Diagnosis of Ocean Mesoscale Eddy Tracer Fluxes

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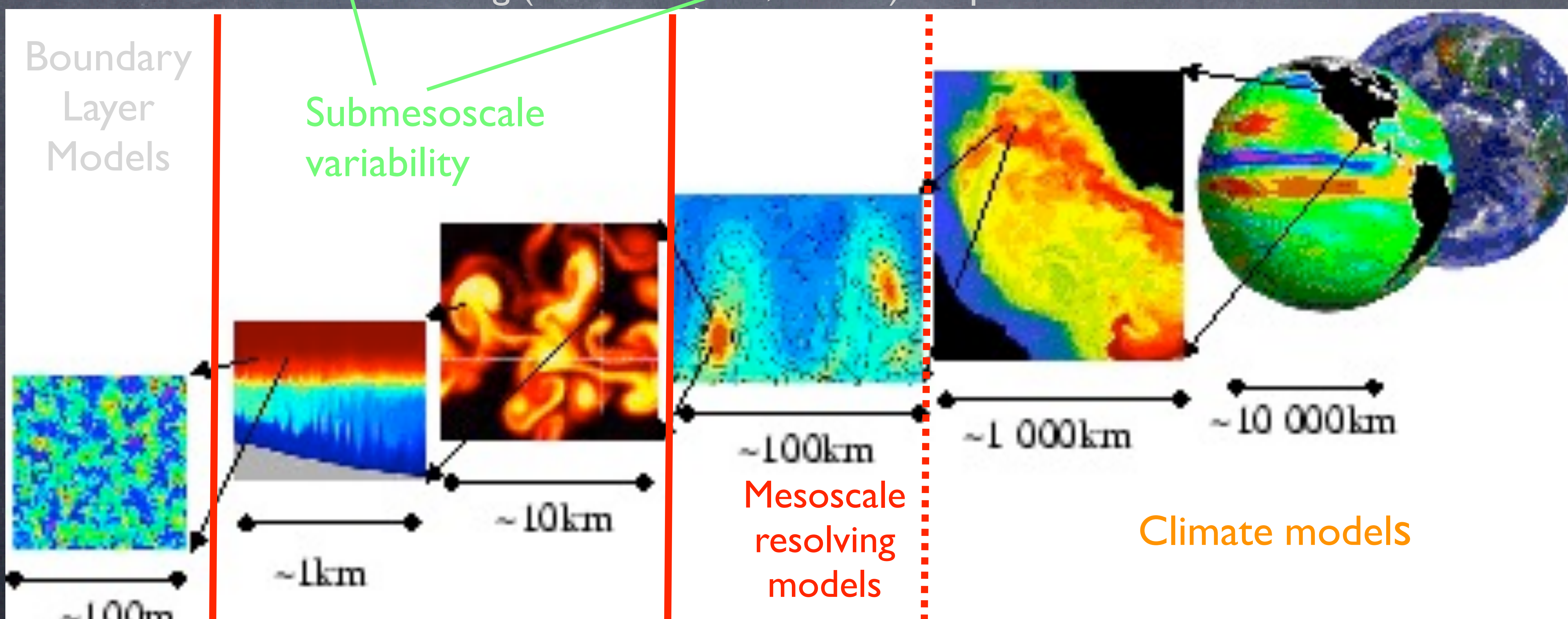
Walsh Cottage, GFD Seminar, Fr. 7/14/2010 2:30PM

Upper Ocean in Climate Models

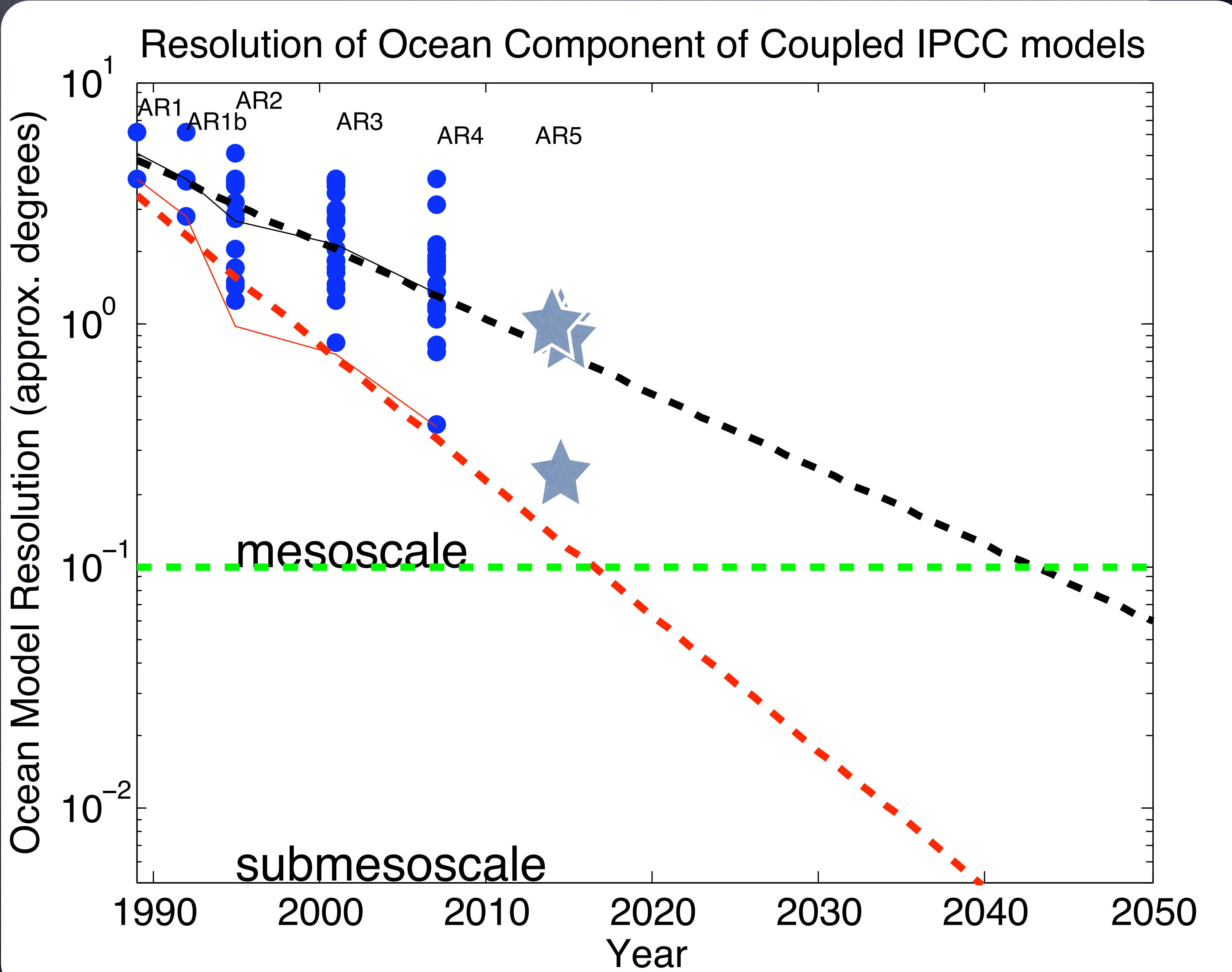
- Large-scale ocean circulation (100 - 10,000 km, yrs->centuries) => resolved
- Mesoscale variability (10 - 100 km, mo -> yrs) => resolved or parameterized
- Submesoscale variability (100 m - 10 km, d -> mo) => ignored until recently
- Internal waves & Langmuir circulations (10-100m, hr -> day) => crudely param.
- Turbulent mixing (10 cm - 100 m, s -> hr) => parameterized



Coupling?

Coupling?

The Future of Resolution



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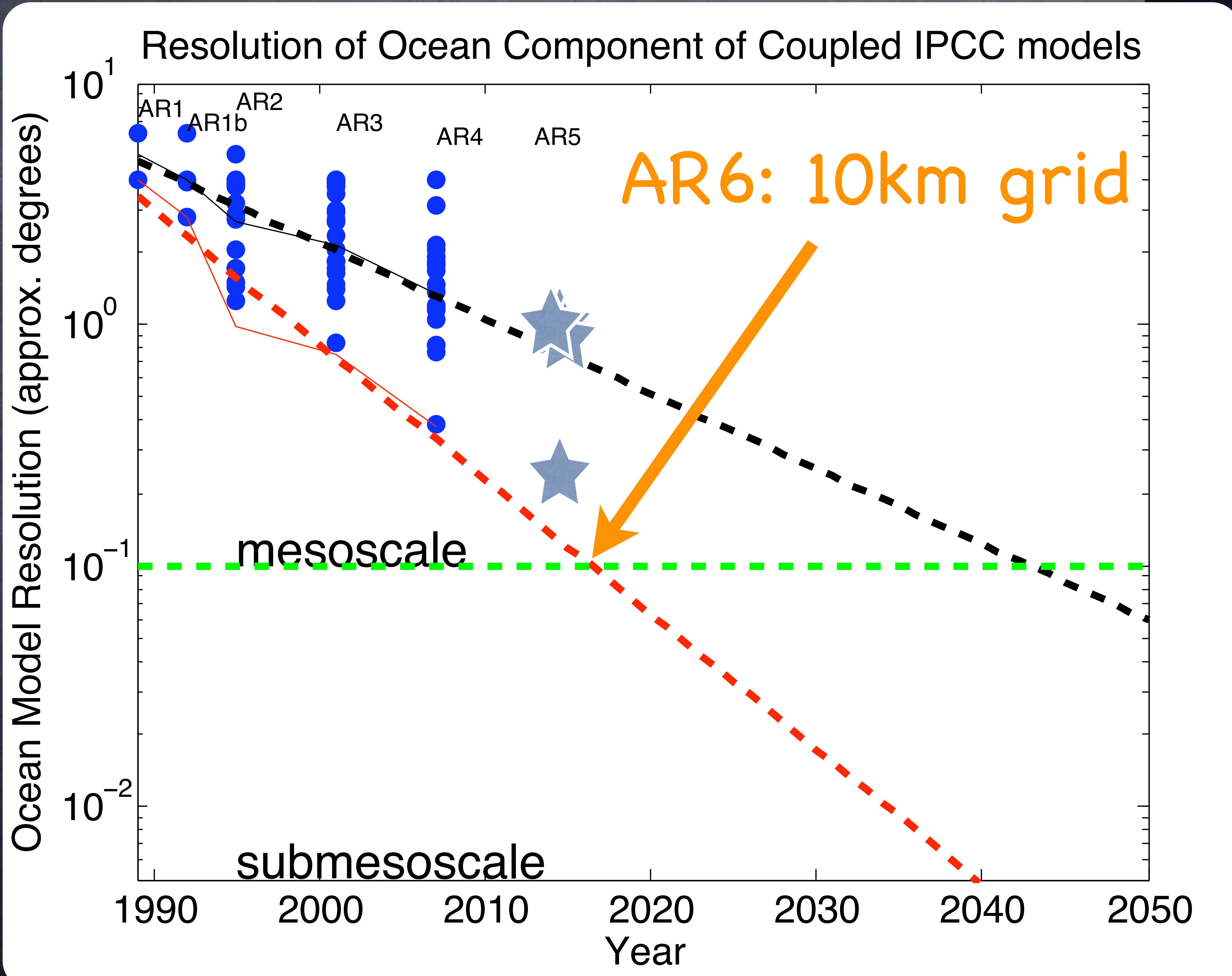

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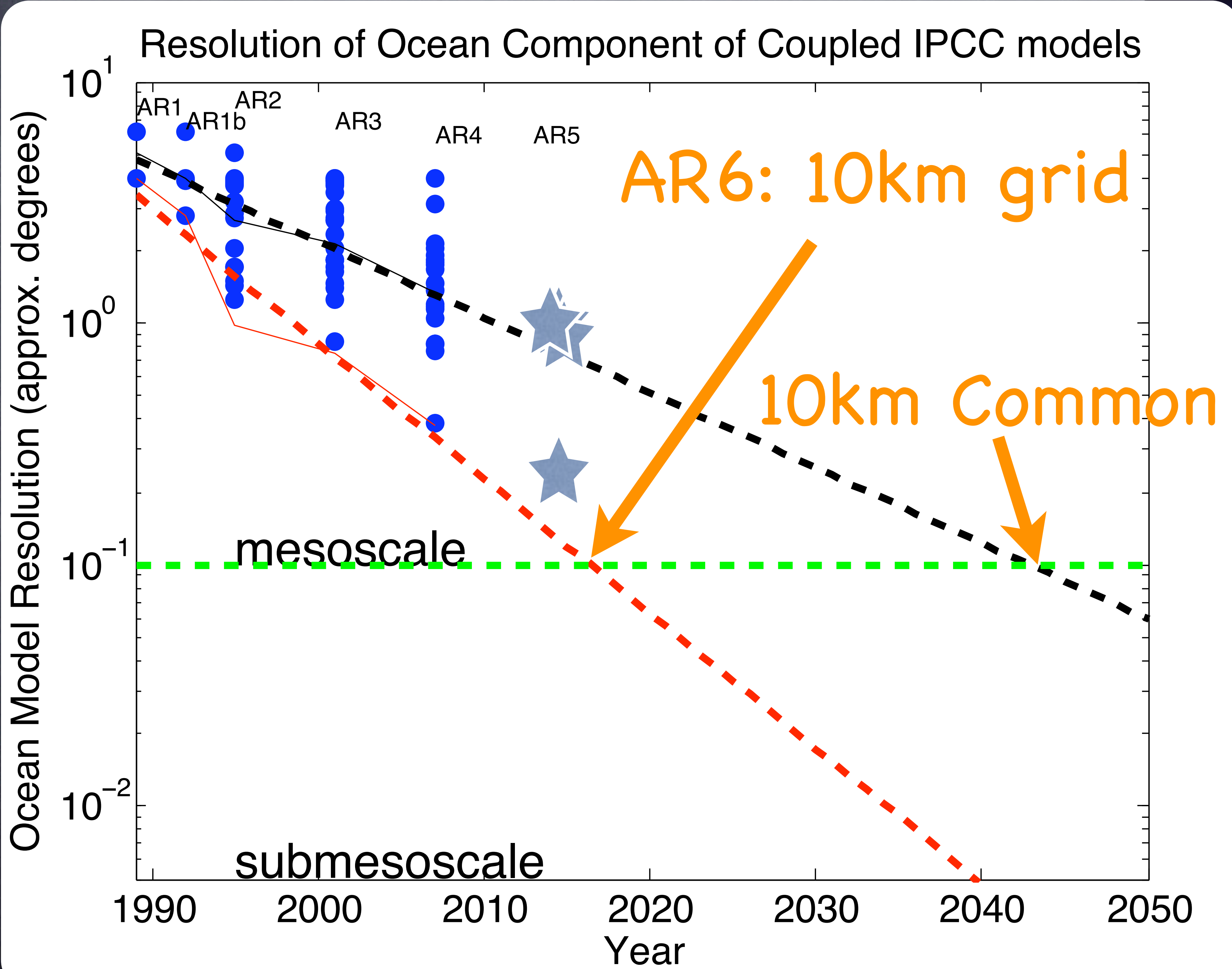

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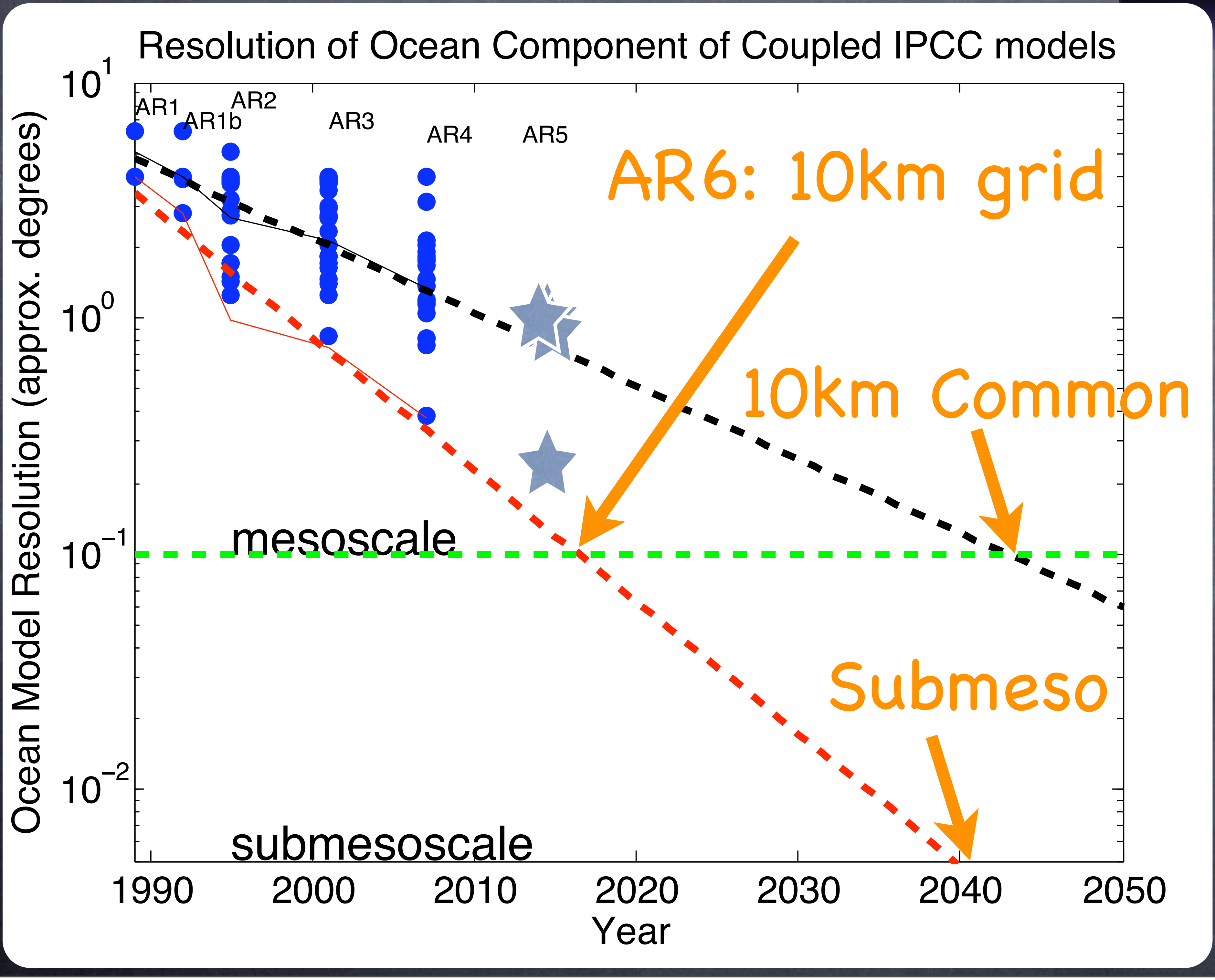

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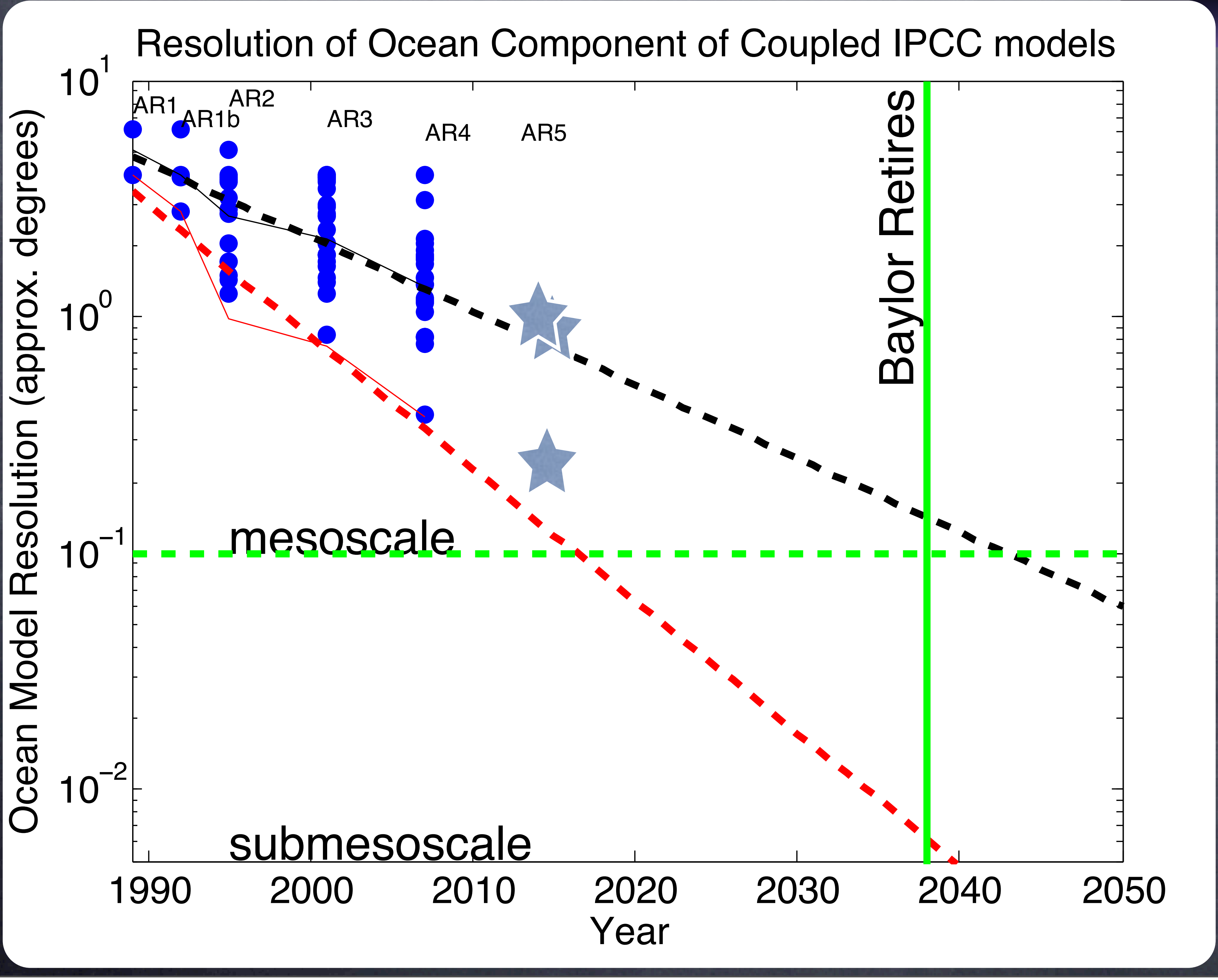
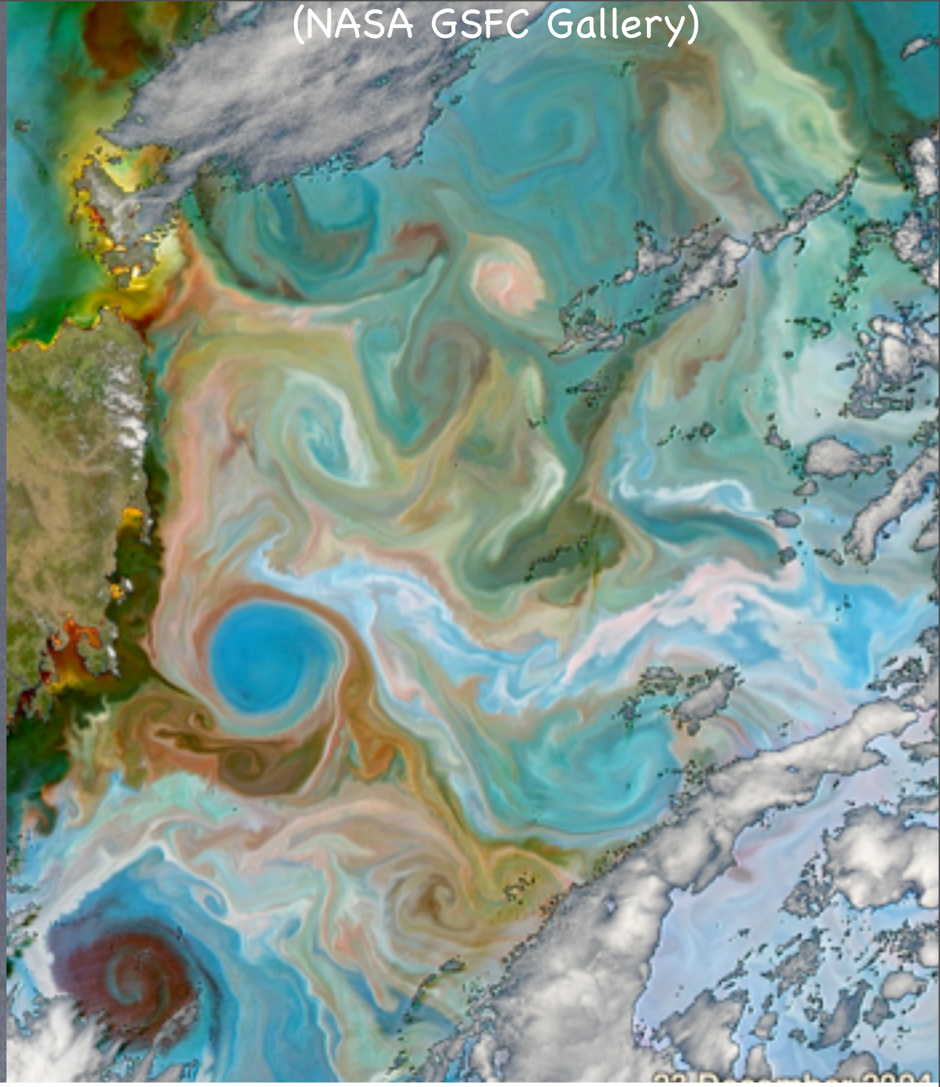


The Character of the Mesoscale

100 km

(NASA GSFC Gallery)



(Capet et al., 2008)

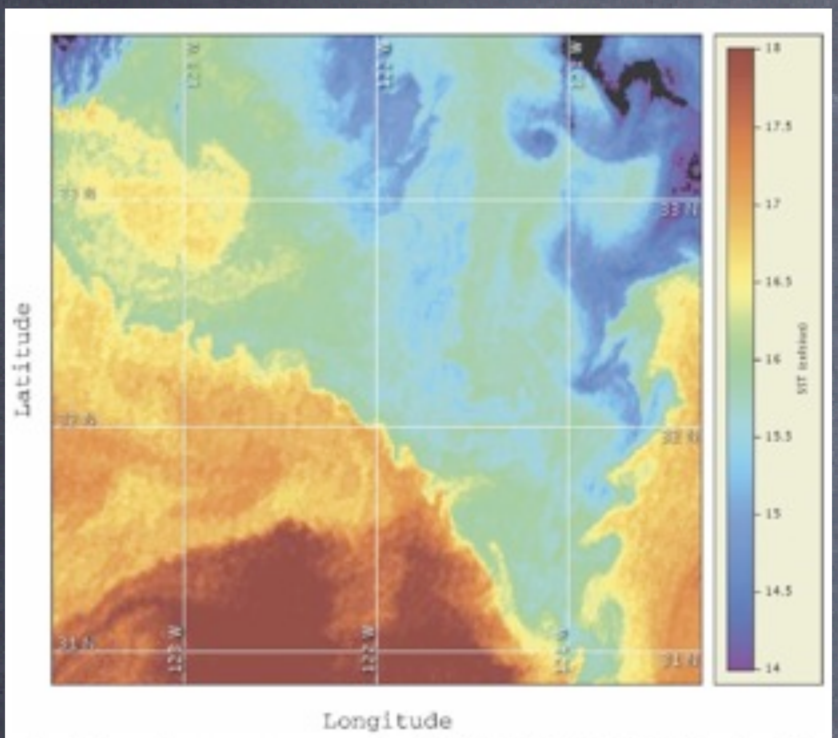
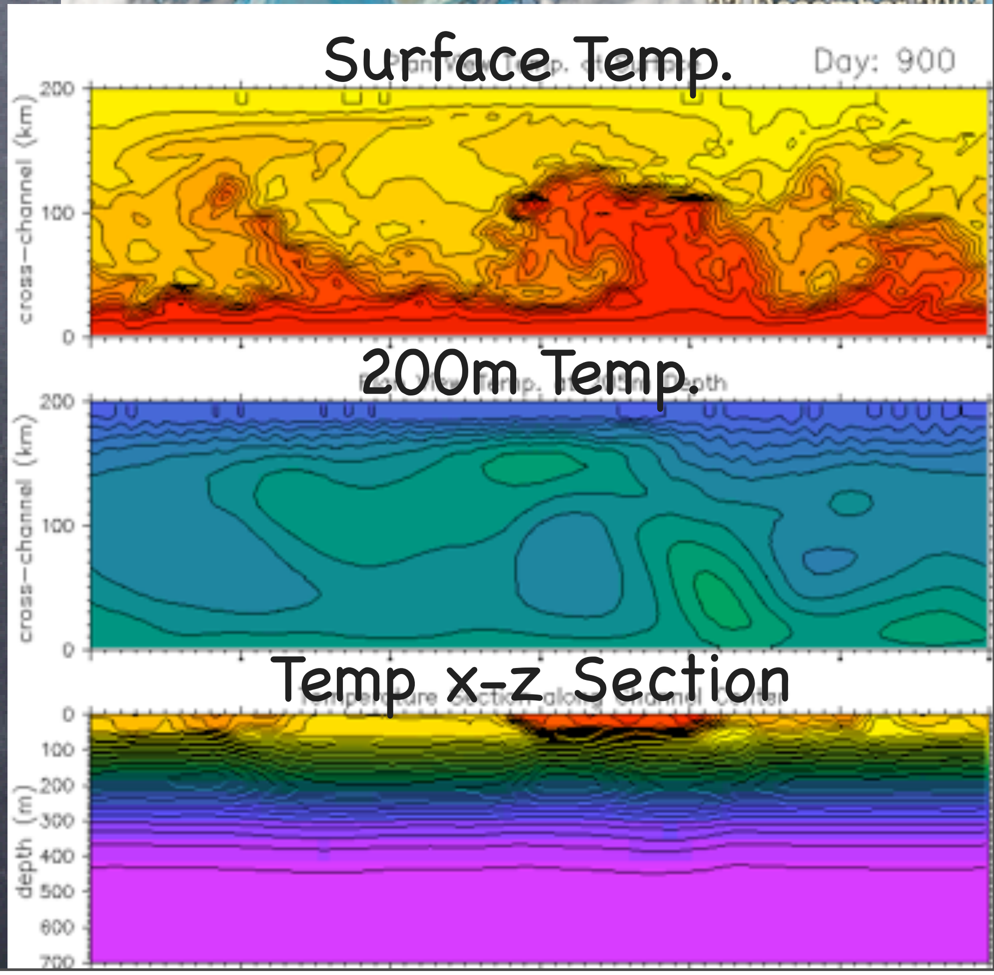


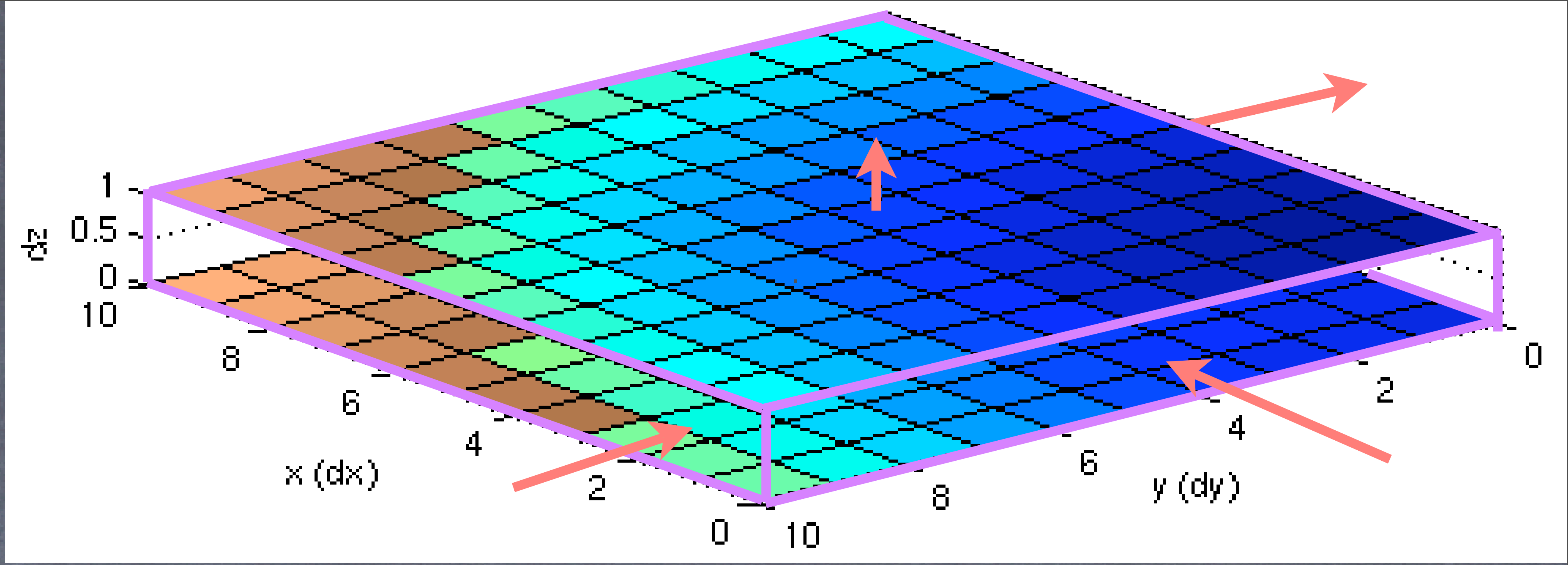
FIG. 16. Sea surface temperature measured at 1832 UTC 3 Jun 2006 off Point Conception in the California Current from CoastWatch (http://coastwatch.pfeg.noaa.gov). The fronts between recently upwelled water (i.e., 15°–16°C) and offshore water (≥17°C) show submesoscale instabilities with wavelengths around 30 km (right front) or 15 km (left front). Images for 1 day earlier and 4 days later show persistence of the instability events.

- Boundary Currents
- Eddies
- Ro=O(0.1)
- Ri=O(1000)
- Full Depth
- Eddies strain to produce Fronts
- 100km, months



Eddy processes mainly baroclinic & barotropic instability. Parameterizations of baroclinic instability (GM, Visbeck...).

What's the issue?



- Ocean climate models are typically horiz.-resolution limited
- Constant horiz. gridscale near 100km:
  - *does not permit even mesoscale instabilities*
- Boussinesq, Hydrostatic, Navier-Stokes equations w/ S, T, tracers.
- Timestep and variable vert. resolution are much finer, relative to structures present
- So, want to close for horiz. coarse-graining, from eddy-resolving to climate model

What is a subgrid parameterization?

