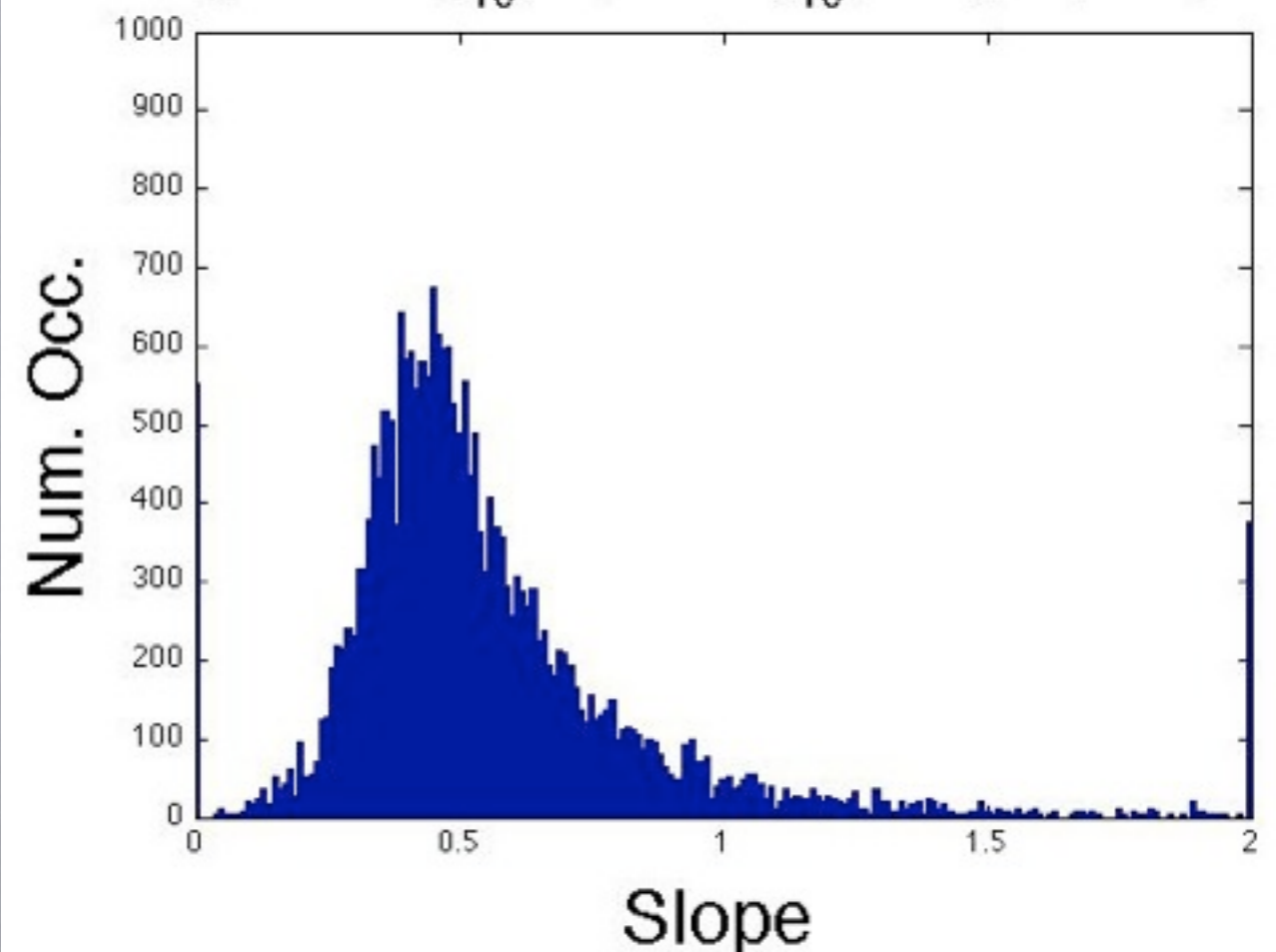
Result:
coarse KE \rightarrow vertical structure of Mixing

