Uncertainty in Ocean General Circulation Model Mixing Tensors

Baylor Fox-Kemper,
University of Colorado Boulder, Brown U. (>Jan. 2013)
Cooperative Insititute for Research in Environmental Sciences and Dept. of Atmospheric and Oceanic Sciences
with
Frank Bryan (NCAR), John Dennis (NCAR),
Scott Bachman (CIRES/ATOC), Jim McWilliams (UCLA),
NCAR Oceanography Section

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Mesoscale Parameterizations

- Researchers have already cast much darkness on this subject and if they continue their investigations we shall soon know nothing at all about it.
  - --Mark Twain

Ocean Equations*:
Boussinesq Fluid on Tangent Plane to a Rotating Sphere

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla_h \mathbf{u} + w \partial_z \mathbf{u} + Ro^{-1} \mathbf{f} \times \mathbf{u} = -\overline{P} \nabla_h p + Re^{-1} \nabla^2 \mathbf{u}$$

$$\partial_t w + \mathbf{u} \cdot \nabla_h w + w \partial_z w = -\overline{P} \partial_z p + \Gamma b \hat{\mathbf{z}} + Re^{-1} \nabla^2 w$$

Buoyancy (or S, T): $$\partial_t b + \mathbf{u} \cdot \nabla_h b + w \partial_z b = Pe^{-1} \nabla^2 b$$

$$\nabla_h \cdot \mathbf{u} + \partial_z w = 0$$

Re, Pe for an affordable gridscale are 10^6 to 10^{11}

Numerics require O(1)

Parameters		Ratios
Rossby	$Ro = \frac{U}{f_0 L}$	$A_\tau = \frac{L}{U\tau^*} = \frac{t^*}{\tau^*}$
Euler	$\overline{P} = \frac{p^*}{\rho_0 U^2}$	$A_h = \frac{L}{L_{pg}}$
Buoyancy	$\Gamma = \frac{BL}{U^2}$	$A_z = \frac{H}{L}$
Reynolds	$Re = \frac{UL}{\nu}$	$A_\beta = \frac{L_{pg}}{R} \tan \varphi_0$
Péclet	$Pe = \frac{UL}{\kappa}$	

*From Grooms, Julien, & F-K, 11

What is a parameterization?

- Express the coarse-grain averages of quantities (including the subgrid effects), e.g.:

$$\overline{\frac{\partial \tau}{\partial t}} \qquad \overline{\frac{\partial u}{\partial x}} \qquad \overline{\frac{\partial u\tau}{\partial x}}$$

- As a function of the resolved coarse-grain fields

$$\overline{\frac{\partial \tau}{\partial t}} = \frac{\partial \overline{\tau}}{\partial t} \qquad \overline{\frac{\partial u}{\partial x}} = \frac{\partial \overline{u}}{\partial x} \qquad \overline{\frac{\partial u\tau}{\partial x}} = \frac{\partial \overline{u}\ \overline{\tau}}{\partial x} + \frac{\partial \overline{u'\tau'}}{\partial x}$$

- Note that nonlinear terms require special treatment
- And Couple different scales, small talks to large!

The Character of the Mesoscale

(Capet et al., 2008)

