

Mixed Layer Eddy Parameterization:
Theory, Impact, and Comparison with
Mesoscale Eddy Parameterizations.

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Tracer Flux-Gradient Relationship

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\bar{\tau}$$

- Most **subgridscale eddy closures** have this form: GM^* , Redi, FFH** submesoscale
- Relates the **eddy flux** to the coarse-grain gradients **locally**
- If we knew the dependence of \mathbf{M} on the coarse-resolution flow, we'd have the **optimal local eddy closure**

*Gent & McWilliams (1990)

**Fox-Kemper, Ferrari, Hallberg (2008)

10 km

The Character of the Submesoscale

(Capet et al., 2008)

- Fronts
 - Eddies
 - $Ro=O(1)$
 - $Ri=O(1)$
 - near-surface
 - 1-10km, days
- Eddy processes mainly **baroclinic instability** (Boccaletti et al '07, Haine & Marshall '98).
 Parameterizations of baroclinic instability **apply?** (GM, Visbeck, FFH).

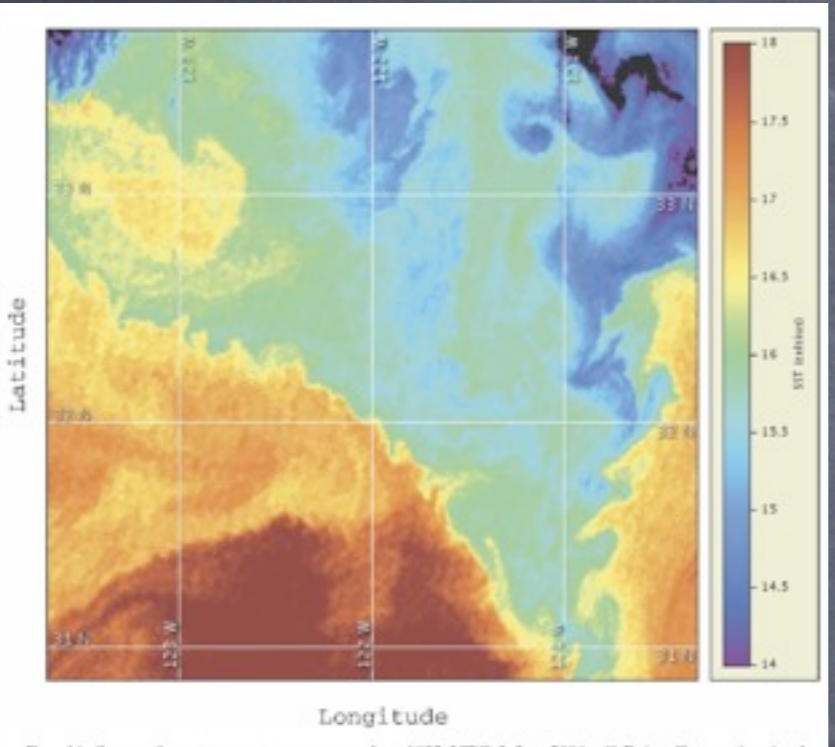
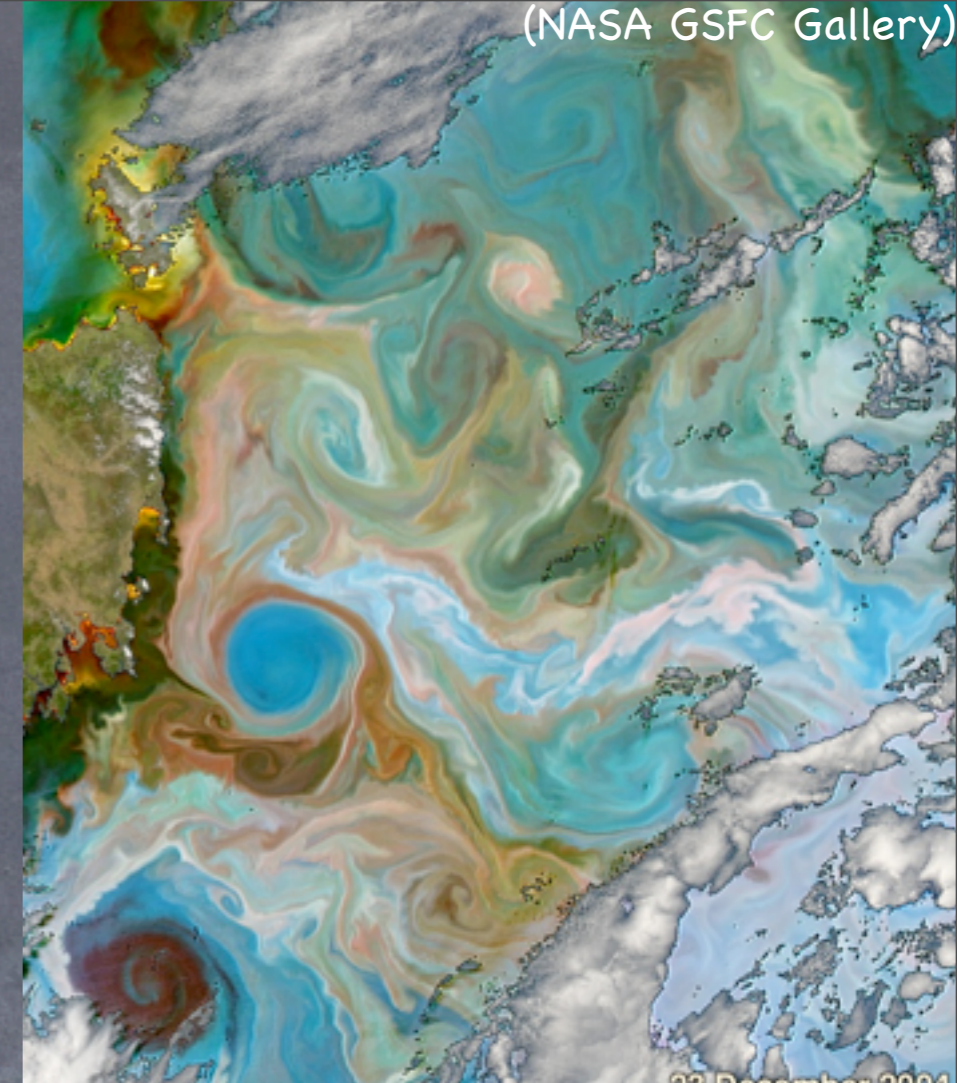
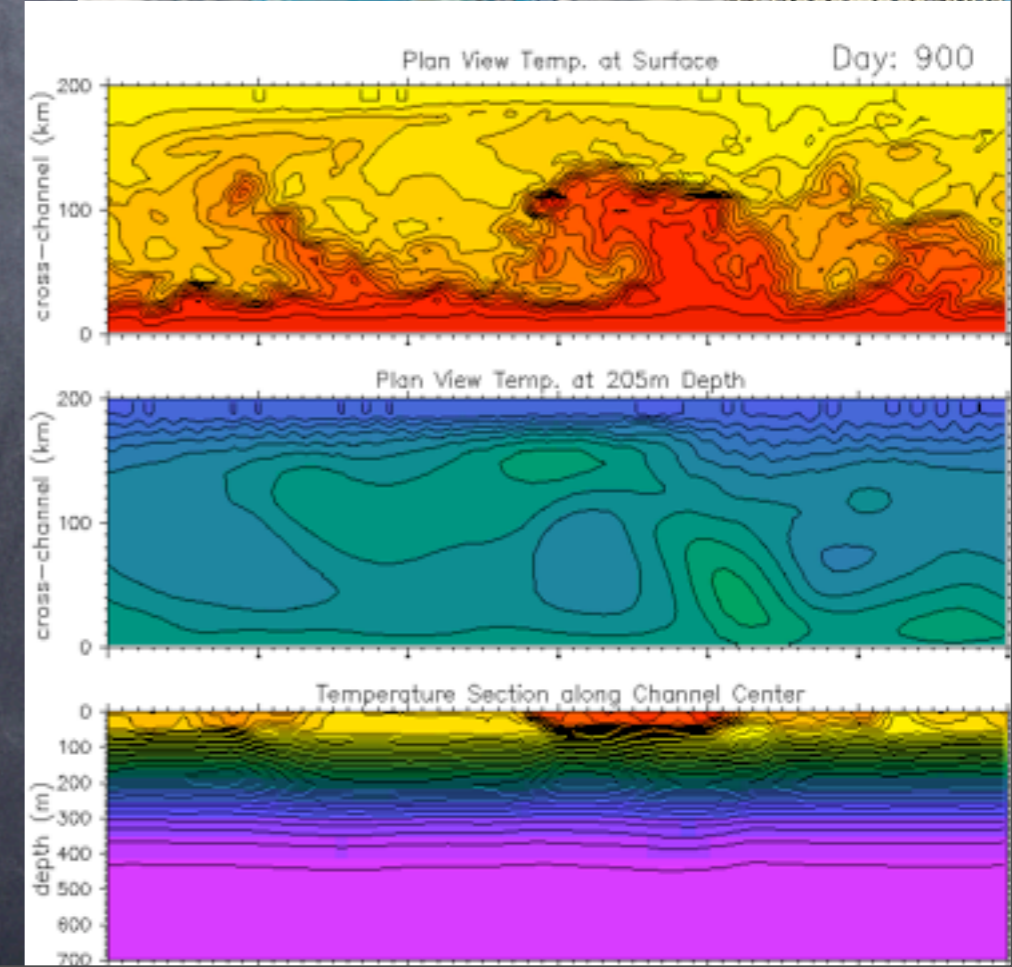
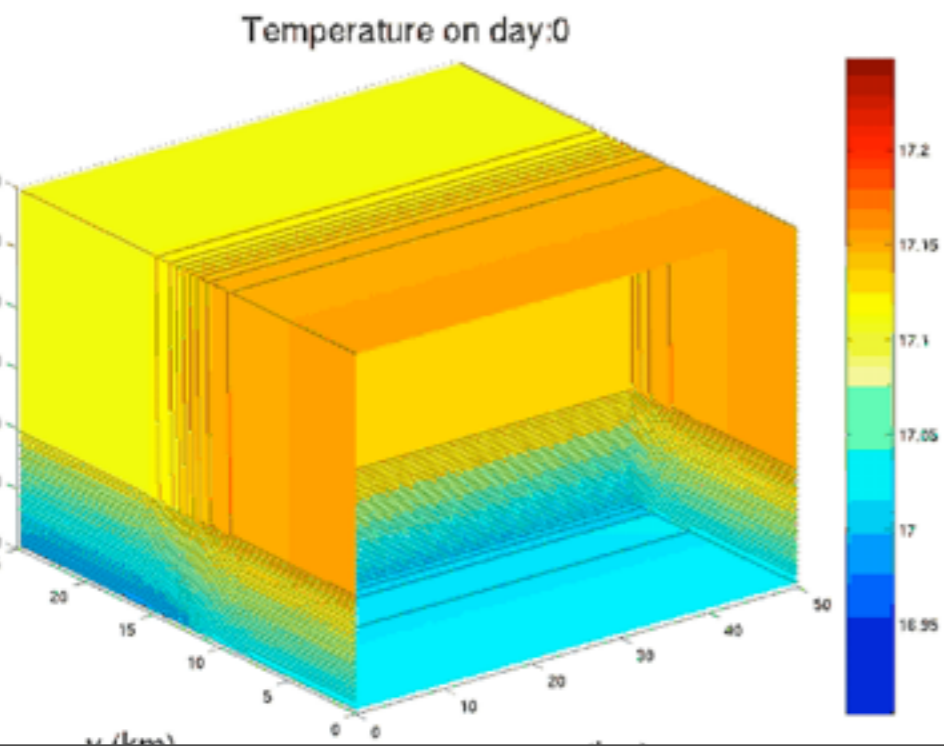


Fig. 16 Sea surface temperature measured at 1833 UTC 1 Jan 2006 off Baja, California in the



A Global Parameterization of Mixed Layer Eddy Restratification

with FLOW DEPENDENT \mathbf{M} validated against simulations

Fox-Kemper, Ferrari, & Hallberg (2008) &
Fox-Kemper, Danabasoglu, Ferrari, & Hallberg (2008)