$$K \propto \sqrt{\langle KE \rangle}$$

Histogram of $\log_{10}(KE)$ vs. $\log_{10}(M \text{ eig. \#1})$ Slope

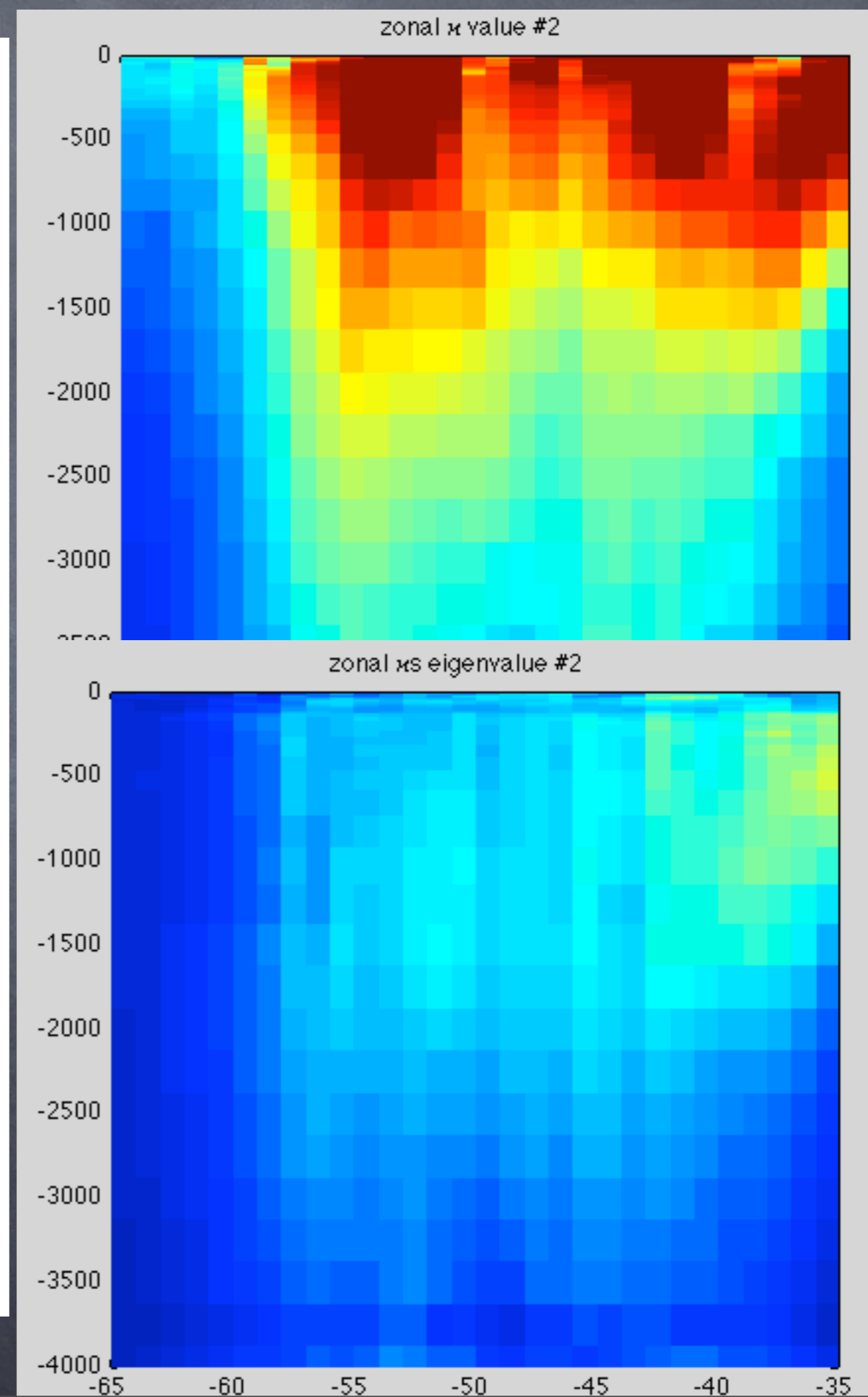
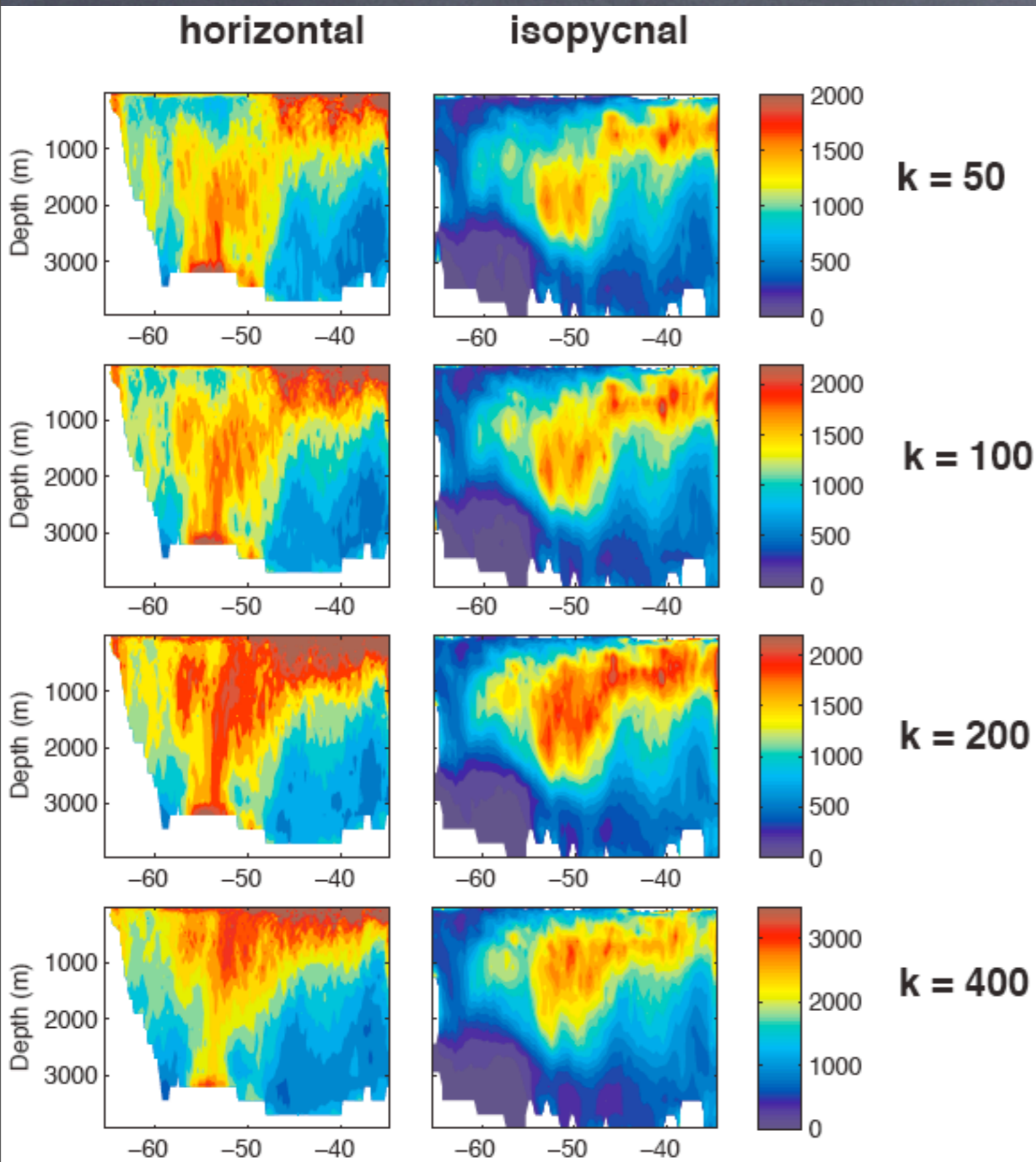