- Express the coarse-grain averages of quantities (including the subgrid effects), e.g.:

$$\overline{\frac{\partial \tau}{\partial t}} \qquad \overline{\frac{\partial u}{\partial x}} \qquad \overline{\frac{\partial u\tau}{\partial x}}$$

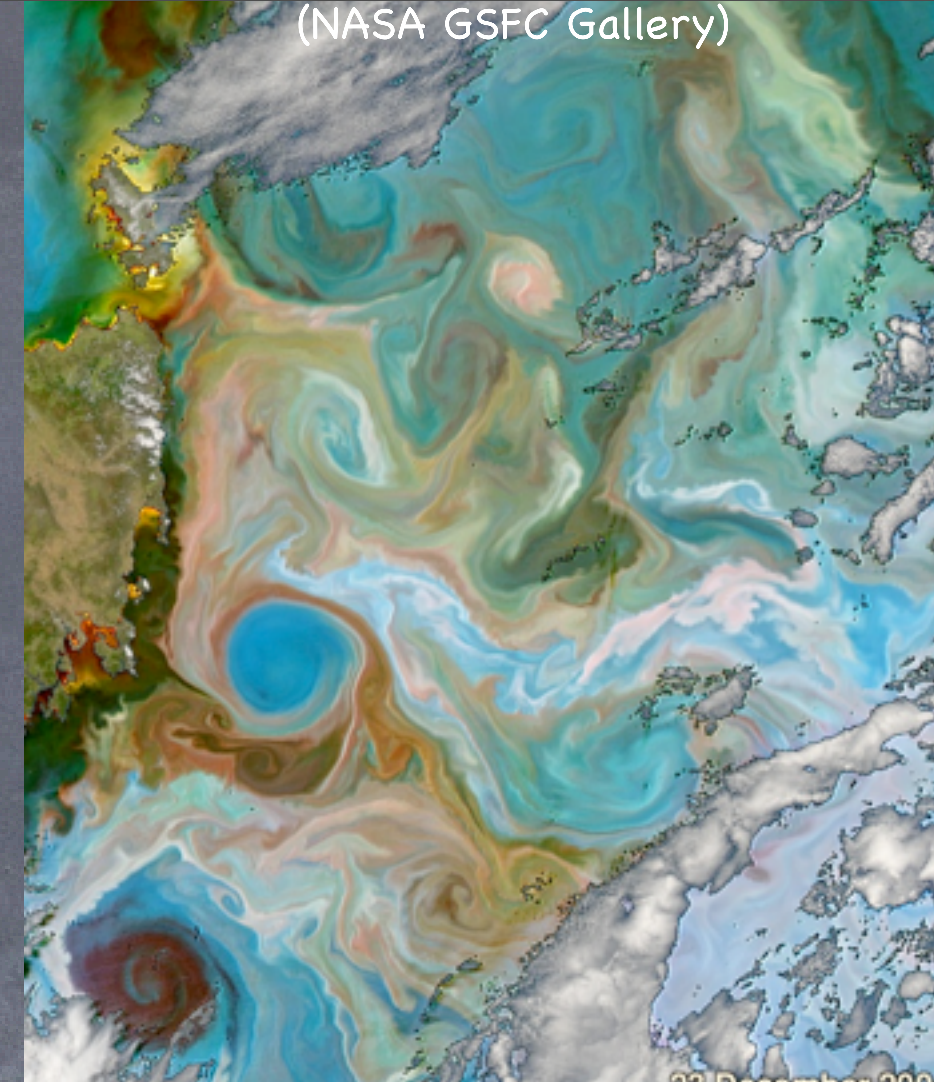
- As a function of the resolved coarse-grain fields

$$\overline{\frac{\partial \tau}{\partial t}} = \frac{\partial \overline{\tau}}{\partial t} \qquad \overline{\frac{\partial u}{\partial x}} = \frac{\partial \overline{u}}{\partial x} \qquad \overline{\frac{\partial u\tau}{\partial x}} = \frac{\partial \overline{u}\ \overline{\tau}}{\partial x} + \frac{\partial \overline{u'\tau'}}{\partial x}$$

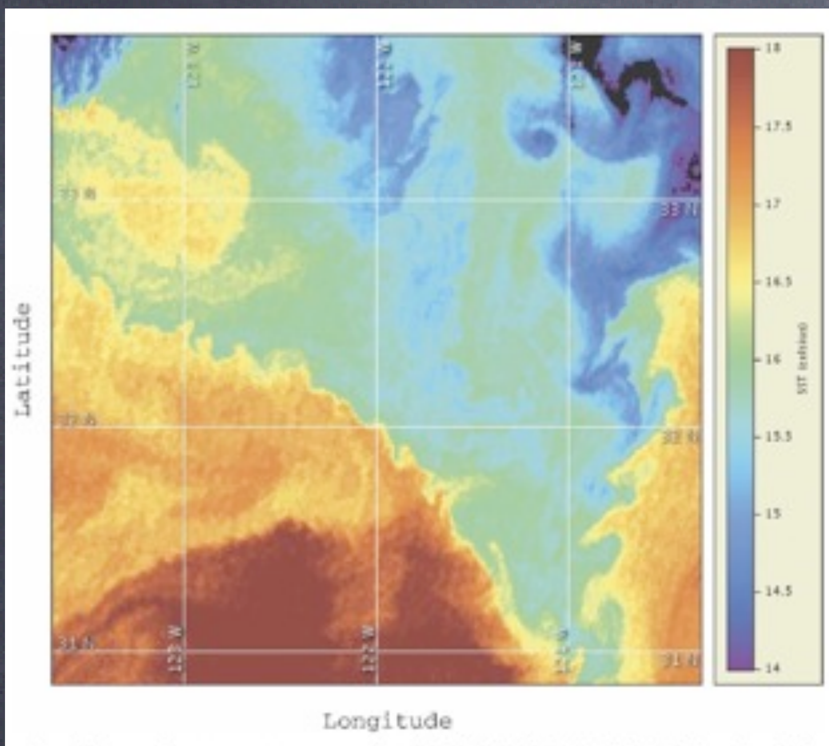
- Note that nonlinear terms require special treatment

The Character of the Submesoscale

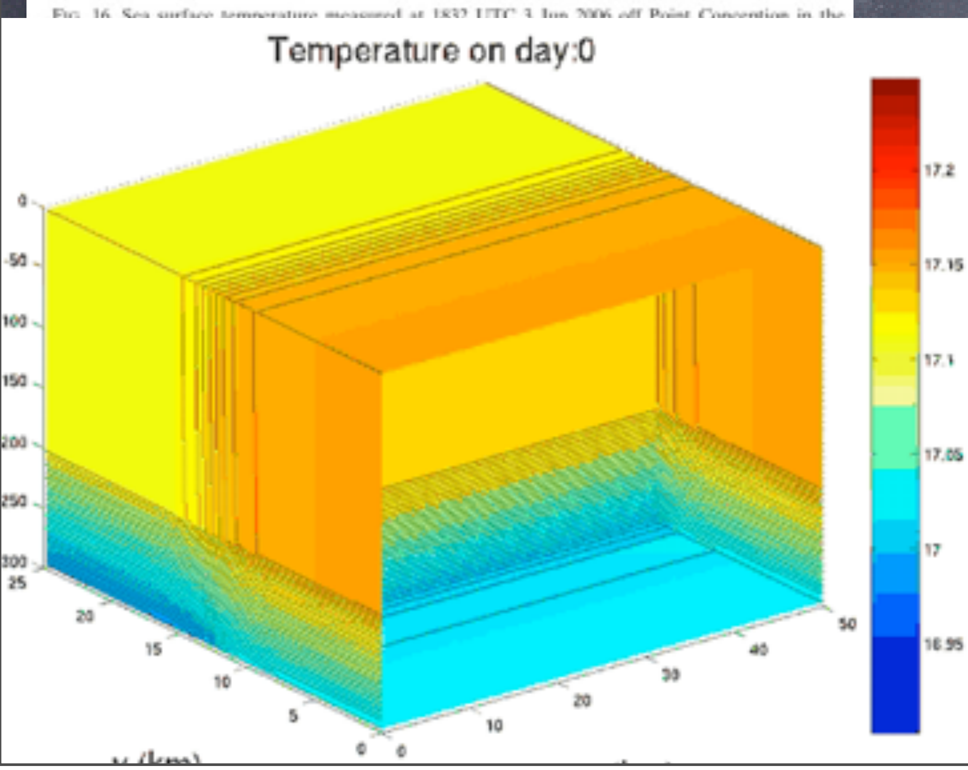
10 km



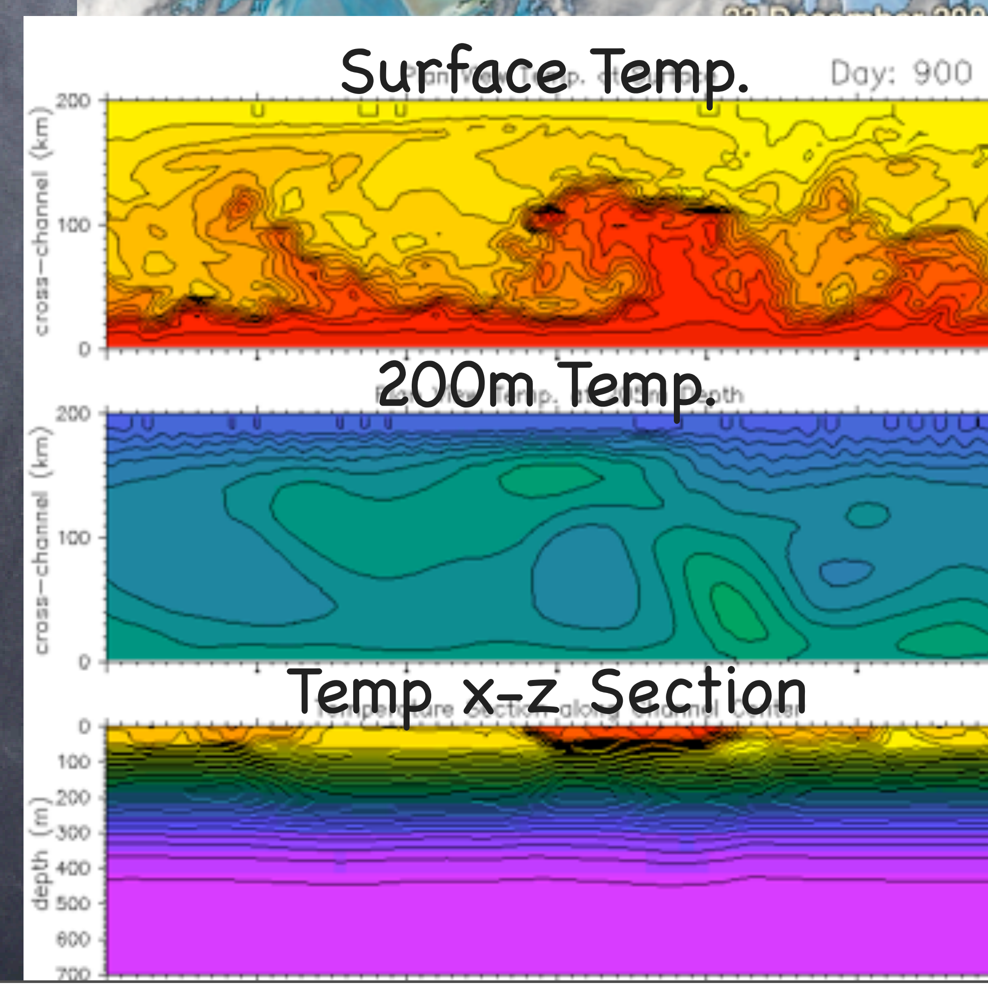
(Capet et al., 2008)



- Fronts
- Eddies
- Ro=O(1)
- Ri=O(1)
- near-surface
- 1-10km, days



Eddy processes mainly baroclinic instability (Boccaletti et al '07, Haine & Marshall '98).
Parameterizations of baroclinic instability?



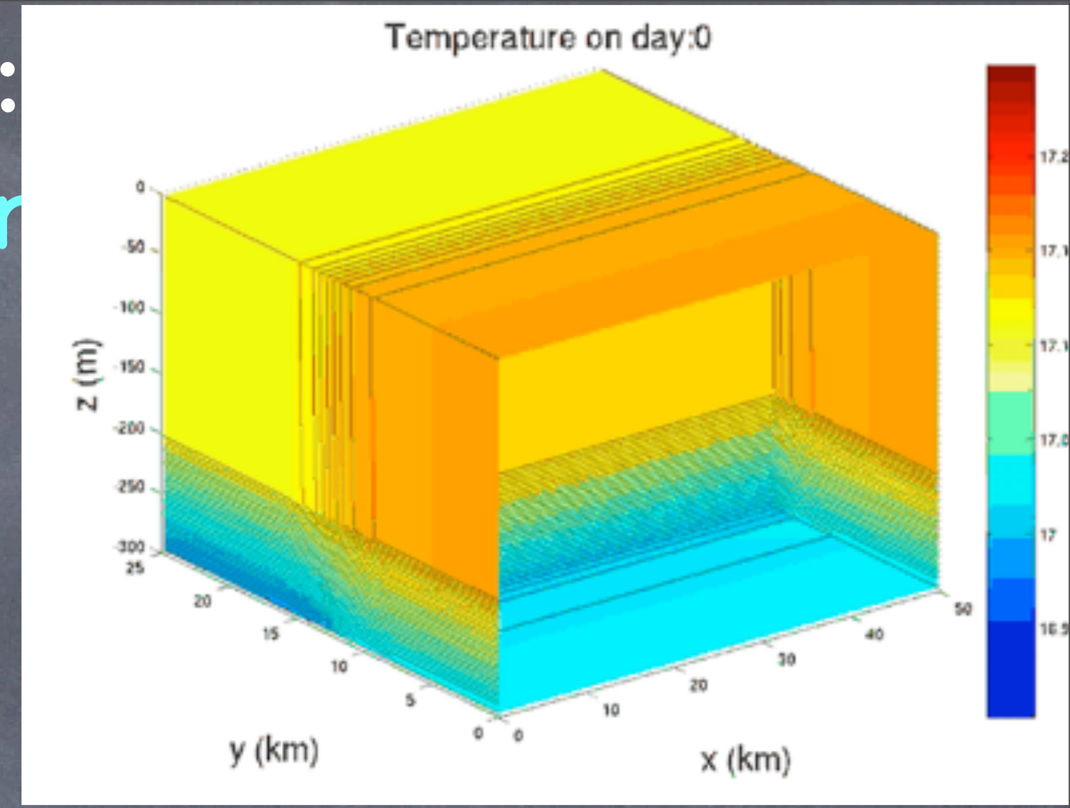
A Special Treatment of Nonlinear: Mixed Layer Eddy Restratification

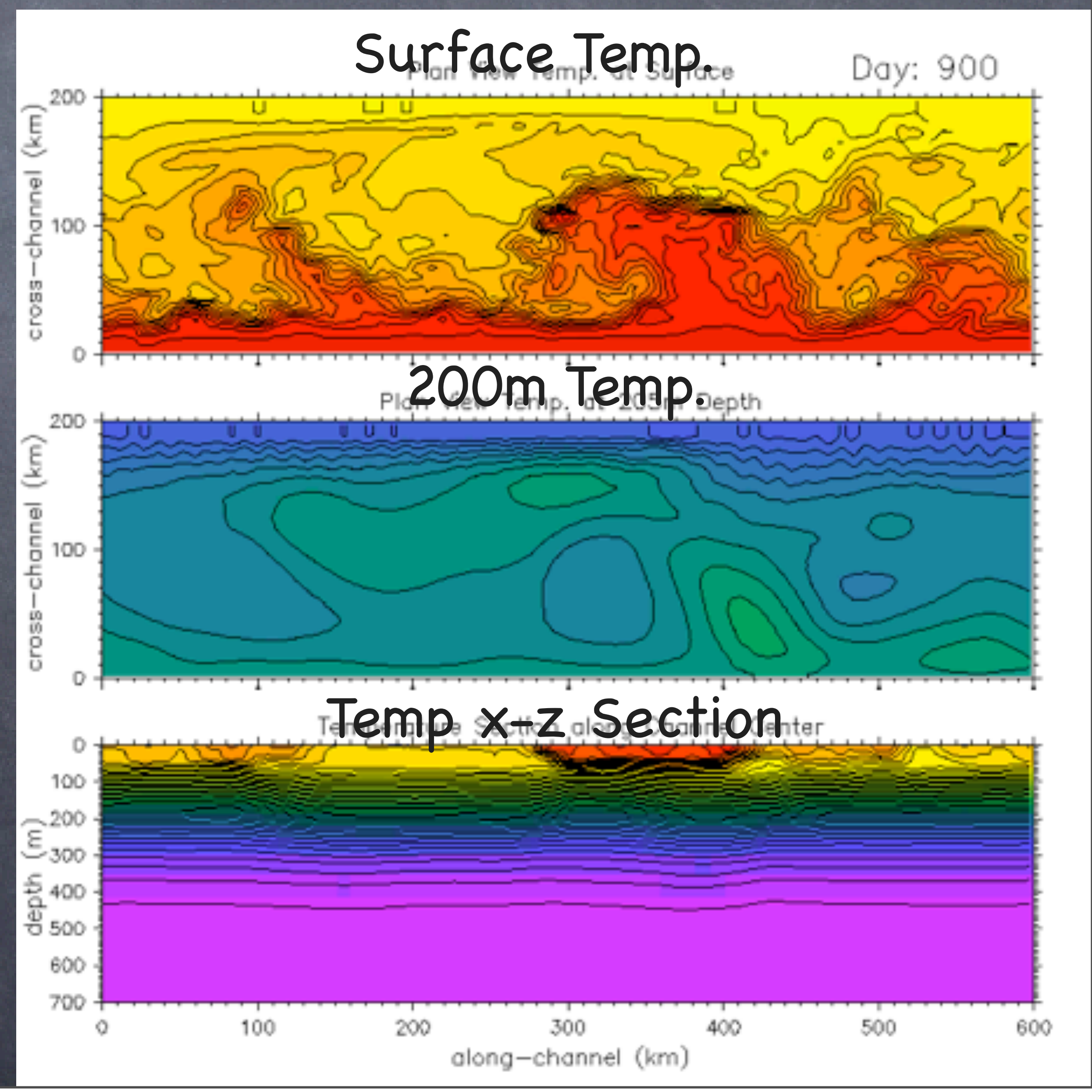
Estimating eddy buoyancy/density fluxes:

$$\overline{\mathbf{u}'b'} \equiv \mathbf{\Psi} \times \nabla\bar{b}$$

A submeso eddy-induced overturning:

$$\mathbf{\Psi} = \frac{C_e H^2 \mu(z)}{|f|} \nabla\bar{b} \times \hat{\mathbf{z}}$$





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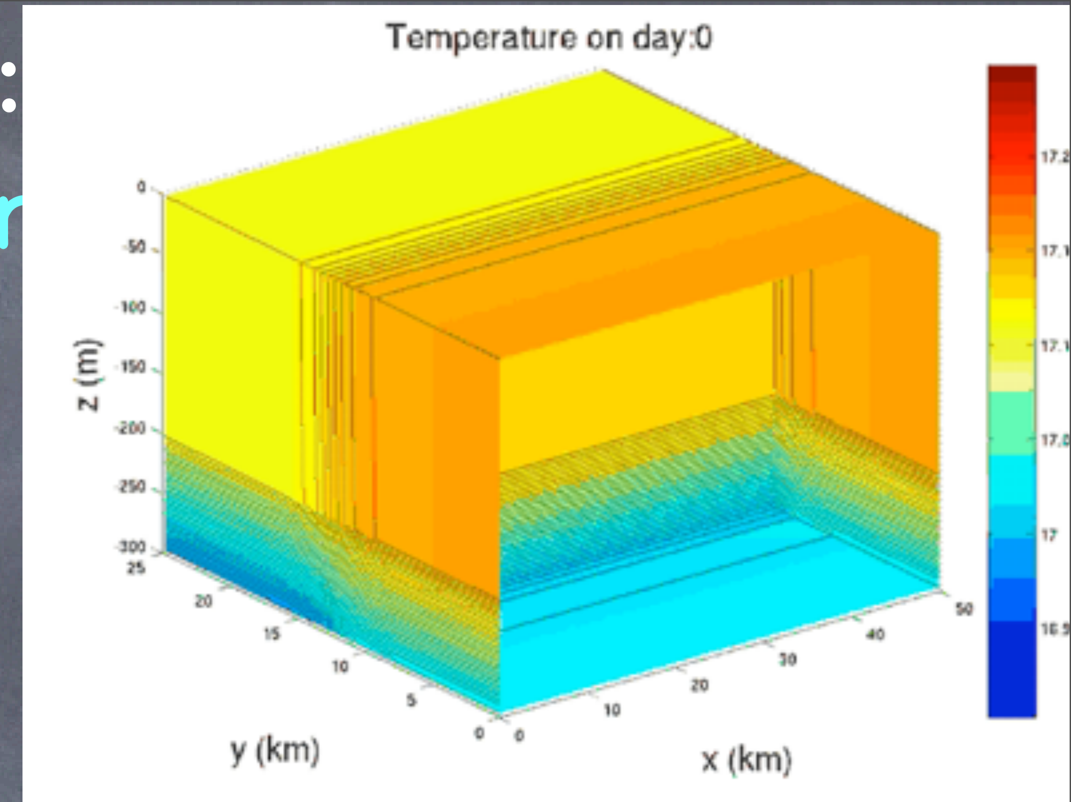
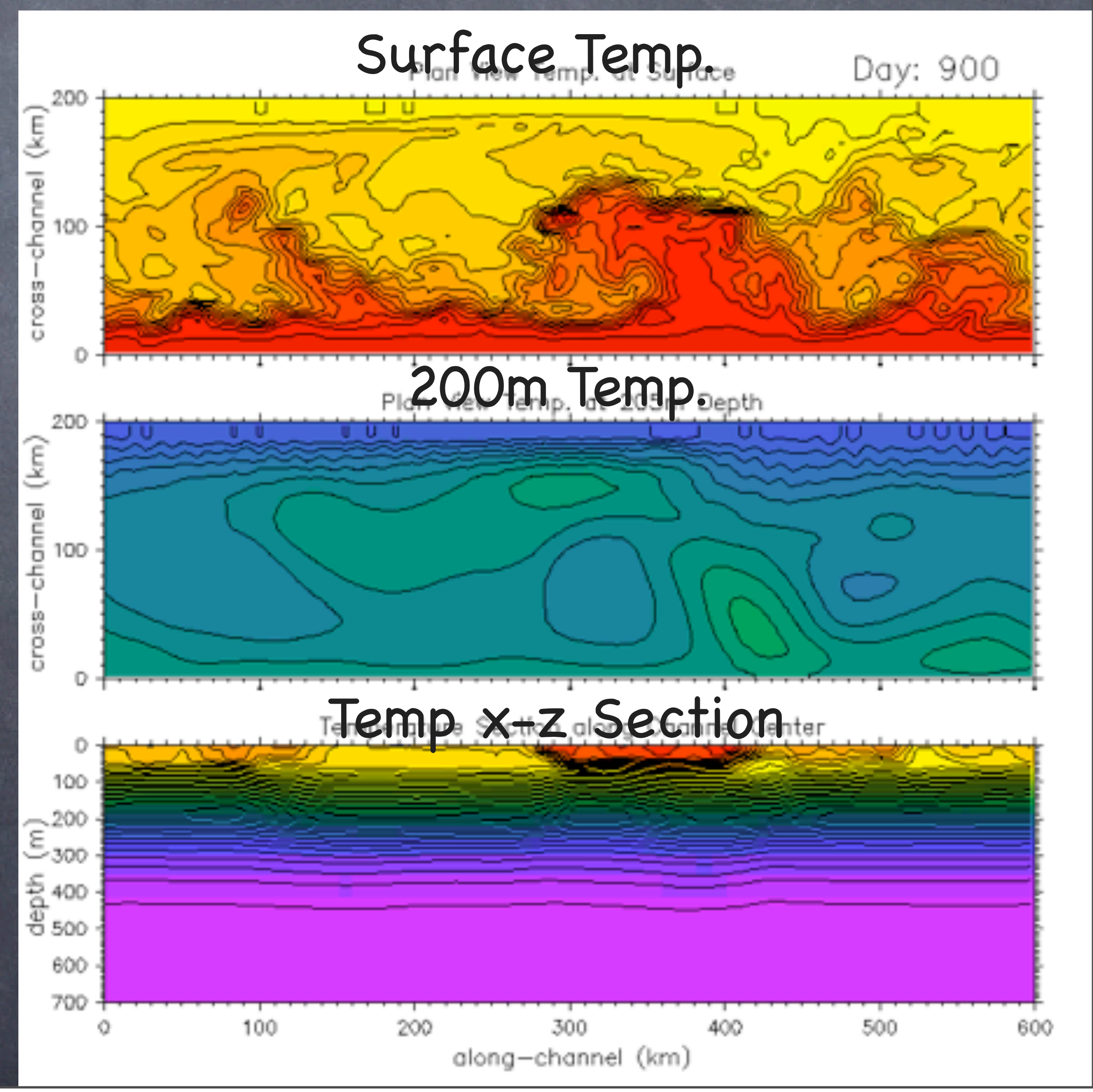
in ML only:

$$\mu(z) = 0 \text{ if } z < -H$$





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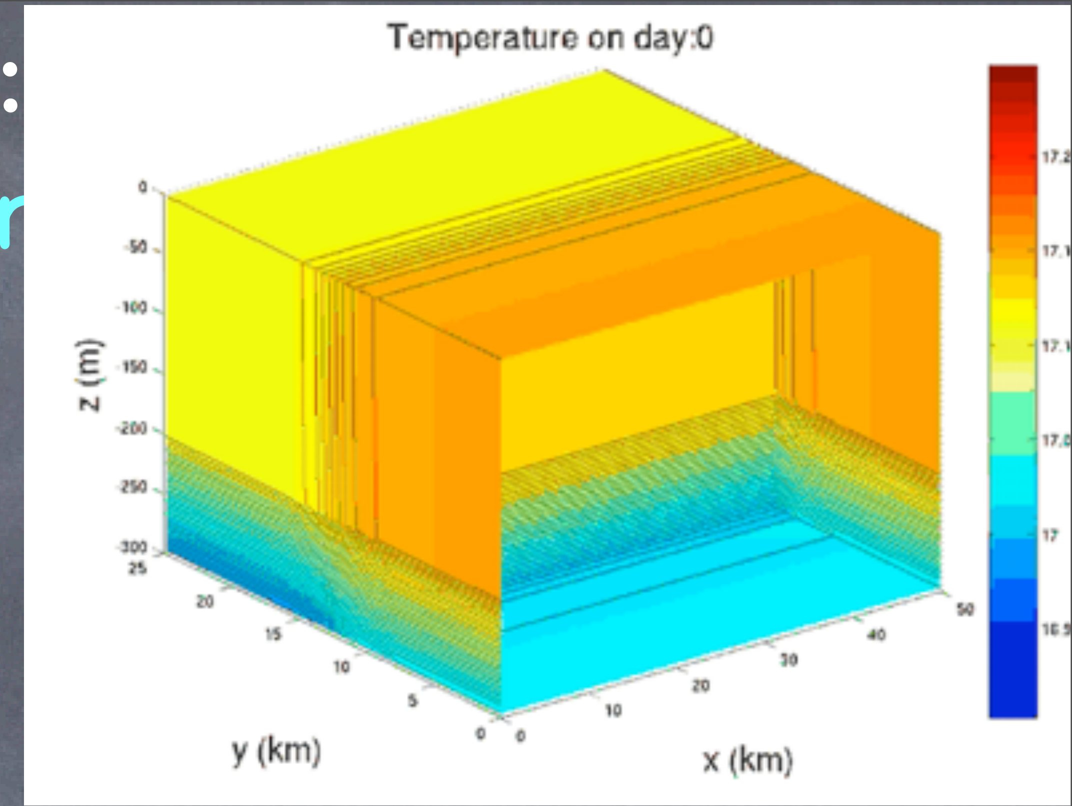

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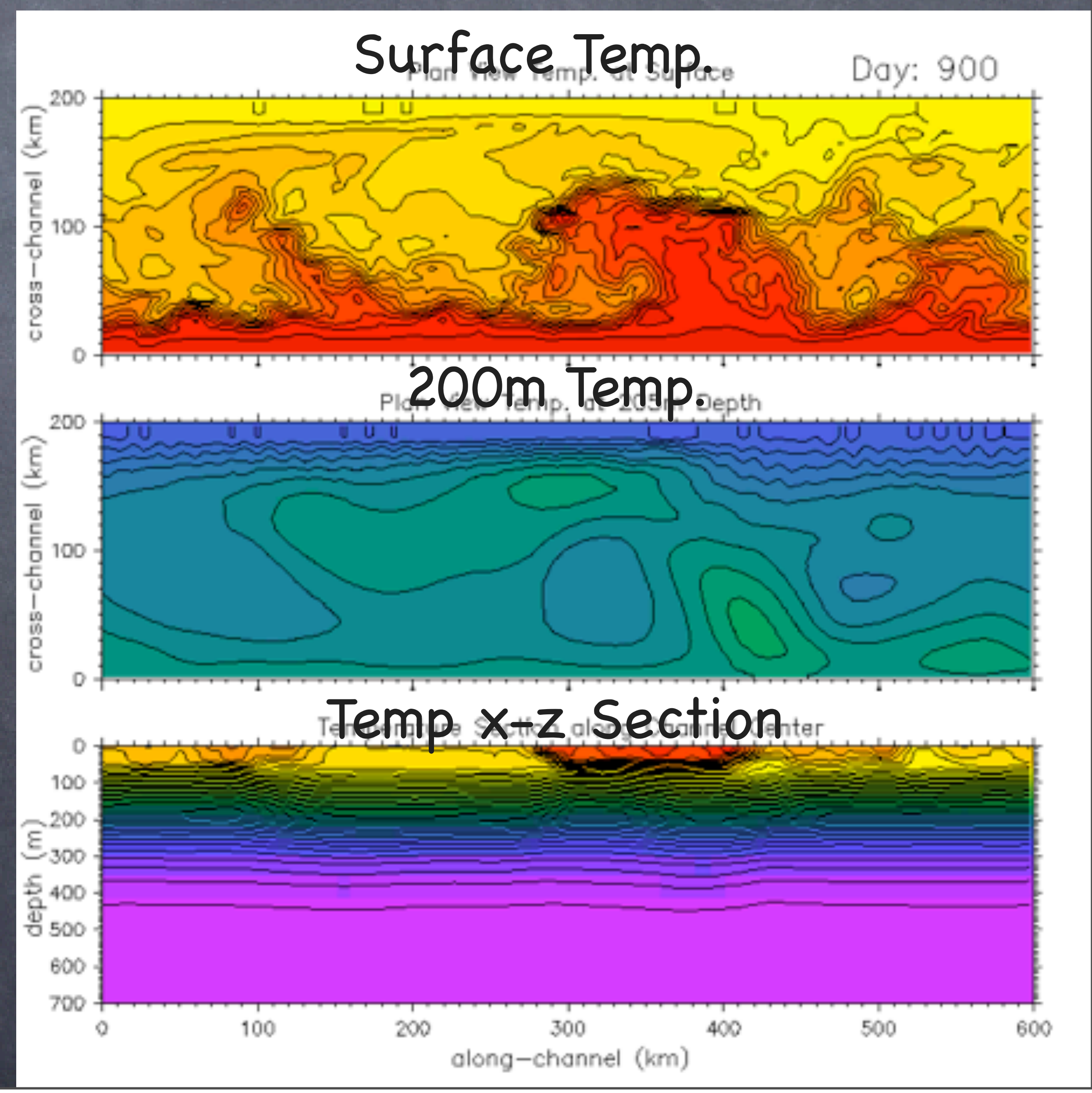
$$\overline{w'b'} \propto \frac{H^2}{|f|} \left| \nabla_H \bar{b} \right|^2$$



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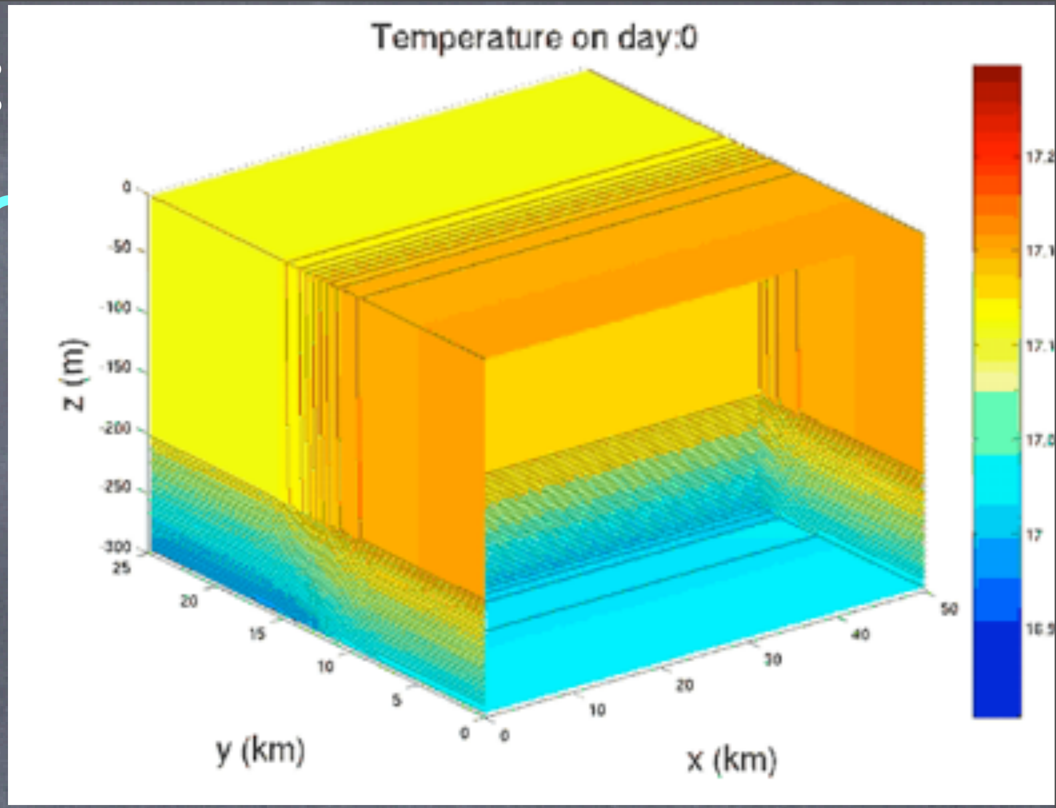
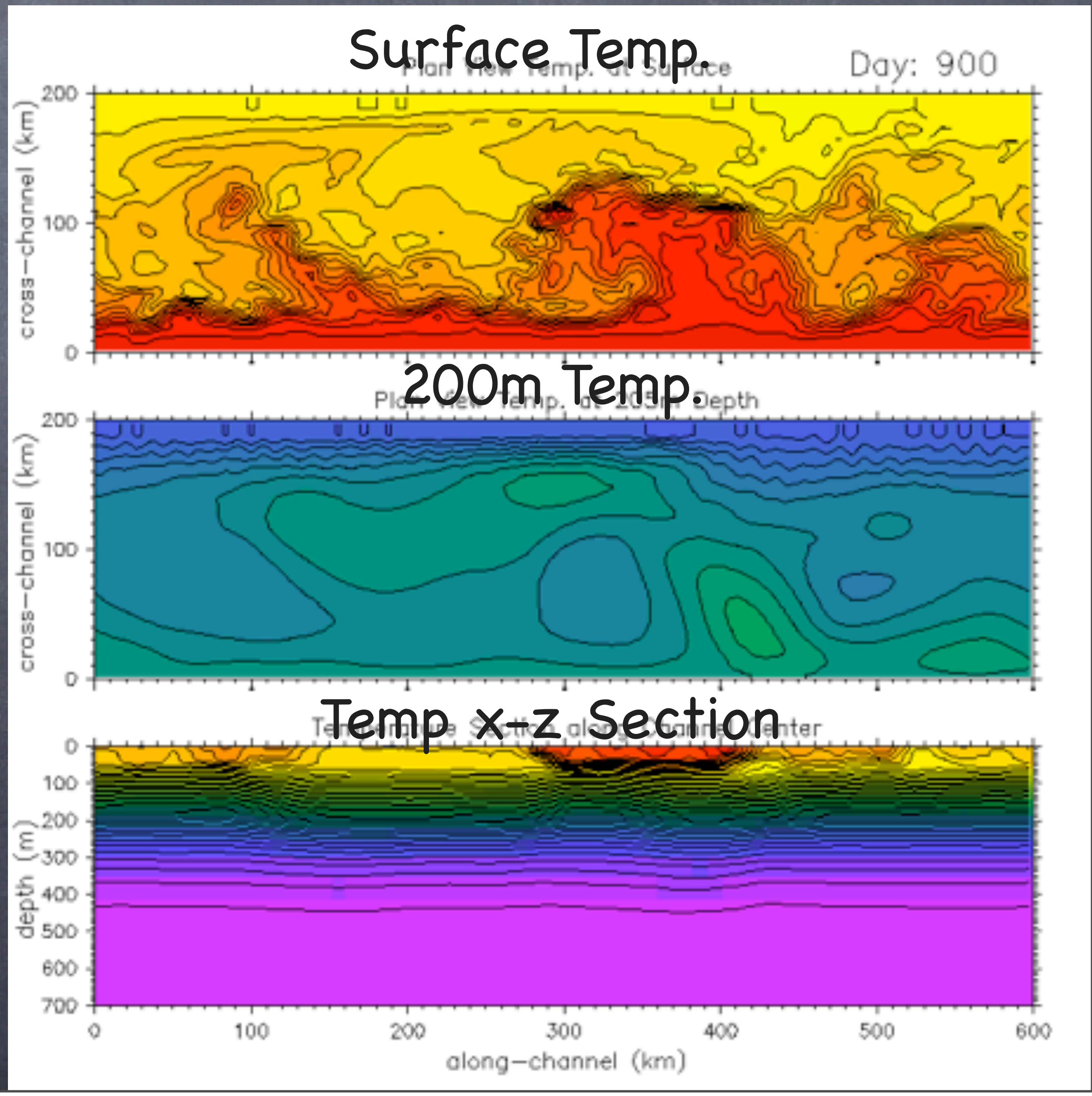
For a consistently restratifying,

$$\overline{w'b'} \propto \frac{H^2}{|f|} \left| \nabla_H \bar{b} \right|^2$$

and horizontally downgradient flux.

$$\overline{\mathbf{u}'_H b'} \propto \frac{-H^2 \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla_H \bar{b}$$





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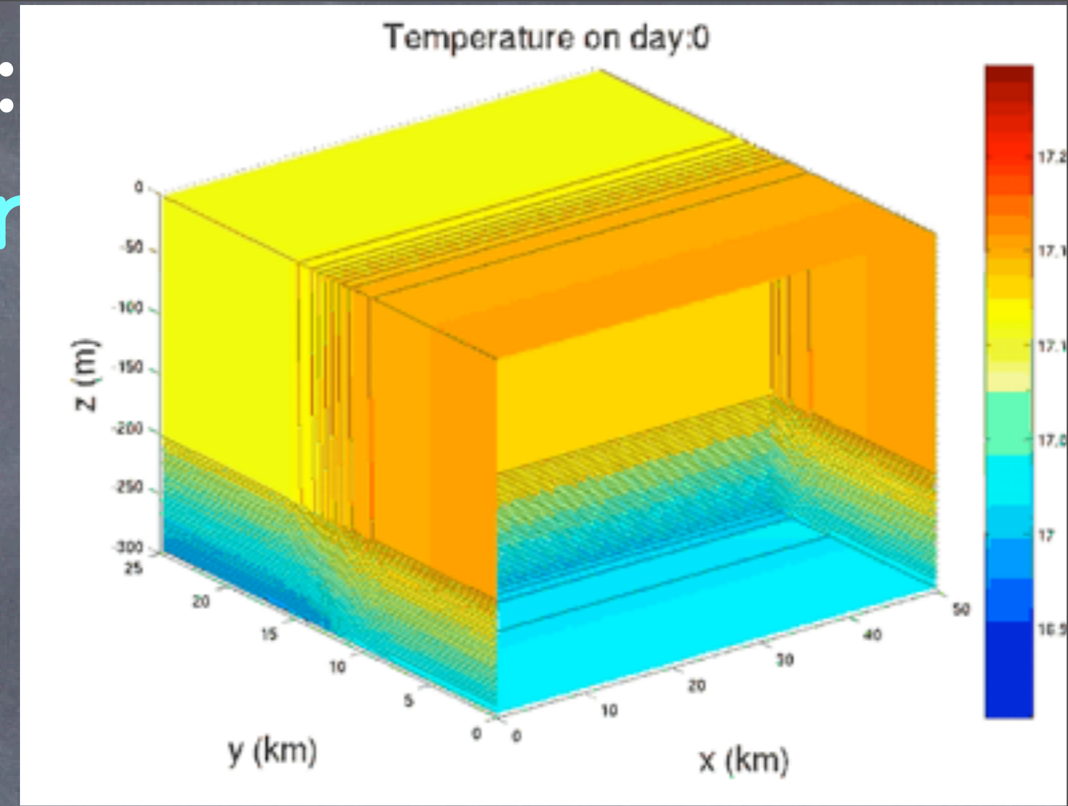

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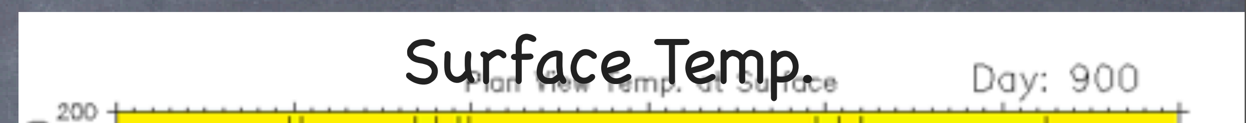

