

# Eddies, Mixing and all that: Ocean Parameterization Developments from 4m to 400km

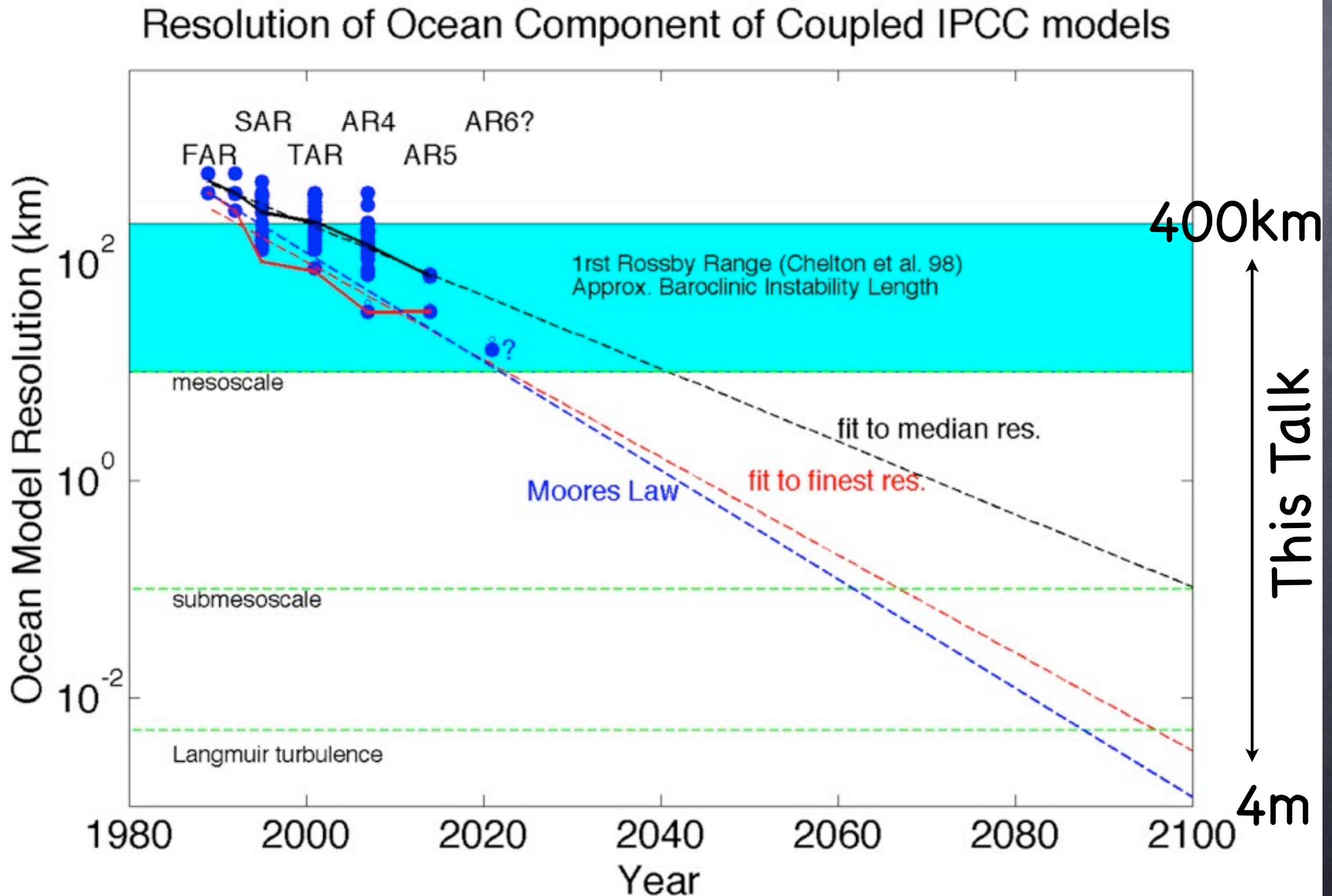
Baylor Fox-Kemper, CIRES and ATOC  
with

Luke Van Roekel (CIRES), Adrean Webb (CIRES/APPM), Scott Bachman (CIRES/ATOC), Andrew Margolin, Ian Grooms, Keith Julien, Raf Ferrari, NCAR Oceanography Section

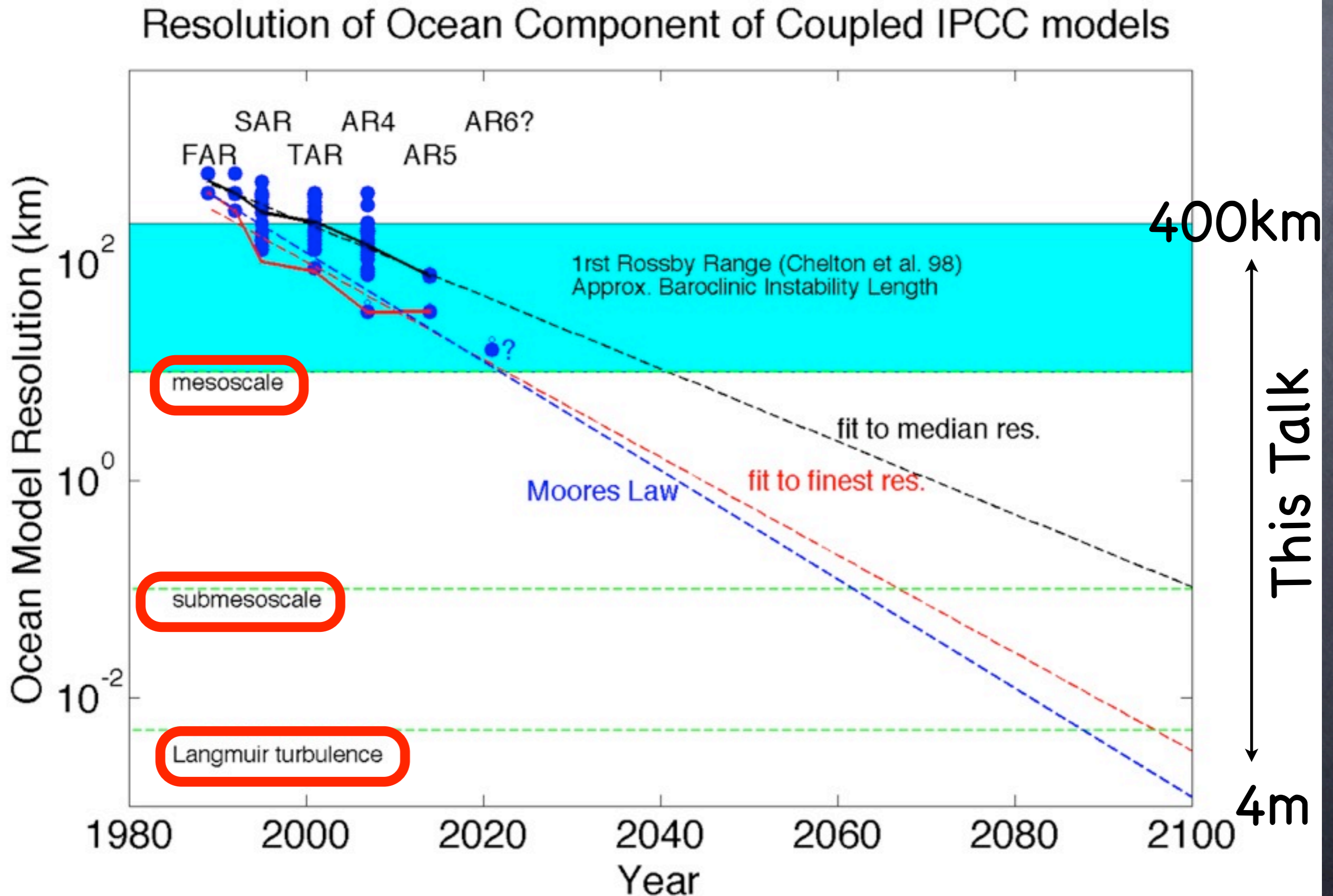
Sponsors:

NSF 0934737, 0855010, 0825614; NASA NNX09AF38G  
TeraGRID, IBM, UCAR, CIRES, CU-Boulder

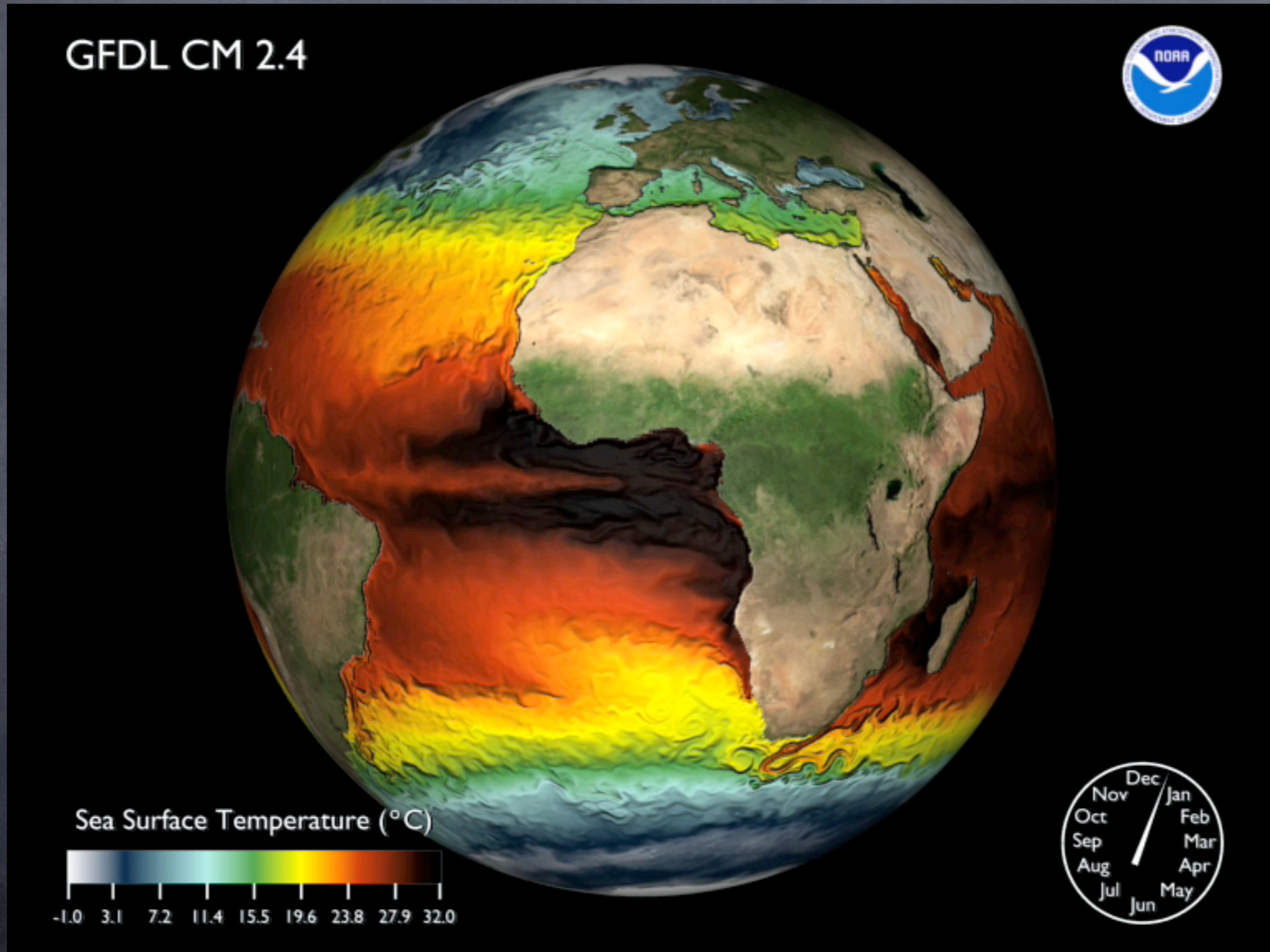
# Climate Forecasts (IPCC/CMIP Runs) have a very coarse ocean gridscale (>100km)



# Climate Forecasts (IPCC/CMIP Runs) have a very coarse ocean gridscale (>100km)



# A Bleeding-Edge Climate Model (in terms of ocean resolution) Has some ocean mesoscale instabilities:



# Ocean Equations\*:

## Boussinesq Fluid on Tangent Plane to a Rotating Sphere

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla_h \mathbf{u} + w \partial_z \mathbf{u} + Ro^{-1} \mathbf{f} \times \mathbf{u} = -\bar{P} \nabla_h p + Re^{-1} \nabla^2 \mathbf{u}$$

$$\partial_t w + \mathbf{u} \cdot \nabla_h w + w \partial_z w = -\bar{P} \partial_z p + \Gamma b \hat{\mathbf{z}} + Re^{-1} \nabla^2 w$$

Buoyancy (or S, T):  $\partial_t b + \mathbf{u} \cdot \nabla_h b + w \partial_z b = Pe^{-1} \nabla^2 b$

$$\nabla_h \cdot \mathbf{u} + \partial_z w = 0$$

Re, Pe for an affordable  
gridscale are  $10^6$  to  $10^{11}$

Numerics require  $O(1)$

\*From Grooms,  
Julien, & F-K, 11

Parameters		Ratios
Rossby	$Ro = \frac{U}{f_0 L}$	$A_\tau = \frac{L}{U \tau^*} = \frac{t^*}{\tau^*}$
Euler	$\bar{P} = \frac{p^*}{\rho_0 U^2}$	$A_h = \frac{L}{L_{pg}}$
Buoyancy	$\Gamma = \frac{BL}{U^2}$	$A_z = \frac{H}{L}$
Reynolds	$Re = \frac{UL}{\nu}$	$A_\beta = \frac{L_{pg}}{R} \tan \varphi_0$
Péclet	$Pe = \frac{UL}{\kappa}$	

# What is a subgrid model?

- Express the **coarse-grain averages** of quantities (including the subgrid effects), e.g.:

$$\overline{\frac{\partial \tau}{\partial t}} \quad \overline{\frac{\partial u}{\partial x}} \quad \overline{\frac{\partial u \tau}{\partial x}}$$

- As a function of the **resolved coarse-grain fields**

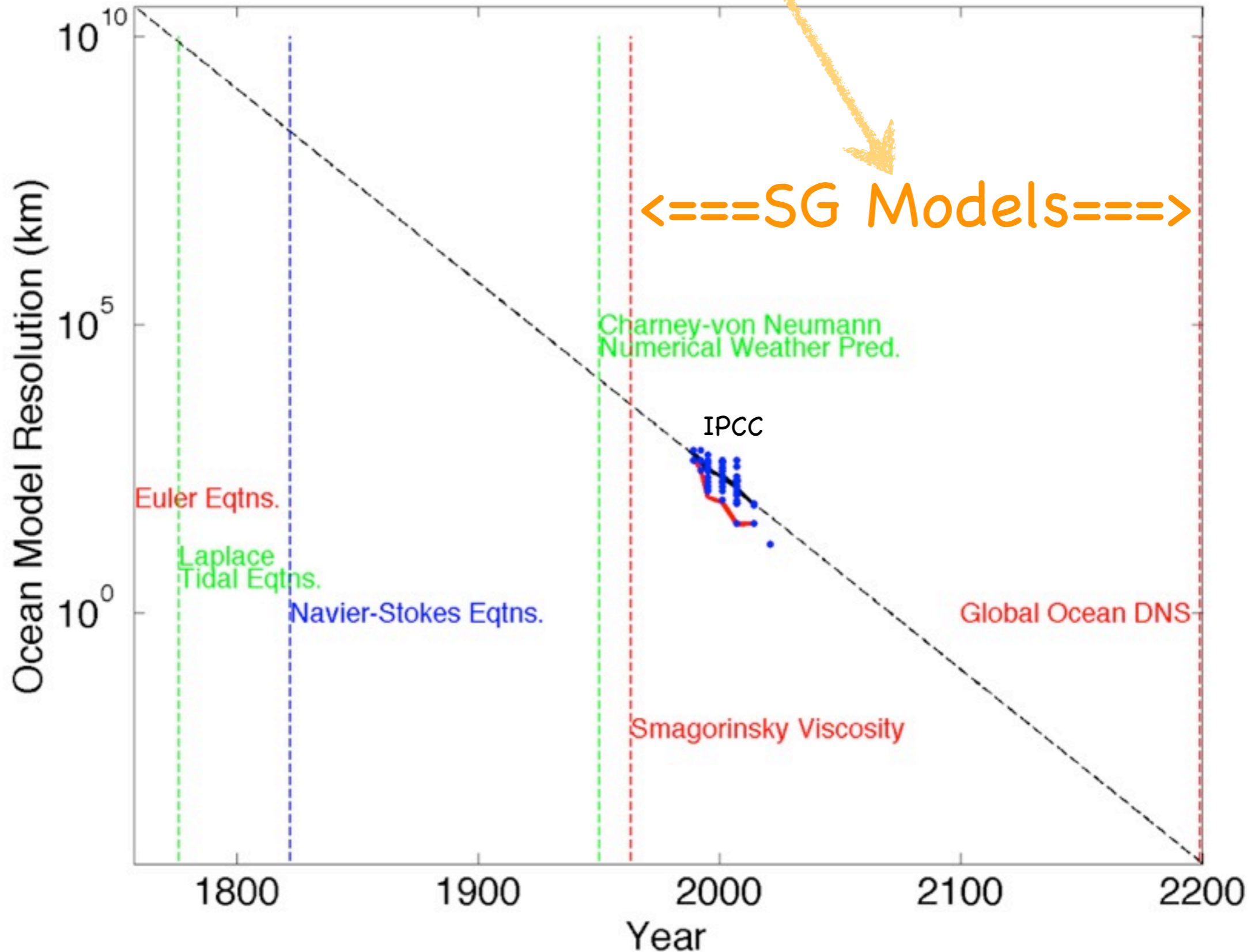
$$\overline{\frac{\partial \tau}{\partial t}} = \frac{\partial \bar{\tau}}{\partial t} \quad \overline{\frac{\partial u}{\partial x}} = \frac{\partial \bar{u}}{\partial x} \quad \overline{\frac{\partial u \tau}{\partial x}} = \frac{\partial \bar{u} \bar{\tau}}{\partial x} + \frac{\partial \overline{u' \tau'}}{\partial x}$$

- Note that **nonlinear** terms require **special treatment**
- And Couple different scales, small talks to large!

# Different Uses, Different Needs

- **MOLES** (e.g., the CM2.4 movie before; grid 5–25km)
  - Mesoscale Ocean Large Eddy Simulation
  - Largest eddies are resolved--need smooth cutoff in mesoscale range
- **MORANS** (e.g., typical IPCC/CMIP models; grid >50km)
  - Mesoscale Ocean Reynolds-Averaged Navier-Stokes
  - Nothing resolved, unresolved to be parameterized
- **SMORANS** (e.g., Fox-Kemper et al., 2011; grid 1–10km)
  - Submesoscale Ocean...
  - Mesoscale resolved, submesoscale unresolved...
- **NOTE:** RANS contains all smaller scales that couple!

# Extrapolate for historical perspective: The Golden Era of Subgrid Modeling is Now!





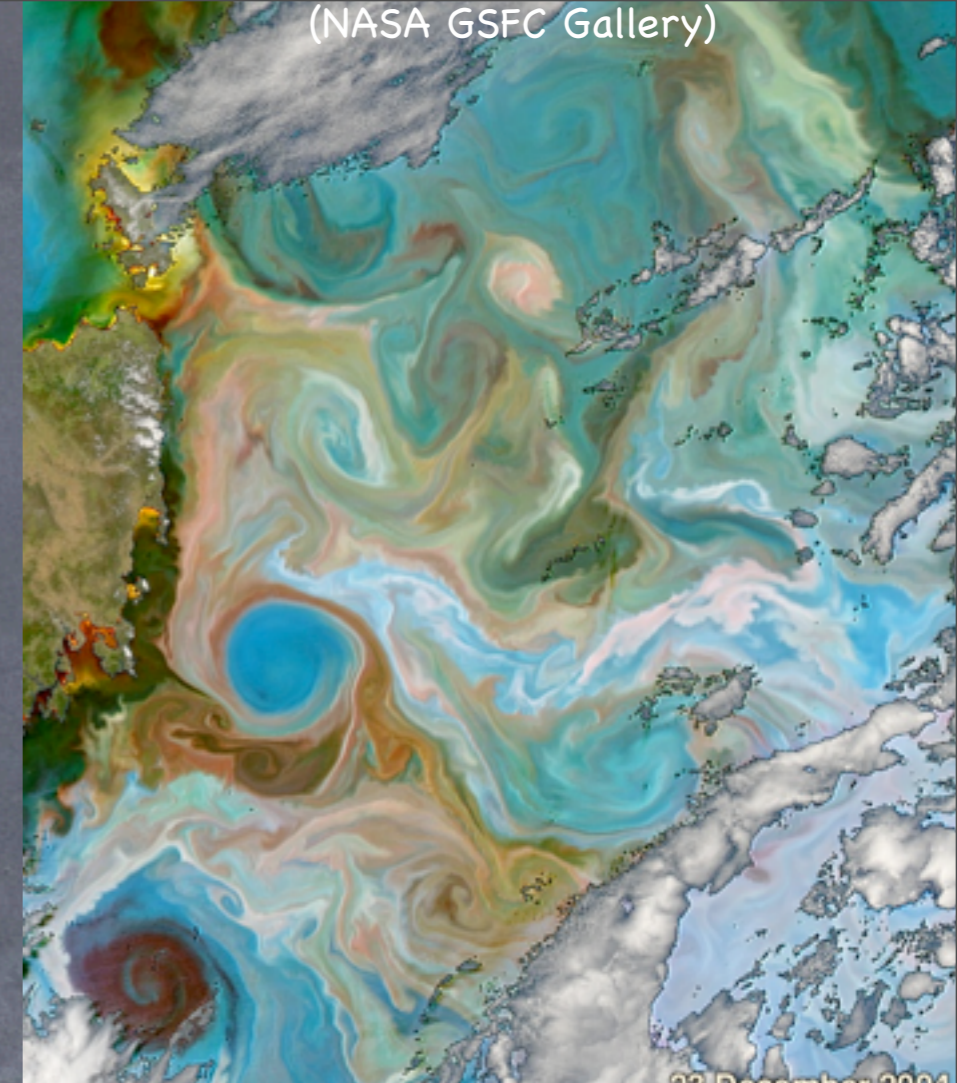
# Mesoscale Parameterizations

- Researchers have already cast much darkness on this subject and if they continue their investigations we shall soon know nothing at all about it.

• --Mark Twain

# The Character of the Mesoscale

←  
100  
km



(Capet et al., 2008)

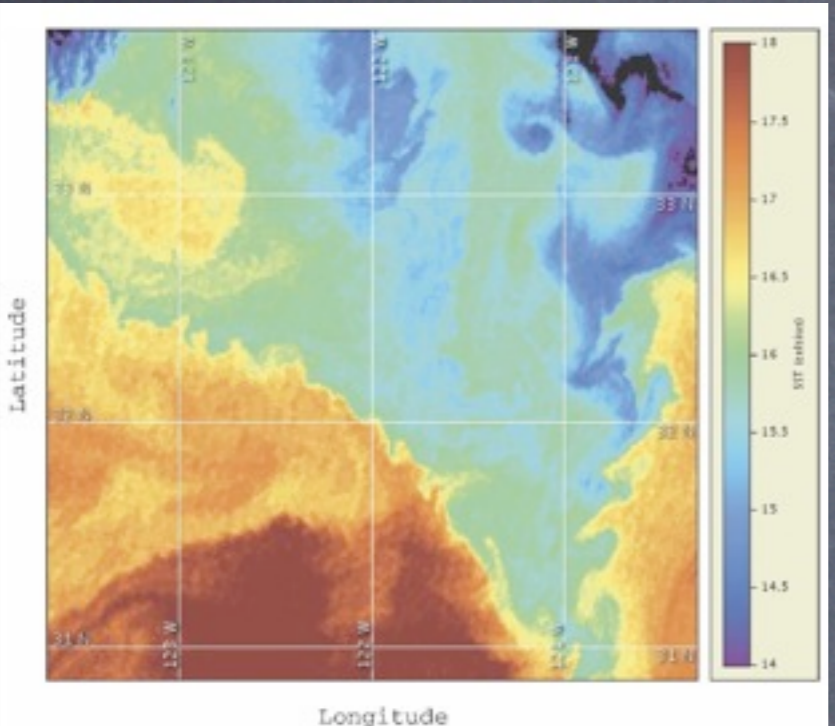
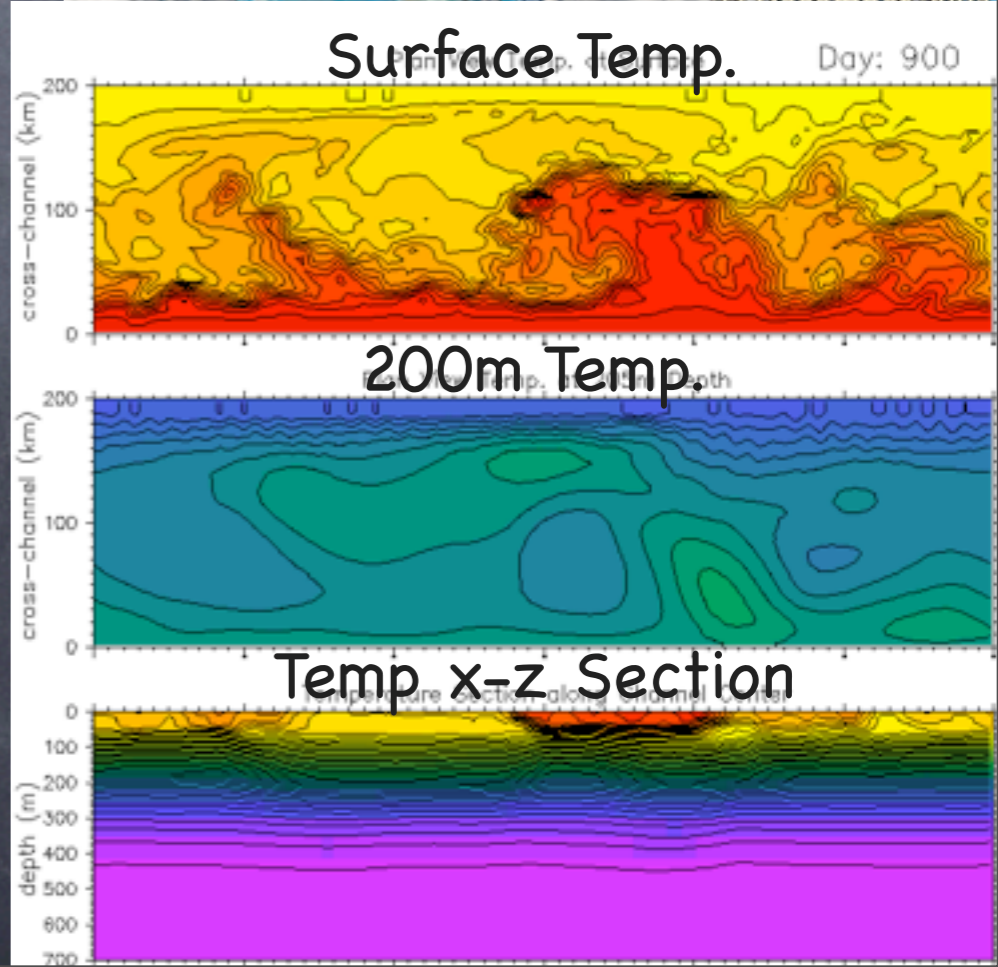


FIG. 16. Sea surface temperature measured at 1832 UTC 3 Jun 2006 off Point Conception in the California Current from CoastWatch (<http://coastwatch.pfeg.noaa.gov>). The fronts between recently upwelled water (i.e., 15°–16°C) and offshore water ( $\geq 17^\circ\text{C}$ ) show submesoscale instabilities with wavelengths around 30 km (right front) or 15 km (left front). Images for 1 day earlier and 4 days later show persistence of the instability events.

- Boundary Currents
- Eddies
- $Ro=O(0.1)$
- $Ri=O(1000)$
- Full Depth
- Eddies strain to produce Fronts
- 100km, months

Eddy processes mainly **baroclinic & barotropic instability**. Parameterizations of baroclinic instability (GM, Visbeck...).

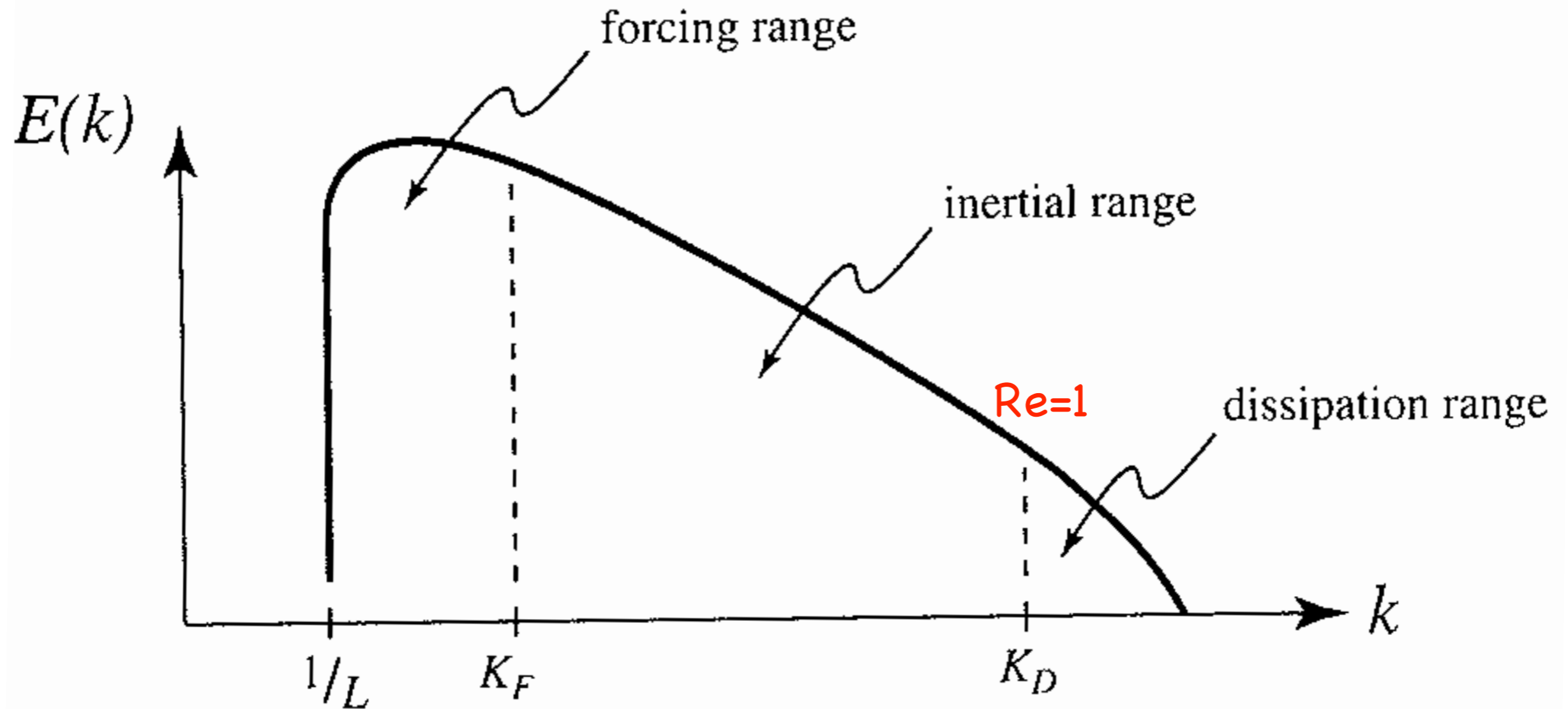


**A MOLES Closure:**  
Smagorinsky & Kolmogorov  
VS.  
Leith & Kraichnan

**Idea:** Replace Eddy Momentum Fluxes with  
Artificially Inflated Viscosity

**Relies on:**  
Energy Source, Dissipation, Flow  
& Dimensional Analysis

# Truncation of Cascades

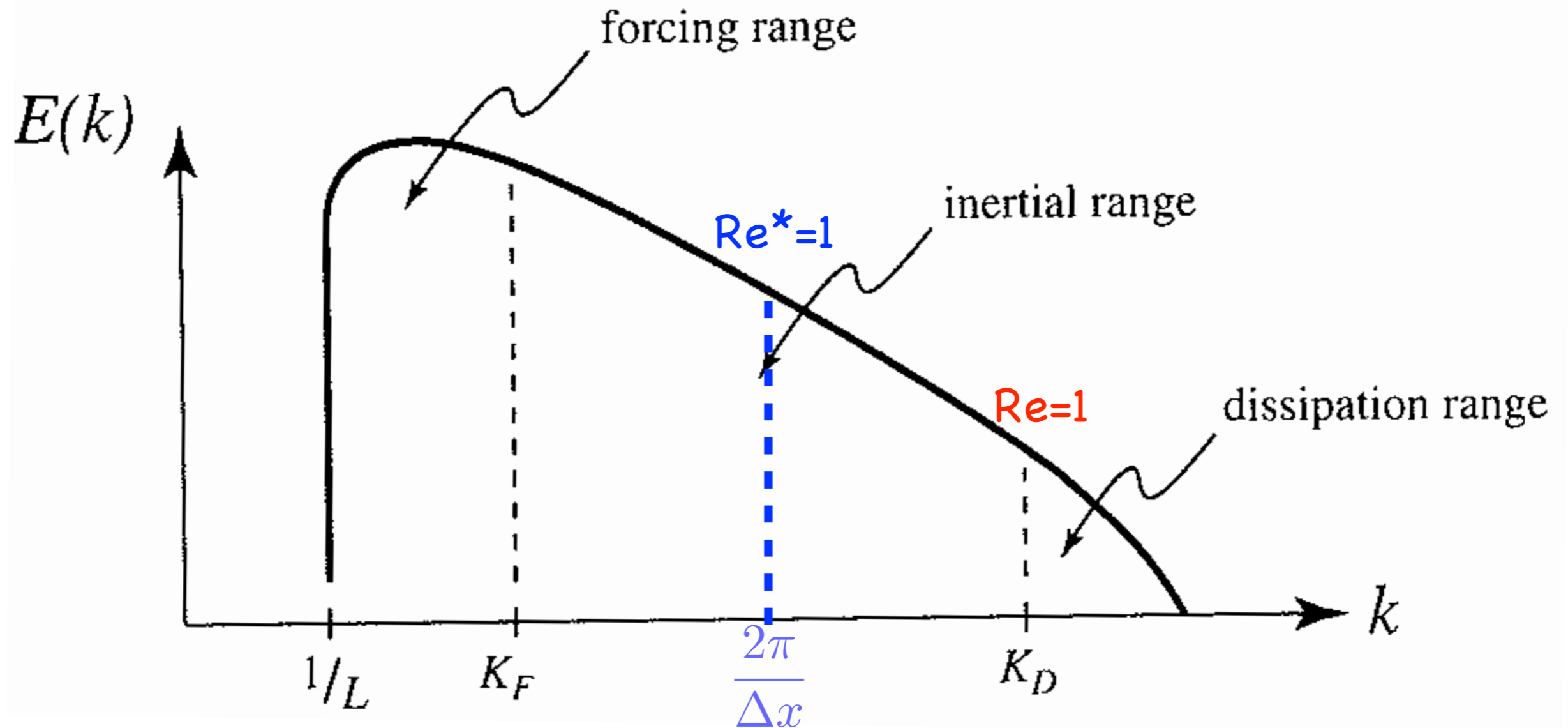


Power Spectrum:  
Energy/Wavenumber

$$\langle E \rangle = \frac{1}{V} \iiint \frac{1}{2} (\mathbf{u} \cdot \mathbf{u}) dV = \int_0^{\infty} E(k) dk.$$

1941: Kolmogorov Envisions the Inertial Range

# Truncation of Cascades



1963: Smagorinsky Devises Viscosity Scaling,

So that the Energy Flow is Preserved,

but order-1 gridscale Reynolds #:  $Re^* = UL/\nu_*$

$$\nu_{*h} = \left( \frac{\Upsilon_h \Delta x}{\pi} \right)^2 \sqrt{\left( \frac{\partial u_*}{\partial x} - \frac{\partial v_*}{\partial y} \right)^2 + \left( \frac{\partial u_*}{\partial y} + \frac{\partial v_*}{\partial x} \right)^2}$$

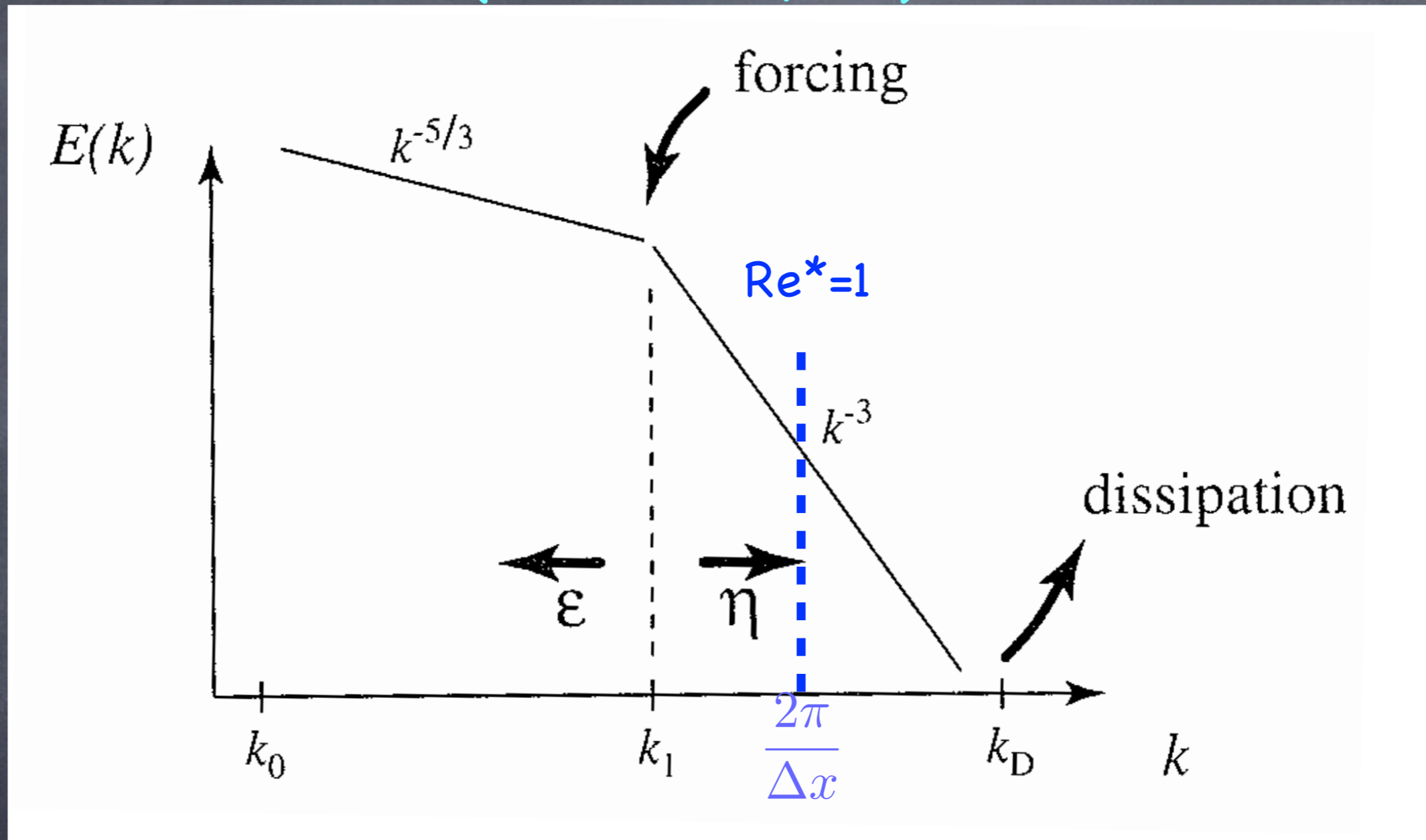
# Except... Ocean Turbulence isn't 3d Turbulence at the Gridscale

- The ocean is wide (10000km)
- But not deep (4km)
- So motions are largely 2d
- The layer of blue paint on a globe has roughly the right aspect ratio!
- MOLES grid aspect is similar



# 2d Turbulence Differs

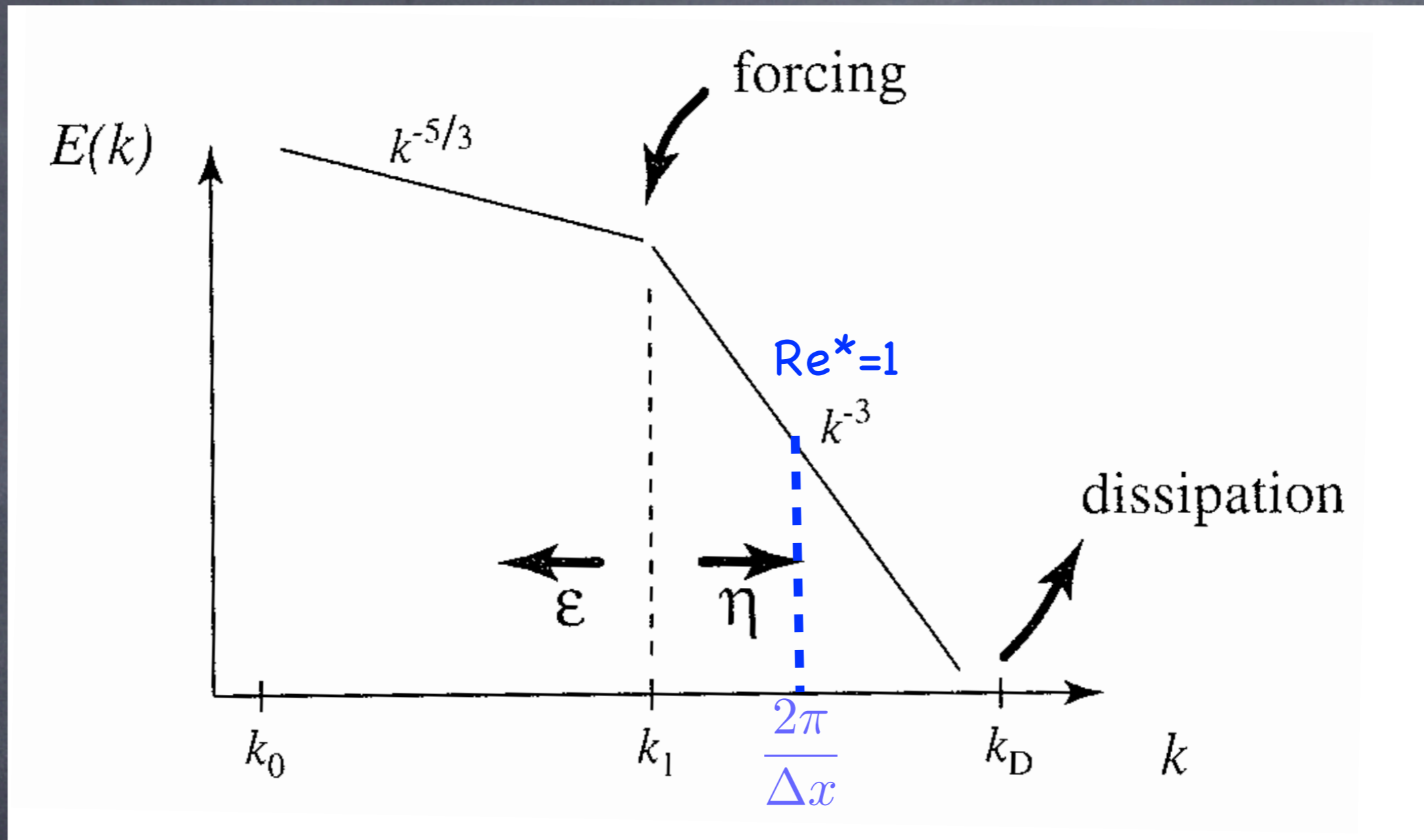
(Kraichnan, 67)



2 Conserved Quantities: Energy and Enstrophy  
(vorticity variance)

Energy Cascades Upscale, Enstrophy Downscale...

# 2d Turbulence Differs



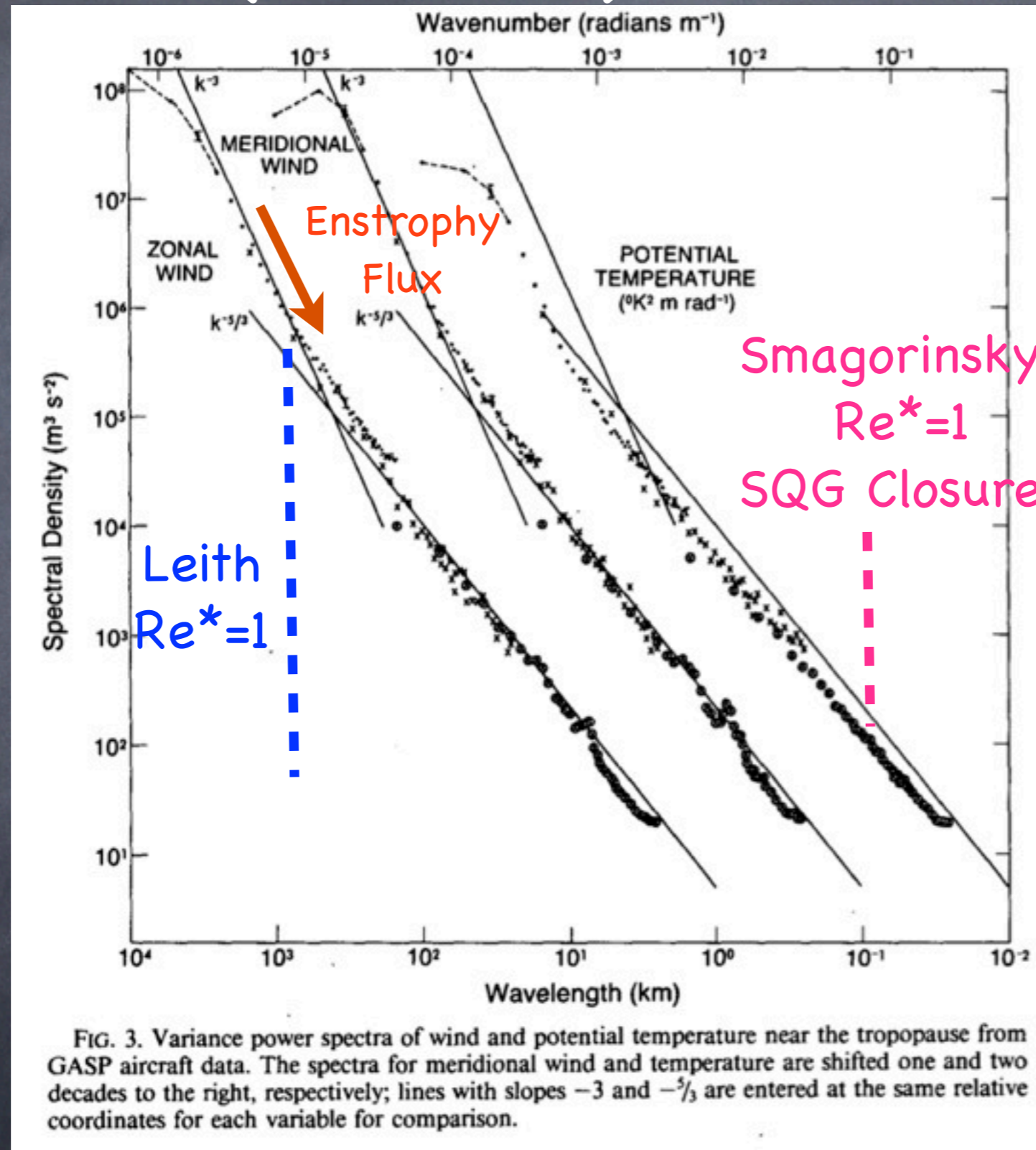
1996: Leith Devises Viscosity Scaling,  
So that the Enstrophy Flow is Preserved

$$\nu_* = \left( \frac{\Lambda \Delta x}{\pi} \right)^3 \left| \nabla_h \left( \frac{\partial u_*}{\partial y} - \frac{\partial v_*}{\partial x} \right) \right|.$$



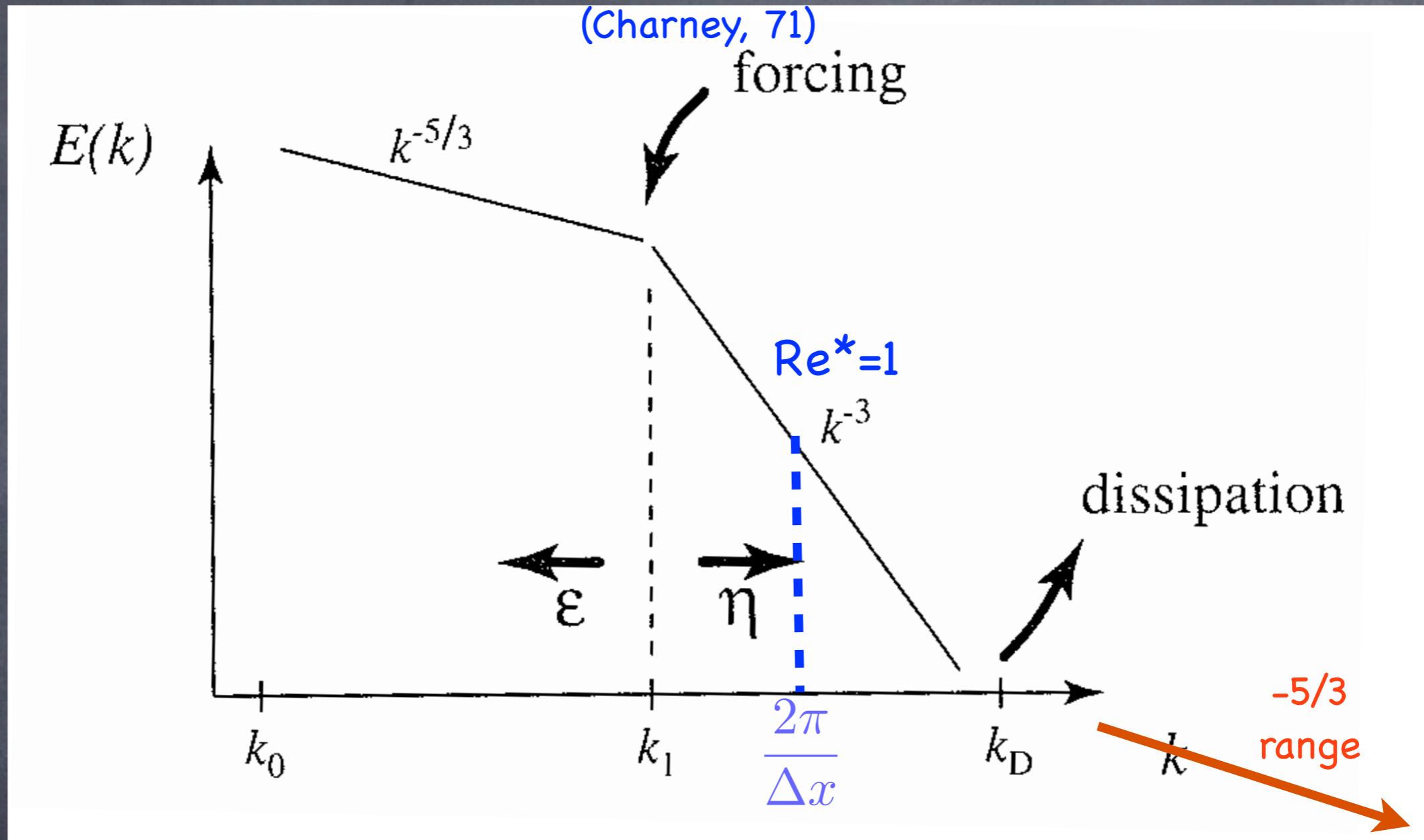
# 2-d Turbulence is different from Atmosphere (Ocean\*?) macro-turbulence

Figure adapted from Nastrom & Gage (85)



\* My student, Katie McCaffrey, is working on ocean spectra from obs.