Histogram of $\log_{10}(KE)$ vs. $\log_{10}(M \text{ eig. \#2})$ Slope



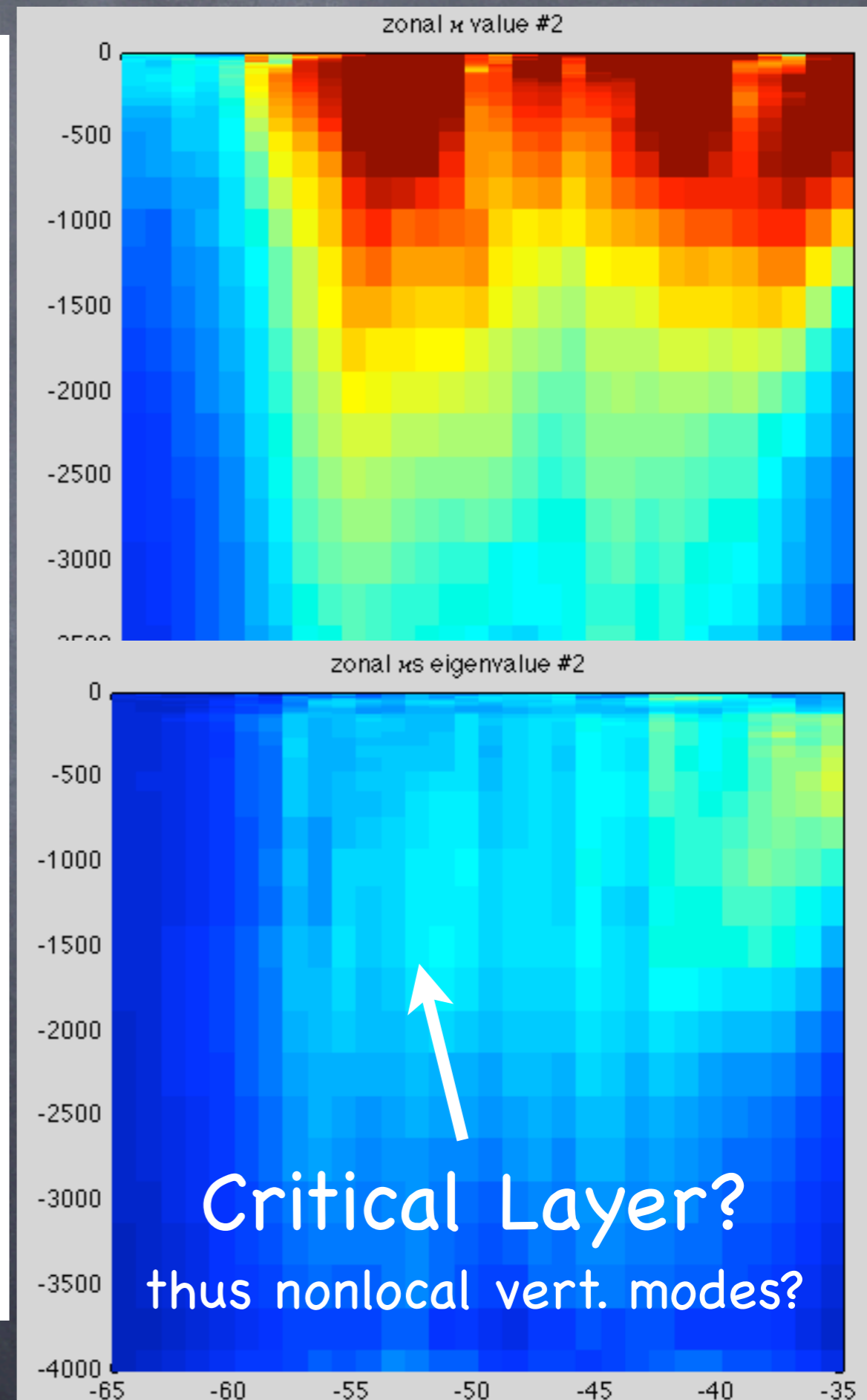
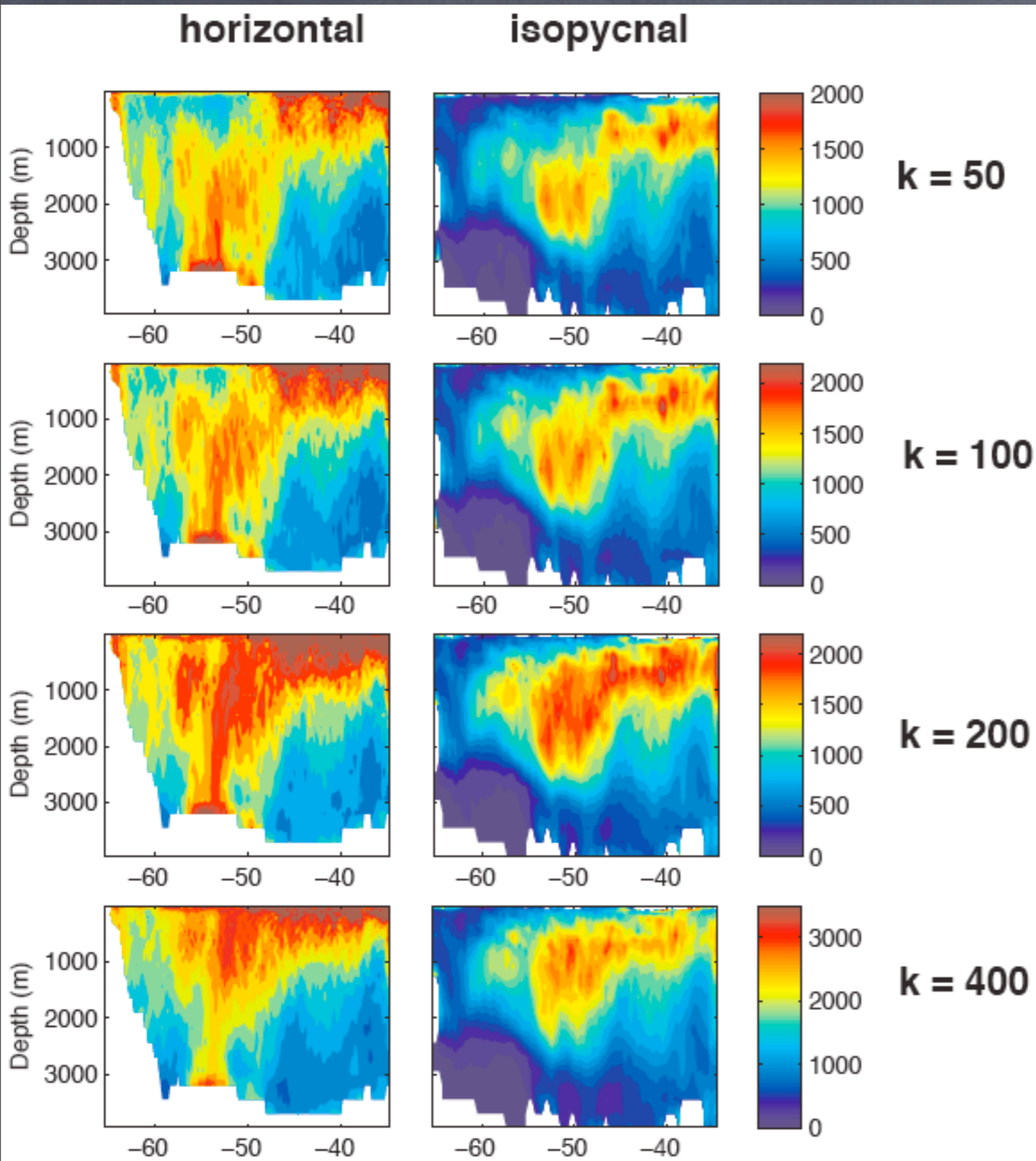
Even better with EKE!
Note--barotropic mode is in there!

Comparisons with Marshall et al.



Abernathy et al 09

Comparisons with Marshall et al.



Abernathy et al 09

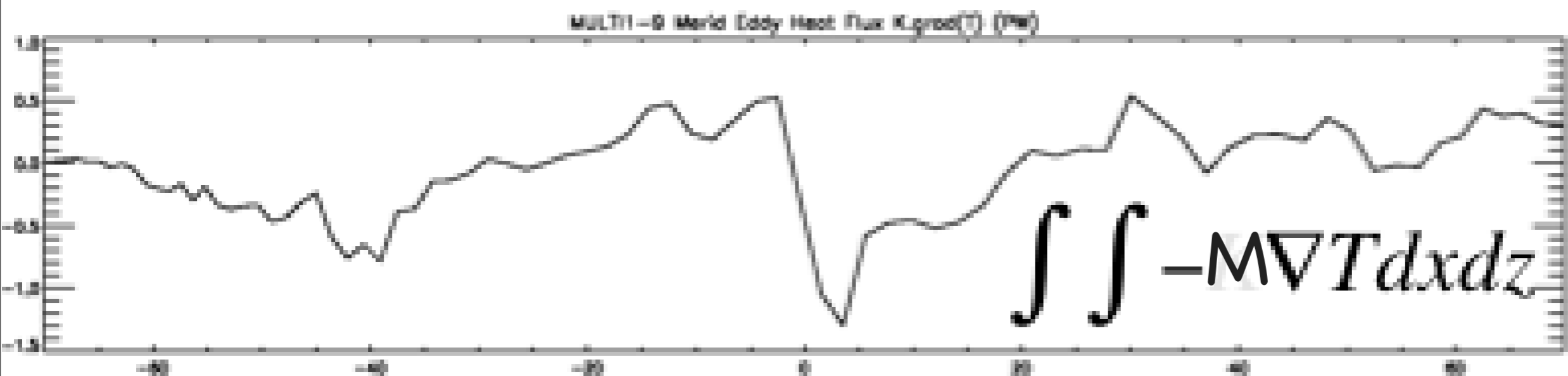
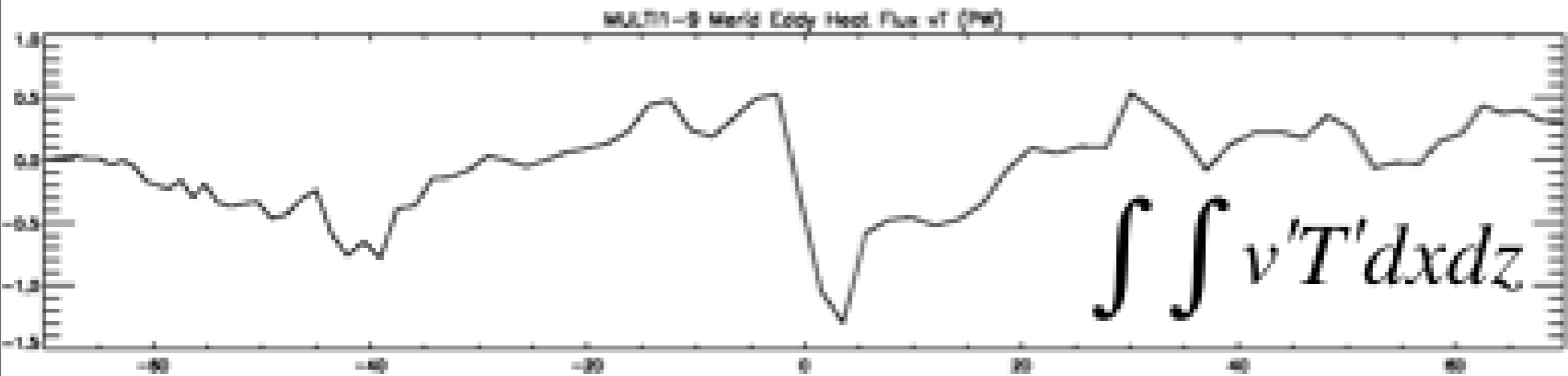
Conclusions

- A method for diagnosing the eddy stirring associated with fluxes represented in a 0.1° model but not a 2° model is presented
- It estimates the tracer-type-independent transport of tracer
- The shape and structure agrees roughly with Griffies (98) and Gent & Smith (04) analyses of GM & Redi isoneutral fluxes with *equal* anisotropic mixing & stirring.
- No gauge/rot. fluxes are needed to eliminate negative spurious eigenvalues

Use a Natural, Mesoscale Eddy Environment to Test Out:

Testing the Diagnosis:

Note: T not used for diagnosis, active tracers are apparently transported as passive ones are!