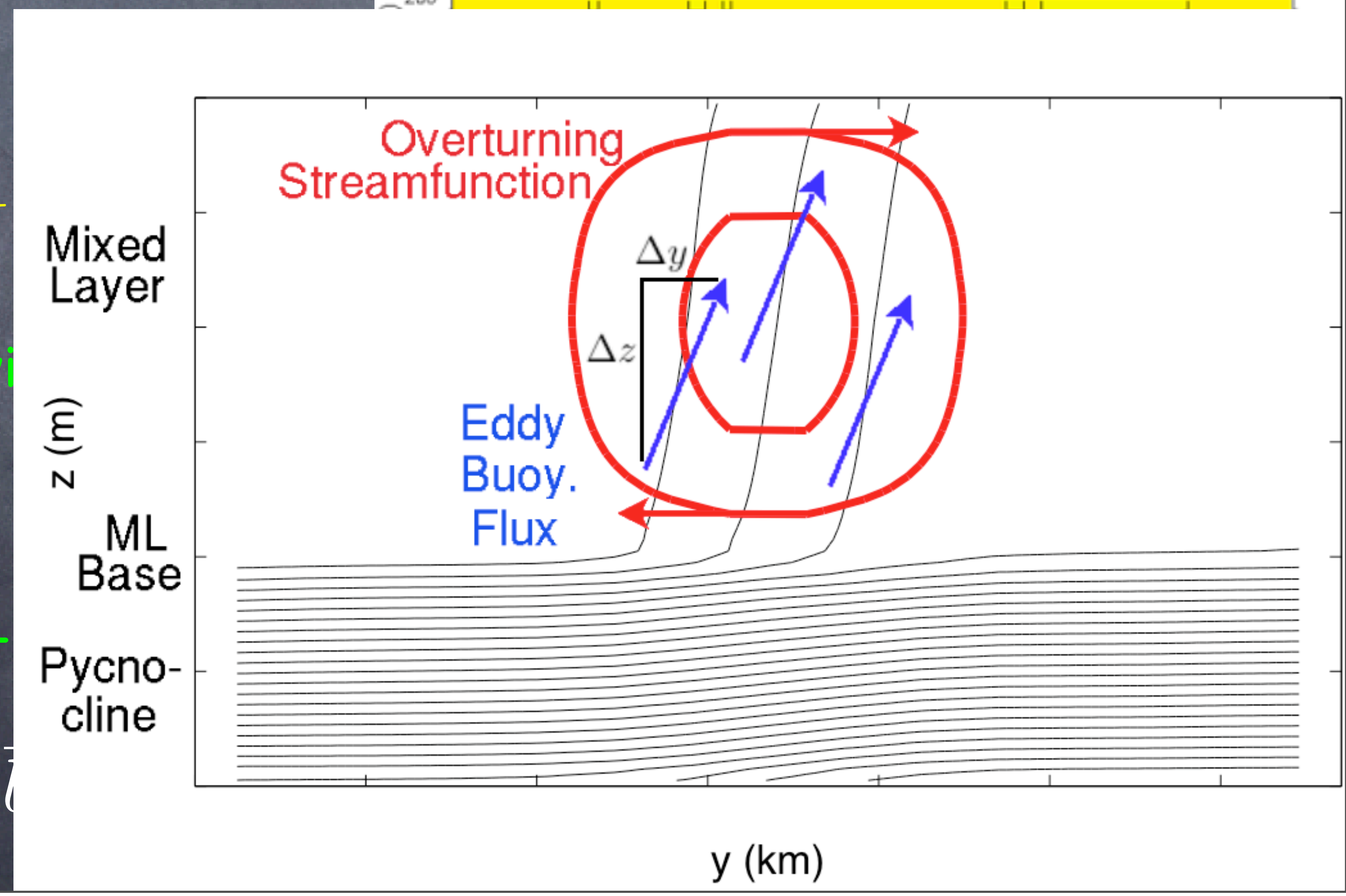

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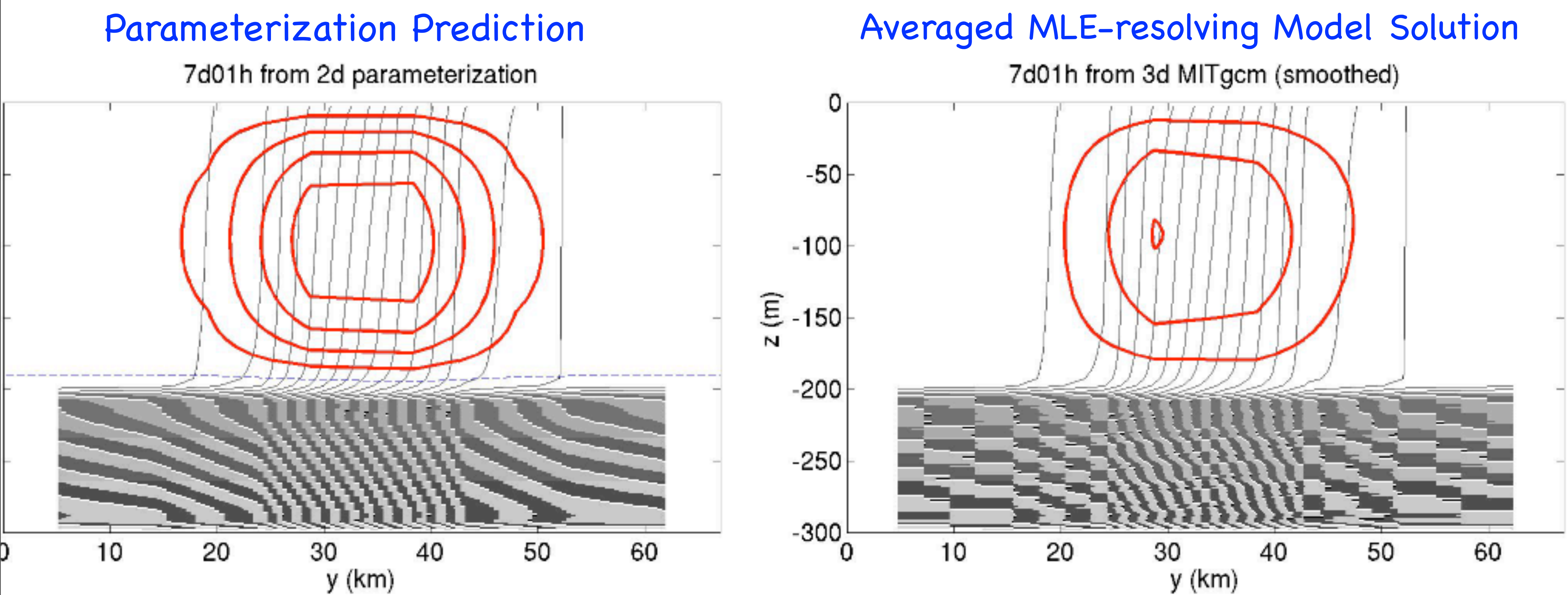
What does eddy restratification look like?



red=streamfunction

black=mean density

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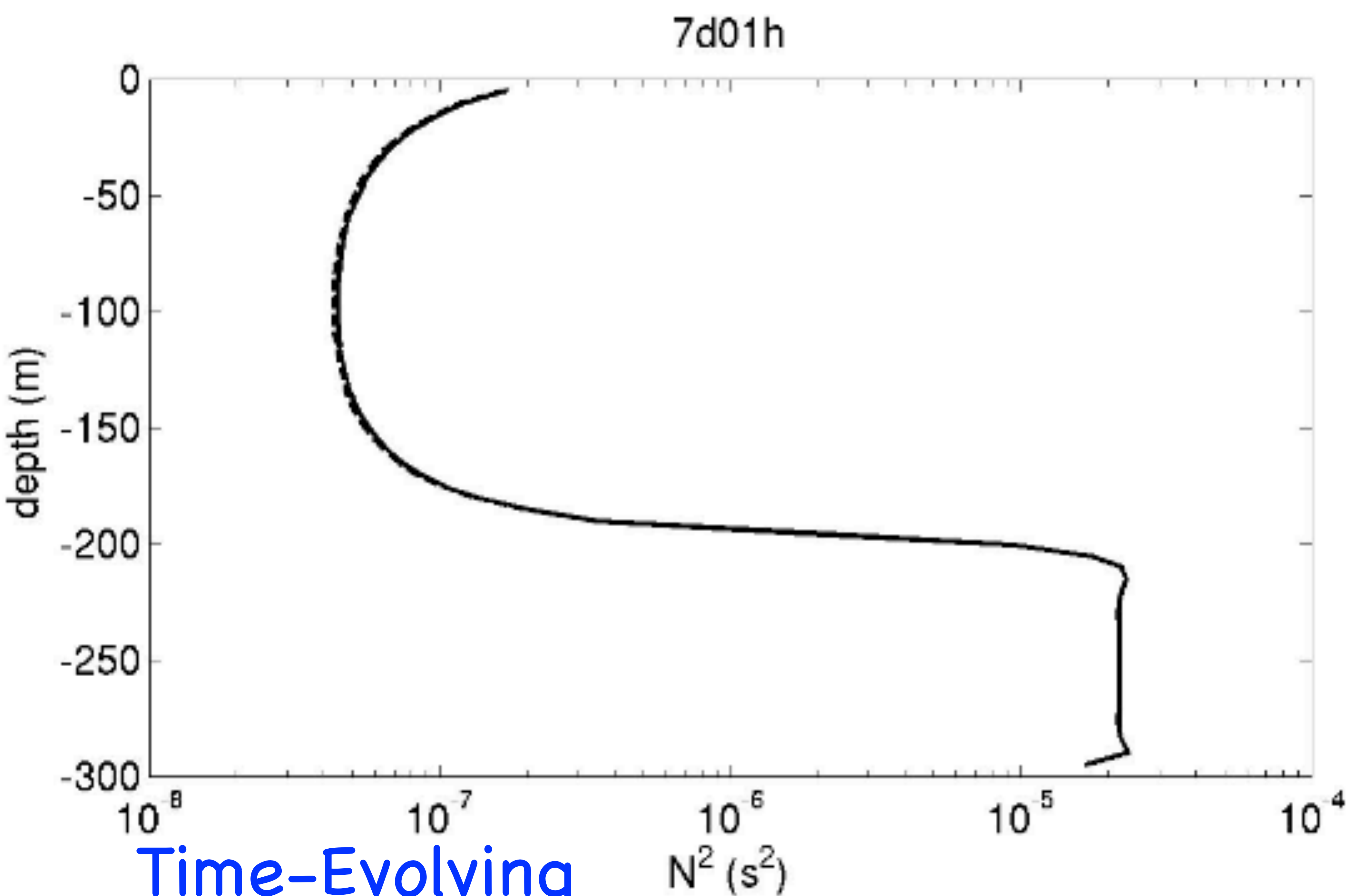
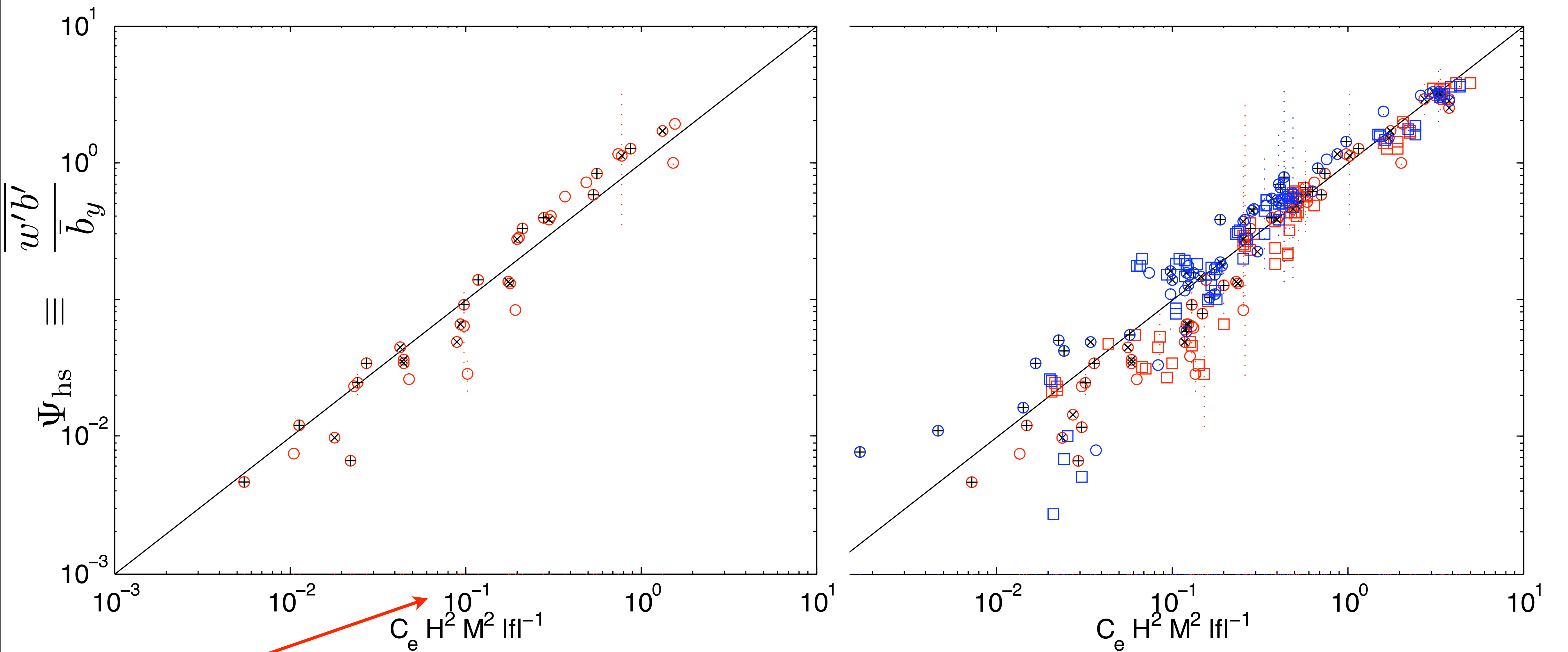


Time-Evolving Stratification (N^2)

It works for Prototype front slumping

Red: No Diurnal

Blue: With Diurnal



>2 orders of magnitude!

Circles: Balanced Initial Cond.

Squares: Unbalanced Initial Cond.

Tracer Flux-Gradient Relationship

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

- Most subgridscale parameterizations have this form: GM*, Redi, FFH** submesoscale
- Relates the eddy flux to the coarse-grain gradients locally
- If we knew the dependence of $\mathbf{M}$ on the coarse-resolution flow, we'd have the optimal local eddy closure

*Gent & McWilliams (1990) **Fox-Kemper, Ferrari, Hallberg (2008)

An example of what a (submeso) subgrid parameterization looks like

with FLOW DEPENDENT $\mathbb{M}$:

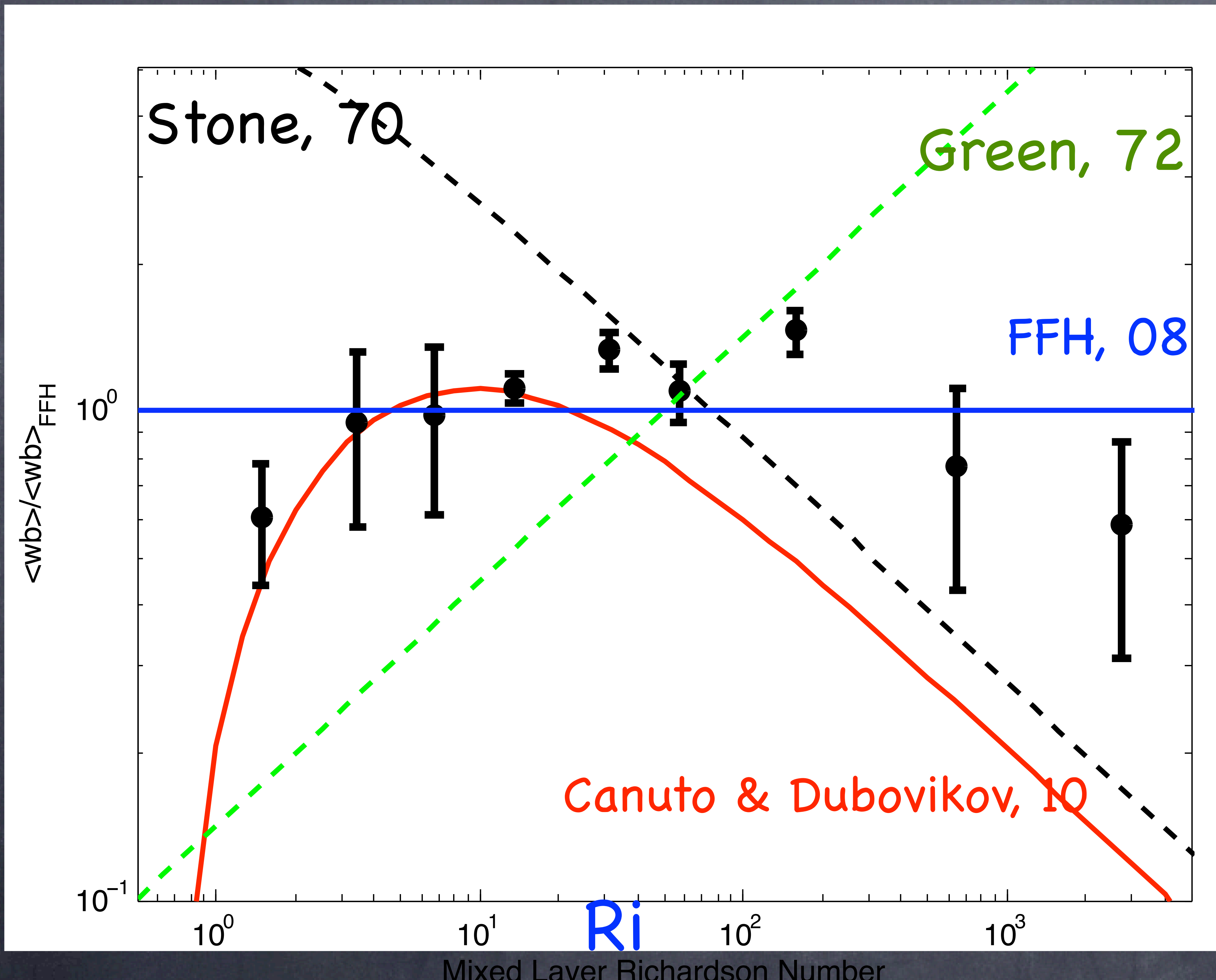
Fox-Kemper, Ferrari, & Hallberg (2008) &
Fox-Kemper, Danabasoglu, Ferrari, & Hallberg (2008)

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\mathbf{\Psi_y} \\ 0 & 0 & \mathbf{\Psi_x} \\ \mathbf{\Psi_y} & -\mathbf{\Psi_x} & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

$$\mathbf{\Psi} = \left[\frac{\Delta x}{L_f}\right] \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla \overline{b} \times \hat{\mathbf{z}}$$

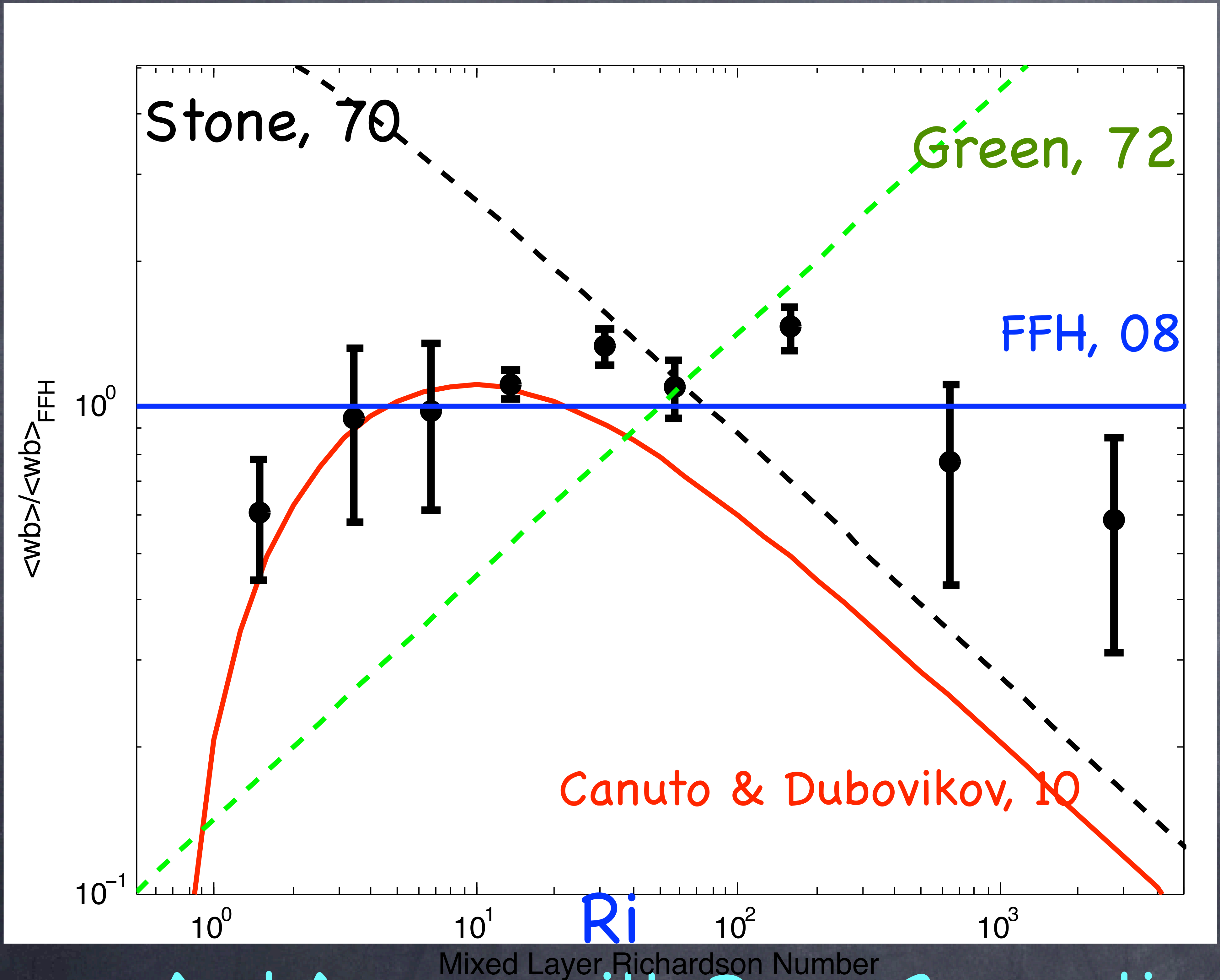
$$\mu(z) = \left[1 - \left(\frac{2z}{H} + 1\right)^2\right] \left[1 + \frac{5}{21}\left(\frac{2z}{H} + 1\right)^2\right]$$

Better than the Competition:



Extends over Ri more mesoscale (9000) than submesoscale (1)

Better than the Competition:



Green equals
Visbeck (97)
Held & Larichev (95)

Extends over
Ri more
mesoscale
(9000)
than
submesoscale
(1)

And Agrees with Deep Convection Studies:
Jones & Marshall (93,97), Haine & Marshall (98)

The Problem is:

The mesoscale equivalent isn't rEady

- Clearly, MLE parameterization is challenged by situations where medium-sized interior PV grads; Big PV grads are equivalent to rigid surfaces and are OK.
- Smith (07) shows Phillips-type (interior PV grads) dominate mesoscale energy extraction
- Also, submesoscale horizontal stirring is negligible, not so for mesoscale where 3d is a must

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$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

F-G General Form

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = -\begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

- 3 equations... 9 degrees of freedom
- Choices!

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

Transformed Eulerian Mean Form

$$\begin{bmatrix} \overline{u'\rho'} \\ \overline{v'\rho'} \\ \overline{w'\rho'} \end{bmatrix} = -\begin{bmatrix} 0 & 0 & -\overline{u'\rho'}/\overline{\rho}_z \\ 0 & 0 & -\overline{v'\rho'}/\overline{\rho}_z \\ 0 & 0 & -\overline{w'\rho'}/\overline{\rho}_z \end{bmatrix} \begin{bmatrix} \overline{\rho}_x \\ \overline{\rho}_y \\ \overline{\rho}_z \end{bmatrix}$$

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- 3 equations... 9 degrees of freedom
- Choices!
- Excitement in TEM community, as 1st contribution is advective and 2nd contribution vanishes for steady, adiabatic flow! Thus it's 'purely diabatic'

Diffusive vs. Advective:

- Diffusion, to me, must be a symmetric tensor, (real) eigenvalues=rate, eigenvectors=direction
- Skew fluxes can be written with a non-divergent velocity in F-G form is an antisymmetric tensor

$$-\ \frac{\mathbf{M}-\mathbf{M}^\dagger}{2}\cdot\nabla\tau=\mathbf{\Psi}\times\nabla\tau$$

- A generic tensor can represent both diffusion and skew fluxes (i.e., arbitrary for nonzero gradient).

Diabatic vs. Adiabatic (and Steady)

- In wave-mean interactions, Andrews & McIntyre (78, Generalized Lagrangian Mean) show that steady, adiabatic flow results in the same form of the equations (i.e., no diabatic or diffusion terms), but with advection replaced with advection by the Lagrangian mean velocity.
- Thus, since TEM has advection only in adiabatic conditions, nonacceleration theorems, etc. pass on
- However, the GLM velocity may be divergent...

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

Transformed Eulerian Mean Form

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Veronis Effect Form

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} -\overline{u'\tau'}/\overline{\tau}_x & 0 & 0 \\ 0 & -\overline{v'\tau'}/\overline{\tau}_y & 0 \\ 0 & 0 & -\overline{w'\tau'}/\overline{\tau}_z \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

- 3 equations... 9 degrees of freedom
- Choices!
- What about for a different tracer? fails.
- In principle, same is true for TEM...fails for different tracer.

A bit of stochastics...

- Dukowicz & Smith (97) and Smith (99) lay out the form of a stochastic, adiabatic relocation of particles
- The resulting Fokker-Planck Equation for the probablility density of the particles gives a K and a nondivergent v, which are closely related to the Lagrangian mean transport and the diffusion of probability.

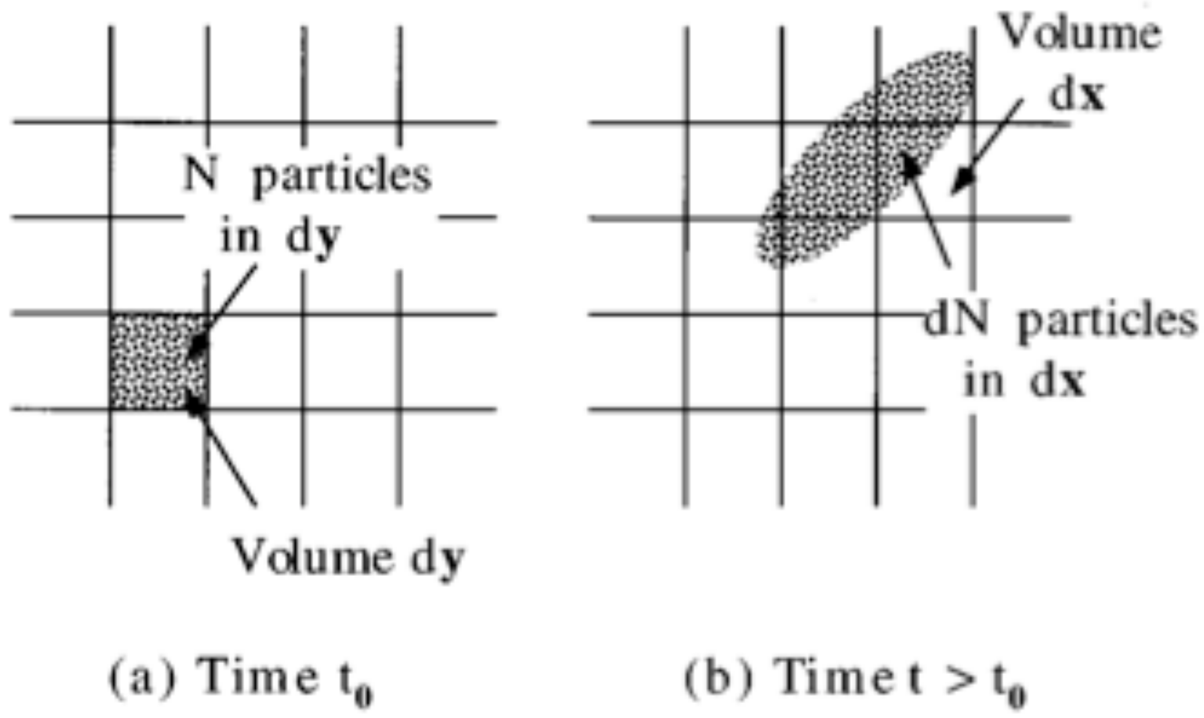


FIG. 1. A schematic illustration of the turbulent transport of a cloud of particles originating in $d\mathbf{y}$ at time t_0, and the fraction dN/N which arrives in $d\mathbf{x}$ at a later time t.

$$\partial_t p(\mathbf{x}, t \mid \mathbf{y}, t_o) + \nabla \cdot \mathbf{U} p(\mathbf{x}, t \mid \mathbf{y}, t_o) = \nabla \cdot \mathbf{K} \cdot \nabla p(\mathbf{x}, t \mid \mathbf{y}, t_o) \quad (50)$$

$$\mathbf{U} = \mathbf{v} - \nabla \cdot \mathbf{K} \quad (51)$$

$$\mathbf{v}(\mathbf{x}, t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int d\mathbf{x}' (\mathbf{x}' - \mathbf{x}) p(\mathbf{x}', t + \Delta t \mid \mathbf{x}, t) \quad (52)$$

$$\mathbf{K}(\mathbf{x}, t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int d\mathbf{x}' \frac{1}{2} (\mathbf{x}' - \mathbf{x})(\mathbf{x}' - \mathbf{x}) p(\mathbf{x}', t + \Delta t \mid \mathbf{x}, t). \quad (53)$$

Since the pdf is positive, it is clear from (53) that **K** is a 2×2 symmetric positive-definite tensor. We will refer to **v** as the Lagrangian mean velocity, although this identification is not exact (see Bennett 1996, p. 7). In (52)

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

Sym Part=Anisotropic* Redi

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = -\begin{bmatrix} K_{xx} & K_{xy} & \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ K_{yx} & K_{yy} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \tilde{\nabla}z\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Yellow $\mathbf{K}$ ‘are‘ horizontal stirring & mixing

Blue factors in Redi (82), Solomon (71) are symmetric and scaled to make eddy mixing along neutral surfaces--No Veronis

*Anistropic form due to Smith & Gent (04)

Monday, June 13, 2011

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

AntiSym Part=Anisotropic* GM

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = -\begin{bmatrix} 0 & 0 & -\hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ 0 & 0 & -\hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Antisymmetric Elements in GM (1990)
are scaled to overturn fronts, make vertical fluxes
extract PE, and restratify the fluid
equivalent to eddy-induced advection--no Veronis
Q: Same horiz. mixing ($\mathbf{K}$) as Redi?

*Anistropic form due to Smith & Gent 04 *Tensor Form (Griffies, 98)

Monday, June 13, 2011

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

Sym Part=Anisotropic* Redi

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ K_{yx} & K_{yy} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \tilde{\nabla}z\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

AntiSym Part=Anisotropic* GM

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ 0 & 0 & -\hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Yellow $\mathbf{K}$ 'are' horizontal stirring & mixing

Need a Natural, Mesoscale Eddy Environment to Test Out:

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

3 equations/tracer

9 unknowns ($\mathbf{M}$ components)

BY USING 3 or MORE TRACERS, can determine $\mathbf{M}$!!!

(a la Plumb & Mahlman '87, Bratseth '98)

No assumptions about symmetry required.

Need a Natural, Mesoscale Eddy Environment to Test Out:

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

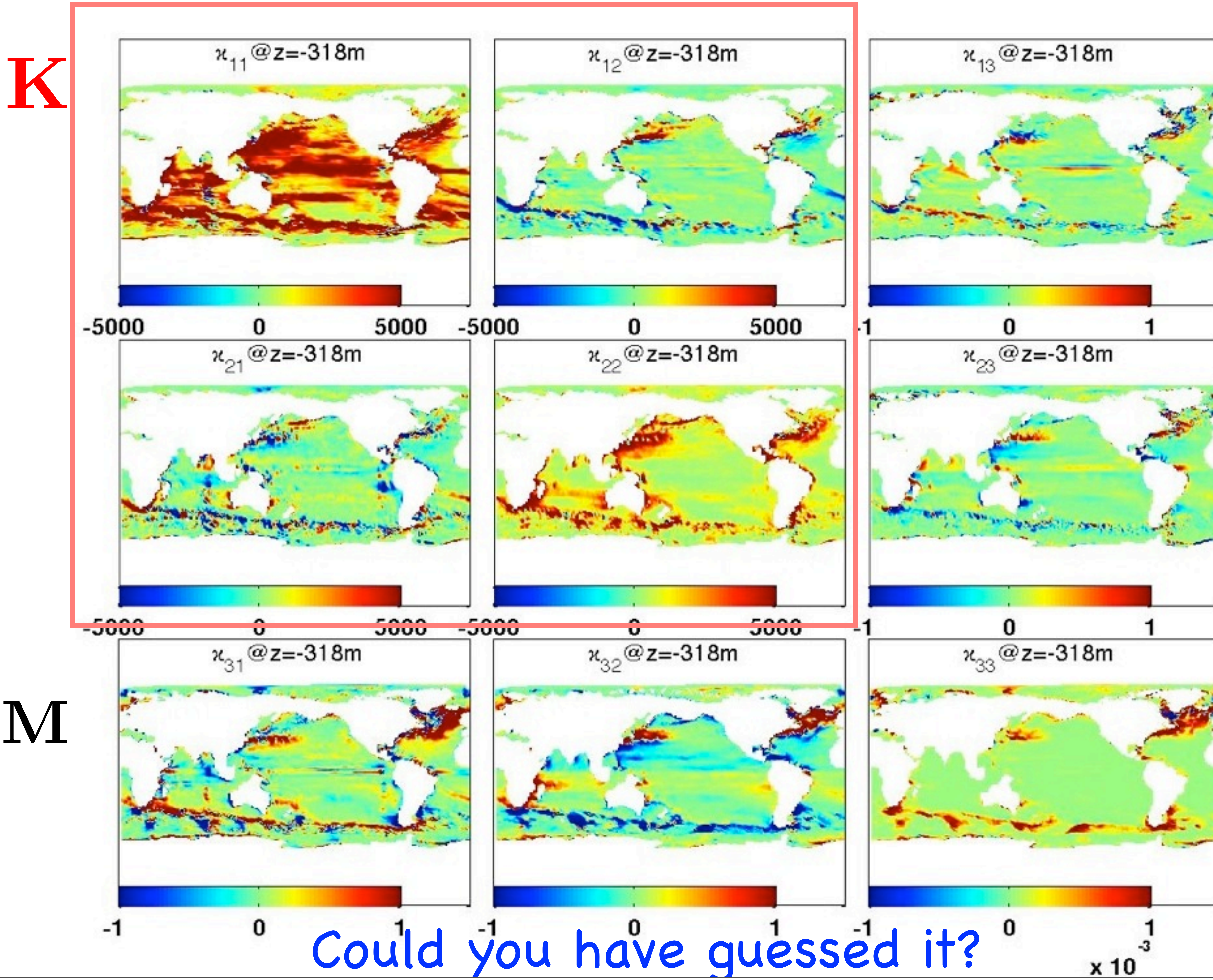
$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = -\begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

With John Dennis & Frank Bryan, we took a POP0.1° Normal-Year forced model (yrs 16-20 for anal.)

Added 9 Passive tracers--restored to x,y,z @ 3 rates

Kept all the eddy fluxes for passive & active tracers

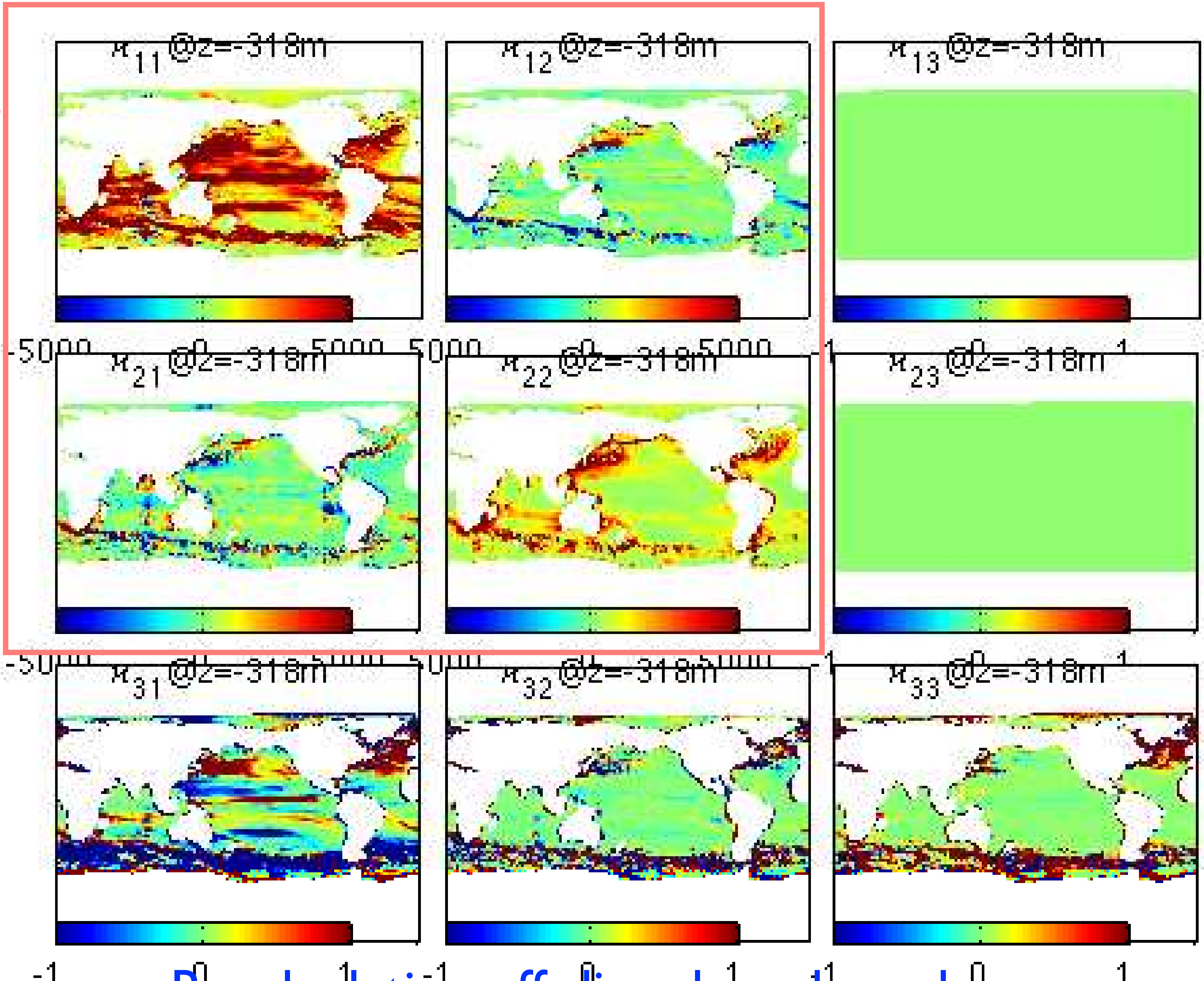
Coarse-grained to 2°, transient eddies, tracers to $\mathbf{M}$



Could you have guessed it?