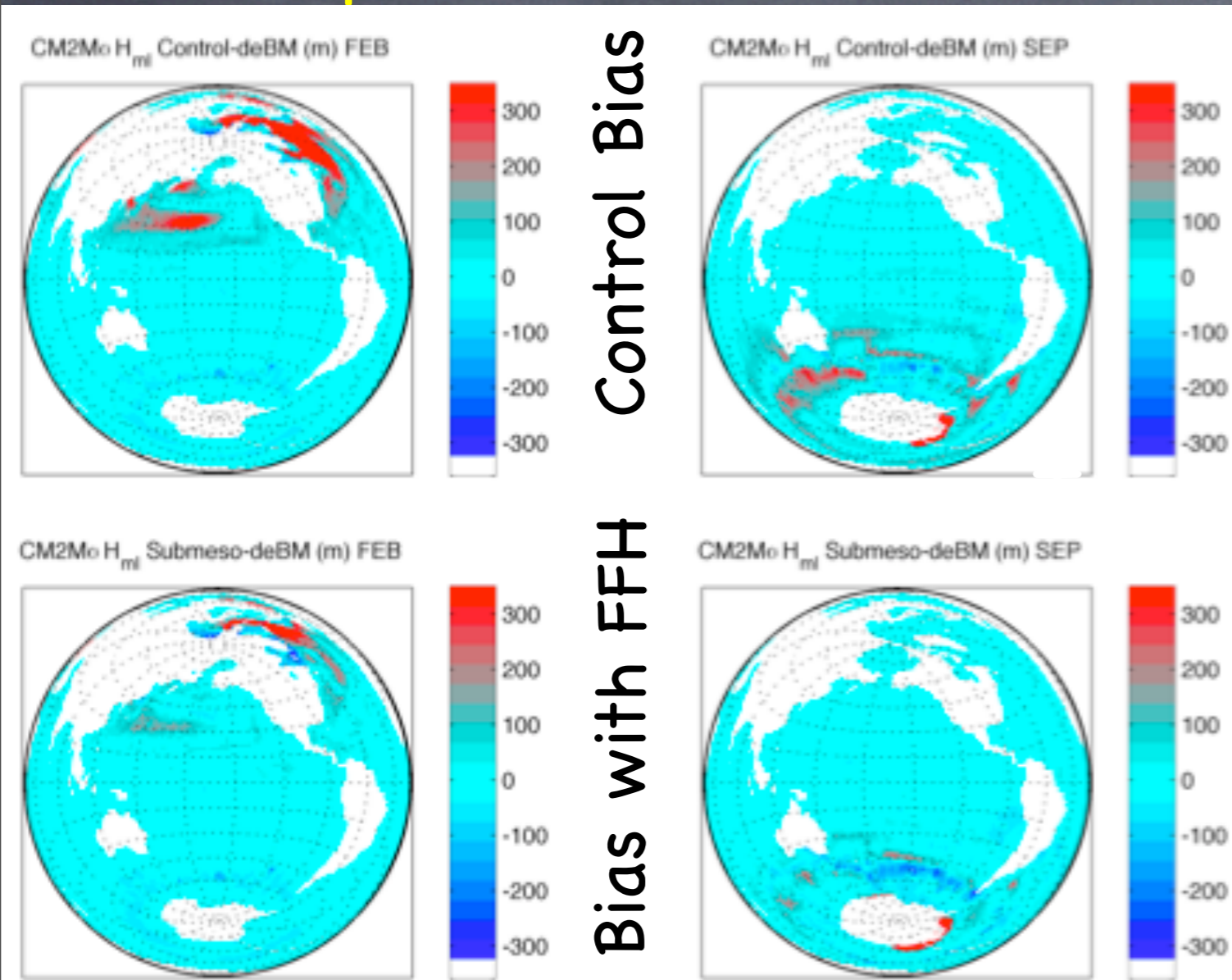
$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\Psi_y \\ 0 & 0 & \Psi_x \\ \Psi_y & -\Psi_x & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

$$\Psi = \left[\frac{\Delta x}{L_f} \right] \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla \bar{b} \times \hat{\mathbf{z}}$$

$$\mu(z) = \left[1 - \left(\frac{2z}{H} + 1 \right)^2 \right] \left[1 + \frac{5}{21} \left(\frac{2z}{H} + 1 \right)^2 \right]$$

Physical Sensitivity of Ocean Climate to Submesoscale Eddy Restratification:

FFH implemented in CCSM (NCAR), CM2M & CM2G (GFDL)



Control Bias

Bias with FFH

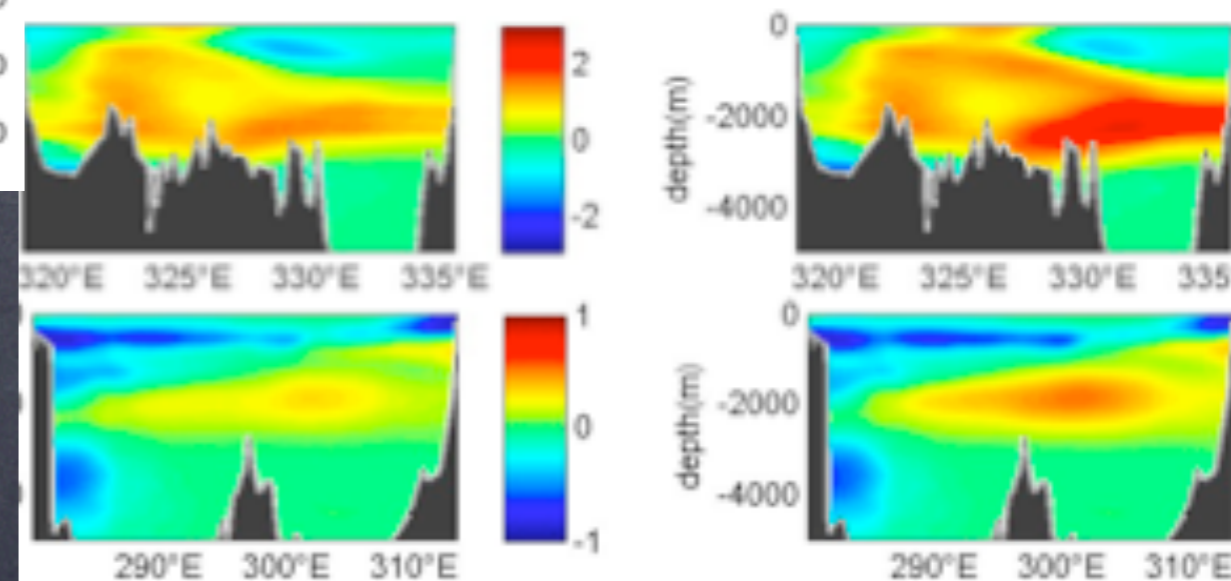
NO RETUNING NEEDED!!!

Improves CFCs

Bias with FFH

Control Bias

Deep ML Bias reduced
From Fox-Kemper et al., in prep



Sensitivity of Climate to Submeso: AMOC & Cryosphere Impacts

May Stabilize AMOC

Maximum AMOC at 45n in coupled MOM

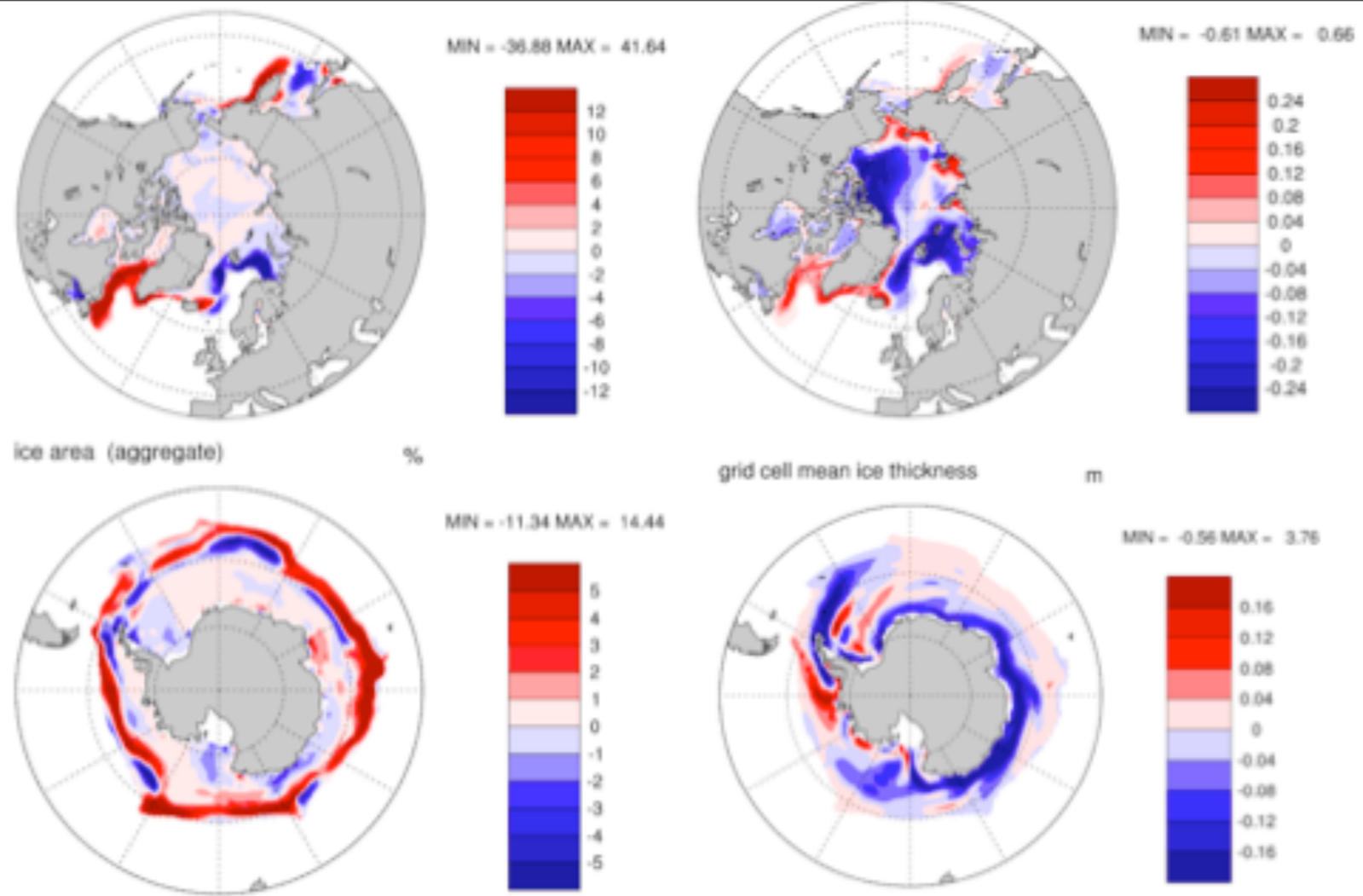
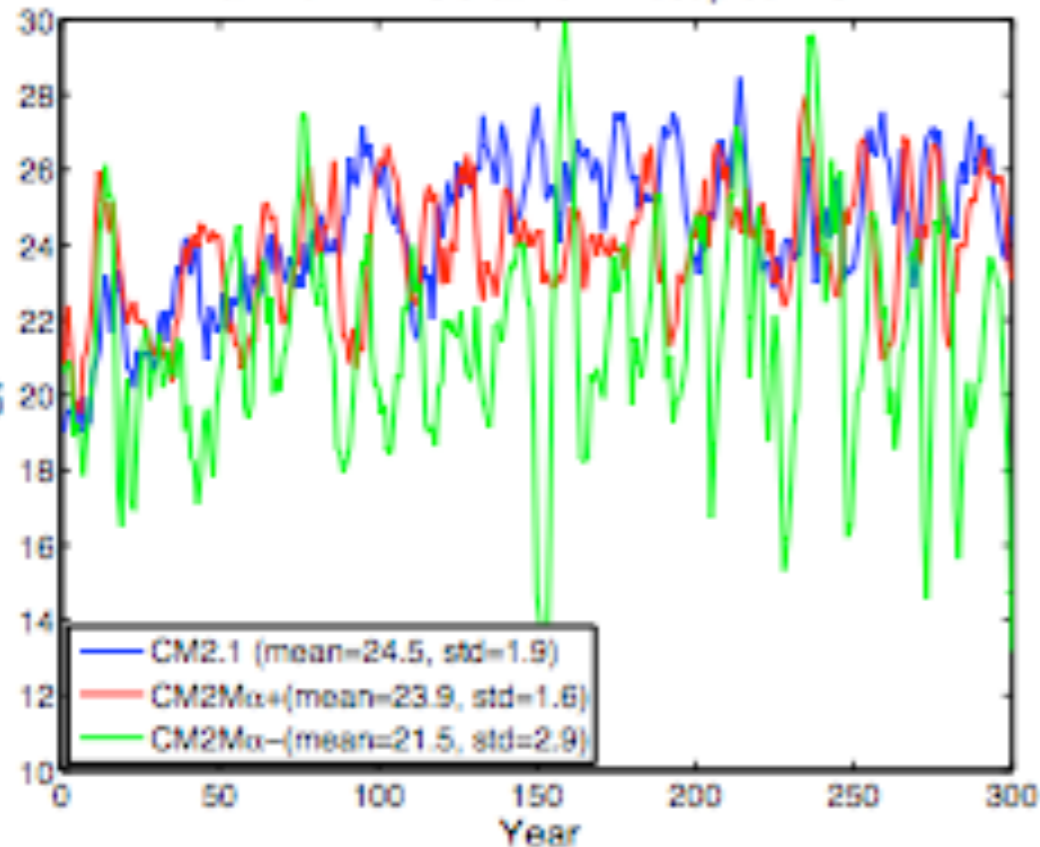


Figure 10: Wintertime sea ice sensitivity to introduction of MLE parameterization (CCSM⁺ minus CCSM⁻): January to March Northern Hemisphere a) ice area and b) thickness and July to September Southern Hemisphere c) ice area and d) thickness.