Difference: Diffusive - Eddy

Use a Natural, Mesoscale Eddy

Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla}_z \\ 0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla}_z \\ \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla}_z & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla}_z & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

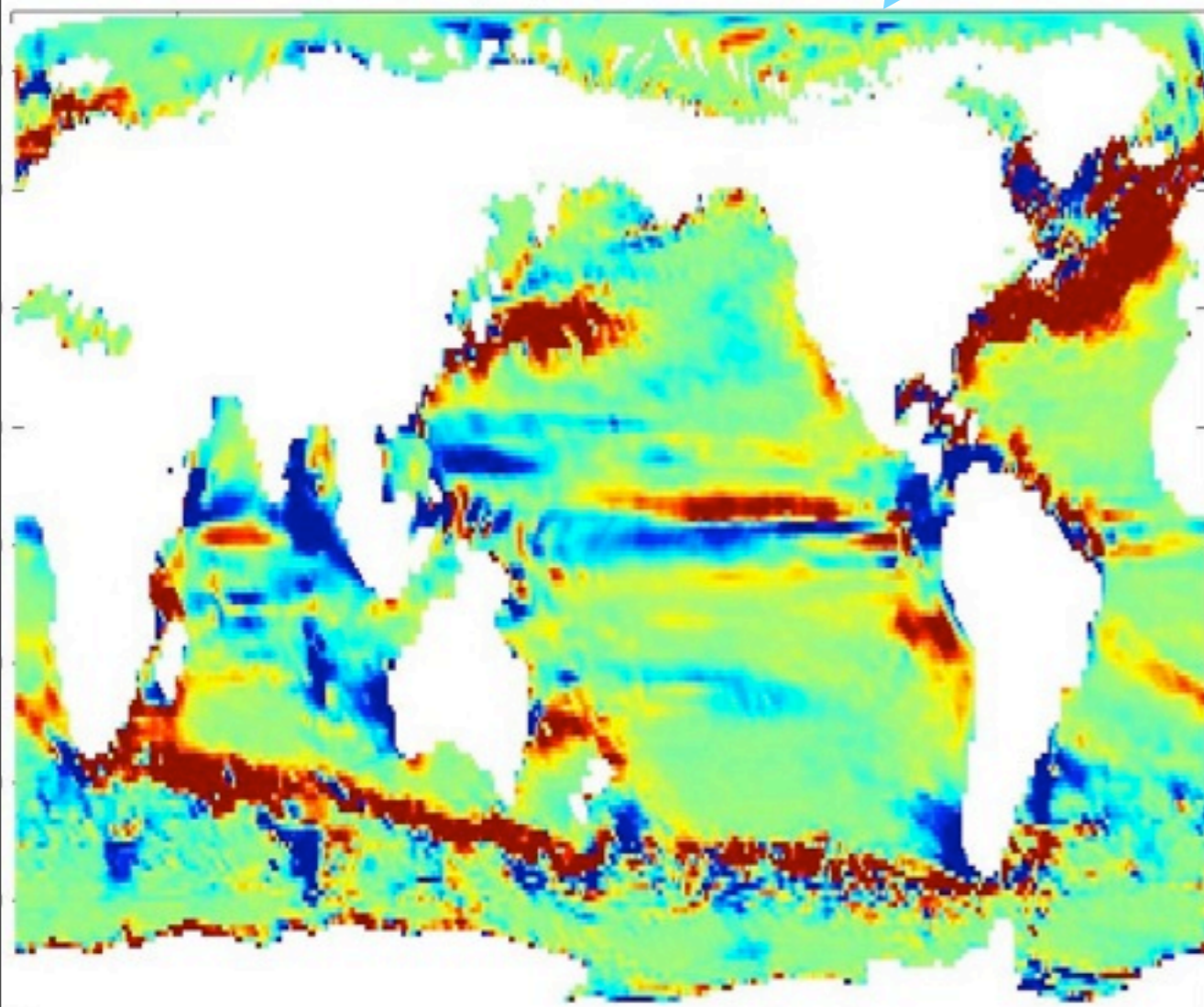
Result 1: Antisymmetric (GM) Elements
scale with
corresponding Symmetric (Redi)
elements in extratropics.

Thus, GM/Redi basic shape of M is
roughly correct
(some detailed validation remains)

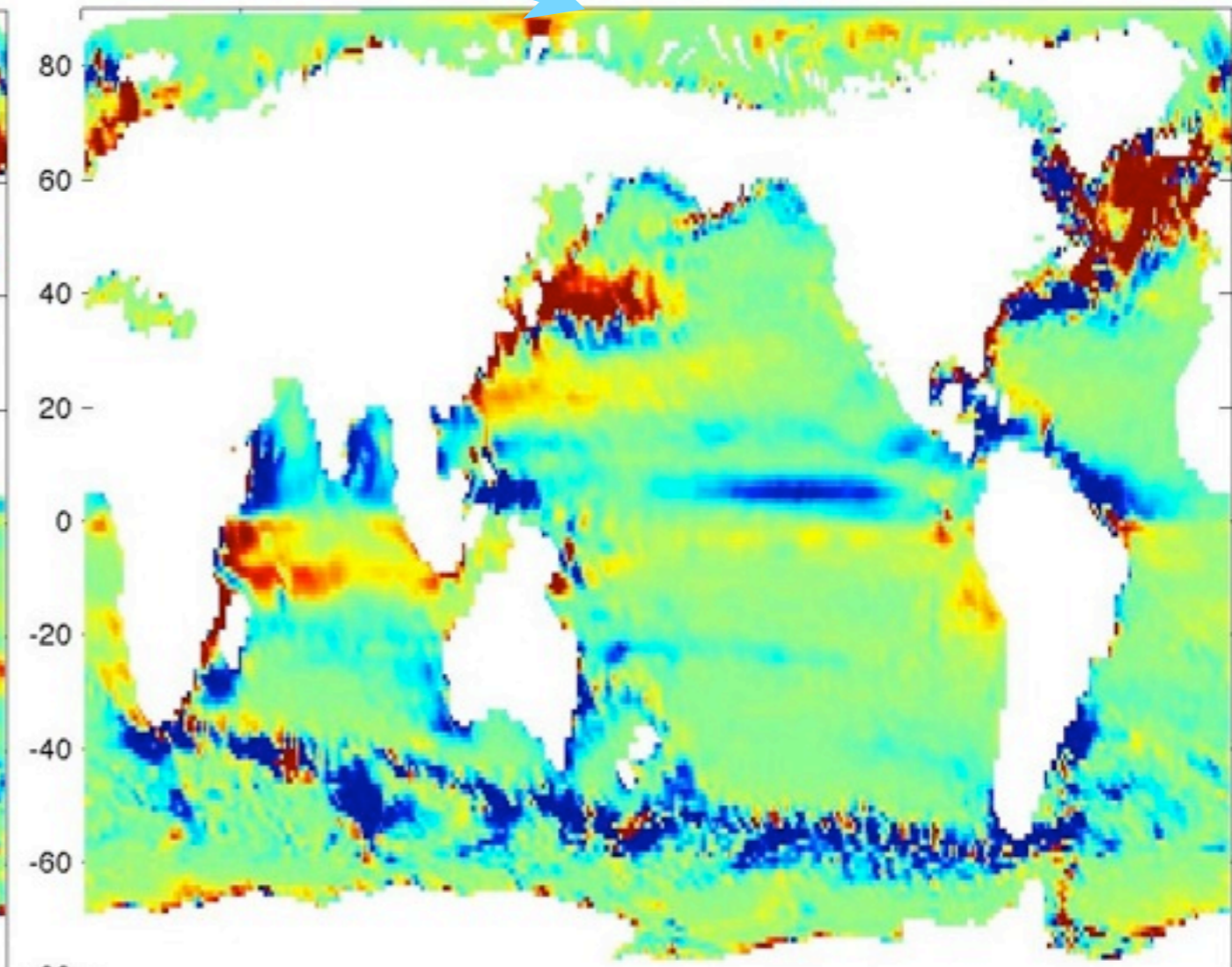
Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ 0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Asym 3,1: GM@z=-149rr