# MOLES Turbulence Like Pot'l Enstrophy cascade, but divergent

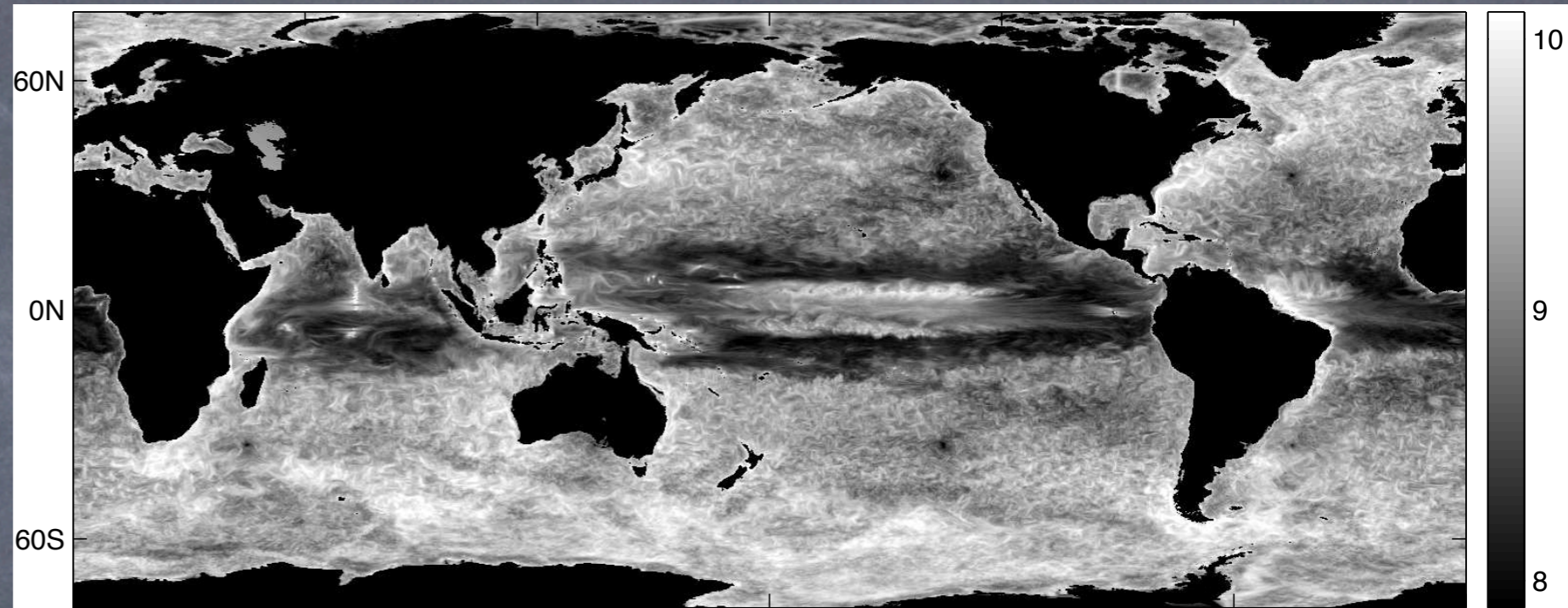


2008: F-K & Menemenlis Revise Leith Viscosity Scaling,  
So that diverging, vorticity-free, modes are also damped

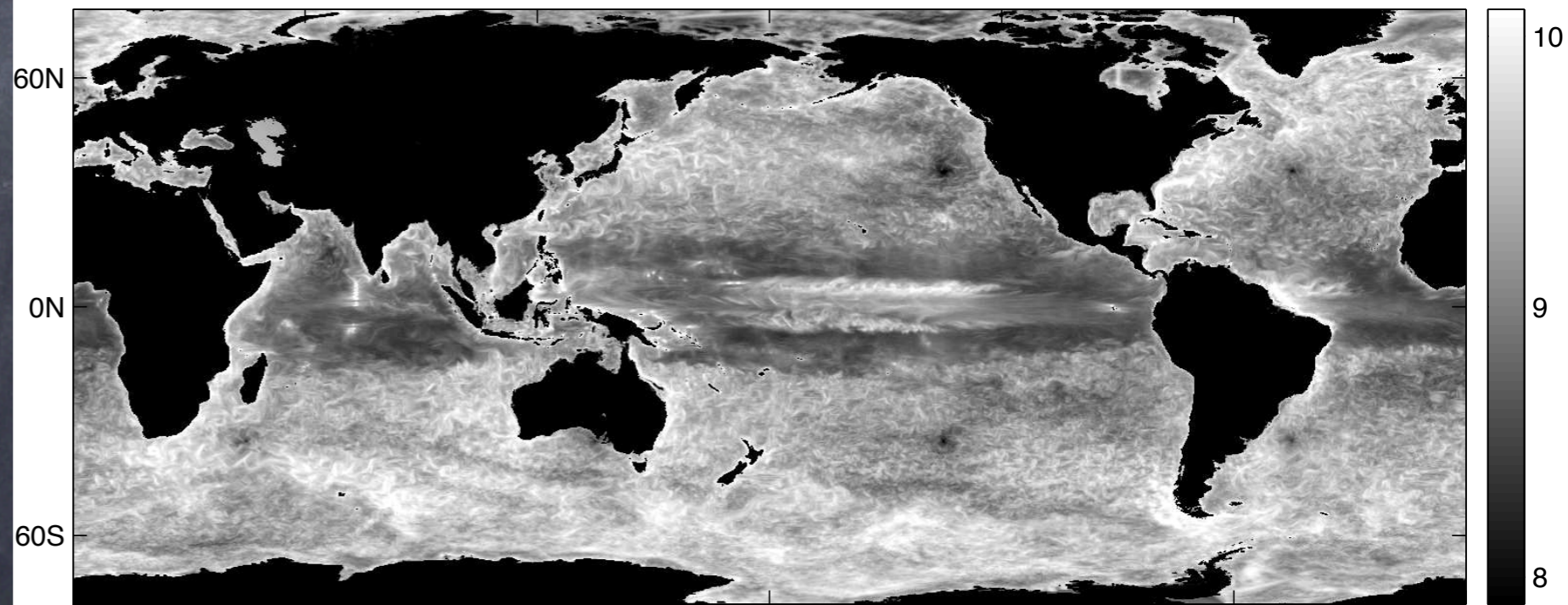
$$\nu_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2}$$

Makes viscosity a bit bigger, especially near Eq.

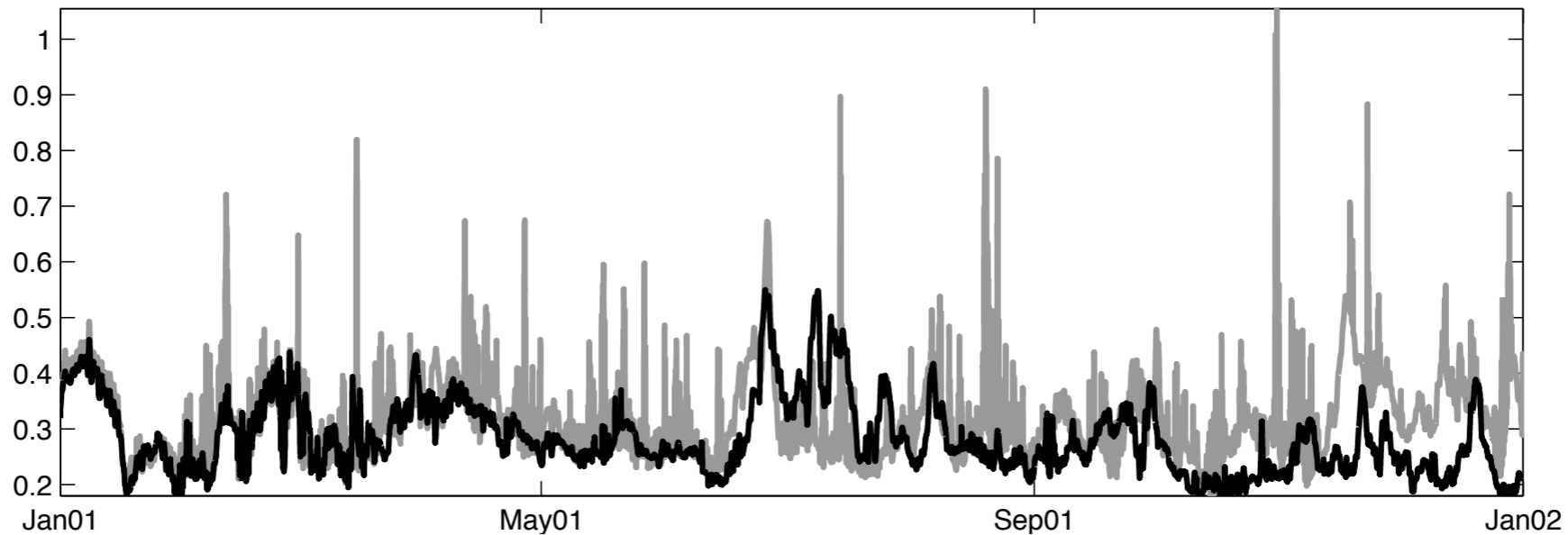
Leith



F-K&M



# But matters a lot for stability!



**Figure 4.** Maximum Courant number,  $w\Delta t/\Delta z$ , for vertical advection. Gray line is from the *LeithOnly* integration and black line is from the *LeithPlus* integration.

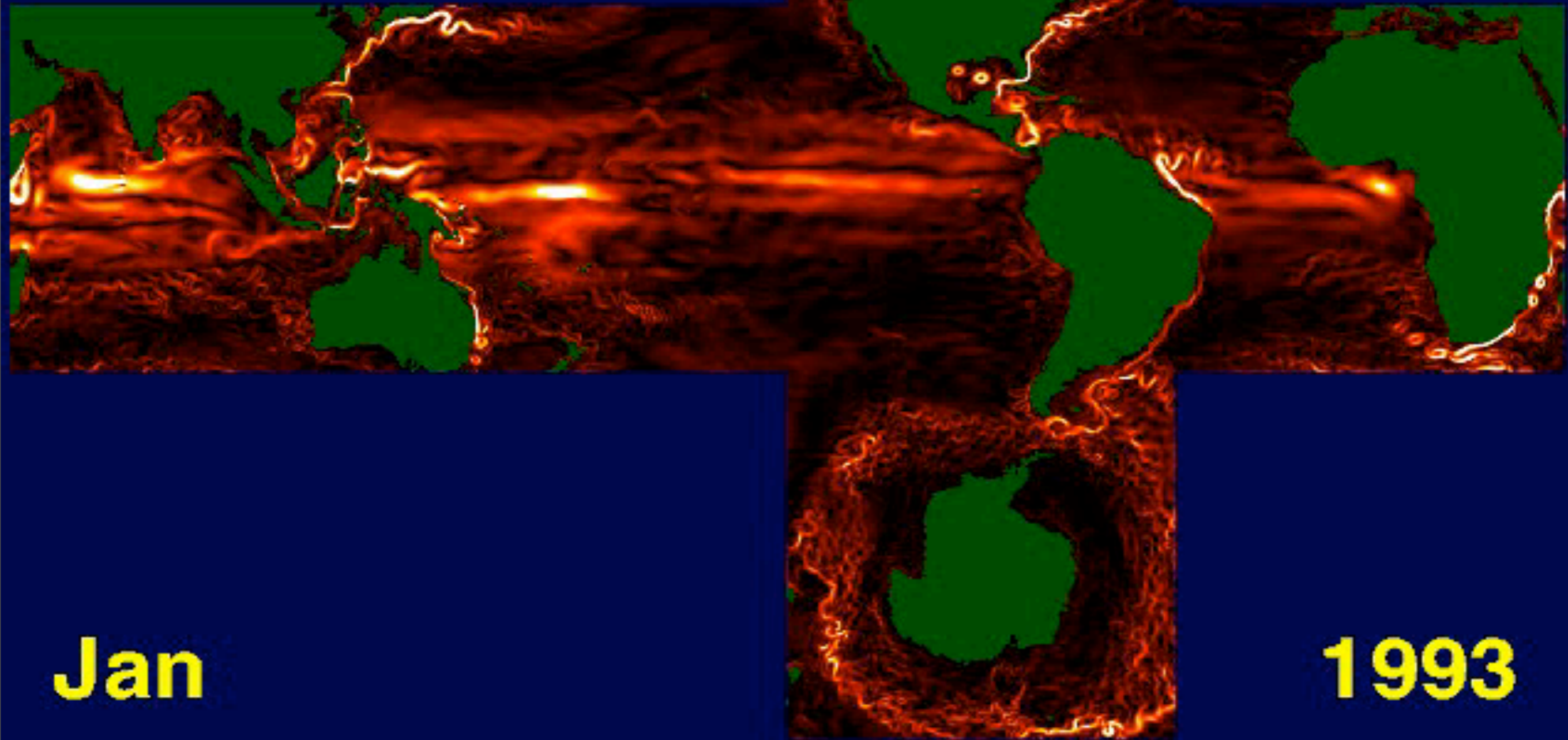
Fox-Kemper & Menemenlis, 2008

It works here!  
Even with irregular grid!



It works here!  
Even with irregular grid!

**|v|@15m  
m/s**



**Jan**

**1993**

ECCO2 (Estimating the Circulation & Climate of the Ocean, Phase 2, [www.ecco2.org](http://www.ecco2.org))

It works here!  
Even with irregular grid!

Spectra & Viscosity are good for MOLES,  
 but... Asymptotics tell us to worry about  
 scalar transport, not momentum for MORANS!

Equations for Large Scale Ocean Dynamics:

$$\begin{aligned}
 (f_0 + \beta Y) \hat{\mathbf{z}} \times \bar{\mathbf{u}} &= -\bar{\nabla}_h \bar{p}, \\
 \partial_z \bar{p} &= \bar{b}, \\
 \bar{\nabla}_h \cdot \bar{\mathbf{u}} + \partial_z \bar{w} &= 0, \\
 \partial_\tau \bar{b} + \bar{\mathbf{u}} \cdot \bar{\nabla}_h \bar{b} + \bar{w} \partial_z \bar{b} + \bar{\nabla}_h \cdot \overline{(\mathbf{u}'b')} + \partial_z \overline{(w'b')} &= \kappa_v \partial_z^2 \bar{b}
 \end{aligned}$$

No more momentum fluxes!, i.e.,

$$\bar{\nabla}_h \cdot \overline{(\mathbf{u}'\mathbf{u}')}$$

Grooms, Julien, & F-K, 2011



# TESTING MORANS Closures:

Validation & Spatial  
variations of

Gent-McWilliams & Redi

**Idea:** Study the fluxes of passive tracers and  
reconstruct the flux-gradient relationship

**Relies on:**

Unique Lagrangian Transport Operator  
for All Tracers

# Mesoscale Eddy Parameterizations

all have the form:

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\bar{\tau}$$

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

With John Dennis & Frank Bryan, we took a  
POP0.1° Normal-Year forced model (yrs 16-20)

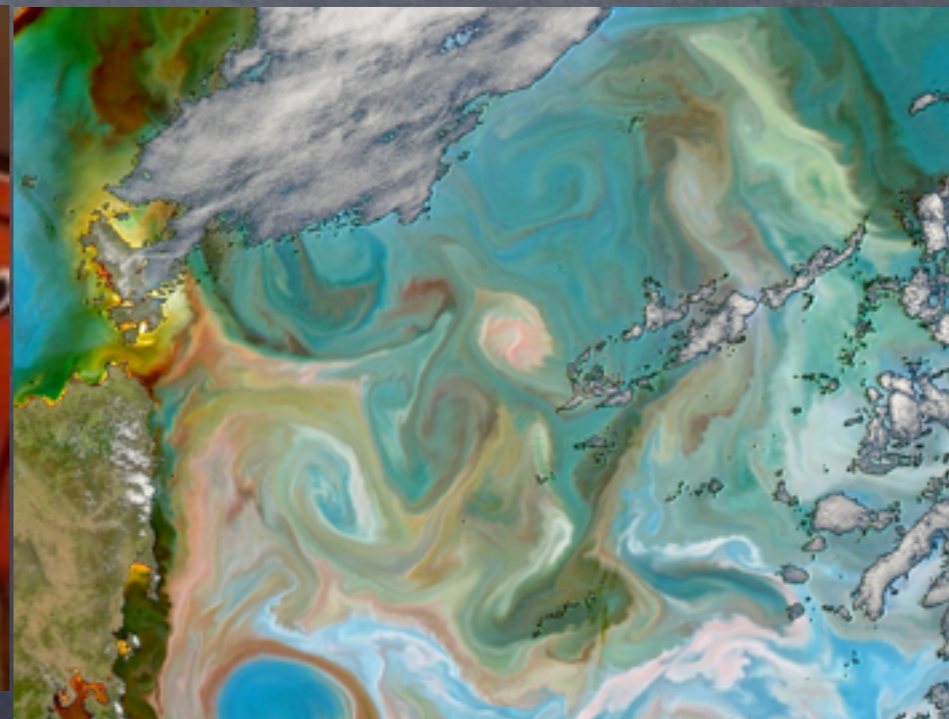
Added 9 Passive tracers--restored x,y,z @ 3 rates

Kept all the eddy fluxes for passive & active

Coarse-grained to 2°, transient eddies, tracers to M

# Does this cover all the degrees of freedom?

- More tracers does provide a just-determined or overdetermined (Moore-Penrose/least squares) problem for  $M$  with a unique answer, but...
- Different tracers will have different fluxes as they feel the subgrid 'nooks and crannies' of the mesoscale eddies!



$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\bar{\tau}$$

Sym Part=Anisotropic\* Redi

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ K_{yx} & K_{yy} & \hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ \hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & \hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & \tilde{\nabla}_z\cdot\mathbf{K}\cdot\tilde{\nabla}_z \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

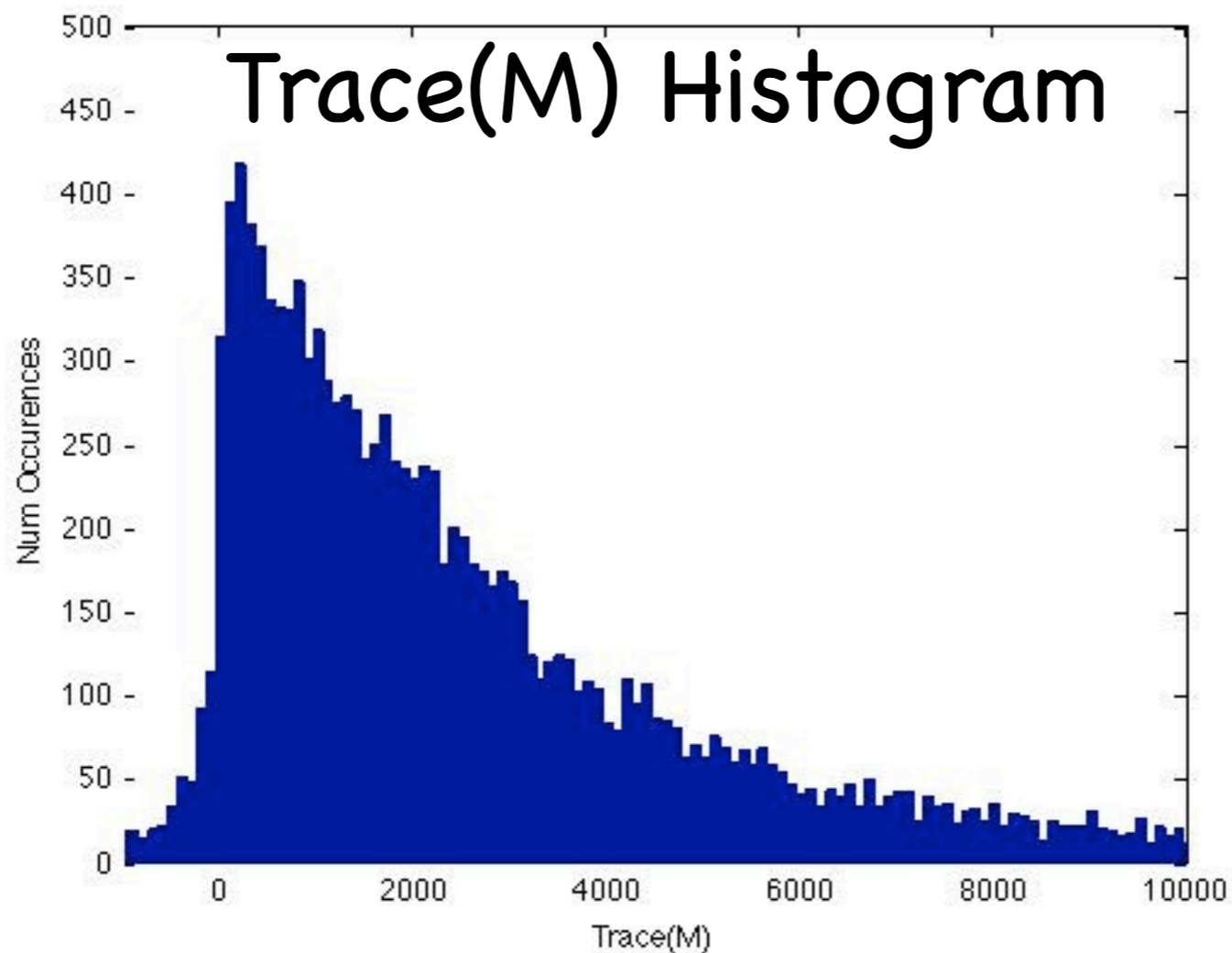
AntiSym Part=Anisotropic\* GM

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ 0 & 0 & -\hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z \\ \hat{x}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & \hat{y}\cdot\mathbf{K}\cdot\tilde{\nabla}_z & 0 \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$

Yellow  $\mathbf{K}$  'are' horizontal stirring & mixing

# Are Diffusivity Values Reasonable?

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} z \\ K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} z \\ \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} z & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} z & \tilde{\nabla} z \cdot \mathbf{K} \cdot \tilde{\nabla} z \end{bmatrix} \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix}$$



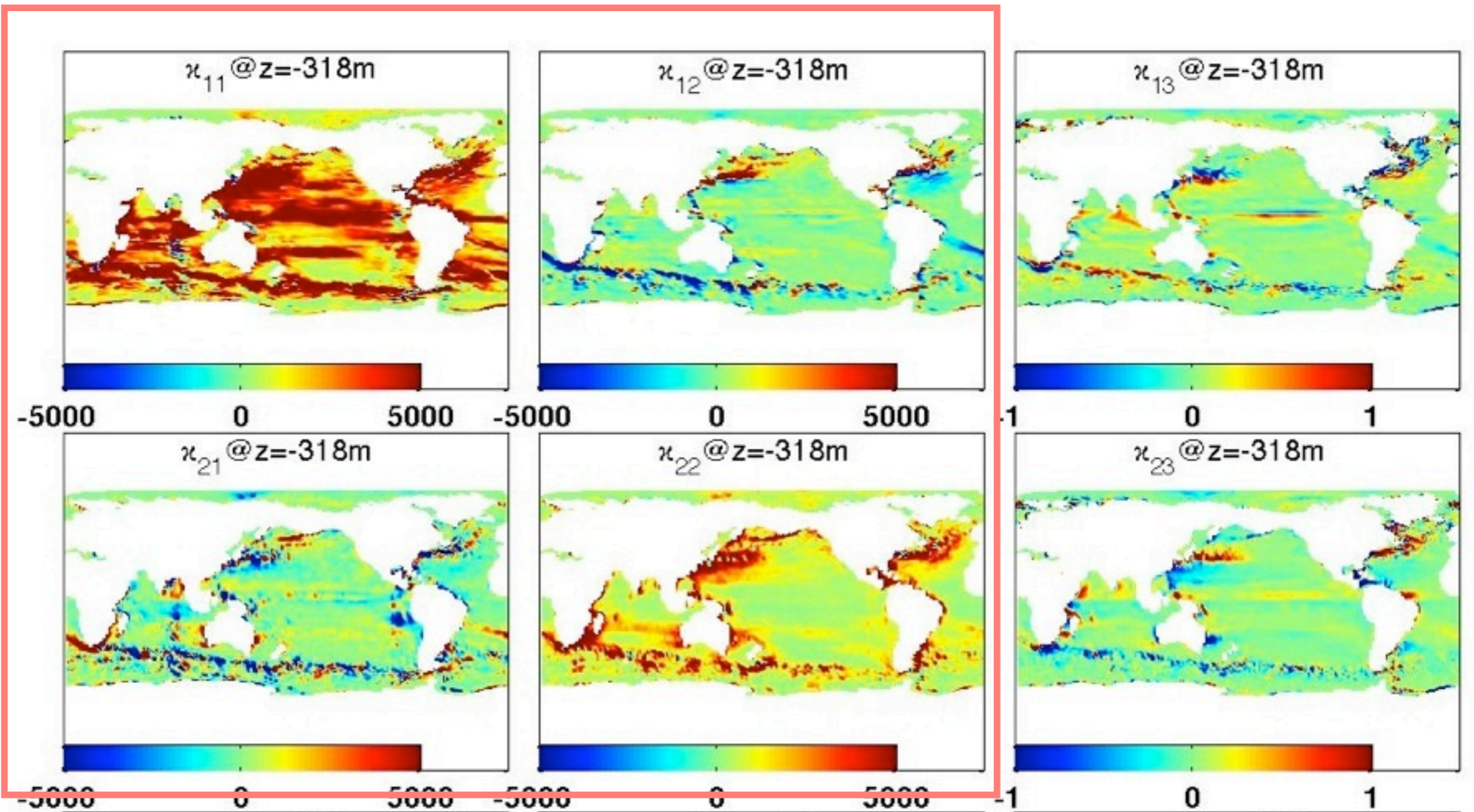
Hor. Diffusivity is roughly  $\text{Trace}(M)/2$

Peak of Diffusivity near  $250 \text{ m}^2/\text{s}$

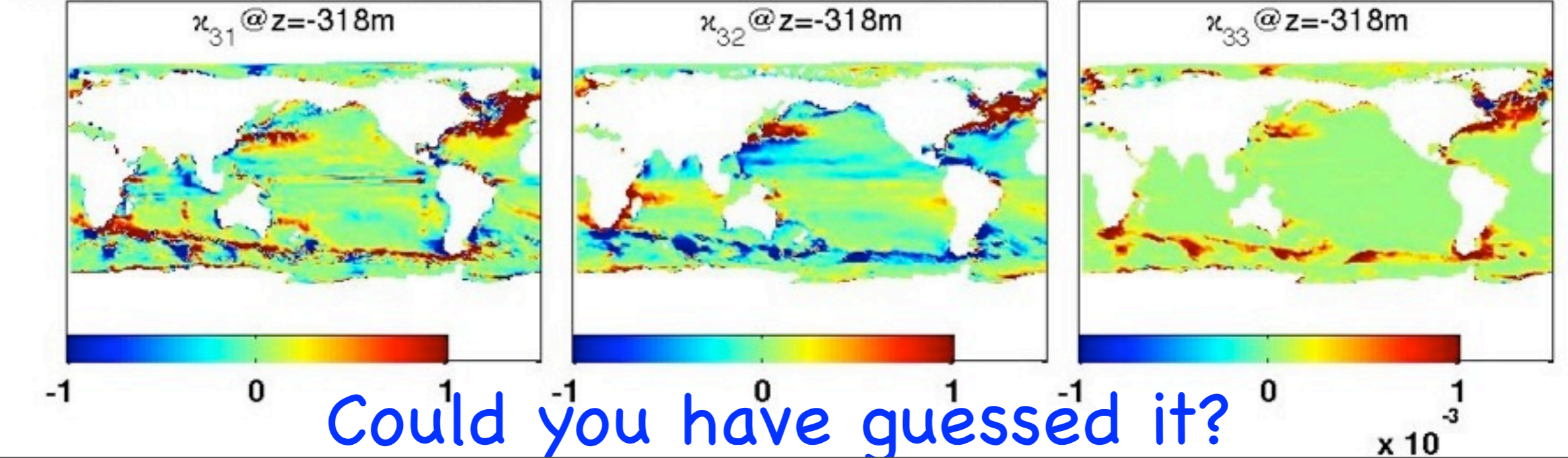
Median Diffusivity near  $1000 \text{ m}^2/\text{s}$

<6% negative

K



M

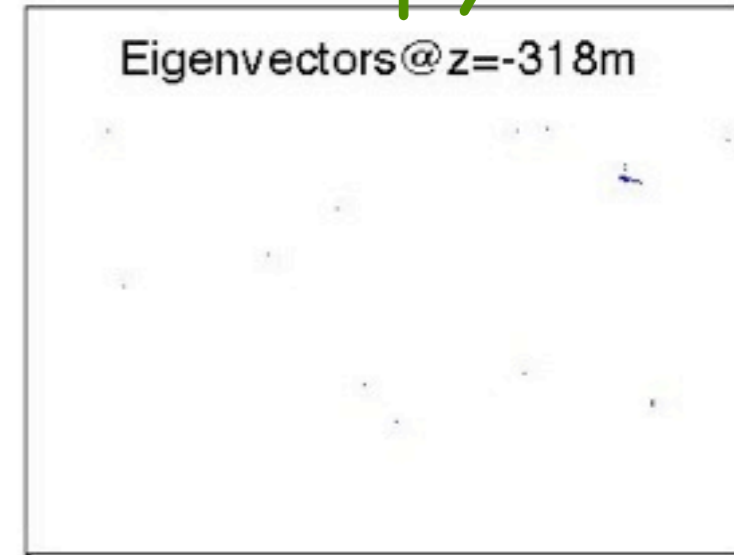
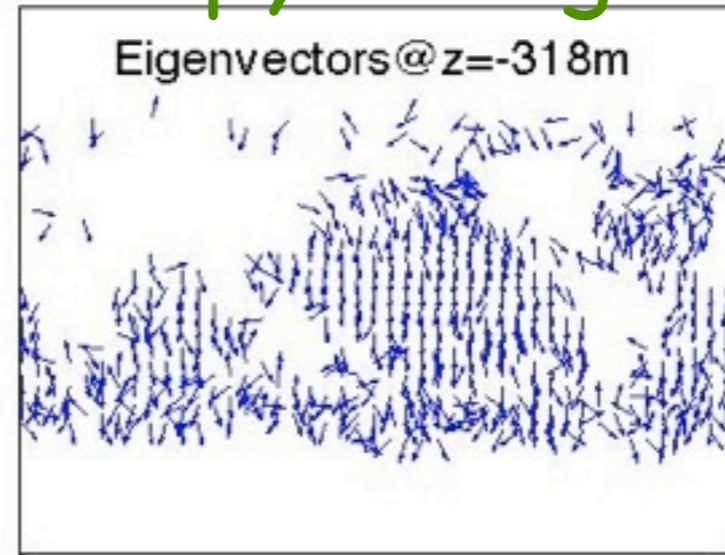
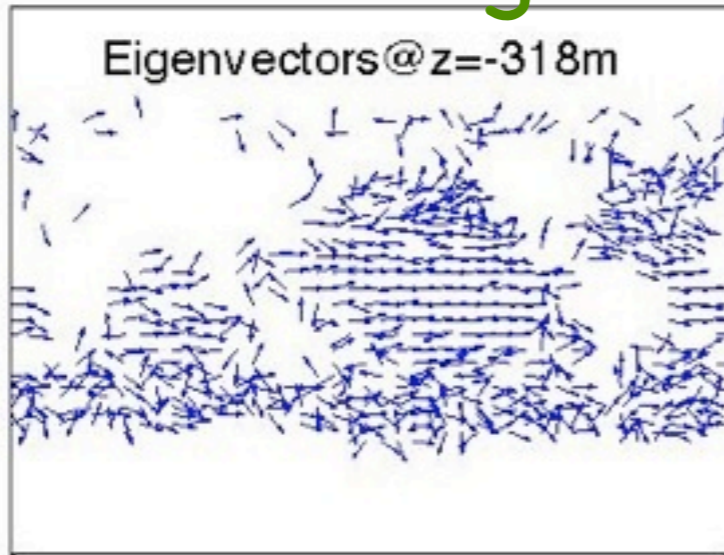


Could you have guessed it?

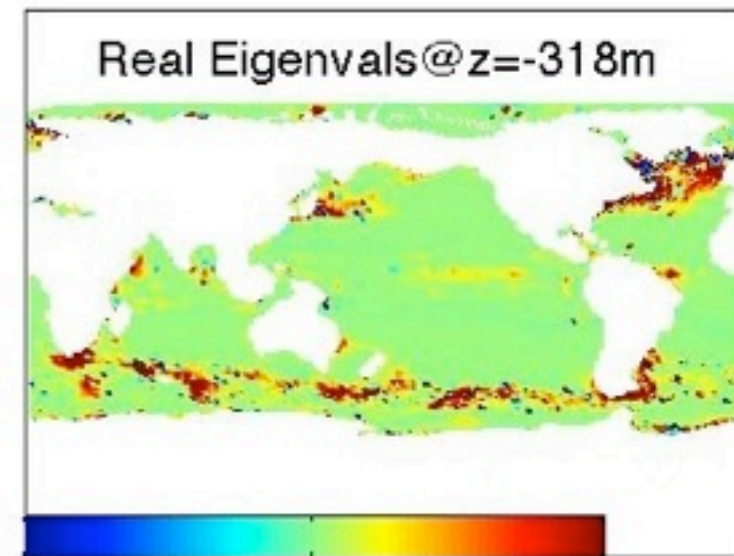
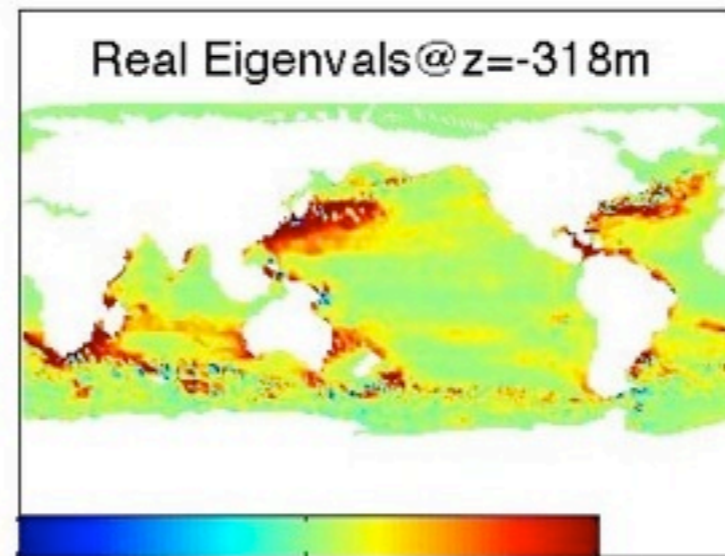
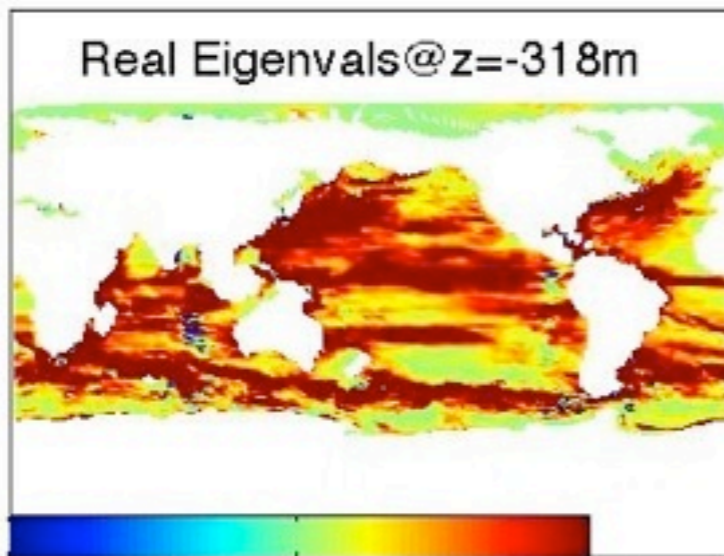
$\times 10^{-3}$

# Result: Strong Anisotropy Along/Across Isopycnals

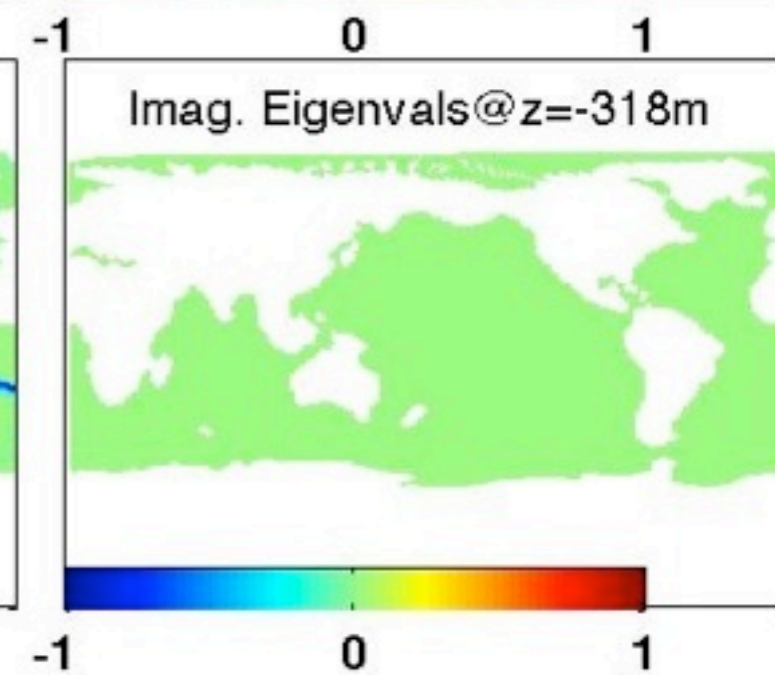
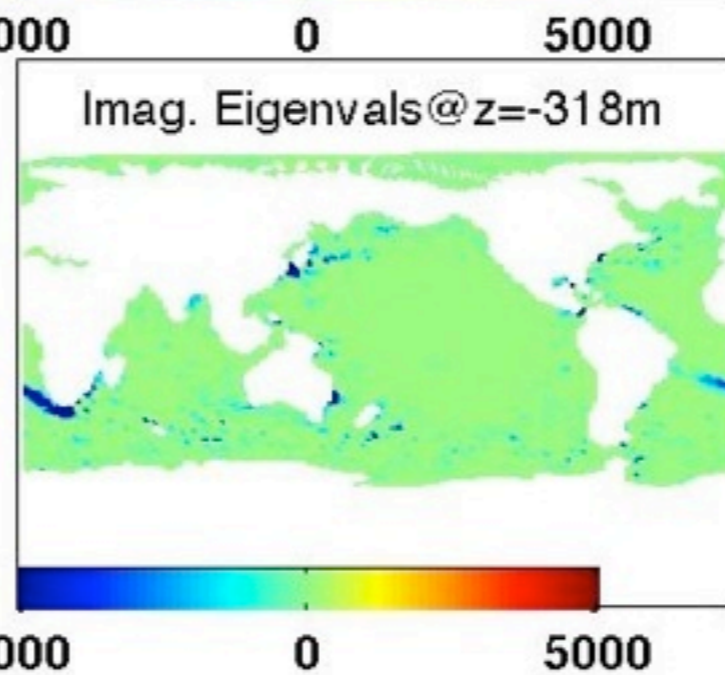
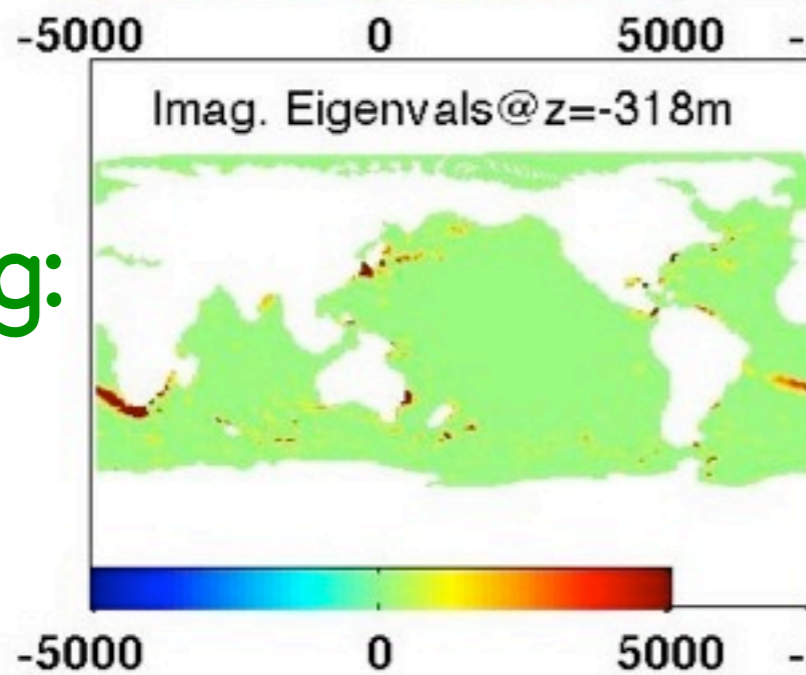
Mixing  
direction



Mixing:



Stirring:

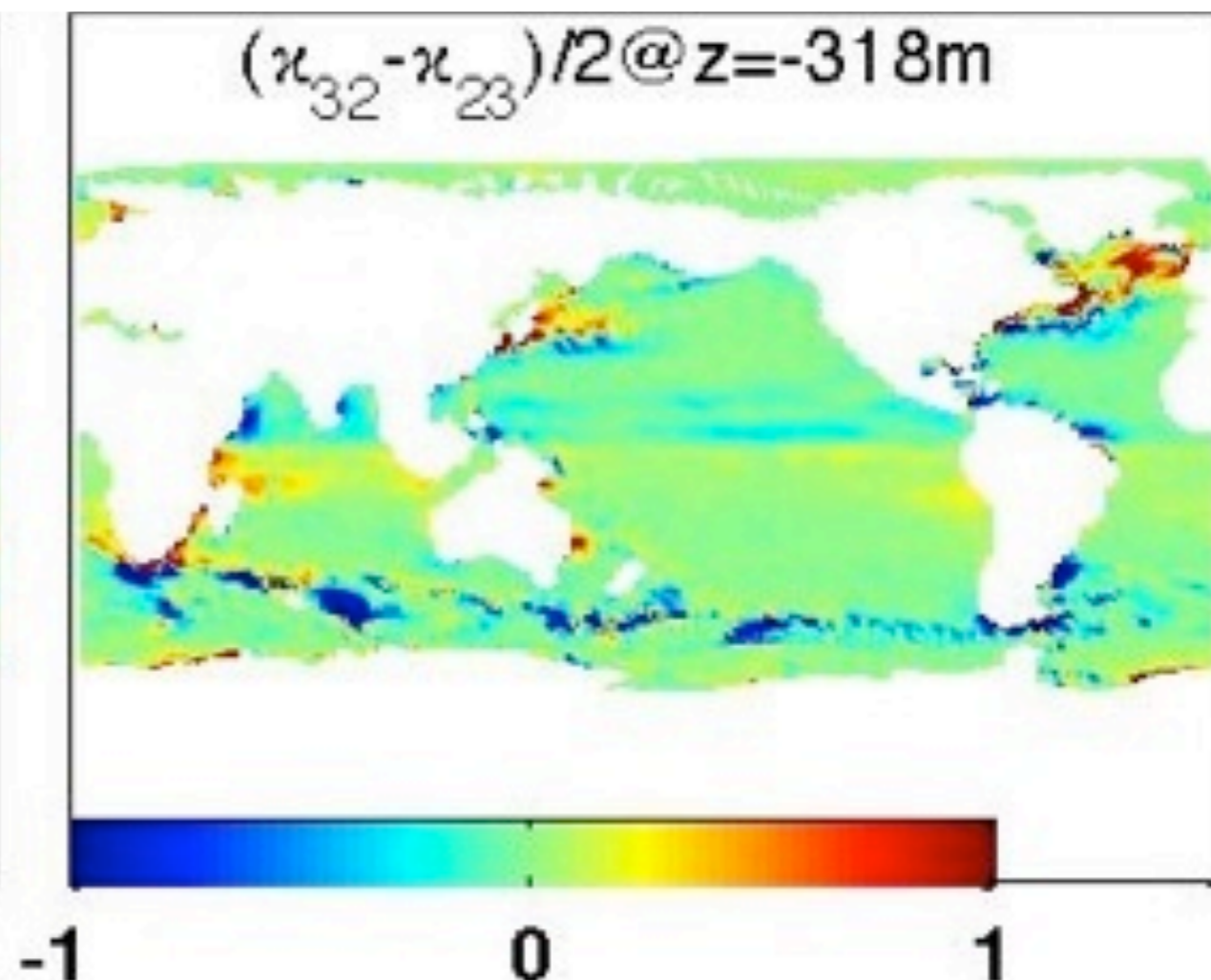
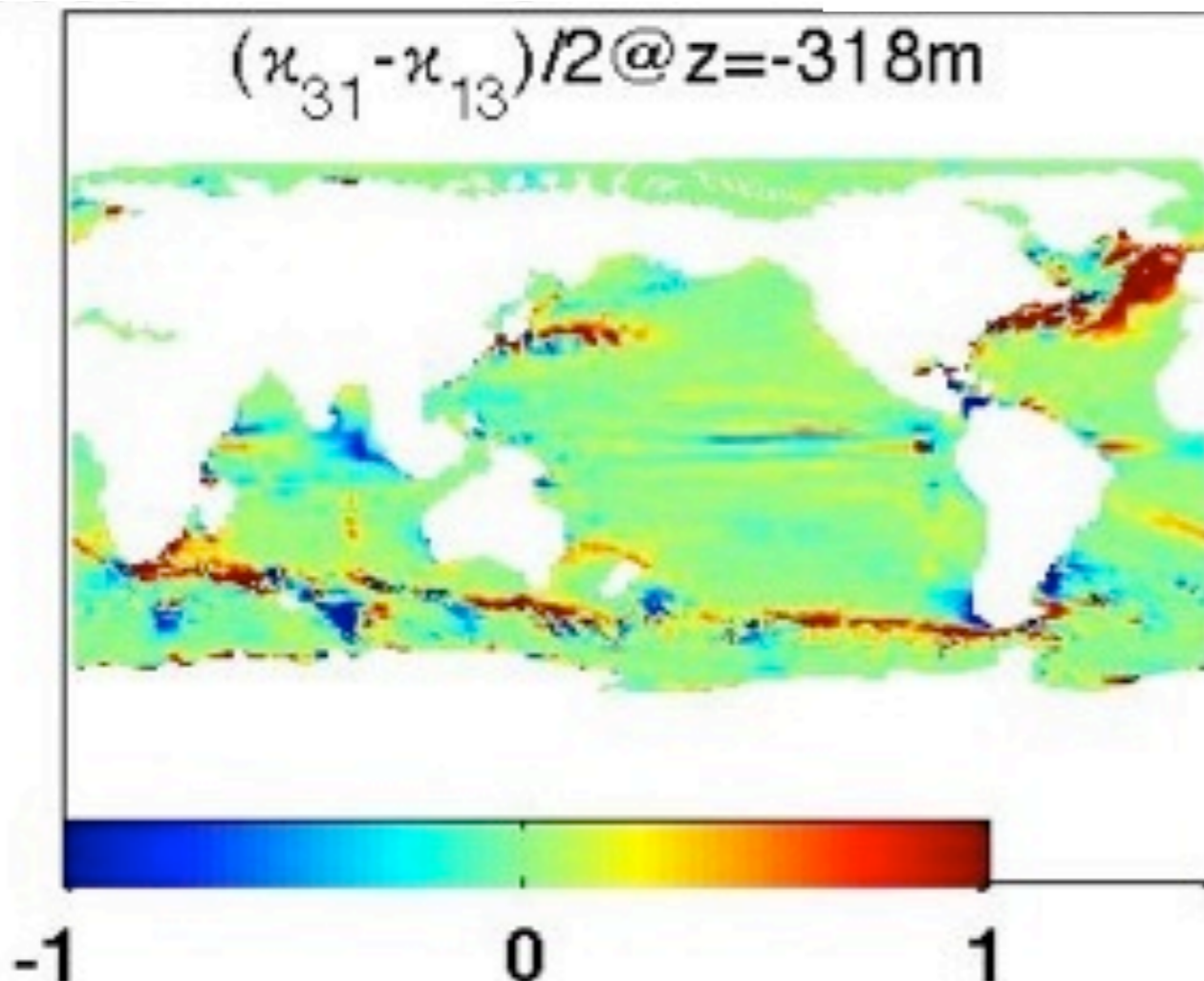
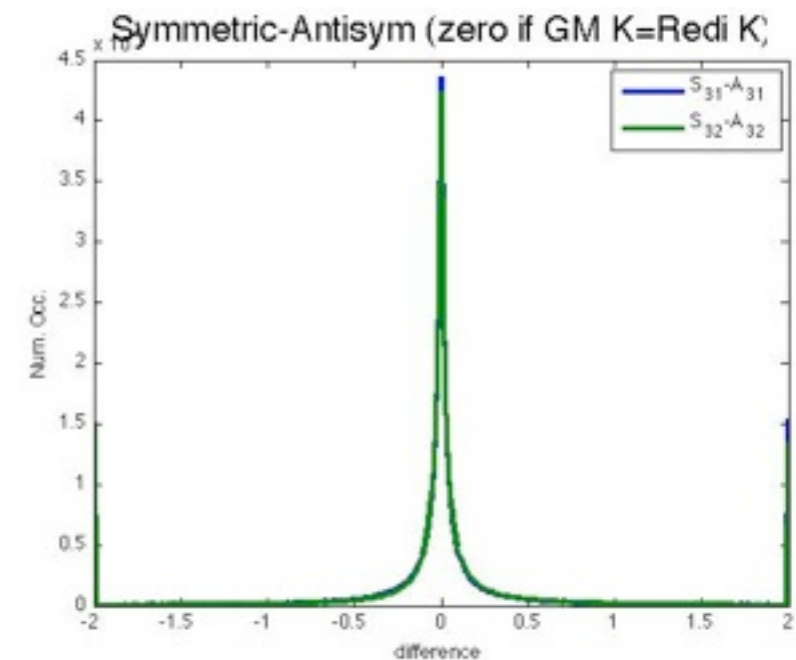


$\times 10^{-3}$

# Result:

Redi  $K=GM$   $K$  (mostly)