Affects sea ice

NO RETUNING
NEEDED!!!

These are impacts:
bias change unknown

Conclusions

Submesoscale

- FFH is implemented in at least 3 IPCC models
- Parameterization reduces bias in CFCs & Mixed Layer Depth
- Parameterization also affects ice & AMOC variability--need truth?
- Flow-dependent, nondimensional scalings validated against simulations *did not require retuning*

The Character of the Mesoscale

← 100 km

(Capet et al., 2008)

- Boundary Currents
- Eddies
- $Ro=O(0.1)$
- $Ri=O(1000)$
- Full Depth
- Eddies strain to produce Fronts
- 100km, months

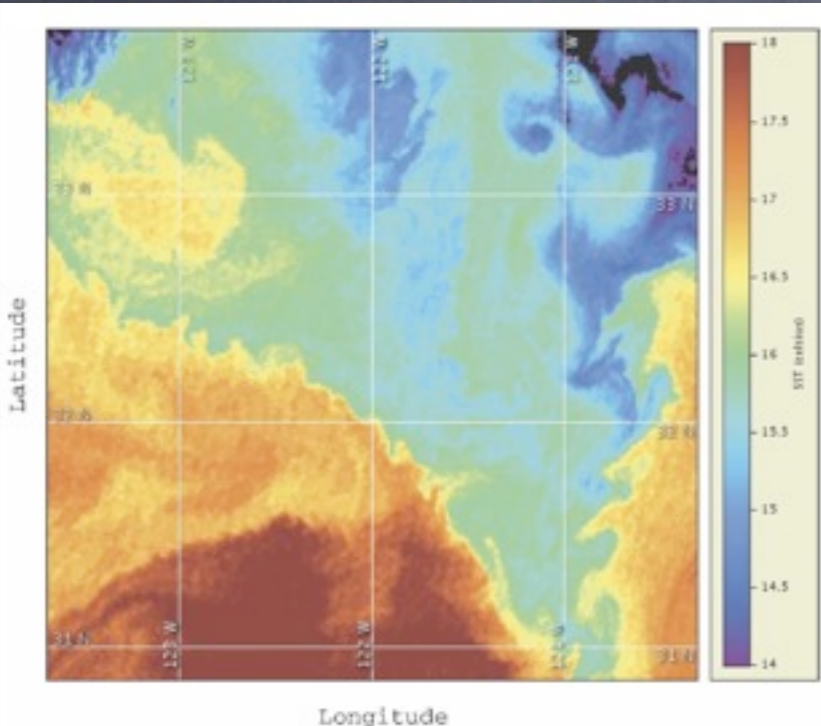
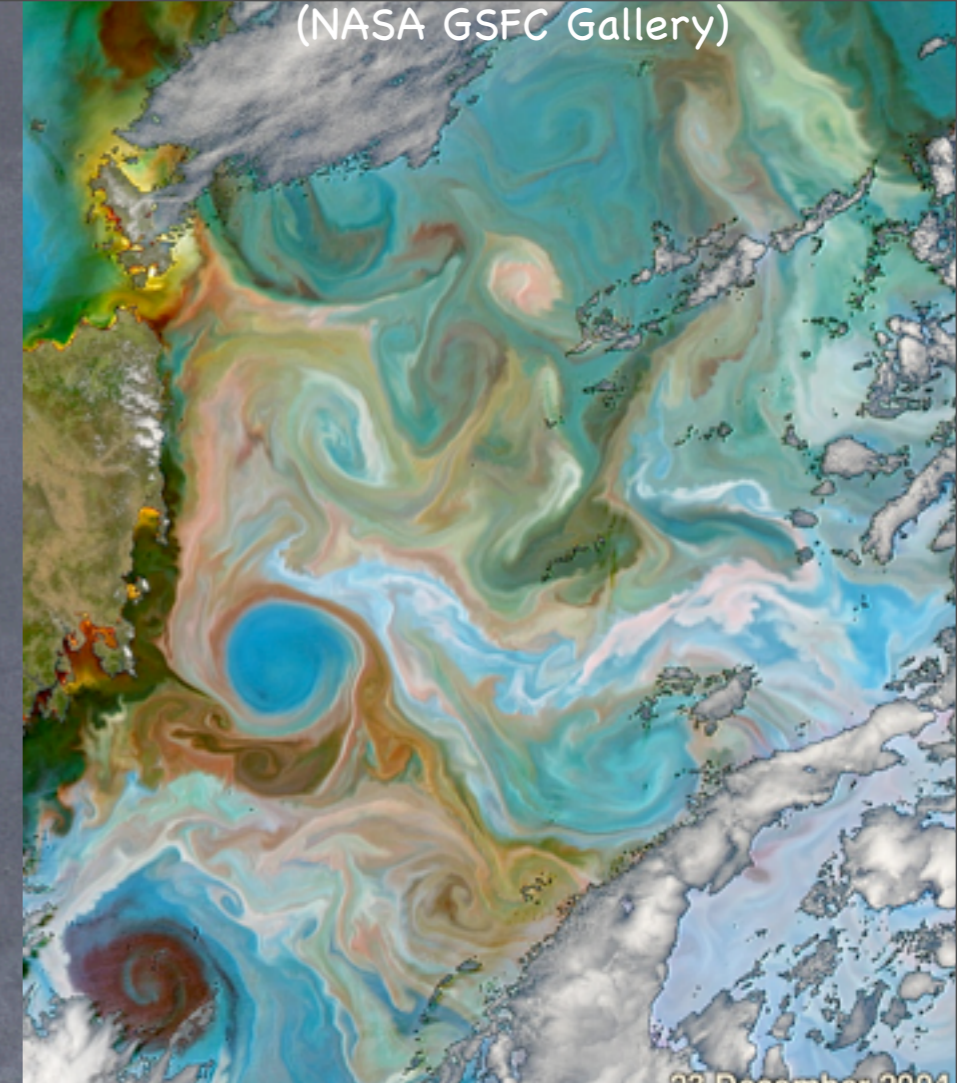
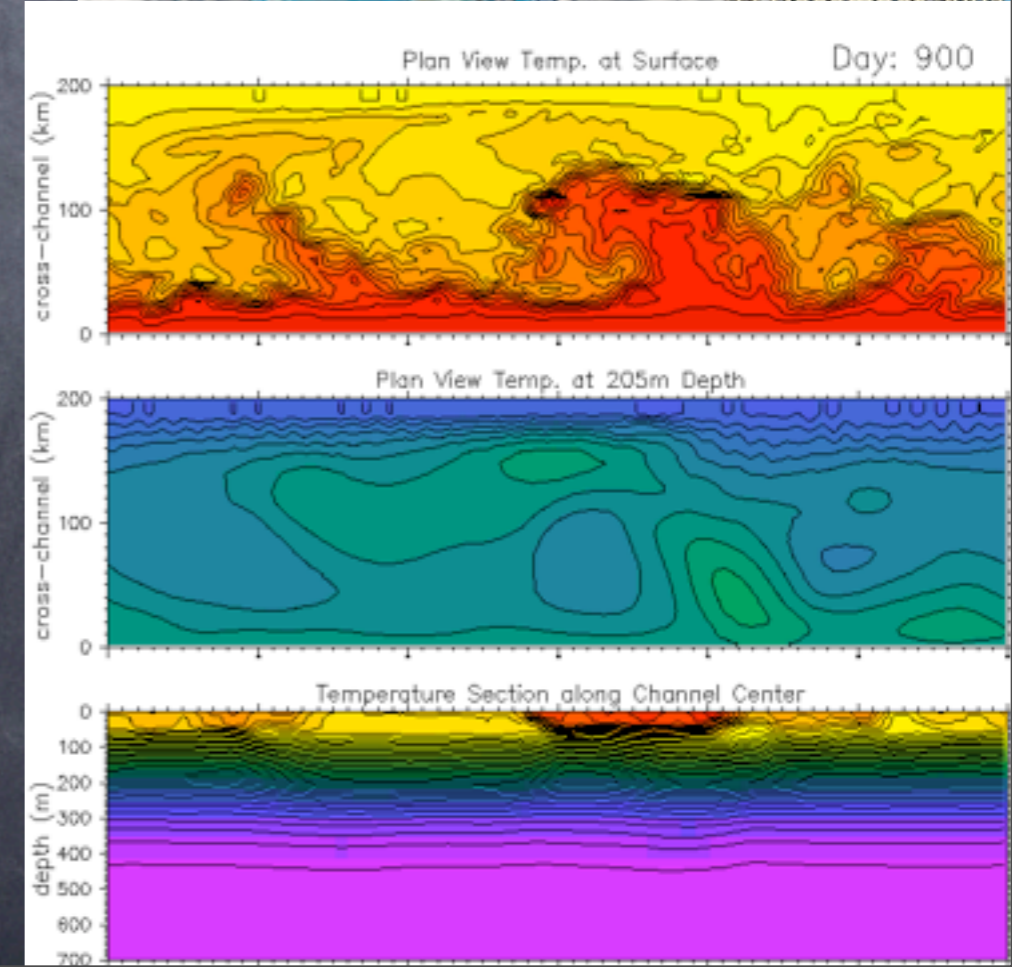


FIG. 16. Sea surface temperature measured at 1832 UTC 3 Jun 2006 off Point Conception in the California Current from CoastWatch (<http://coastwatch.pfeg.noaa.gov>). The fronts between recently upwelled water (i.e., 15°–16°C) and offshore water ($\geq 17^\circ\text{C}$) show submesoscale instabilities with wavelengths around 30 km (right front) or 15 km (left front). Images for 1 day earlier and 4 days later show persistence of the instability events.



Eddy processes still **baroclinic & barotropic instability.**

Parameterizations (GM, Visbeck, Eden).

Tracer Flux-Gradient Relationship Diagnosis

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\bar{\tau}$$

- Virtually **all subgridscale eddy closures** may be written as: GM, Redi, FFH Submesoscale
- Relates the **eddy flux** to the coarse-grain gradients $\nabla\bar{\tau}$ locally
- If we knew the dependence of **M** on the coarse-resolution flow, we'd have the **optimal local eddy closure**

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\bar{\tau}$$

General Form

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

Assume same \mathbf{M} for all tracers:

3 equations per tracer

9 unknowns (components)+rot-parts (2/tracer)

BY USING 3 or MORE TRACER FLUXES, determine it!!!
(a la Plumb & Mahlman '87, Bratseth '98)

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\bar{\tau}$$

Sym Part=Anisotropic* Redi

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ K_{yx} & K_{yy} & \hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ \hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & \hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & \tilde{\nabla}_z\cdot\mathbf{K}\cdot\tilde{\nabla}_z \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

Yellow \mathbf{K} 'are' horizontal stirring & mixing

Blue factors in Redi (1982) are symmetric
and scaled to make

eddy mixing along neutral surfaces

*Anisotropic form due to Smith & Gent 04

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\bar{\tau}$$

AntiSym Part=Anisotropic* GM

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ 0 & 0 & -\hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ \hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & \hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & 0 \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

Antisymmetric Elements in GM (1990)