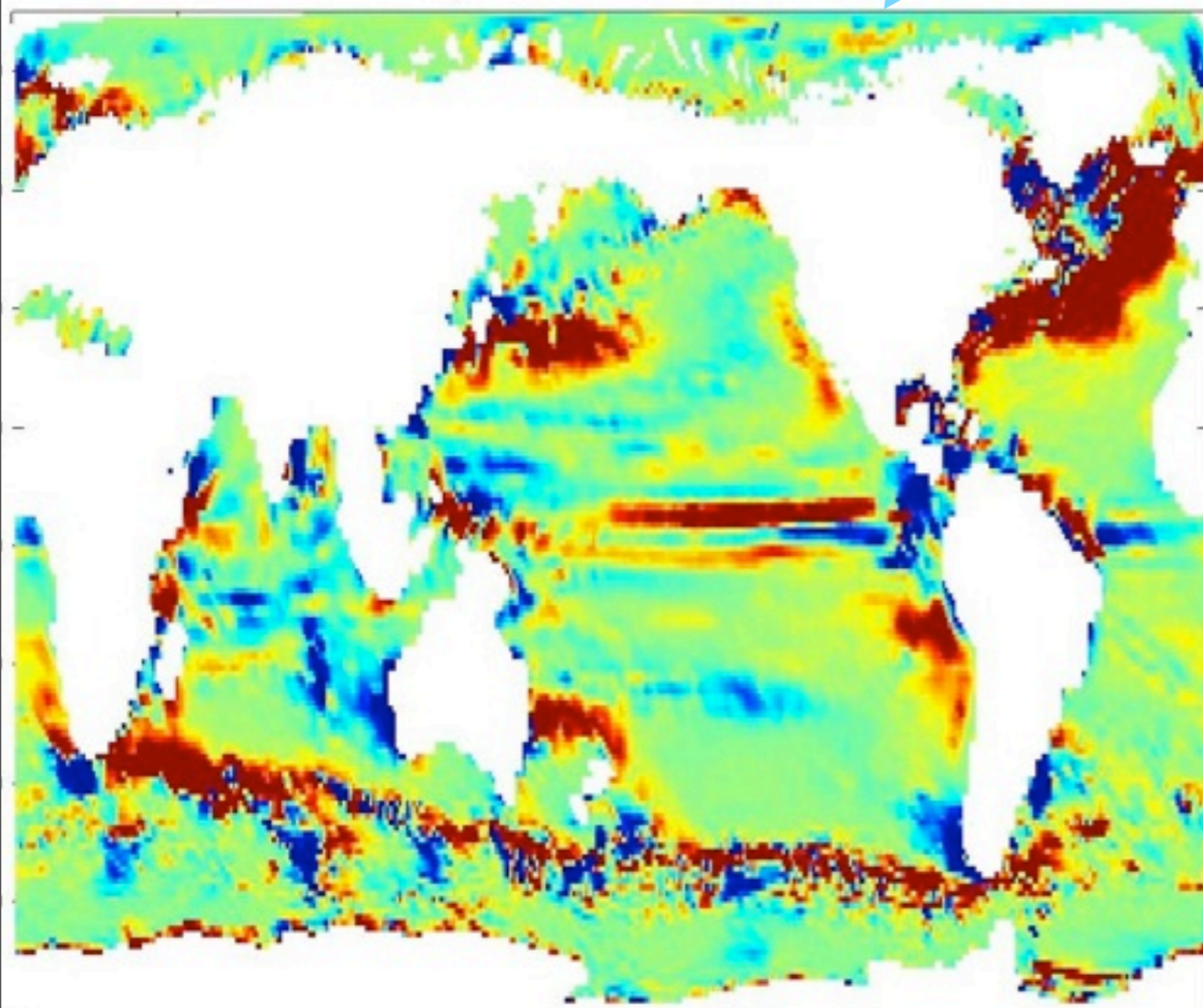
Asym 3,2: GM@z=-149rr



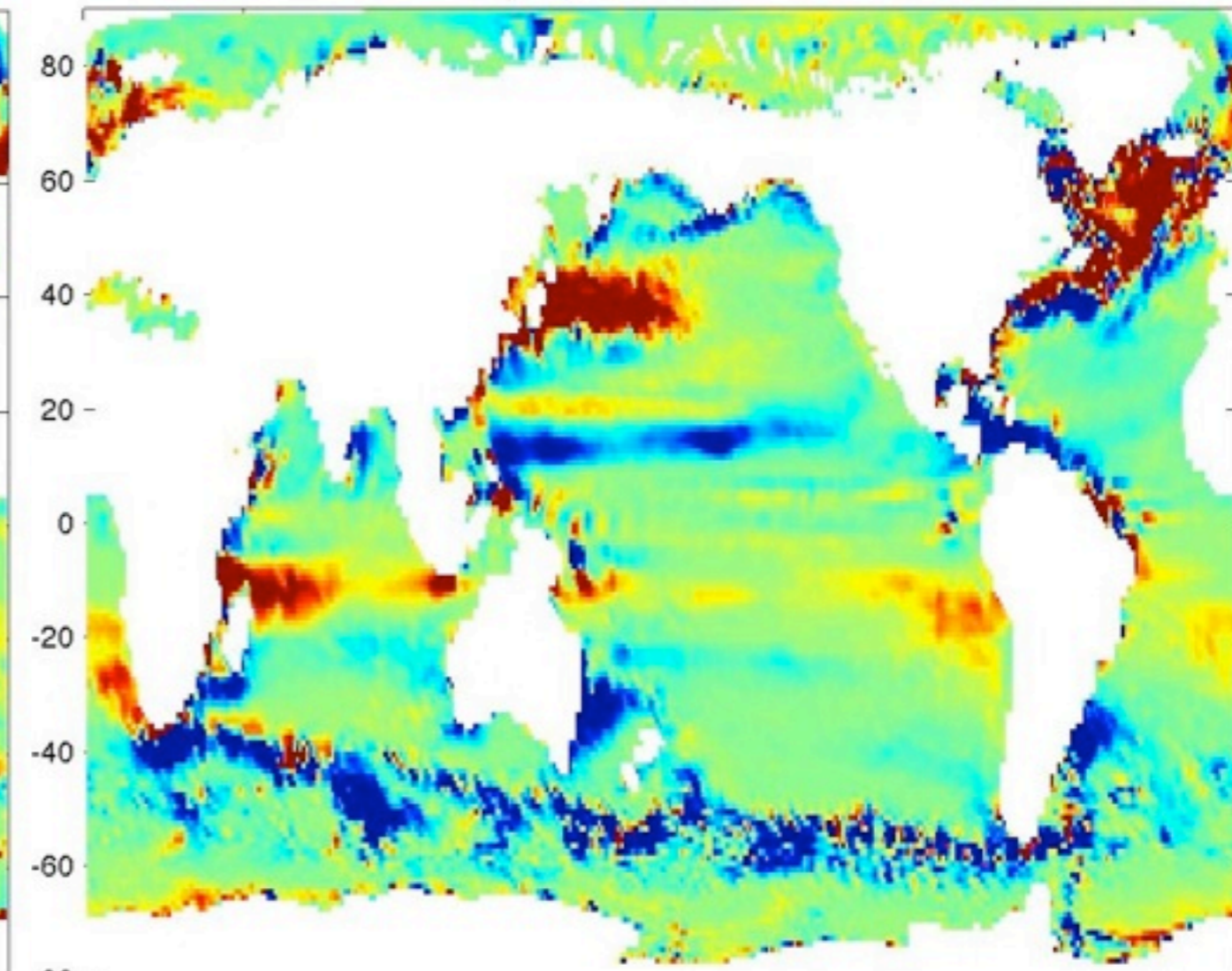
Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & \tilde{\nabla} \mathbf{z} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Sym 3,1: Redi@z=-149m