K

M

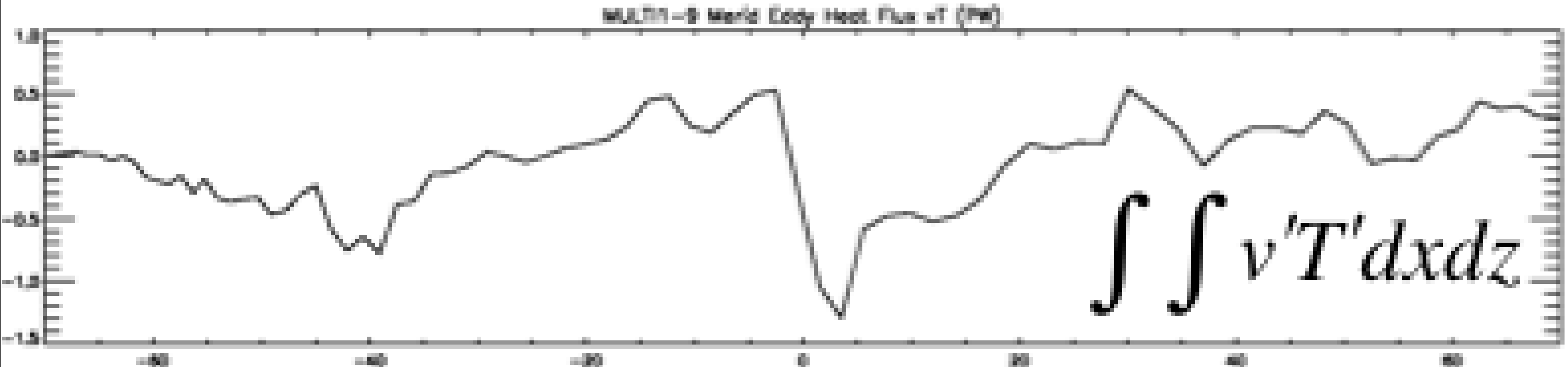


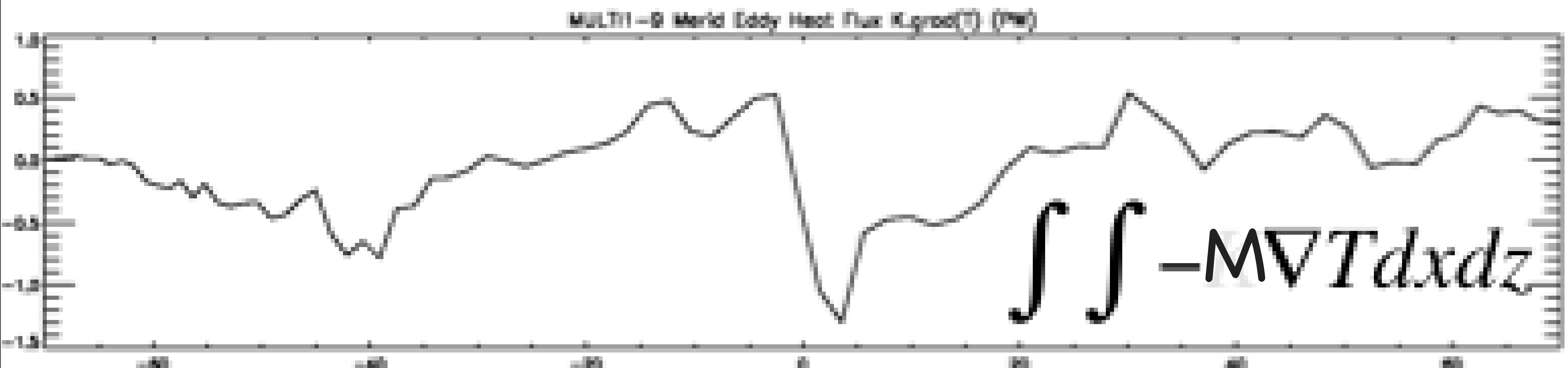
Recalculating off-diags based on slope

Use a Natural, Mesoscale Eddy Environment to Test Out:

Testing the Diagnosis:

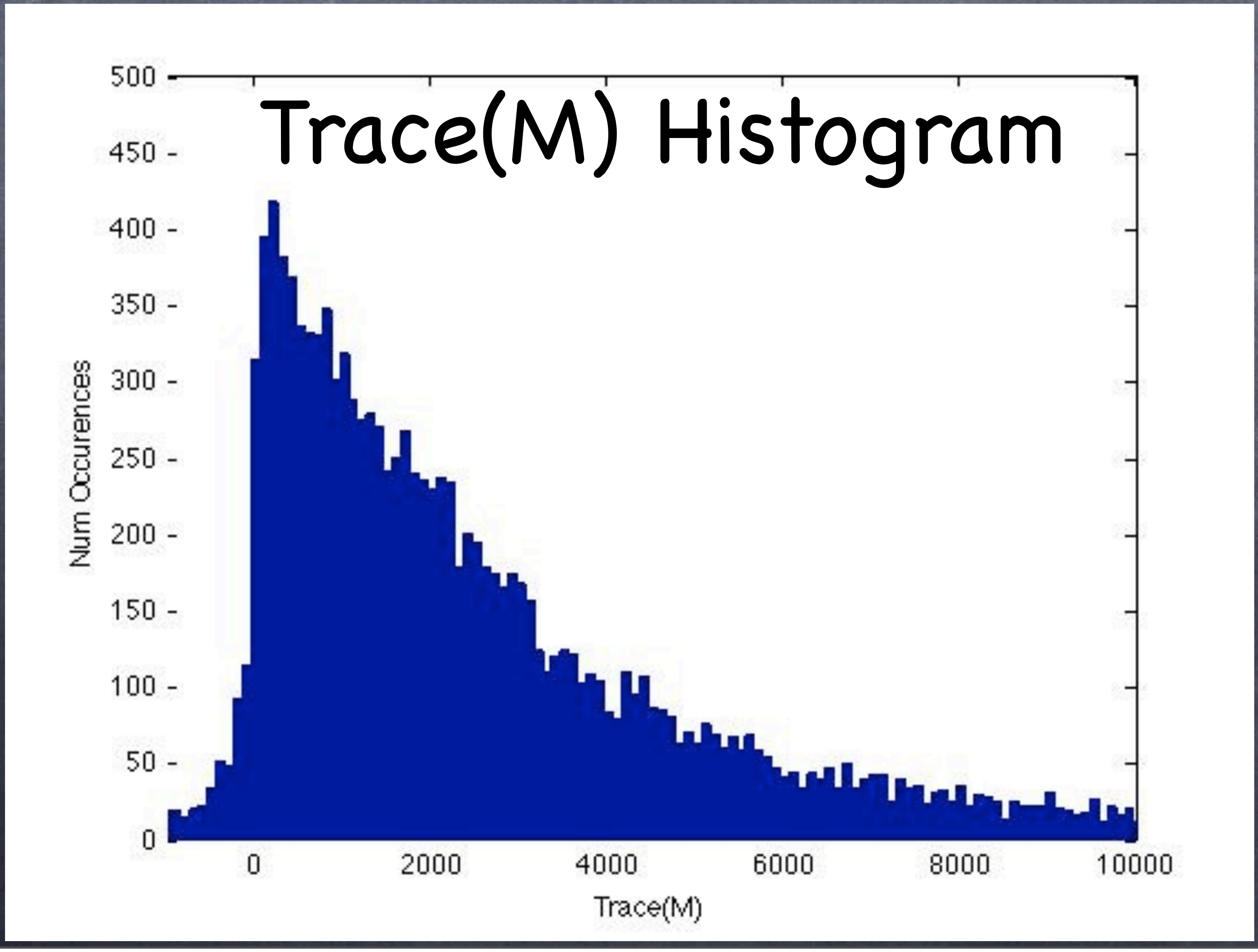
Note: T not used for diagnosis, active tracers are apparently transported as passive are!





Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ K_{yx} & K_{yy} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \tilde{\nabla}z\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$



Hor. Diffusivity is roughly Trace(M)/2

Peak of Diffusivity near 250 m^2/s

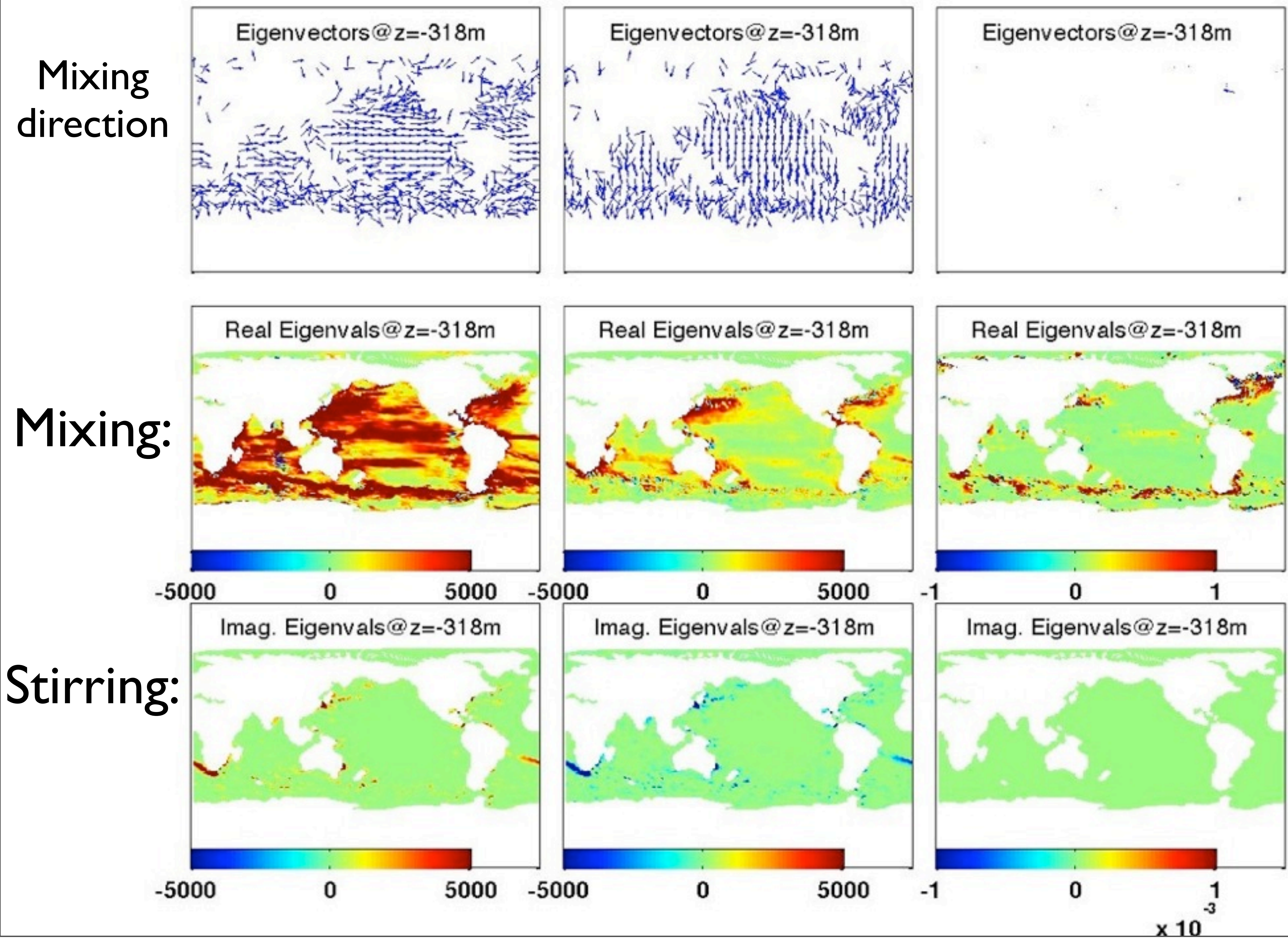
Median Diffusivity near 1000m^2/s

<6% negative

Interpretation?

- Isoneutral diffusion or 'mixing': symmetric $\mathbf{K}$ with real, positive eigenvalues (neg->nonlocal)
- The eigenvalues of $\mathbf{M}$ are related, except there is one more involving the neutral to z coordinate conversion (in S&G theory, at least)
- The eigenvectors give the direction of the mixing associated with each eigenvalue
- Antisymmetric $\mathbf{K}$ & $\mathbf{M}$ are stirring/ overturning by an eddy-induced (quasi-stokes) streamfunction--non-orthogonal eigenvects and imaginary eigenvalues possible!

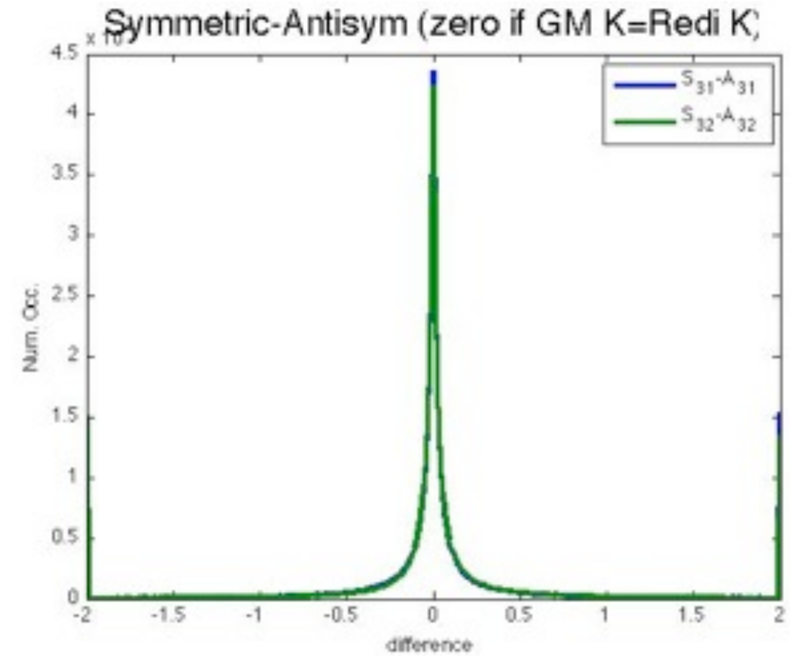
Result: Strong Anisotropy Along/Across Isopycnals

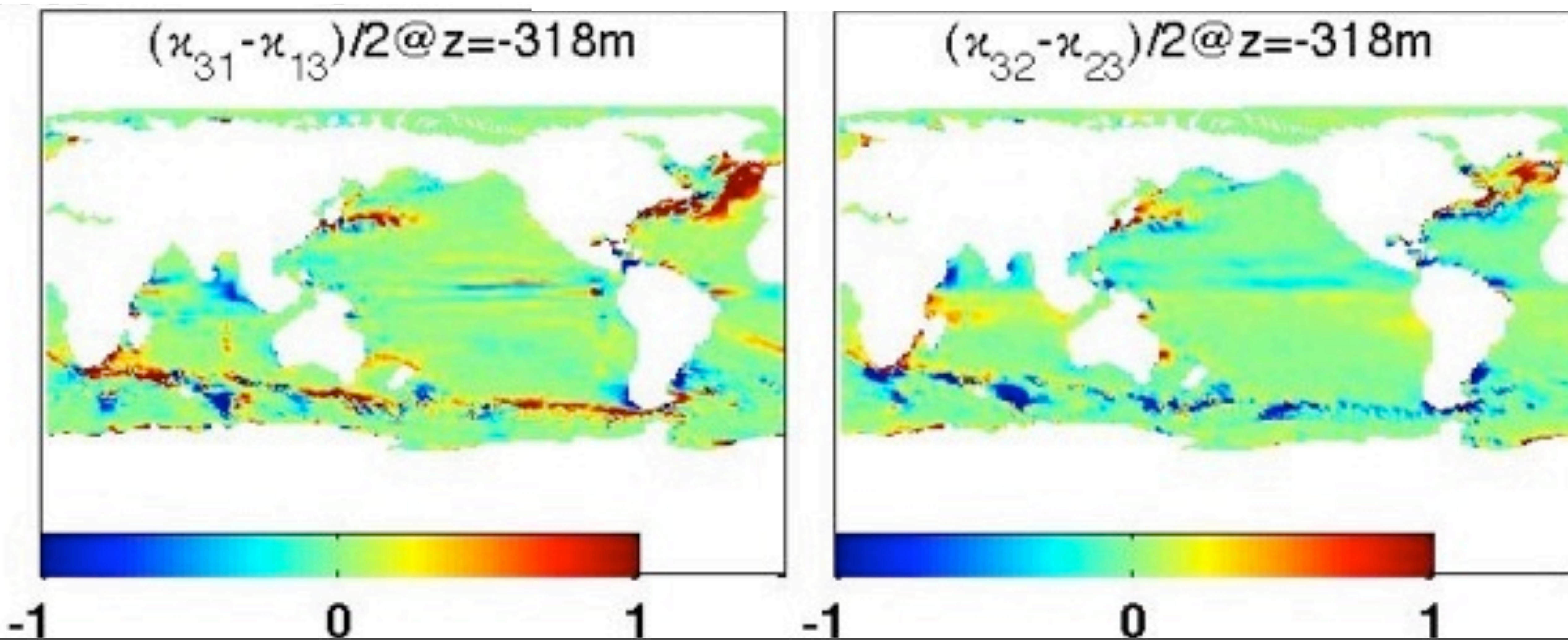


Result:

Redi K=GM K(mostly)

If so these 2 components should match in Sym & Antisym M





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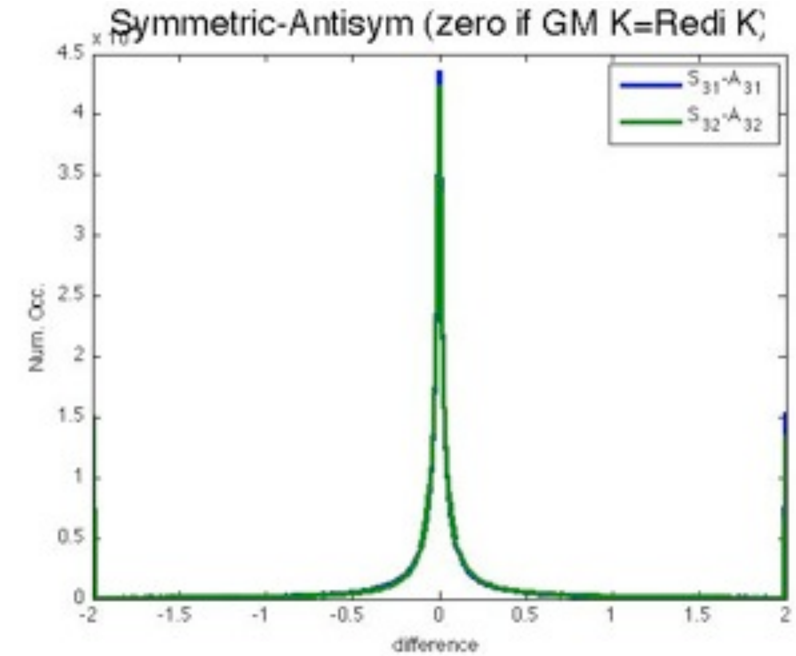
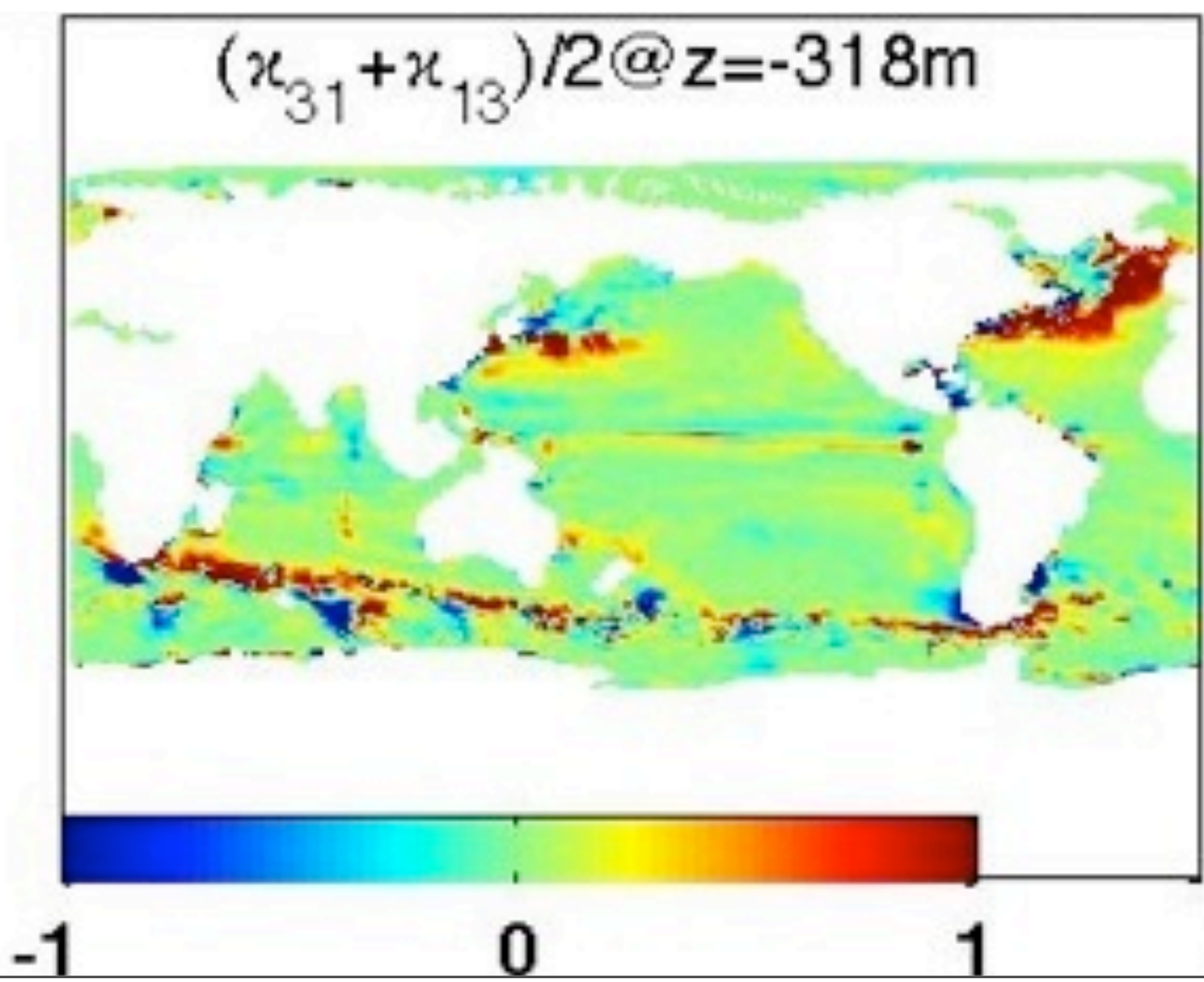
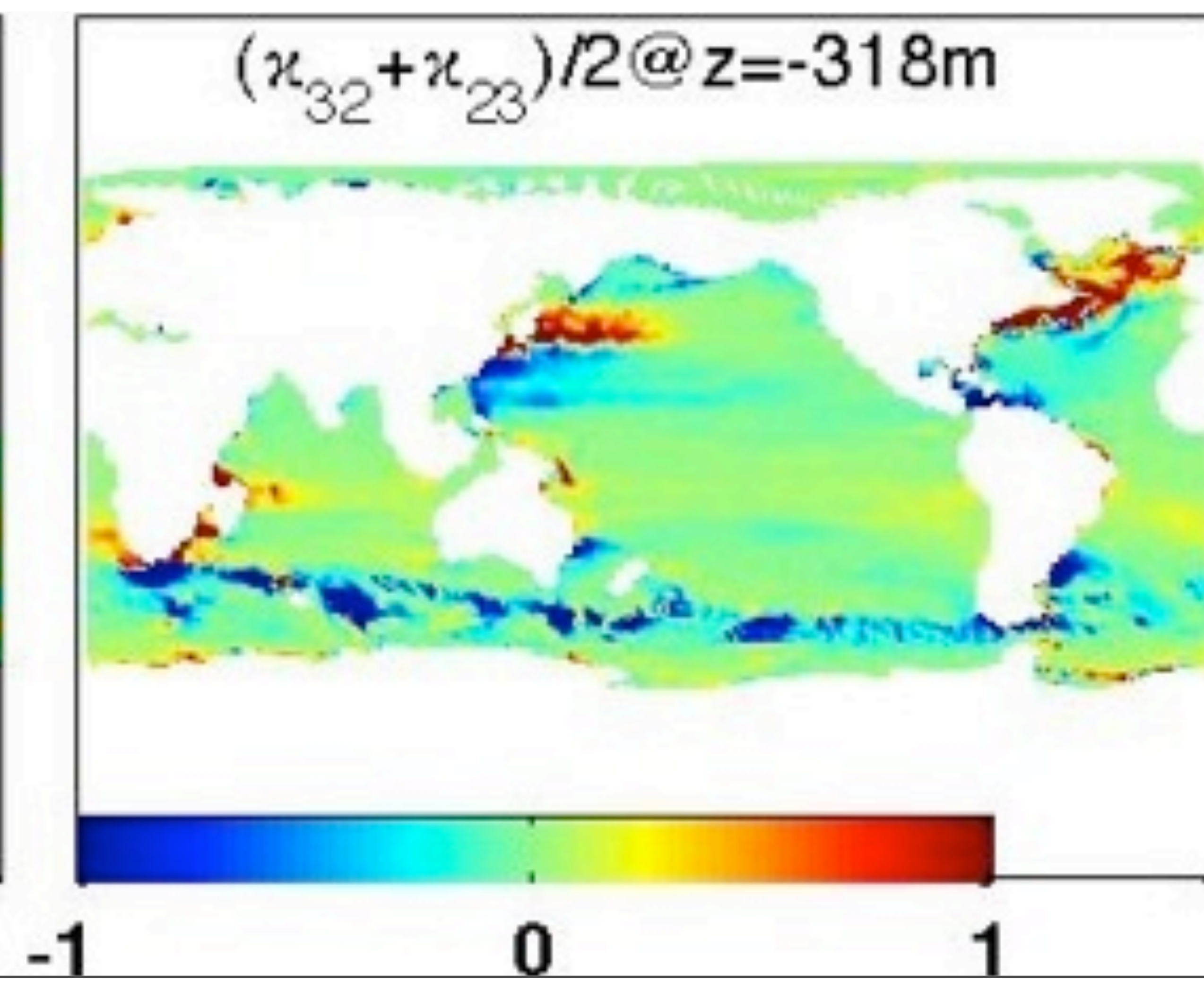




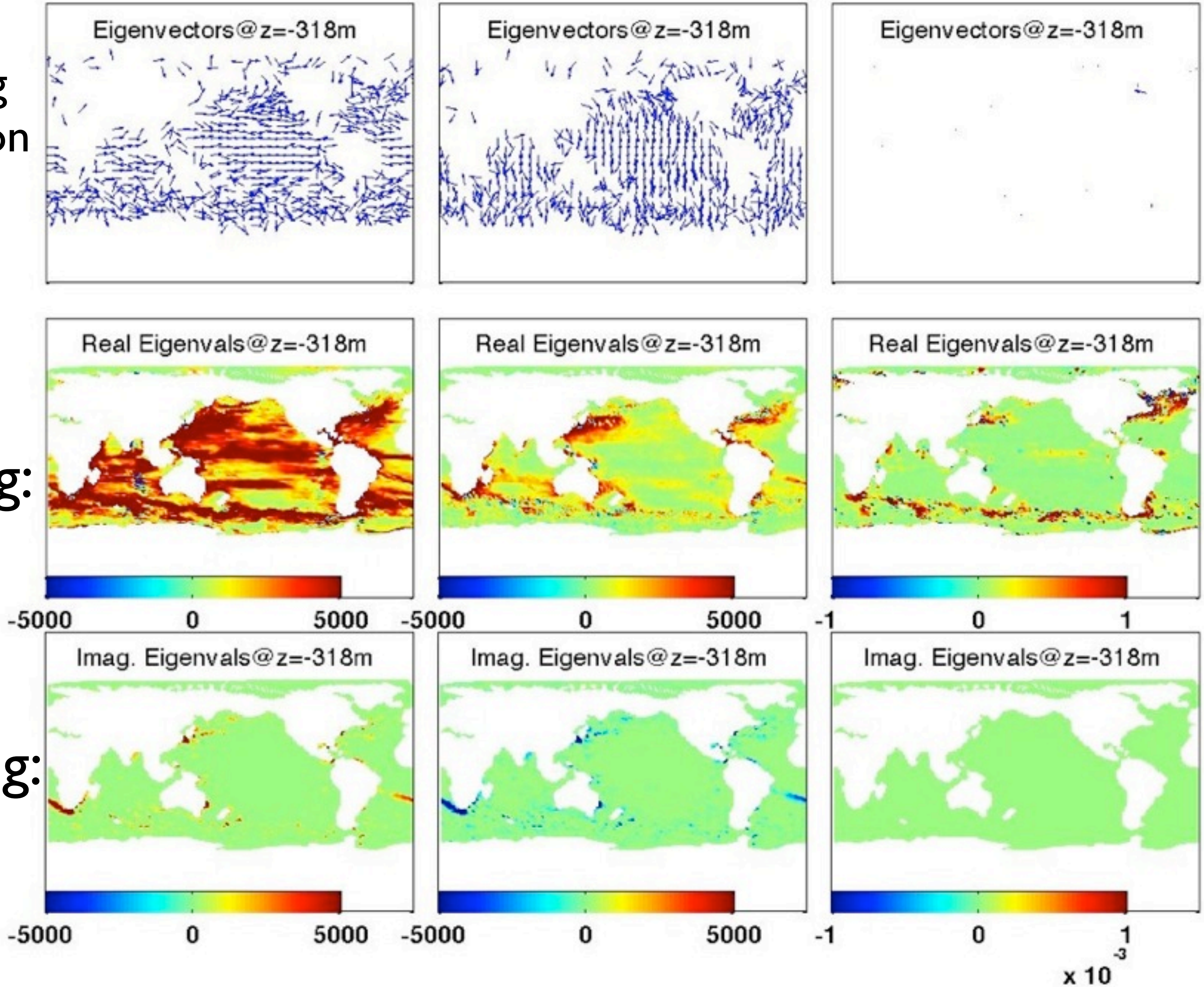


Result: Strong Anisotropy Along/Across PV Grads.

Mixing direction

Mixing:

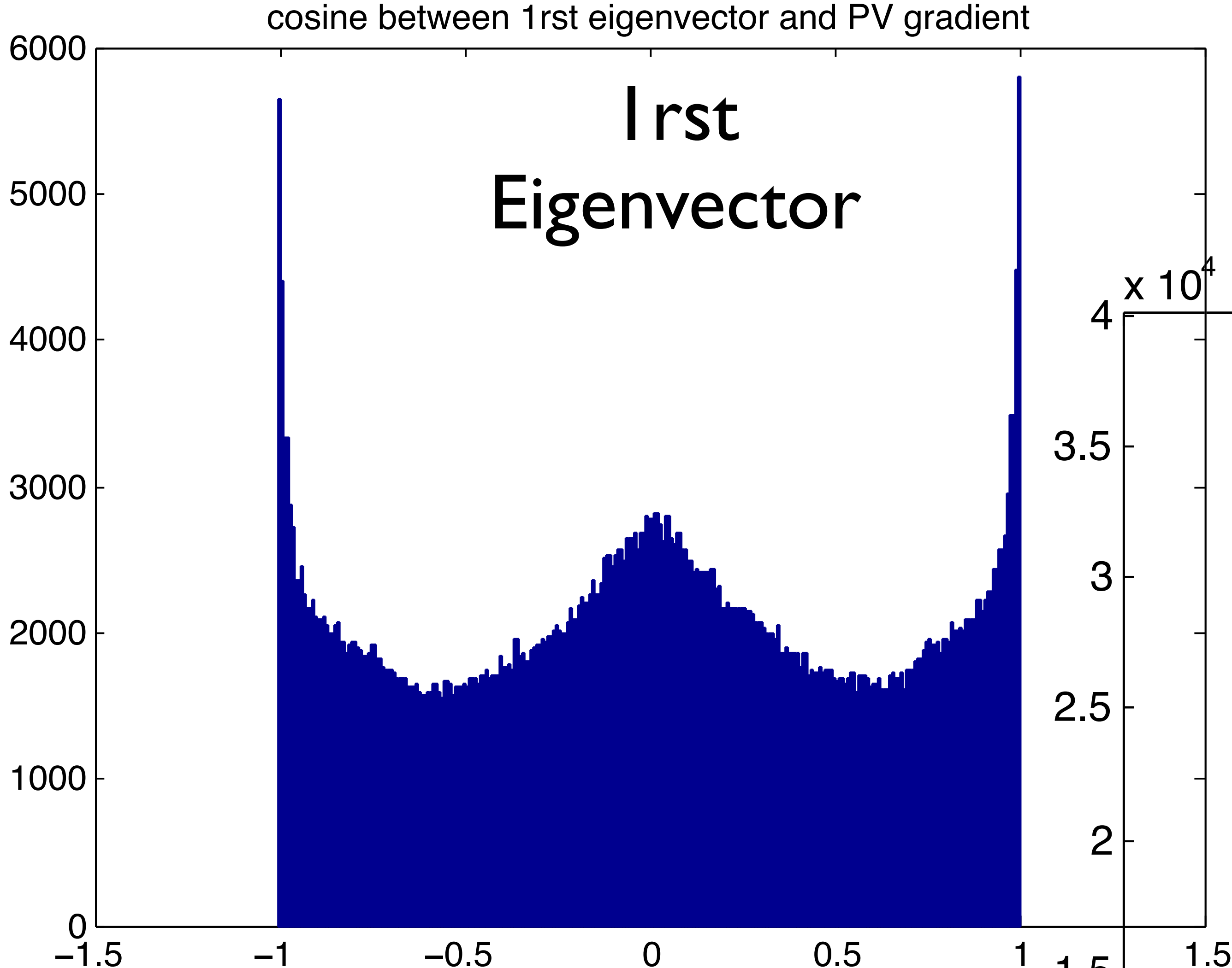
Stirring:



Result: Strong Anisotropy Along/Across PV Grads.

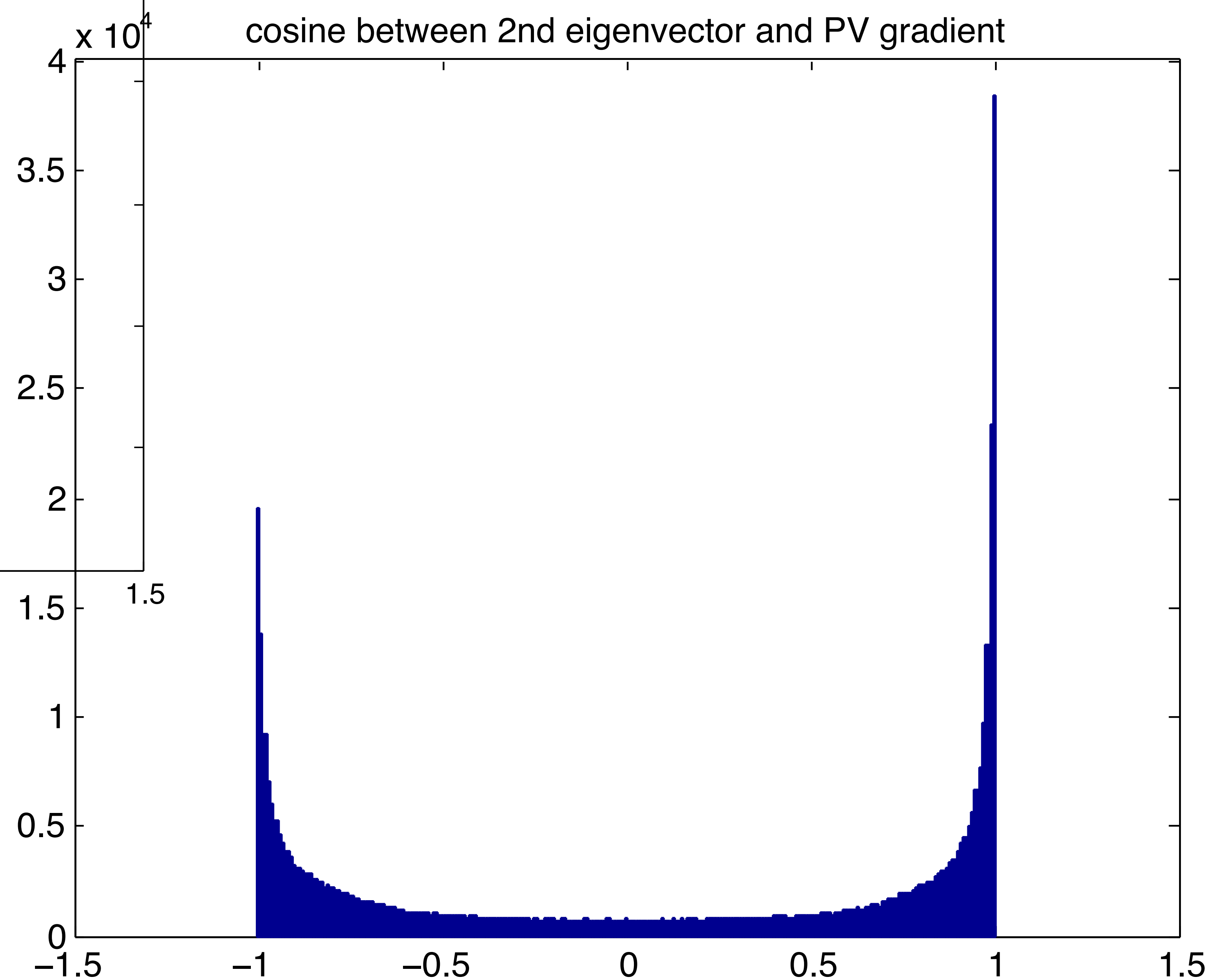
Mixing direction

Either along PV contours or across



2nd Eigenvector

Across PV contours



Comparisons with Marshall et al.

Realistic negative eigs. vs. spurious?