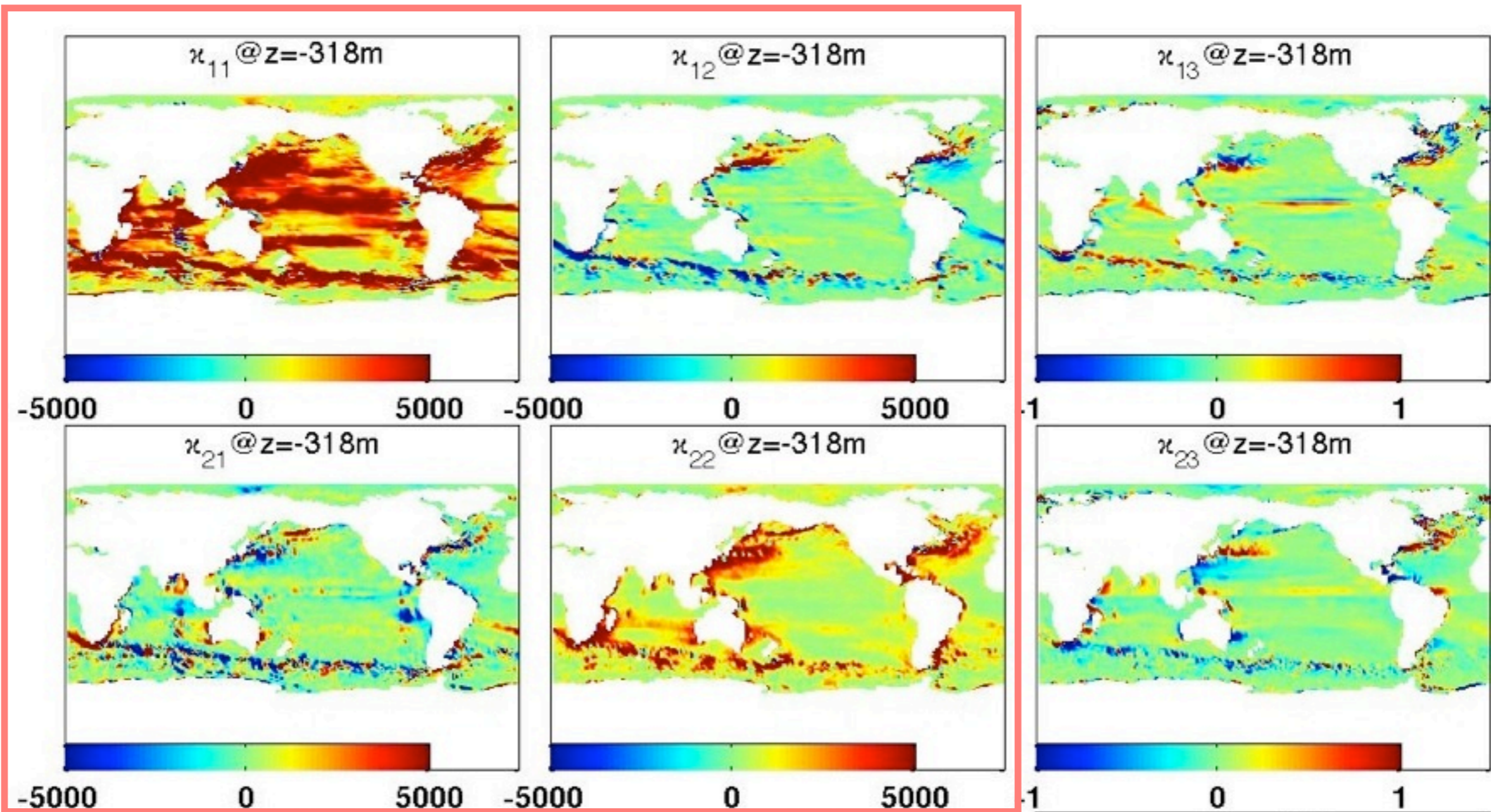
are scaled to overturn fronts, make vertical fluxes

extract PE, and restratify the fluid
equivalent to eddy-induced advection

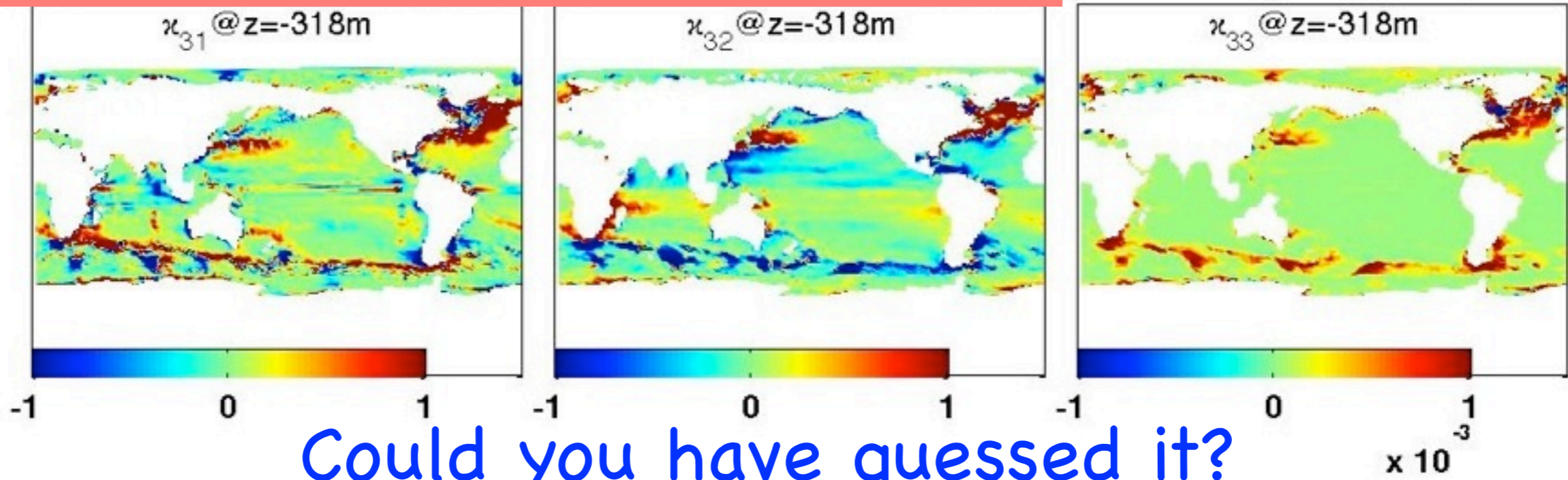
Q: Same horiz. mixing (\mathbf{K}) as Redi?

*Anisotropic form due to Smith & Gent 04 *Tensor Form (Griffies, 98)

K



M



Could you have guessed it?

Validation: **M** Reproduces T-flux w/o negative eigs.

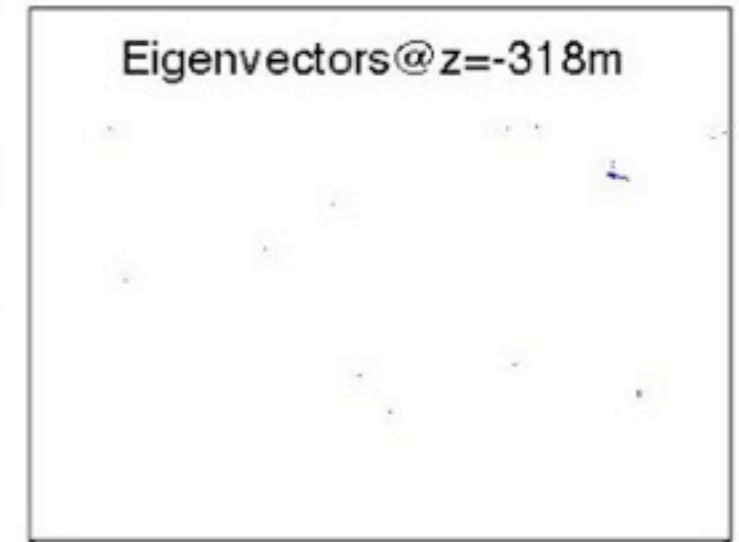
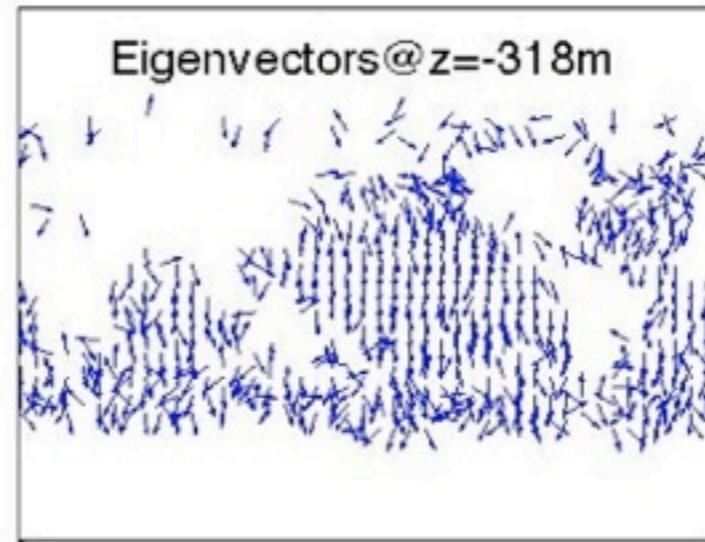
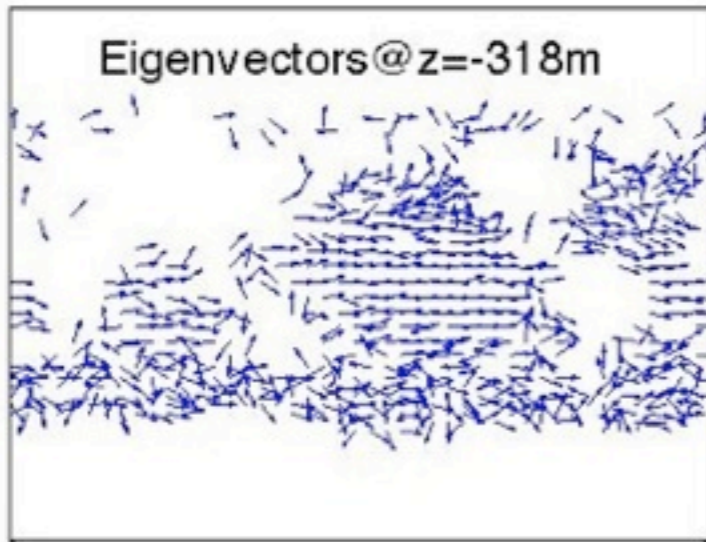
- Even though Temp not used as tracer to find **M**

$$\overline{\mathbf{v}'T'} = -\mathbf{M}\nabla\overline{T} + O(0.1\% \text{error})$$

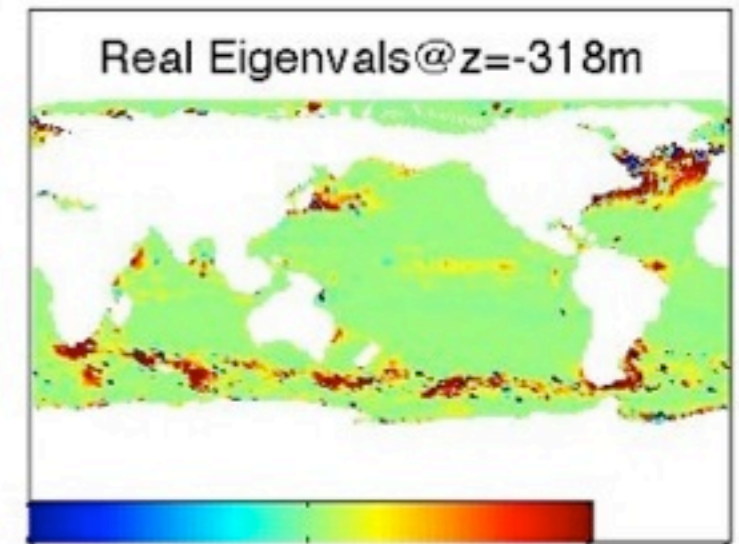
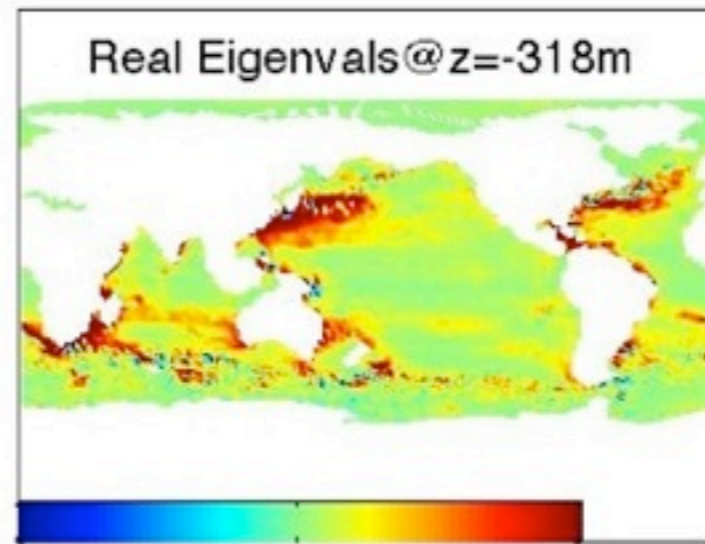
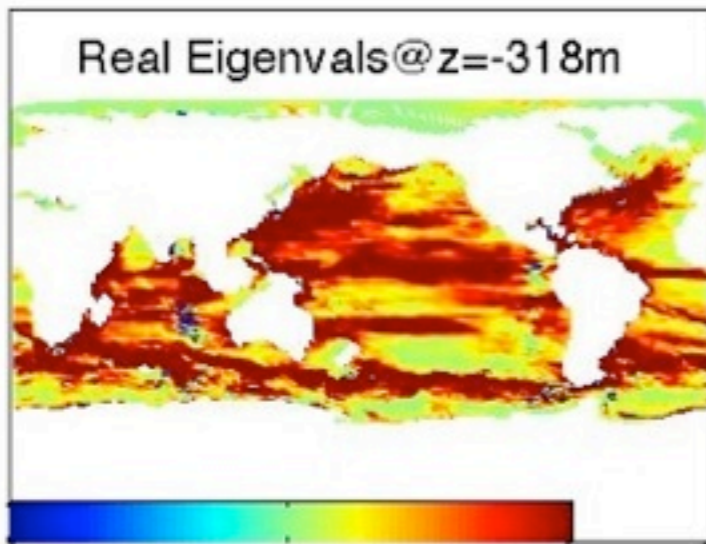
- Typically, diagnoses have problem with **K < 0**
- Here, below the mixed layer **only 6%** of gridpoints have **negative eigenvalues**
- These **few negative values** are consistent with **true nonlocal eddy fluxes**