Sym 3,2: Redi@z=-149m

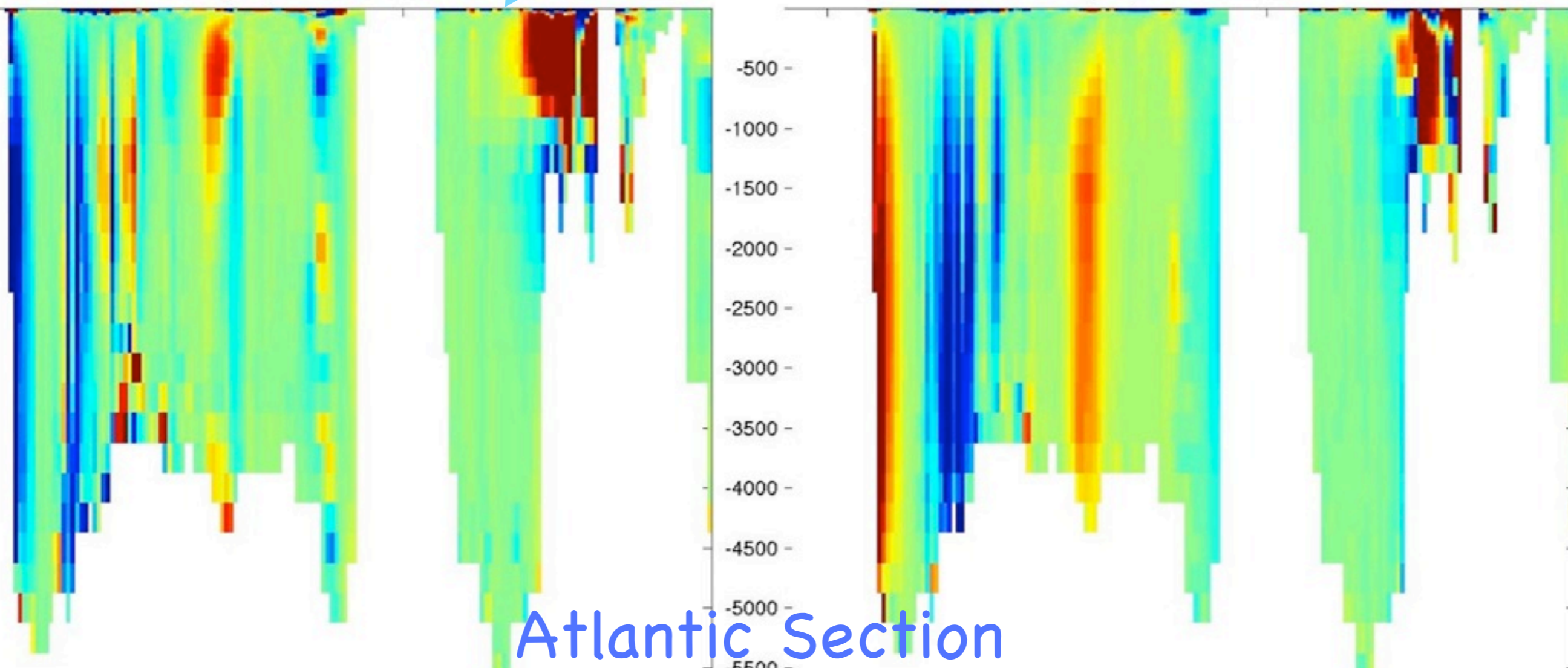


Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ 0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Asym 3,1: GM@lon=345E

Asym 3,2: GM@lon=345E

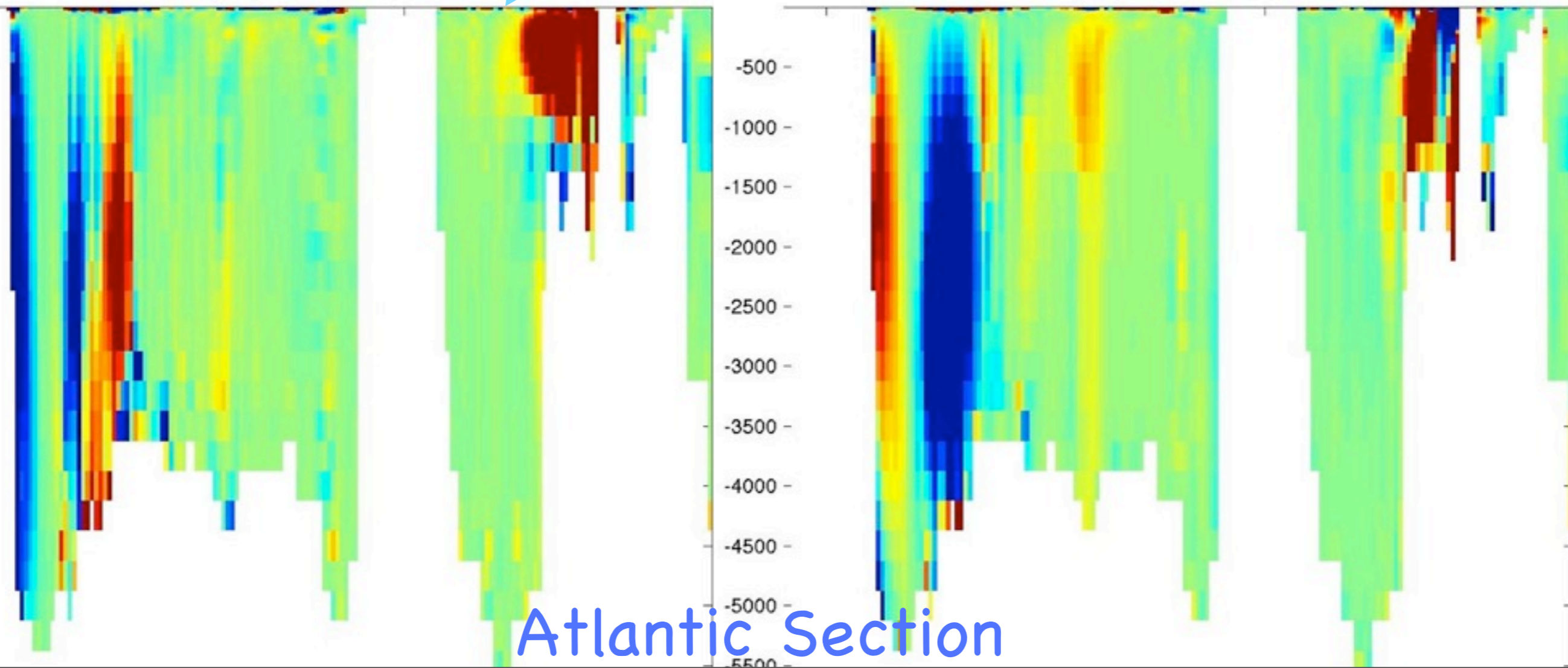


Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & \tilde{\nabla} \mathbf{z} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Sym 3,1: Redi@lon=345E