If so these 2 components should match in Sym & Antisym M

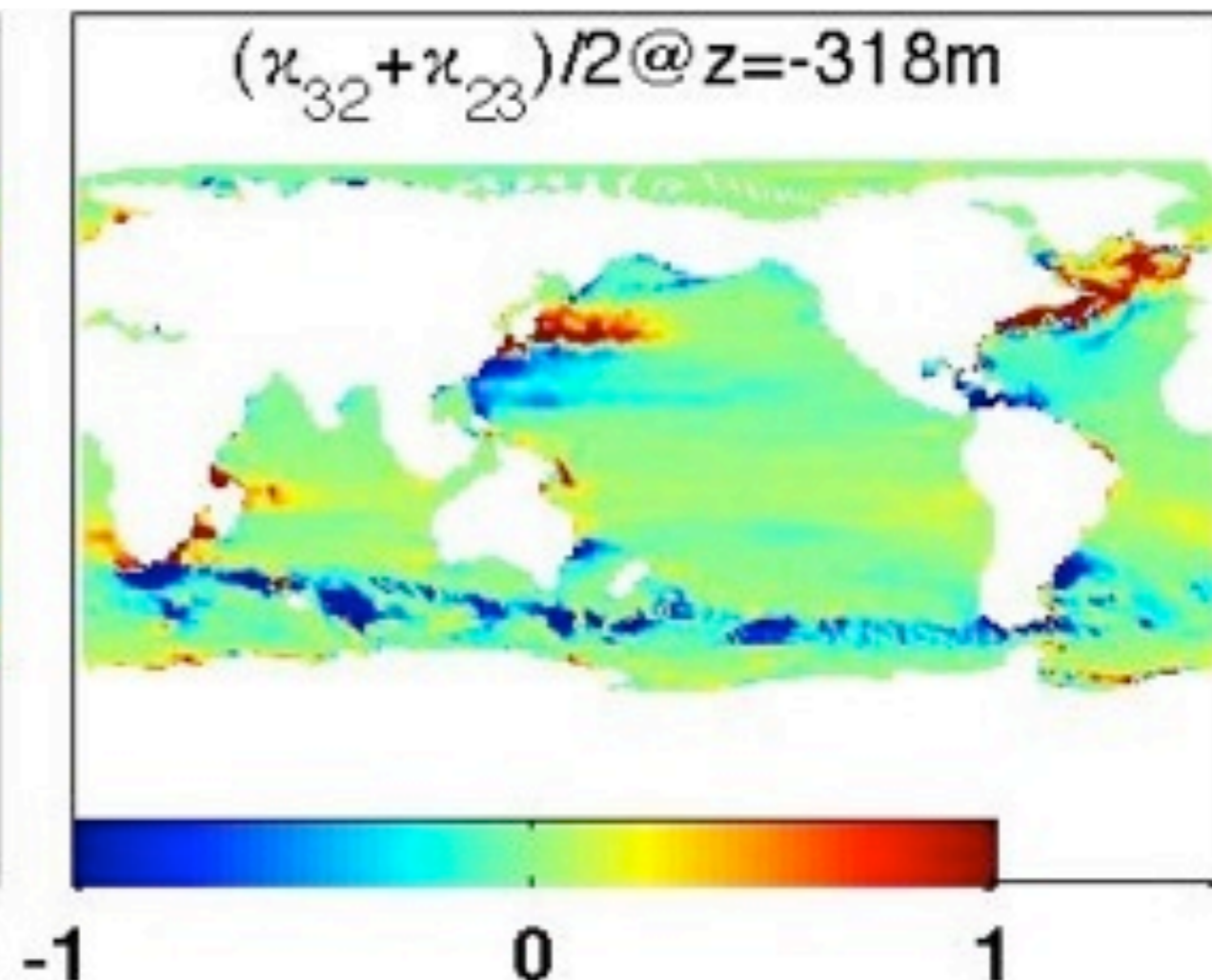
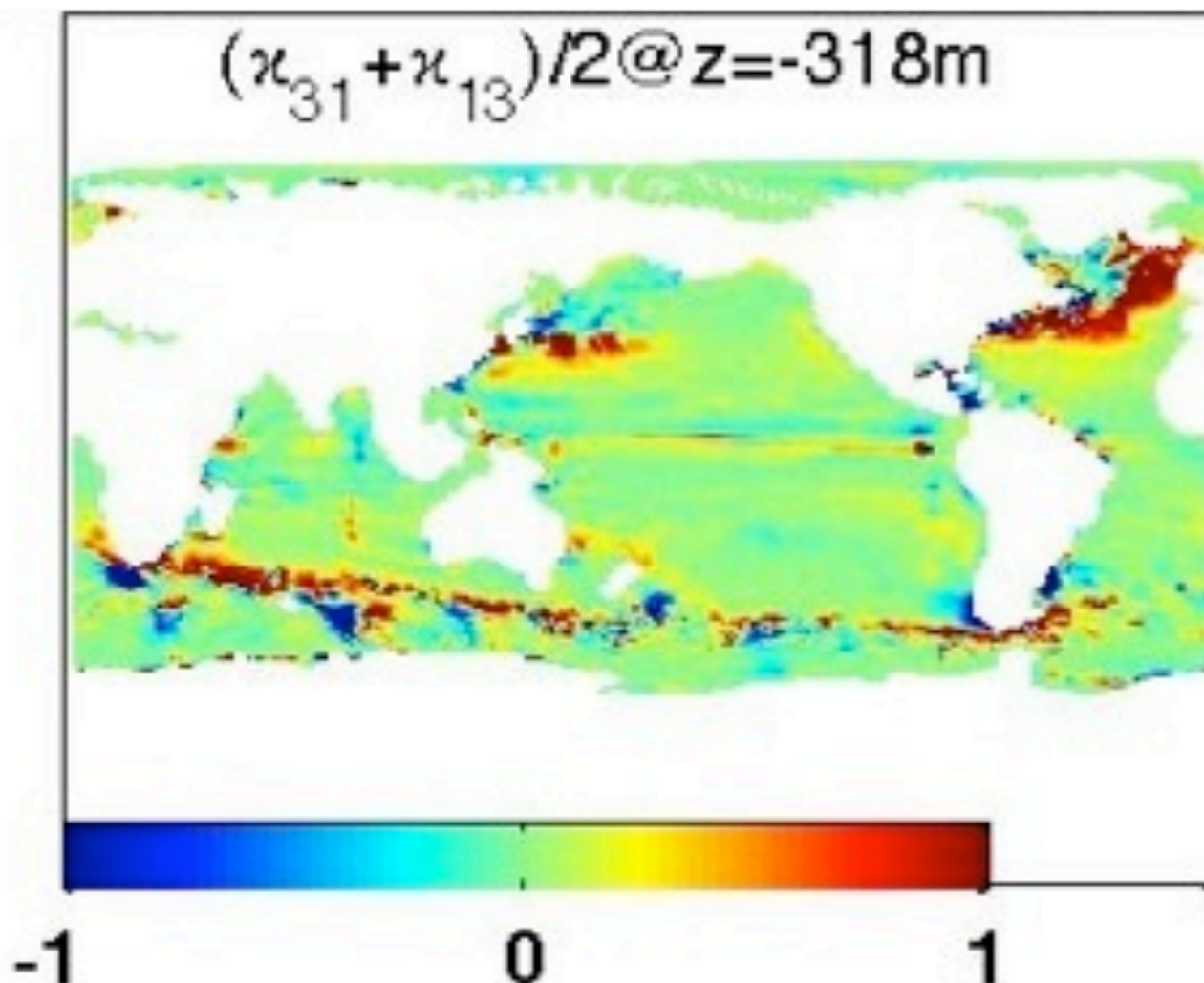
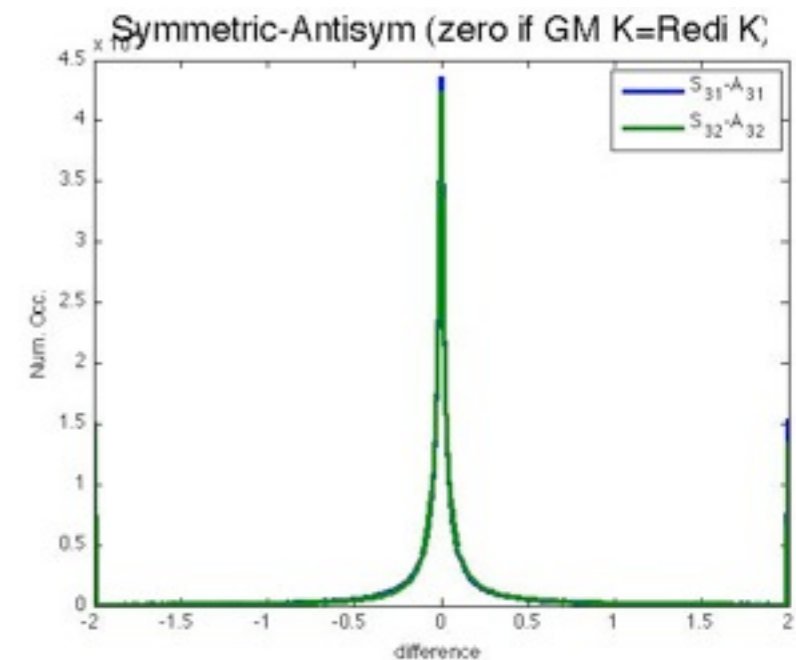




# Result:

Redi  $K=GM K$  (mostly)

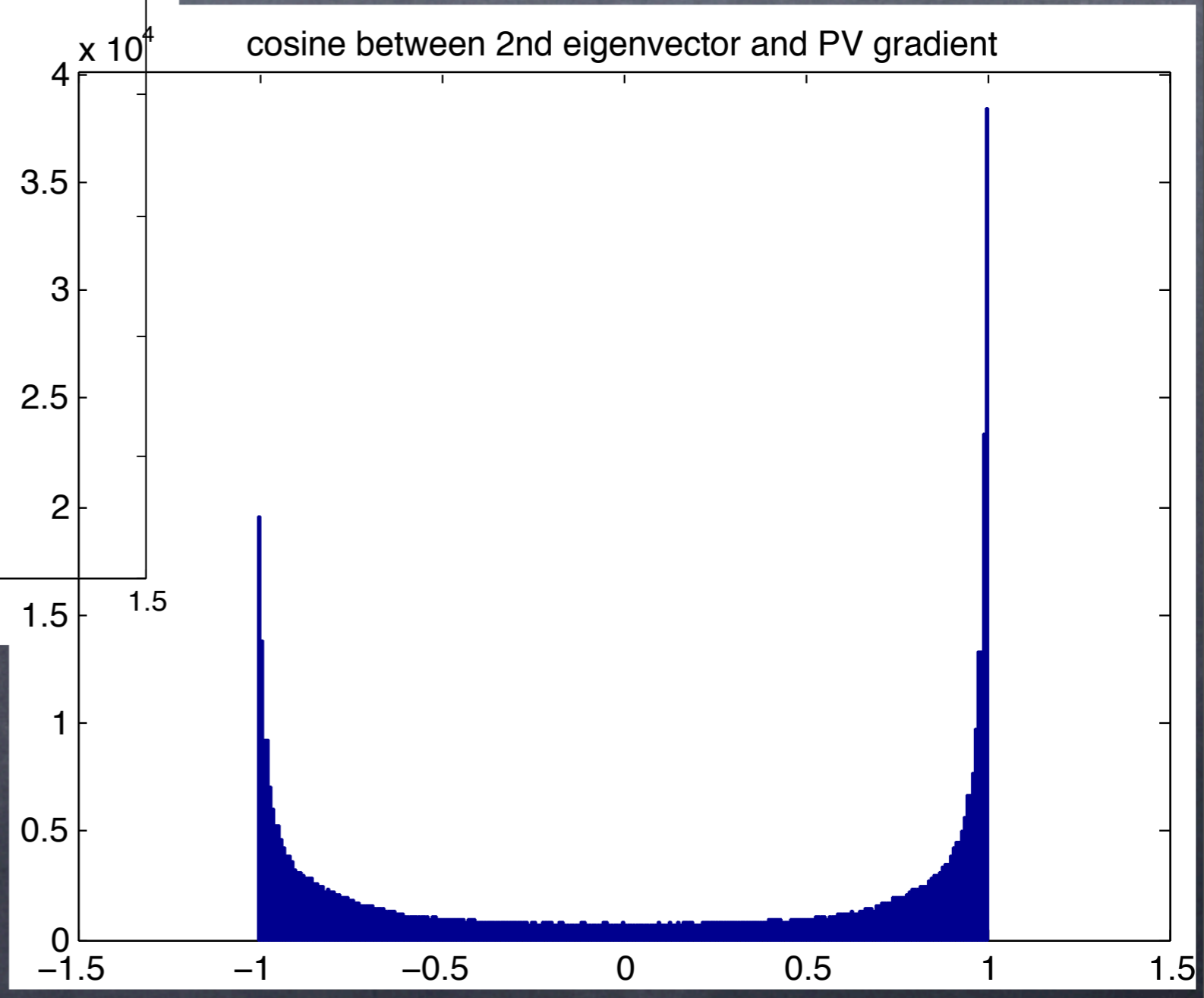
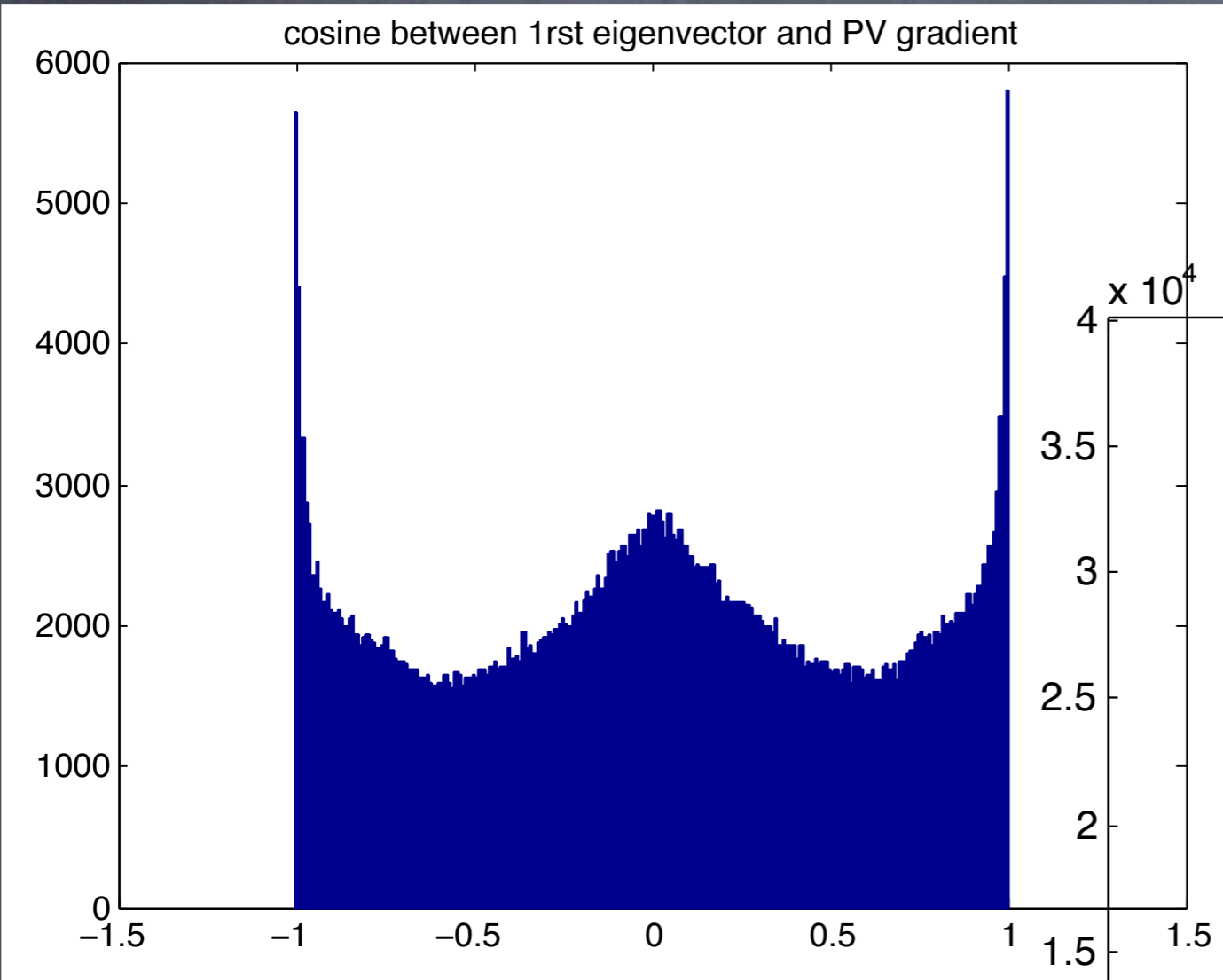
If so these 2 components should match in  $Sym$  & Antisym  $M$



# Result: Strong Anisotropy Along/Across PV Grads.

Mixing direction  
Either along PV contours or across

2nd Eigenvector  
Across PV contours



1rst Eigenvector

# Compare with Eden, Jochum, Danabasoglu compilation of present parameterizations

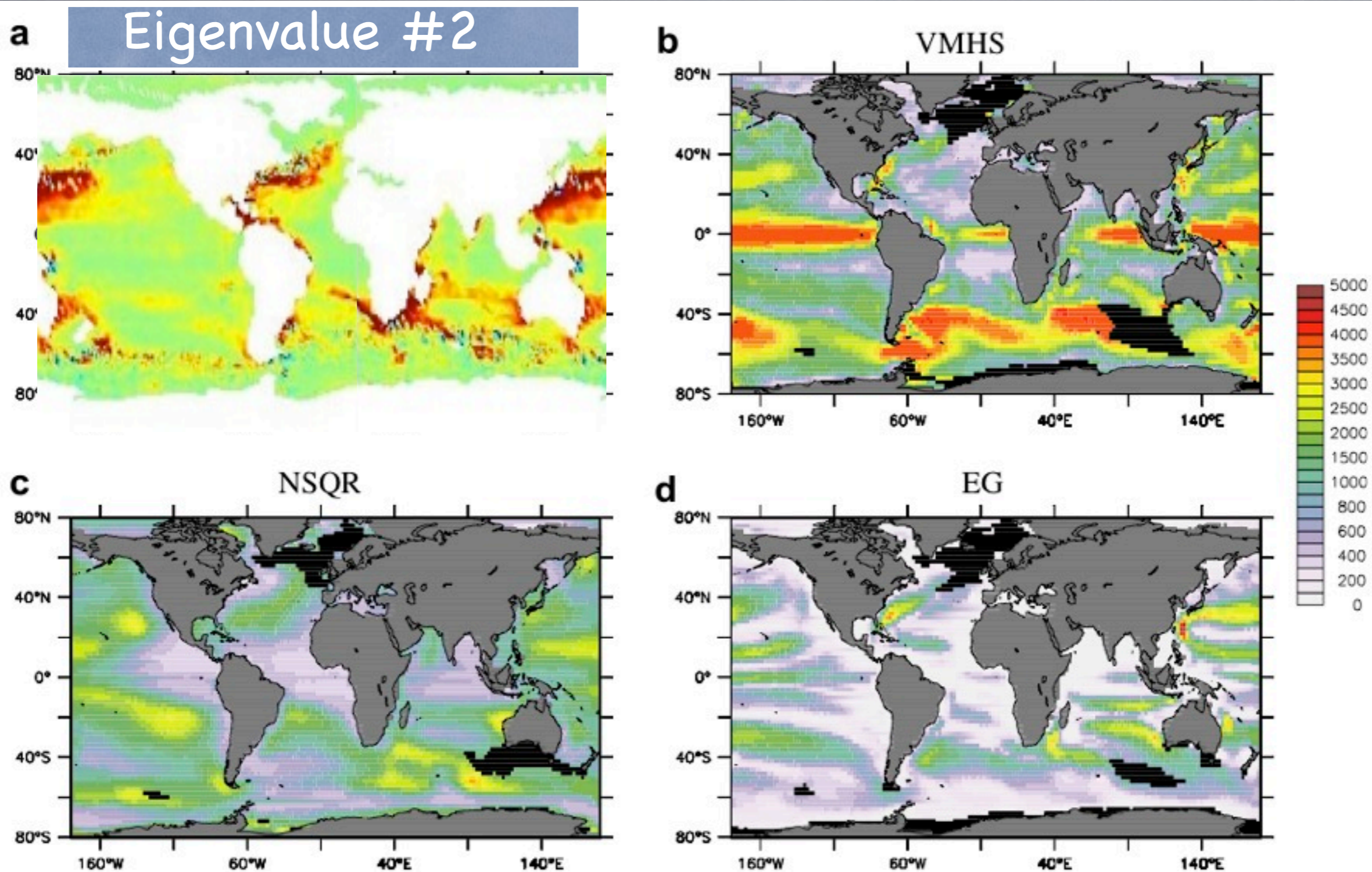
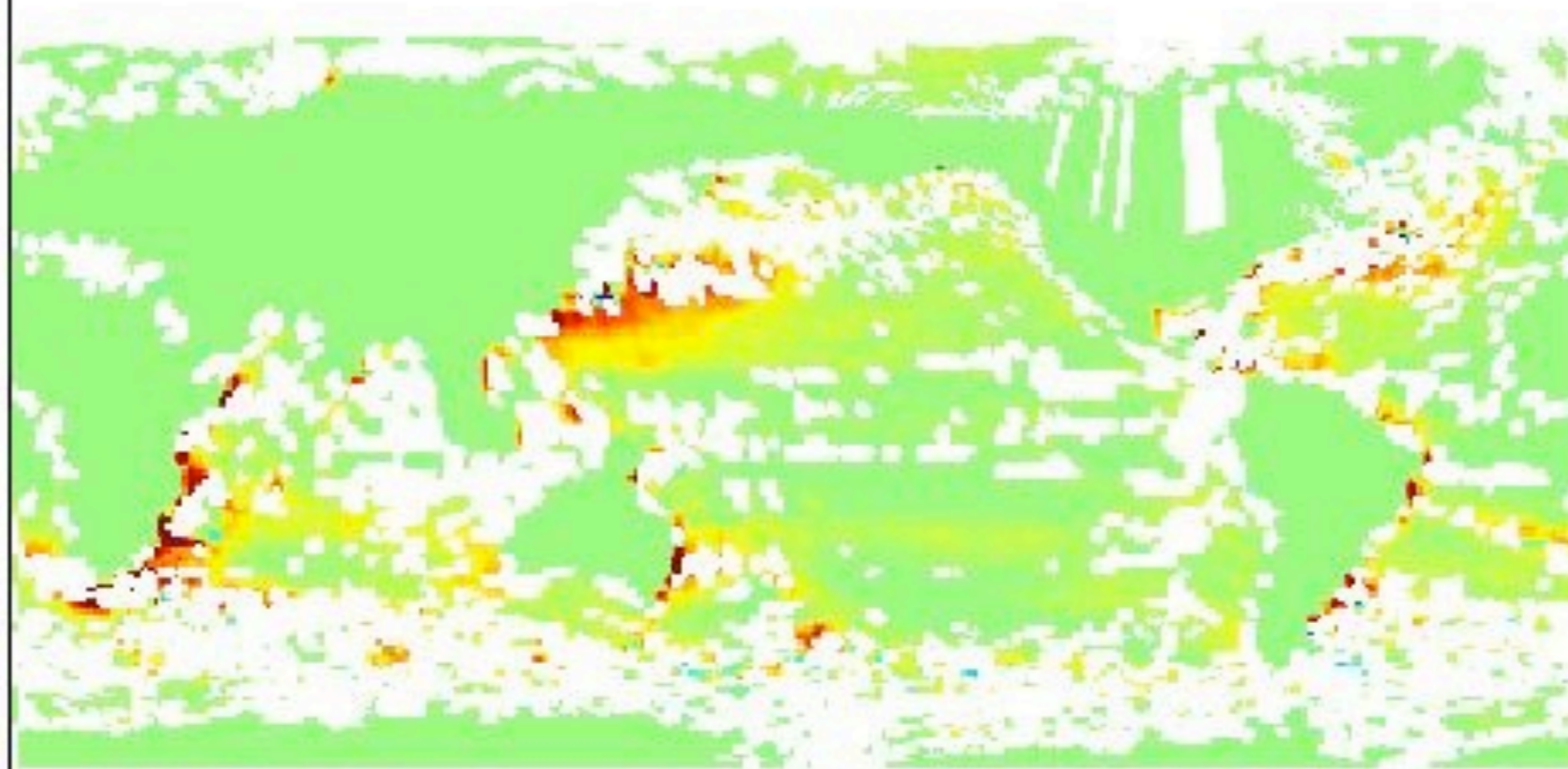


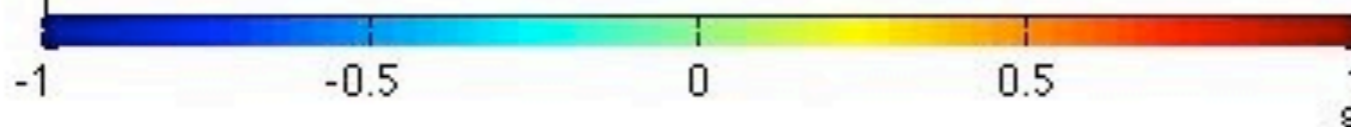
Fig. 1. Annual mean thickness diffusivity ( $K$ ) in  $\text{m}^2/\text{s}$  at 300 m depth in experiment CONST (a), VMHS (b), NSQR (c) and EG (d) after 500 years integration. Values of  $K$  are shown for the interior region only, i.e. values of  $K$  in the (seasonal maximum) diabatic surface and transition layer are not shown and shaded black. Note the non-linear colour scale for the thickness diffusivity. Note also that the data have been interpolated from the model grid to a regular rectangular grid of similar resolution prior to plotting. The land mask in the figure (taken from Smith and Sandwell (1997)) differs therefore slightly from the model's land mask.

But, how well does it work? Suppose we only plot values where different tracer sets agree...

Not so many trustworthy values!

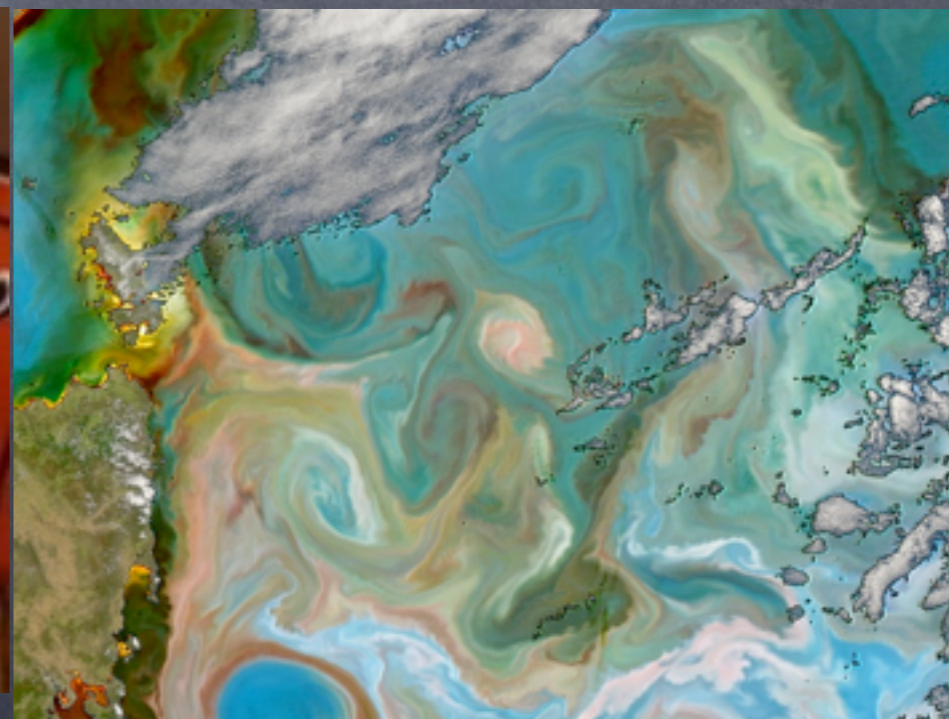


Can't reject params!



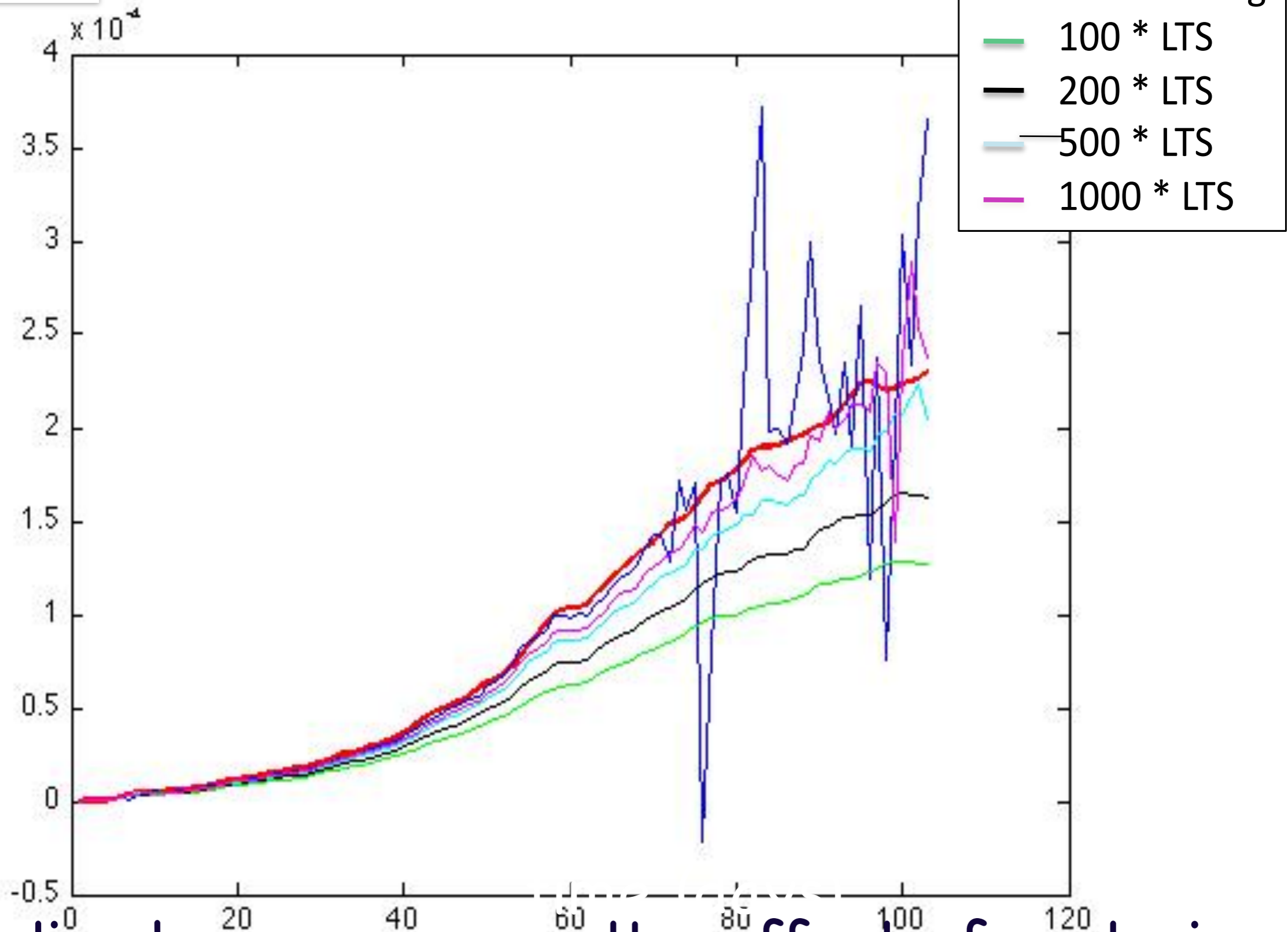
# Does this cover all the degrees of freedom?

- More tracers does provide a just-determined or overdetermined (Moore-Penrose/least squares) problem for  $M$  with a unique answer, but...
- Different tracers will have different fluxes as they feel the subgrid 'nooks and crannies' of the mesoscale eddies!



$$\frac{d\tau}{dt} = -\lambda(\tau - \tau_0)$$

$$\overline{v'b'}_{\text{rec}} = -\mathbf{M}\nabla\bar{b}$$

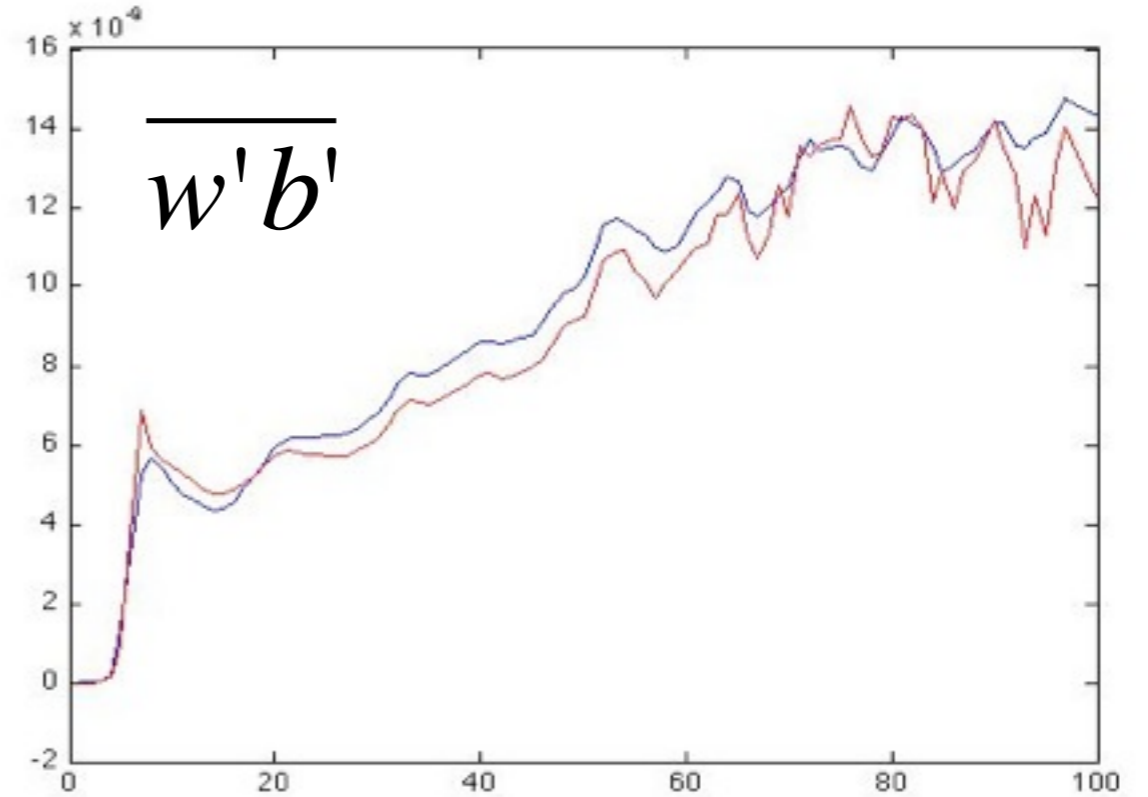
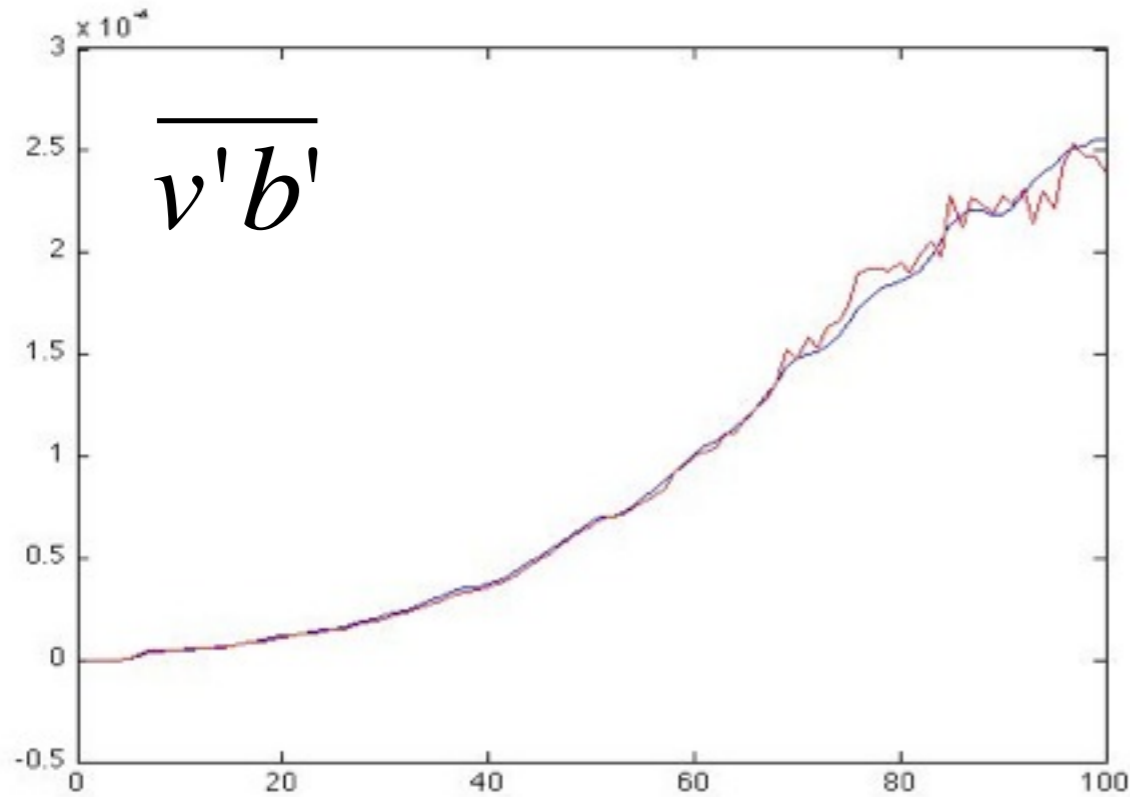


In idealized runs, can see the effect of restoring.  
Whatever we do, we need to get buoyancy right!

In idealized setting, can do better

Reconstruction of eddy buoyancy fluxes

Original fluxes  
Reconstructed fluxes



Using specially-tailored non-restored tracers improves estimate (error is now  $< 10\%$ )... but not feasible in realistic diagnosis.

In realistic diagnosis, we can improve the estimate a bit by approximating restoring effect

# Sub-Mesoscale Parameterizations

- Anyone who doesn't take truth seriously in small matters cannot be trusted in large ones either.

• --Albert Einstein

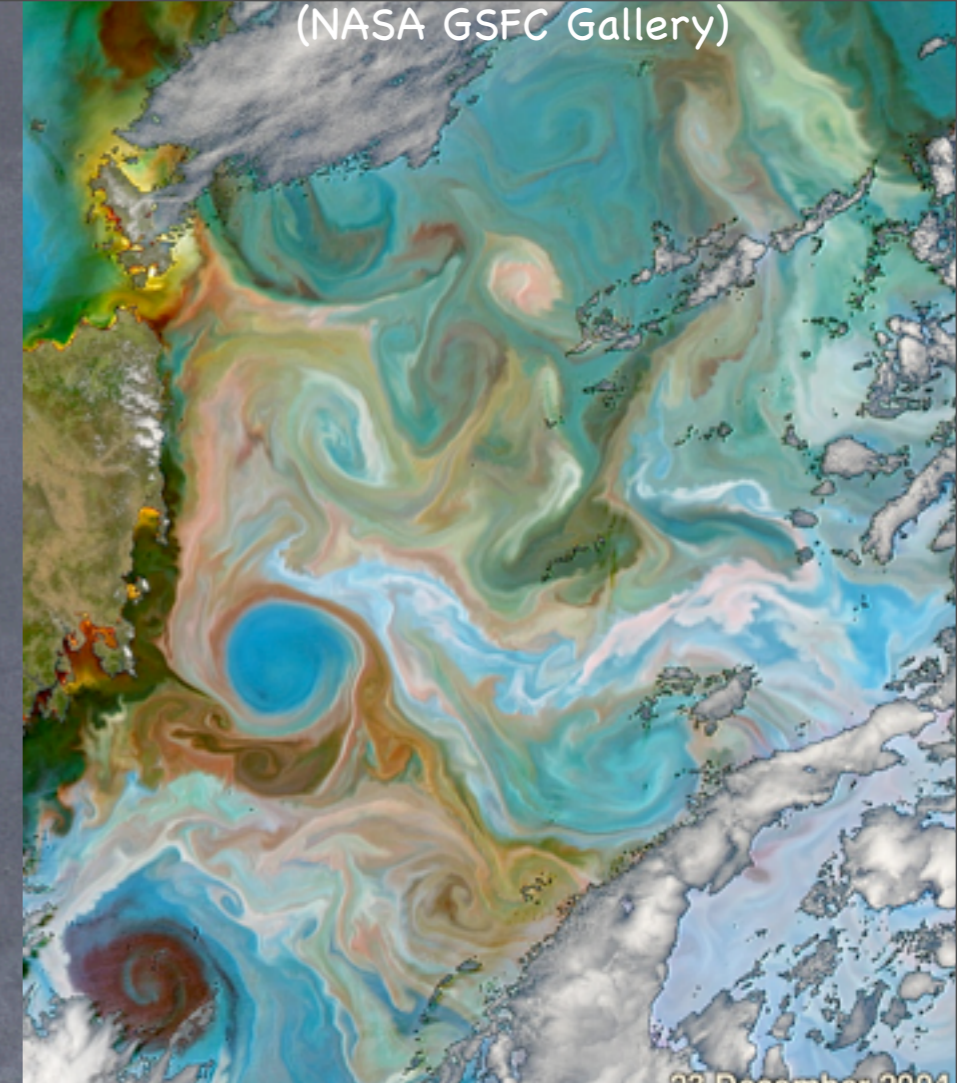


# The Character of the Submesoscale

(Capet et al., 2008)

10 km

(NASA GSFC Gallery)



- Fronts
- Eddies
- $Ro=O(1)$
- $Ri=O(1)$
- near-surface
- 1-10km, days

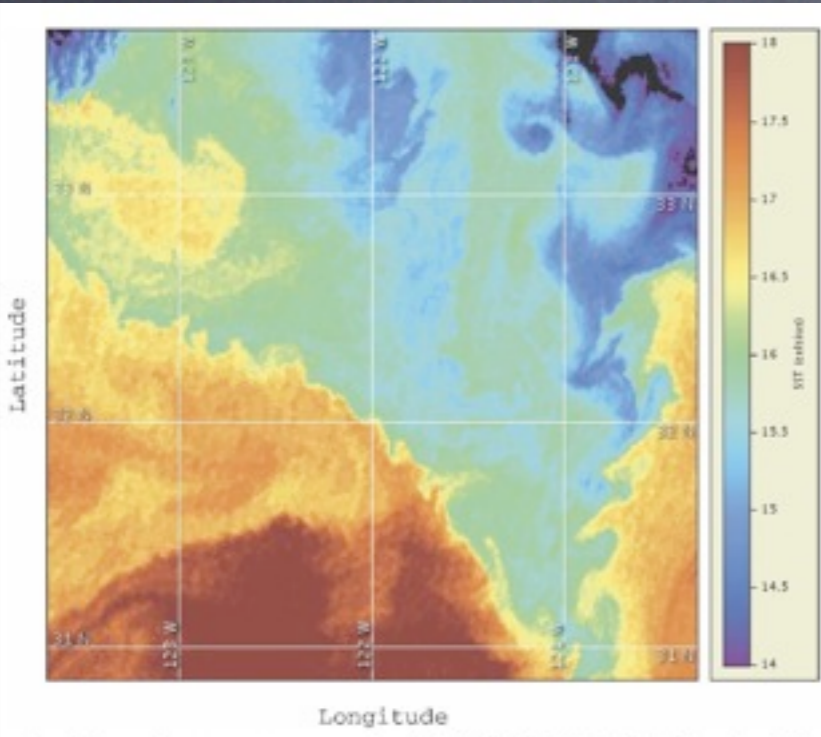
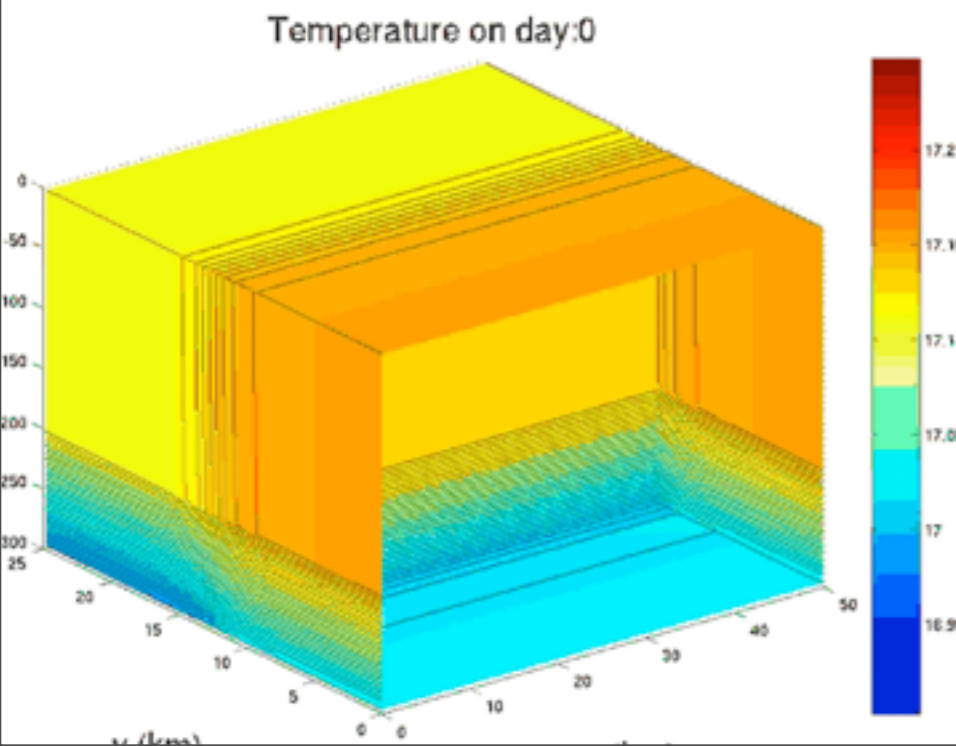
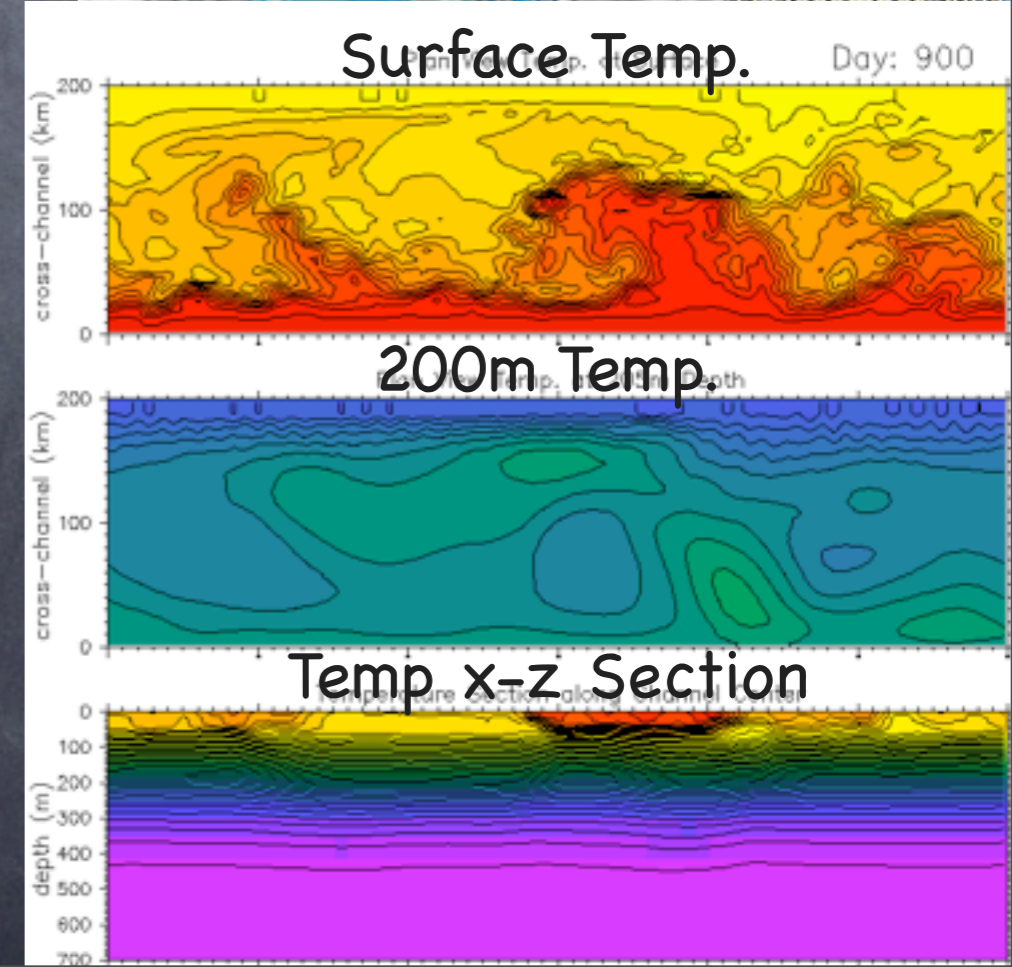


Fig. 16 Sea surface temperature measured at 1833 UTC 1 Jan 2006 off Point Conception in the



Eddy processes mainly **baroclinic instability** (Boccaletti et al '07, Haine & Marshall '98).  
**Parameterizations of baroclinic instability?**



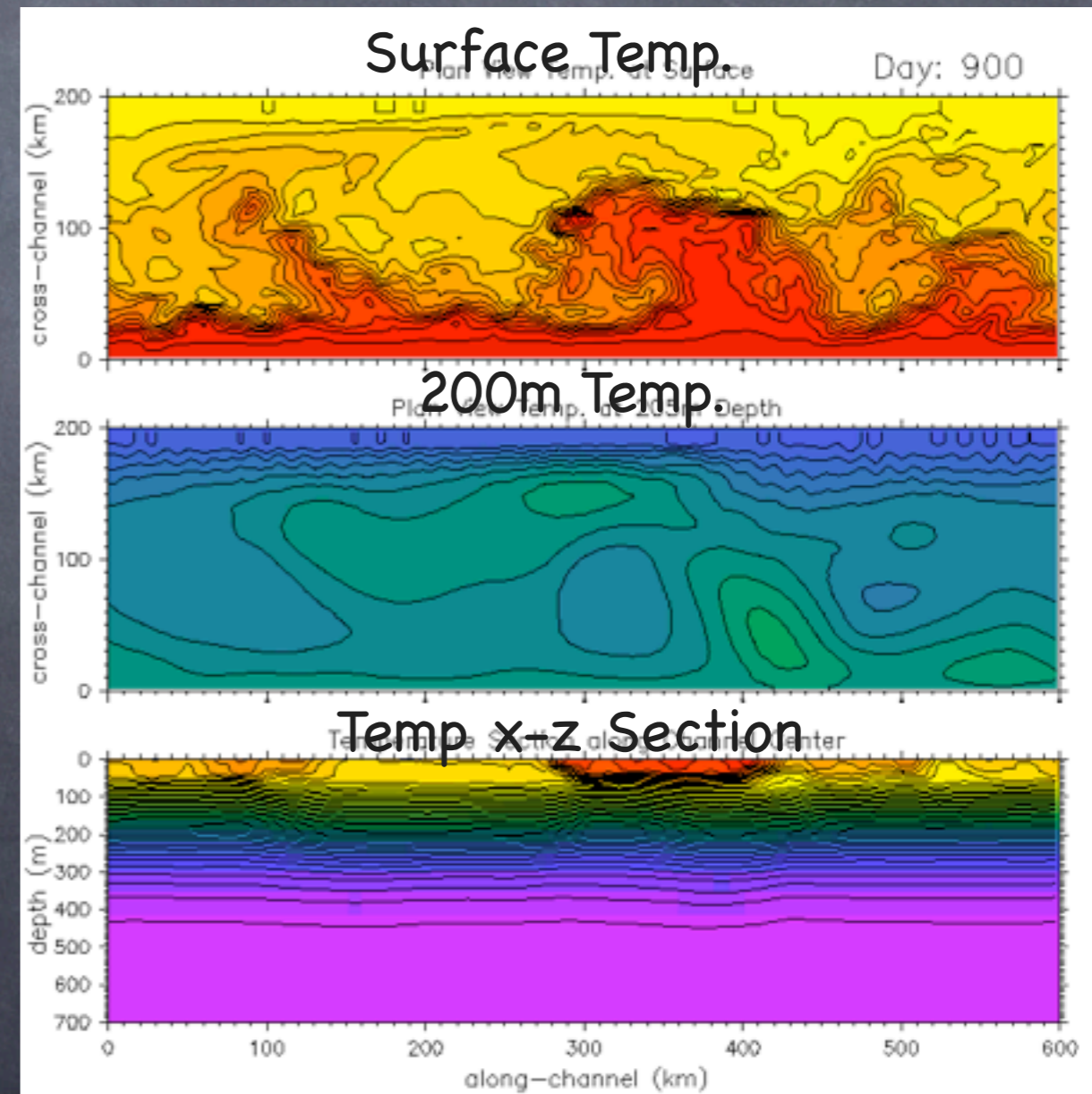
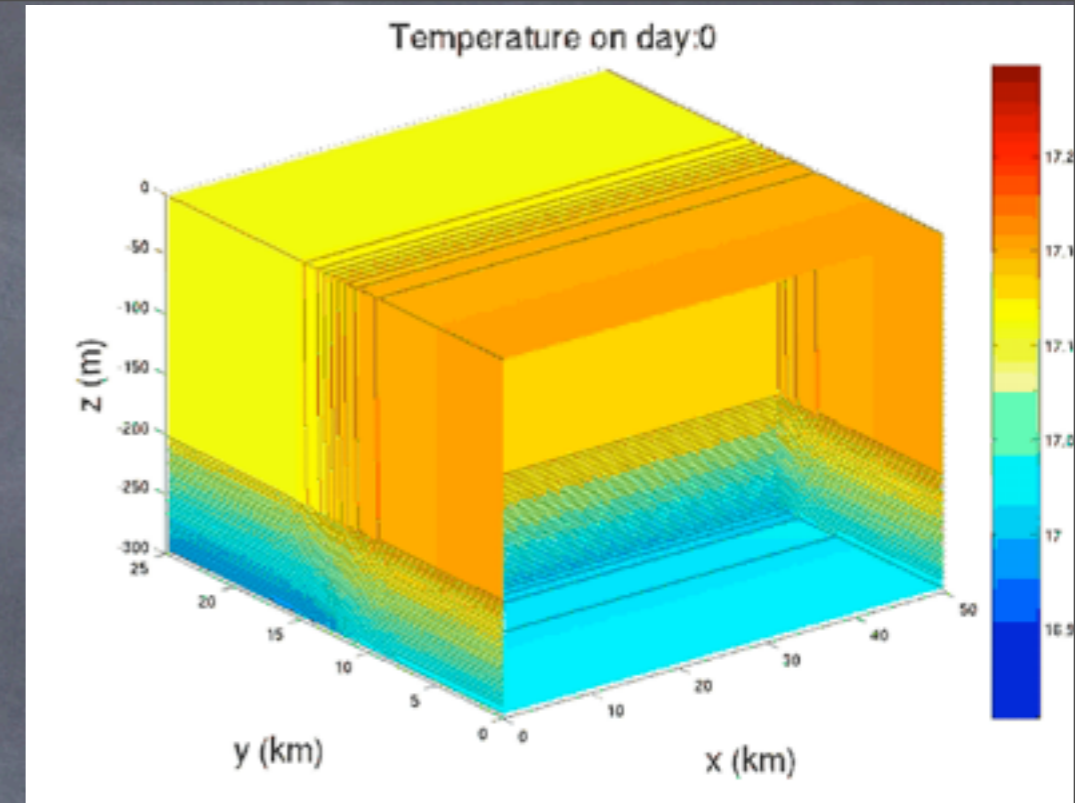
# Mixed Layer Eddy Restratification