Result: Strong Anisotropy Along/Across Isopycnals

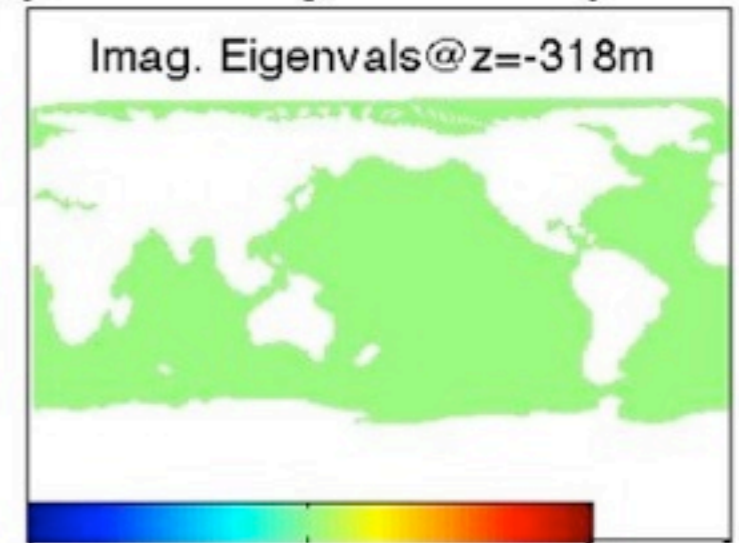
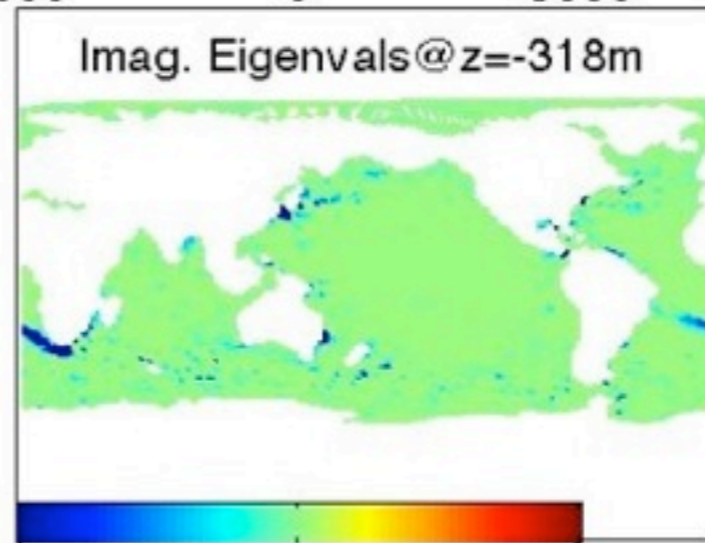
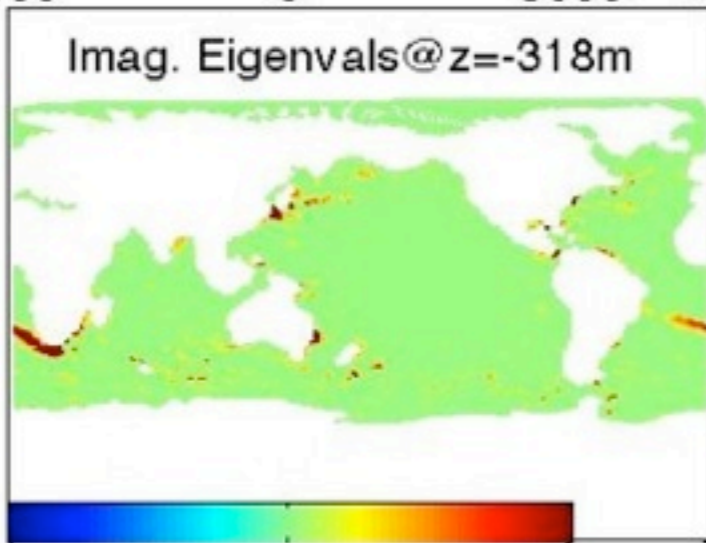
Mixing direction



Mixing:



Stirring:

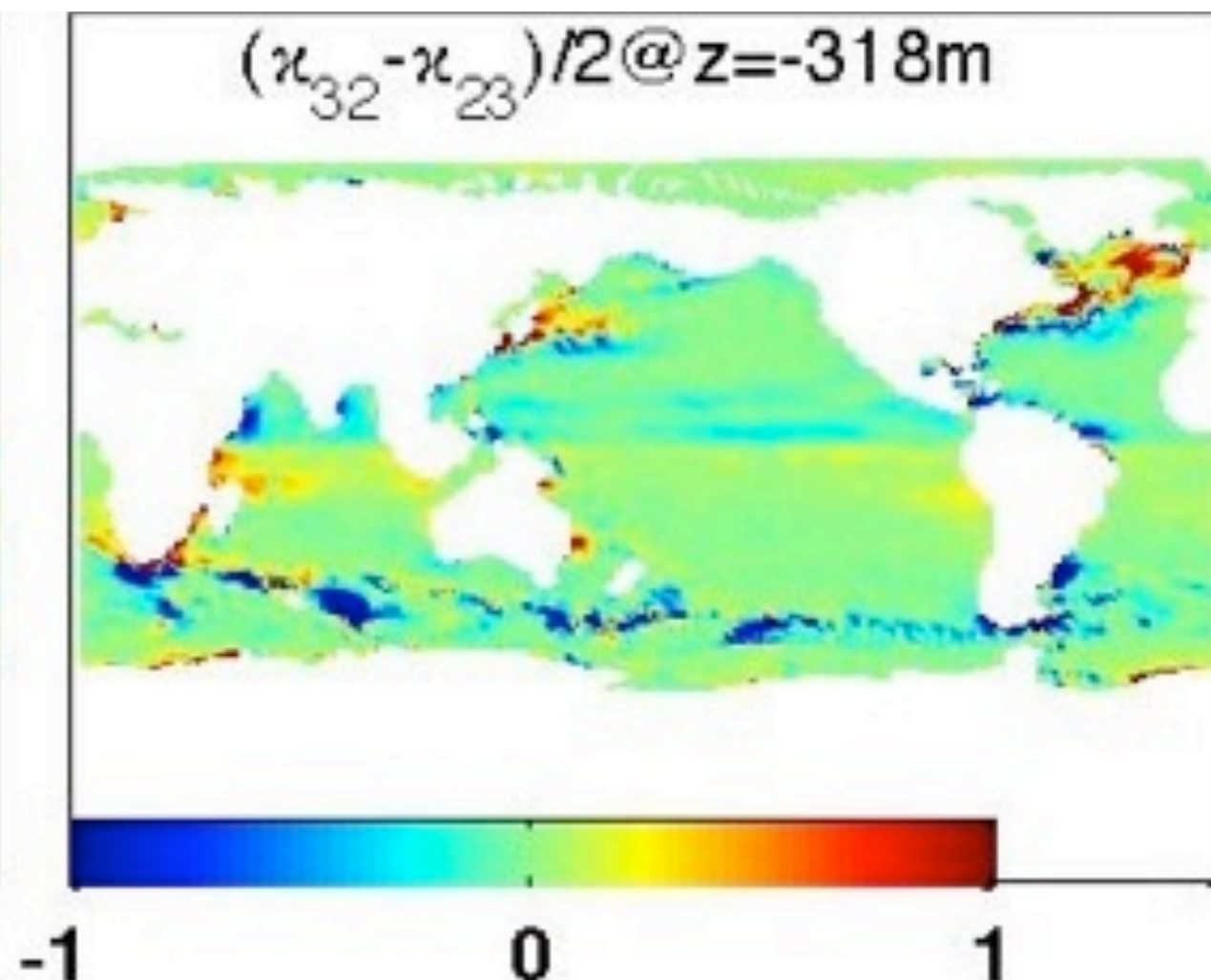
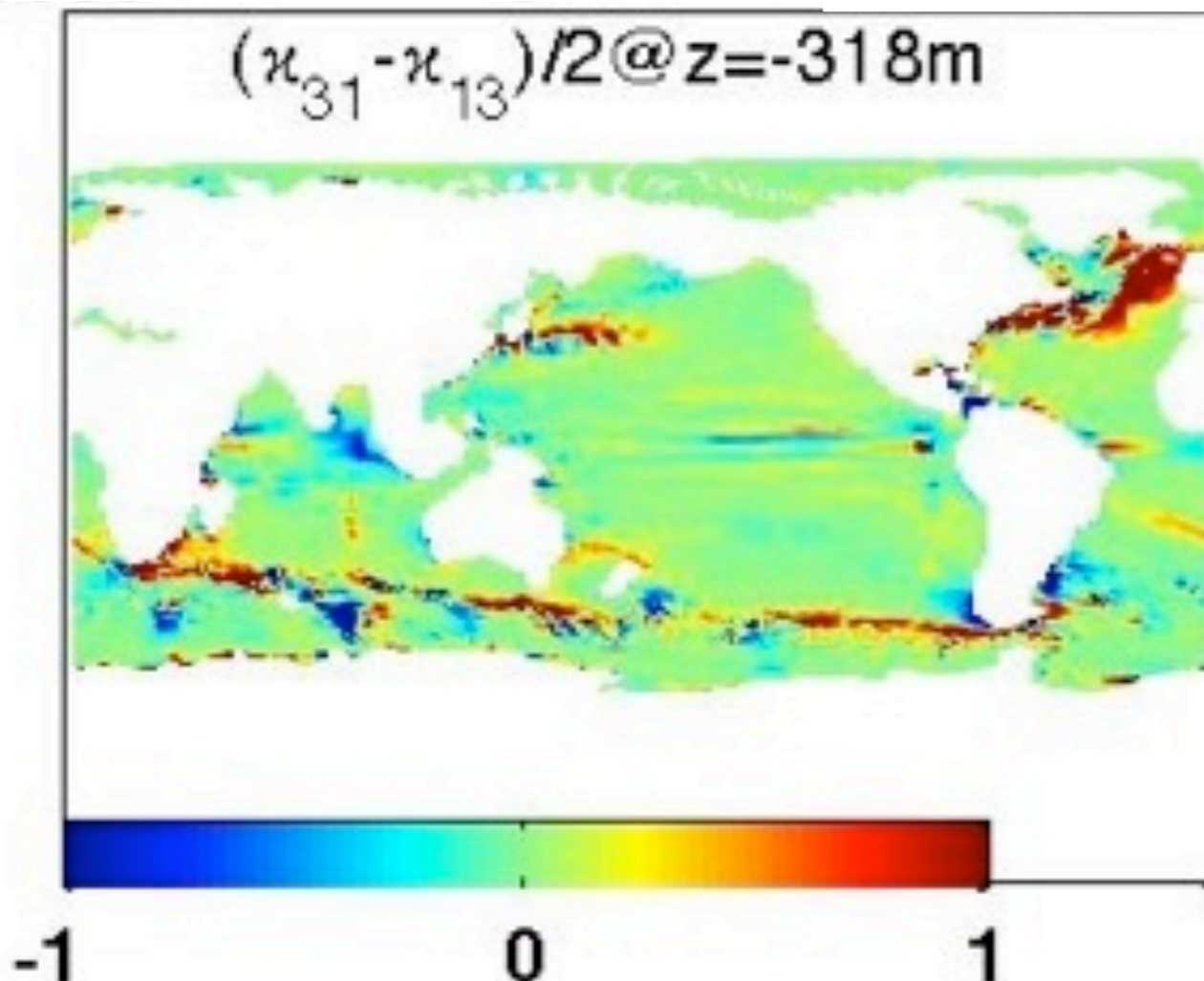
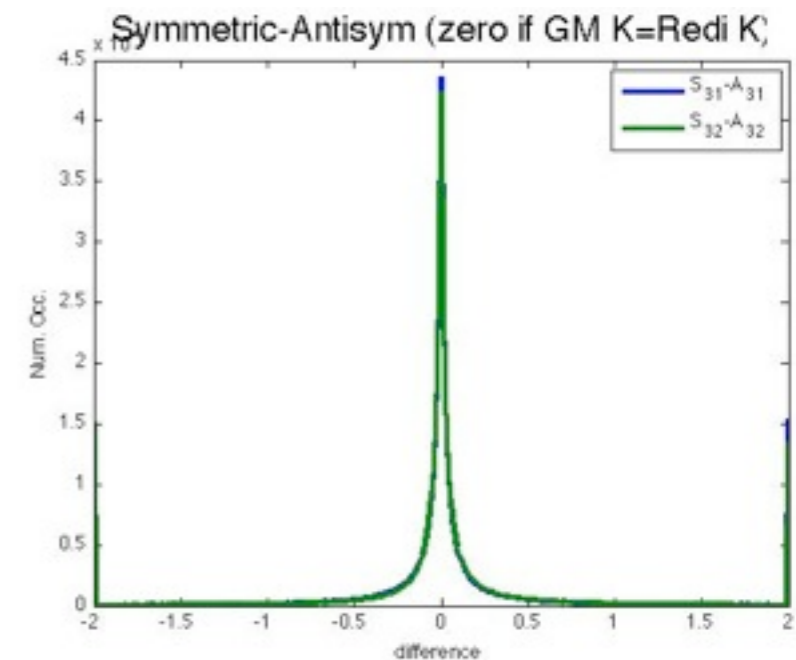