Sym 3,2: Redi@lon=345E

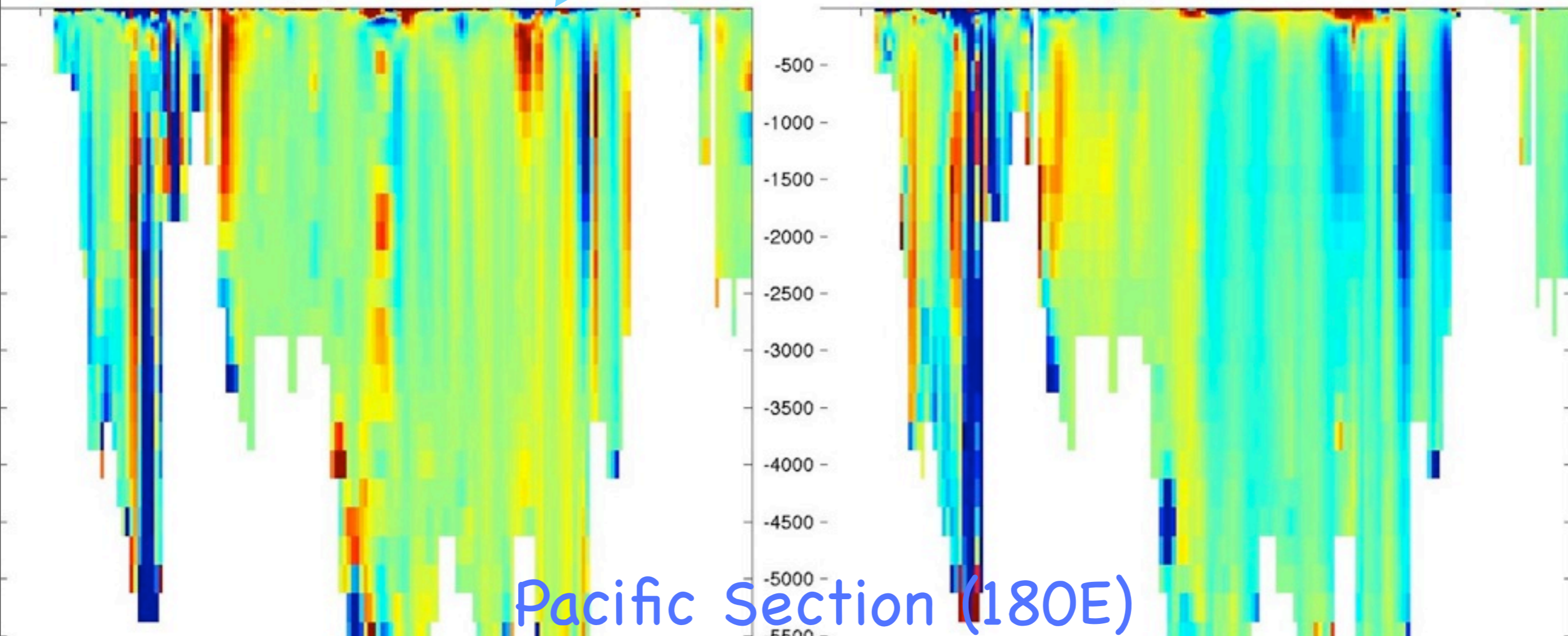


Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla}_z \\ 0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla}_z \\ \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla}_z & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla}_z & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Asym 3,1: GM@lon=180E

Asym 3,2: GM@lon=180E

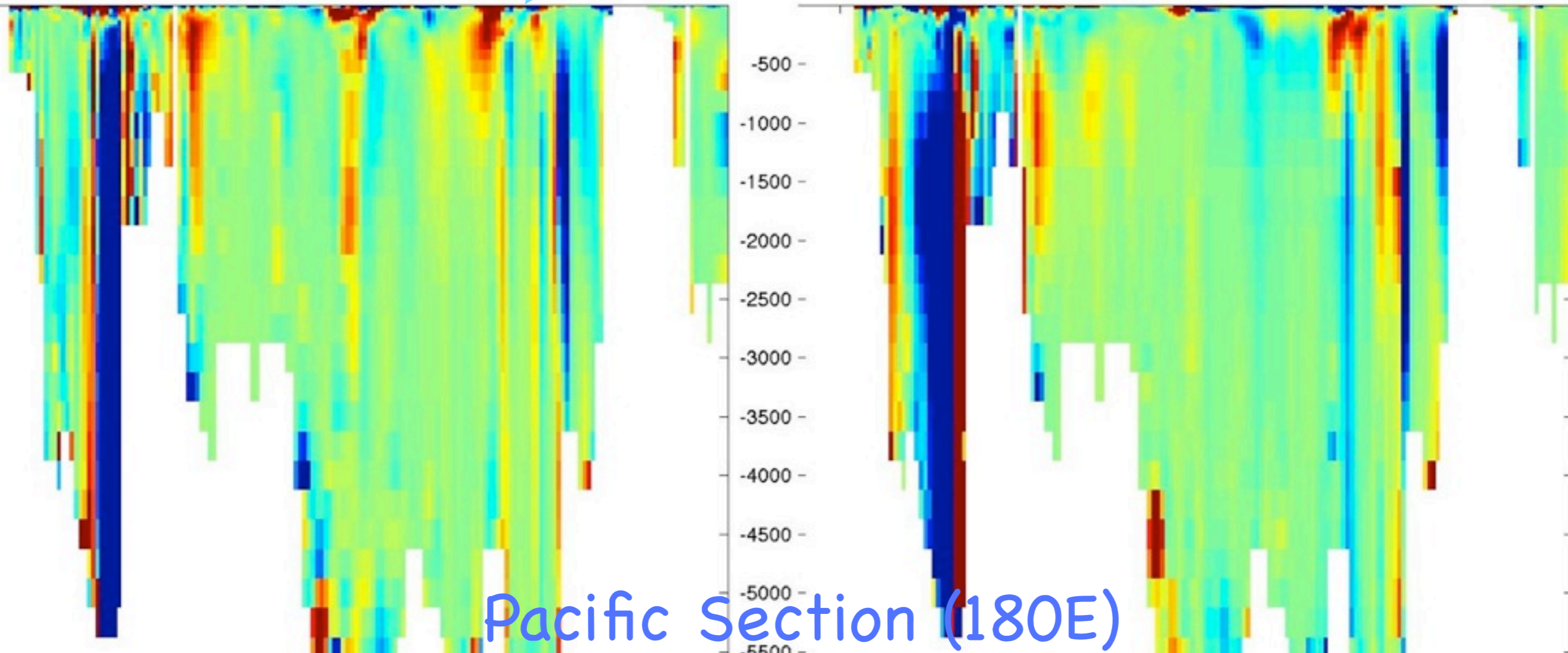


Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & \tilde{\nabla} \mathbf{z} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Sym 3,1: Redi@lon=180E

Sym 3,2: Redi@lon=180E



NSEF & Diabatic/ Transition Layer

- Danabasoglu & Marshall
- Danabasoglu, Ferrari & McWilliams
- Ferrari, McWilliams, Canuto, Dubovikov
- Surface-intensified GM, no boundary condition issues, no over-restratification of Mixed Layer by Eddies