Estimating eddy buoyancy/density fluxes:

$$\overline{\mathbf{u}'b'} \equiv \Psi \times \nabla \bar{b}$$

A submeso eddy-induced overturning:

$$\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{\mathbf{z}}$$



# Mixed Layer Eddy Restratification

Estimating eddy buoyancy/density fluxes:

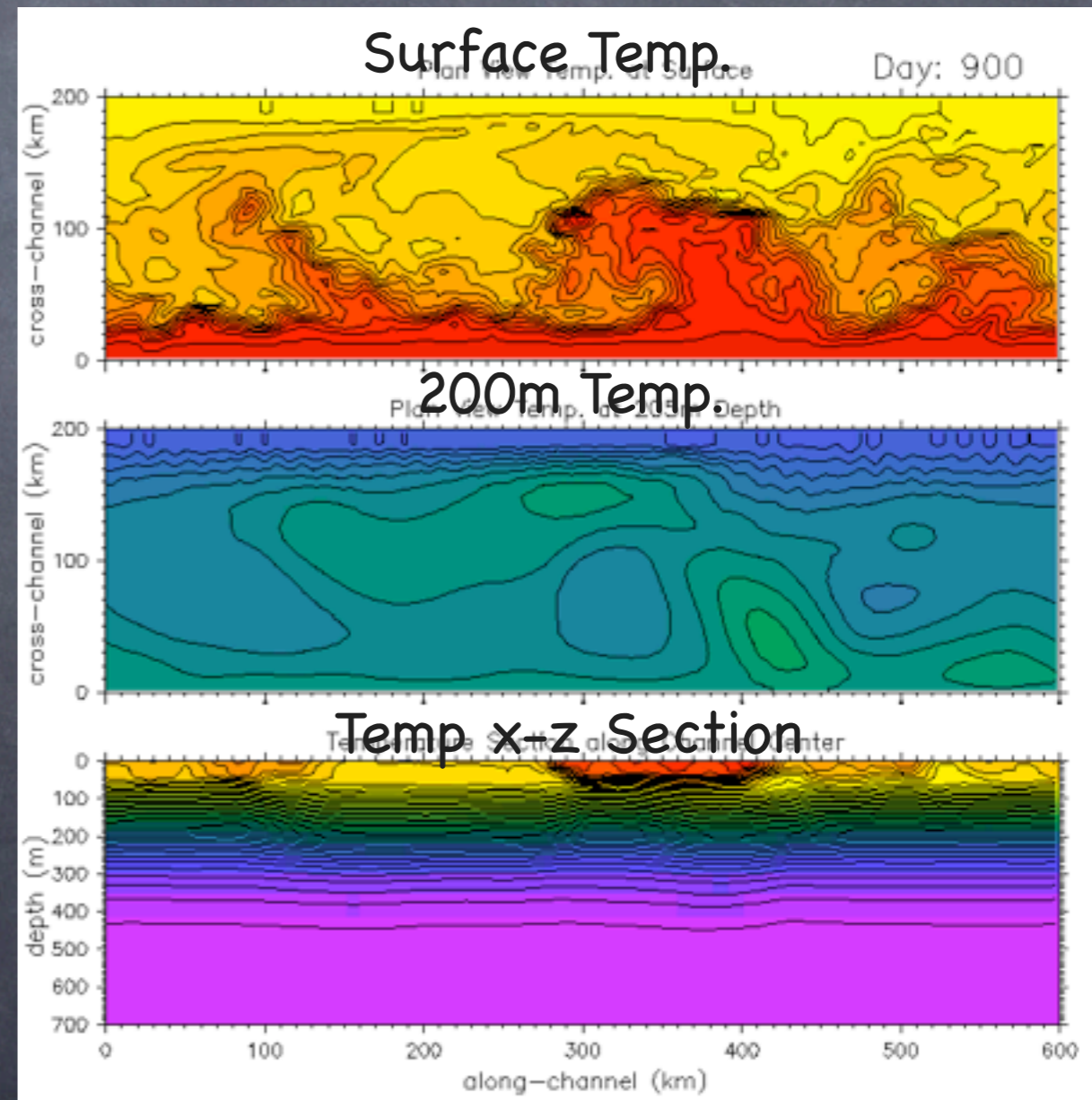
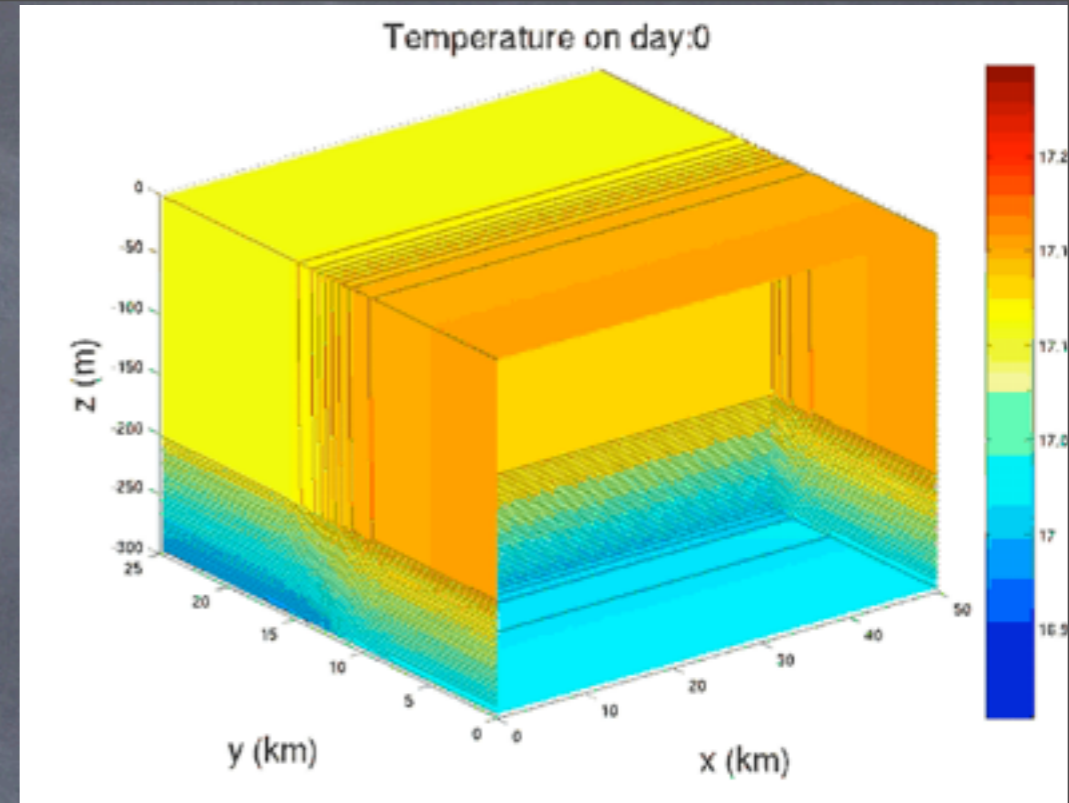
$$\overline{\mathbf{u}'b'} \equiv \Psi \times \nabla \bar{b}$$

A submeso eddy-induced overturning:

$$\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{\mathbf{z}}$$

in ML only:

$$\mu(z) = 0 \text{ if } z < -H$$



# Mixed Layer Eddy Restratification

Estimating eddy buoyancy/density fluxes:

$$\overline{\mathbf{u}'b'} \equiv \Psi \times \nabla \bar{b}$$

A submeso eddy-induced overturning:

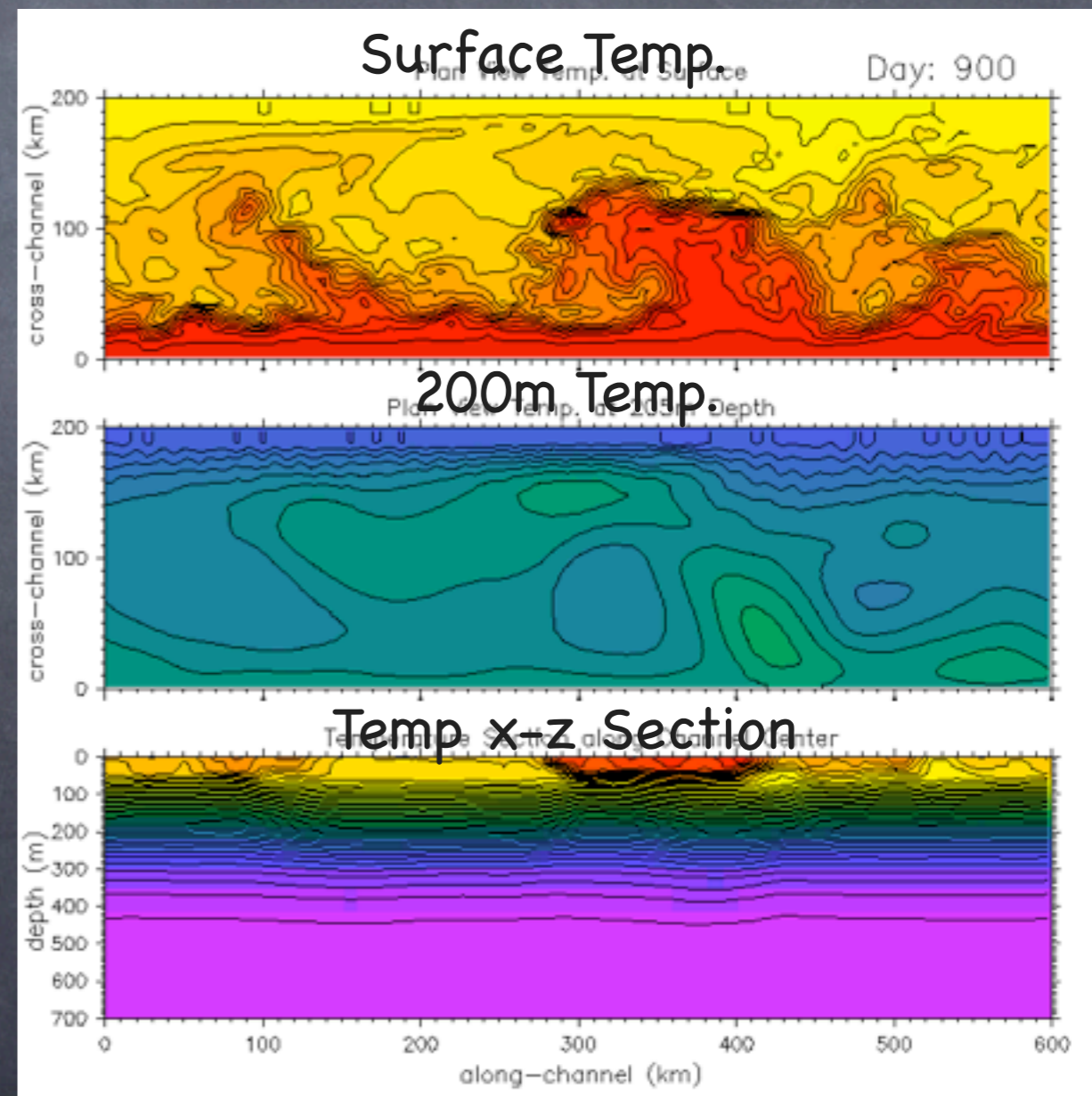
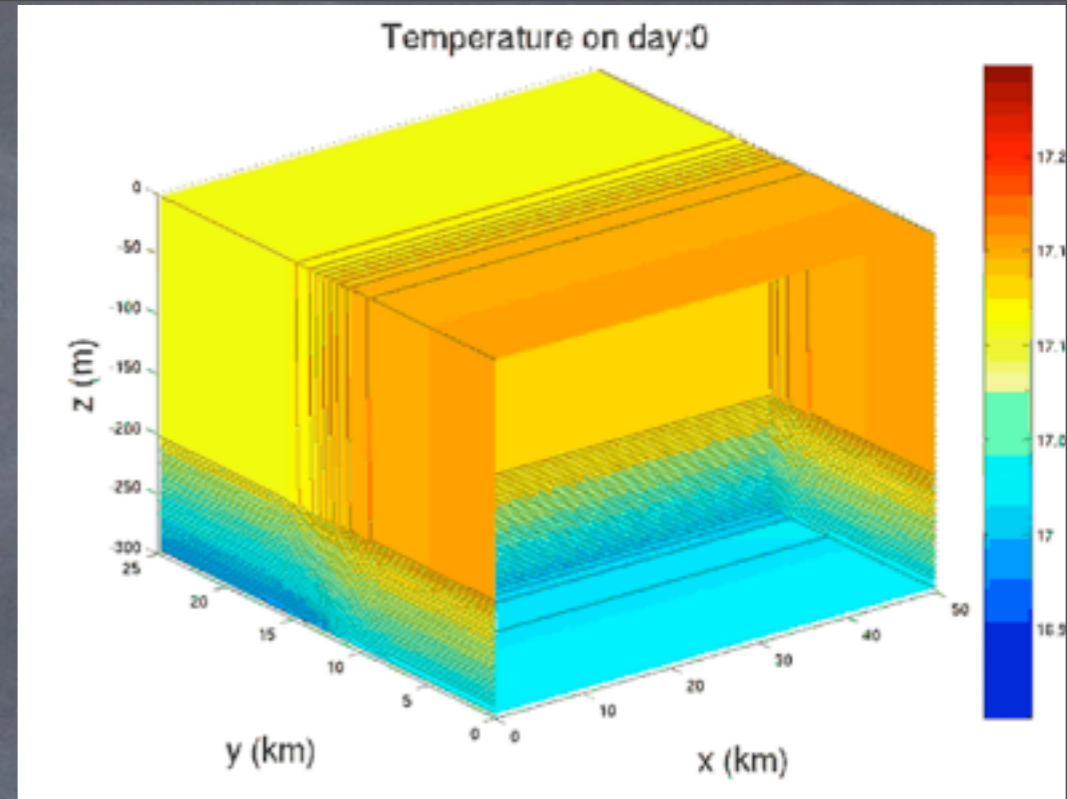
$$\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{\mathbf{z}}$$

in ML only:

$$\mu(z) = 0 \text{ if } z < -H$$

For a consistently restratifying,

$$\overline{w'b'} \propto \frac{H^2}{|f|} |\nabla_H \bar{b}|^2$$



# Mixed Layer Eddy Restratification

Estimating eddy buoyancy/density fluxes:

$$\overline{\mathbf{u}'b'} \equiv \Psi \times \nabla \bar{b}$$

A submeso eddy-induced overturning:

$$\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{\mathbf{z}}$$

in ML only:

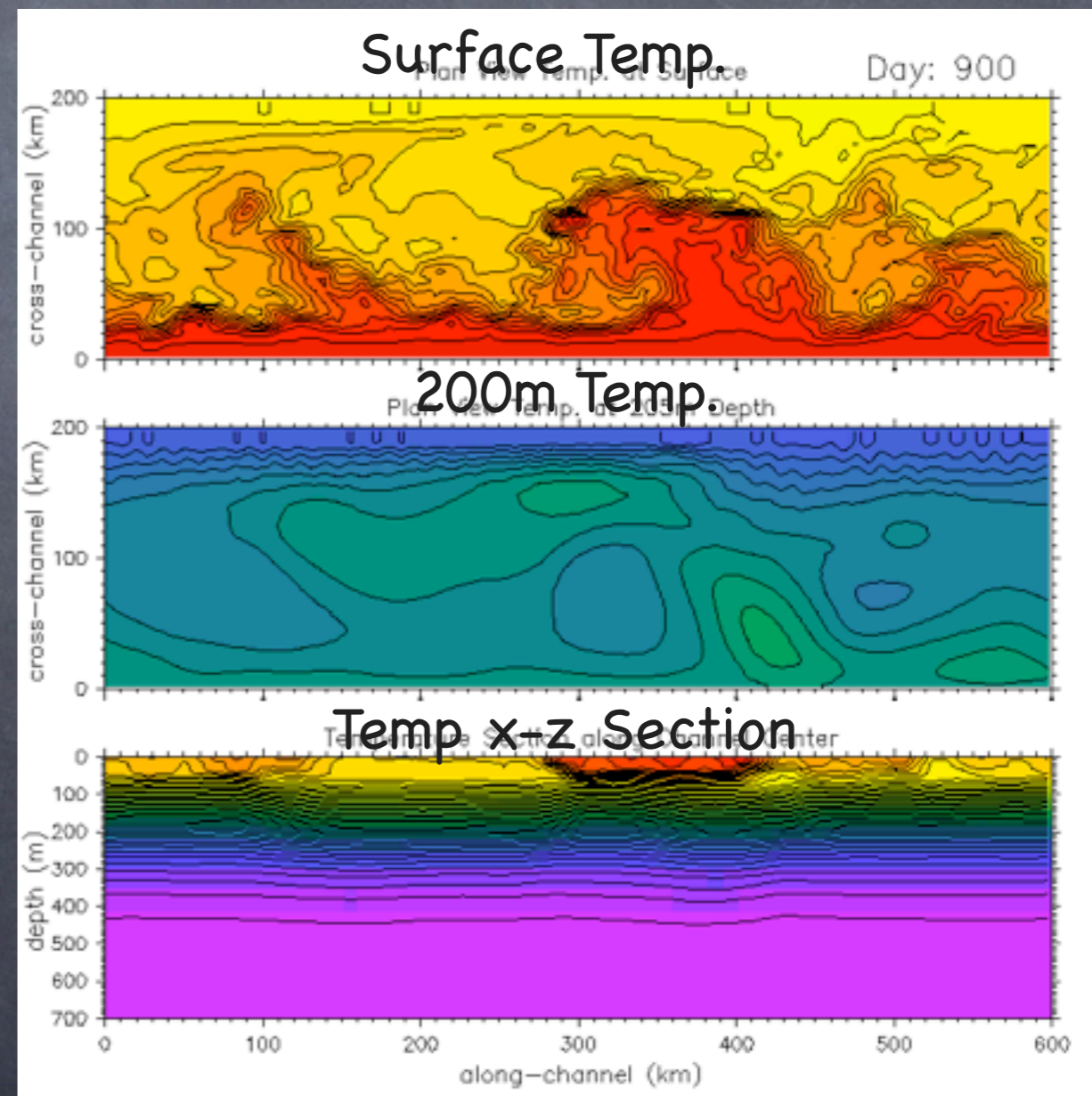
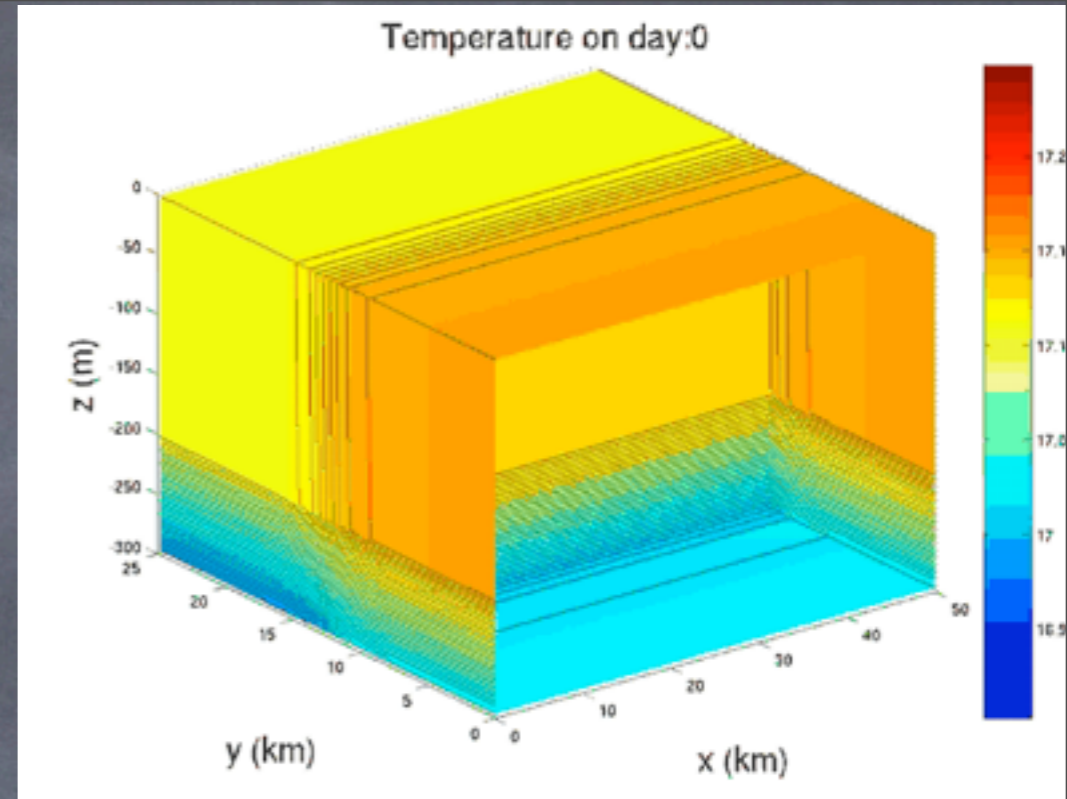
$$\mu(z) = 0 \text{ if } z < -H$$

For a consistently restratifying,

$$\overline{w'b'} \propto \frac{H^2}{|f|} |\nabla_H \bar{b}|^2$$

and horizontally downgradient flux.

$$\overline{\mathbf{u}'_H b'} \propto \frac{-H^2 \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla_H \bar{b}$$



# Mixed Layer Eddy Restratification

Estimating eddy buoyancy/density fluxes:

$$\overline{\mathbf{u}'b'} \equiv \Psi \times \nabla \bar{b}$$

A submeso eddy-induced overturning:

$$\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{\mathbf{z}}$$

in ML only:

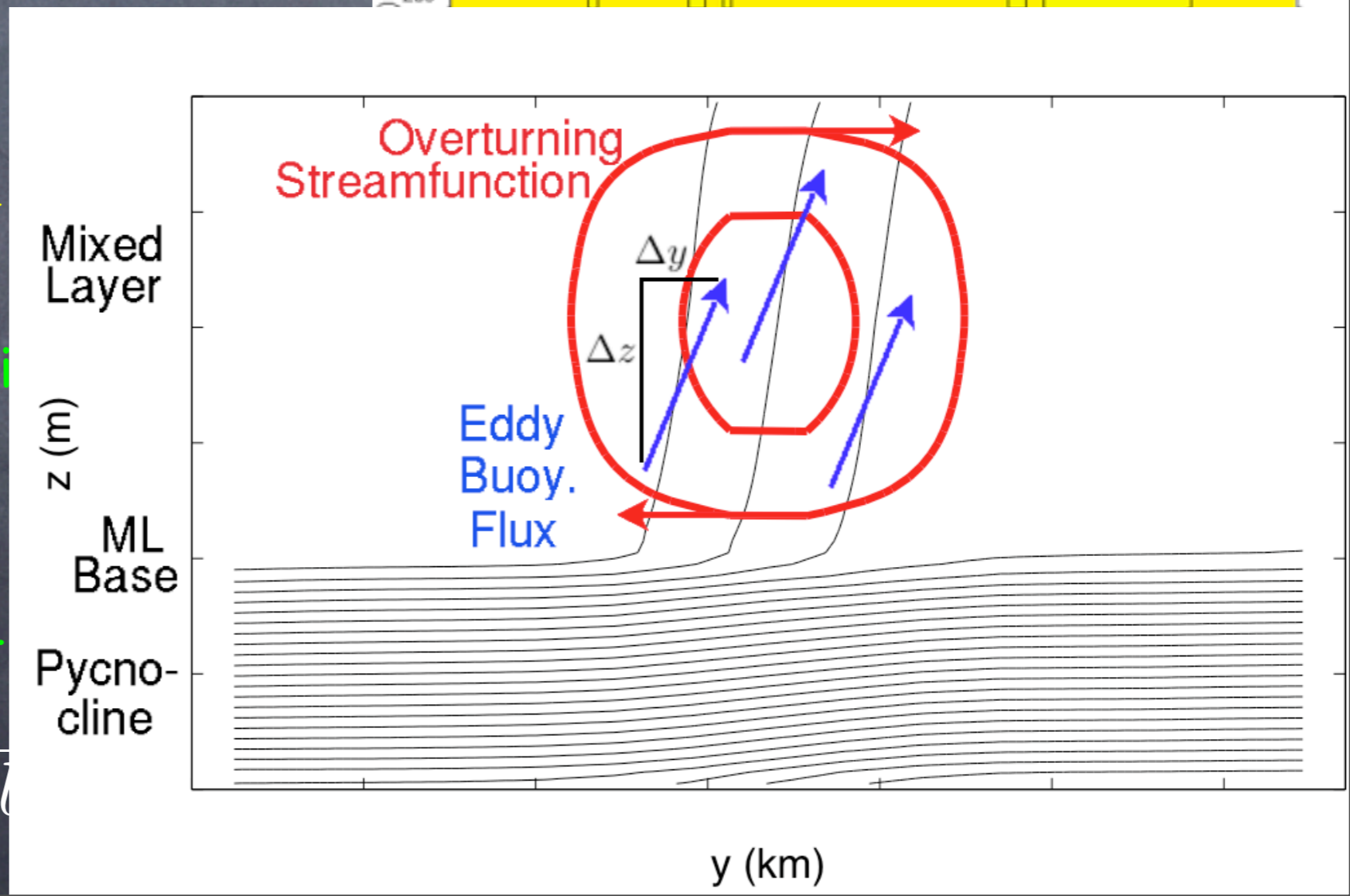
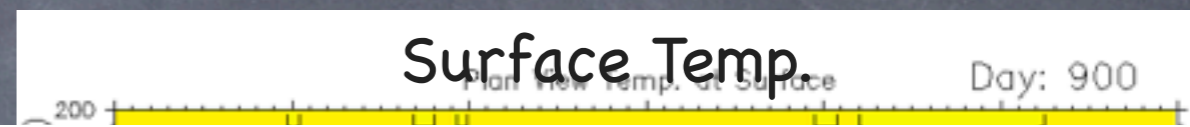
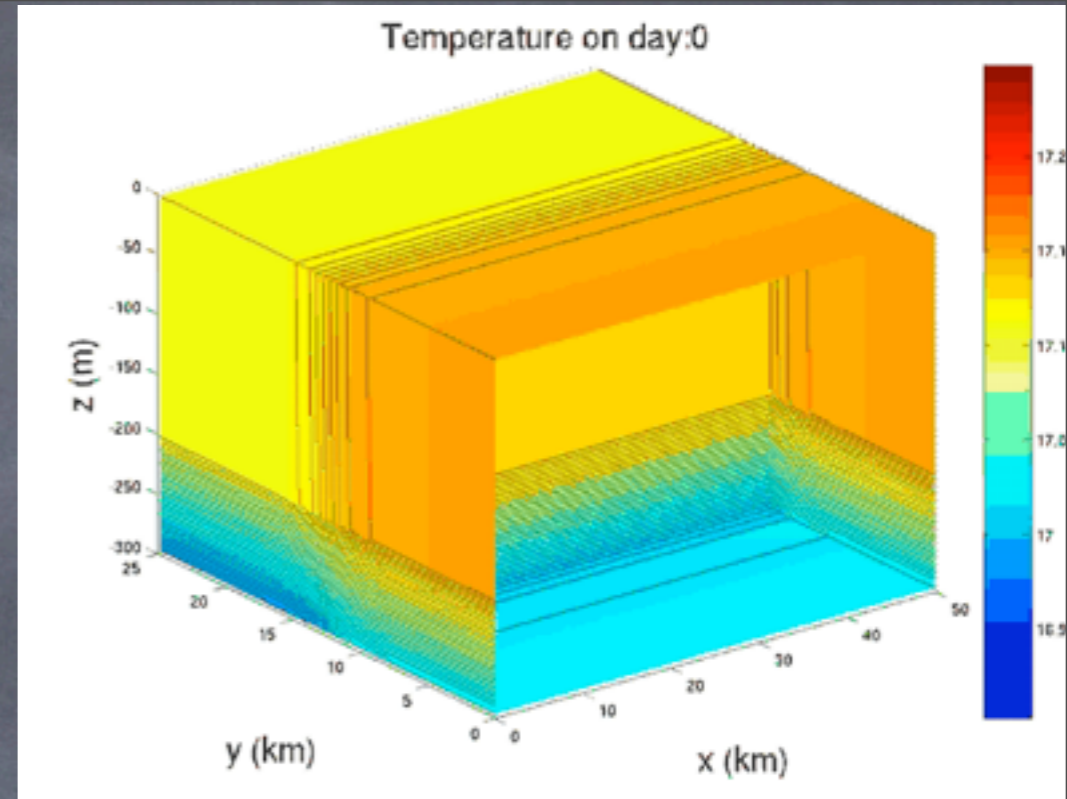
$$\mu(z) = 0 \text{ if } z < -H$$

For a consistently restratifying

$$\overline{w'b'} \propto \frac{H^2}{|f|} |\nabla_H \bar{b}|^2$$

and horizontally downgradient

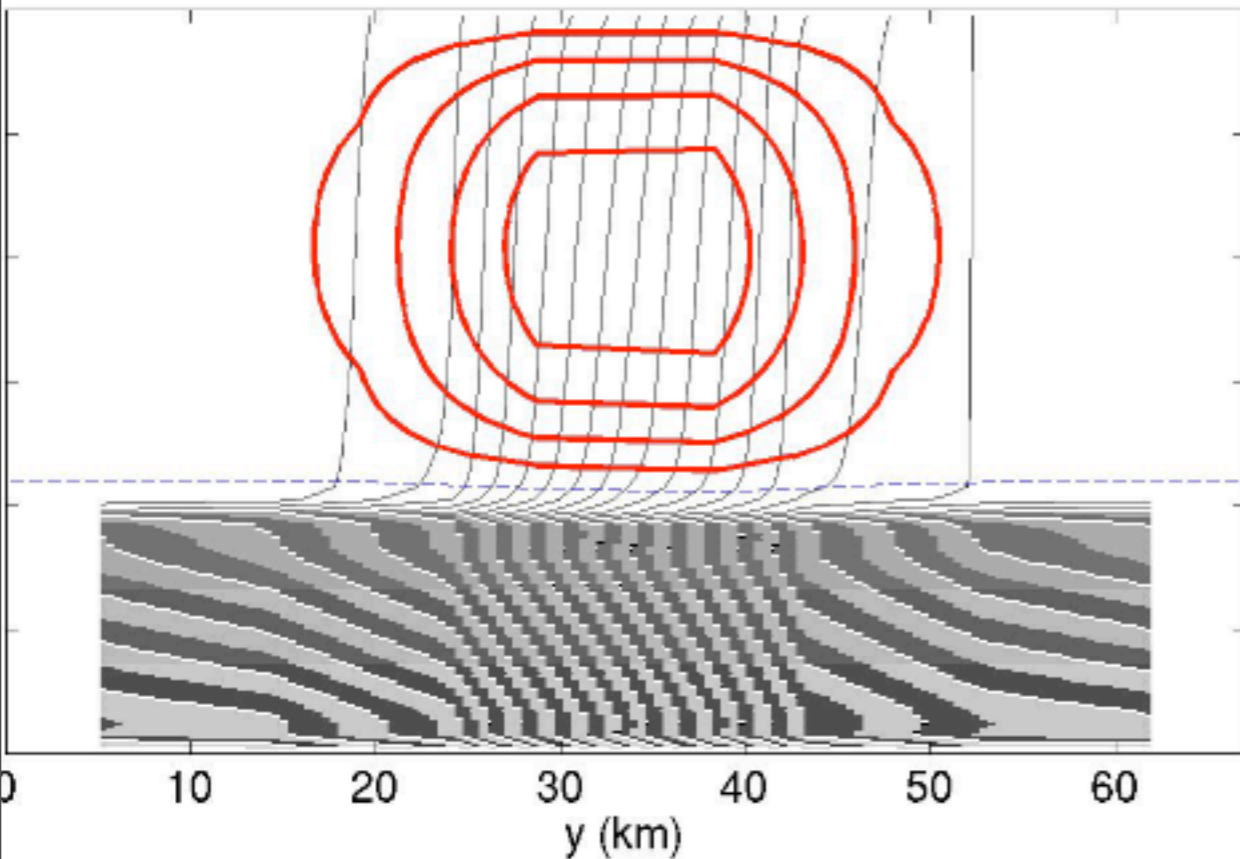
$$\overline{\mathbf{u}'_H b'} \propto \frac{-H^2 \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla_H \bar{b}$$



# What does eddy restratification look like?

## Parameterization Prediction

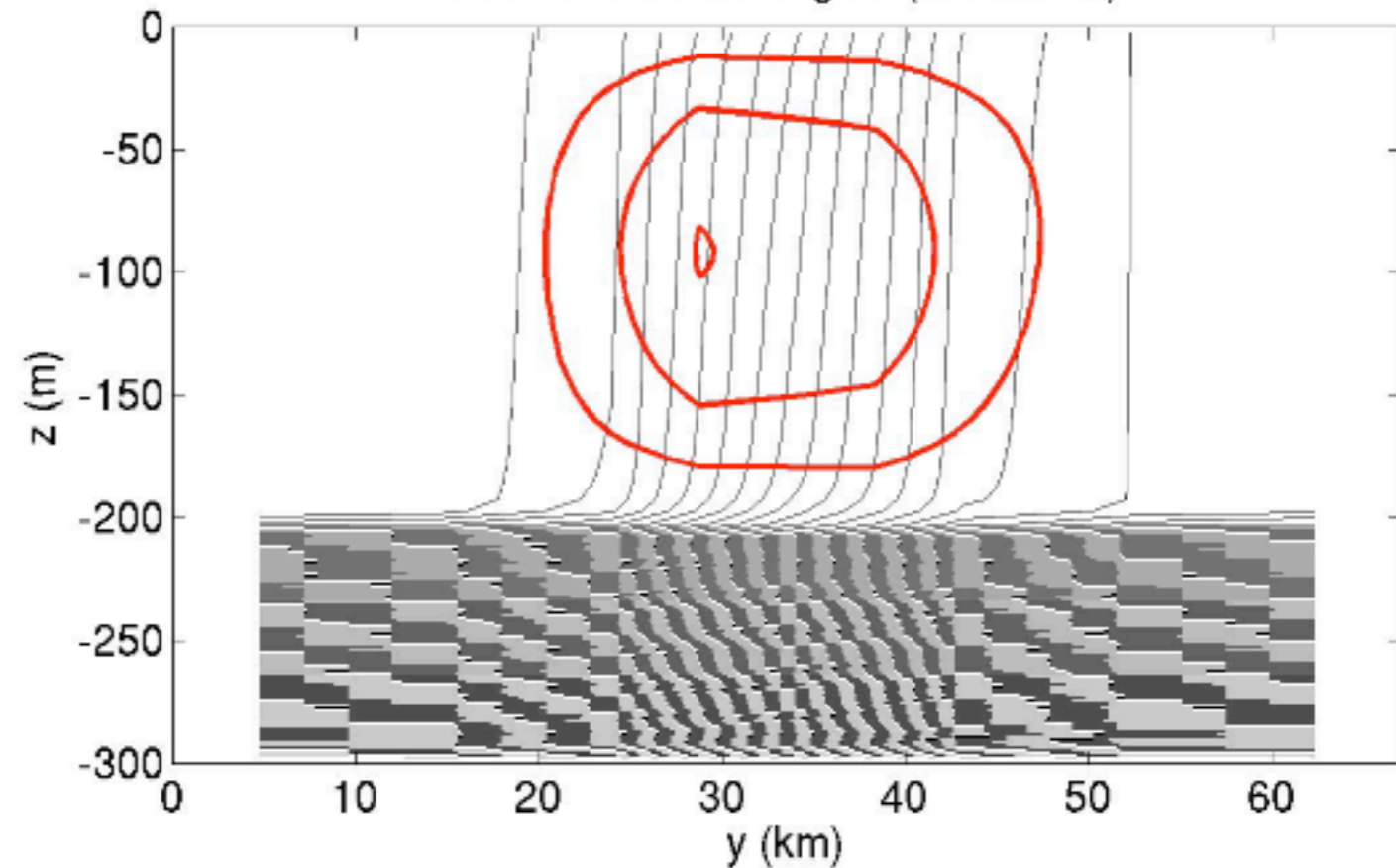
7d01h from 2d parameterization



red=streamfunction

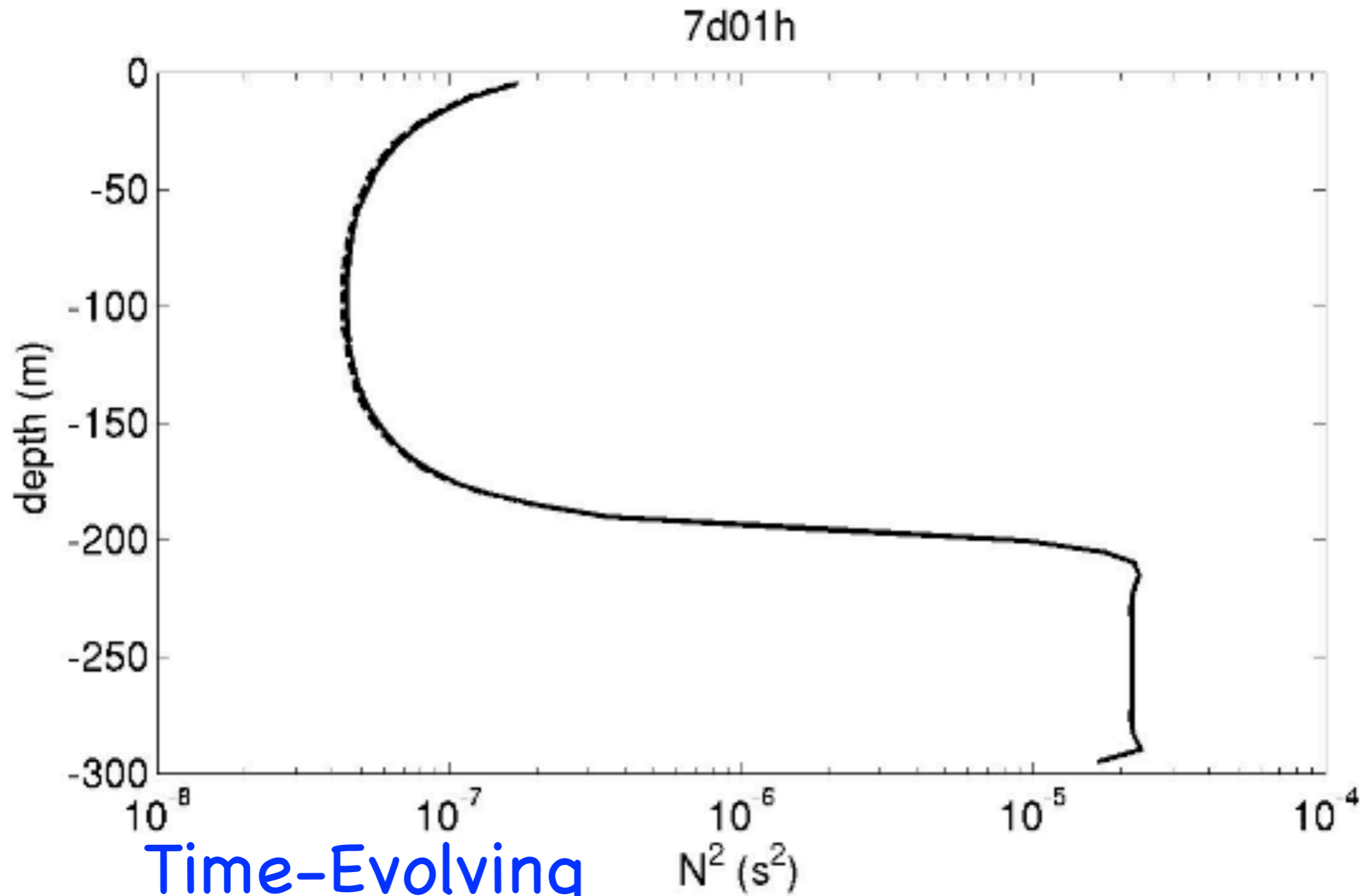
## Averaged MLE-resolving Model Solution

7d01h from 3d MITgcm (smoothed)



black=mean density

# What does eddy restratification look like?

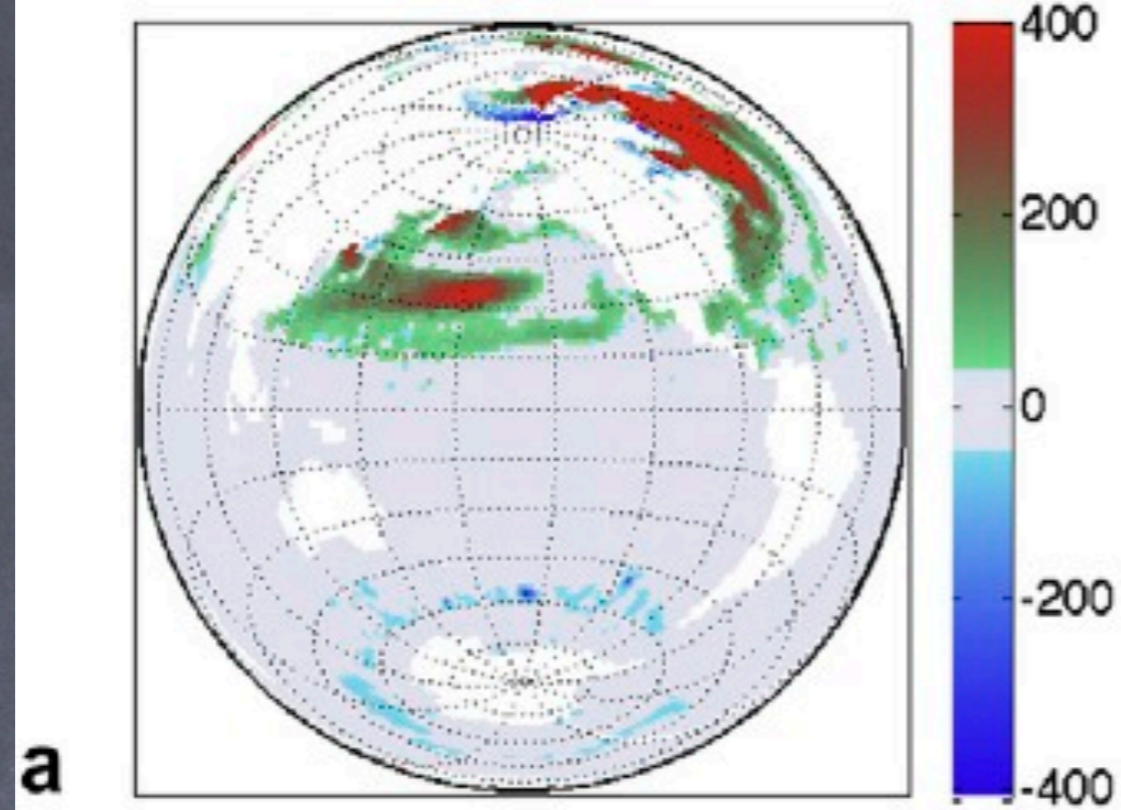


Time-Evolving  
Stratification ( $N^2$ )



Mixed Layer  
Depth Bias  
Versus  
Observations  
(No MLE,  
Control)