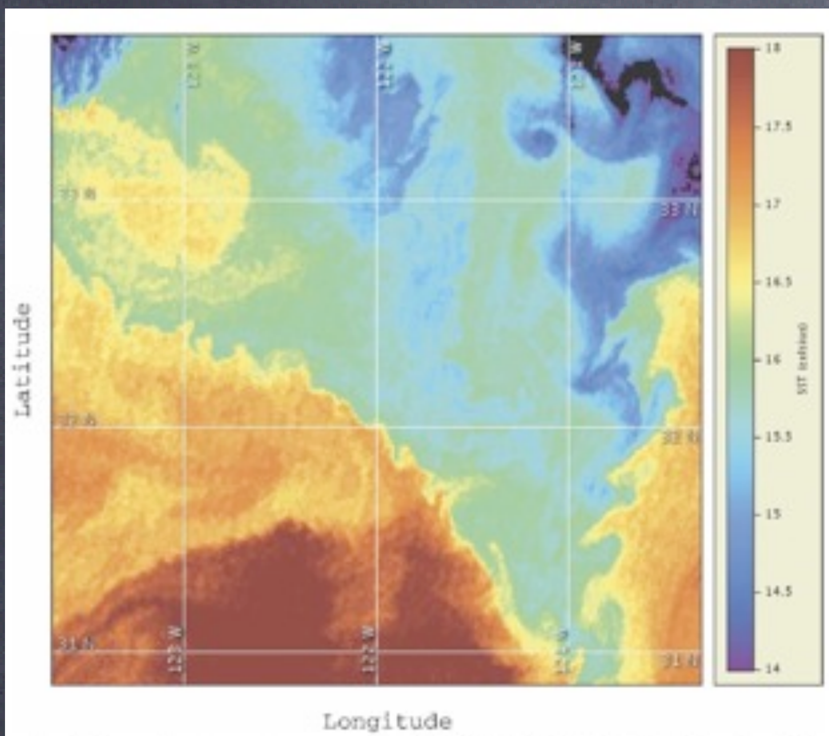


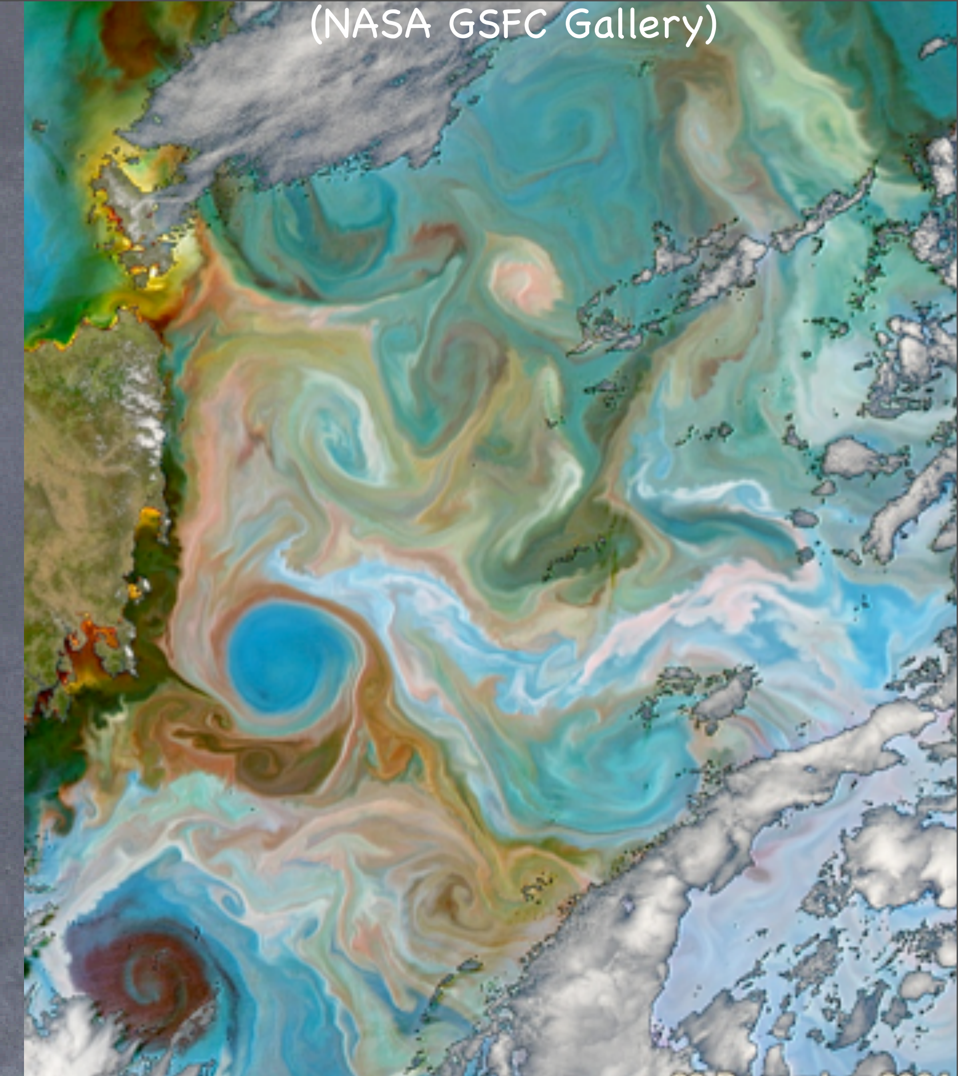
Fig. 16. Sea surface temperature measured at 1832 UTC 3 Jun 2006 off Point Conception in the California Current from CoastWatch (http://coastwatch.pfeg.noaa.gov). The fronts between recently upwelled water (i.e., 15°–16°C) and offshore water (≥17°C) show submesoscale instabilities with wavelengths around 30 km (right front) or 15 km (left front). Images for 1 day earlier and 4 days later show persistence of the instability events.