Ferreira, Marshall, Heimbach 05

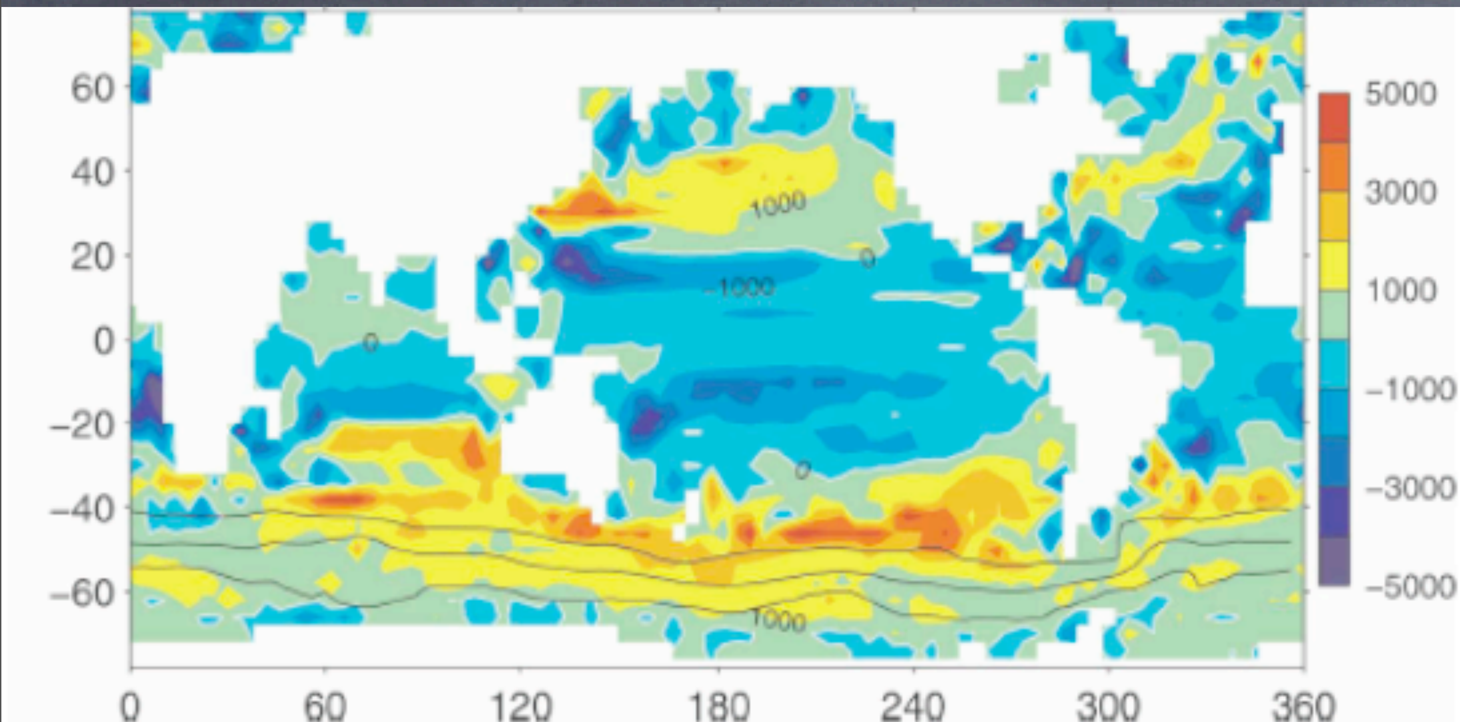
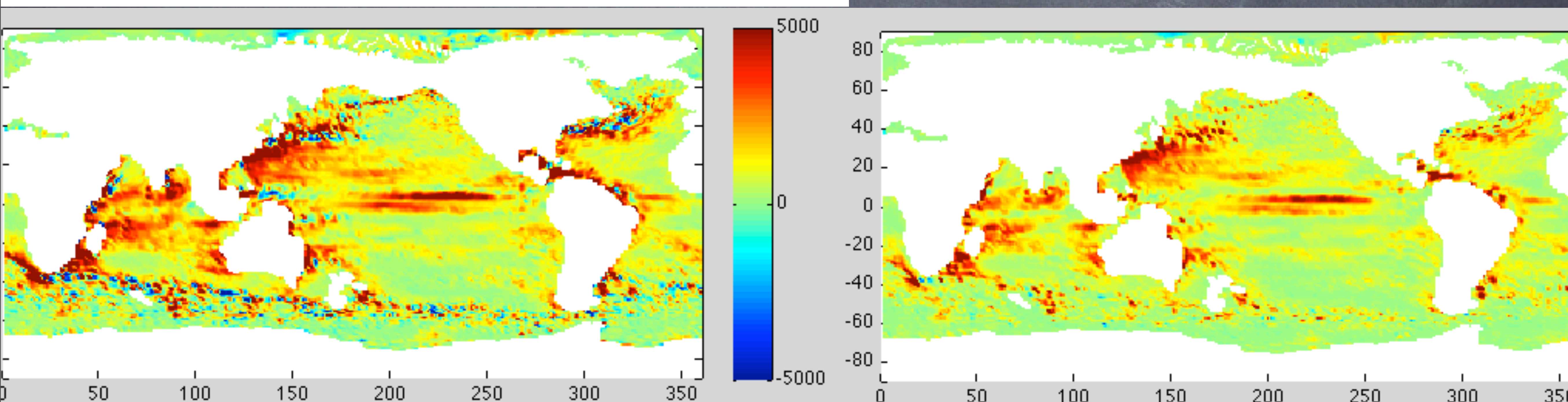


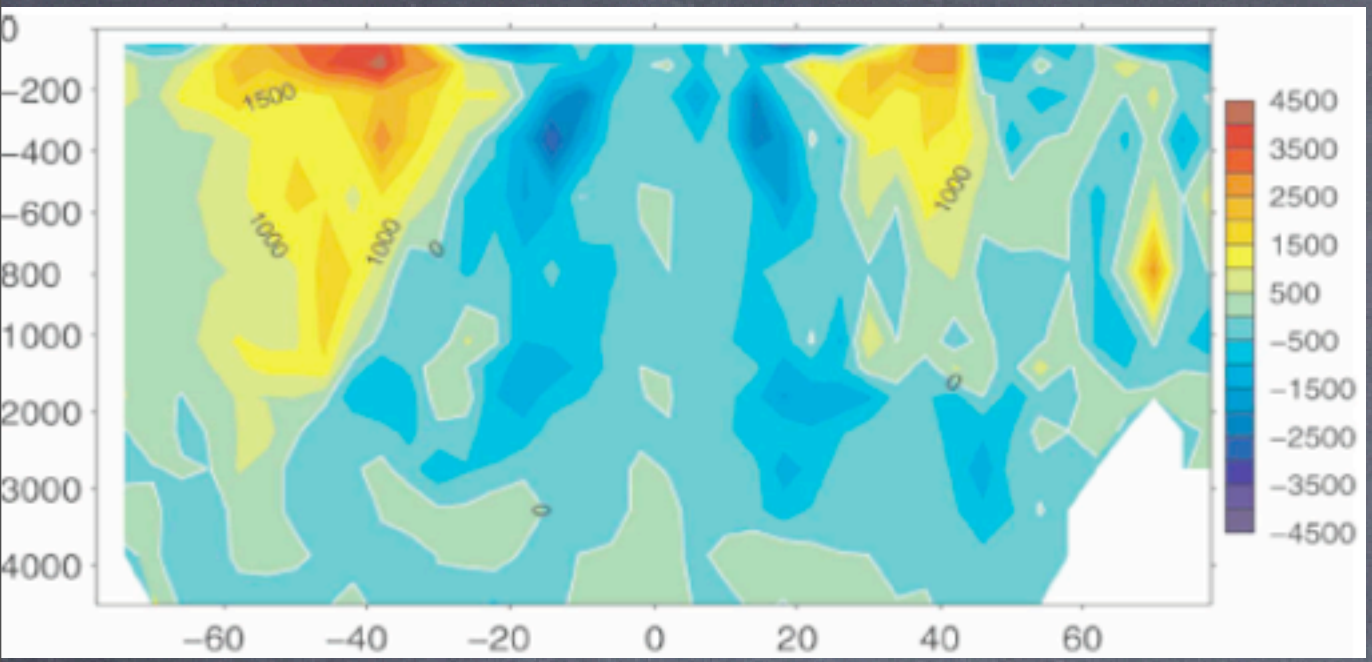
FIG. 12. Inferred horizontal eddy diffusivity κ ($m^2 s^{-1}$): (top) zonal mean and (bottom) vertical mean over the thermocline (0–1200 m). The contour intervals are (top) 500 and (bottom) 1000 $m^2 s^{-1}$. The thick line indicates the zero contour. Also indicated in the bottom panel are the 10-, 70-, and 130-Sv contours of the barotropic streamfunction.



Re(2nd eigenvalue)

(2nd eigenvalue of symmetric M)

Comparisons with Marshall et al.



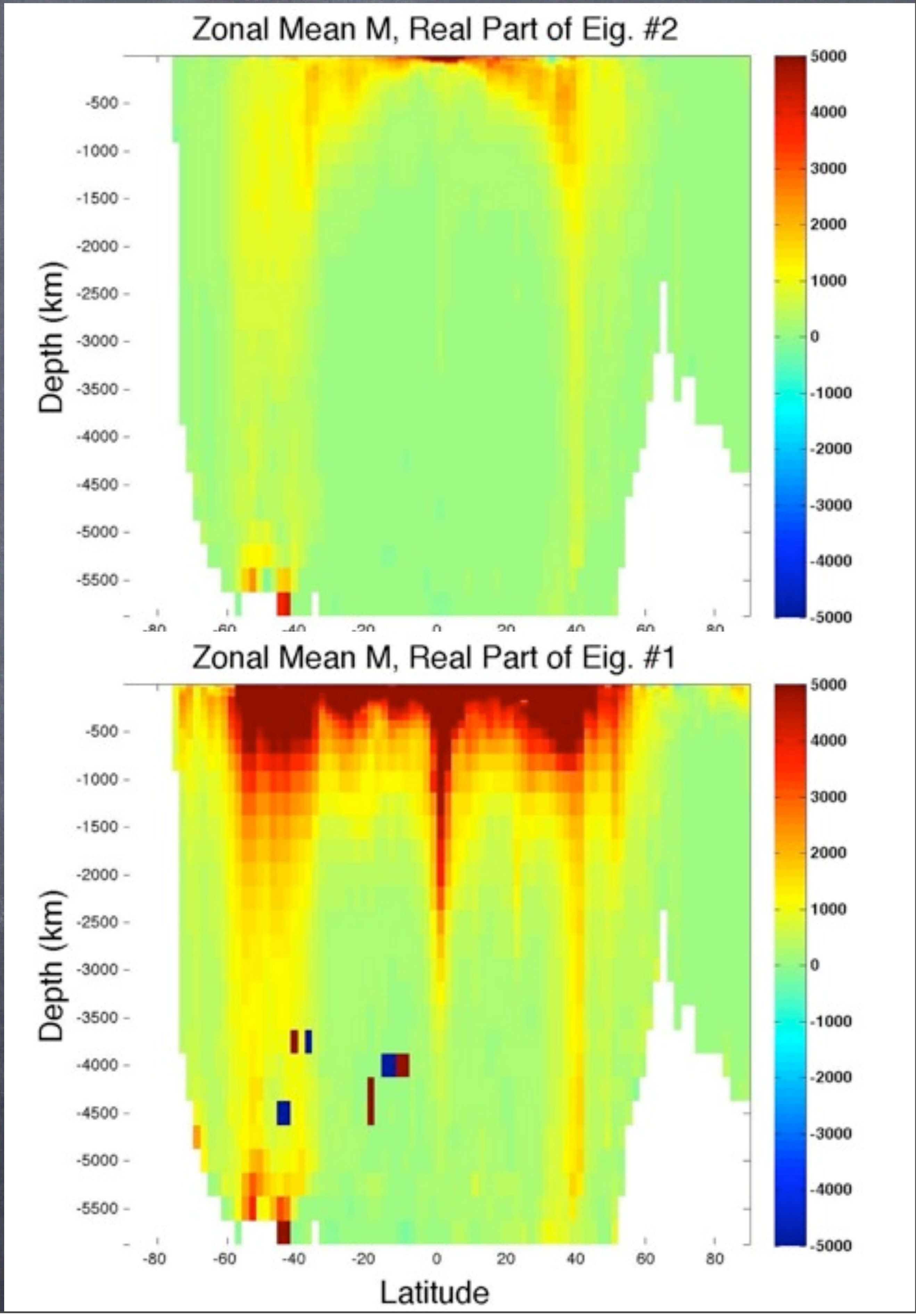
Ferreira, Marshall, Heimbach 05

Zonal mean (scalar) diffusivity

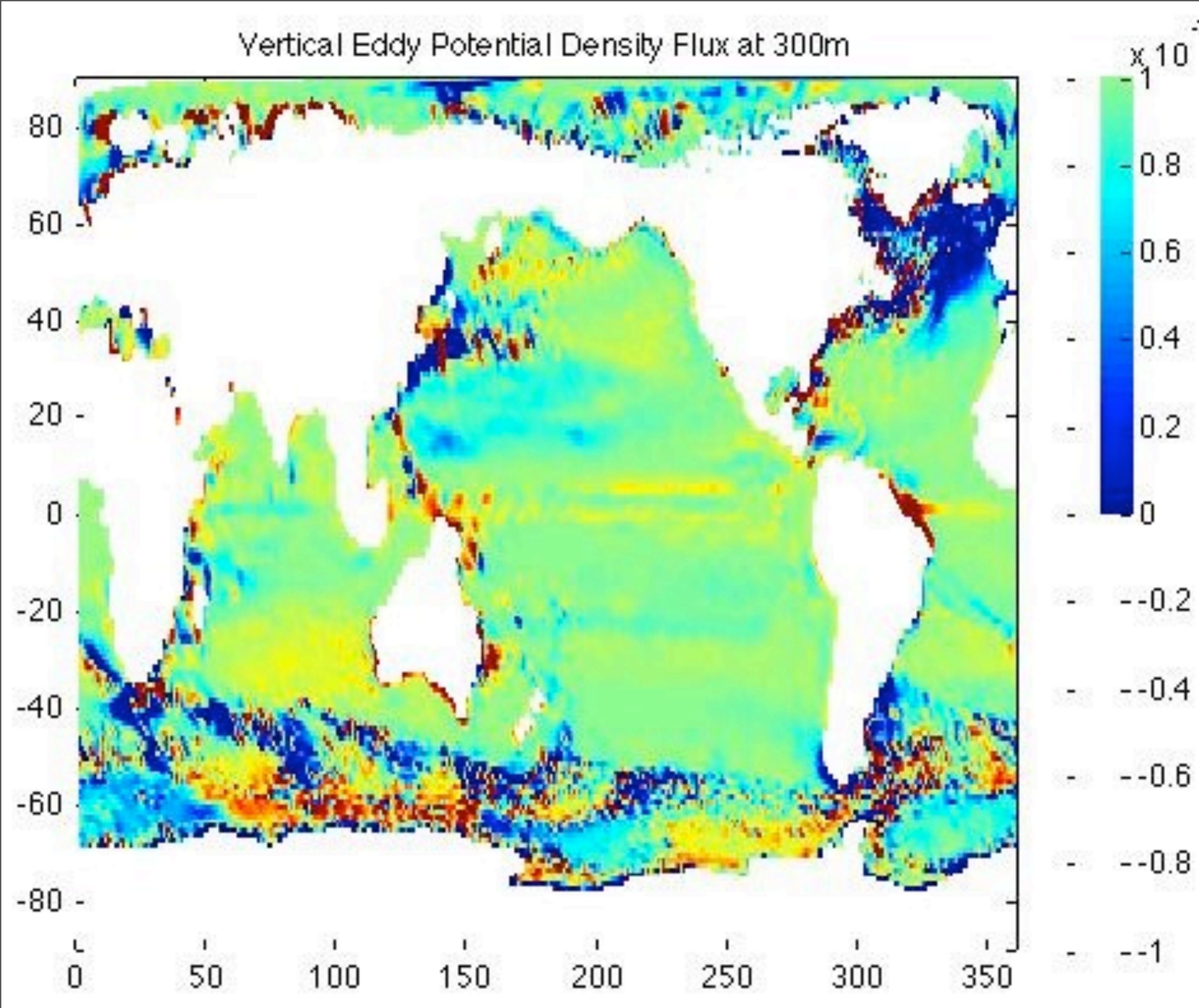
vs.

Eigenvalues of $\mathbf{M}$

Same shape--few negatives!

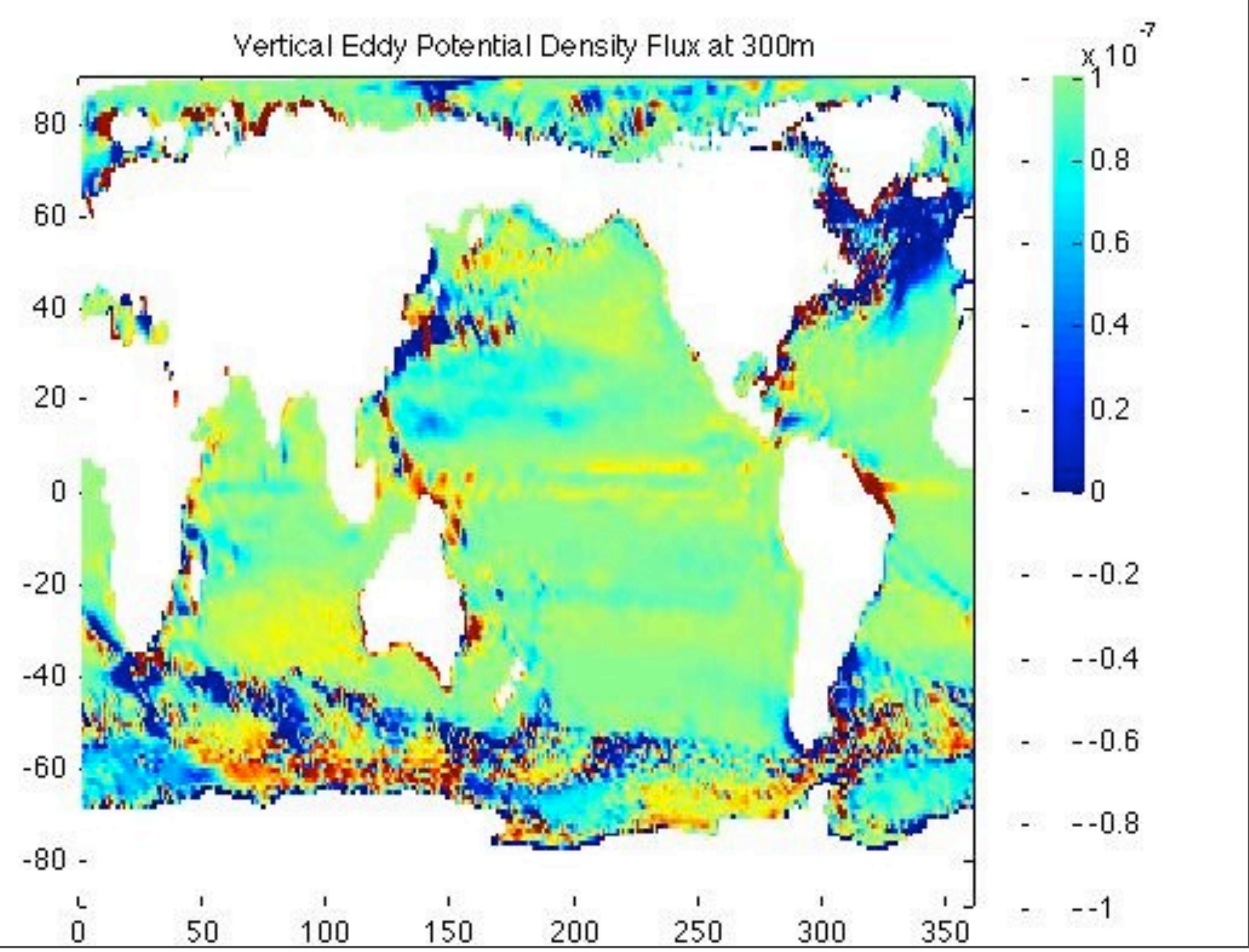


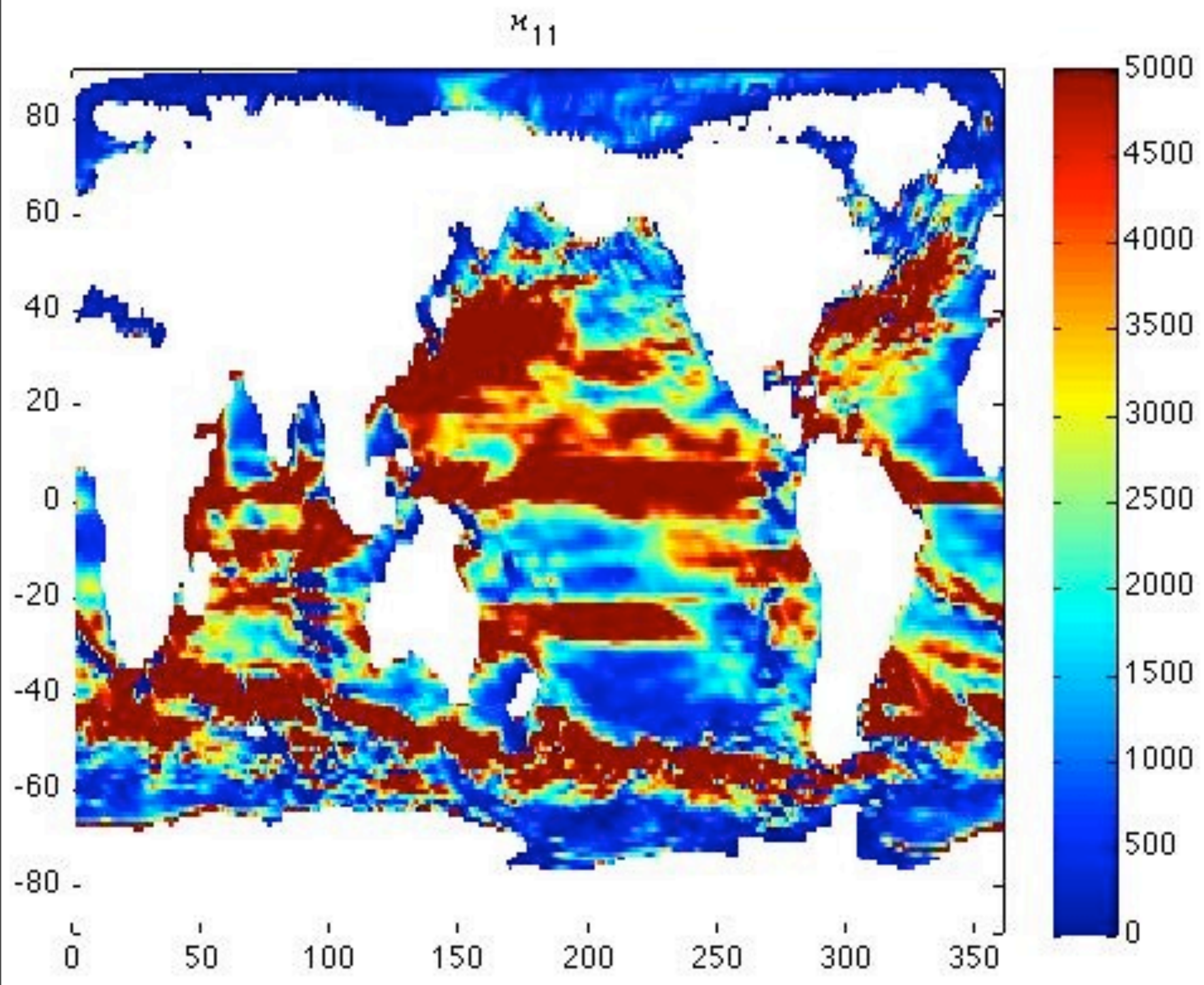
How do we explain the Horizontal Variations of K?



Compare to vertical eddy density flux (PE Extraction)

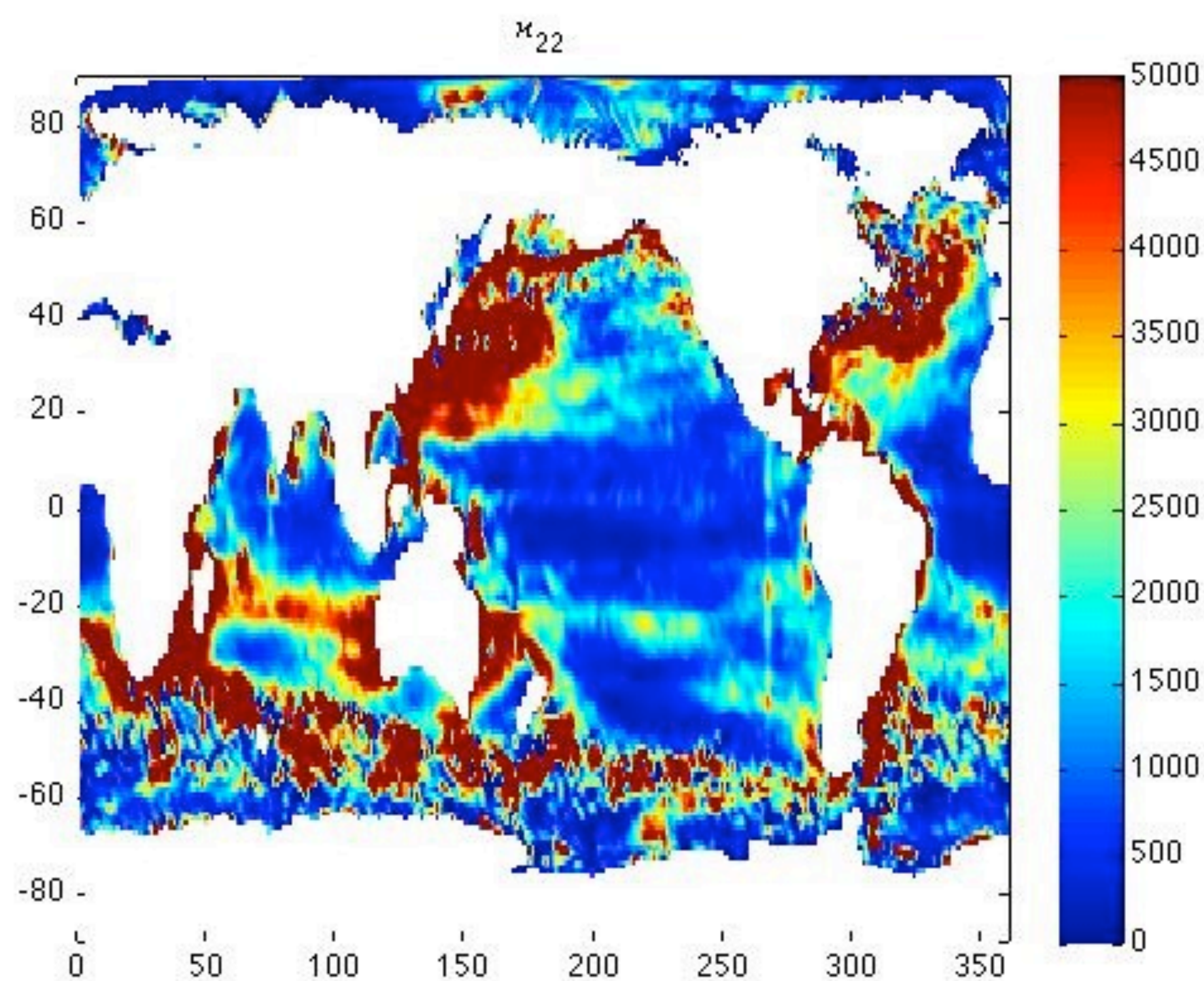
- Eden&Greatbatch (+others) propose that baroclinic instability's production of EKE from PE should guide M magnitude



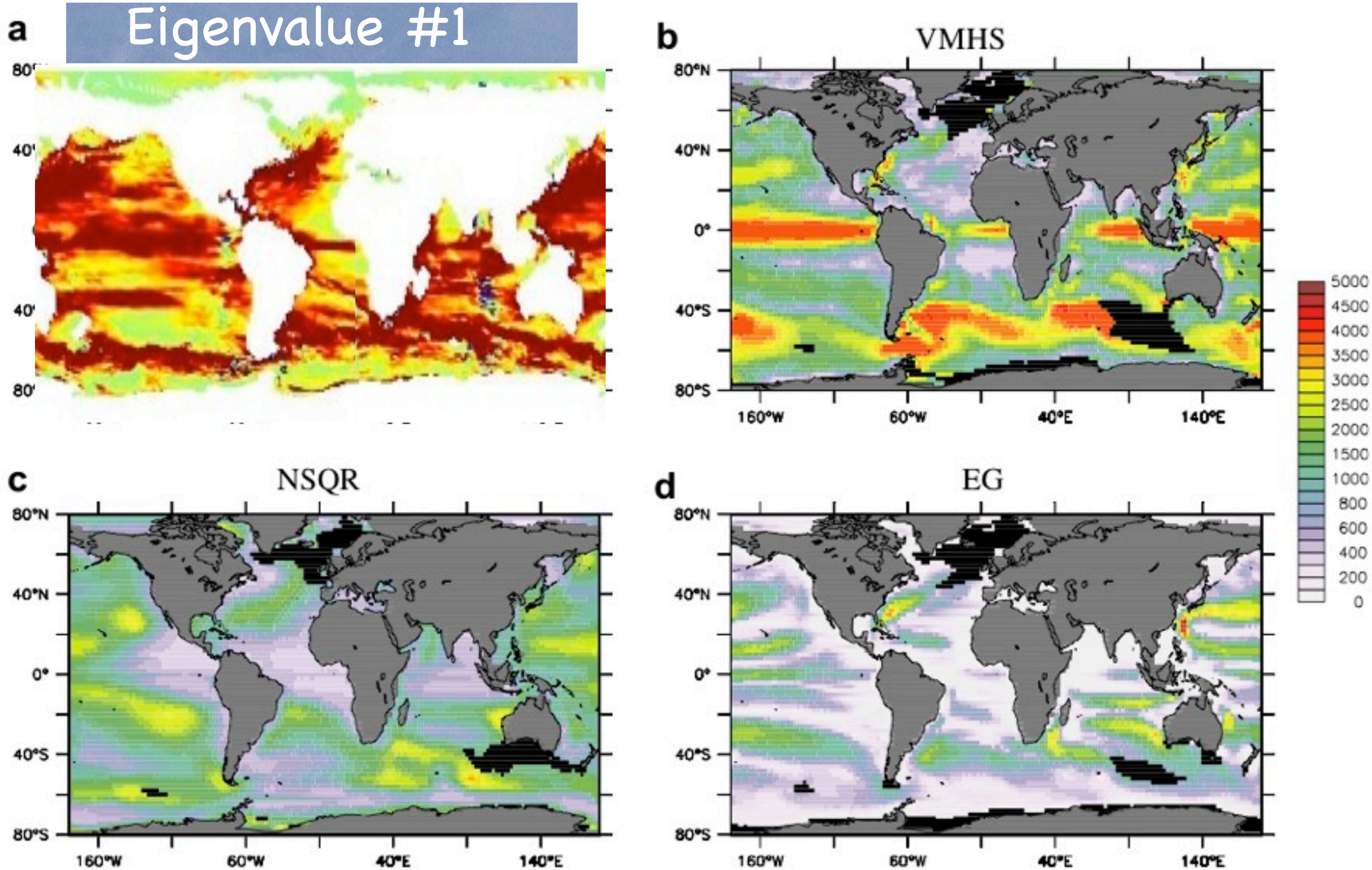


Locations of PE extraction are

Locations of large eigs of **K**

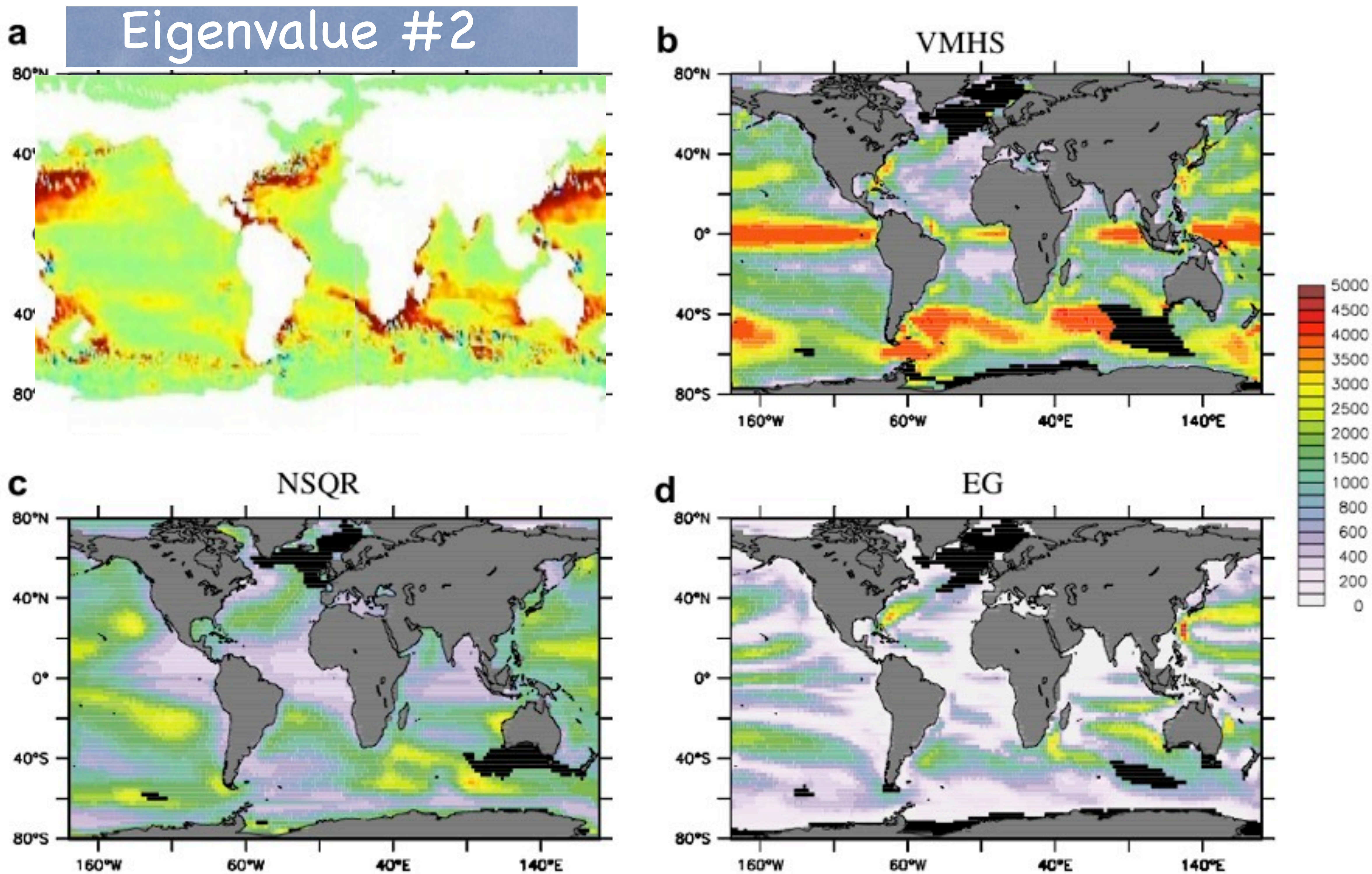


Compare with Eden, Jochum, Danabasoglu compilation of present parameterizations



g. 1. Annual mean thickness diffusivity (K) in m^2/s at 300 m depth in experiment CONST (a), VMHS (b), NSQR (c) and EG (d) after 500 years integration. Values of K are own for the interior region only, i.e. values of K in the (seasonal maximum) diabatic surface and transition layer are not shown and shaded black. Note the non-linear colour ale for the thickness diffusity. Note also that the data have been interpolated from the model grid to a regular rectangular grid of similar resolution prior to plotting. The nd mask in the figure (taken from Smith and Sandwell (1997)) differs therefore slightly from the model's land mask.

Compare with Eden, Jochum, Danabasoglu compilation of present parameterizations



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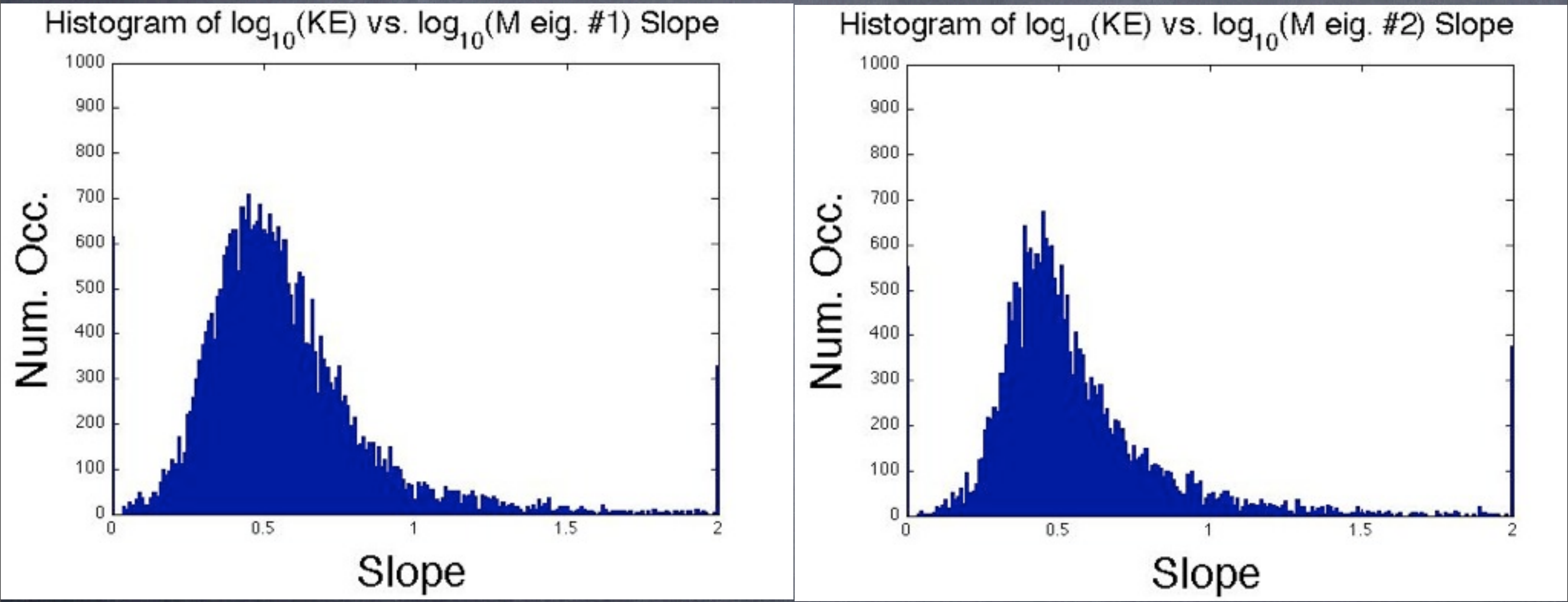
How do we explain the Vertical Variations of K?