$\times 10^{-3}$

Result:

Redi $K=GM$ K (mostly)

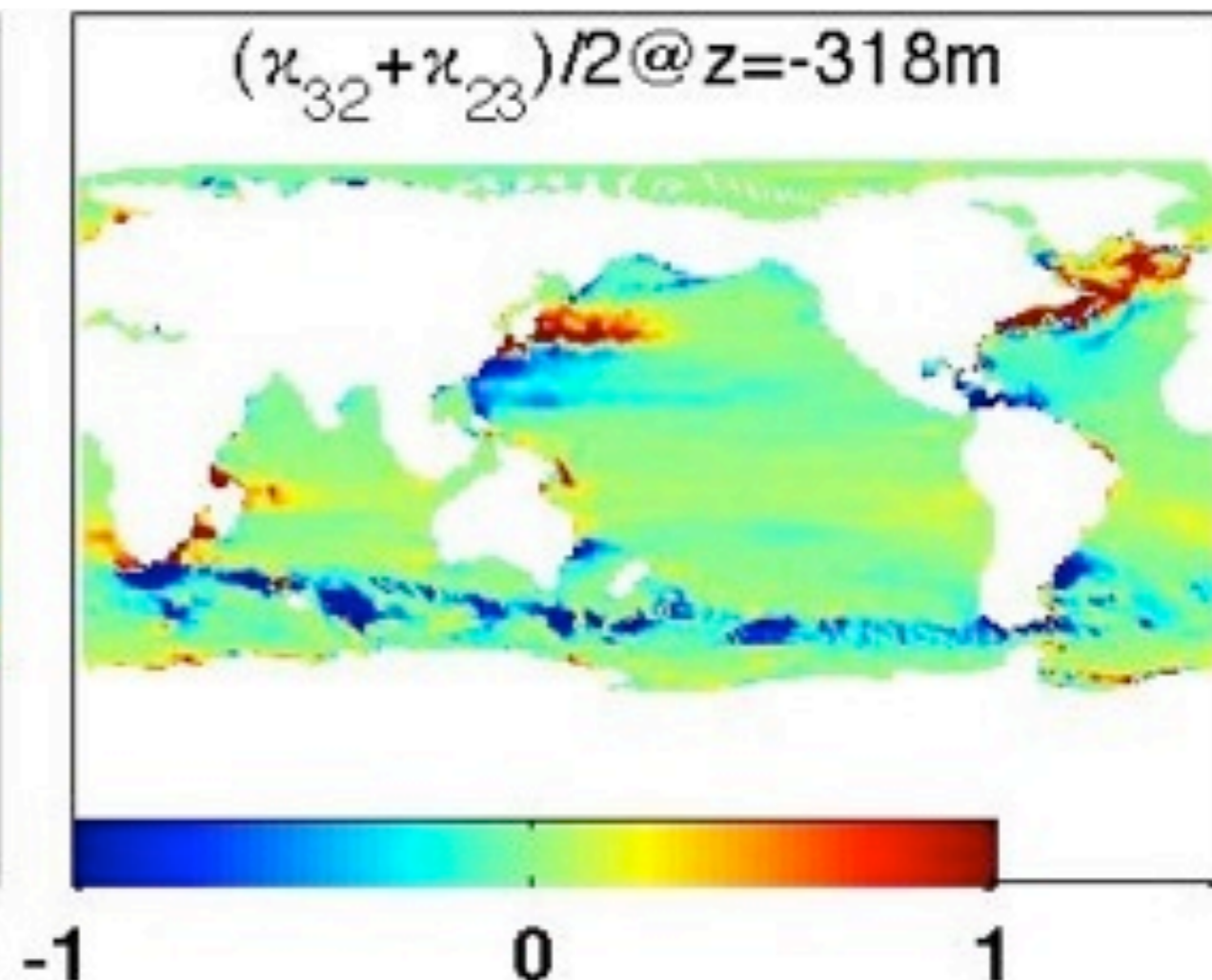
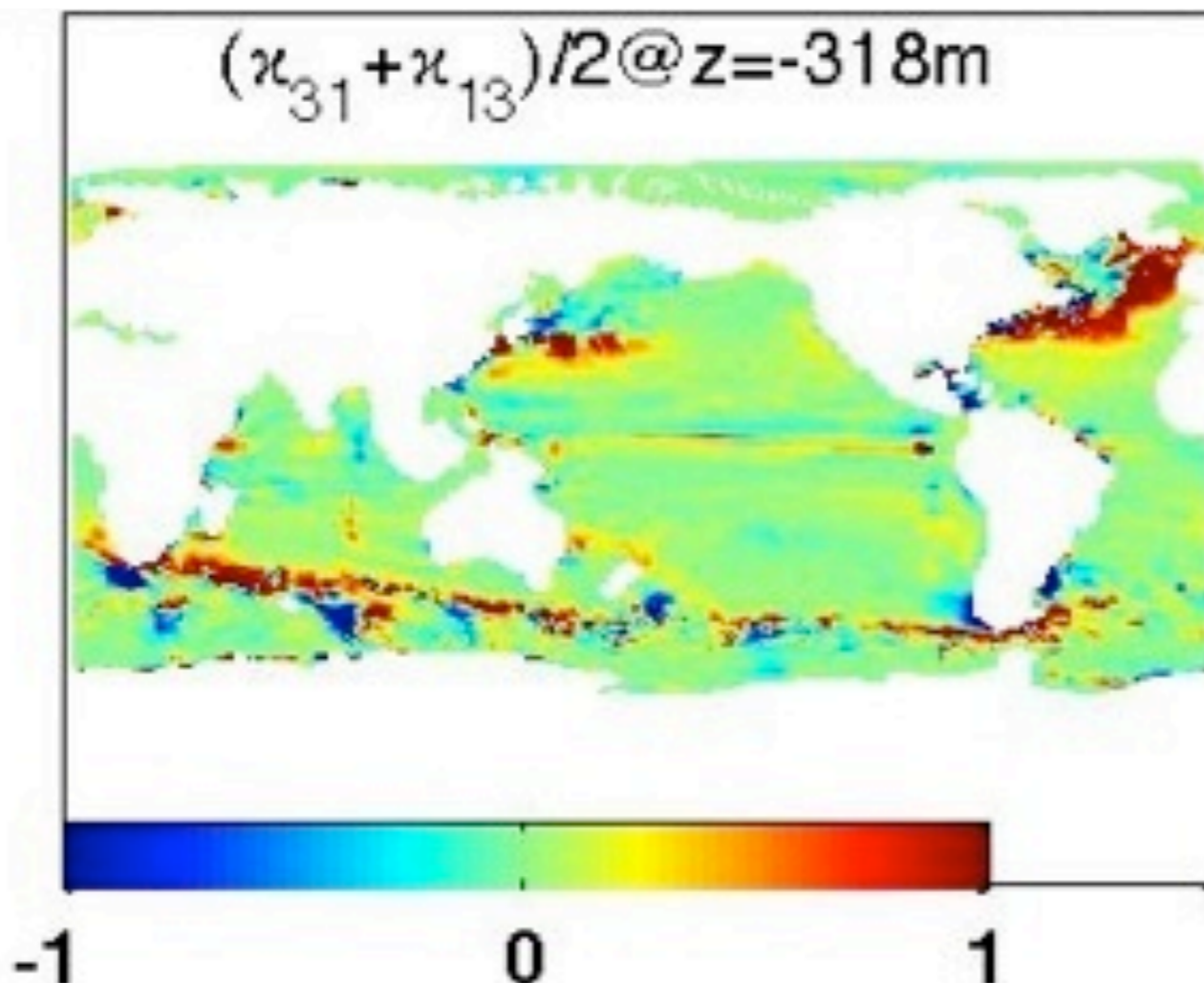
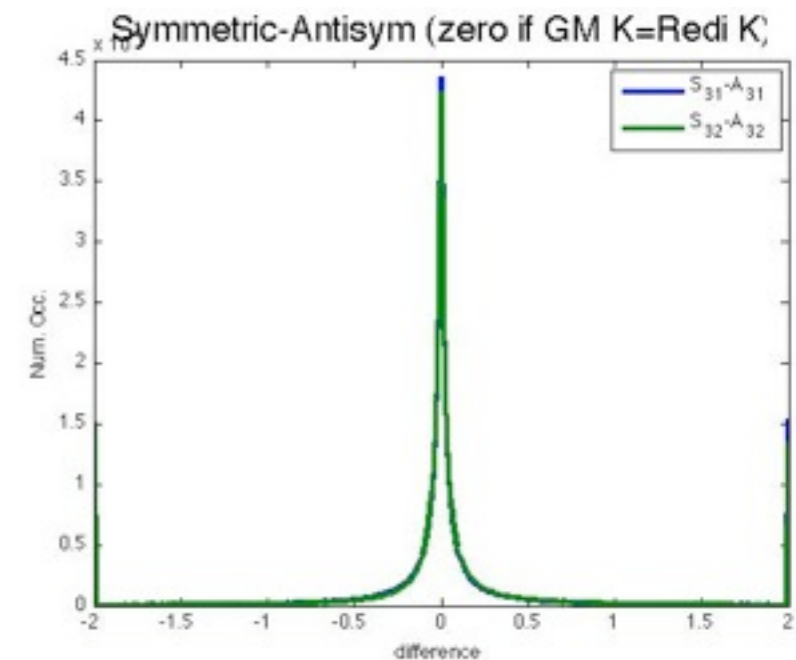
If so these 2 components should match in Sym & Antisym M



Result:

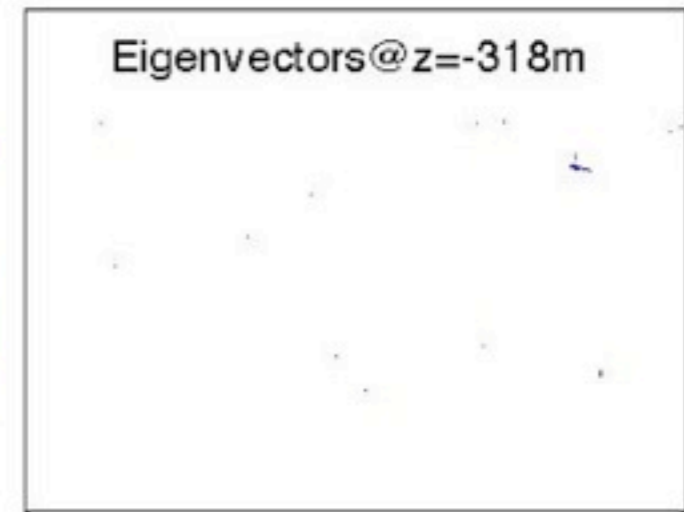
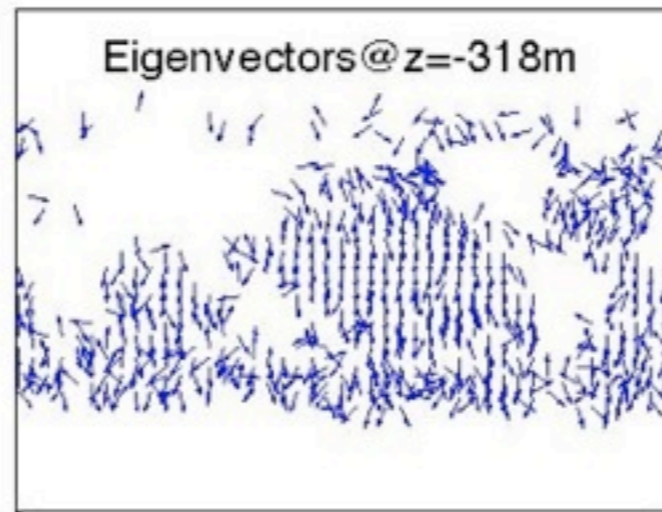
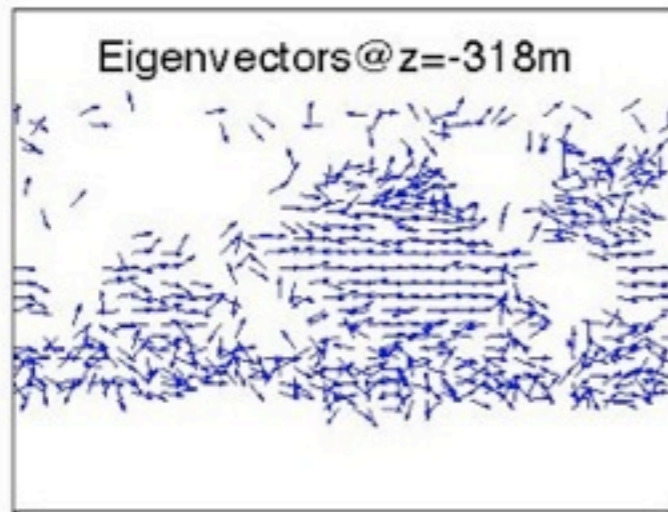
Redi $K=GM$ K (mostly)