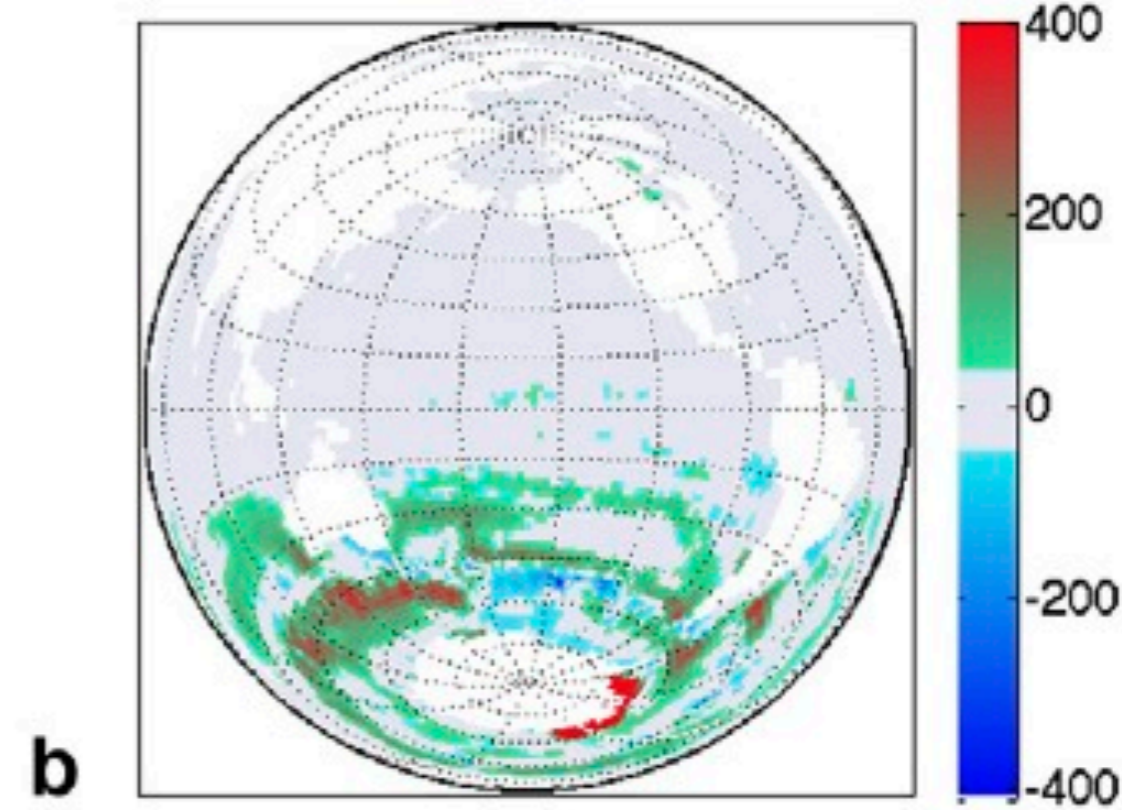
CM2M  $H_{ml}$  Control-deBM (m) FEB



**a**

max=2528m, min=-1560m

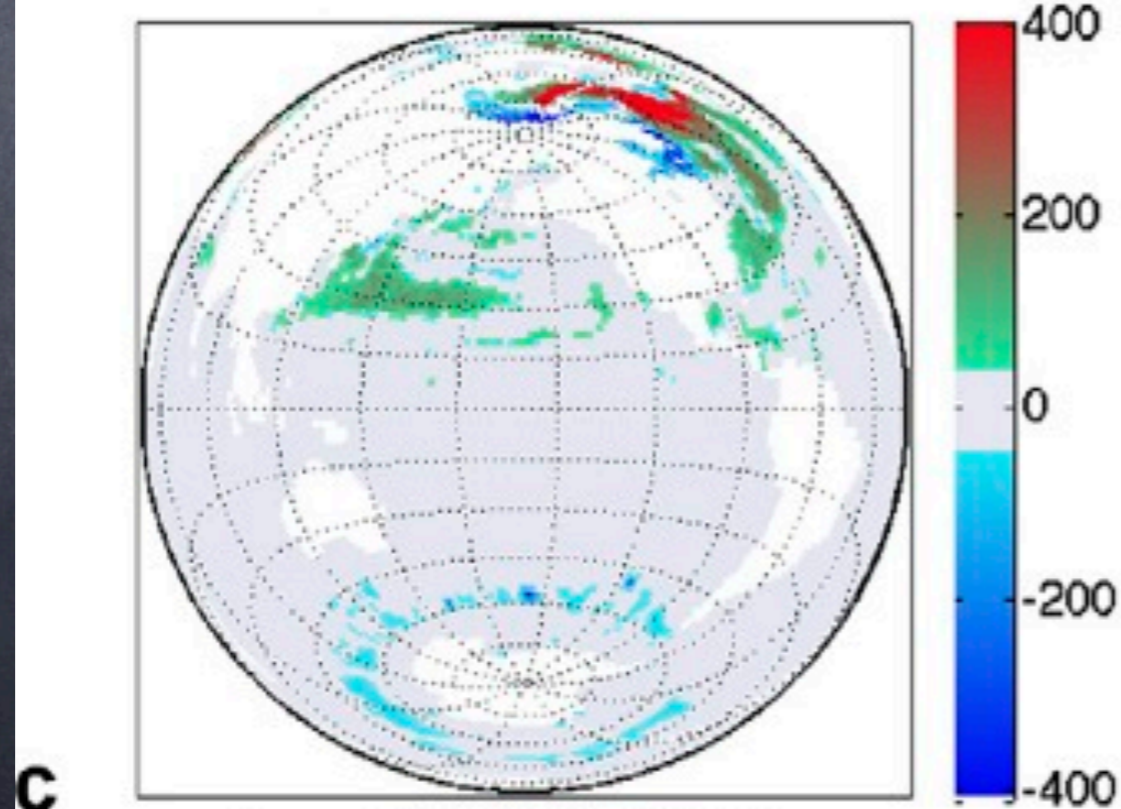
CM2M  $H_{ml}$  Control-deBM (m) SEP



**b**

max=2050m, min=-320m

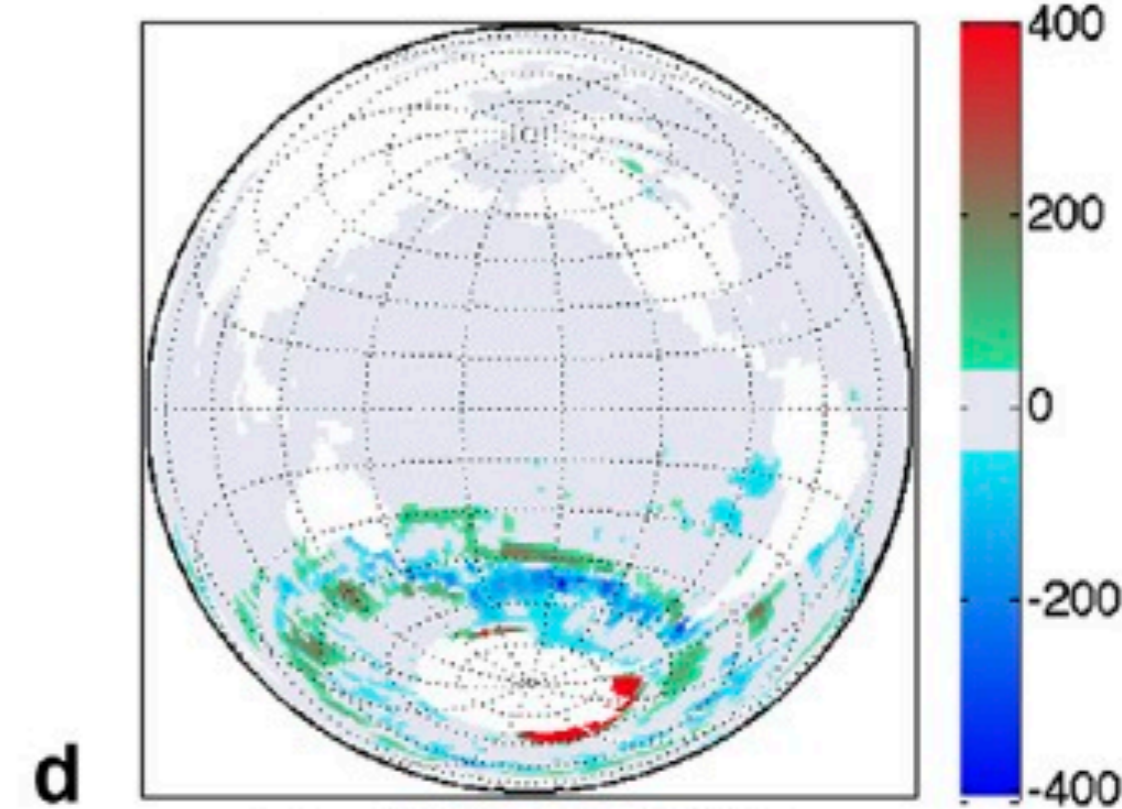
CM2M  $H_{ml}$  Submeso-deBM (m) FEB



**c**

max=1422m, min=-1600m

CM2M  $H_{ml}$  Submeso-deBM (m) SEP



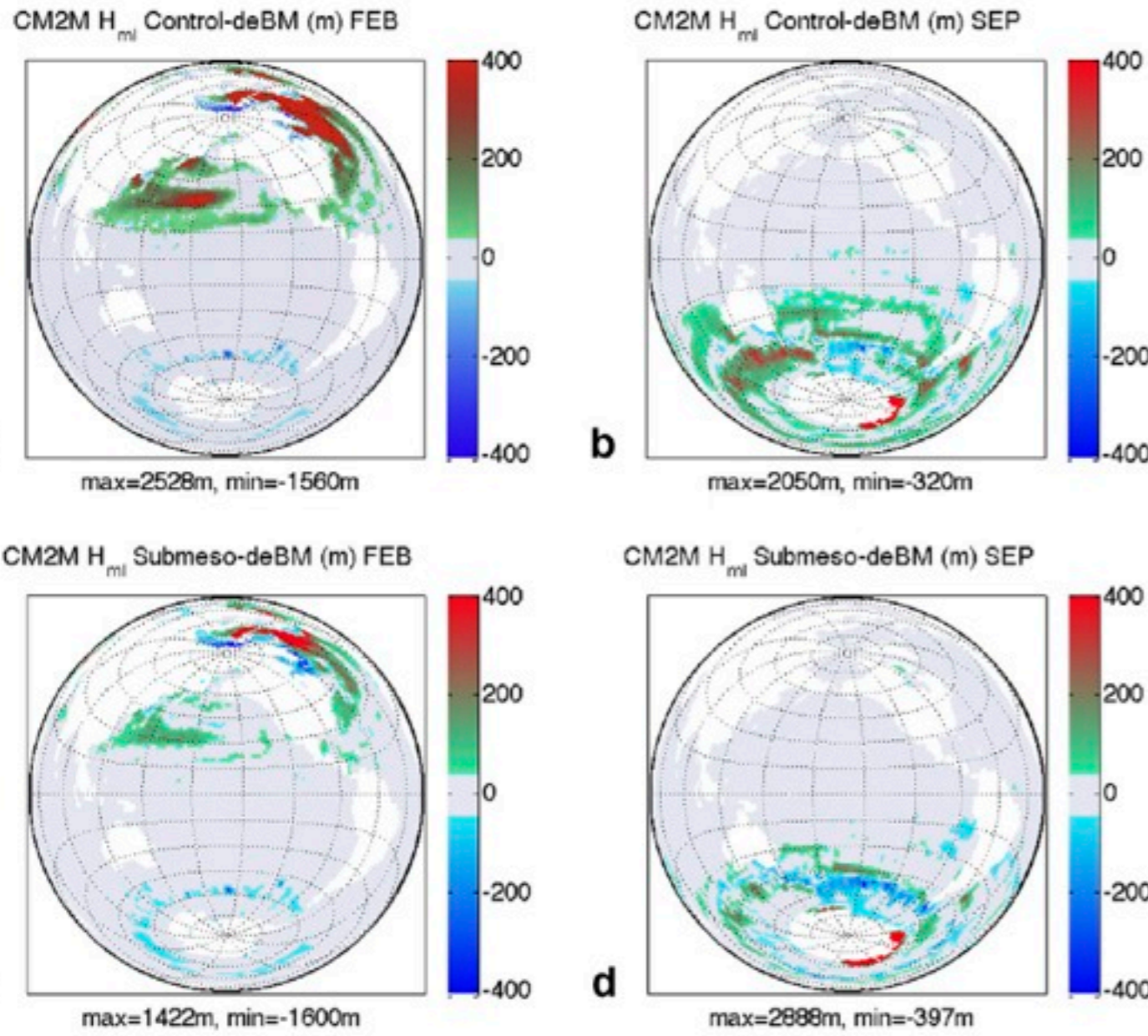
**d**

max=2888m, min=-397m

Mixed Layer  
Depth Bias  
Versus  
Observations  
(With MLE)

# Physical Sensitivity of Ocean Climate to Submesoscale Eddy Restructuring:

FFH implemented in CCSM (NCAR), CM2M & CM2G (GFDL)

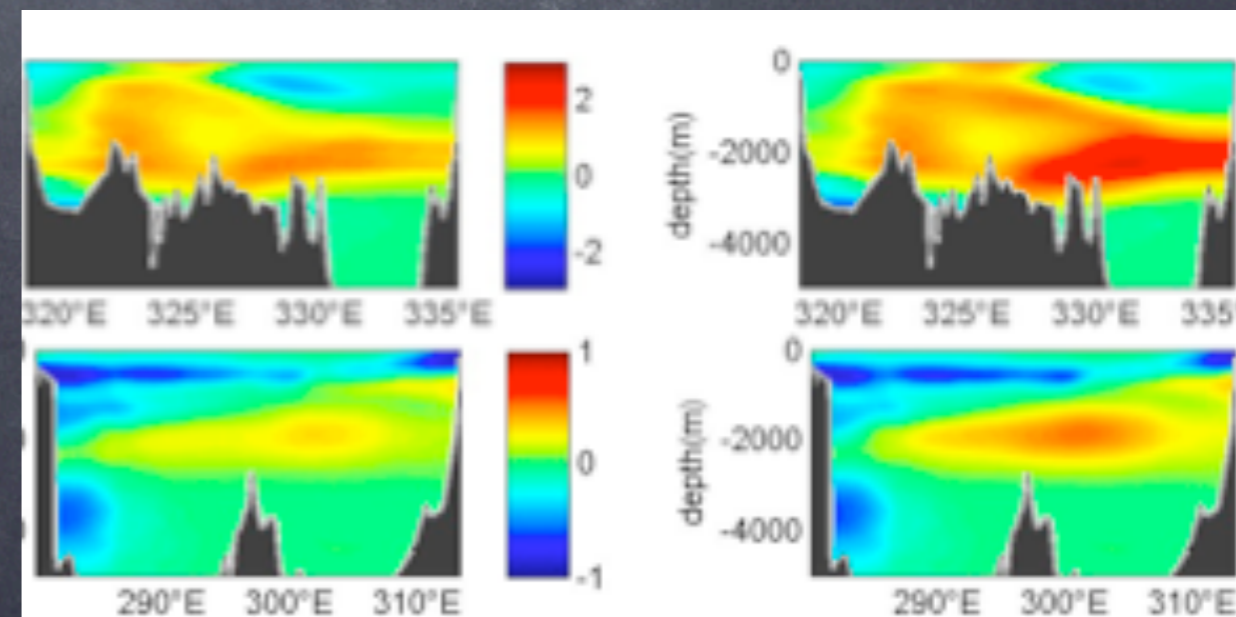


NO RETUNING NEEDED!!!

Improves CFCs  
Passive tracer

Bias with MLE

Bias w/o MLE



Deep ML Bias reduced  
From Fox-Kemper et al., 2011

# Sensitivity of Climate to Submeso: AMOC & Cryosphere Impacts

May Stabilize AMOC

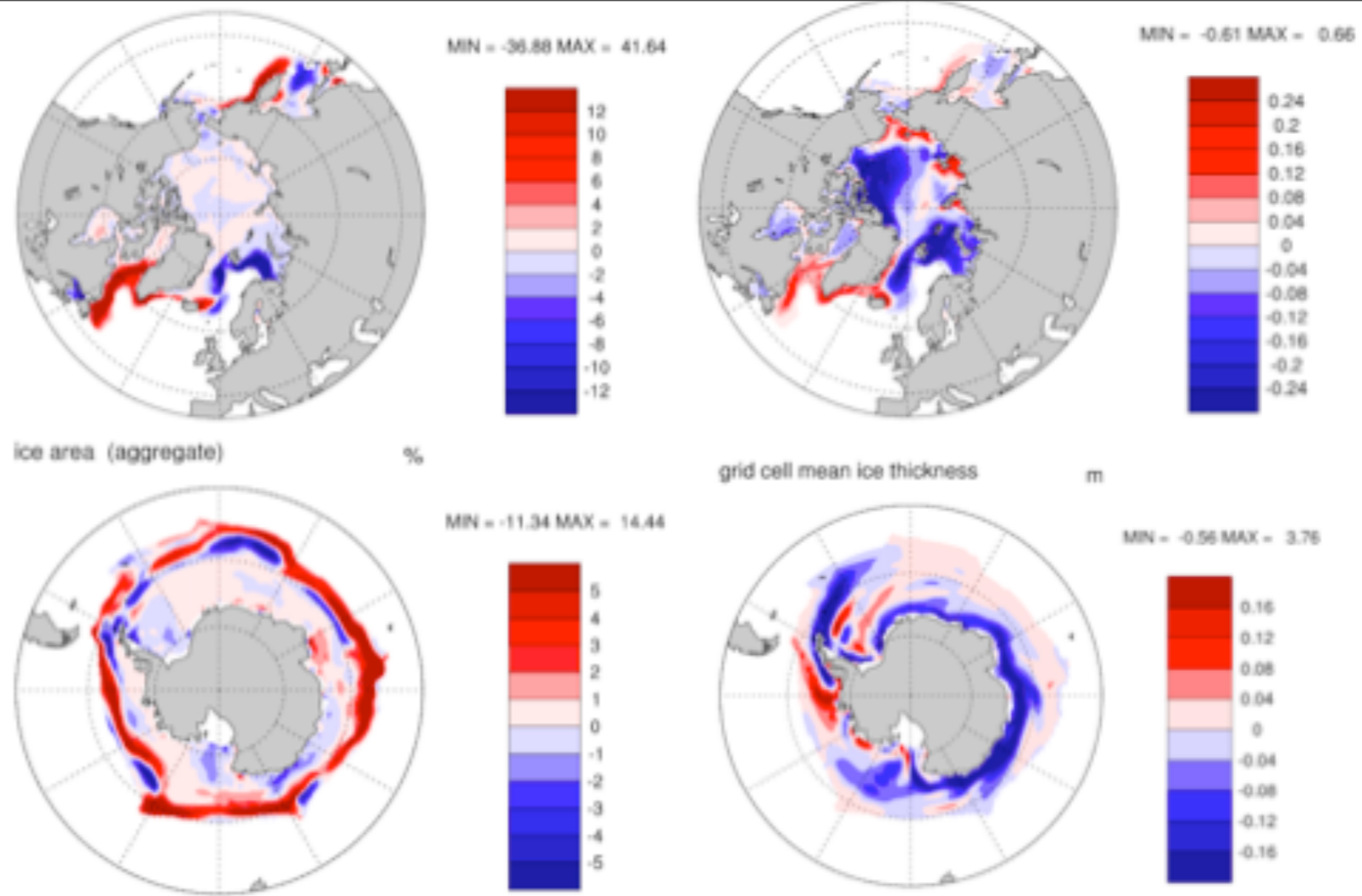


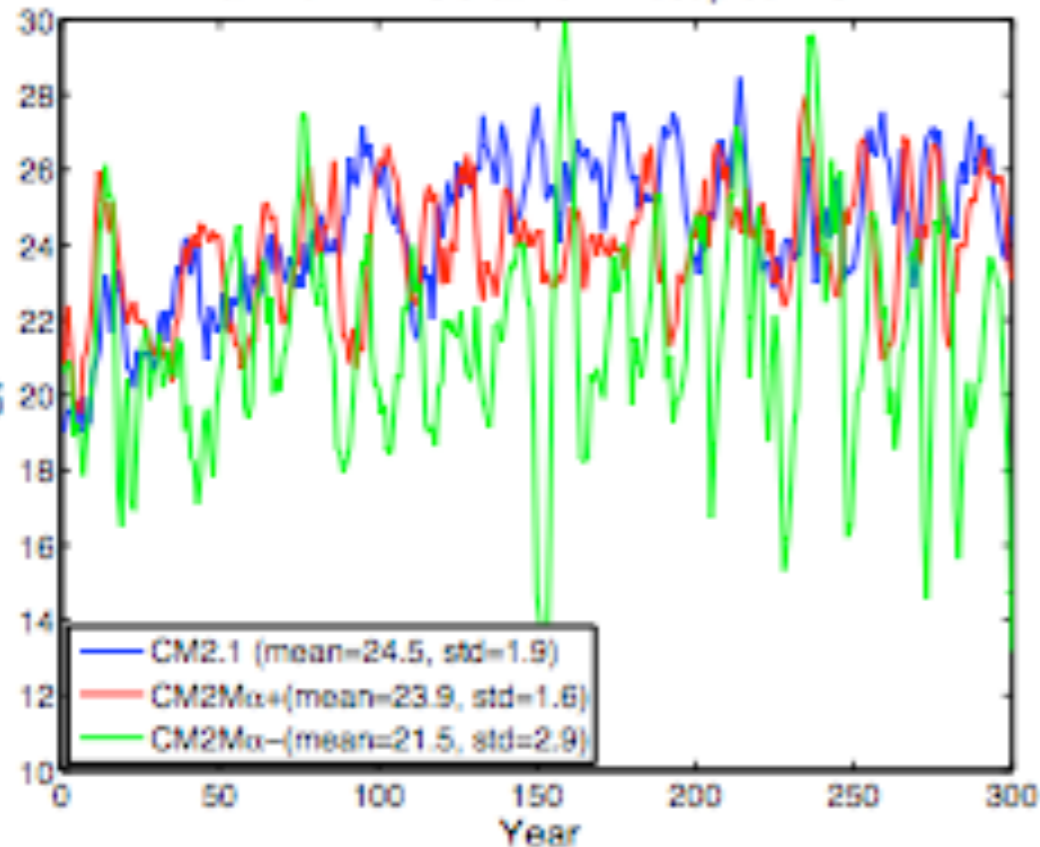
Figure 10: Wintertime sea ice sensitivity to introduction of MLE parameterization (CCSM<sup>+</sup> minus CCSM<sup>-</sup>): January to March Northern Hemisphere a) ice area and b) thickness and July to September Southern Hemisphere c) ice area and d) thickness.

Affects sea ice

**NO RETUNING  
NEEDED!!!**

These are impacts:  
bias change unknown

Maximum AMOC at 45n in coupled MOM



# Langmuir Turbulence Parameterizations

- On a list of the 50 most important things to fix in the ocean model, Langmuir is number 51.
  - --Bill Large

# The Character of the Langmuir Scale

- Near-surface
- Langmuir Cells & Langmuir Turb.
- $Ro \gg 1$
- $Ri < 1$ : Nonhydro
- 10–100m
- mins, hours
- $w, u = O(20\text{cm/s})$
- Stokes drift
- Eqtns: Craik–Leibovich
- unused params exist

image:  
Leibovich, 83

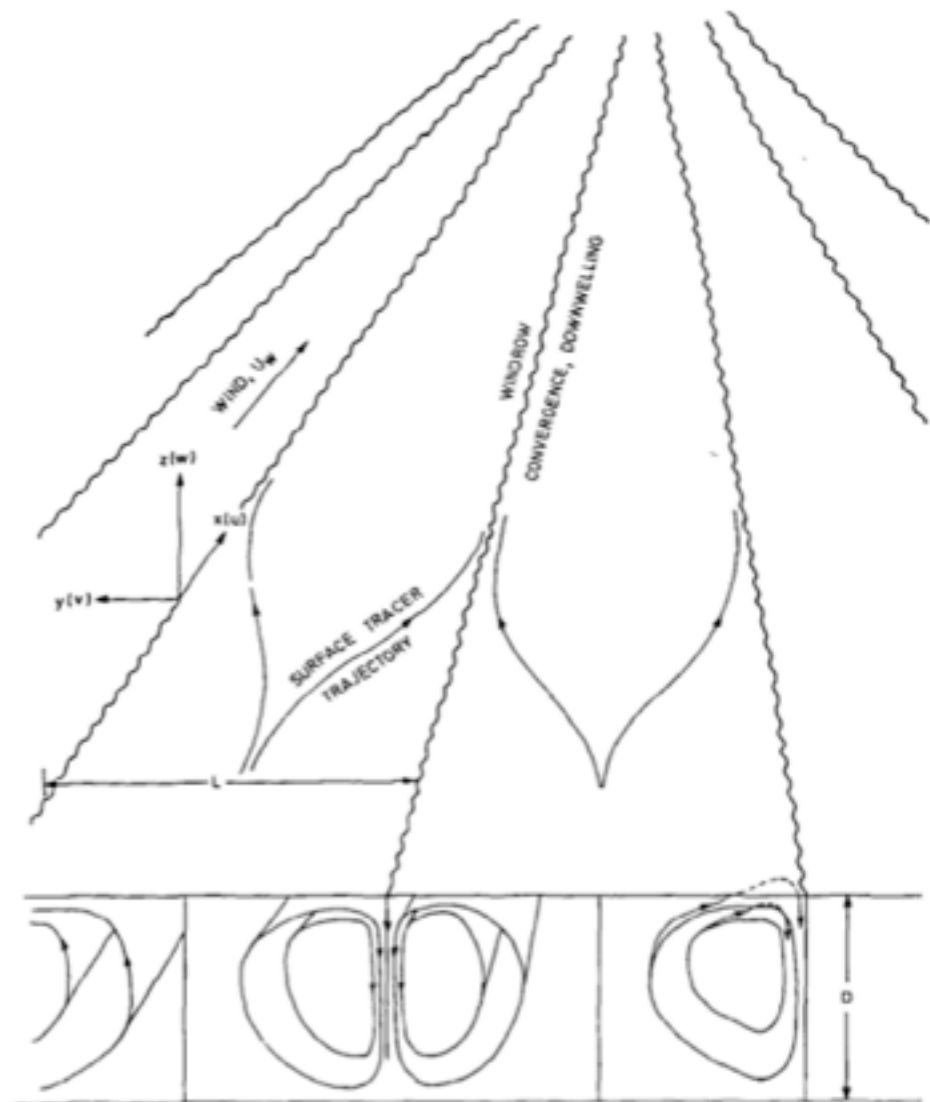


Figure 1a Illustration of Langmuir circulations showing notation used in this review and surface and subsurface motions.

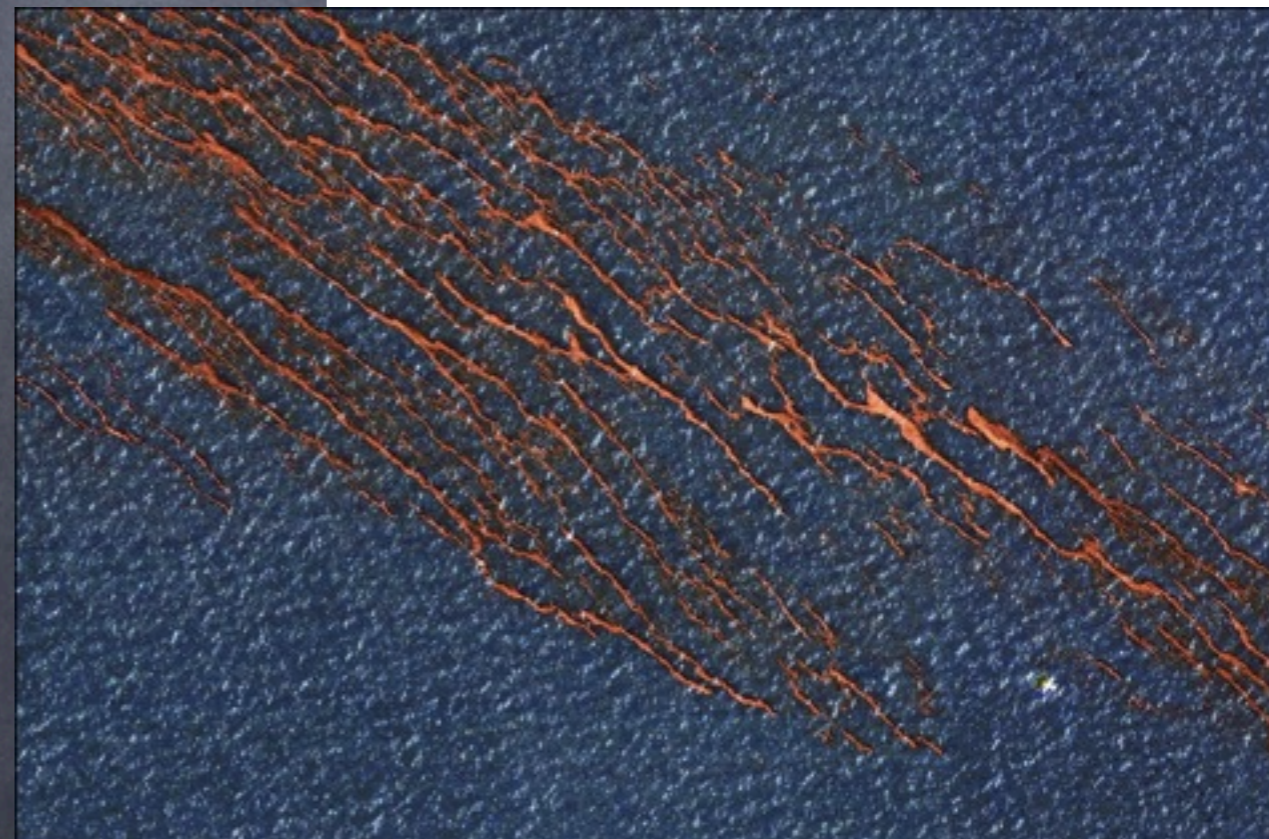


Image: NPR.org,  
Deep Water  
Horizon Spill

# An Immature Improvement to Air-Sea BL



Shuga Ice

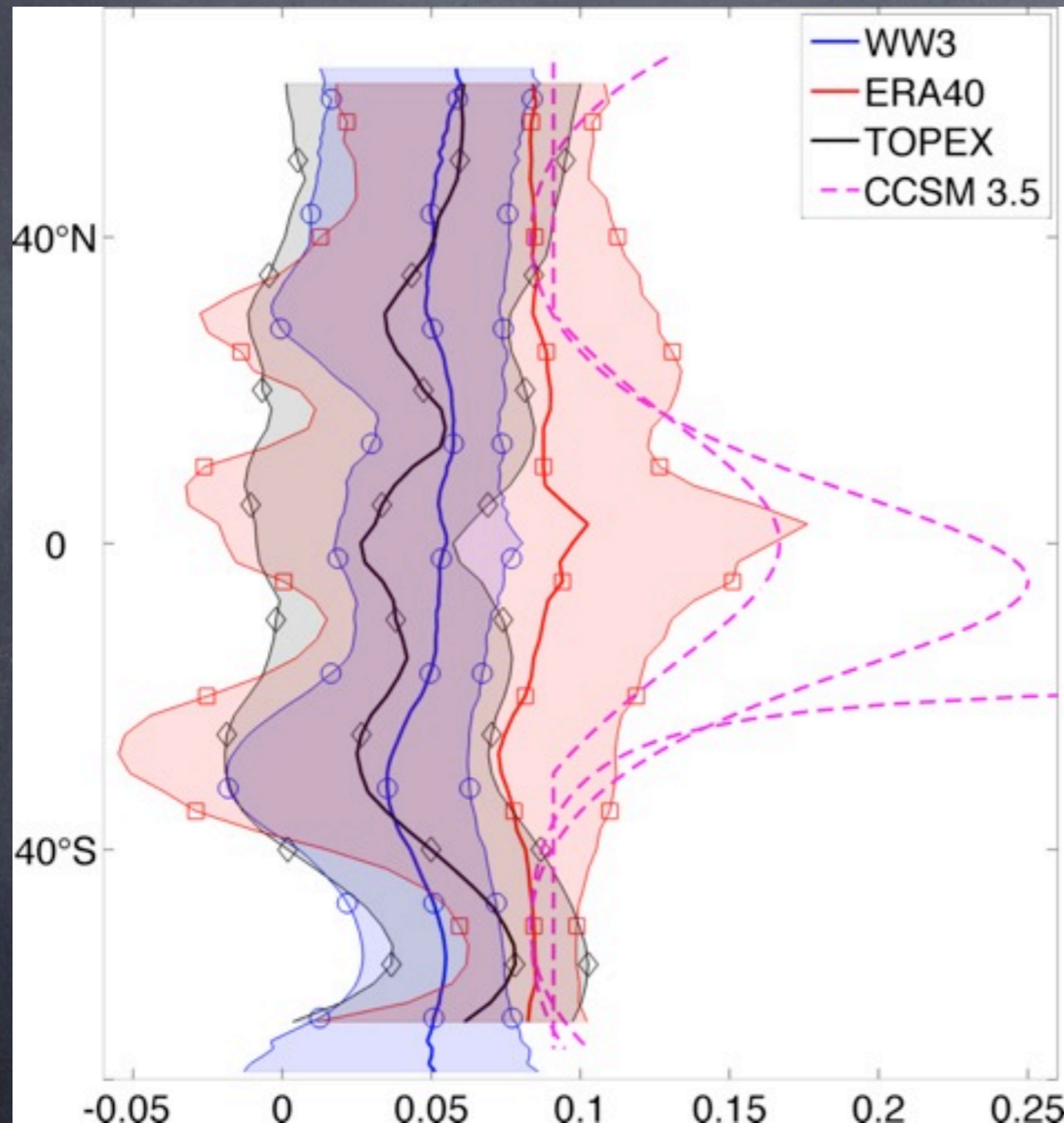
Image: aspect.aq

Image: NPR.org, Deep Water Horizon Spill

- ◉ Mixing by Langmuir Turbulence
  - ◉ Forced by wind and waves
    - ◉ i.e., Stokes drift & Eulerian Shear
  - ◉ Scalings from LES, Observations disagree
- ◉ We used a 2-part approximation
  - ◉ 1) McWilliams & Sullivan (01) additional OBL mixing (within mixed layer)
  - ◉ 2) Li & Garrett (98) Langmuir mixing depth (entrainment)
  - ◉ Roughly comparable to other schemes, but crude & incompletely validated
  - ◉ Needs only  $u^*$ ,  $u_s$  to work

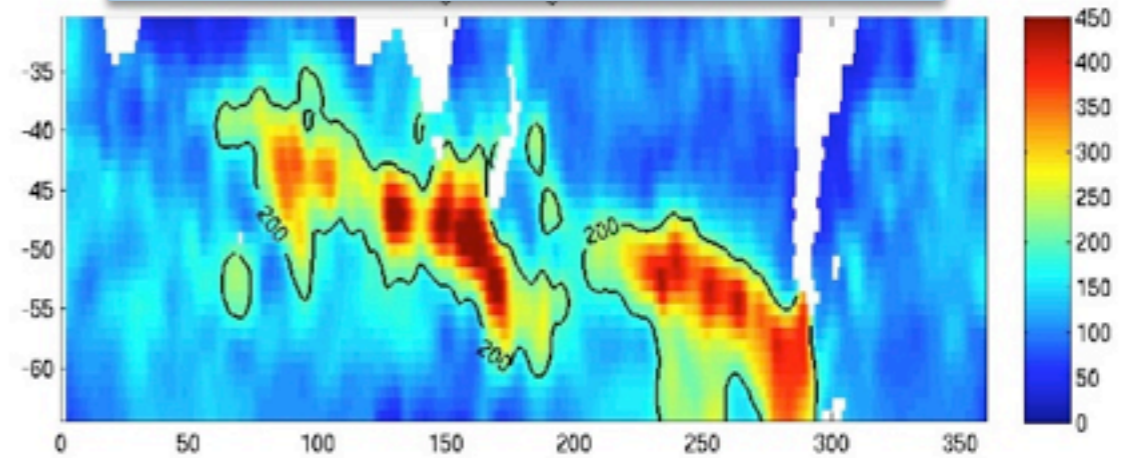


# Langmuir Mixing Forced by Climatology

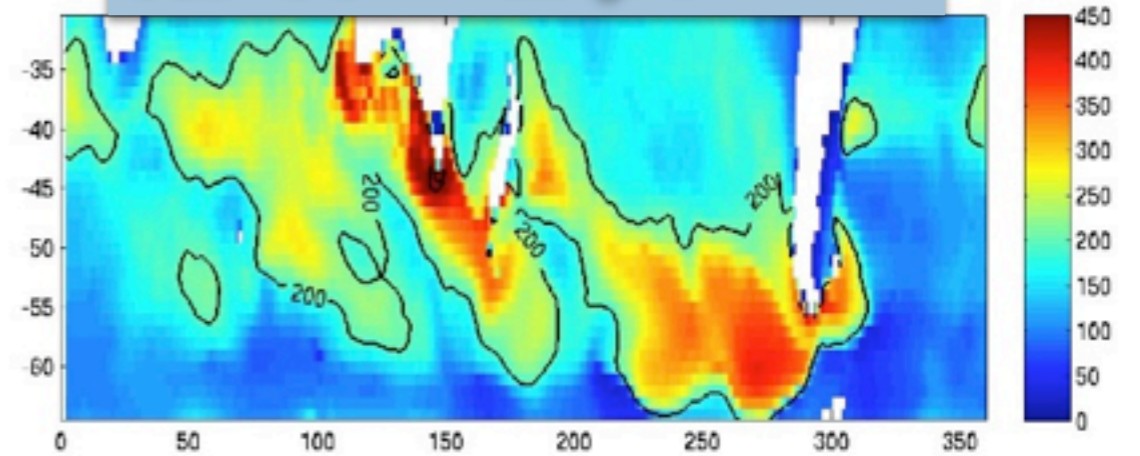


(Generalized Turbulent Langmuir)<sup>2</sup>  
Projection of  $u^*$ ,  $u_s$  into Langmuir Direction

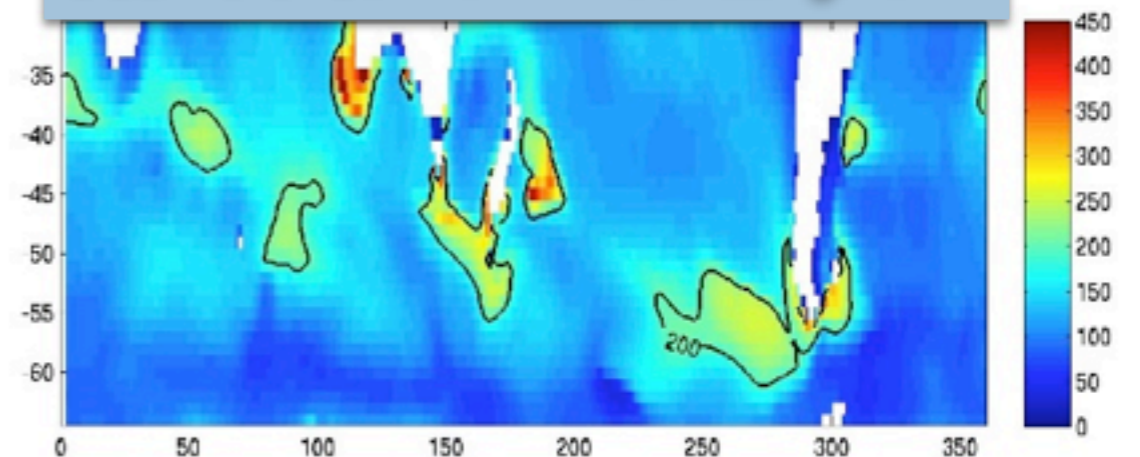
Dong et al. Observations



CCSM3.5 with Langmuir

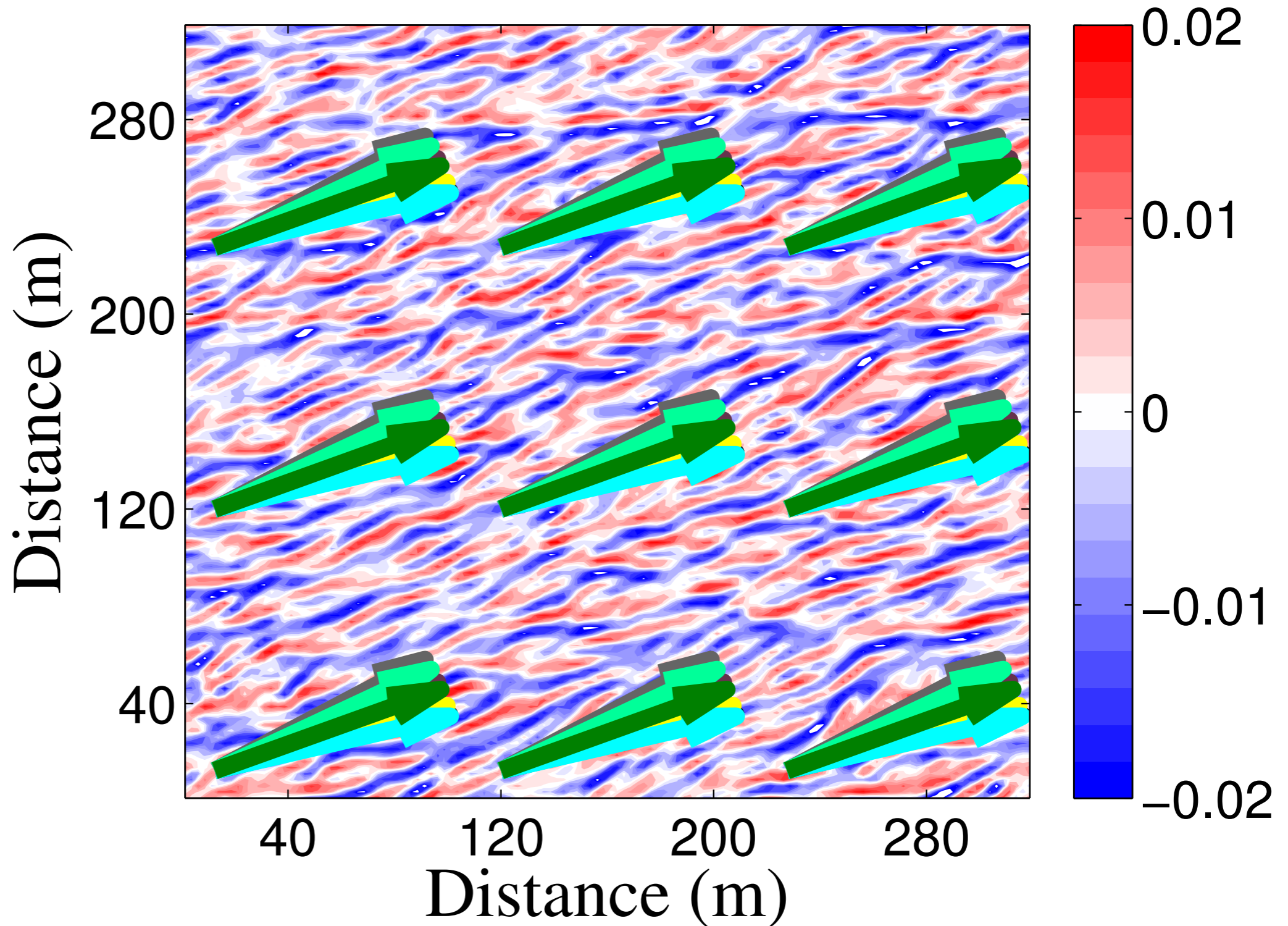


CCSM3.5 Control without Langmuir



$$La_t^2 = \frac{|u^*|}{|u_s|} \left[ \frac{|u^*| + |u_s| \cos \theta}{|u_s| + |u^*| \cos \theta} \right]$$

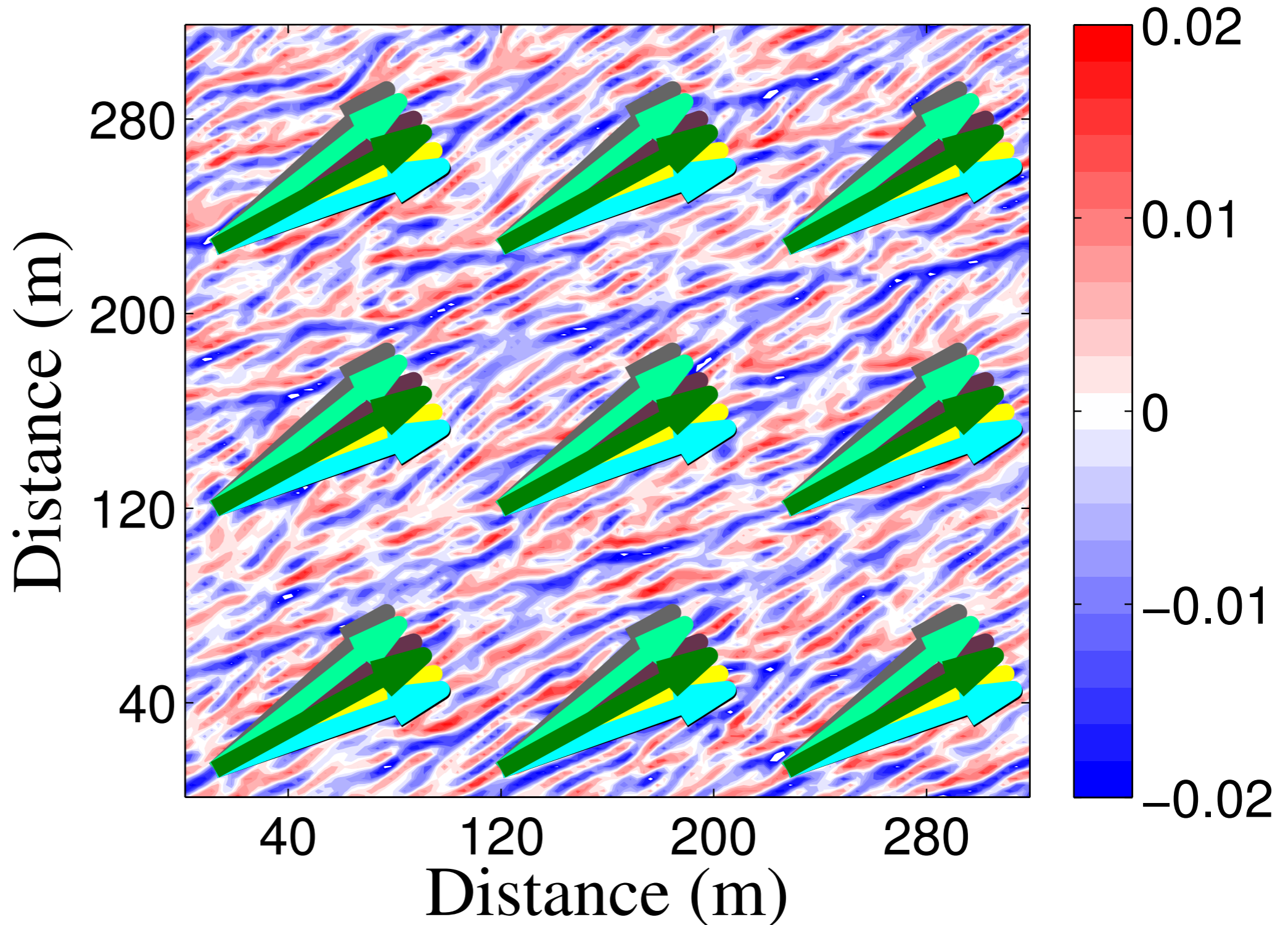
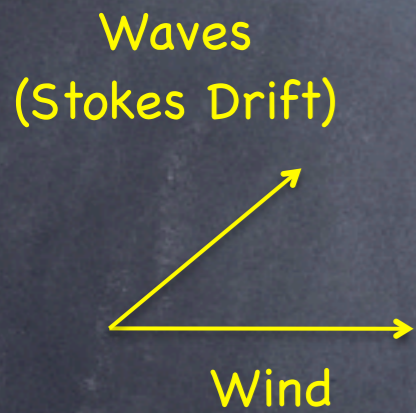
# Tricky: Misaligned Wind & Waves



Van Roekel et al. 2011  
(in prep)

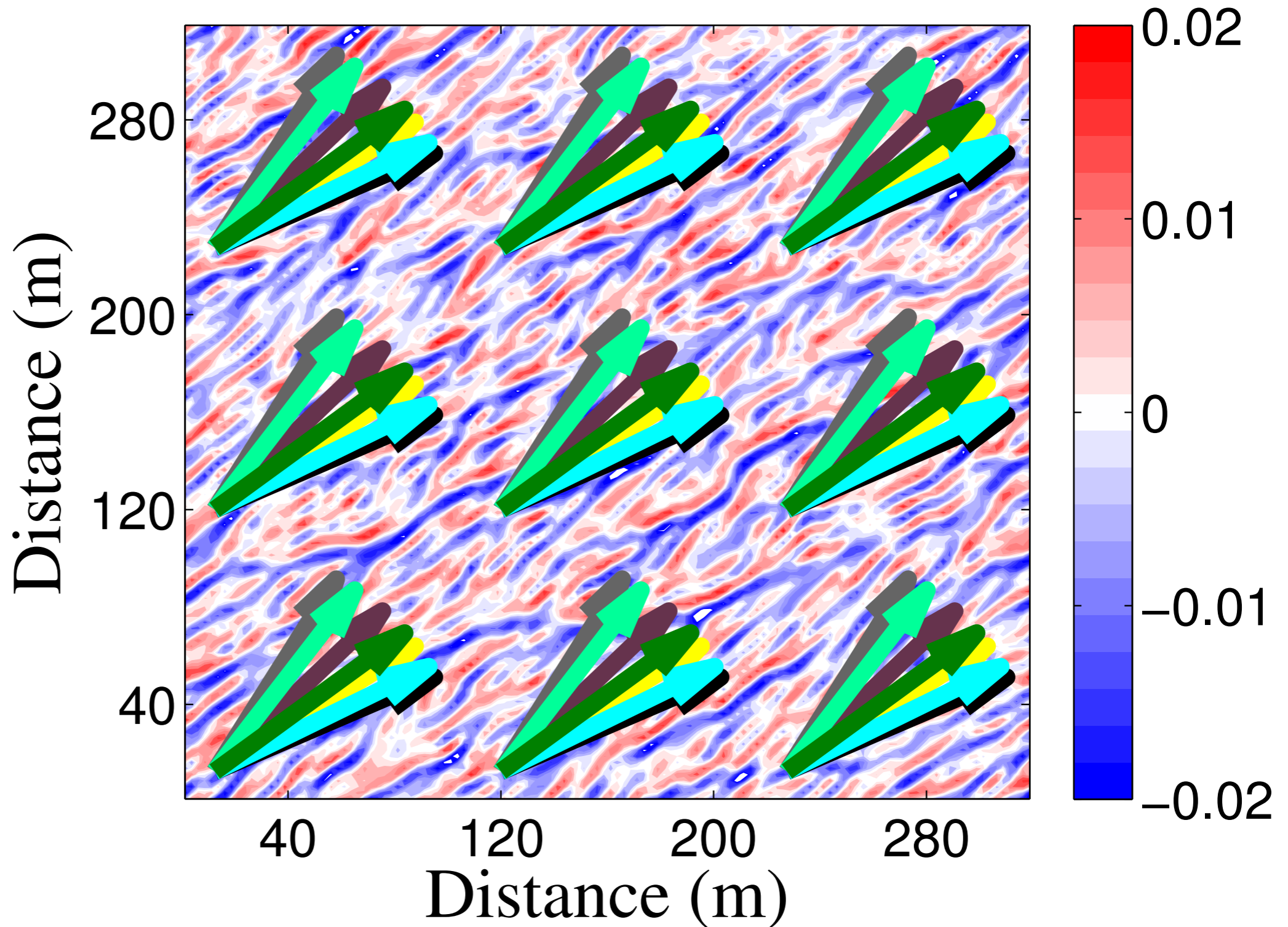
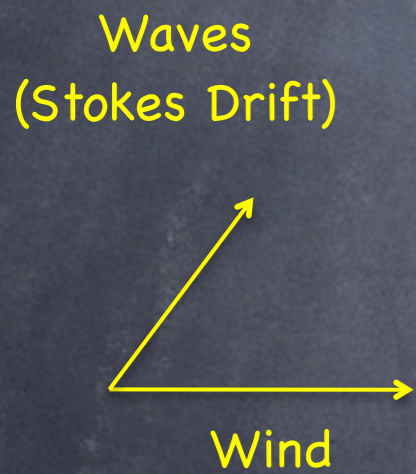


# Tricky: Misaligned Wind & Waves



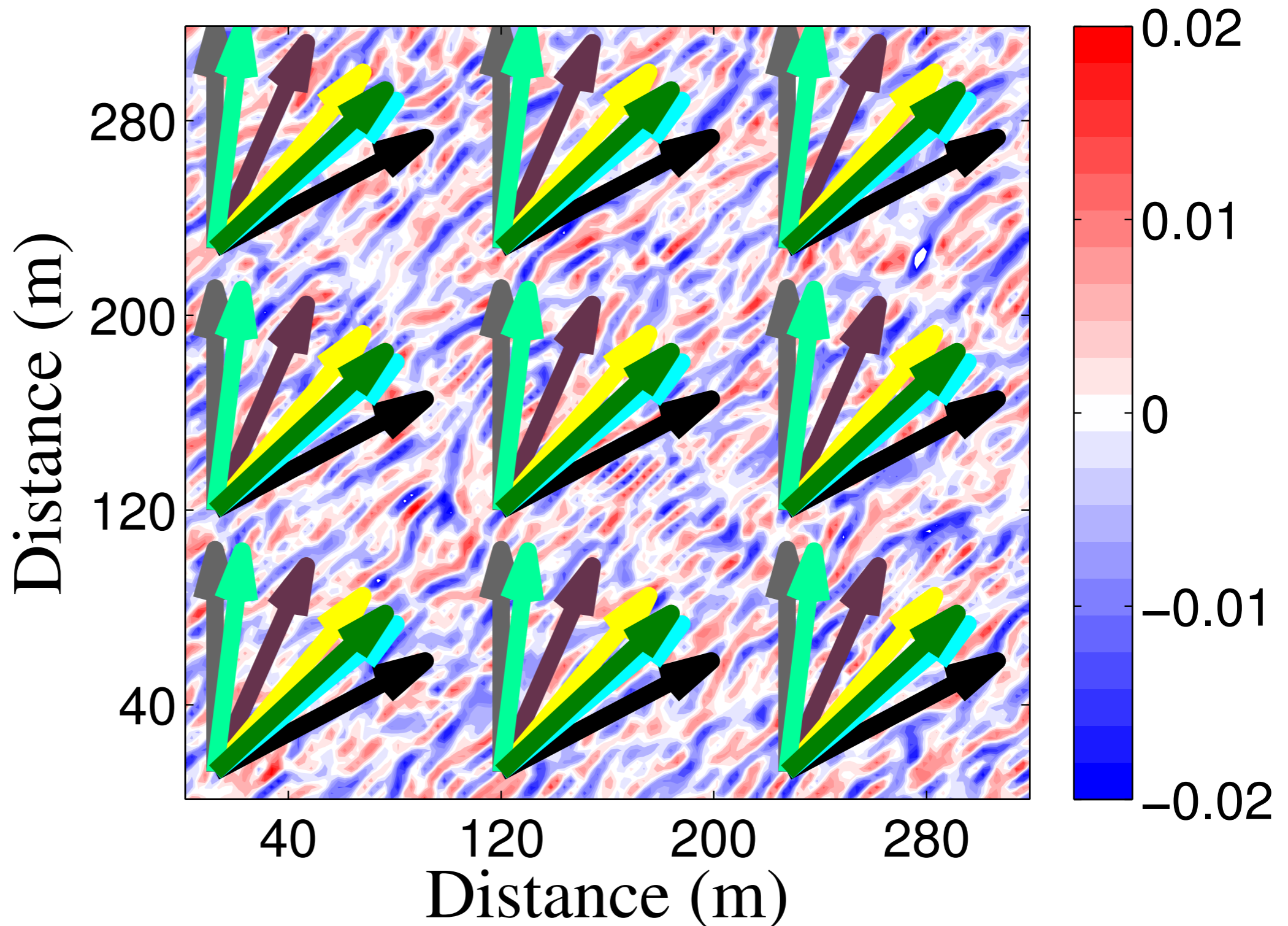
Van Roekel et al. 2011  
(in prep)

# Tricky: Misaligned Wind & Waves



Van Roekel et al. 2011  
(in prep)

# Tricky: Misaligned Wind & Waves



Van Roekel et al. 2011  
(in prep)

# Coupling between Langmuir and Submeso?



• Together?