Result:

coarse KE-> vertical structure of Mixing

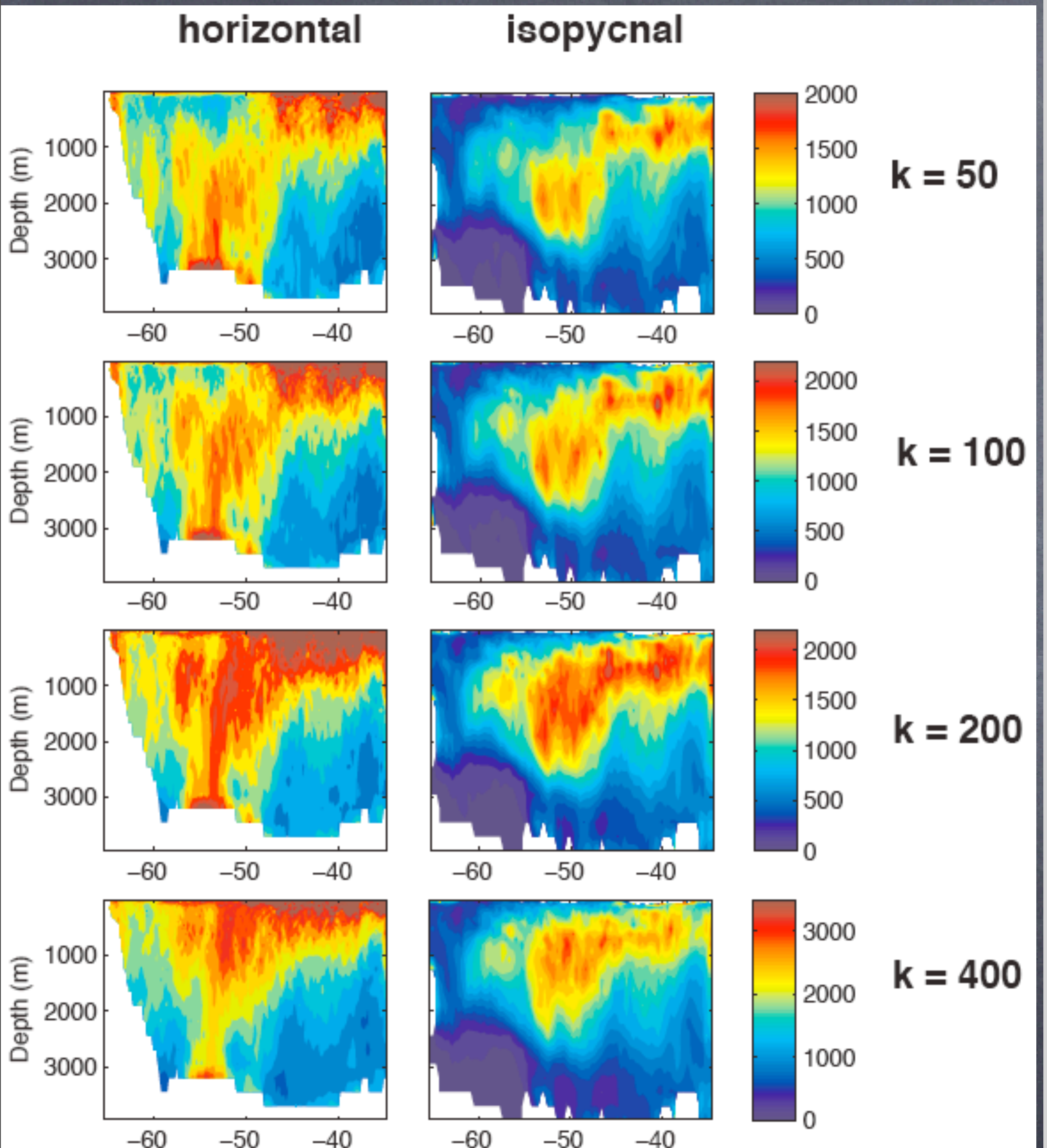
$$\mathbf{K} \propto \sqrt{\langle \mathbf{KE} \rangle}$$



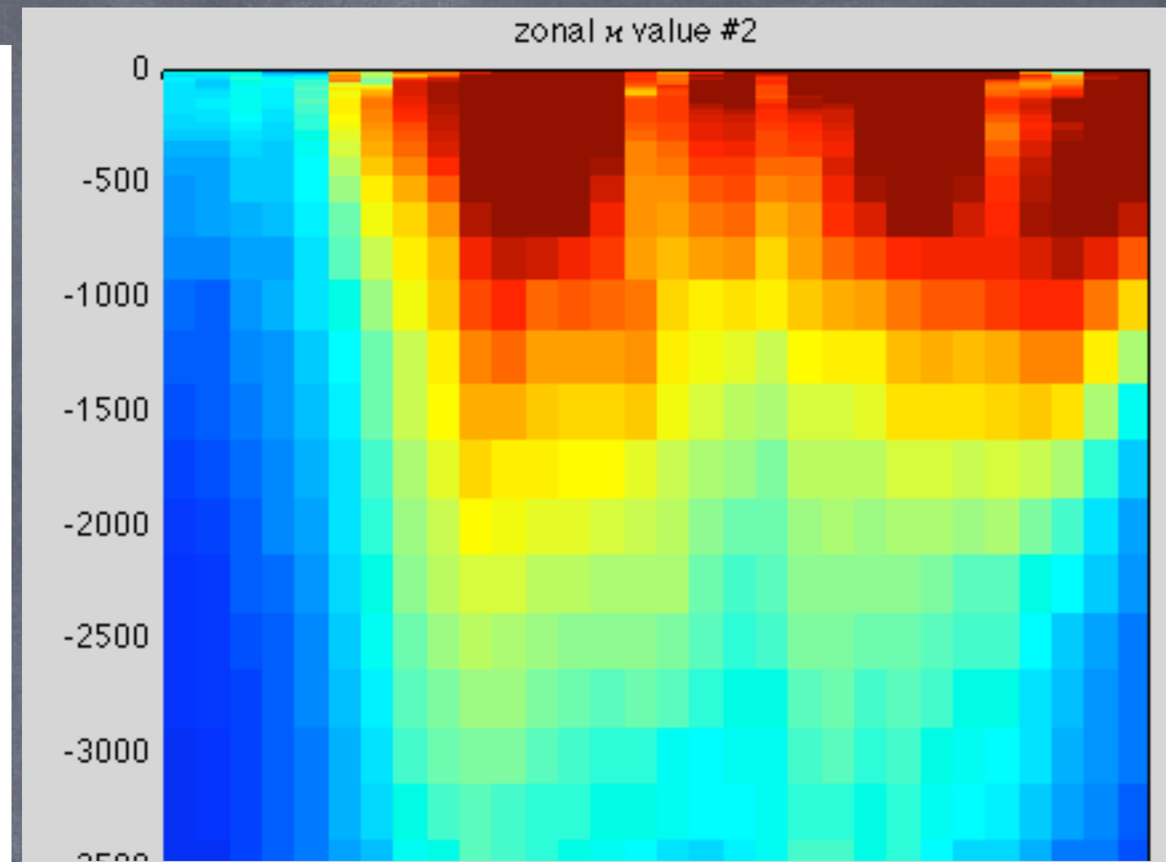
Even better with EKE!

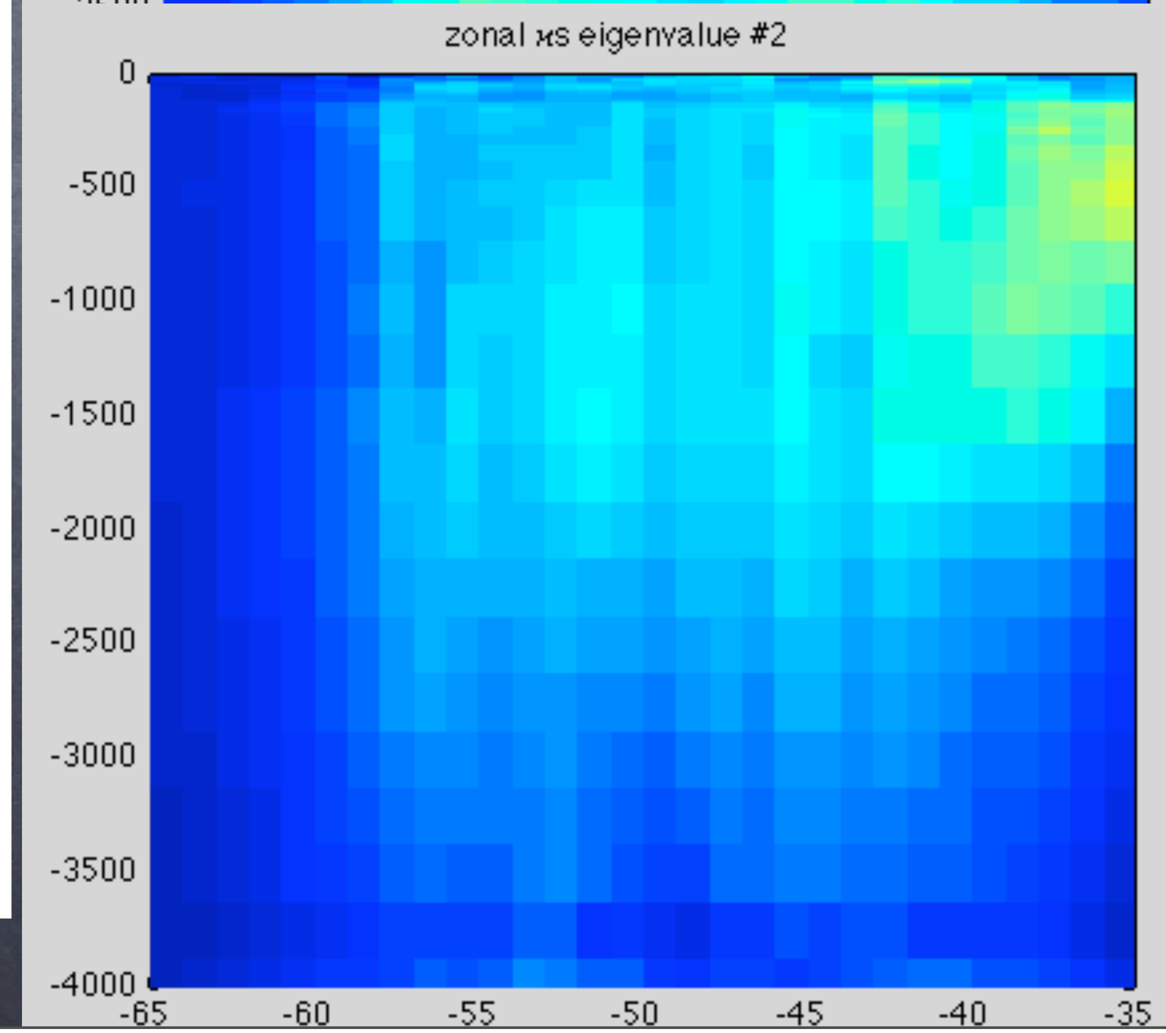
Note--barotropic mode is in there!

Comparisons with Marshall et al.

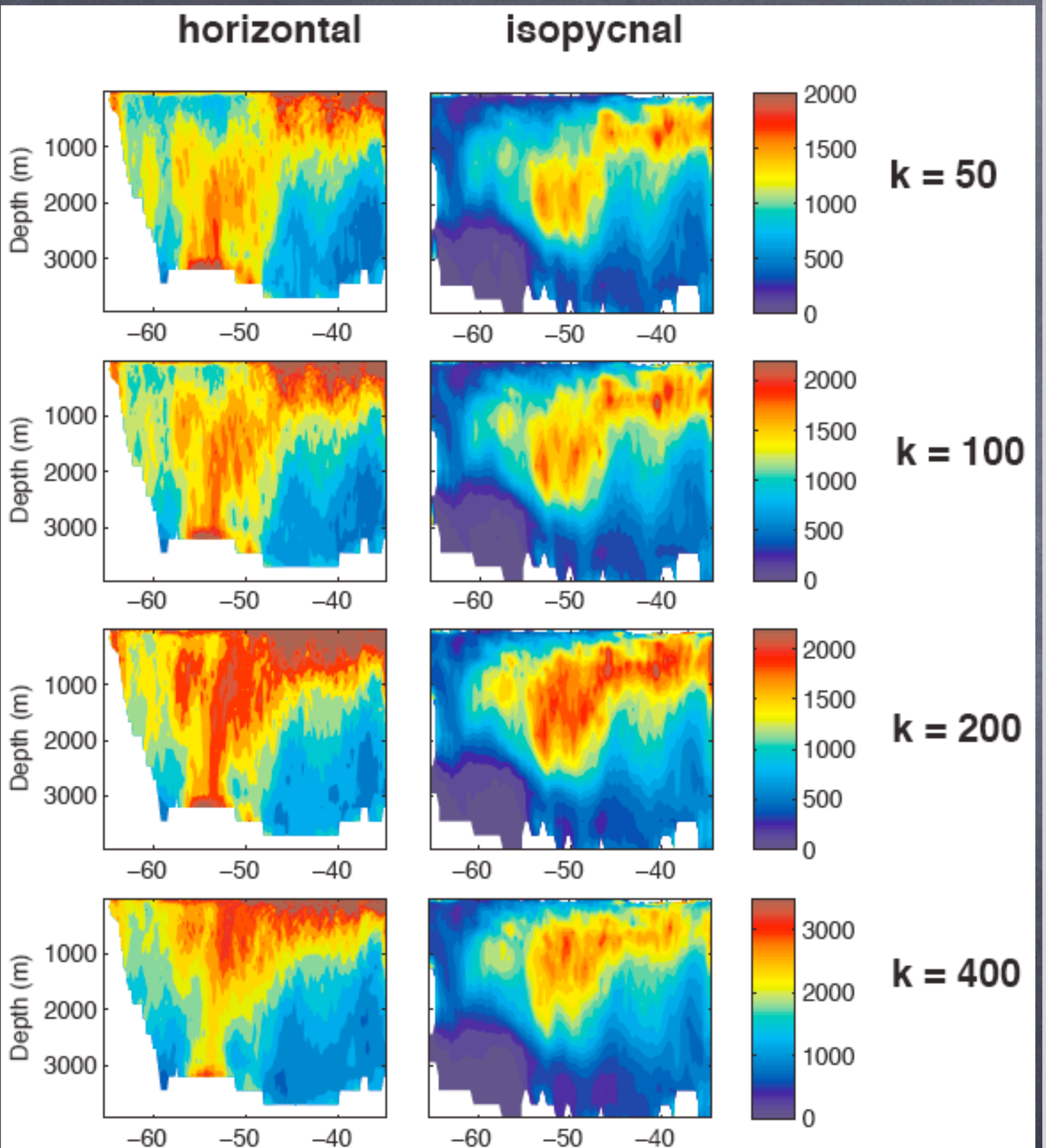


Abernathy et al 09

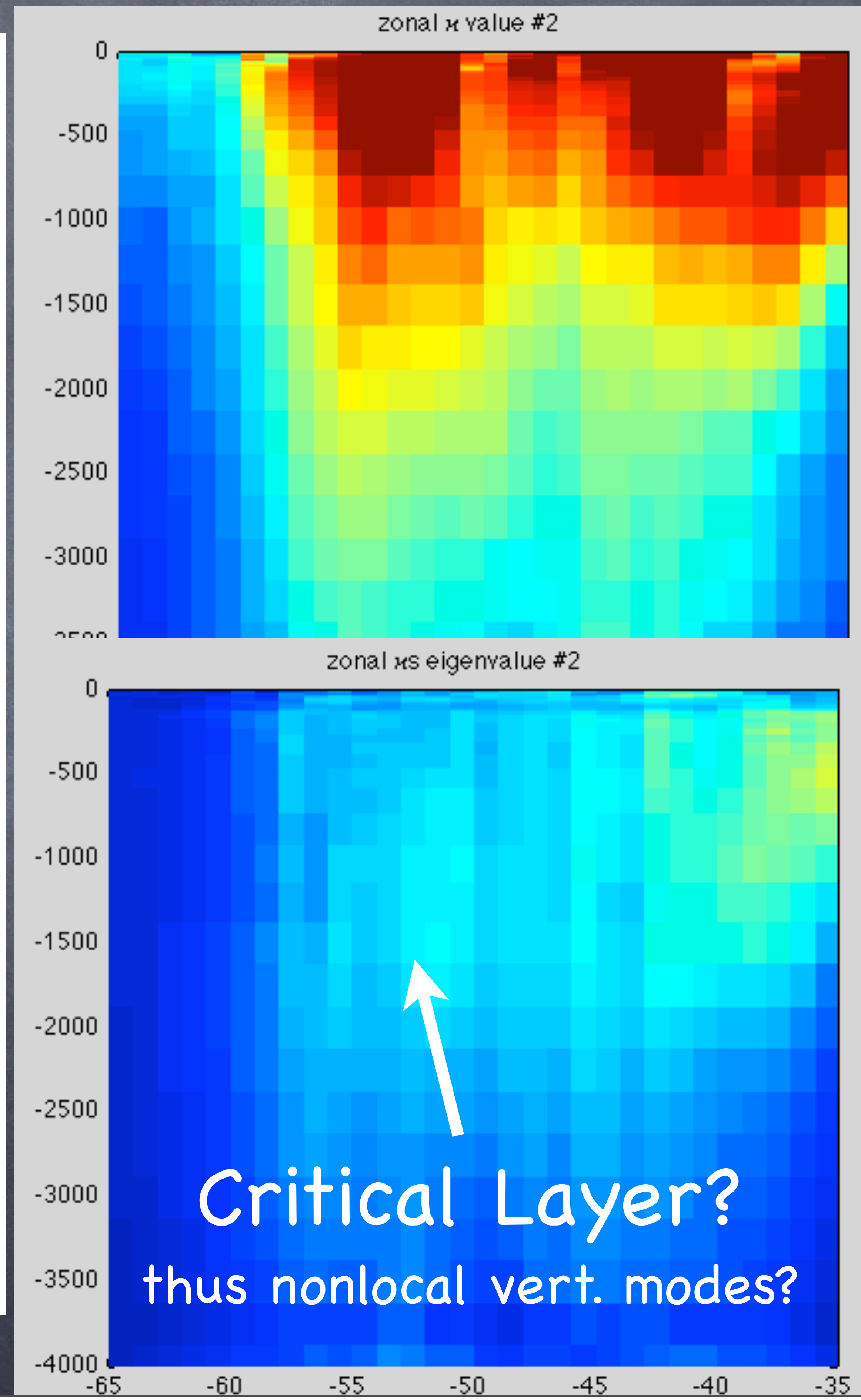




Comparisons with Marshall et al.



Abernathy et al 09



Conclusions

- A method for diagnosing the eddy stirring associated with fluxes represented in a 0.1° model but not a 2° model is presented
- It estimates the tracer-type-independent transport of tracer uniquely
- The shape and structure agrees roughly with Griffies (98) and Gent & Smith (04) analyses of GM & Redi isoneutral fluxes with *equal* anisotropic mixing & stirring.
- No gauge/rot. fluxes are needed to eliminate negative spurious eigenvalues

Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ 0 & 0 & -\hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Result 1: Antisymmetric (GM) Elements
scale with
corresponding Symmetric (Redi)
elements in extratropics.

Thus, GM/Redi basic shape of M is
roughly correct
(some detailed validation remains)

Use a Natural, Mesoscale Eddy Environment to Test Out:

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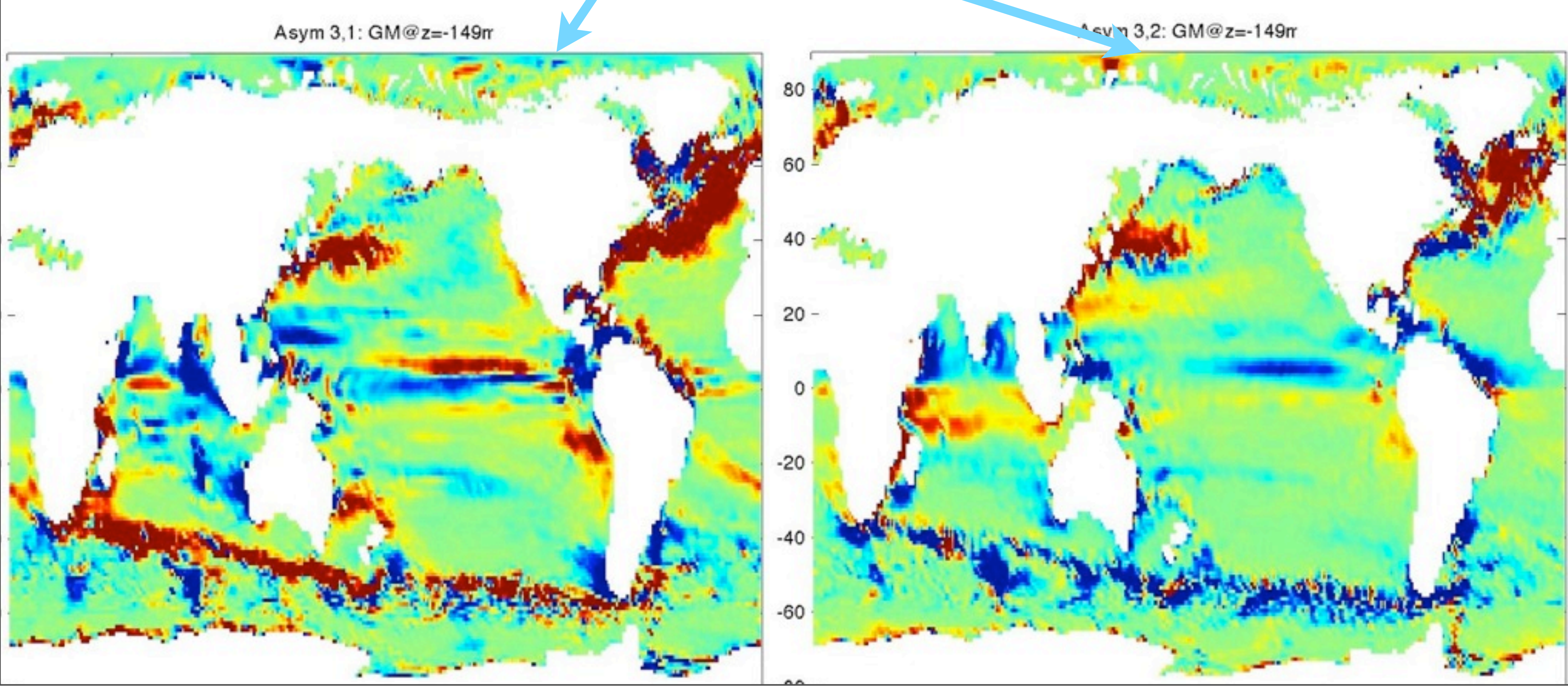

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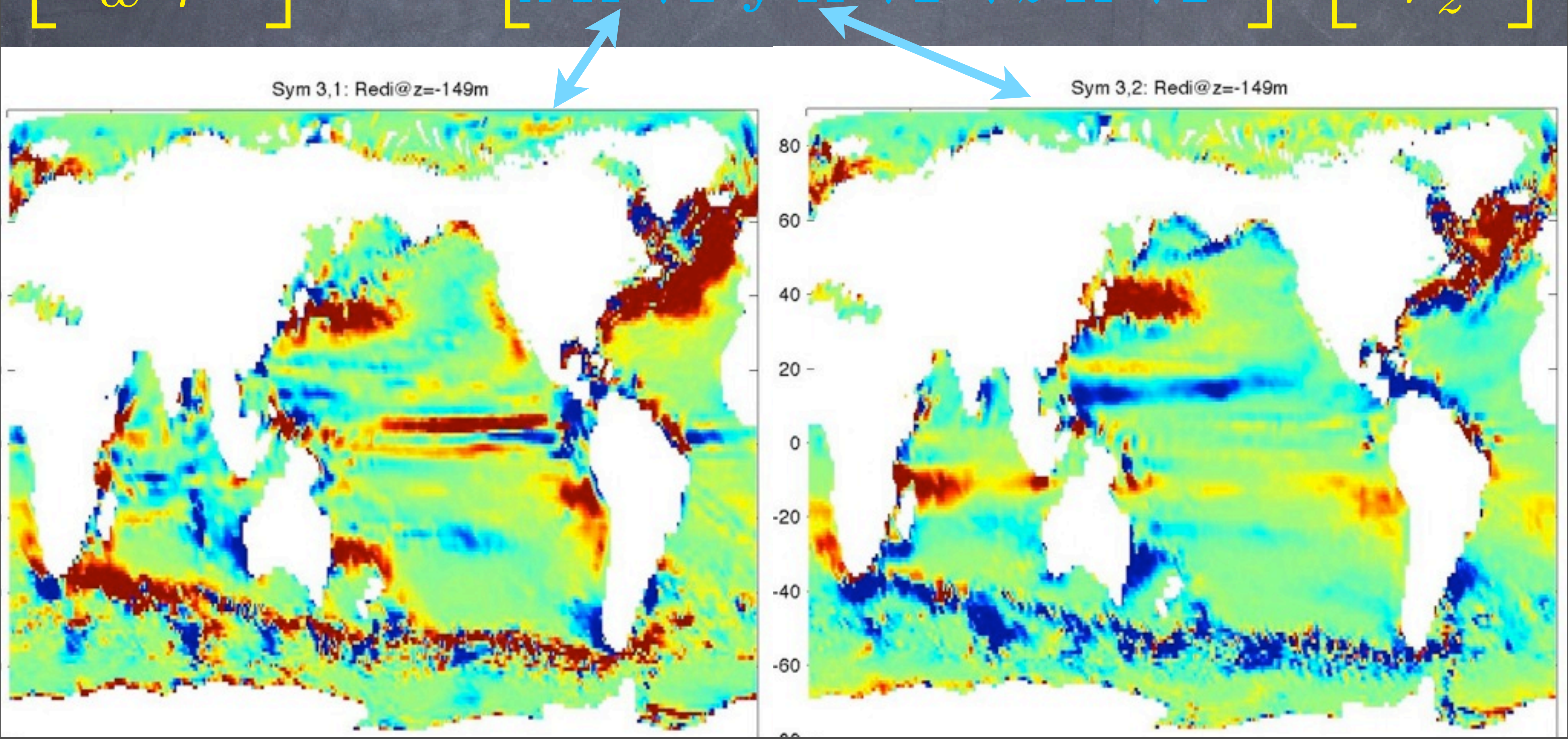

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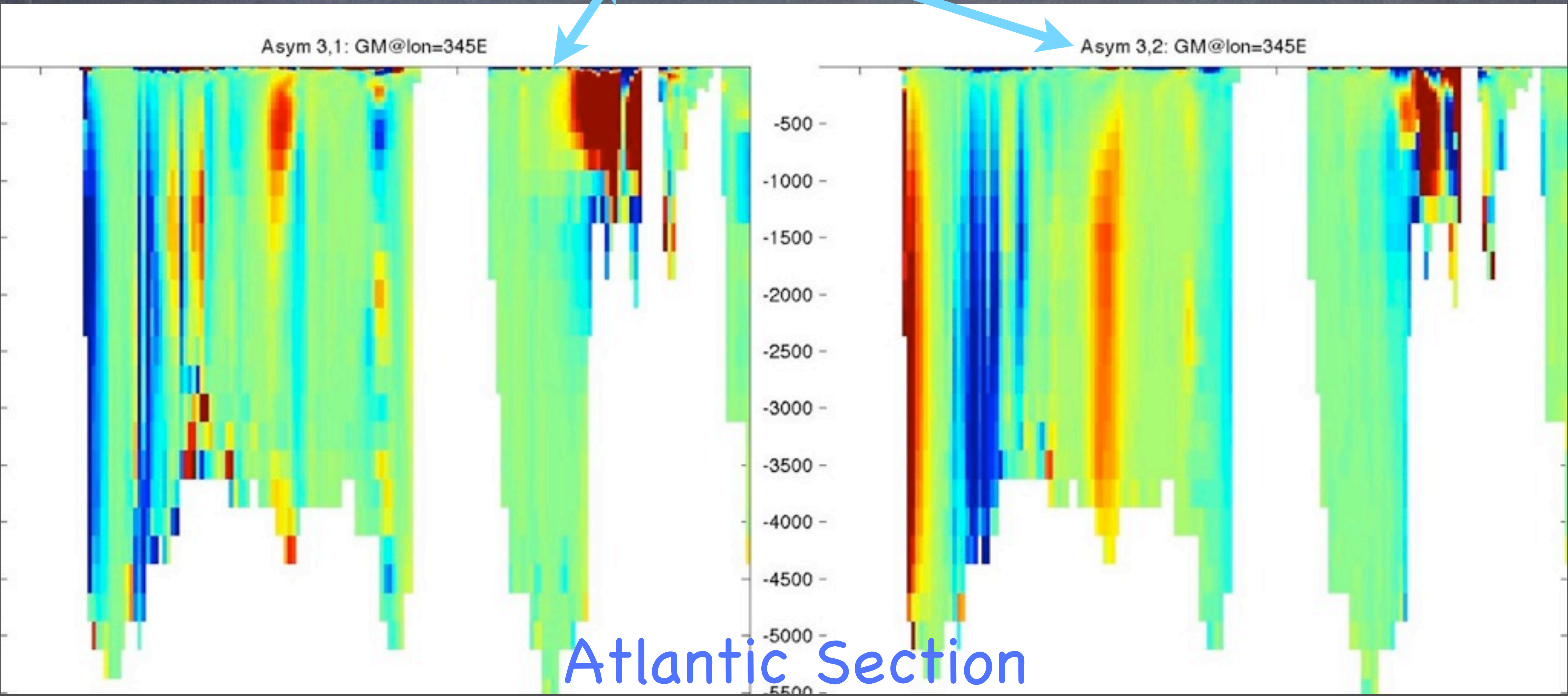

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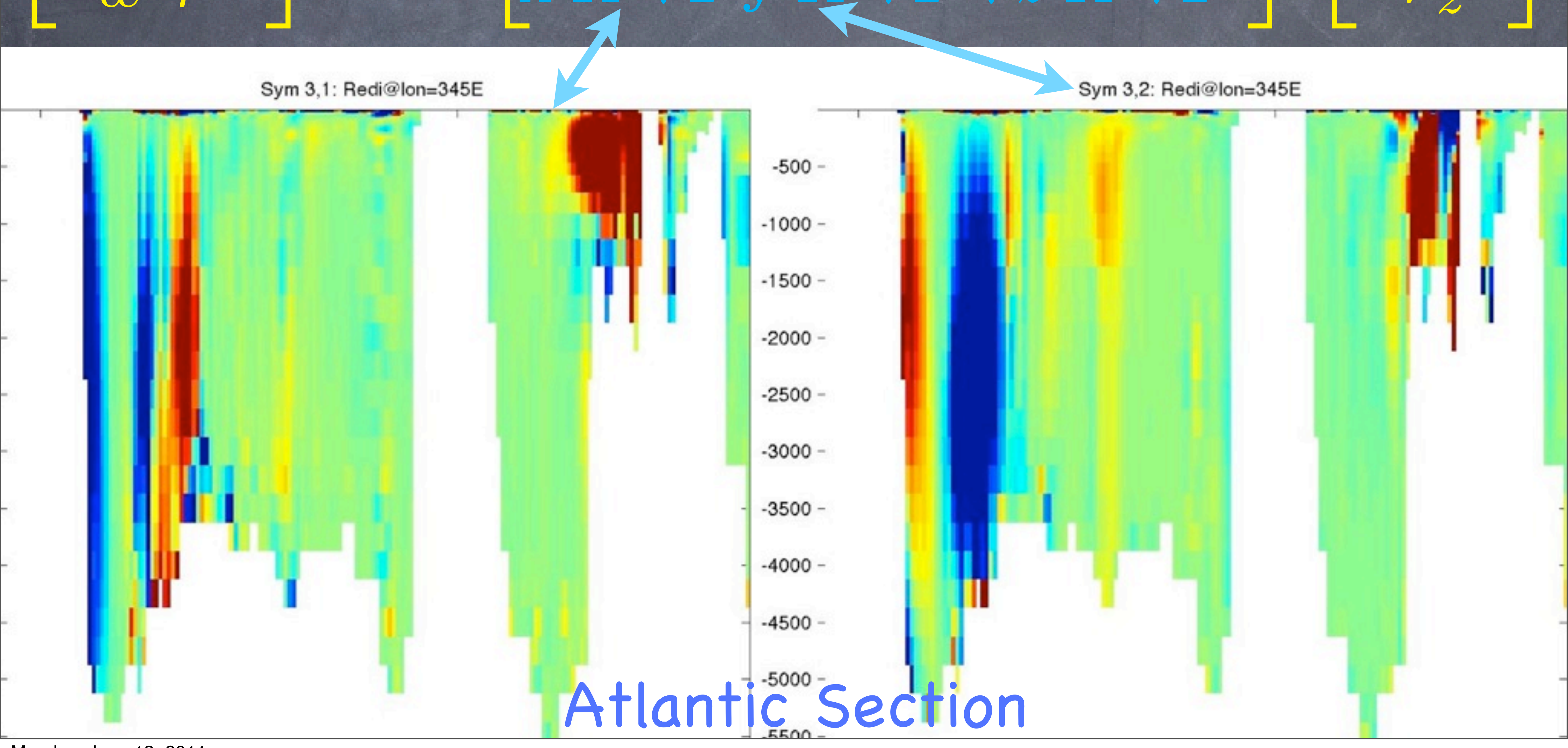

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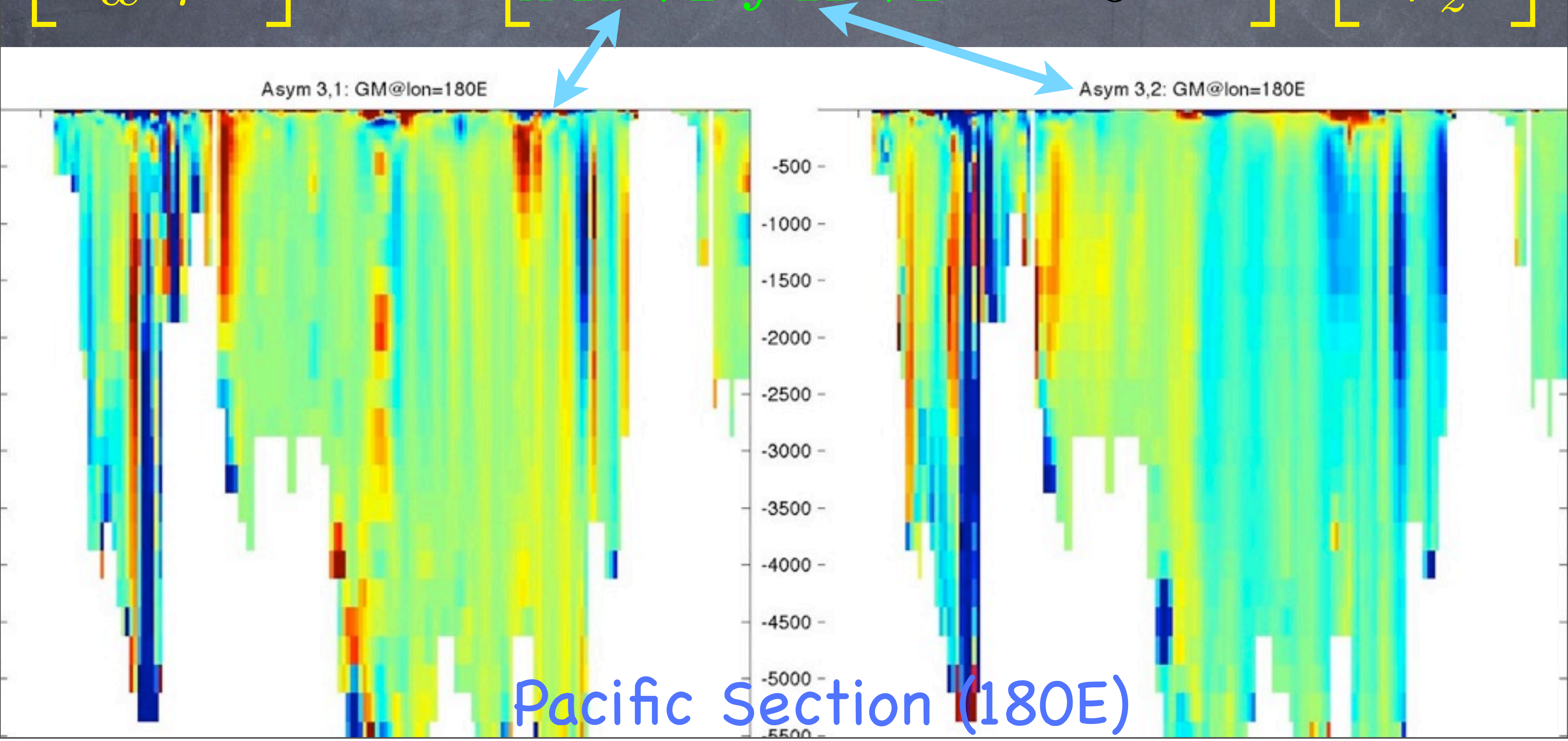

Pacific Section (180E)

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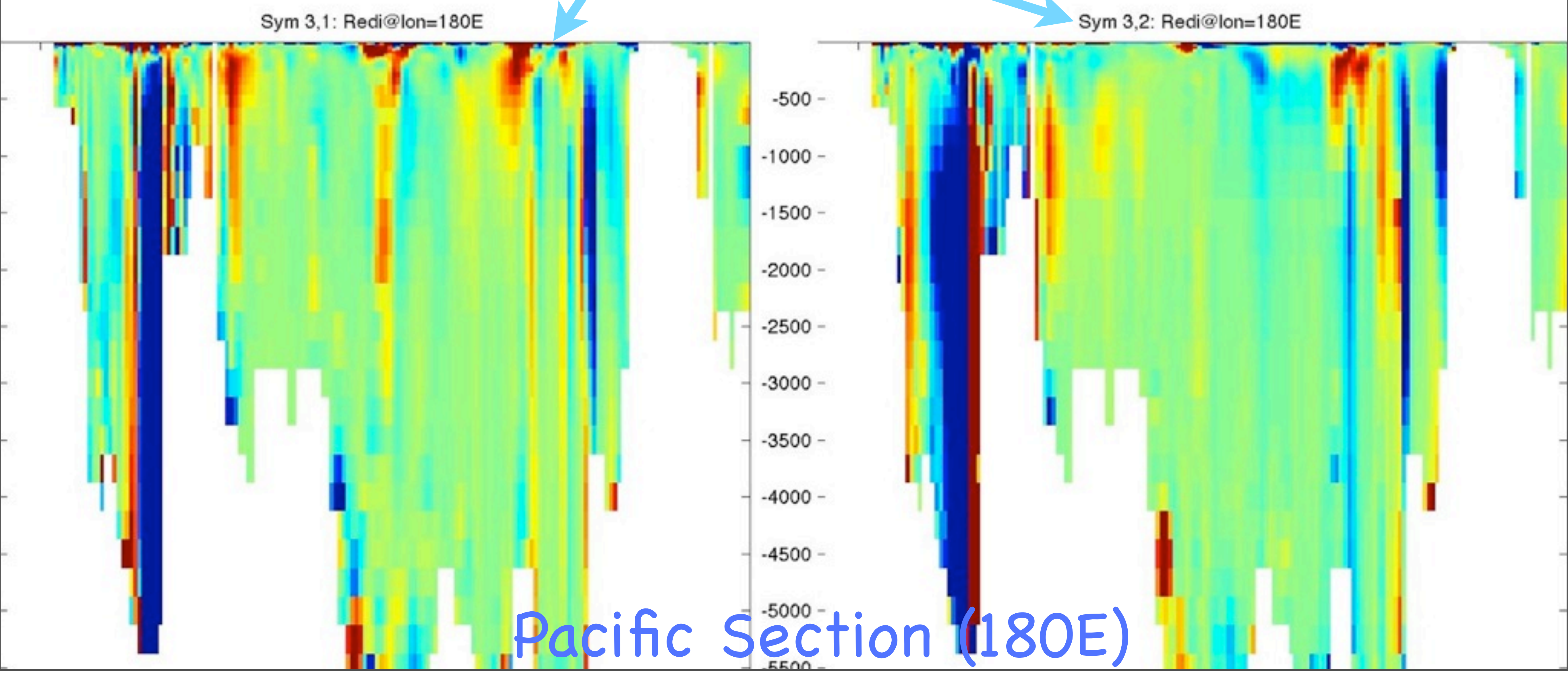