If so these 2 components should match in **Sym** & Antisym M

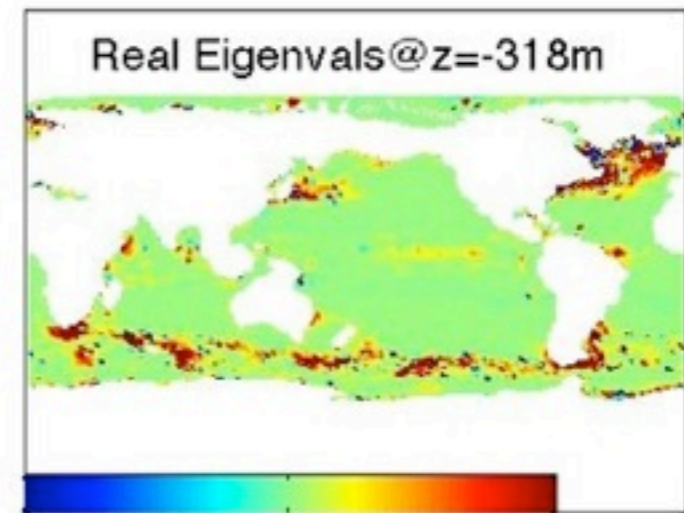
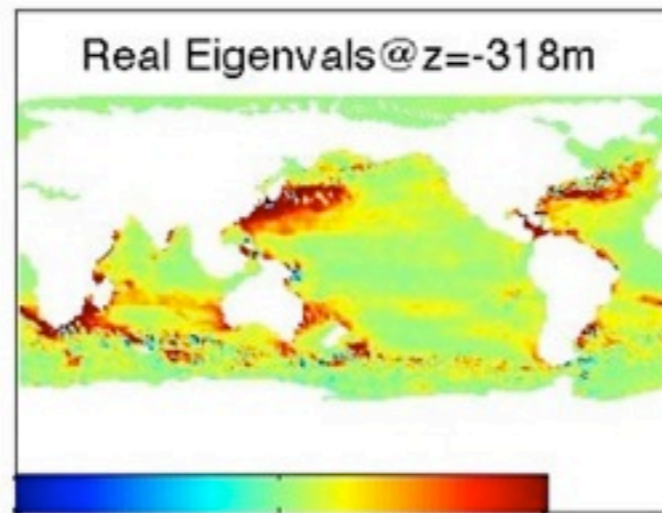
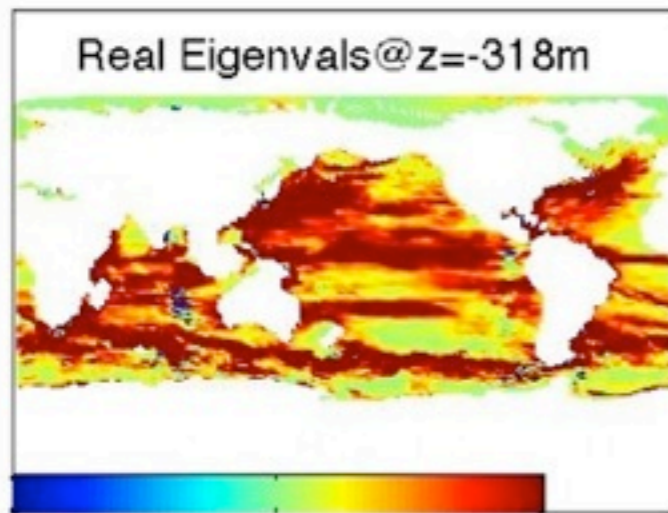


Result: Strong Anisotropy Along/Across PV Grads.

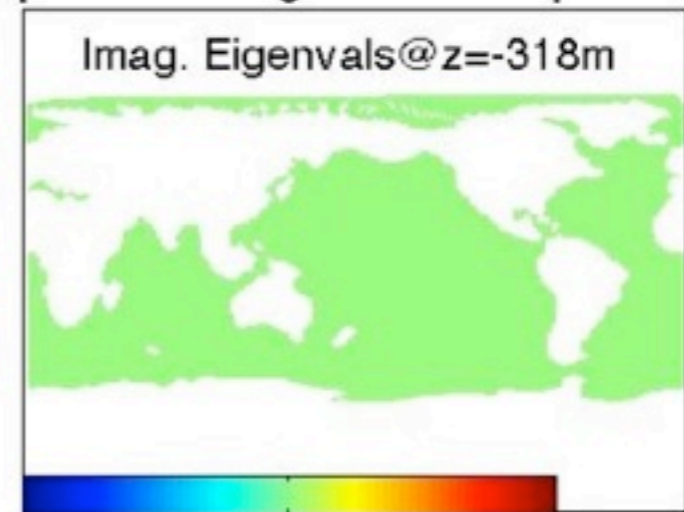
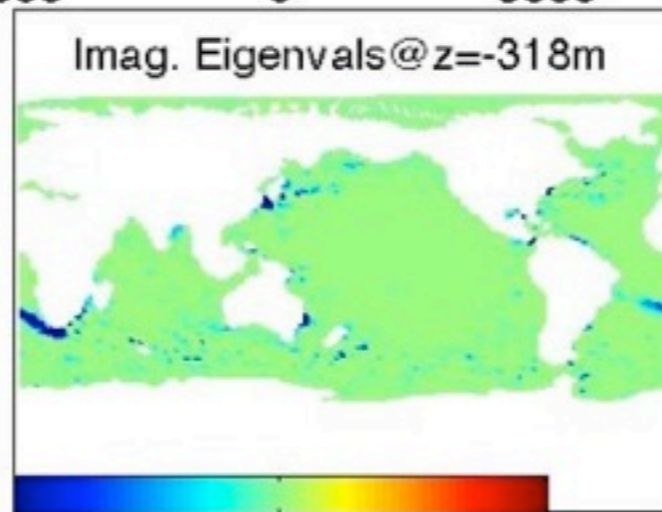
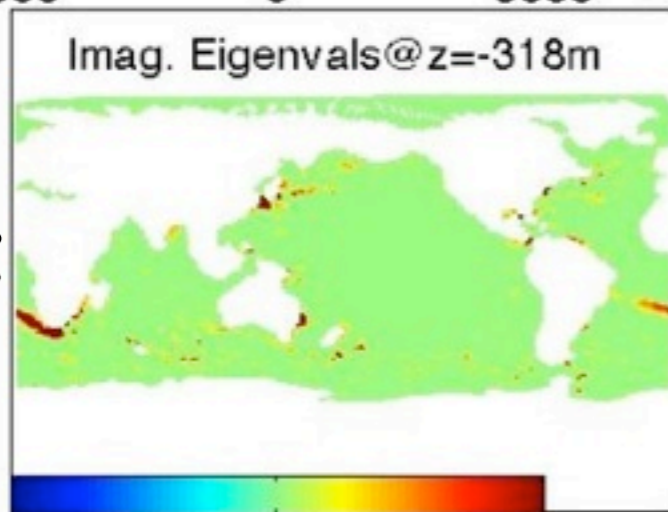
Mixing
direction



Mixing:



Stirring:



$\times 10^{-3}$

Result: eddy KE \rightarrow vertical
power law w/ M eigs?