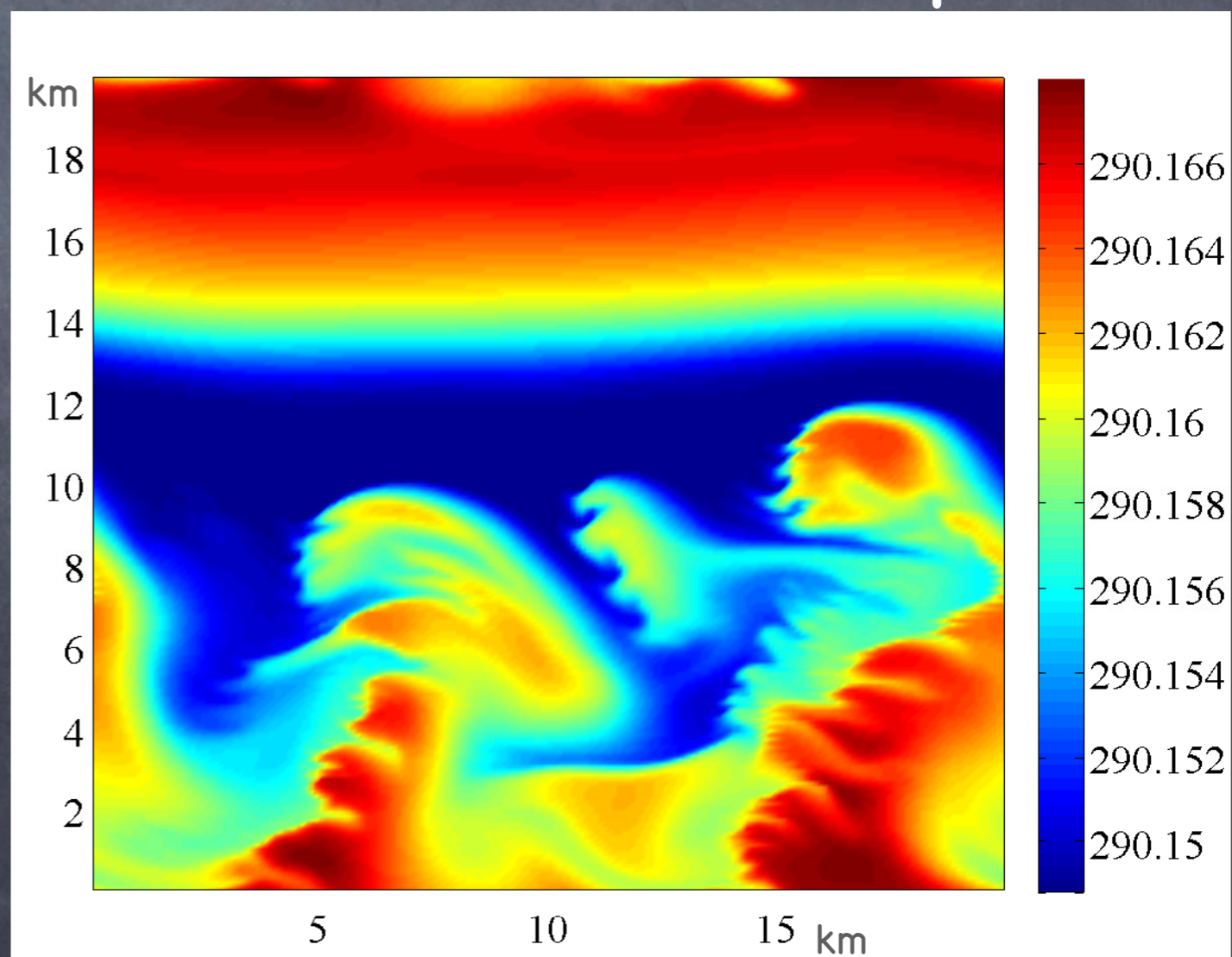
• Separate?



# The Game

- Spin up a submeso-resolving, but not Langmuir resolving model
  - 20kmx20kmx0.1km
  - Grid 384x384x20
  - 52m resolution
- Interpolate down to Langmuir resolving LES
  - 20kmx20kmx0.3km
  - Grid 4096x4096x128
  - 5m resolution
- Run for 2 more days, then...

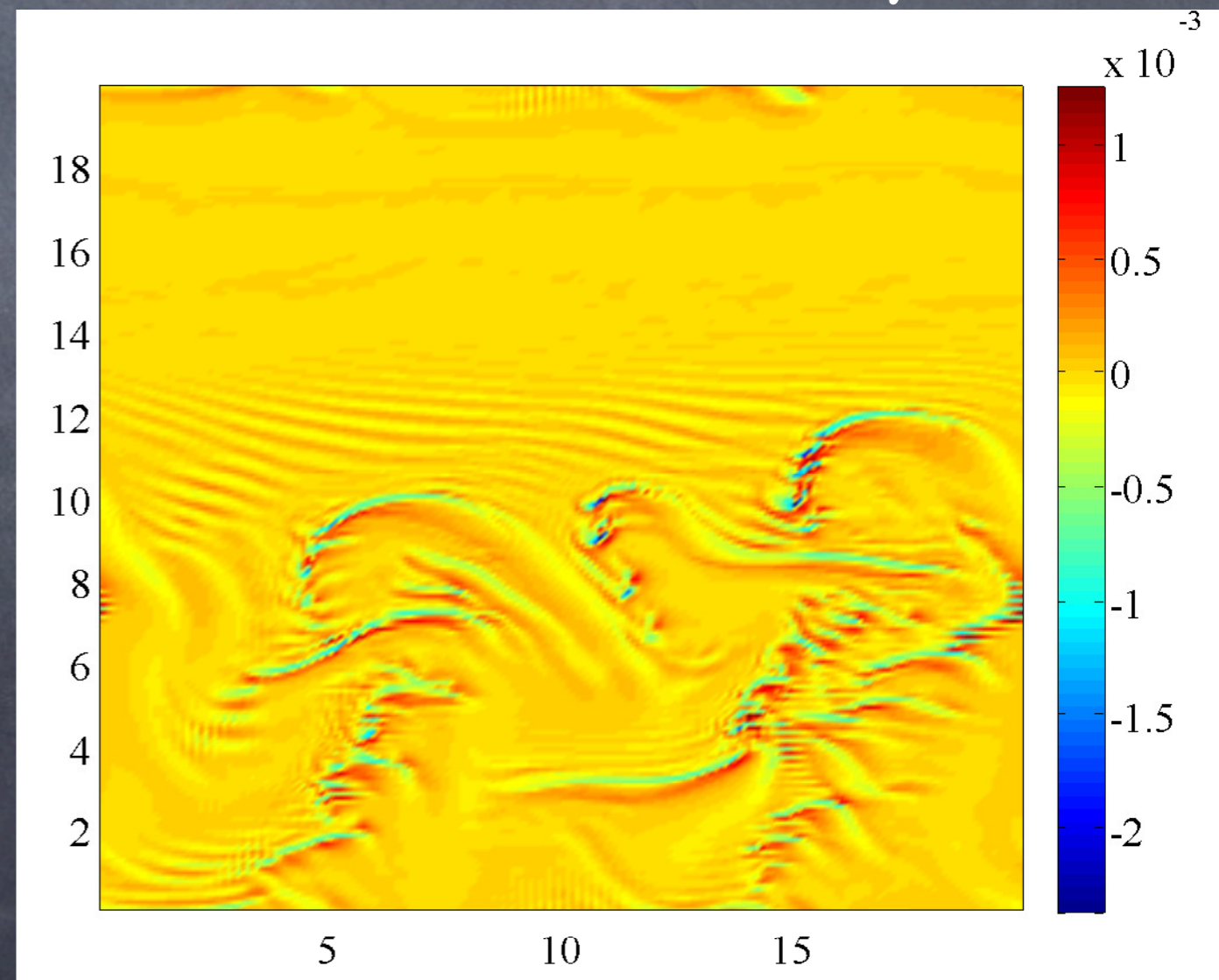
Day 6.5 of a Submeso  
Resolving run  
Near Surf. Temp



# The Game

- Spin up a submeso-resolving, but not Langmuir resolving model
  - 20kmx20kmx0.1km
  - Grid 384x384x20
  - 52m resolution
- Interpolate down to Langmuir resolving LES
  - 20kmx20kmx0.3km
  - Grid 4096x4096x128
  - 5m resolution
- Run for 2 more days, then...

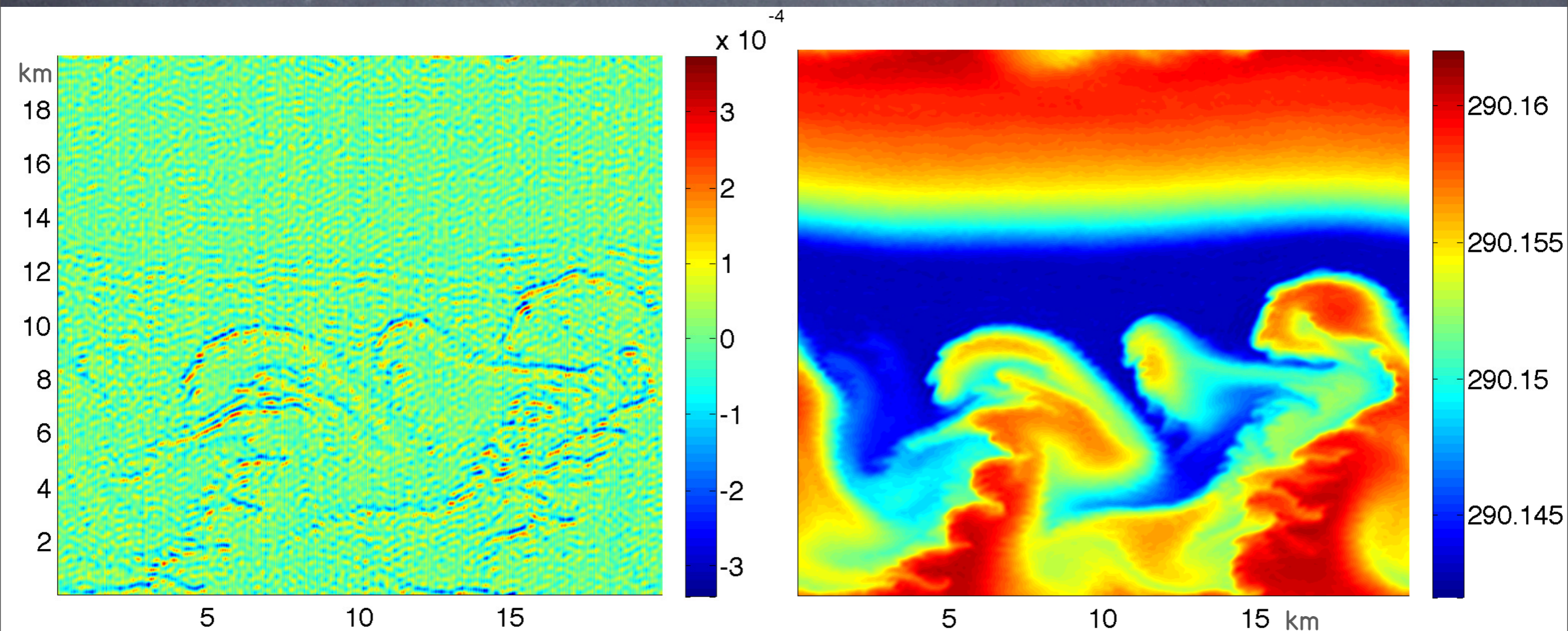
Day 6.5 of a Submeso  
Resolving run  
Vert. Velocity



# Coupling Langmuir to Submesoscale?

Near-Surf Vert. Vel.  
With Stokes Drift

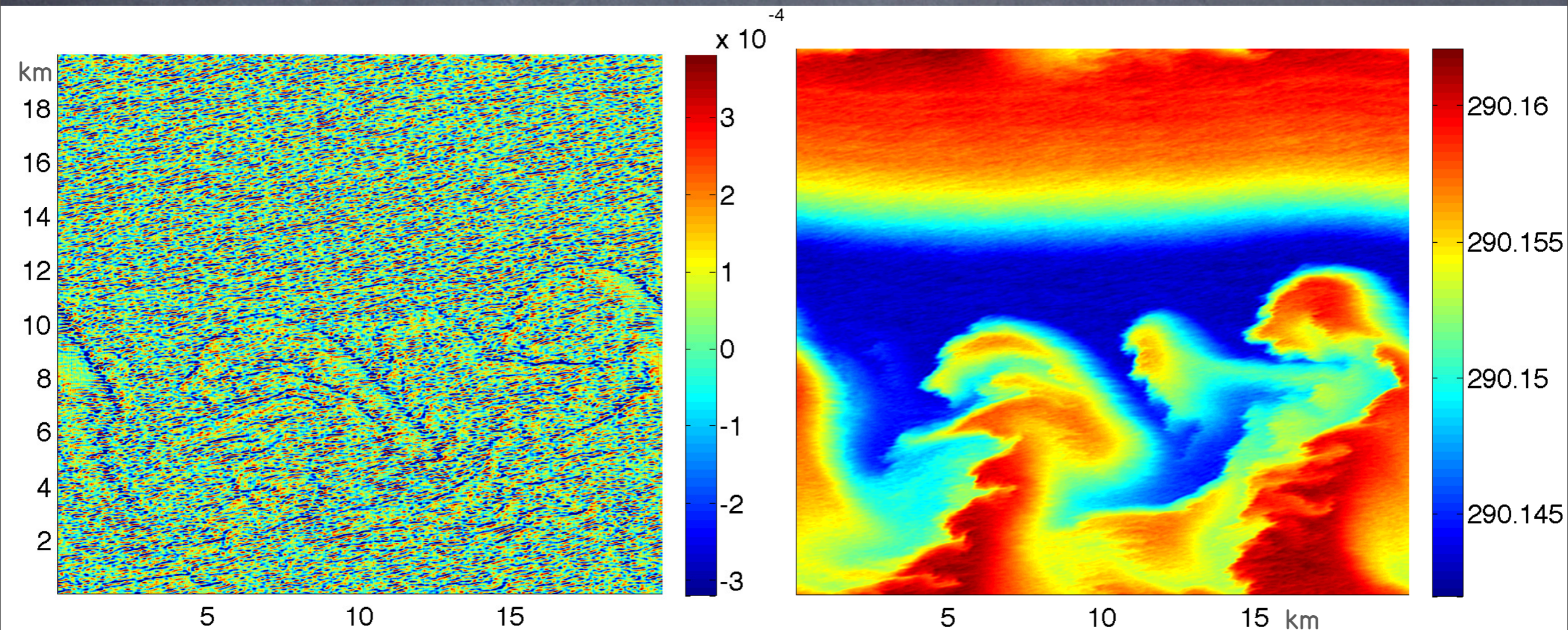
Near-Surf Temp.  
With Stokes Drift



# Coupling Langmuir to Submesoscale?

Vertical Velocity  
No Stokes Drift

Near-Surf. Temp.  
No Stokes Drift



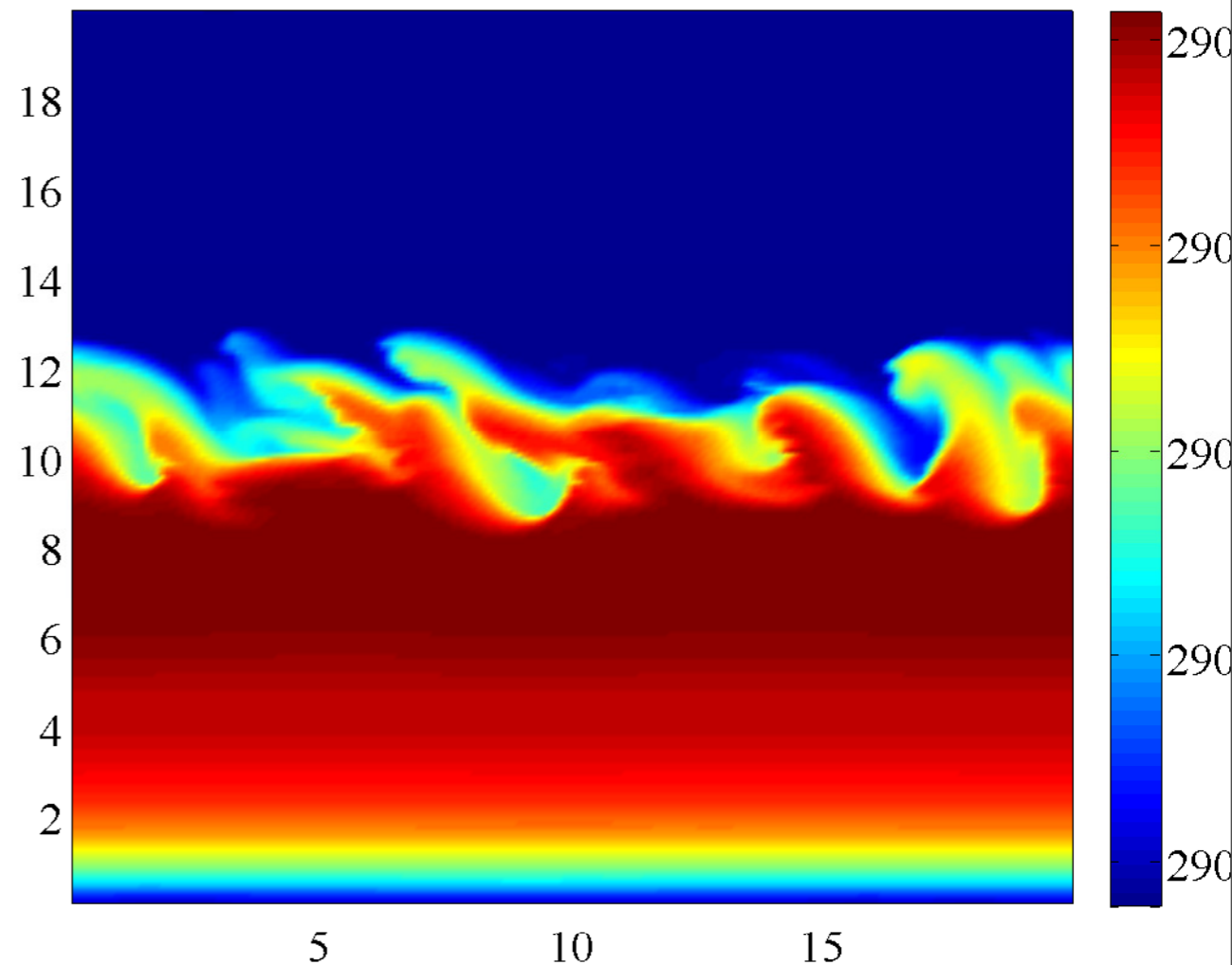
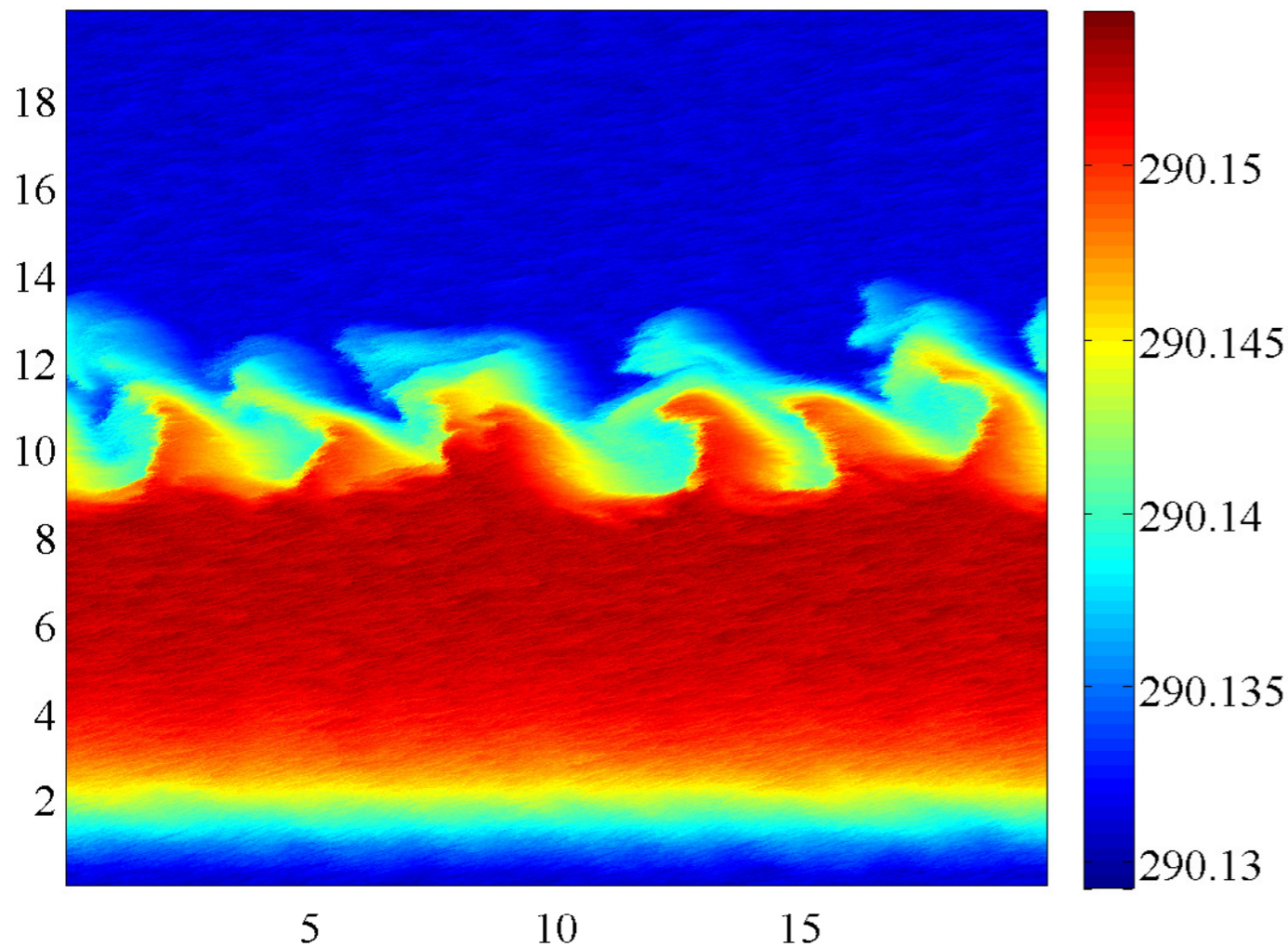


# Coupling Langmuir to Submesoscale?

From Scratch... No interpolation!

Near-Surf. Temp.  
No Stokes Drift

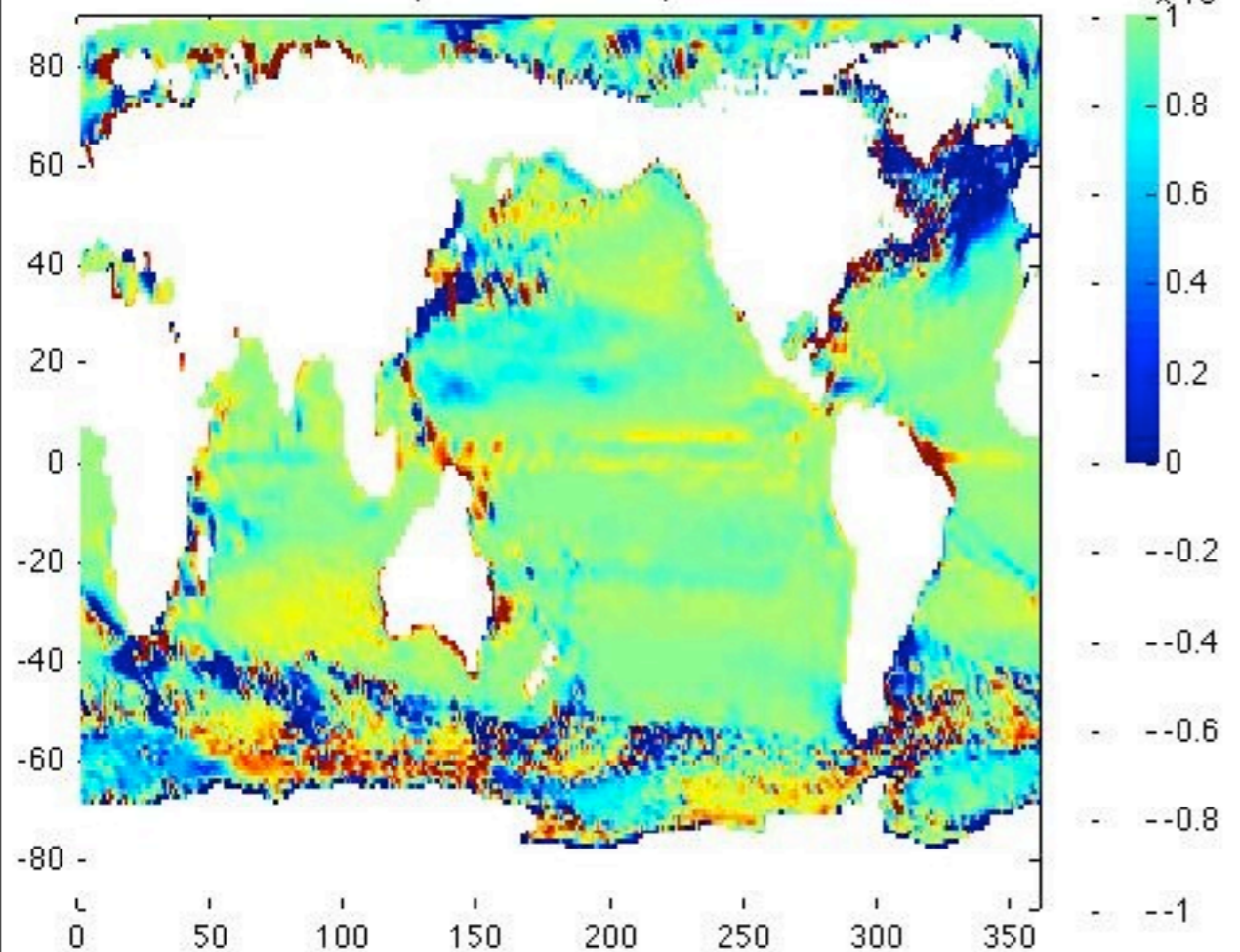
Near-Surf. Temp.  
Submeso-Only Res.



# Conclusions

- Mesoscale, Submesoscale, and Langmuir scale phenomena all have a nontrivial affect on the global climate, thus need accurate parameterizations
- Parameterizations are developed by comparison to higher-resolution models, with careful diagnosis of interesting terms
- These high resolution models reveal primary balances and spatiotemporal dependence that should be approximated by the parameterizations.

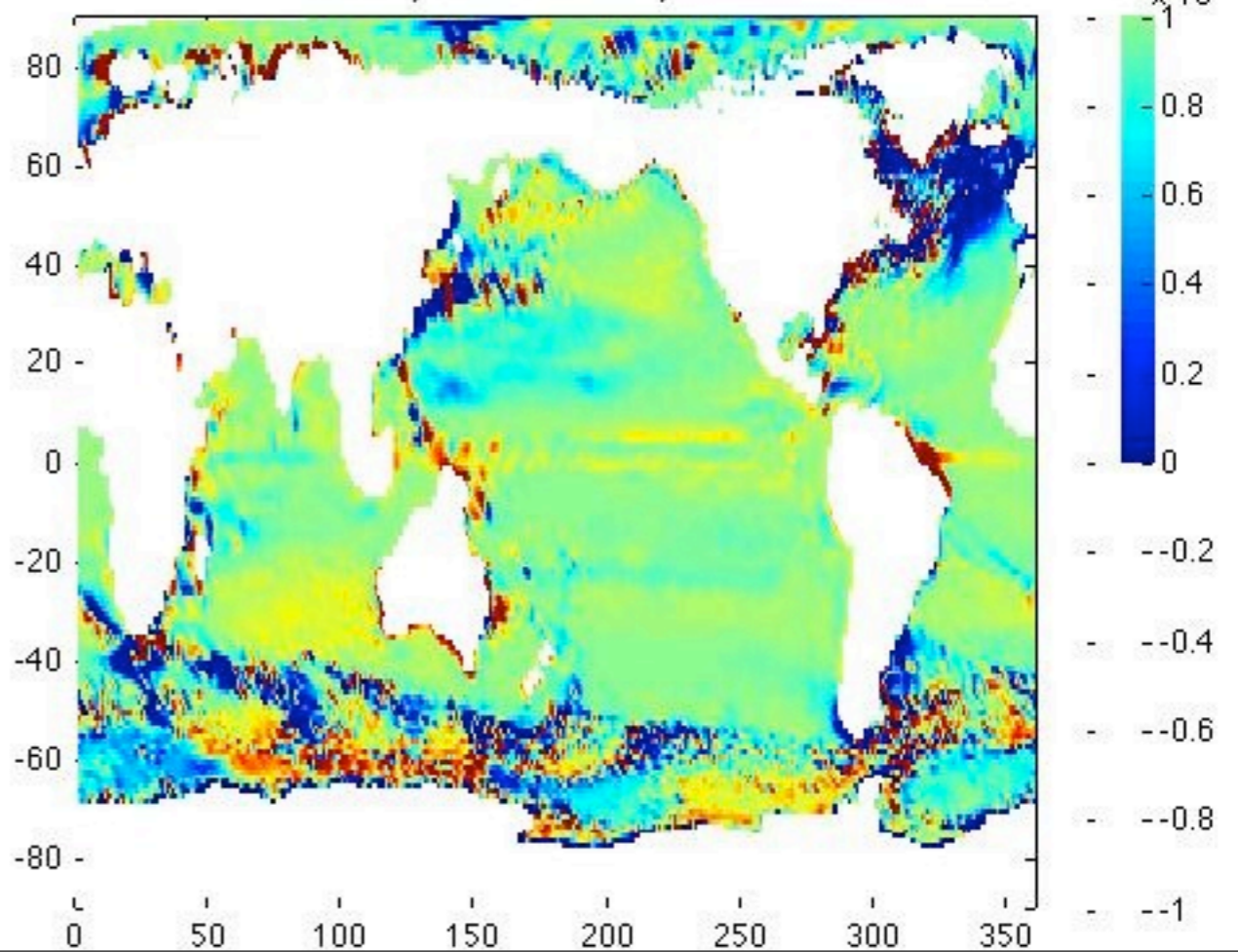
Vertical Eddy Potential Density Flux at 300m



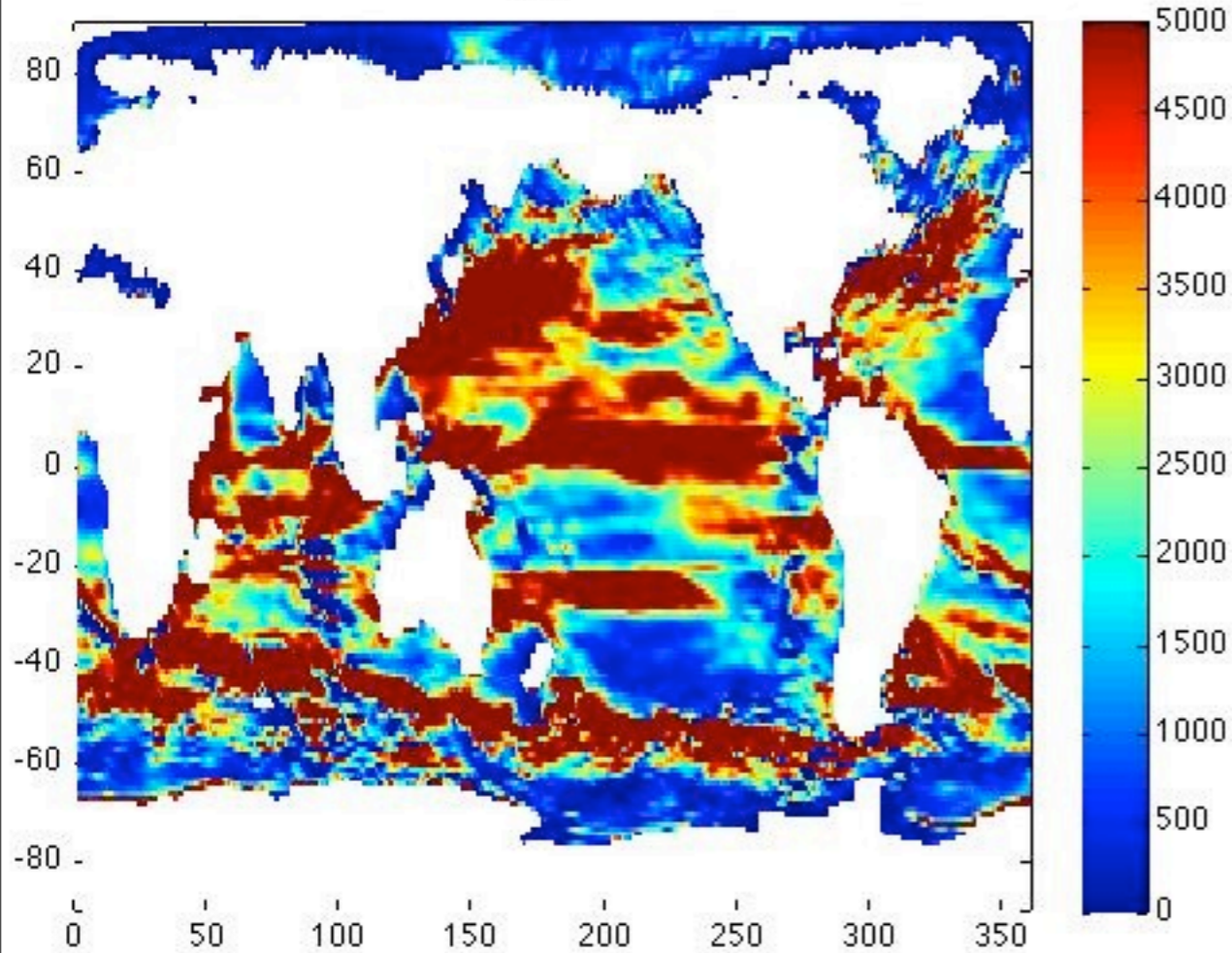
Compare to  
vertical eddy  
density flux  
(PE Extraction)

Eden&Greatbatch (+others) propose that baroclinic instability's production of EKE from PE should guide M magnitude

Vertical Eddy Potential Density Flux at 300m



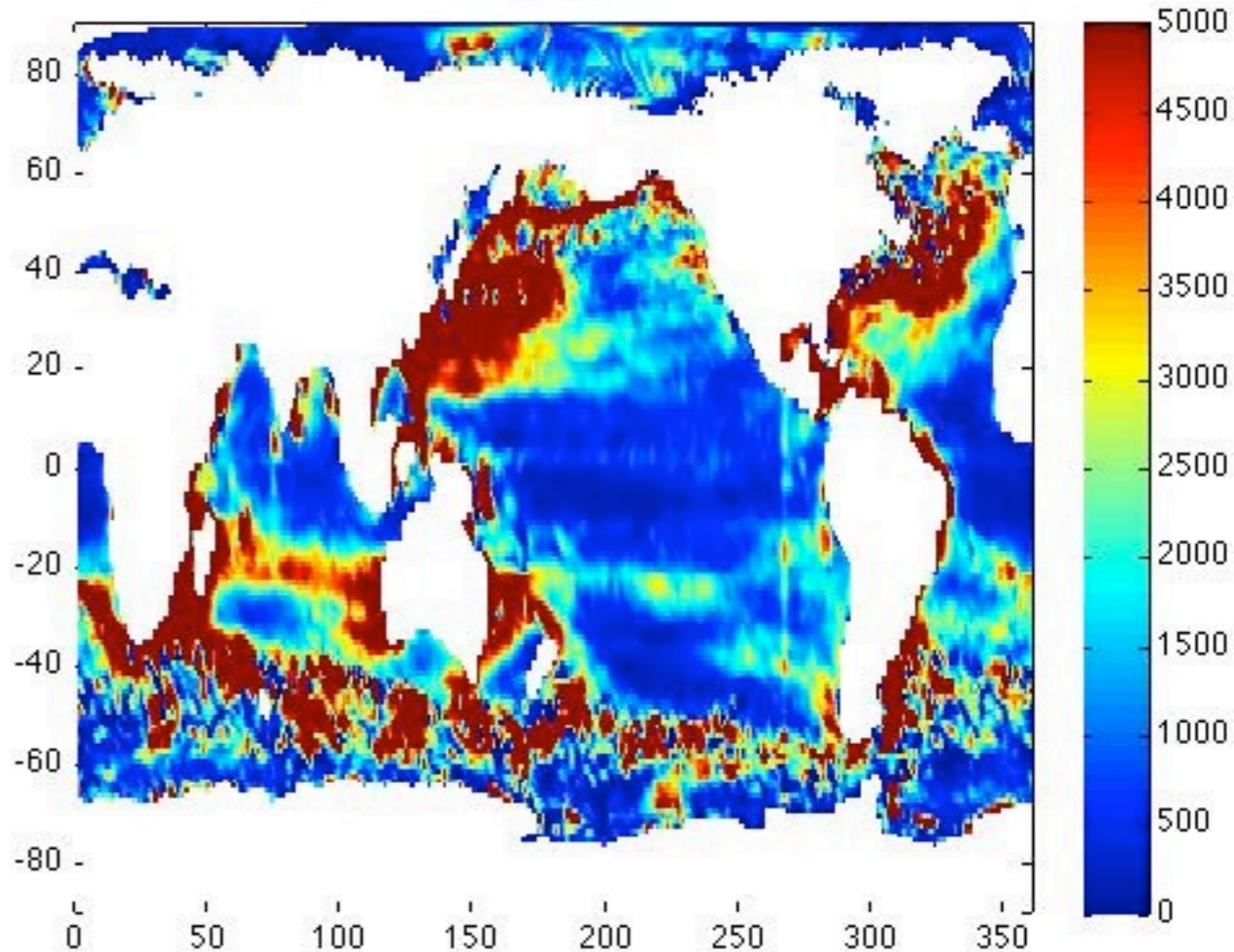
$\kappa_{11}$



Locations of  
PE extraction  
are

Locations of  
large eigs of  
**K**

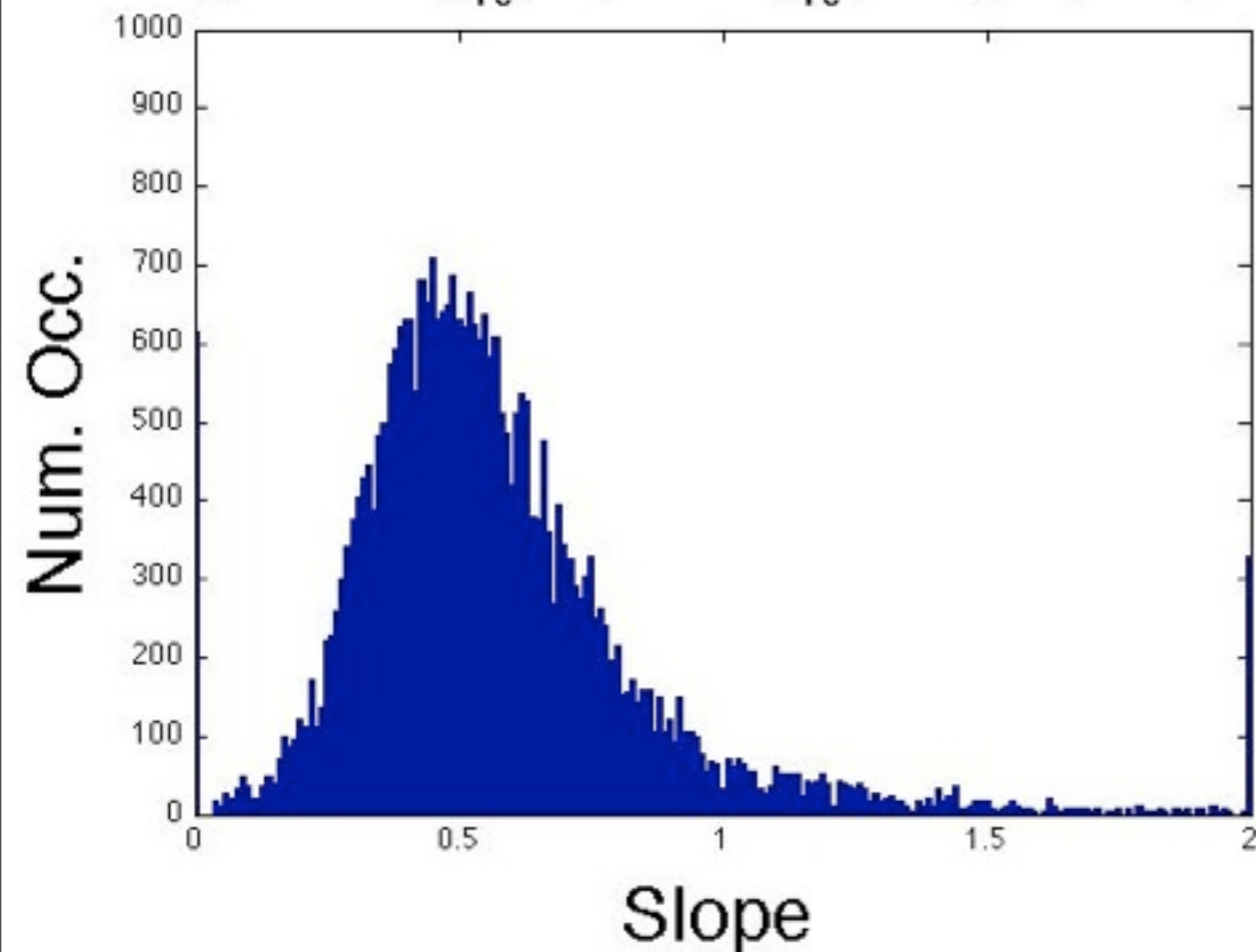
$\kappa_{22}$



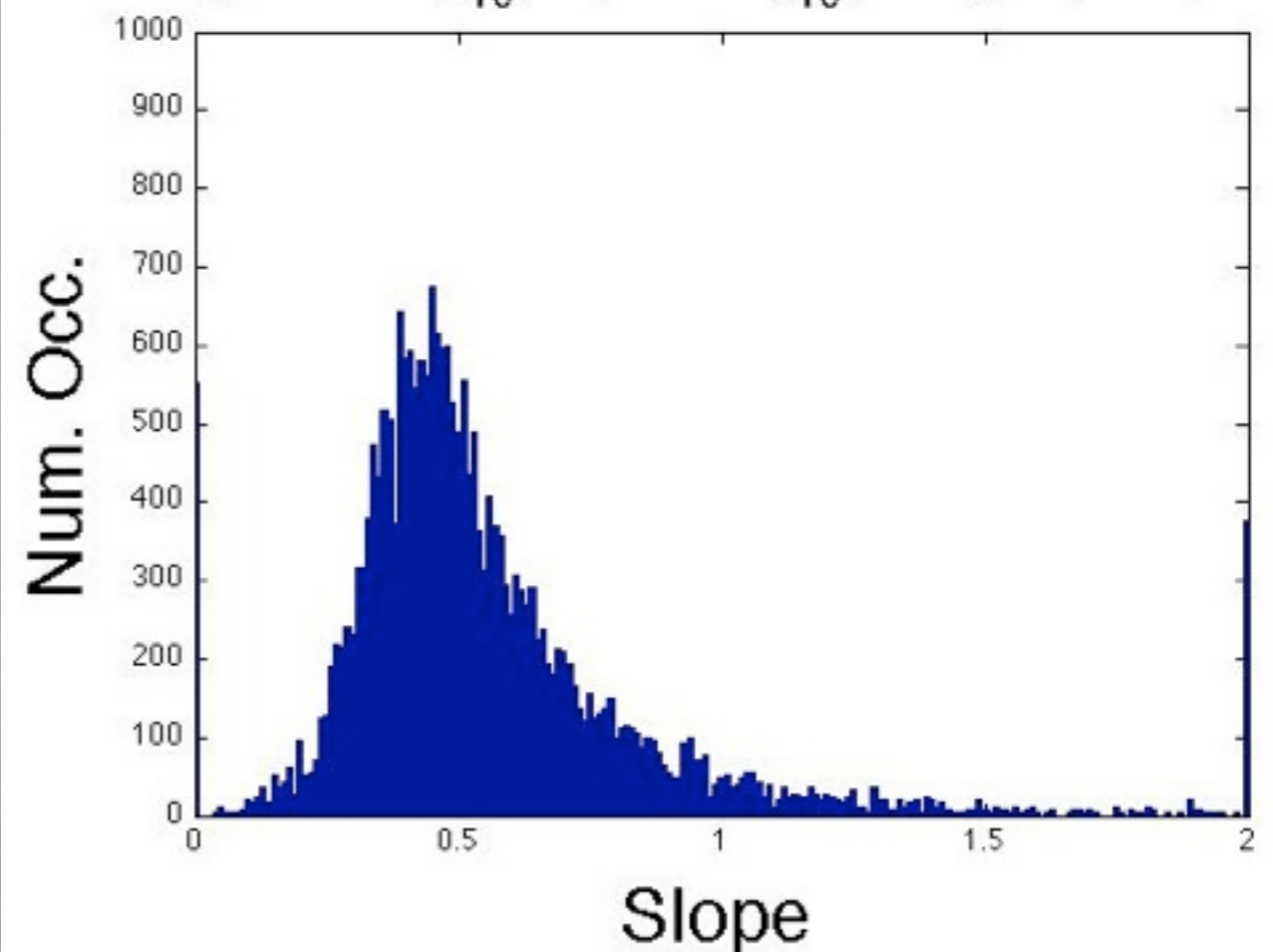
Result:  
coarse KE  $\rightarrow$  vertical structure of Mixing

$$K \propto \sqrt{\langle KE \rangle}$$

Histogram of  $\log_{10}(KE)$  vs.  $\log_{10}(M \text{ eig. \#1})$  Slope

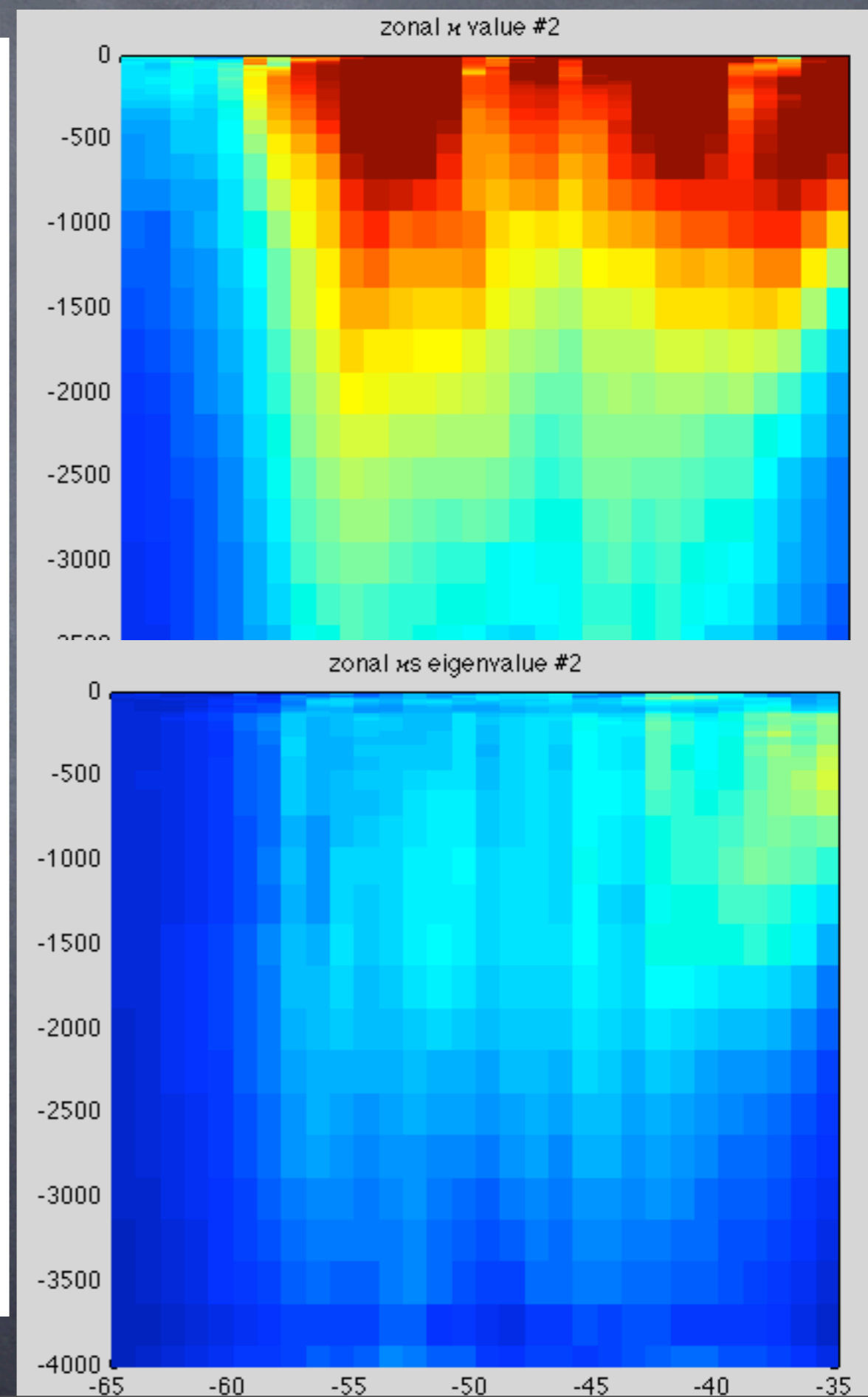
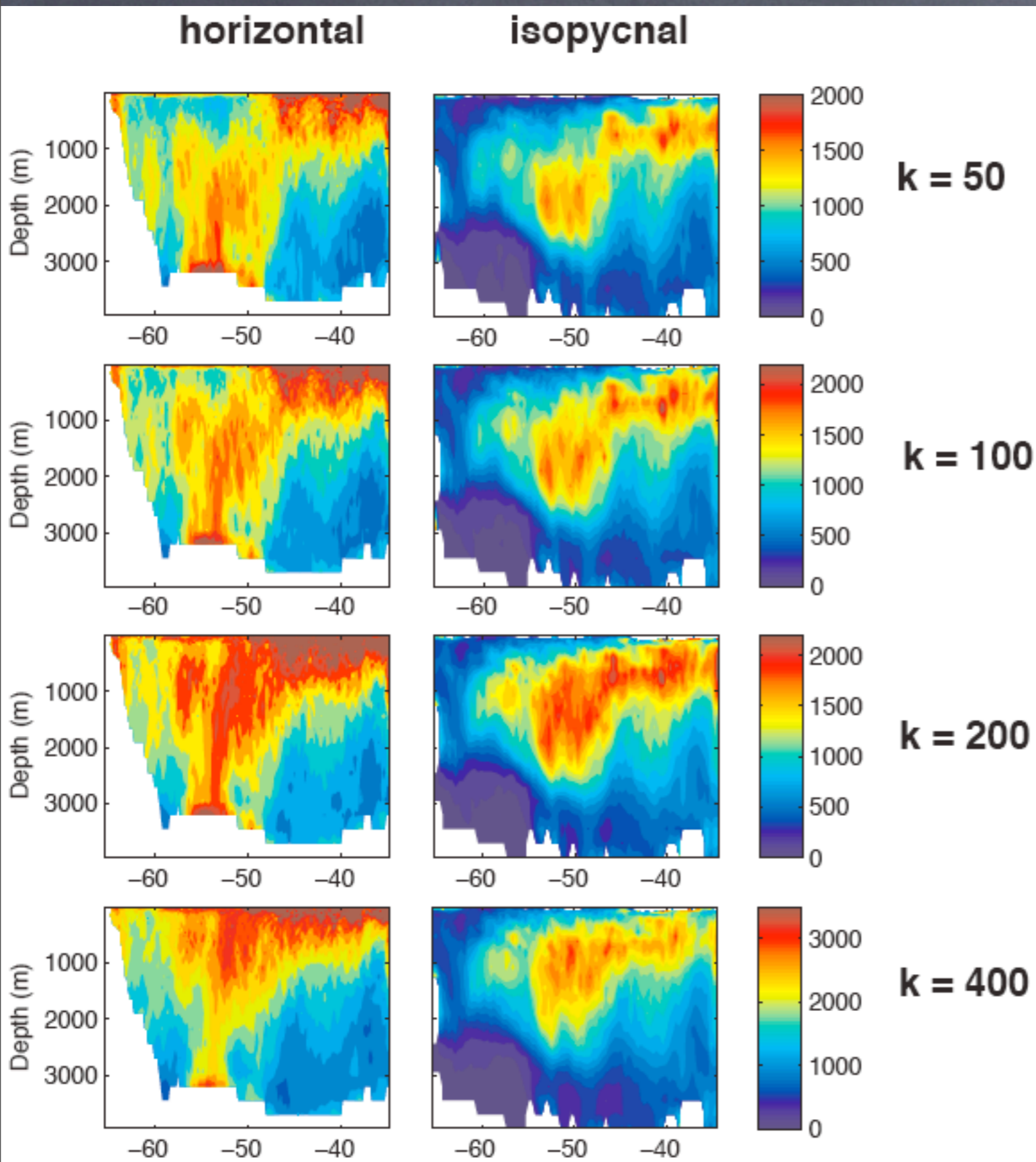