Pacific Section (180E)

NSEF & Diabatic/ Transition Layer

- Danabasoglu & Marshall
- Danabasoglu, Ferrari & McWilliams
- Ferrari, McWilliams, Canuto, Dubovikov
- Surface-intensified GM, no boundary condition issues, no over-restratificiation of Mixed Layer by Eddies

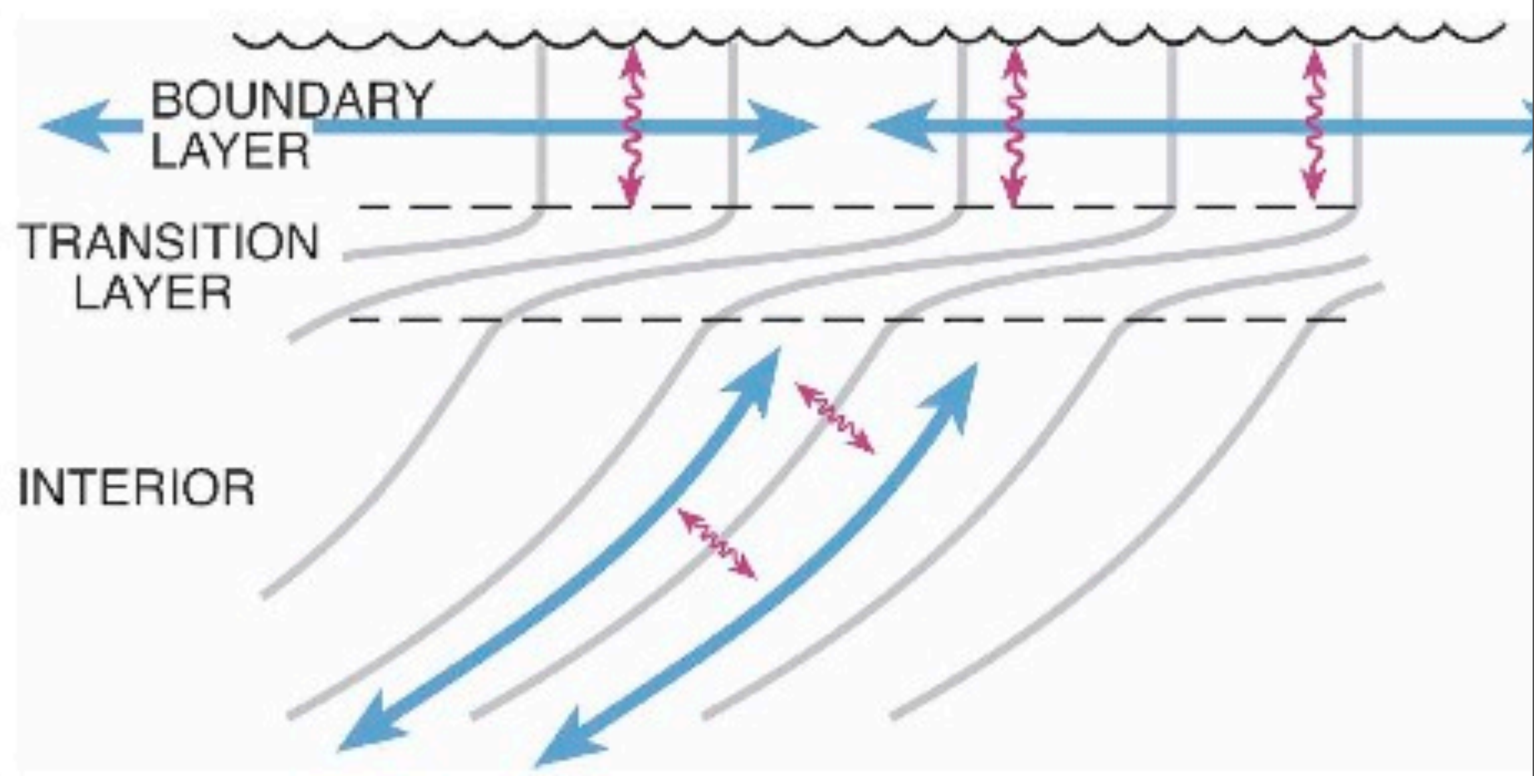
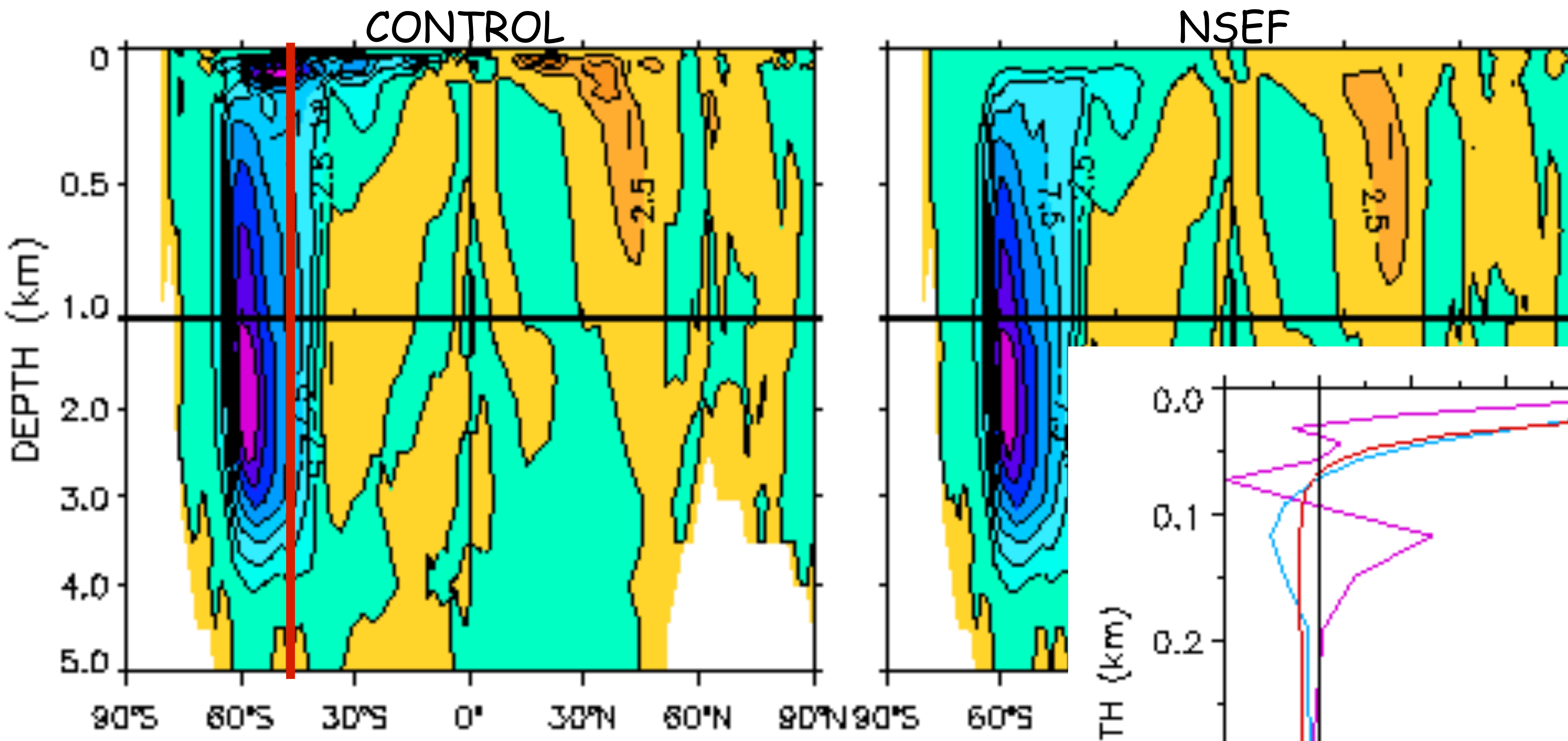


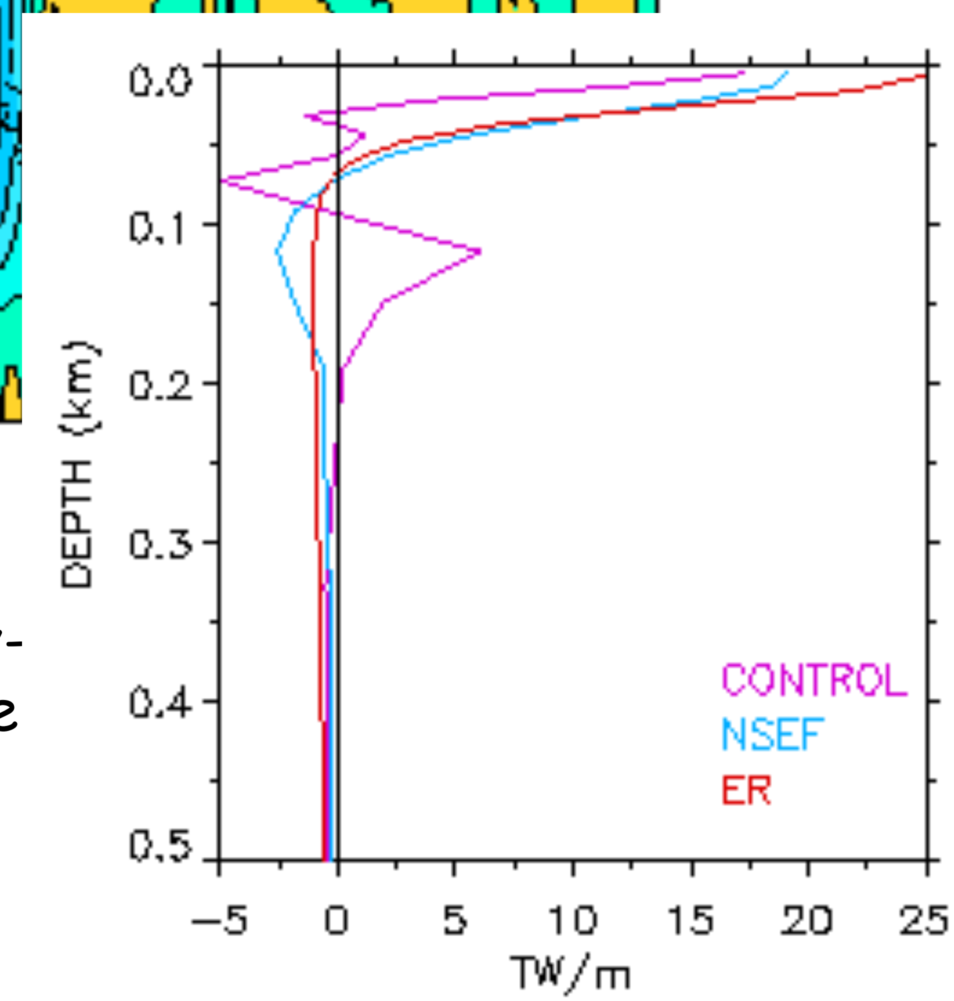
FIG. 2. A conceptual model of eddy fluxes in the upper ocea
Mesoscale eddy fluxes (blue arrows) act to both move isopycn
surfaces and stir materials along them in the oceanic *interior*, b
the fluxes become parallel to the boundary and cross density su
faces within the *BL*. Microscale turbulent fluxes (red arrows) m

Near-surface eddy flux scheme (Ferrari, McWilliams, Canuto, Dubovikov)

EDDY-INDUCED MERIDIONAL OVERTURNING (GLOBAL)



Vertical profile of zonally-integrated total advective heat flux at 49°S



A new eddy parameterization (Ferrari, Griffies, Nurser & Vallis)

- The eddy streamfunction is given by the elliptic problem

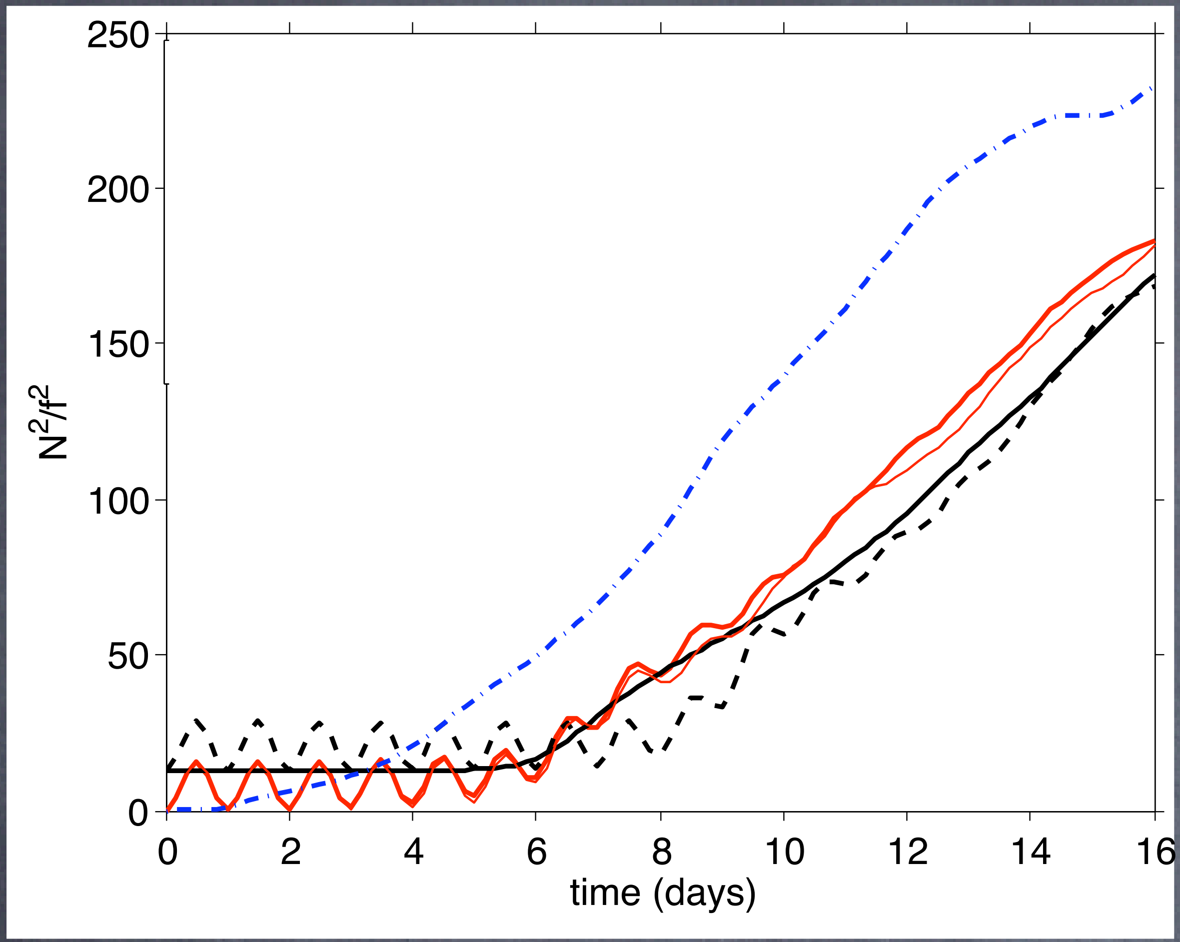
$$\left(c^2 \frac{\mathrm{d}^2}{\mathrm{d}z^2} - N^2\right)\widetilde{\boldsymbol{\Psi}} = -\kappa \nabla \bar{b}$$

$$\widetilde{\boldsymbol{\Psi}} = 0, \quad z = 0, -H$$

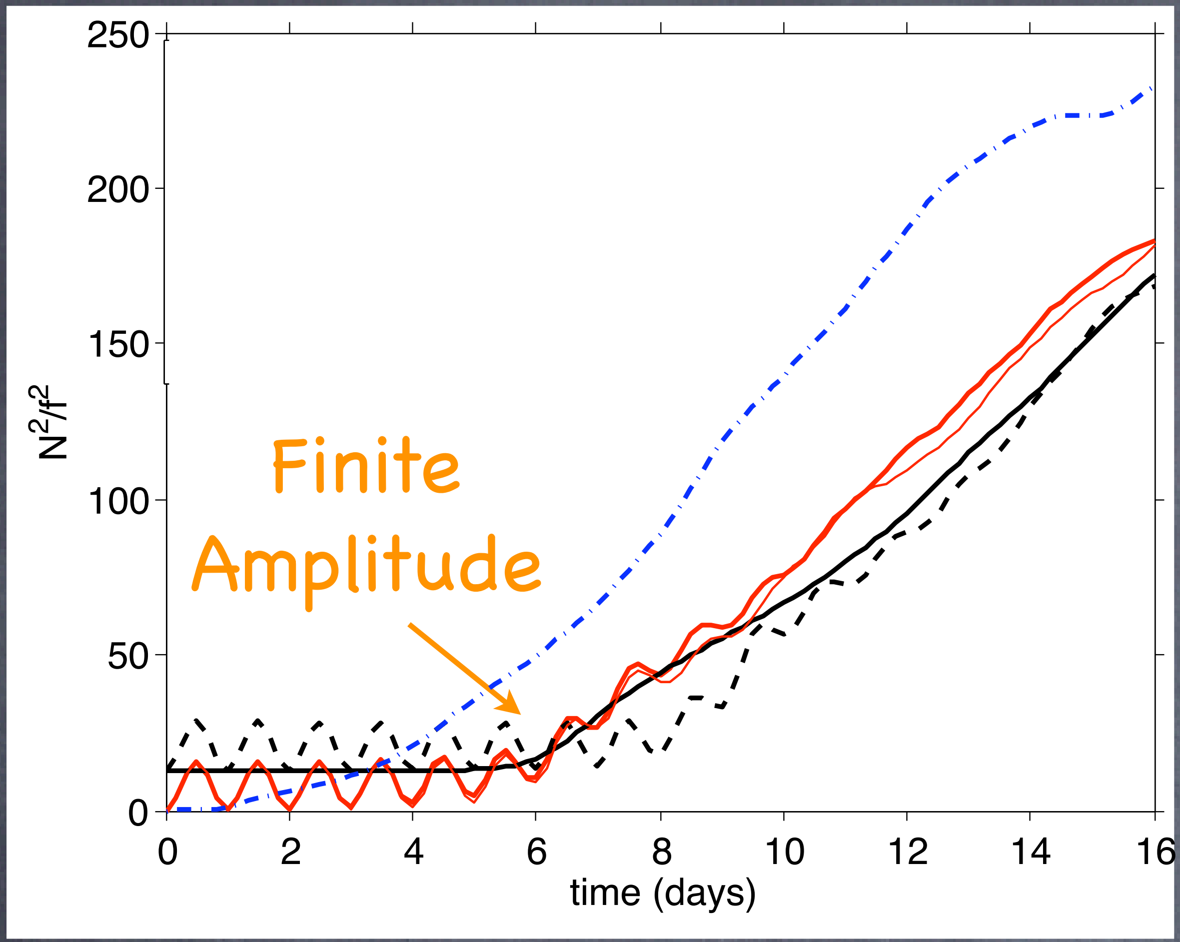
Properties of the new parameterization

- releases mean available potential energy
- the eddy transport vanishes at the ocean boundaries
- the eddy transport is dominated by the first baroclinic mode (if c is set to speed of first baroclinic mode)
- does not require any tapering function
- reduces to GM for c=0

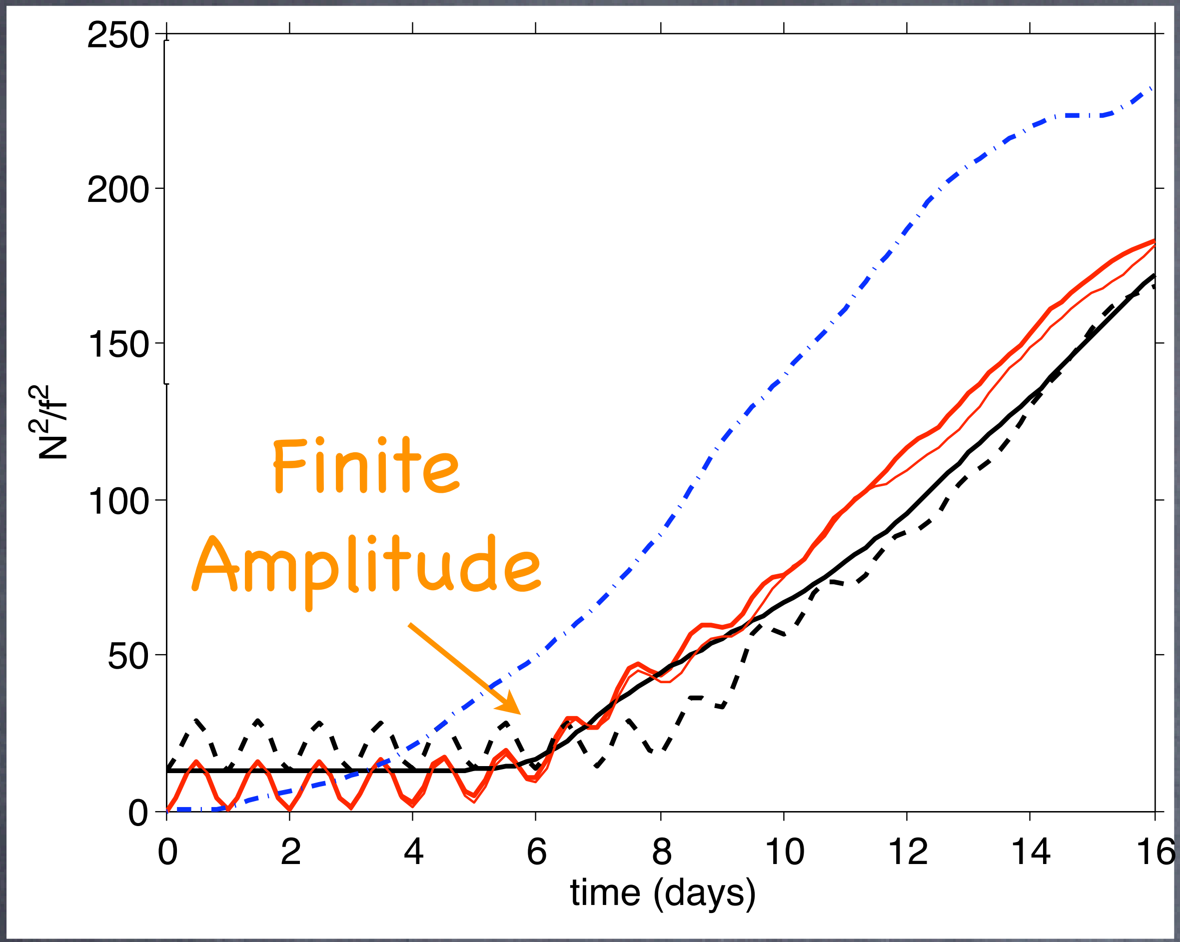
Parameterization of Finite Amp. Eddies: Ingredients

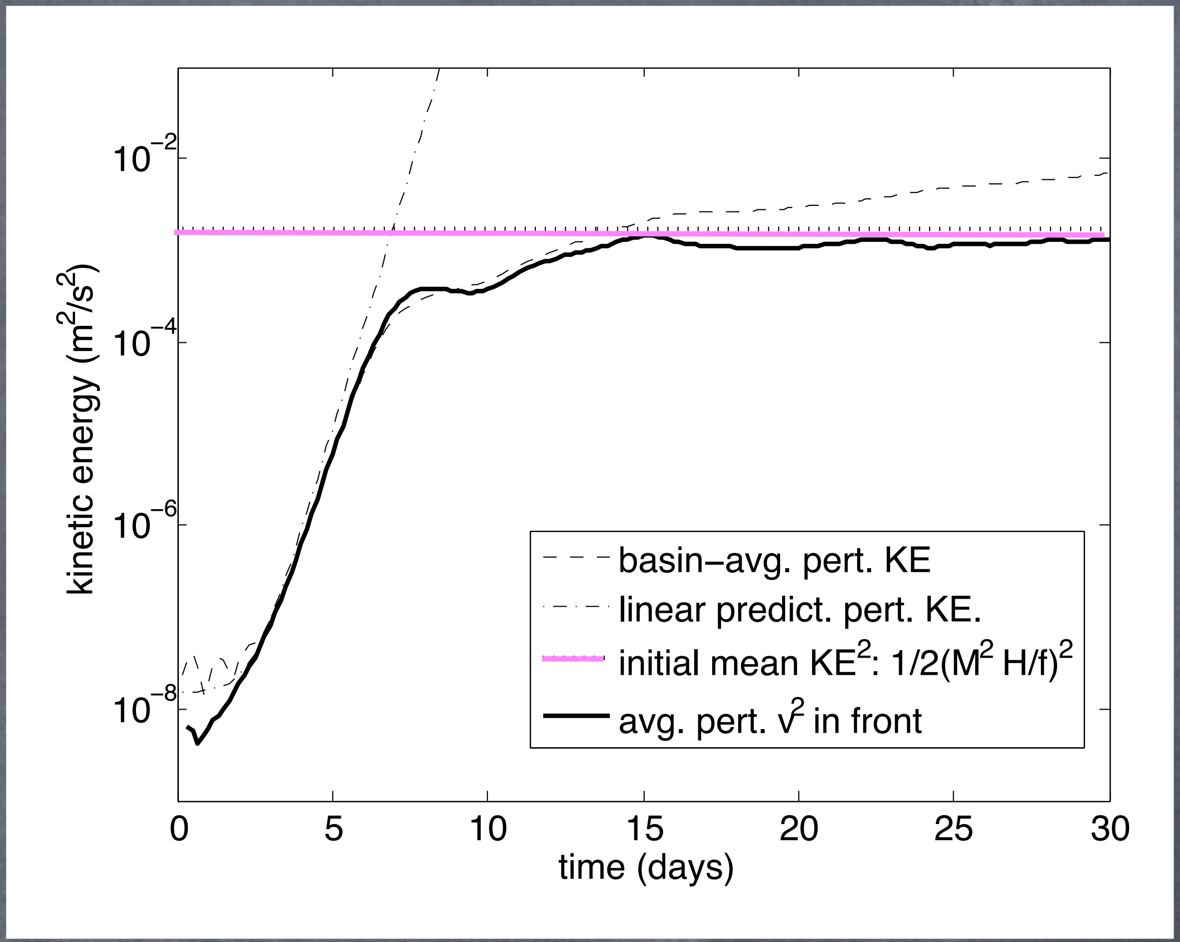


Parameterization of Finite Amp. Eddies: Ingredients

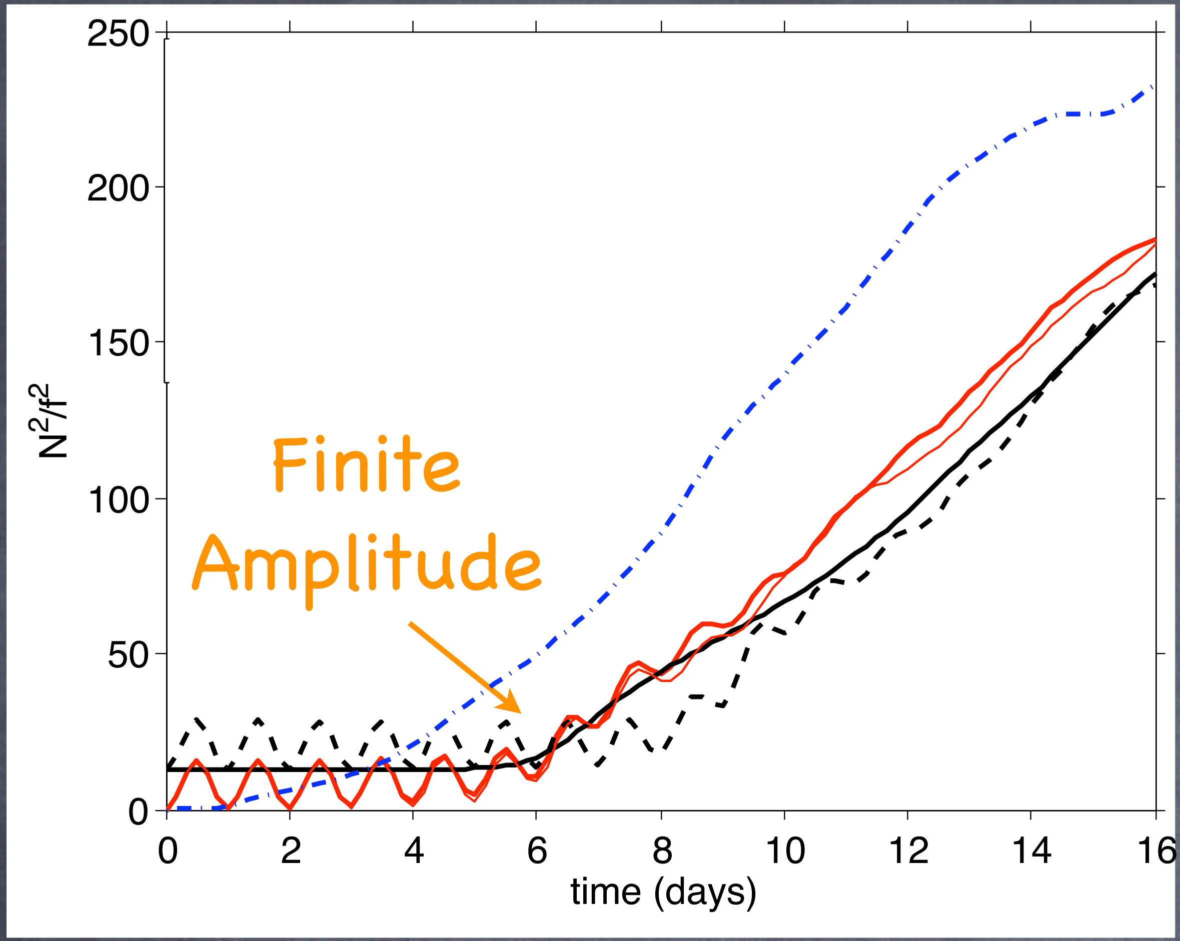


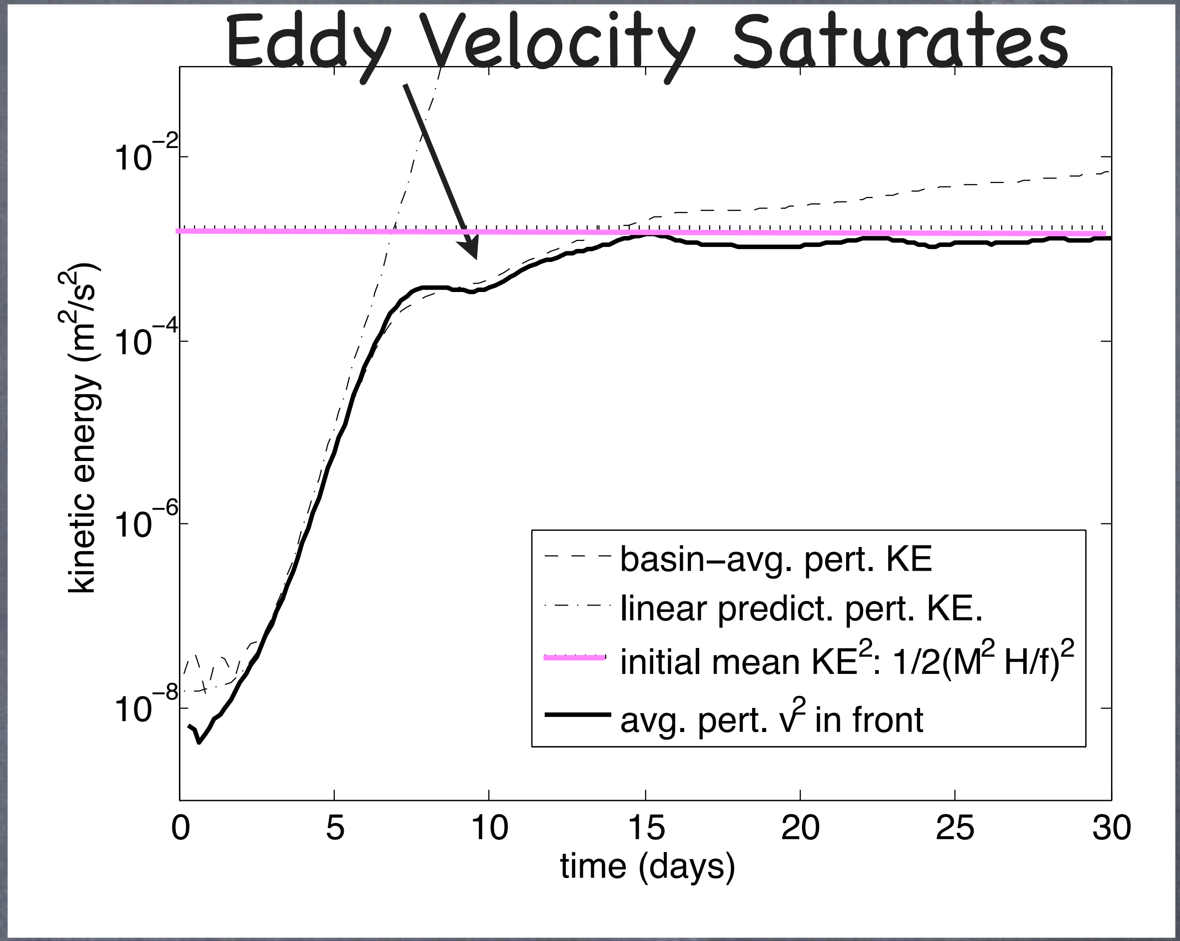
Parameterization of Finite Amp. Eddies: Ingredients



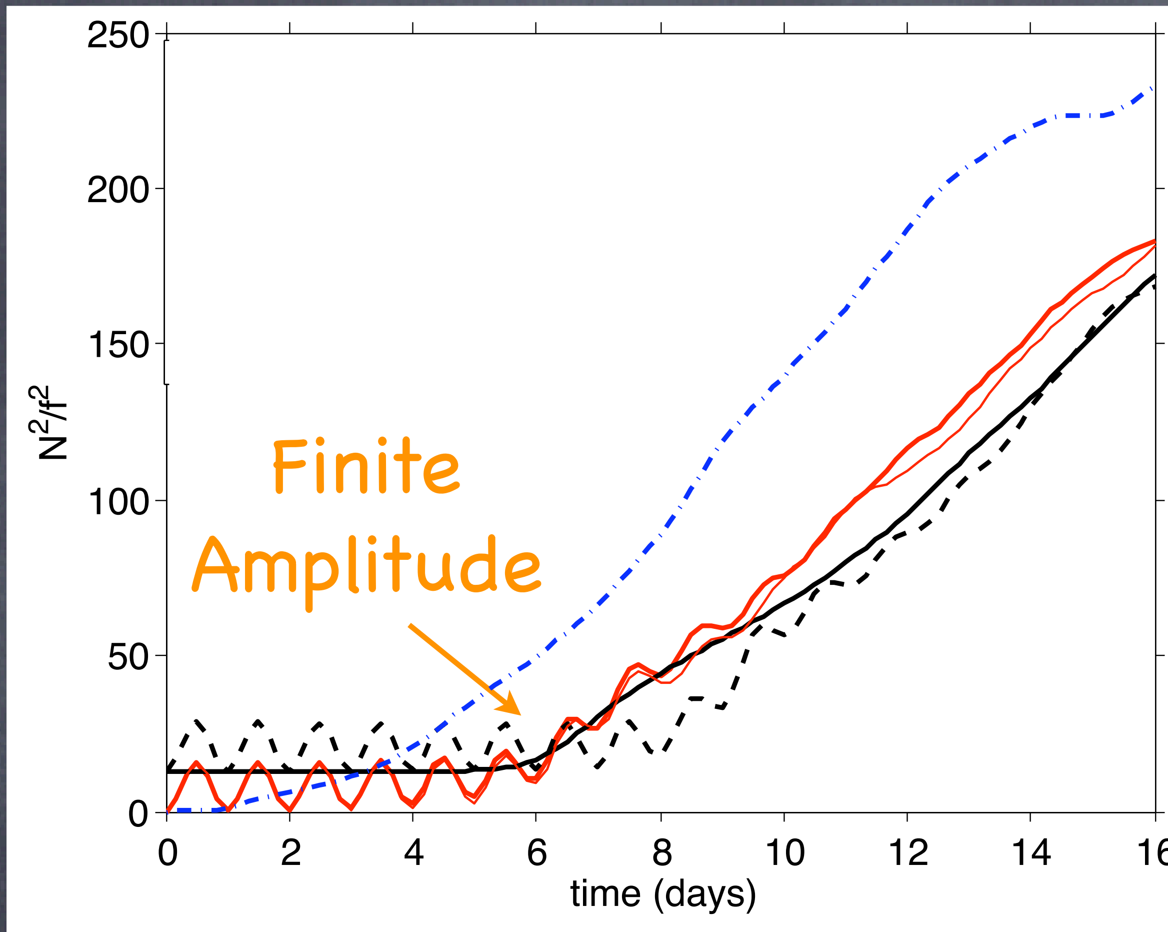


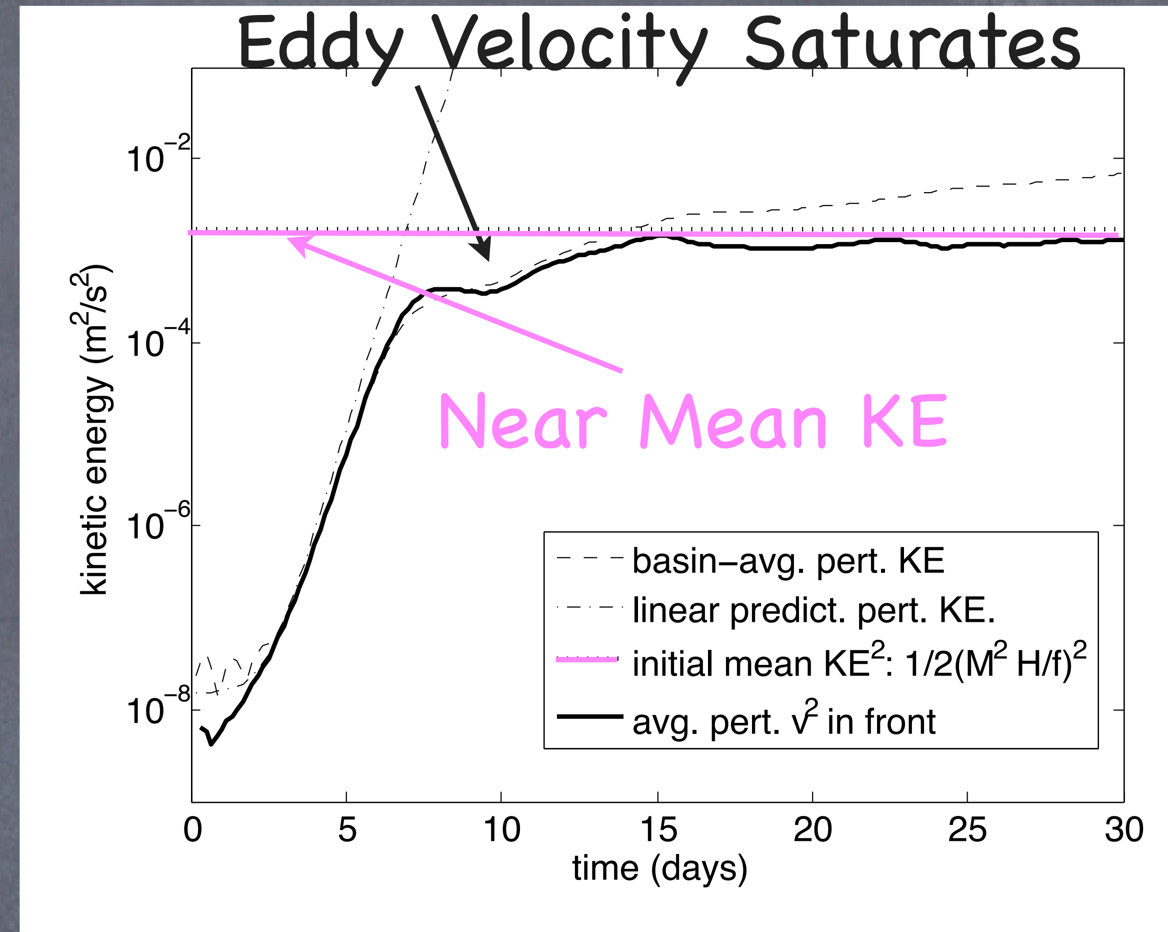
Parameterization of Finite Amp. Eddies: Ingredients