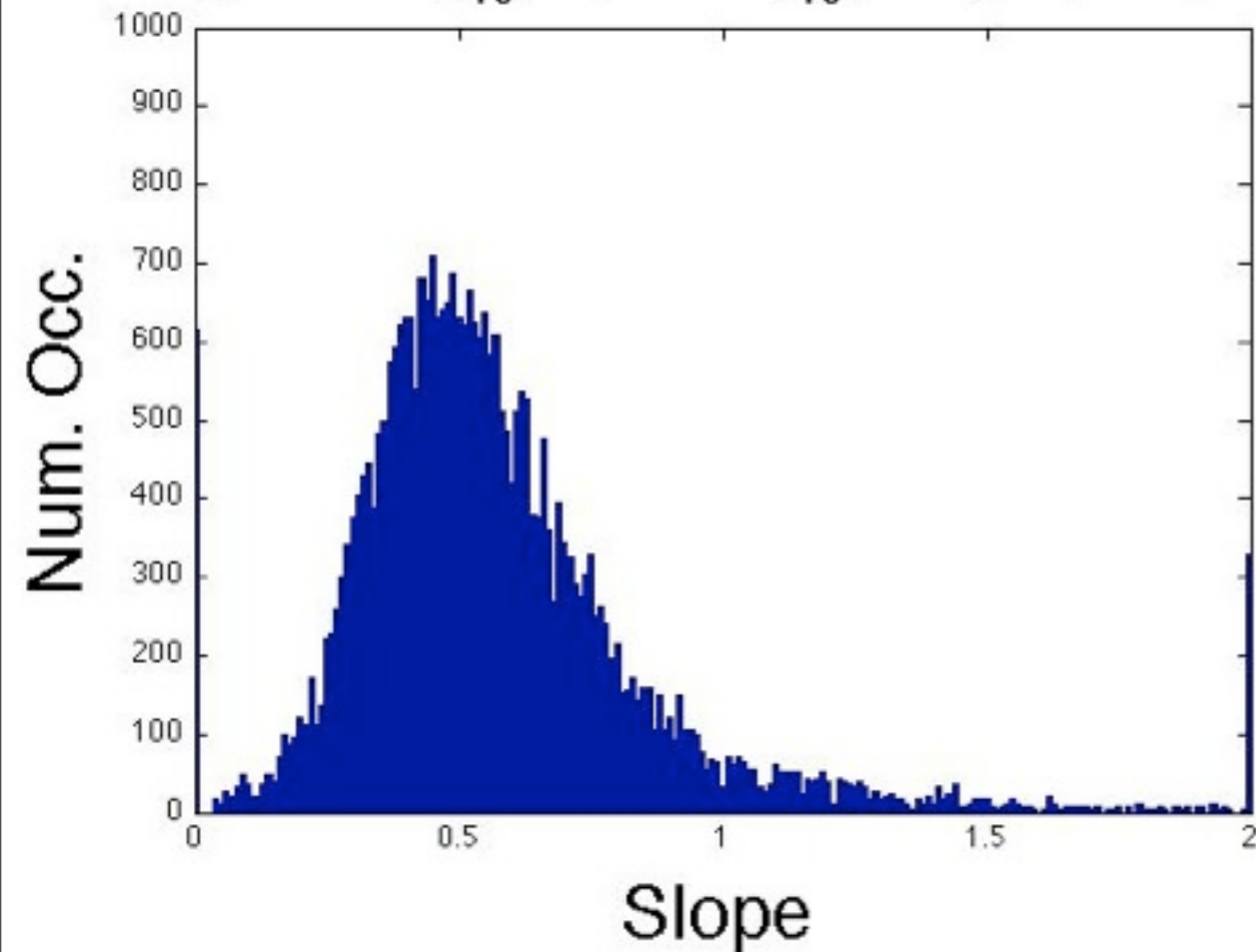
We expect: $K \propto \sqrt{EKE}$

But what about: $K \propto \sqrt{\langle KE \rangle}$

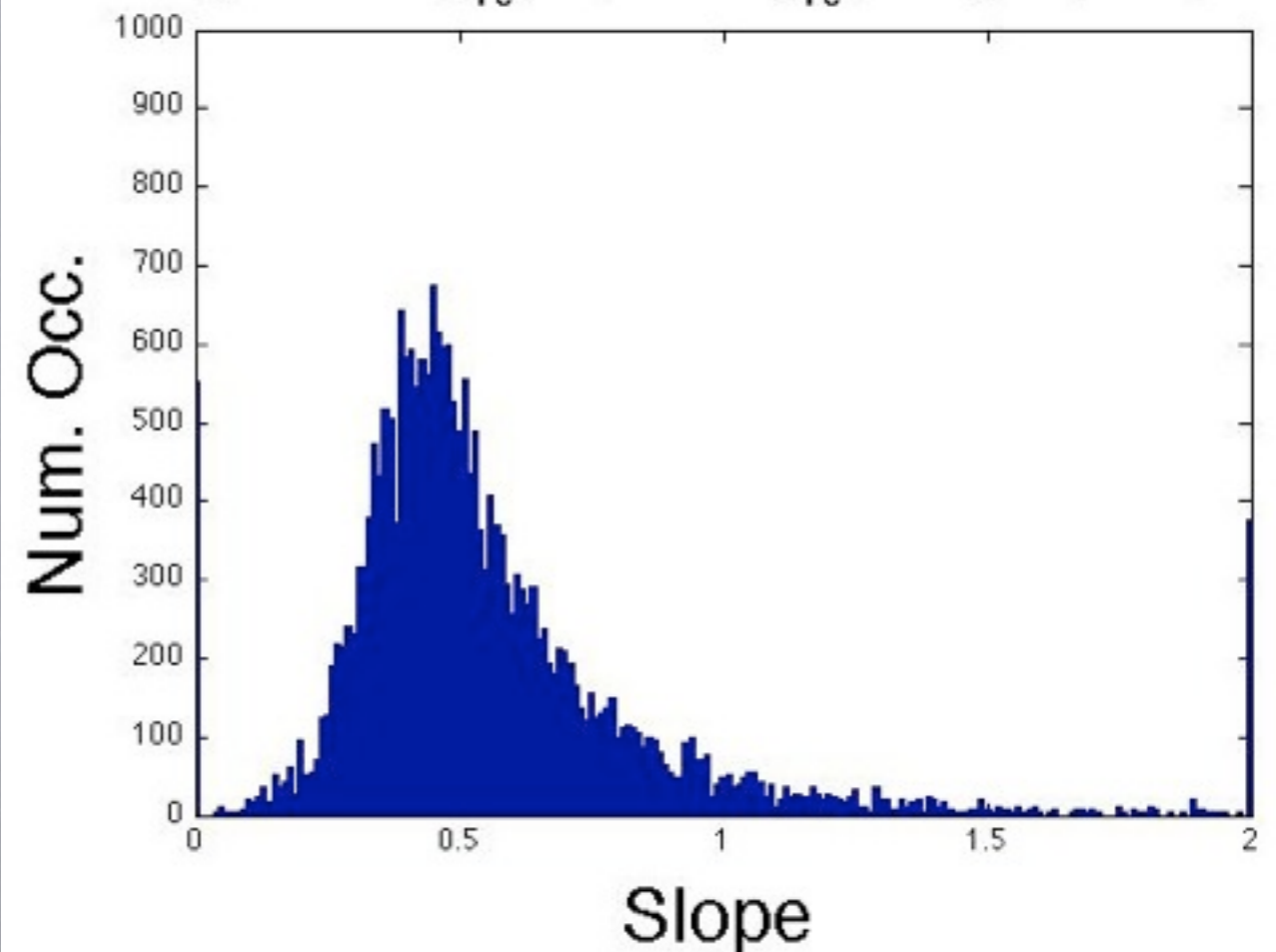
Result:
coarse KE \rightarrow vertical structure of Mixing

$$K \propto \sqrt{\langle KE \rangle}$$

Histogram of $\log_{10}(KE)$ vs. $\log_{10}(M \text{ eig. \#1})$ Slope



Histogram of $\log_{10}(KE)$ vs. $\log_{10}(M \text{ eig. \#2})$ Slope

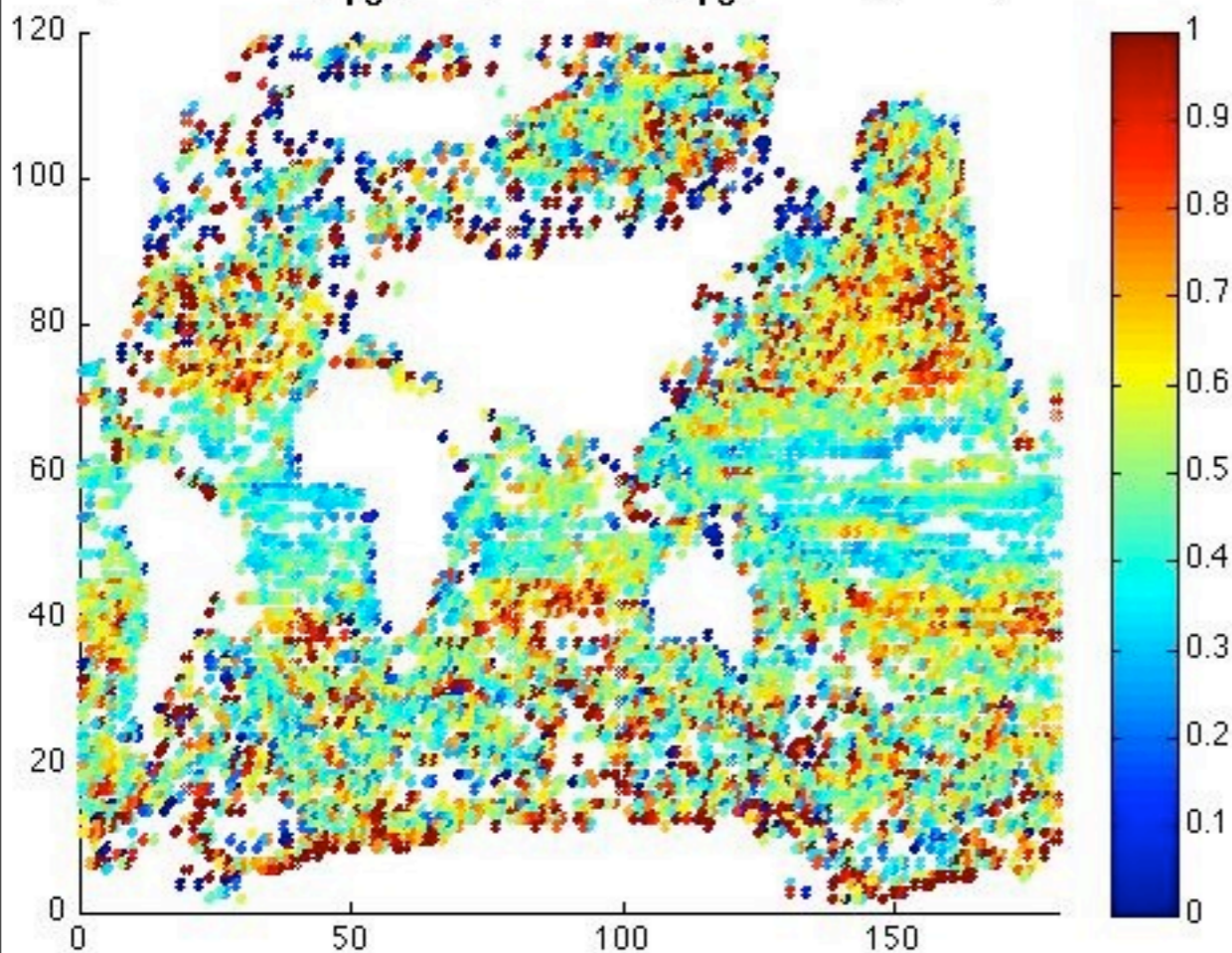


You don't need to know EKE!

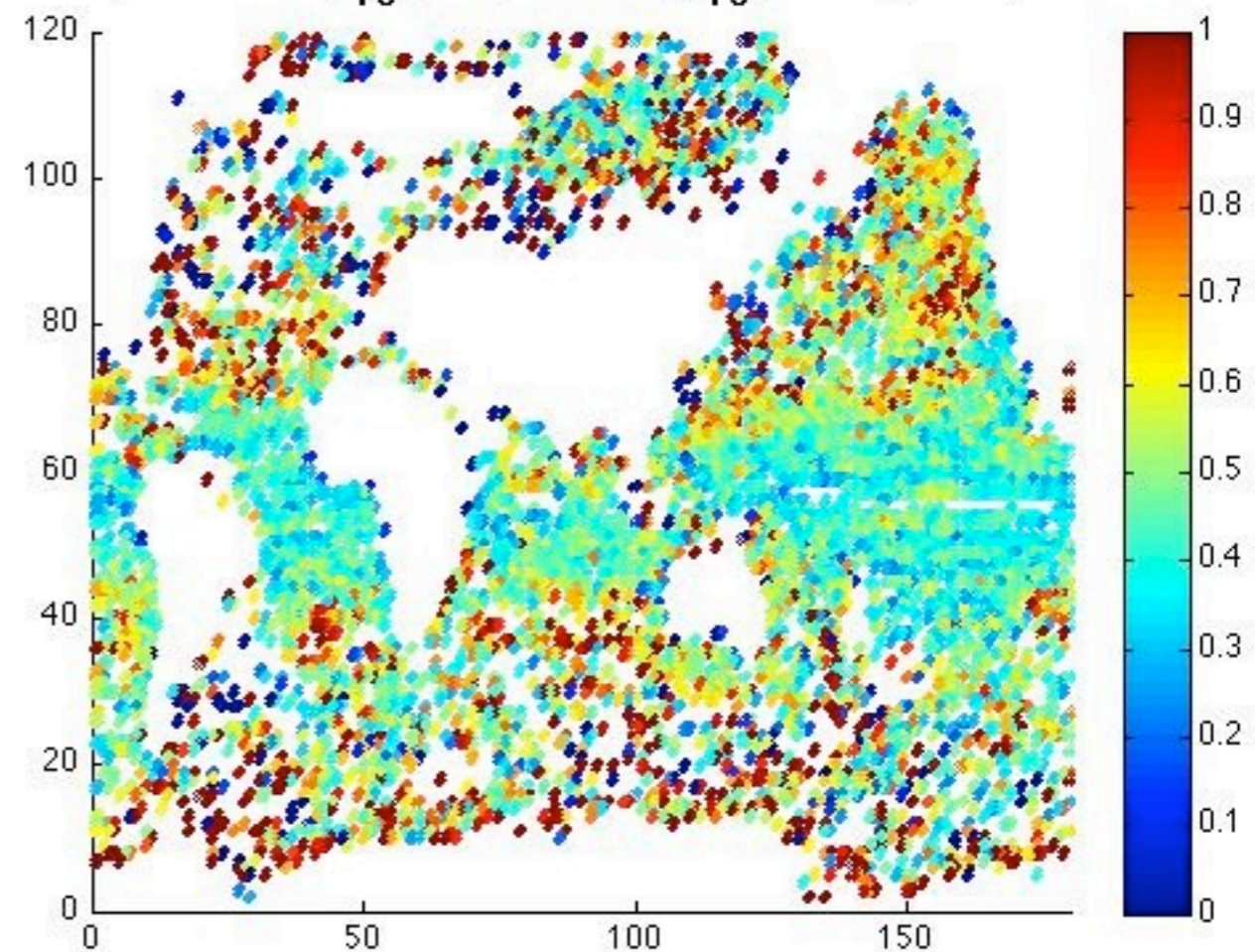
Result:
power law not 'random'

$$K \propto \sqrt{\langle KE \rangle}$$

Slope of $\log_{10}(KE)$ vs. $\log_{10}(M \text{ eig. \#1})$



Slope of $\log_{10}(KE)$ vs. $\log_{10}(M \text{ eig. \#2})$



However, can probably do better!
Slopes not random.

Coarse-graining-- A matter of philosophy

- It would be nicest if when we diagnosed M it agreed with a theory
- However, if theory requires, e.g., scale separation, then it likely won't agree
- But, the approach here gives us the answer we need (M), even if it's not the answer we want.
- Plumb & Mahlman's work suffers from the same theoretical issues--McDougall is working on it!

Conclusions Mesoscale

- Direct diagnosis of \mathbf{M} is a valuable tool
- Gives validated tracer fluxes without negative eigenvalues or rotational issues
- Still, unfamiliar interpretation
- No clean comparison to theory (GLM? Scale Separation? Ensemble? Stochastics?)
- More to come!
 - (e.g., Ferrari et al '08 vs. Ferrari et al. '10)