Histogram of  $\log_{10}(KE)$  vs.  $\log_{10}(M \text{ eig. \#2})$  Slope



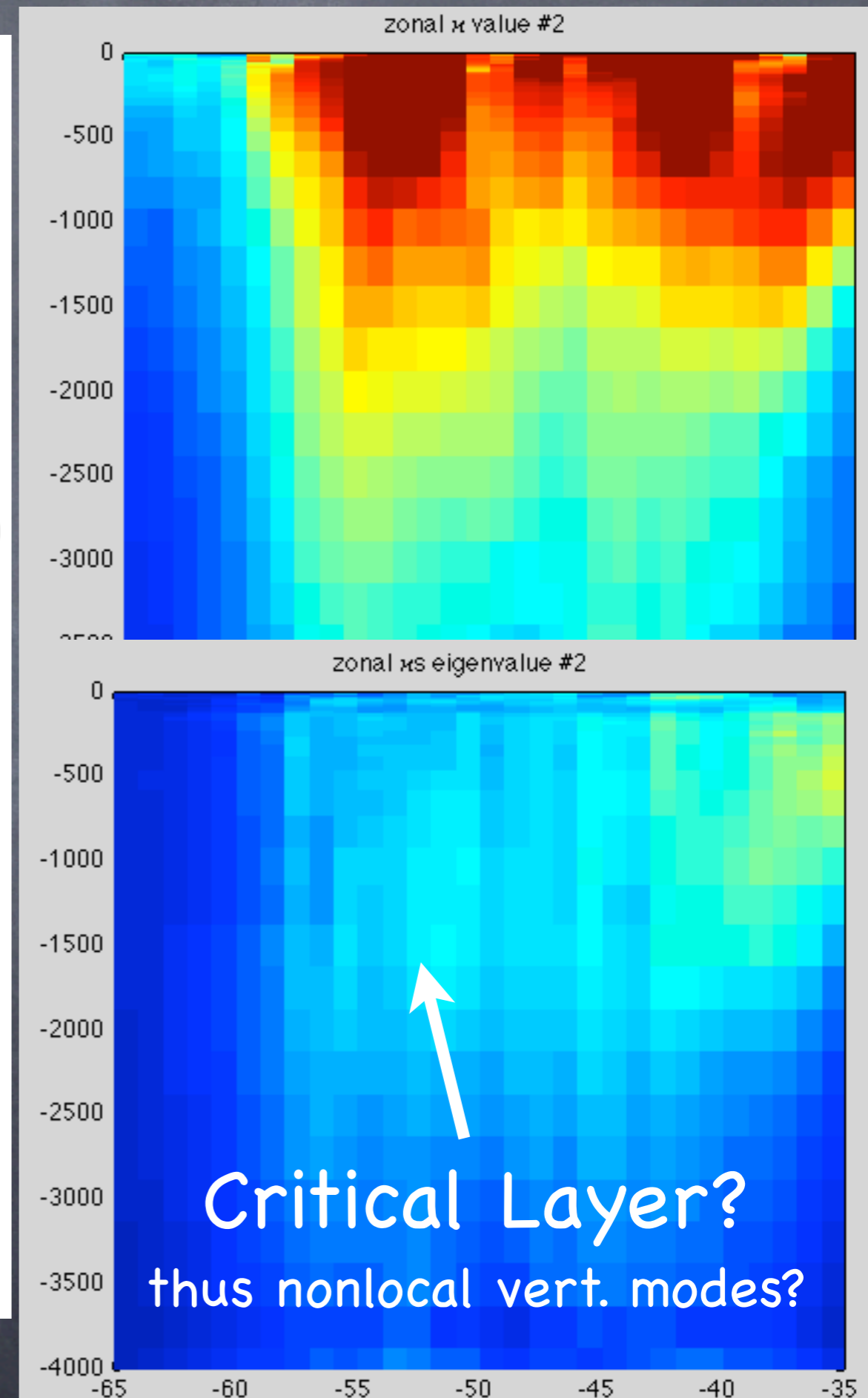
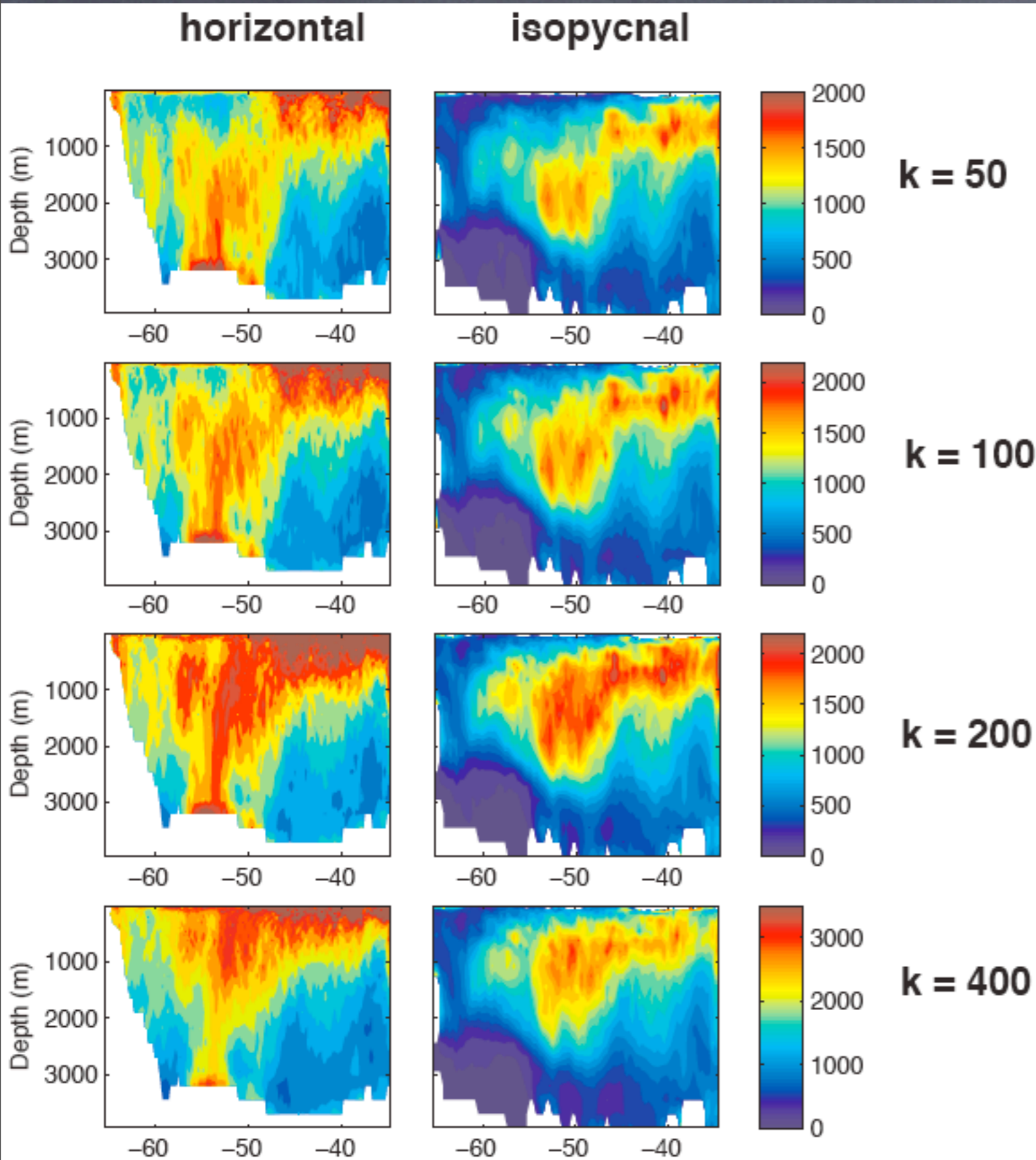
Even better with EKE!  
Note--barotropic mode is in there!

# Comparisons with Marshall et al.



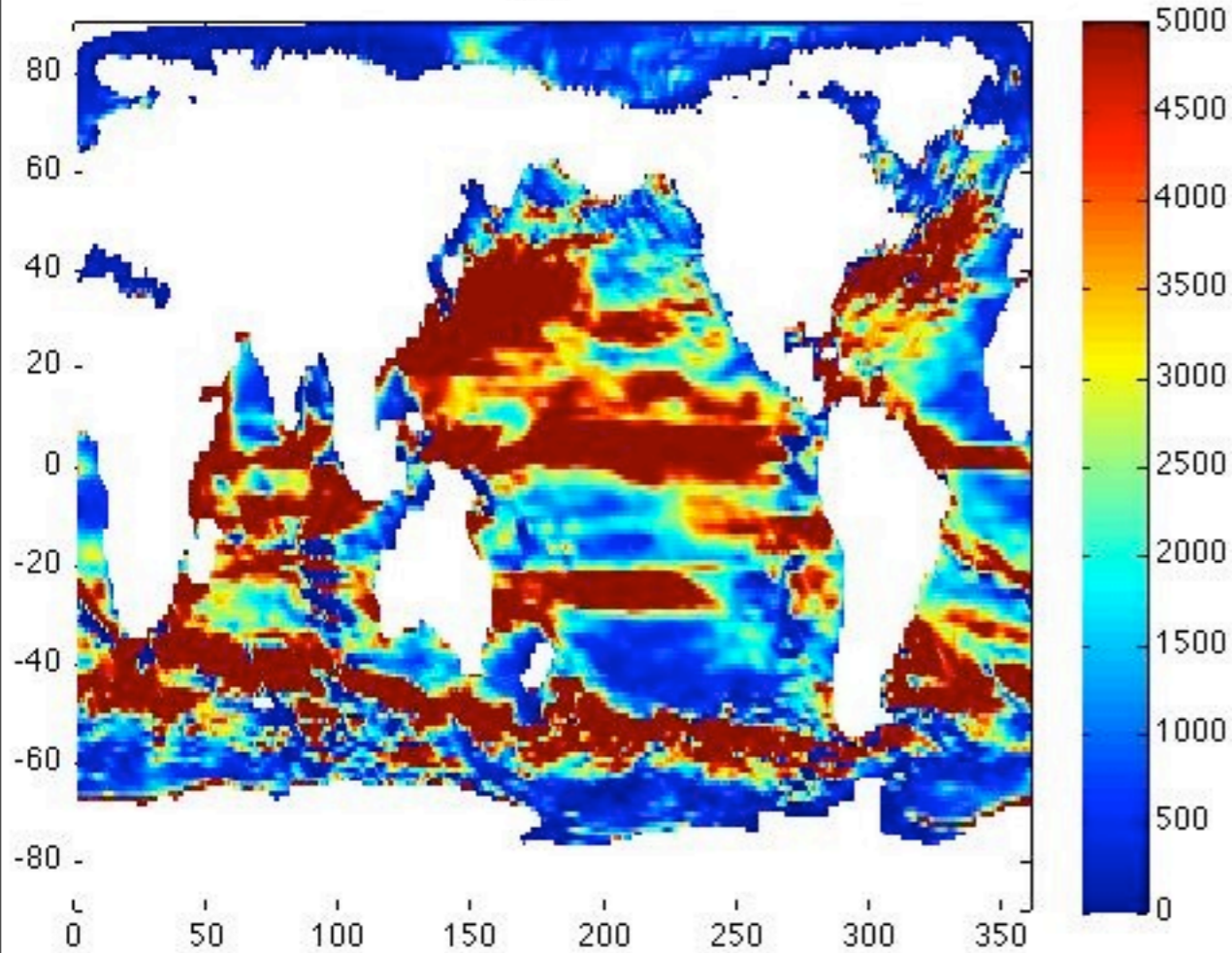
Abernathy et al 09

# Comparisons with Marshall et al.



Abernathy et al 09

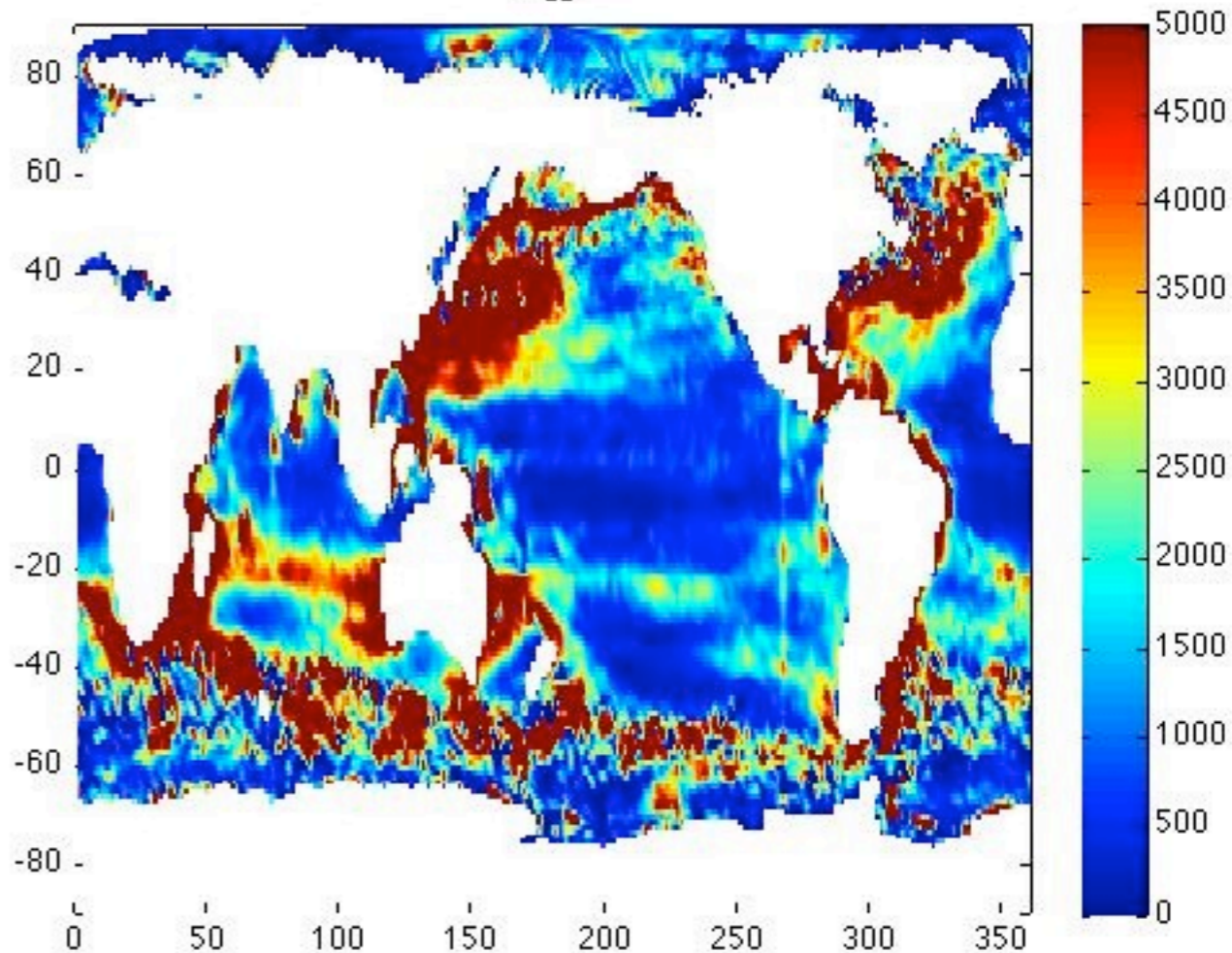
$\kappa_{11}$



Locations of  
PE extraction  
are

Locations of  
large eigs of  
**K**

$\kappa_{22}$



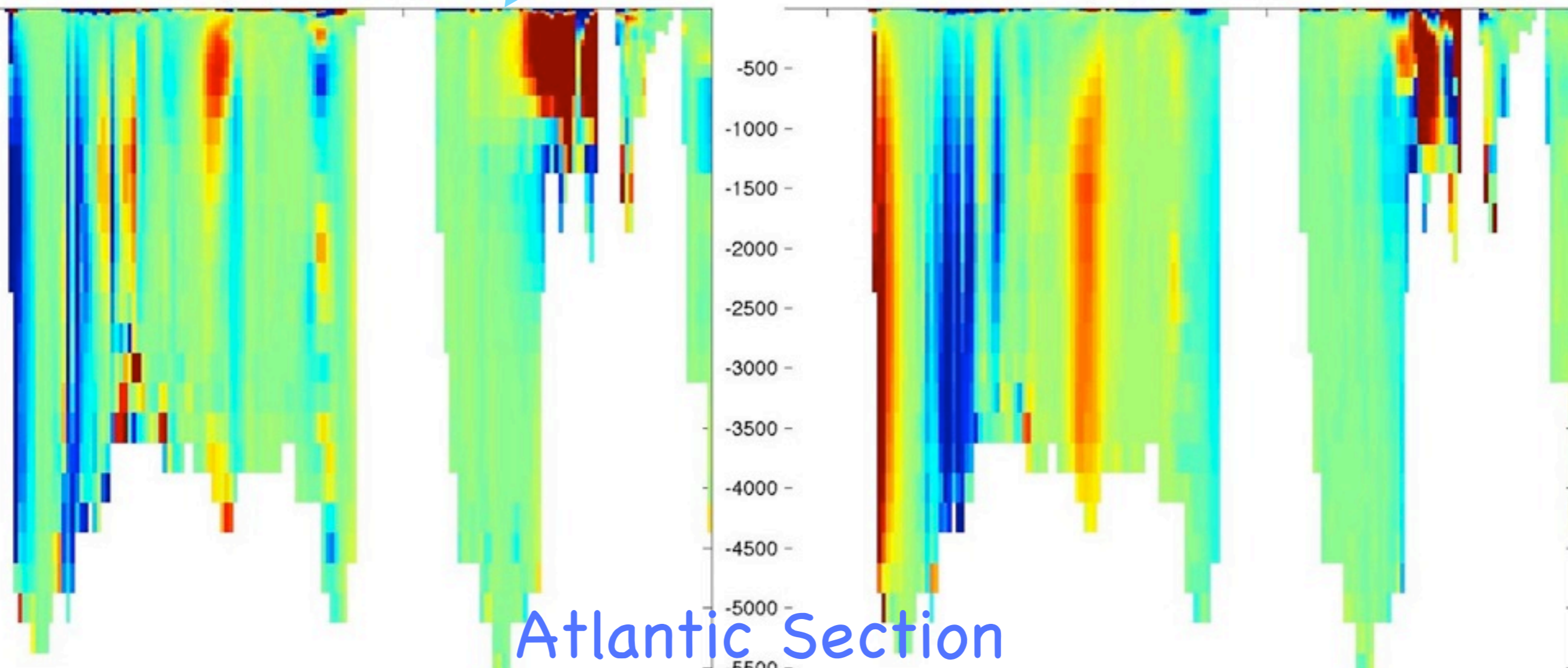


# Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla}_z \\ 0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla}_z \\ \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla}_z & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla}_z & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Asym 3,1: GM@lon=345E

Asym 3,2: GM@lon=345E



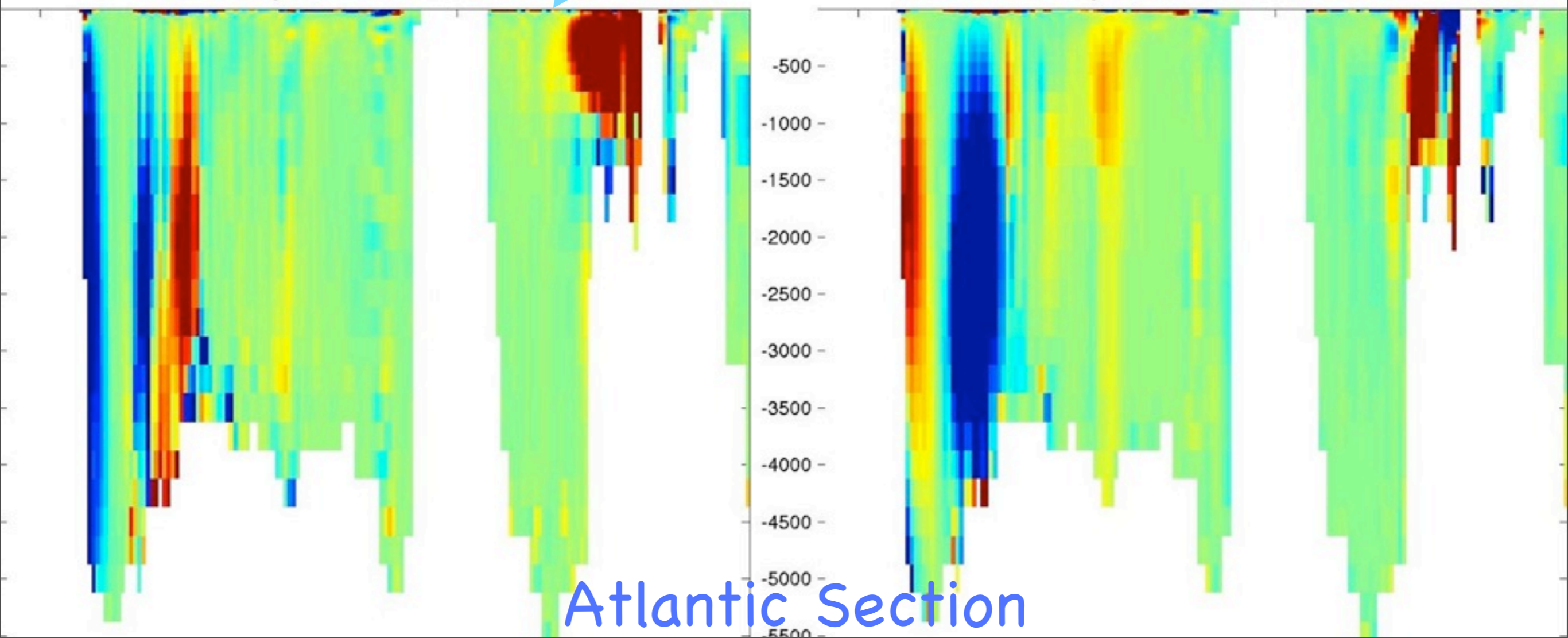
Atlantic Section

# Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & \tilde{\nabla} \mathbf{z} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Sym 3,1: Redi@lon=345E

Sym 3,2: Redi@lon=345E



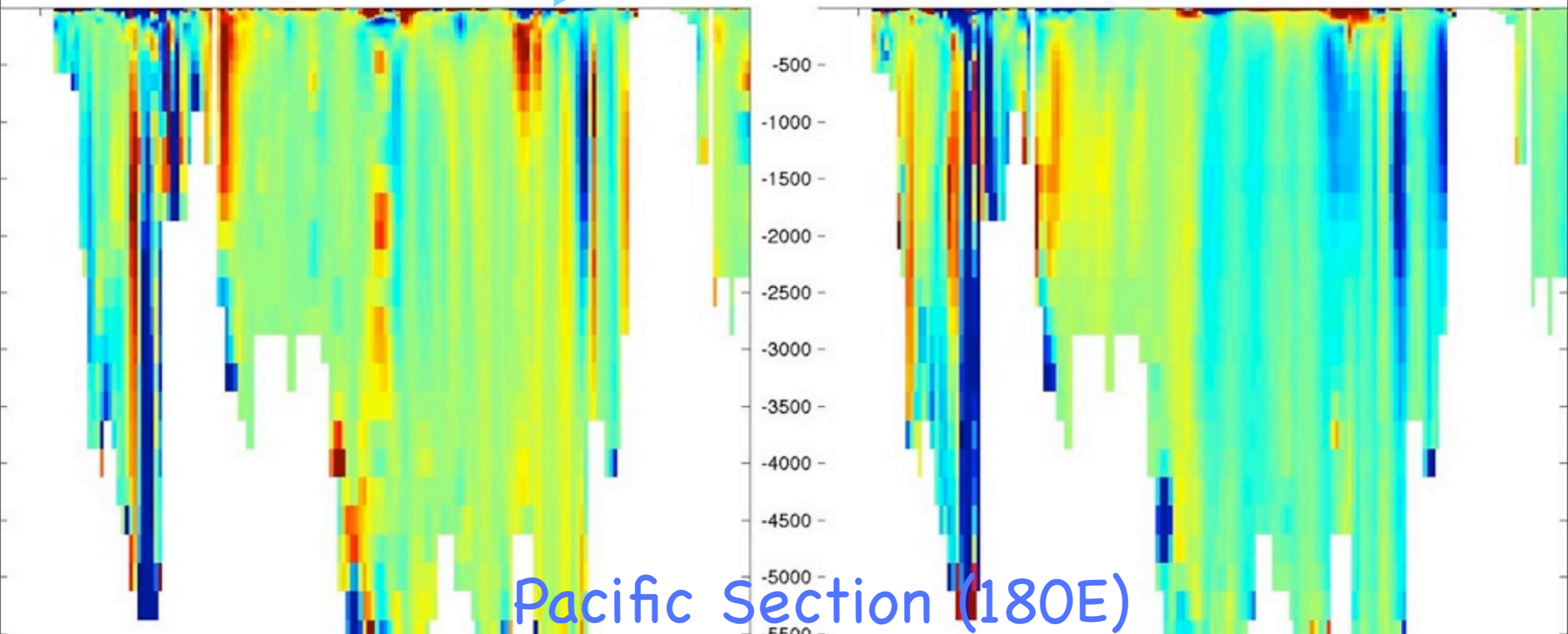
Atlantic Section

# Use a Natural, Mesoscale Eddy Environment to Test Out:

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla}_z \\ 0 & 0 & -\hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla}_z \\ \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla}_z & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla}_z & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Asym 3,1: GM@lon=180E

Asym 3,2: GM@lon=180E



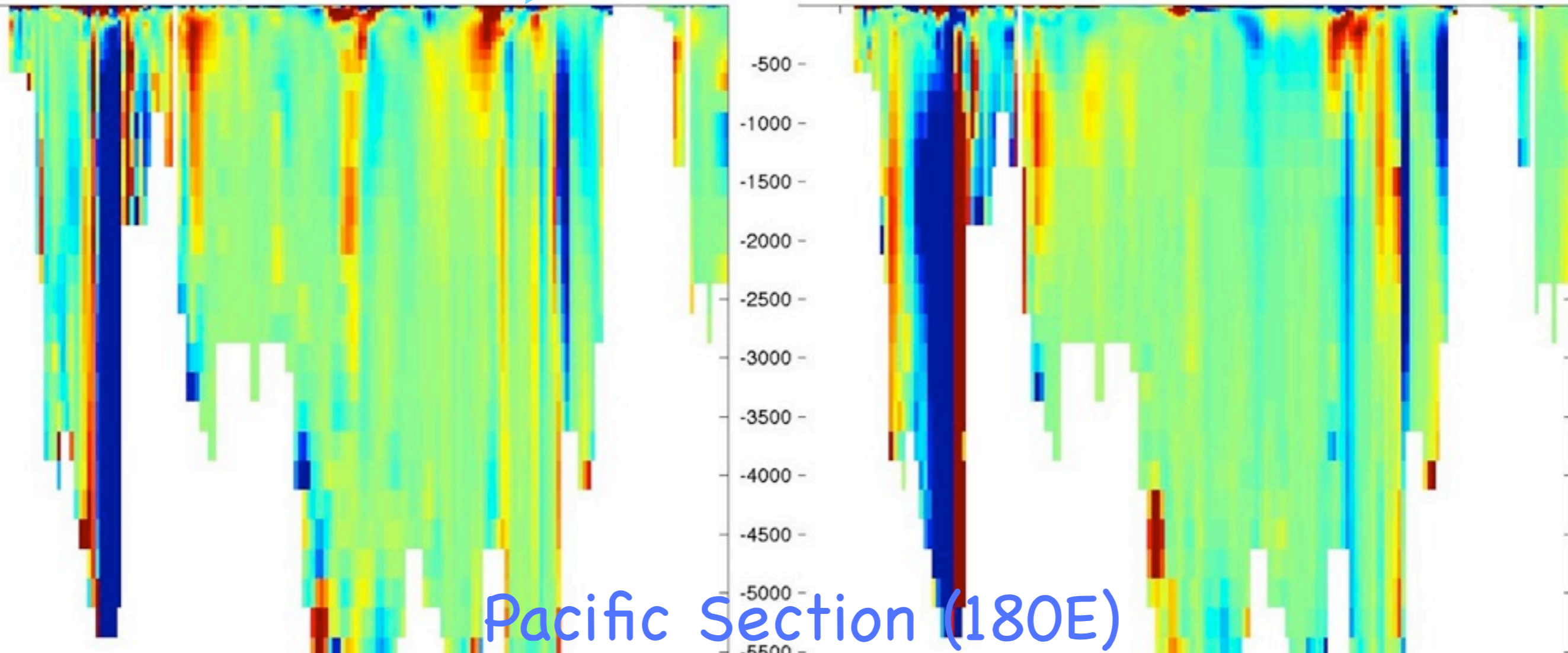
Pacific Section (180E)

# Use a Natural, Mesoscale Eddy Environment to Test Out:

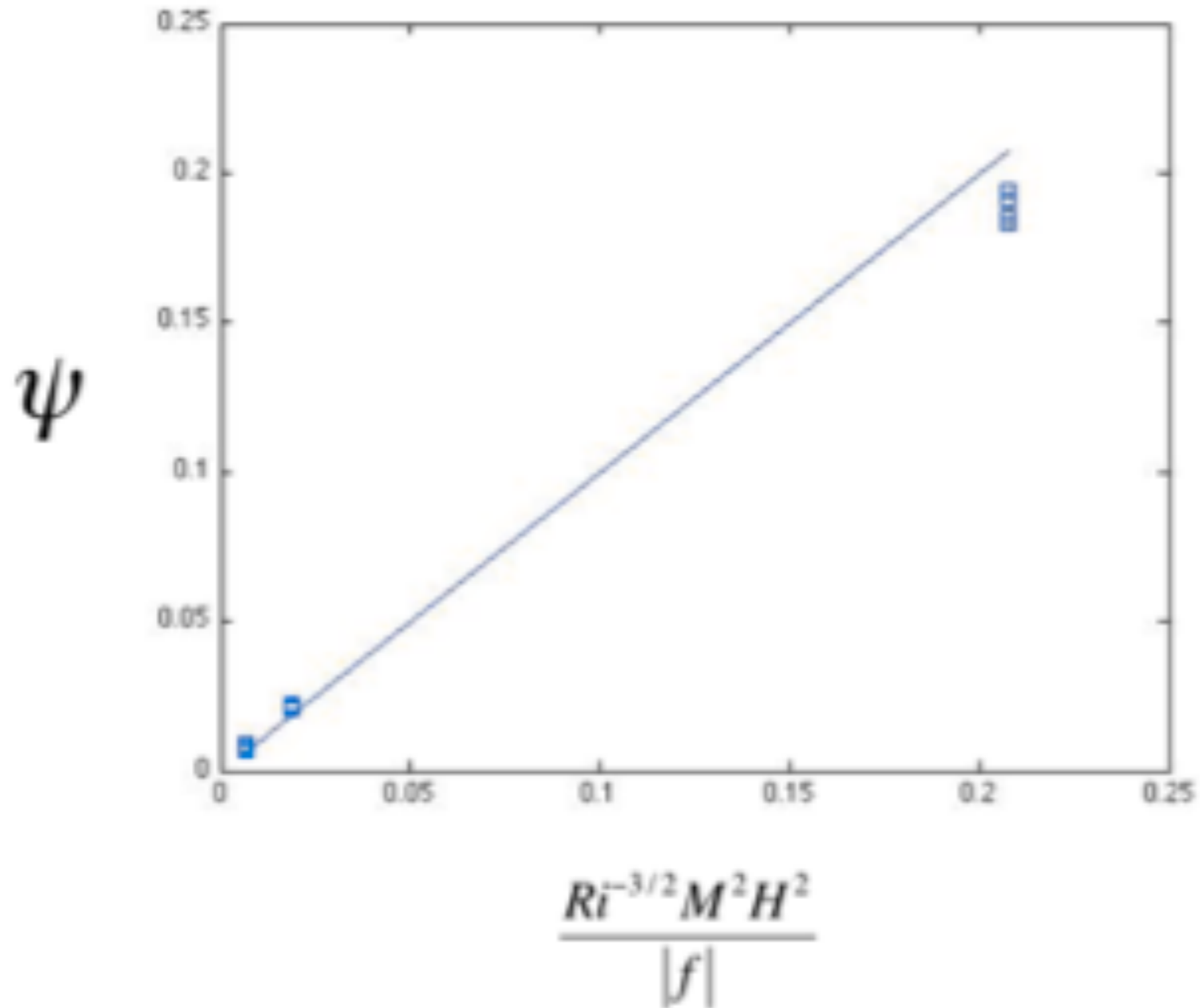
$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ K_{yx} & K_{yy} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \\ \hat{x} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & \hat{y} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} & \tilde{\nabla} \mathbf{z} \cdot \mathbf{K} \cdot \tilde{\nabla} \mathbf{z} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Sym 3,1: Redi@lon=180E

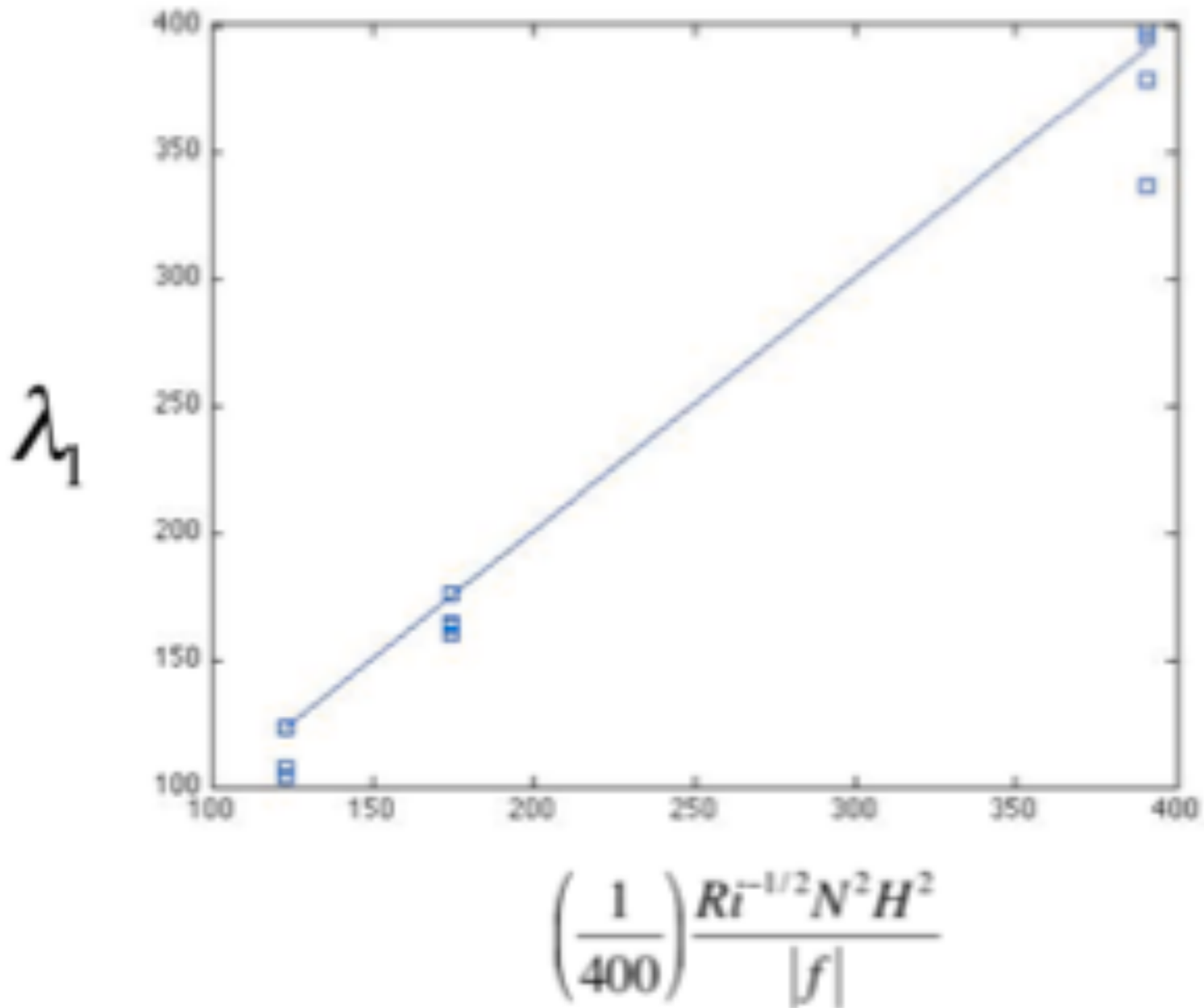
Sym 3,2: Redi@lon=180E



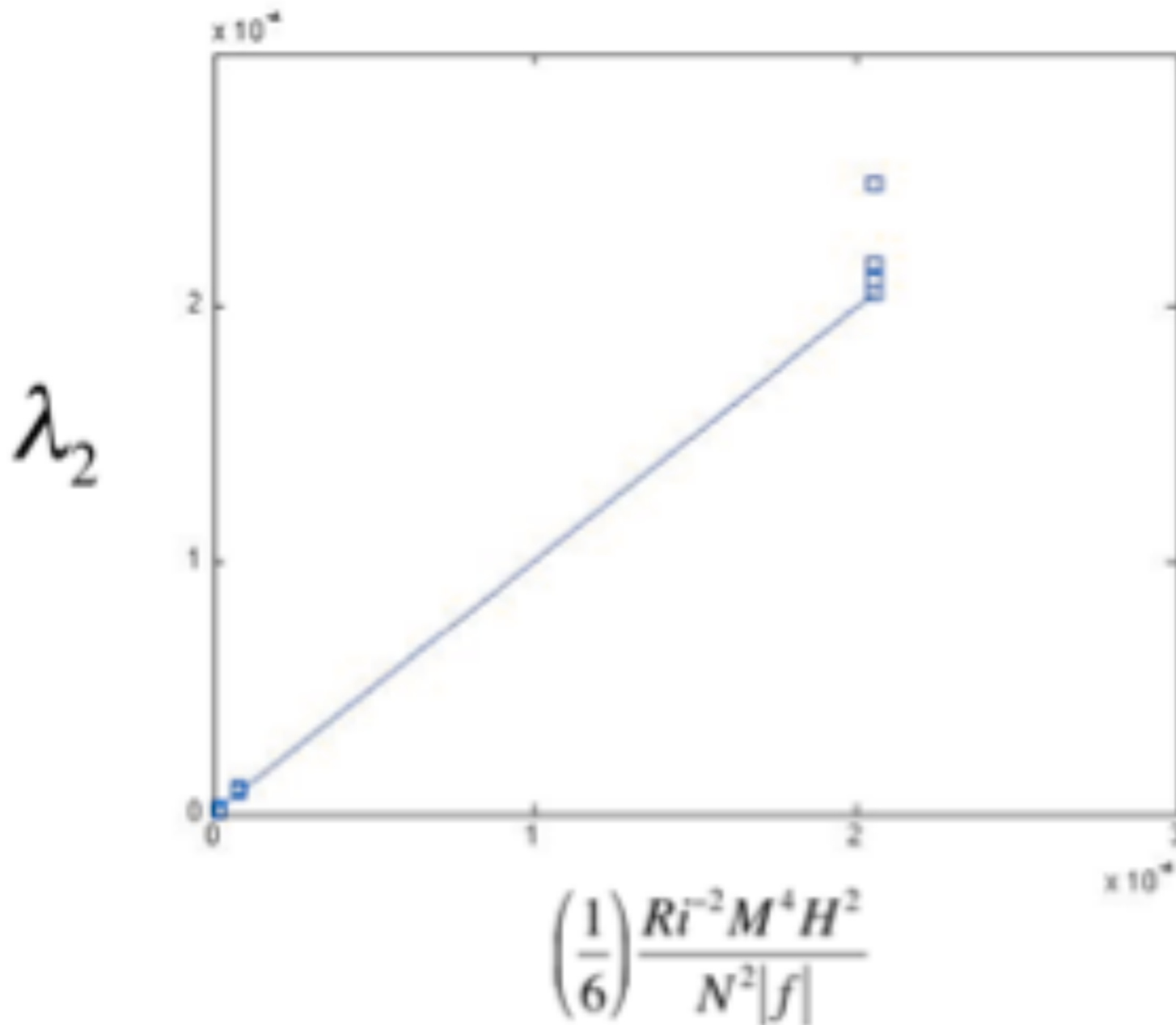
# Scaling: Antisymmetric part



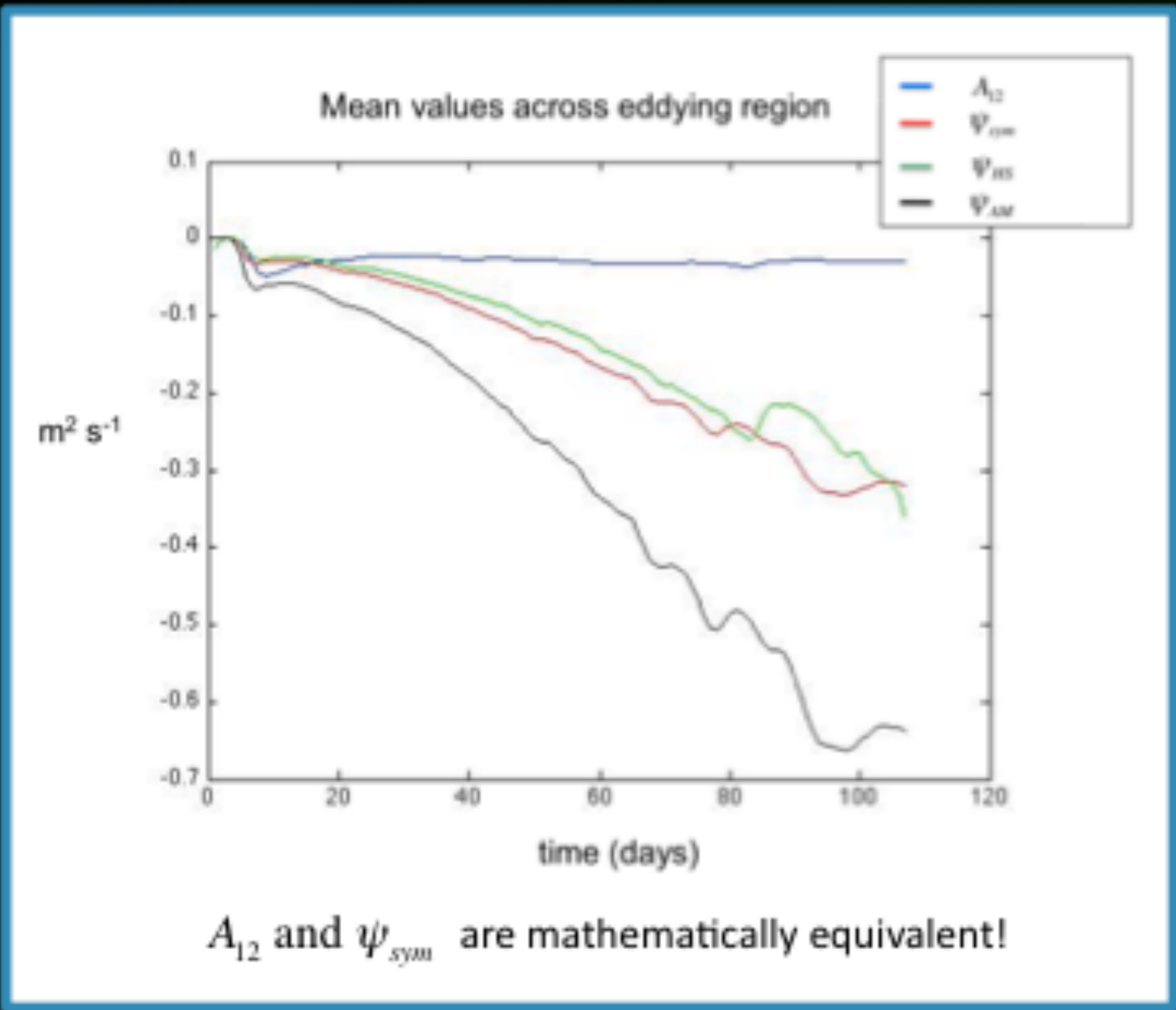
# Scaling: Larger symmetric eigenvalue



# Scaling: Smaller symmetric eigenvalue

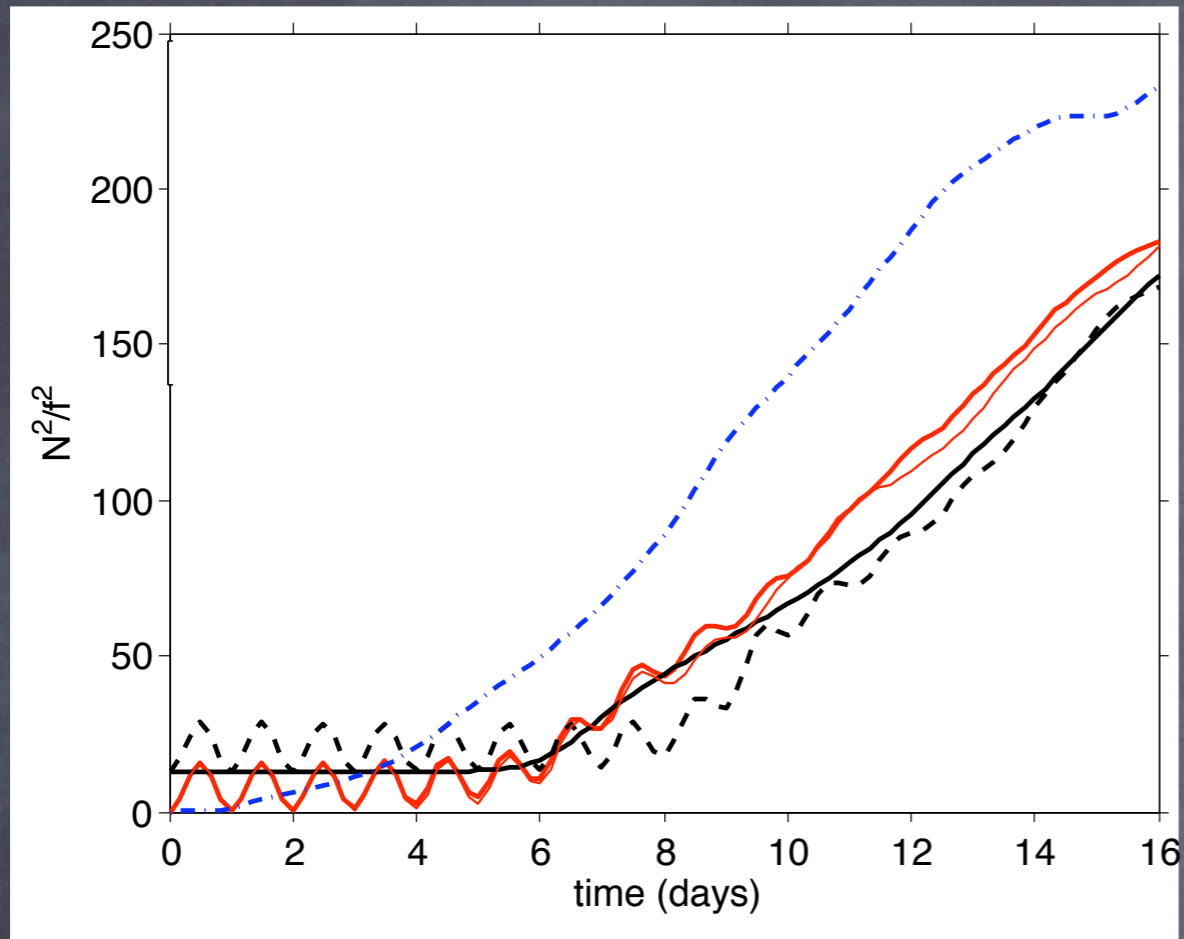


# TEM..? Overdetermined vs. Underdetermined:

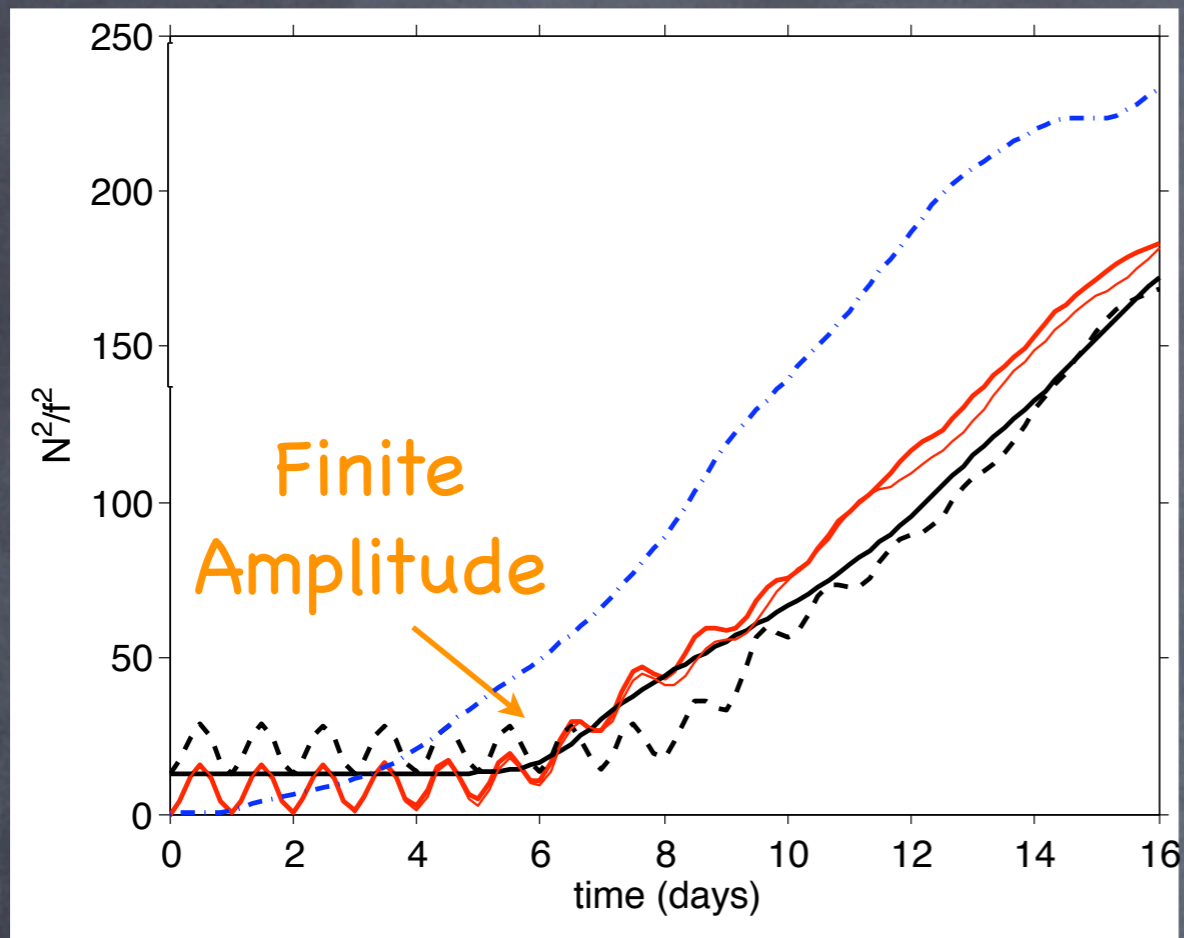




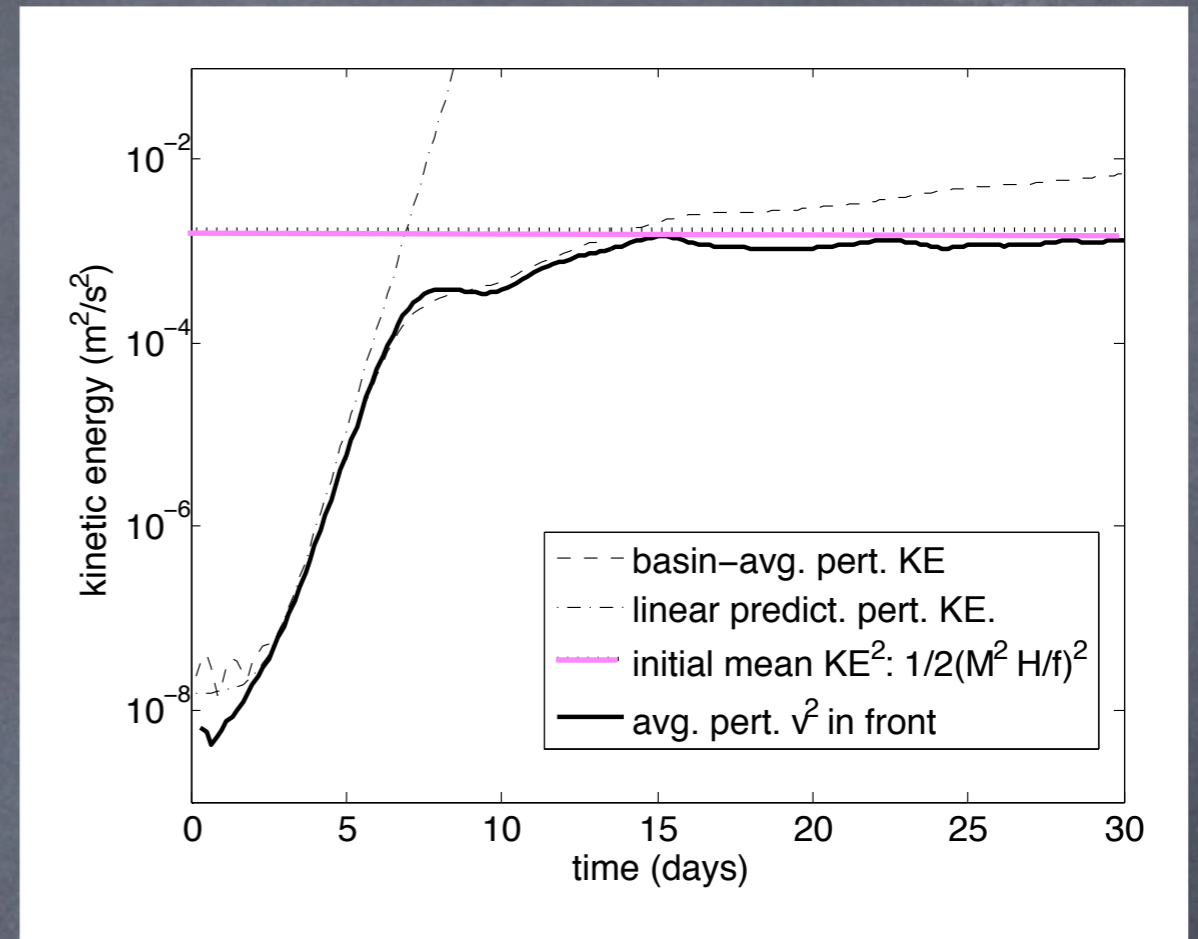
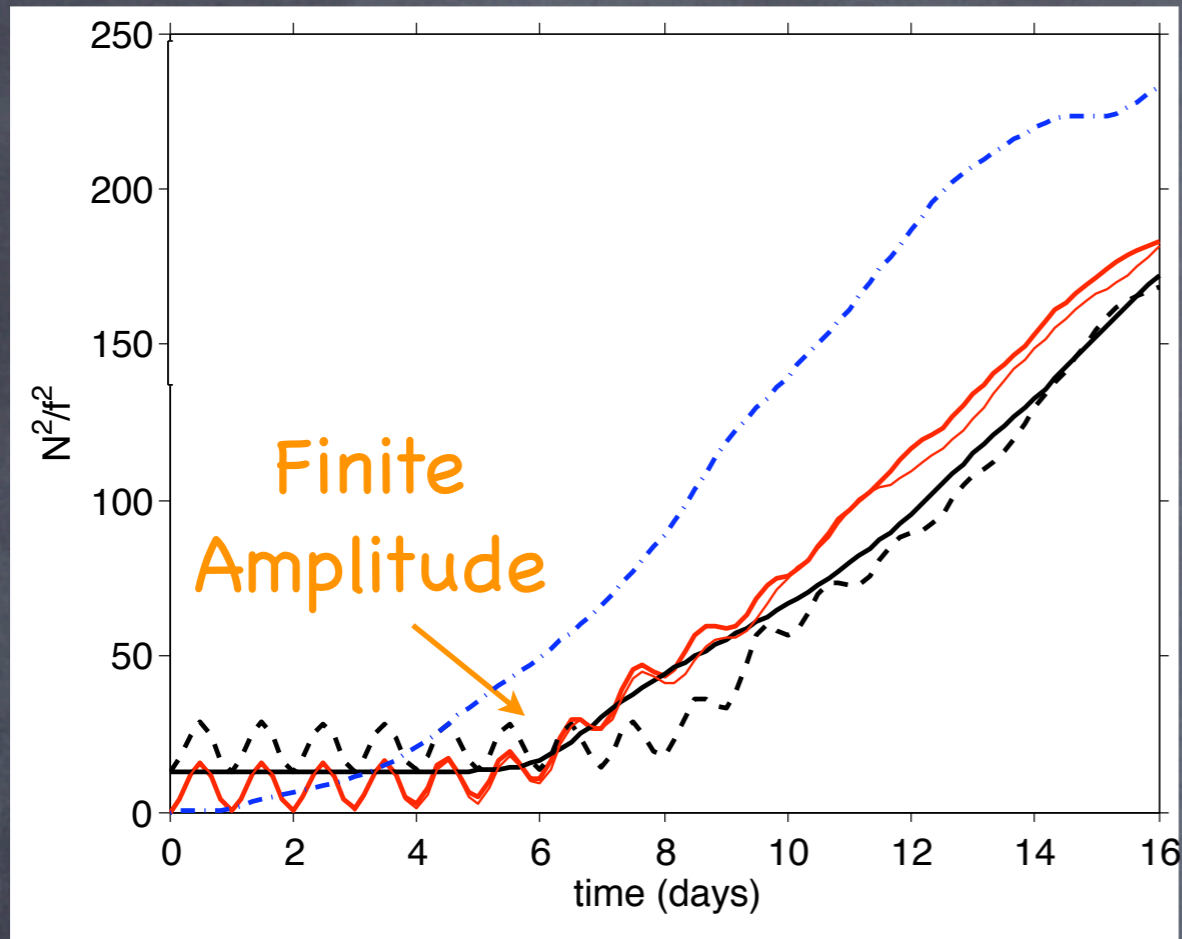
# Parameterization of Finite Amp. Eddies: Ingredients