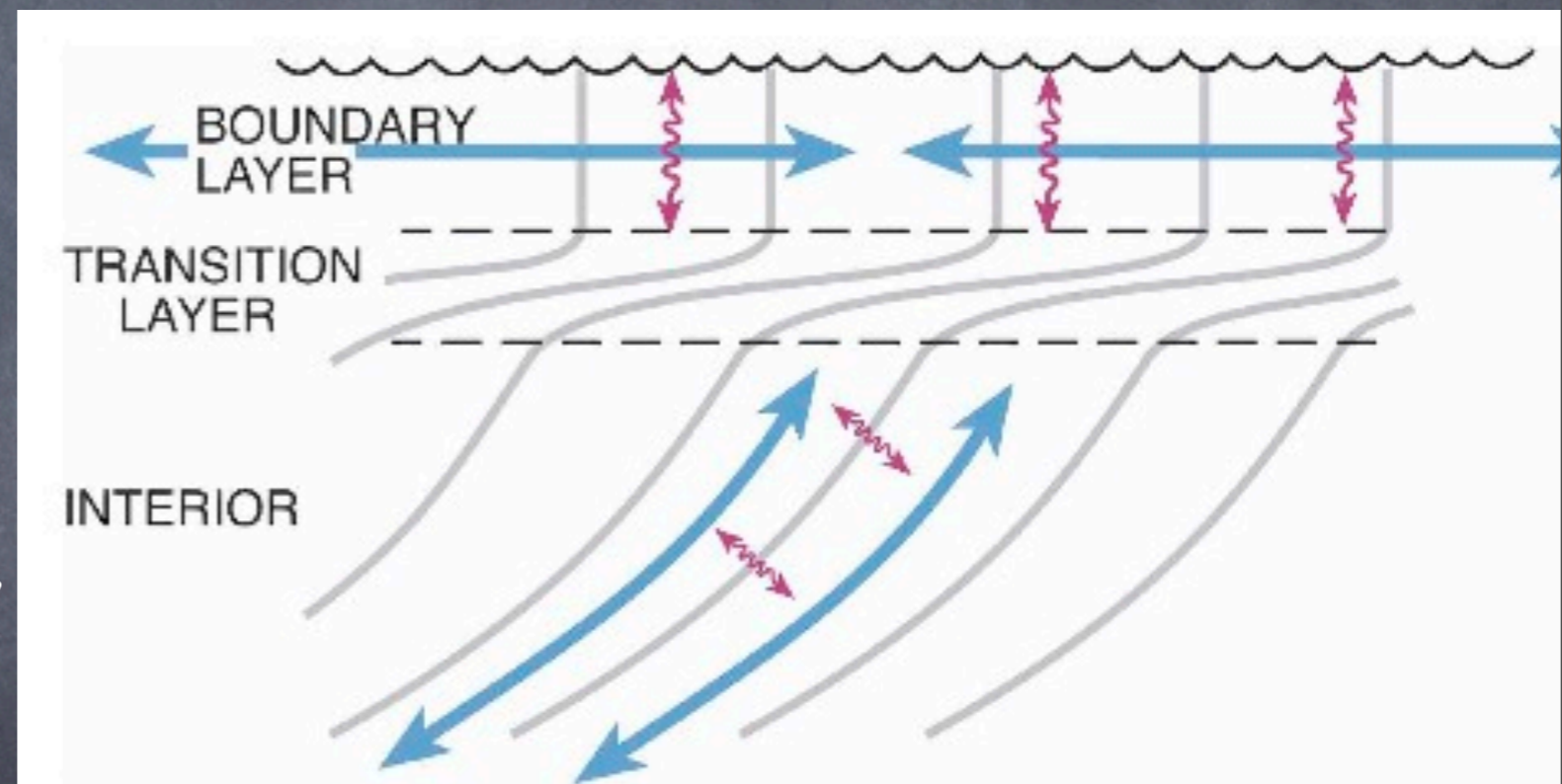
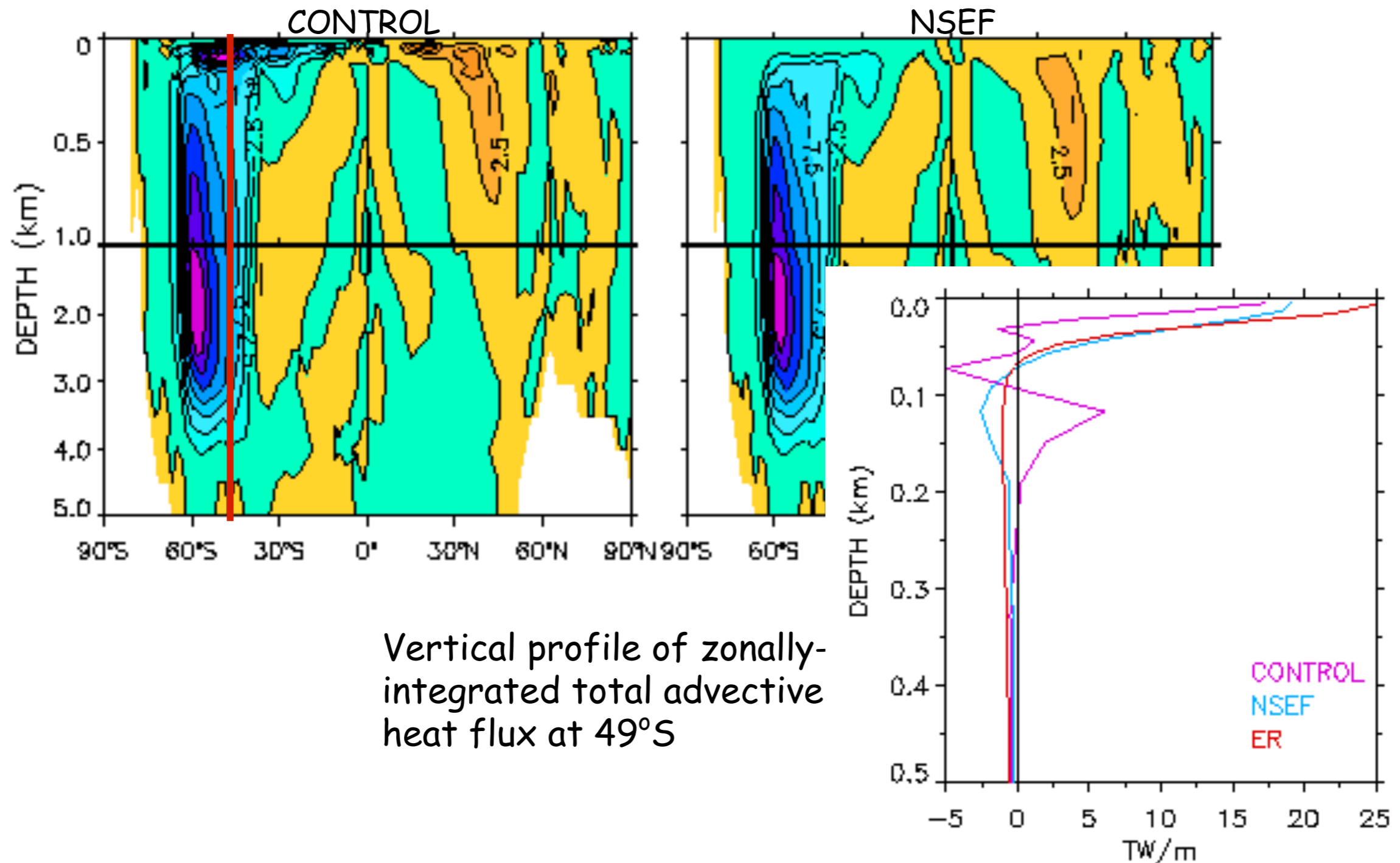


FIG. 2. A conceptual model of eddy fluxes in the upper ocean. Mesoscale eddy fluxes (blue arrows) act to both move isopycnal surfaces and stir materials along them in the oceanic *interior*, but the fluxes become parallel to the boundary and cross density surfaces within the *BL*. Microscale turbulent fluxes (red arrows) move

Near-surface eddy flux scheme (Ferrari, McWilliams, Canuto, Dubovikov)

EDDY-INDUCED MERIDIONAL OVERTURNING (GLOBAL)



A new eddy parameterization (Ferrari, Griffies, Nurser & Vallis)

- The eddy streamfunction is given by the elliptic problem

$$\left(c^2 \frac{d^2}{dz^2} - N^2 \right) \tilde{\Psi} = -\kappa \nabla \bar{b}$$
$$\tilde{\Psi} = 0, \quad z = 0, -H$$

Properties of the new parameterization

- releases mean available potential energy
- the eddy transport vanishes at the ocean boundaries
- the eddy transport is dominated by the first baroclinic mode (if c is set to speed of first baroclinic mode)
- does not require any tapering function
- reduces to GM for $c=0$