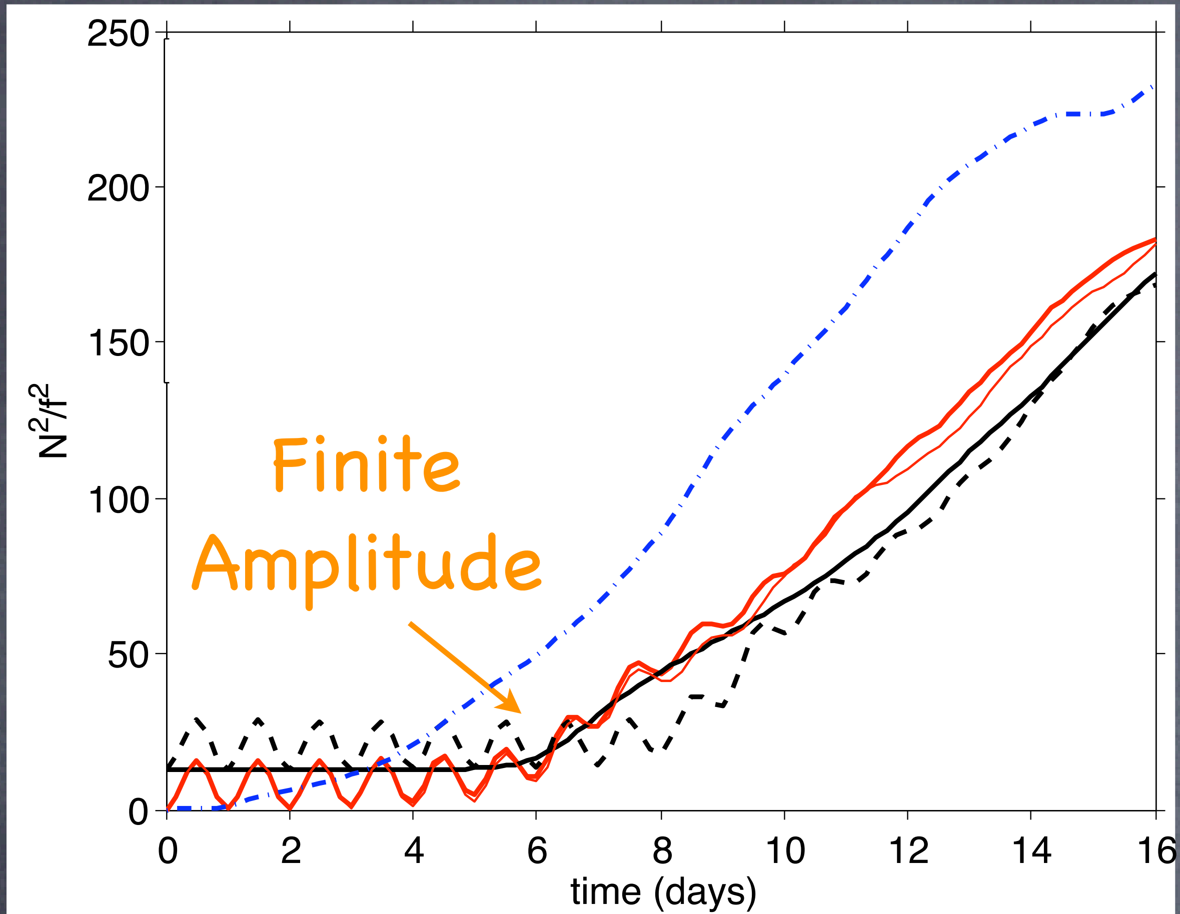


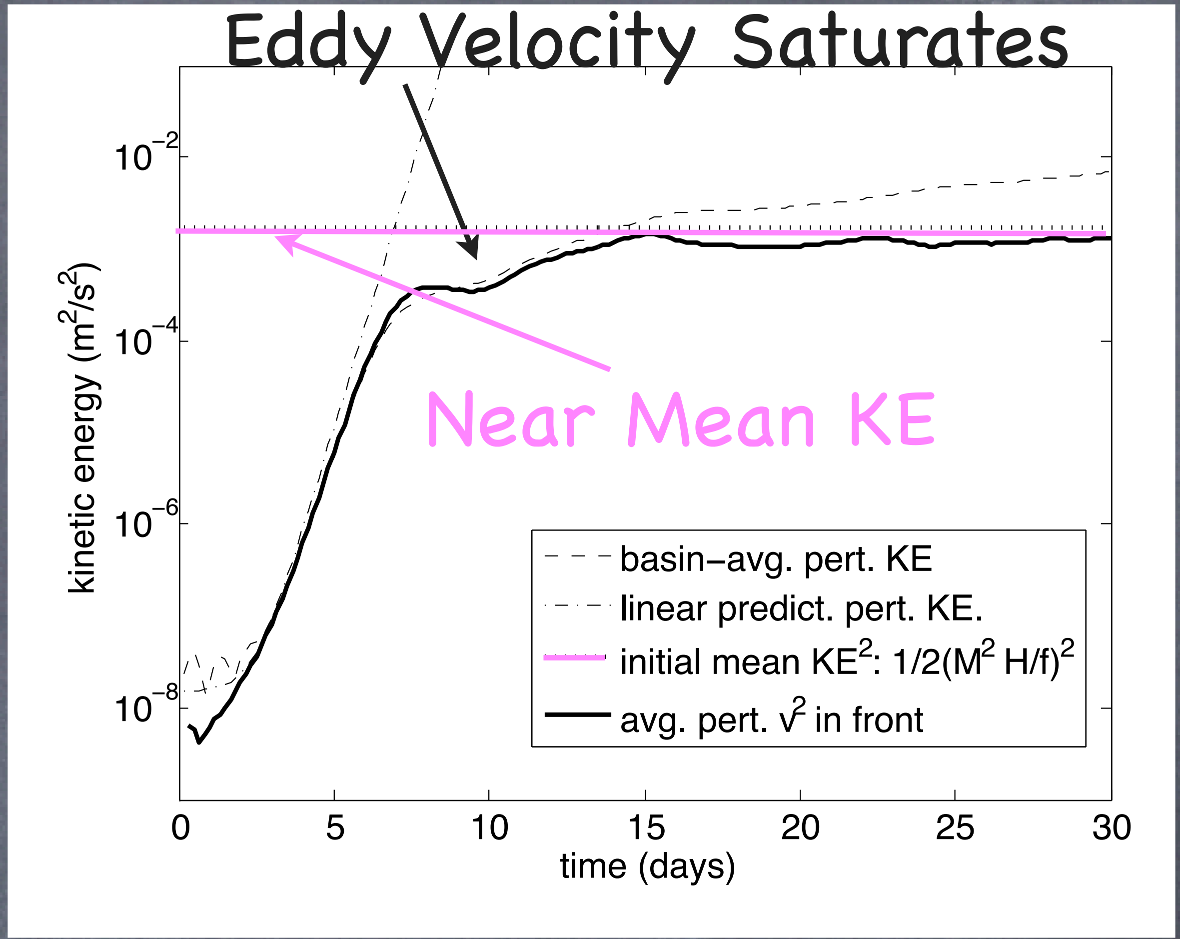
Parameterization of Finite Amp. Eddies: Ingredients

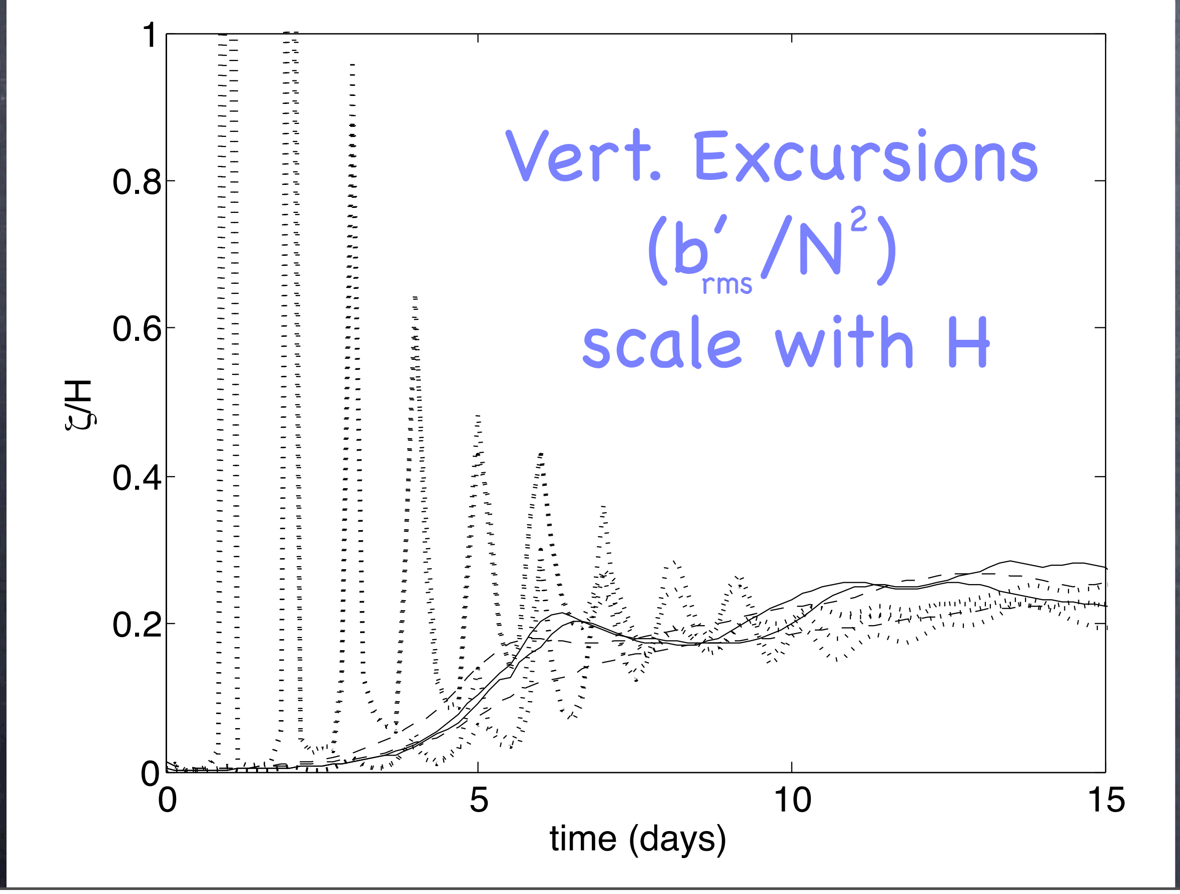




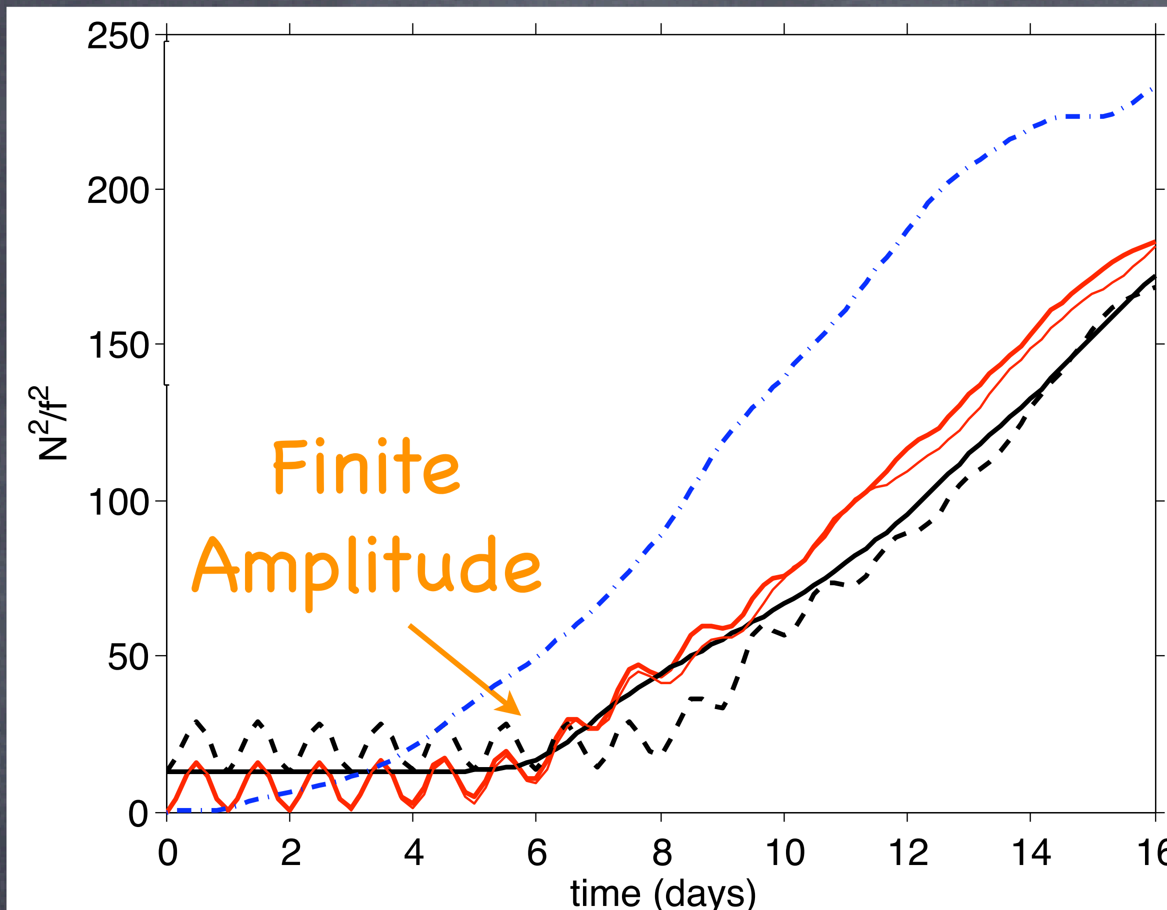
Parameterization of Finite Amp. Eddies: Ingredients

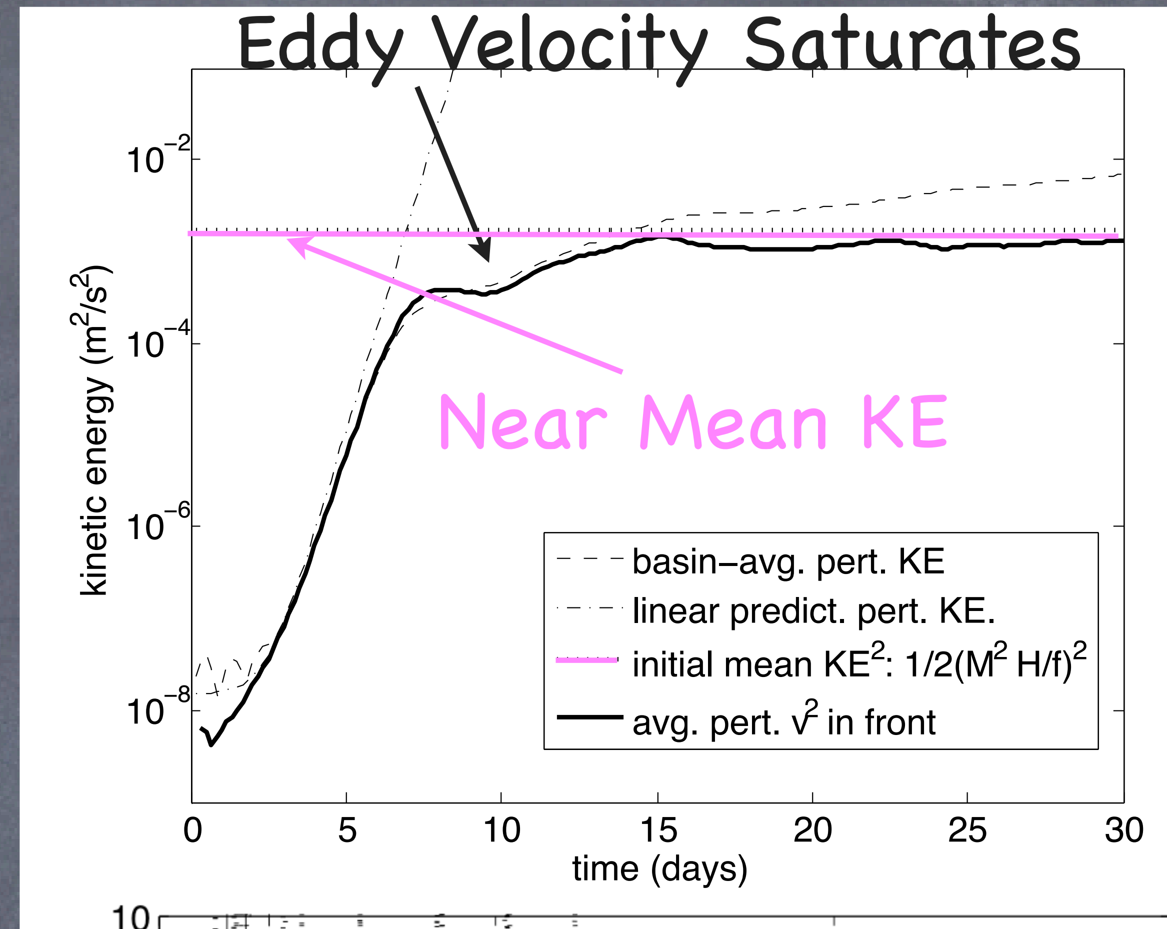


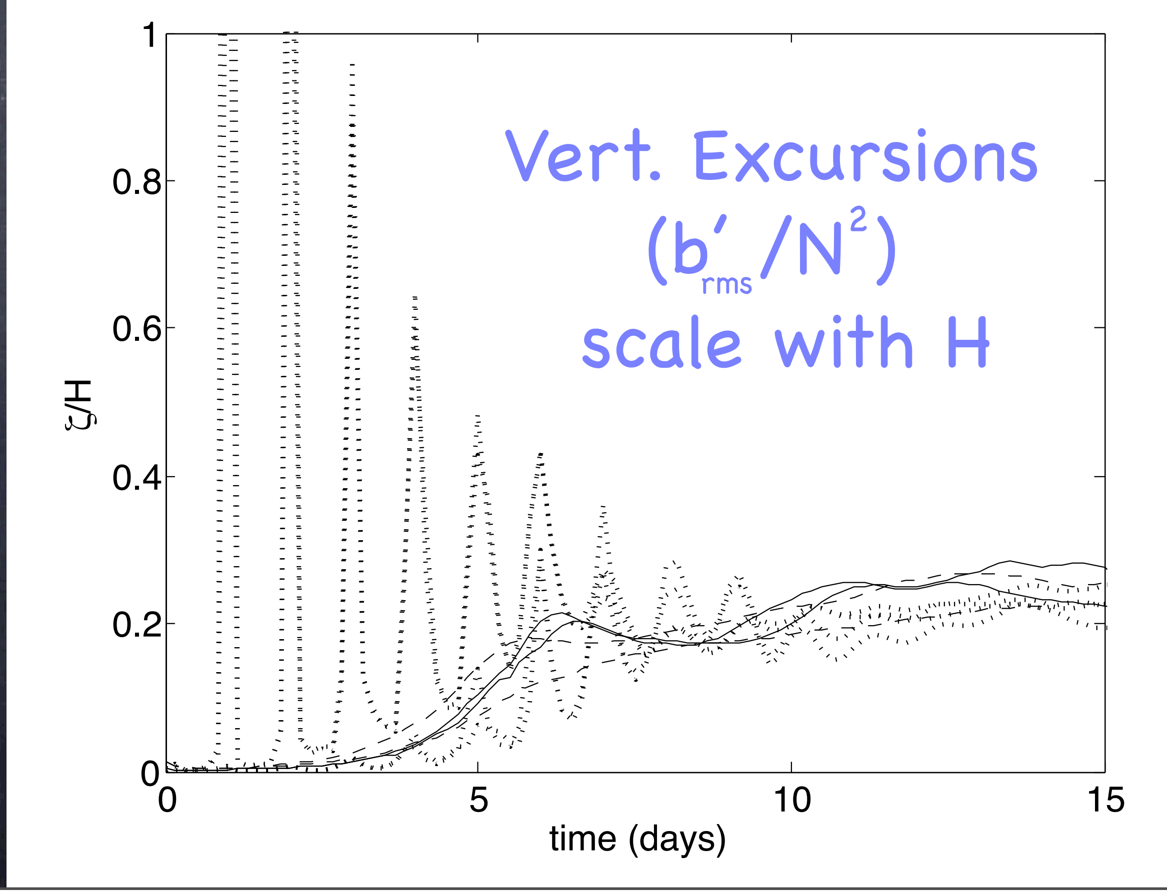


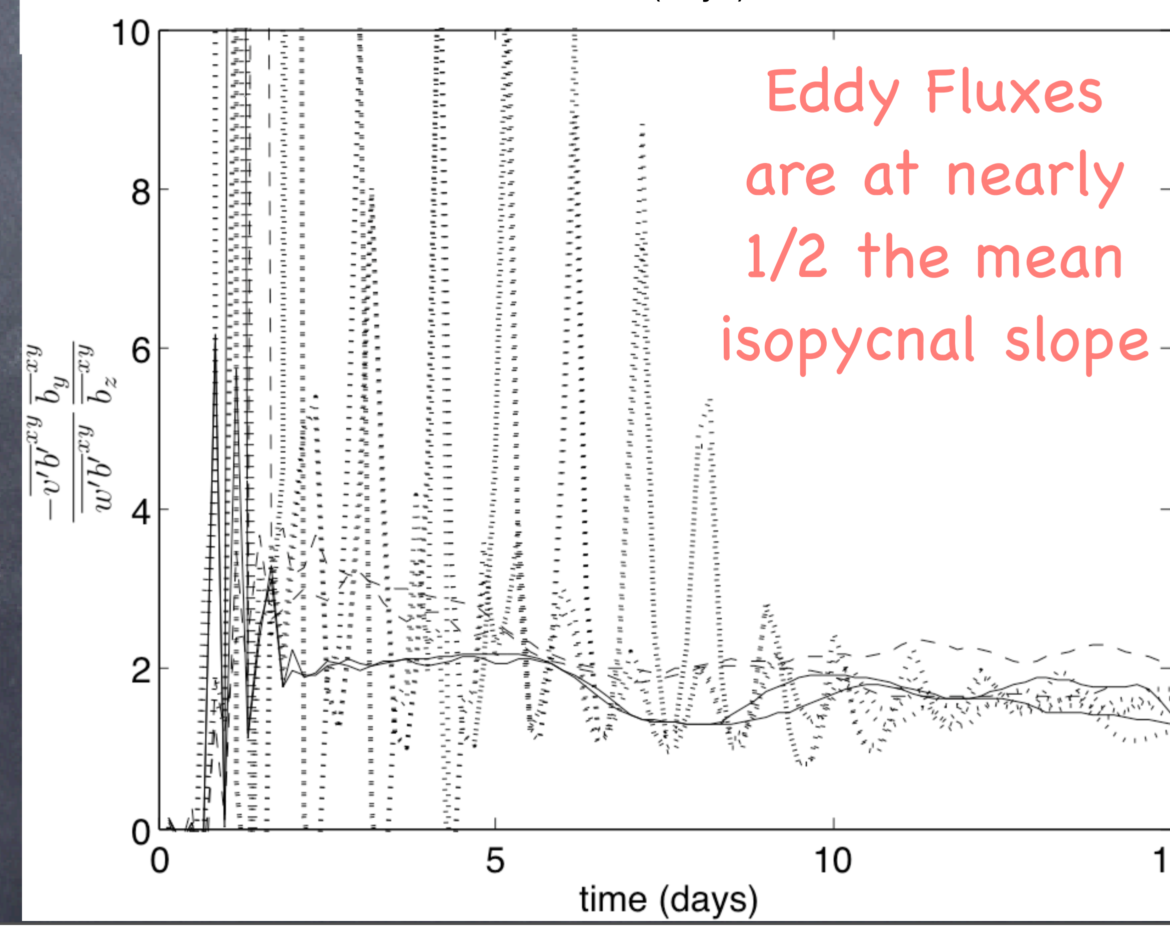


Parameterization of Finite Amp. Eddies: Ingredients

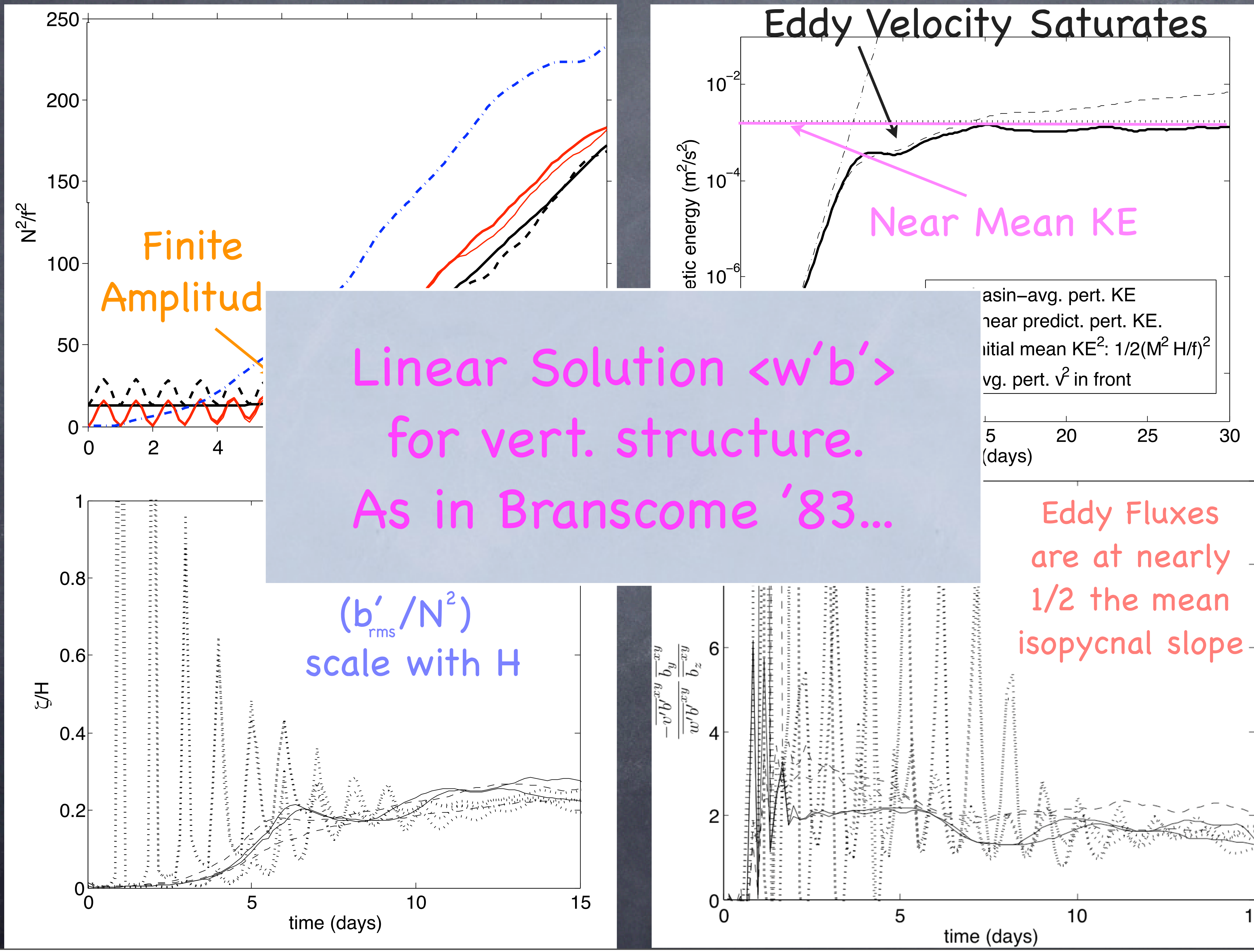






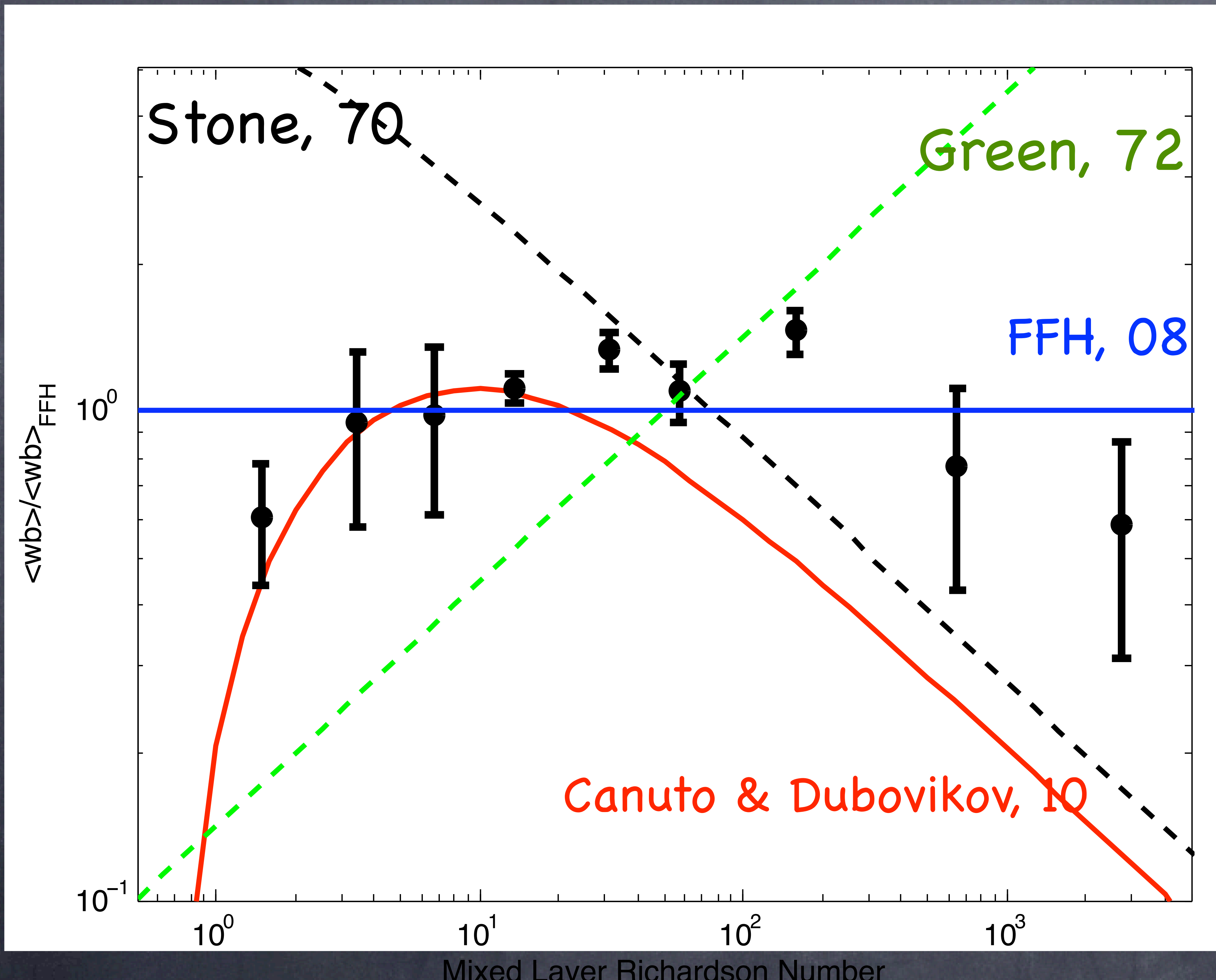


Parameterization of Finite Amp. Eddies: Ingredients



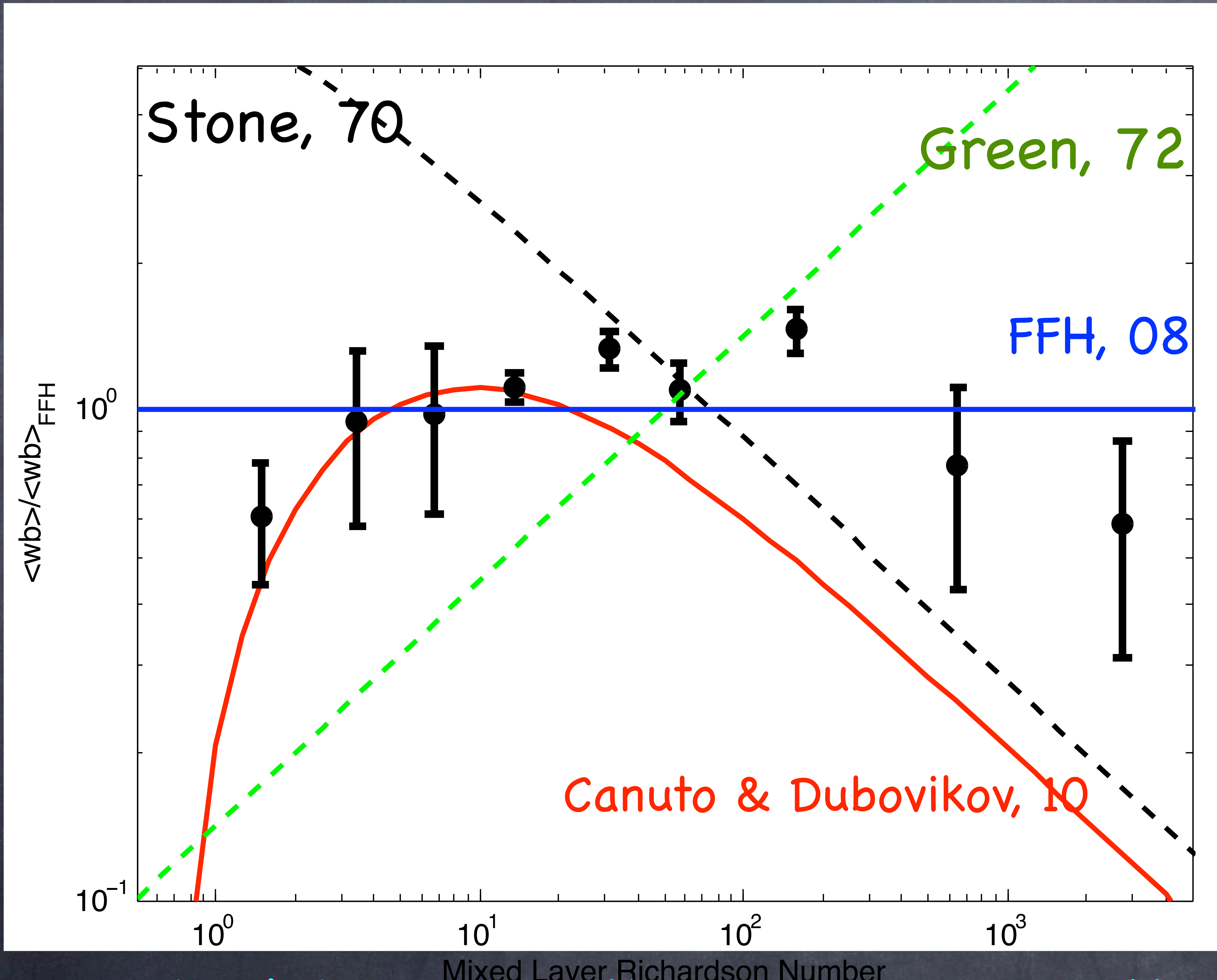
Linear Solution <w'b'>
for vert. structure.
As in Branscome '83...

Better than the Competition:



Extends over Ri more mesoscale (9000) than submesoscale (1)

Better than the Competition:



Green equals Visbeck (97) Held & Larichev (95)

Extends over Ri more mesoscale (9000) than submesoscale (1)

And Agrees with Deep Convection Studies:

Jones & Marshall (93,97), Haine & Marshall (98)