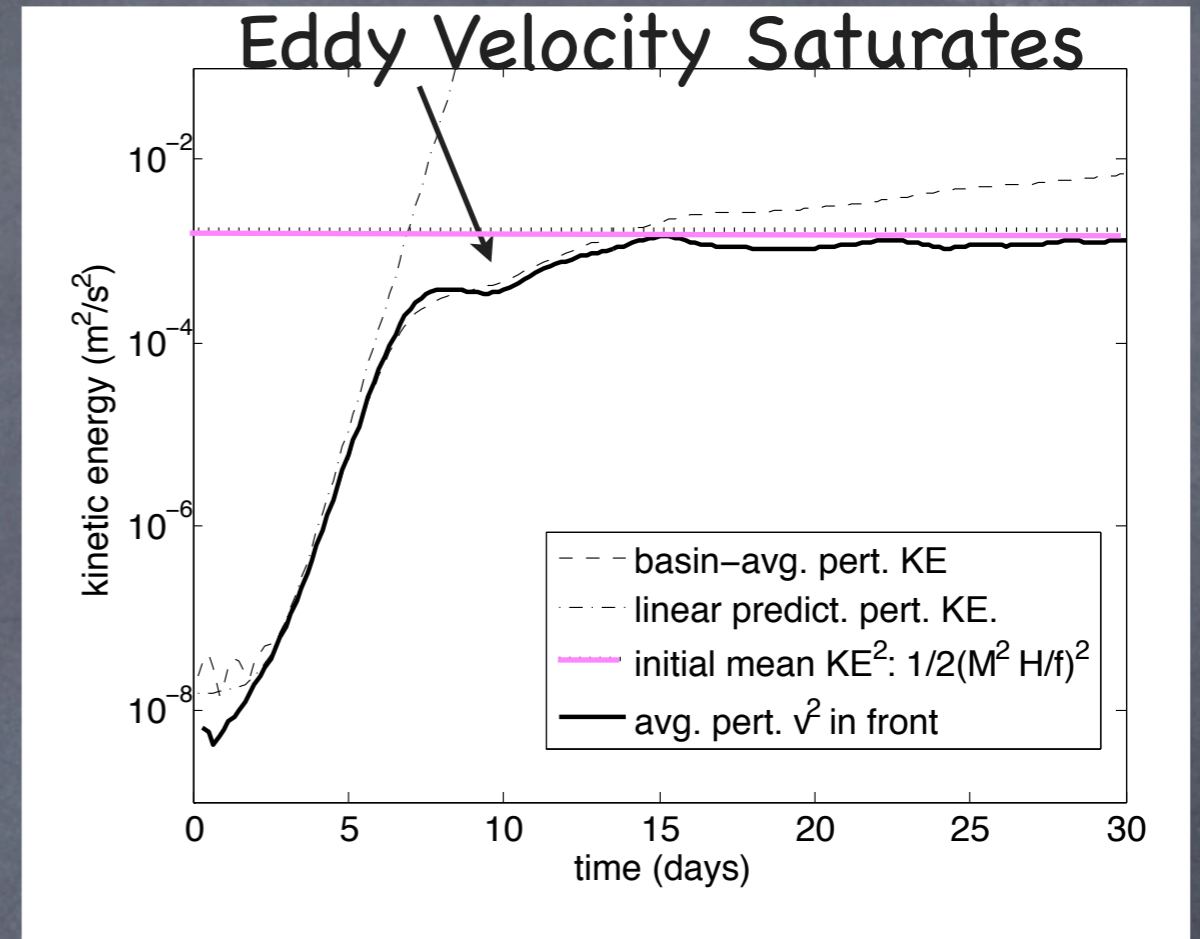
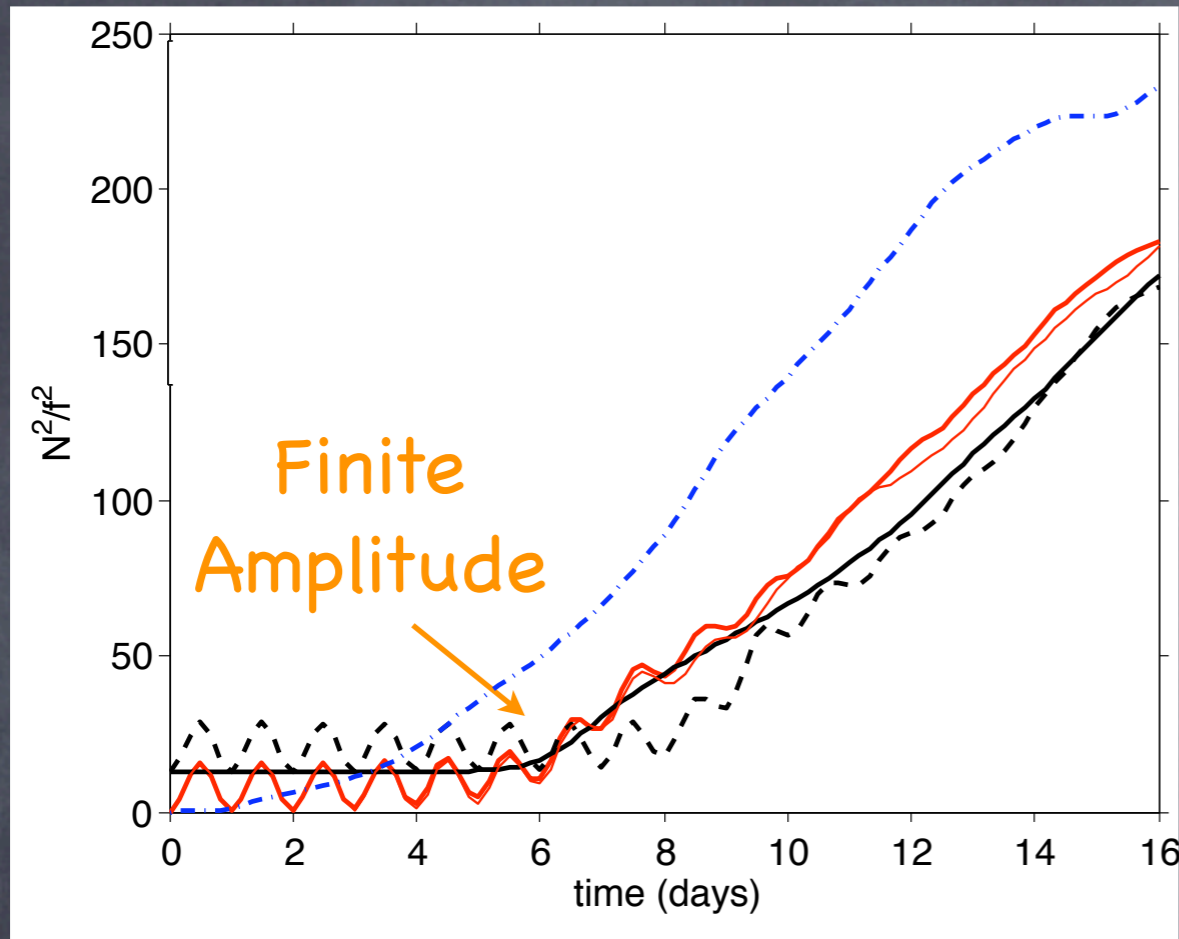
# Parameterization of Finite Amp. Eddies: Ingredients



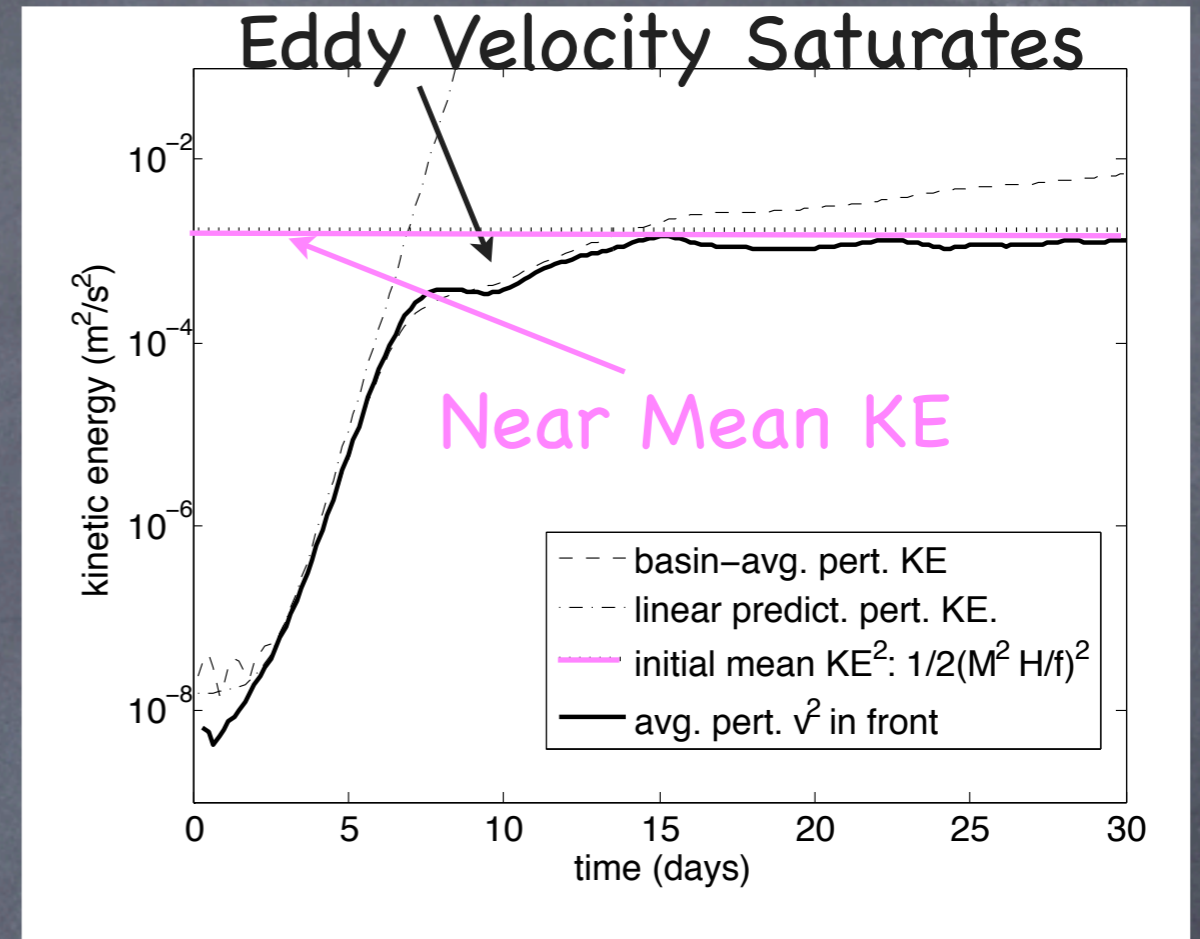
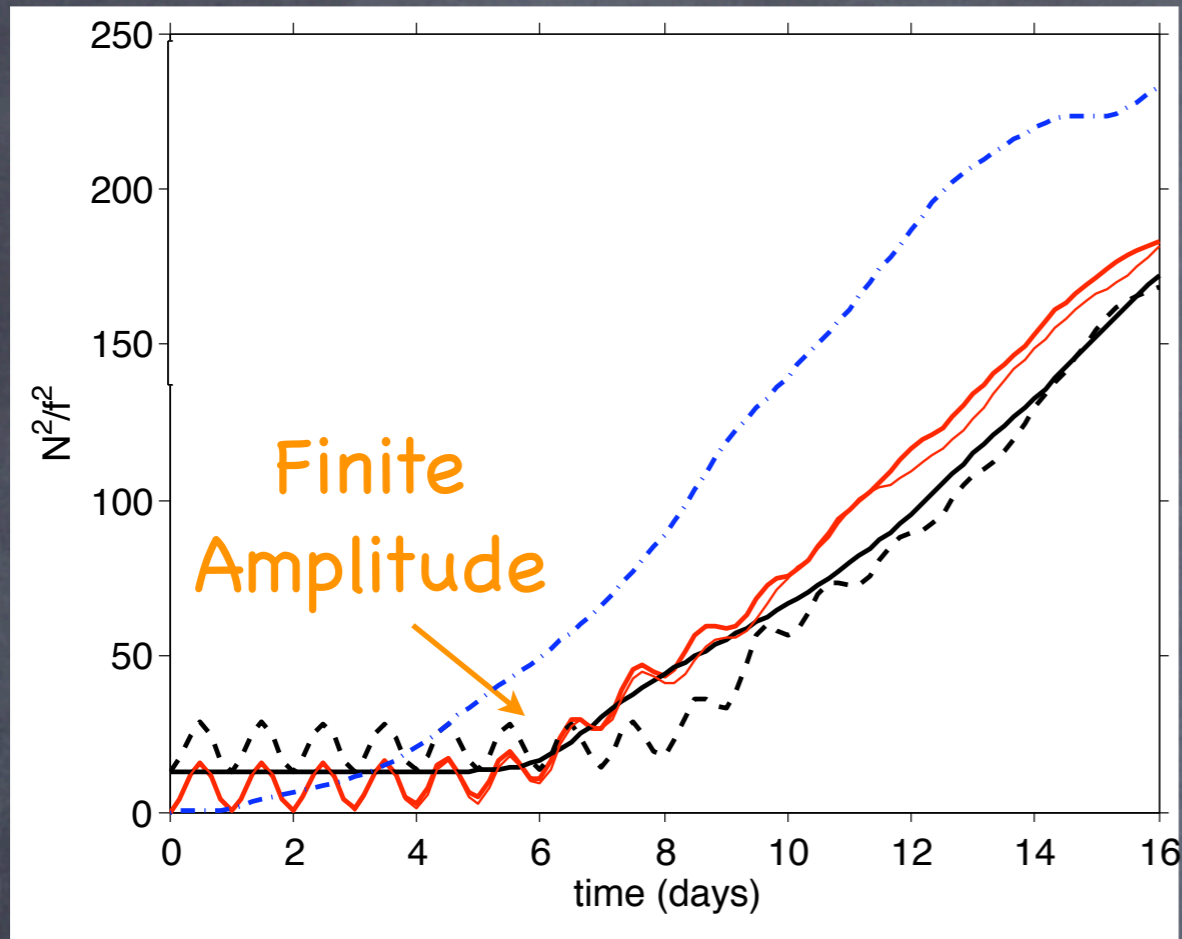
# Parameterization of Finite Amp. Eddies: Ingredients



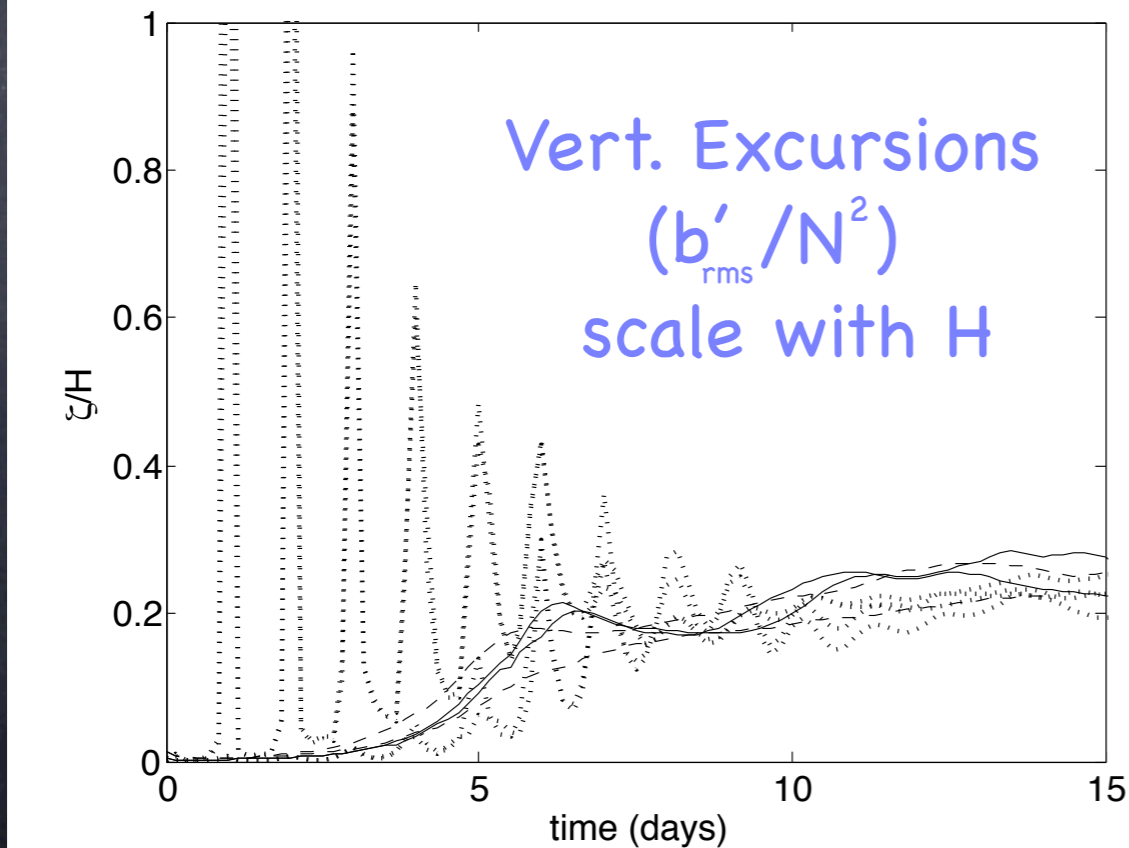
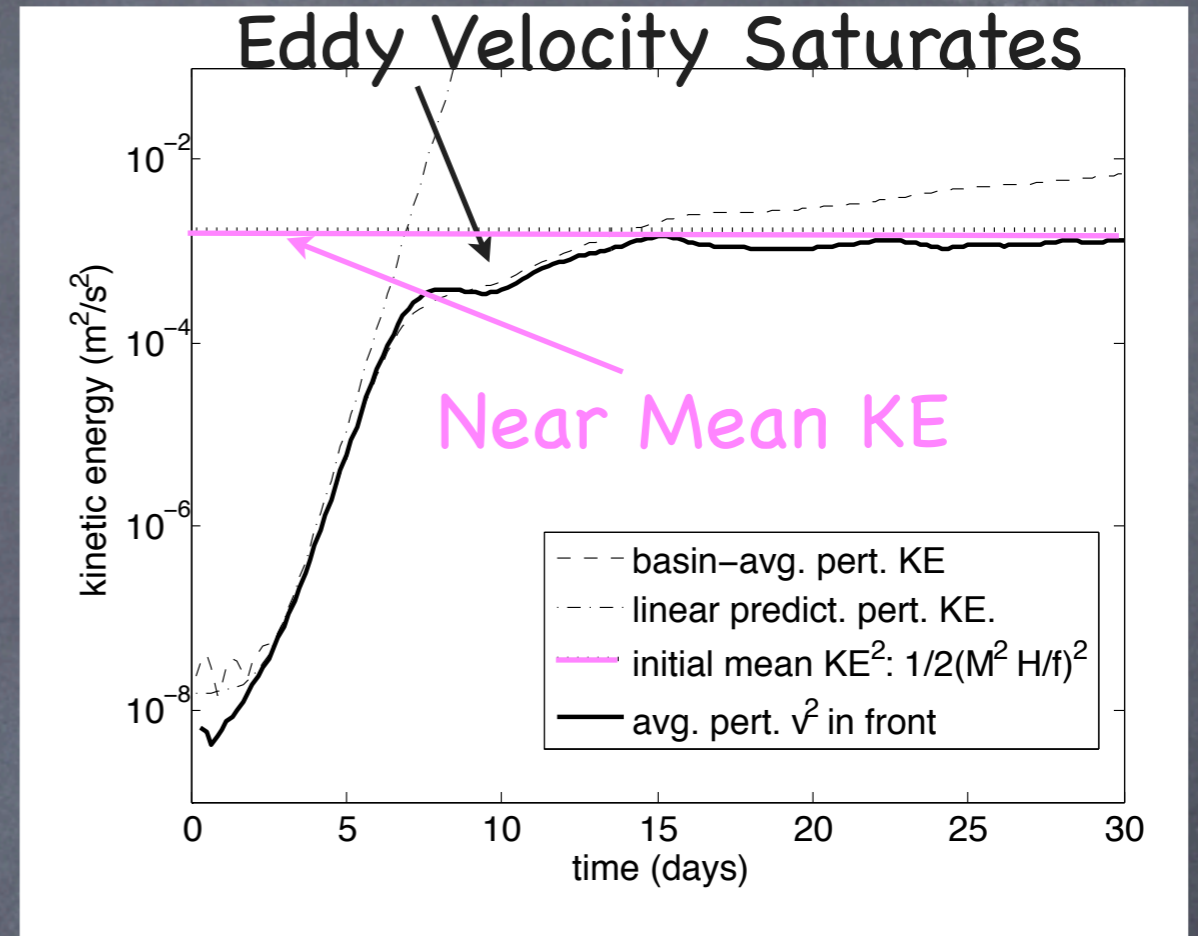
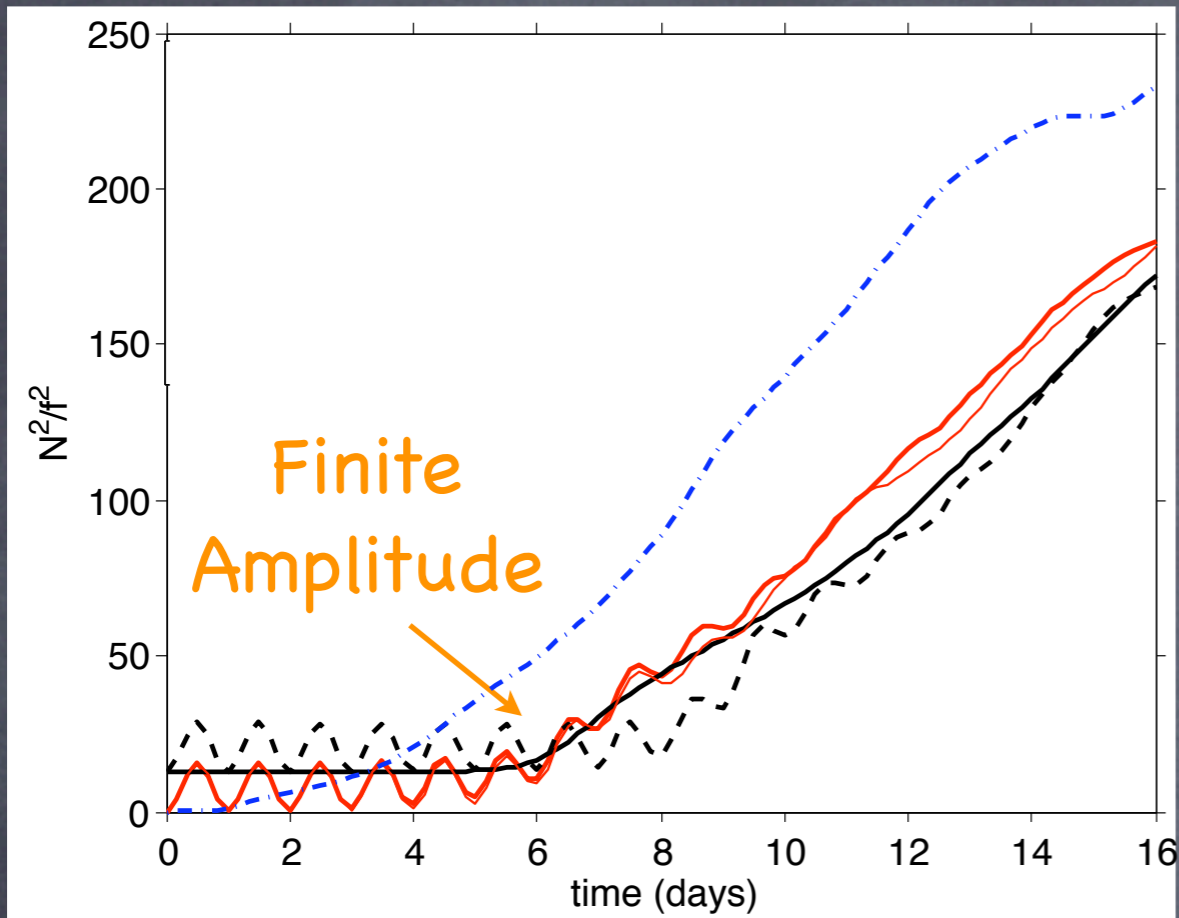
# Parameterization of Finite Amp. Eddies: Ingredients



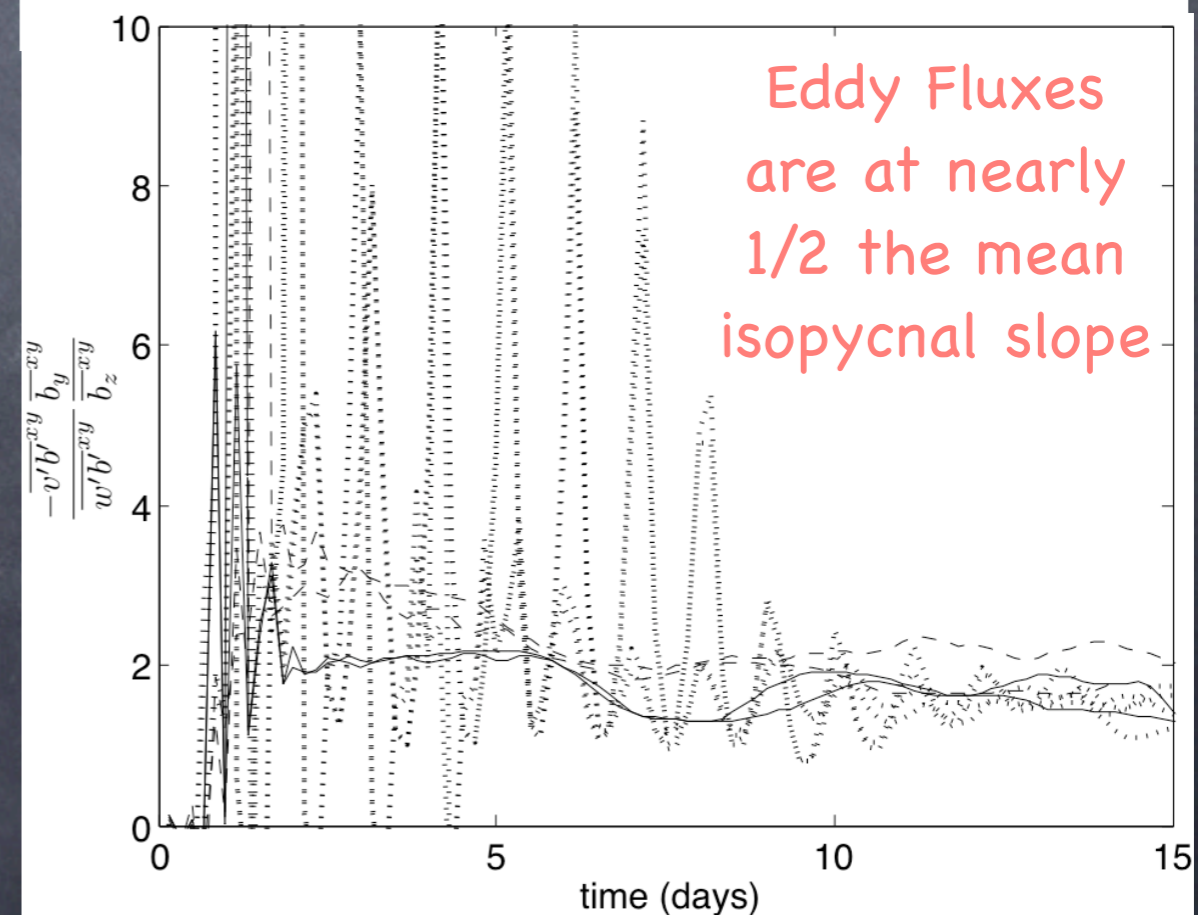
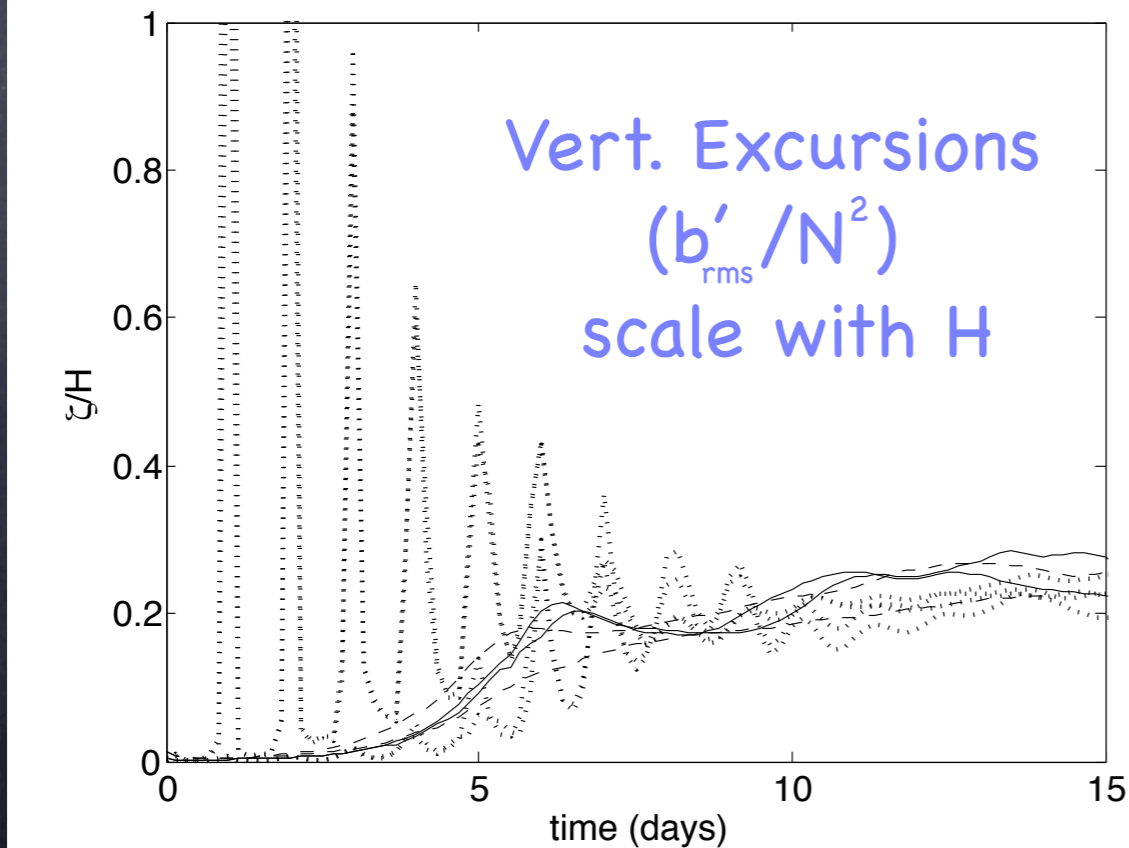
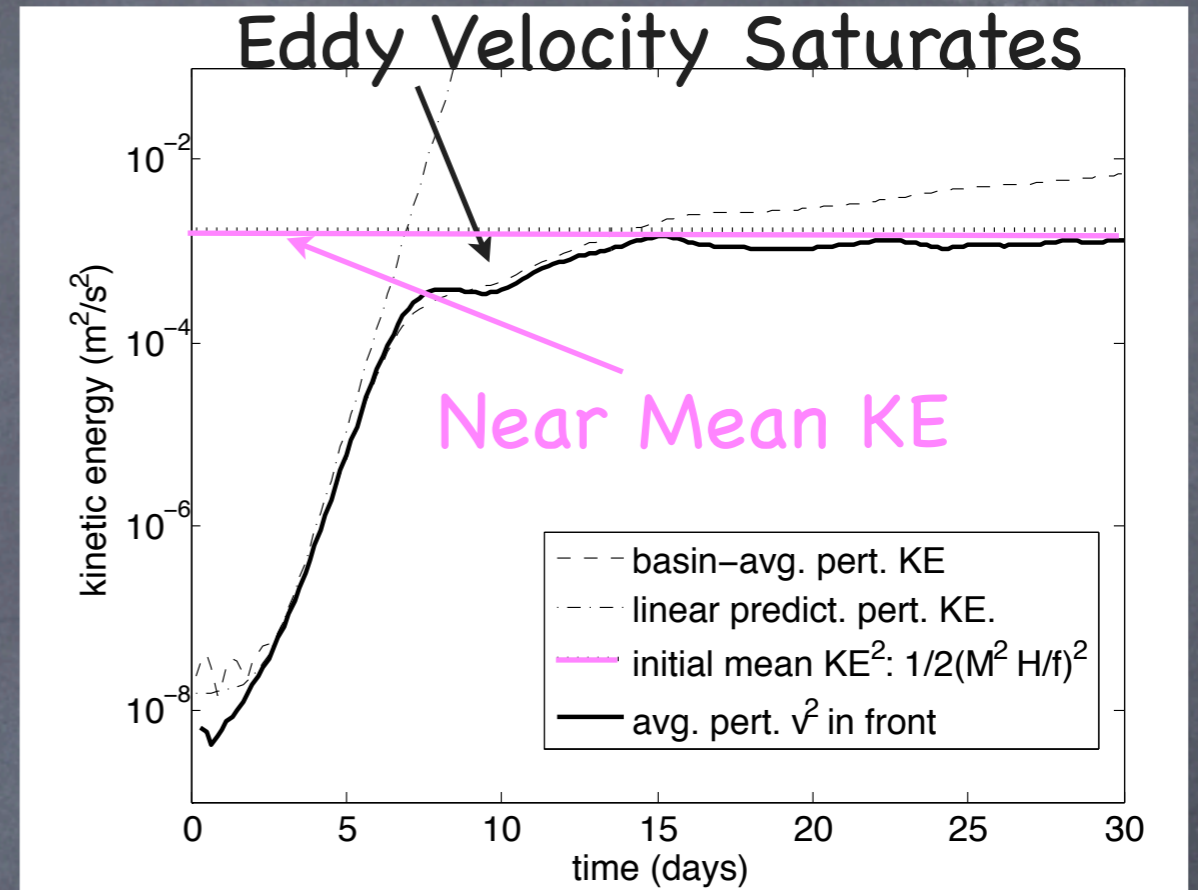
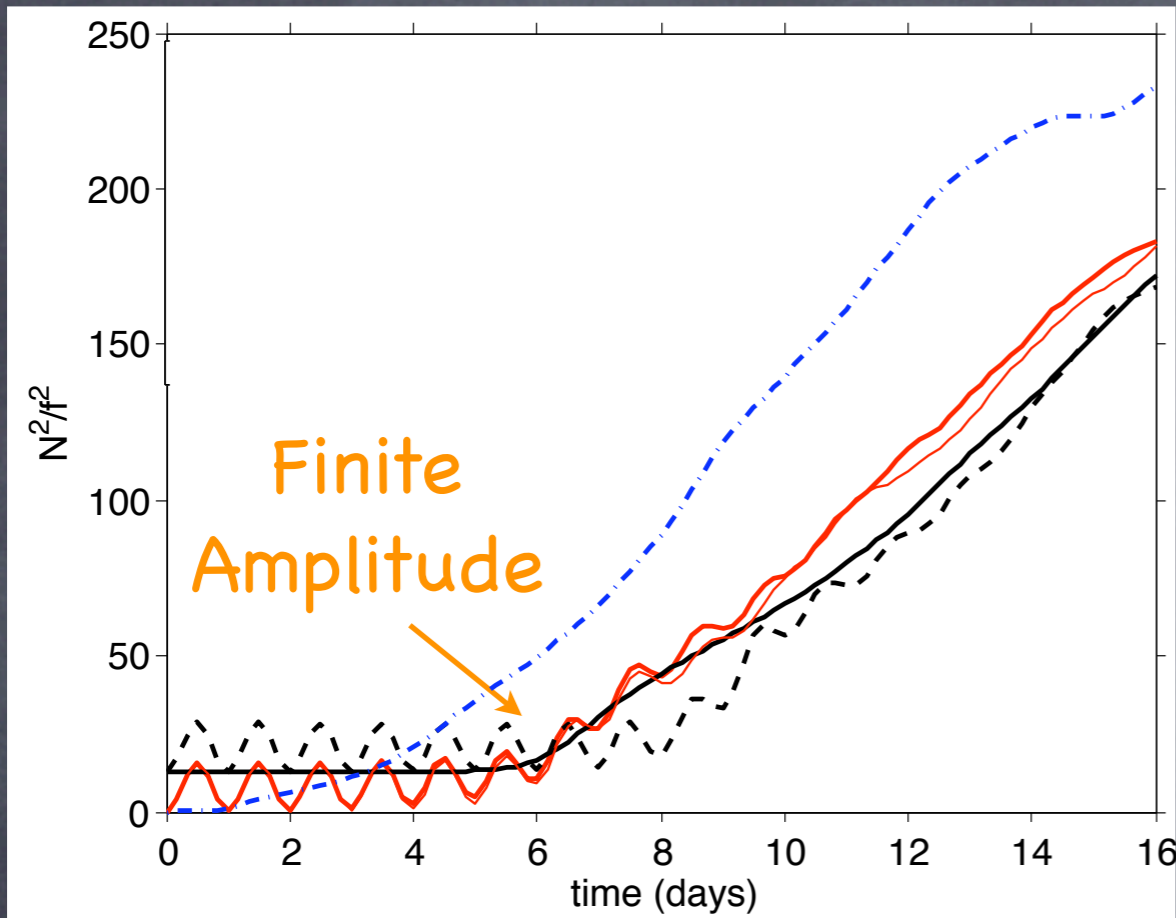
# Parameterization of Finite Amp. Eddies: Ingredients



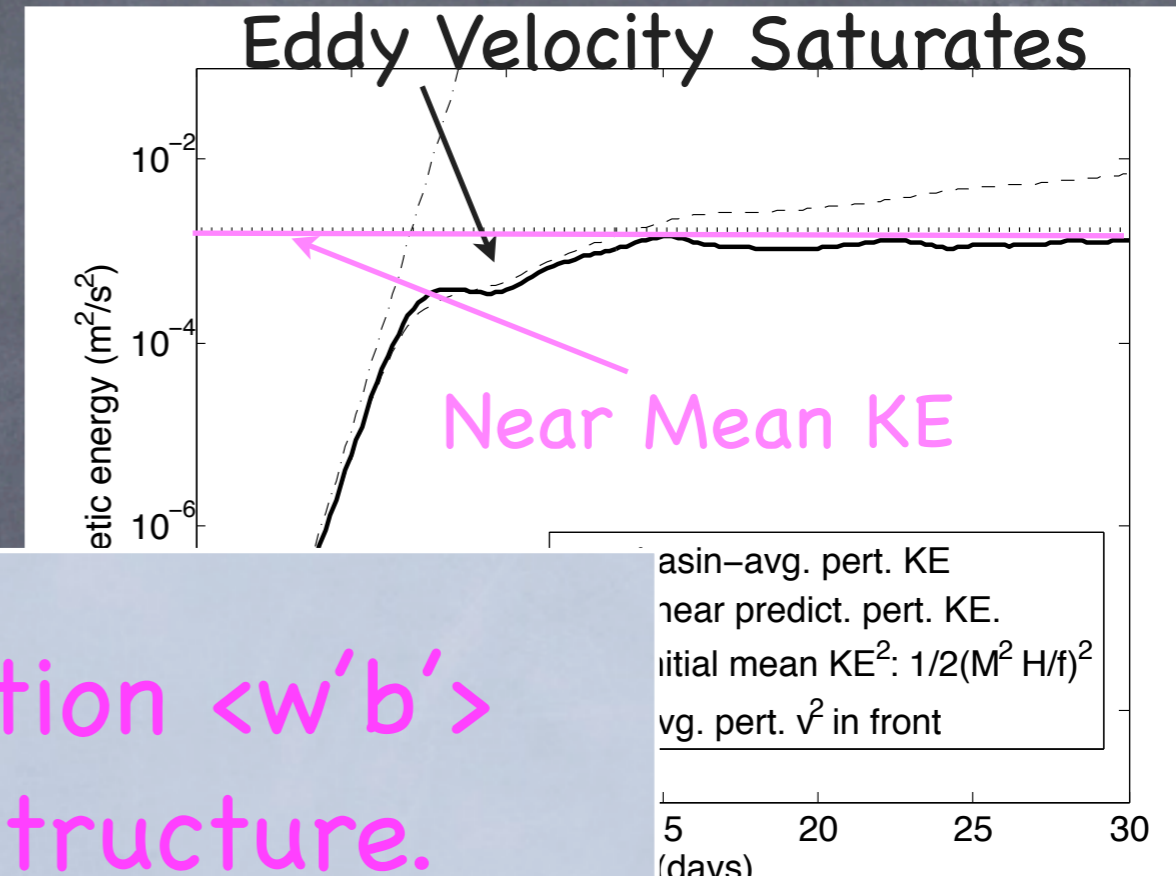
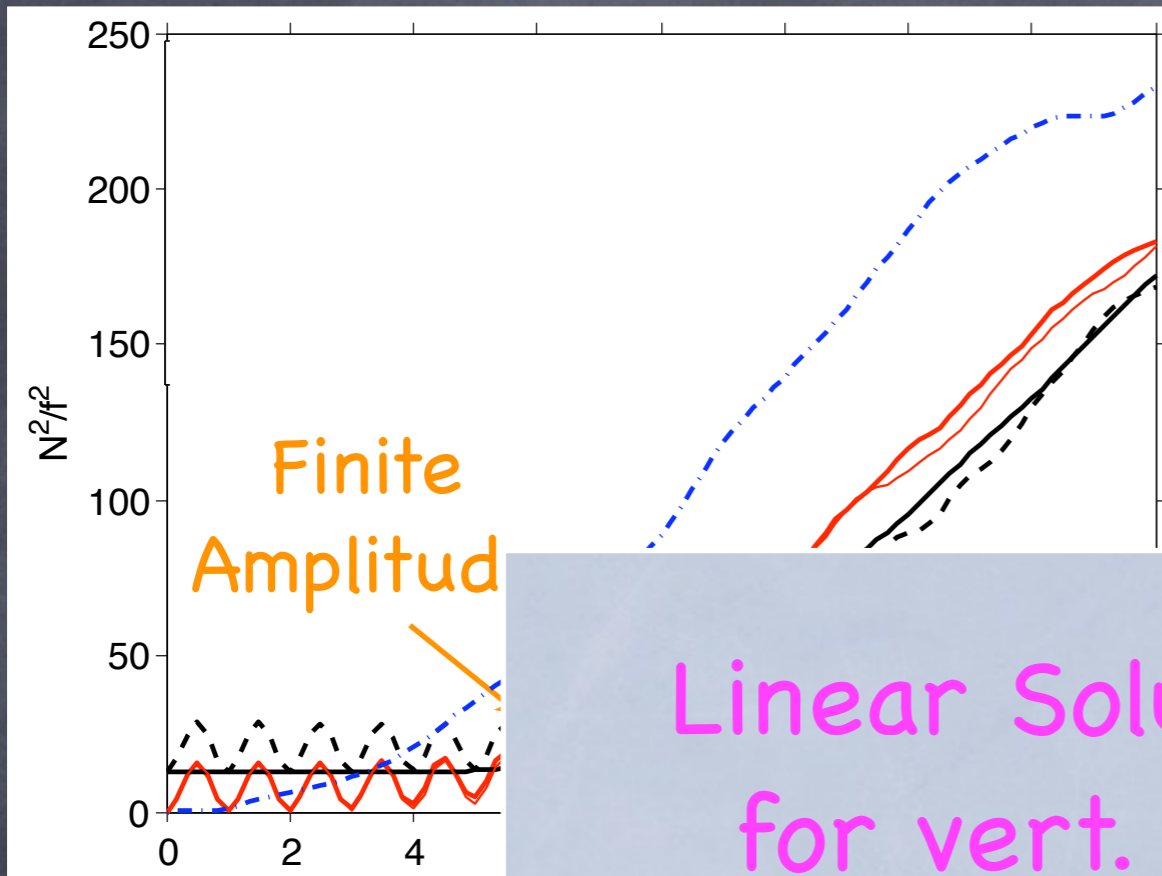
# Parameterization of Finite Amp. Eddies: Ingredients



# Parameterization of Finite Amp. Eddies: Ingredients



# Parameterization of Finite Amp. Eddies: Ingredients



Linear Solution  $\langle w'b' \rangle$   
for vert. structure.  
As in Branscome '83...