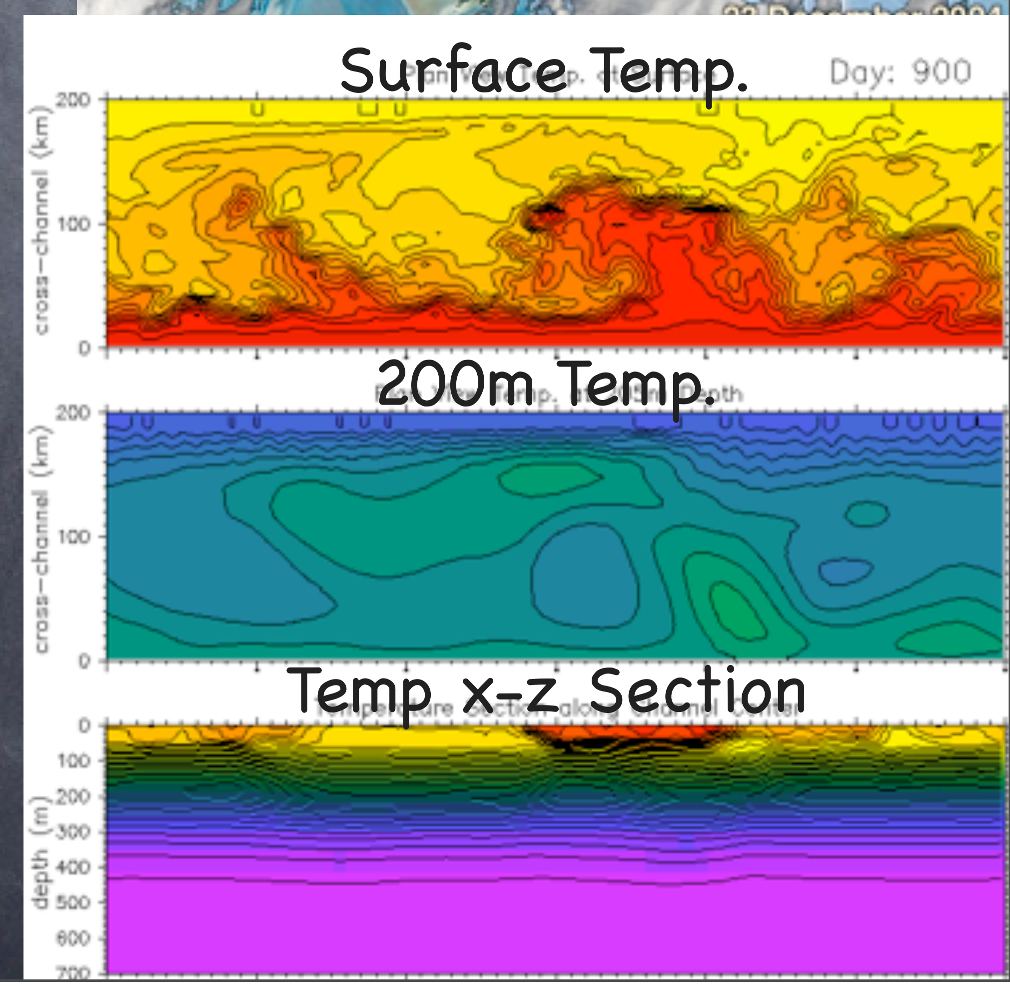
- Boundary Currents
- Eddies
- Ro=O(0.1)
- Ri=O(1000)
- Full Depth
- Eddies strain to produce Fronts
- 100km, months

Eddy processes mainly baroclinic & barotropic instability. Parameterizations of baroclinic instability (GM, Visbeck...).

← 100 km

(NASA GSFC Gallery)





Mesoscale Eddy Parameterizations all have the form:

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Count Degrees of Freedom:

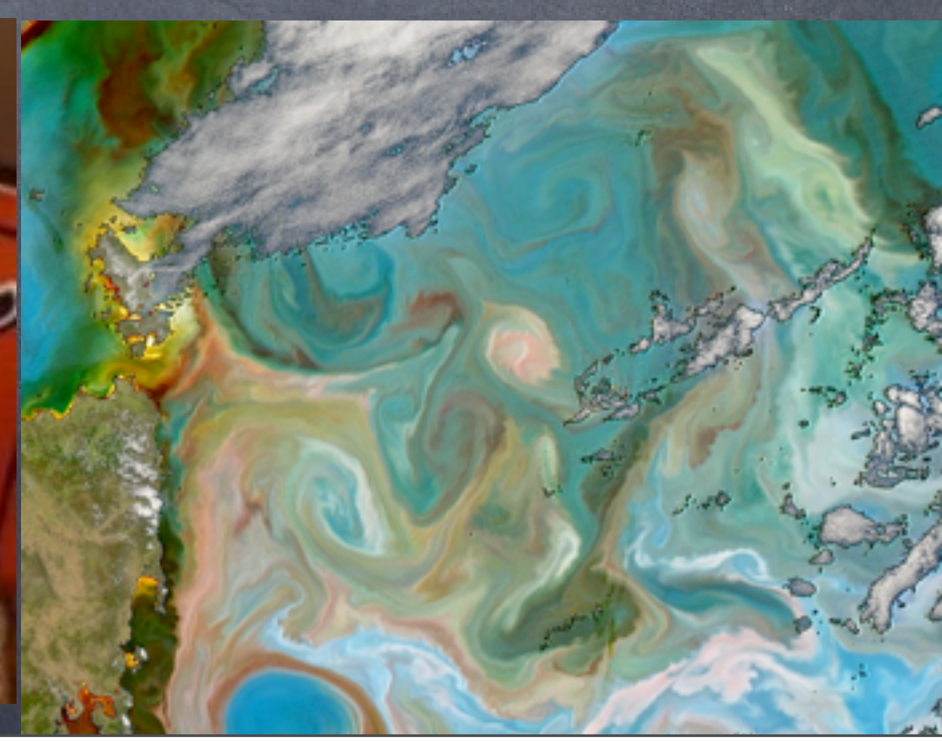
3 tracer flux components

3 tracer gradient components

9 tensor elements!

Does this cover all the degrees of freedom?

- More tracers does provide a just-determined or overdetermined (Moore-Penrose/least squares) problem for M with a unique answer, but...
- Different tracers will have different fluxes as they feel the subgrid 'nooks and crannies' of the mesoscale eddies!



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With John Dennis & Frank Bryan, we took a POP0.1° Normal-Year forced model (yrs 16-20)

Added 9 Passive tracers--restored x,y,z @ 3 rates

Kept all the eddy fluxes for passive & active

Coarse-grained to 2°, transient eddies, tracers to $\mathbf{M}$

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

Sym Part=Anisotropic* Redi

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ K_{yx} & K_{yy} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \tilde{\nabla}z\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

AntiSym Part=Anisotropic* GM

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -\hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ 0 & 0 & -\hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & 0 \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$

Yellow $\mathbf{K}$ ‘are‘ horizontal stirring & mixing

Underdetermined

$$\begin{bmatrix} \overline{u\tau_1} - \bar{u}\bar{\tau}_1 \\ \overline{v\tau_1} - \bar{v}\bar{\tau}_1 \\ \overline{w\tau_1} - \bar{w}\bar{\tau}_1 \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_{1,x} \\ \bar{\tau}_{1,y} \\ \bar{\tau}_{1,z} \end{bmatrix}$$

Just-determined

$$\begin{bmatrix} \overline{u\tau_1} - \bar{u}\bar{\tau}_1 & \overline{u\tau_2} - \bar{u}\bar{\tau}_2 & \overline{u\tau_3} - \bar{u}\bar{\tau}_3 \\ \overline{v\tau_1} - \bar{v}\bar{\tau}_1 & \overline{v\tau_2} - \bar{v}\bar{\tau}_2 & \overline{v\tau_3} - \bar{v}\bar{\tau}_3 \\ \overline{w\tau_1} - \bar{w}\bar{\tau}_1 & \overline{w\tau_2} - \bar{w}\bar{\tau}_2 & \overline{w\tau_3} - \bar{w}\bar{\tau}_3 \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_{1,x} & \bar{\tau}_{2,x} & \bar{\tau}_{3,x} \\ \bar{\tau}_{1,y} & \bar{\tau}_{2,y} & \bar{\tau}_{3,y} \\ \bar{\tau}_{1,z} & \bar{\tau}_{2,z} & \bar{\tau}_{3,z} \end{bmatrix}$$

Overdetermined

$$\begin{bmatrix} \overline{u\tau_1} - \bar{u}\bar{\tau}_1 & & \overline{u\tau_N} - \bar{u}\bar{\tau}_N \\ \overline{v\tau_1} - \bar{v}\bar{\tau}_1 & \dots & \overline{v\tau_N} - \bar{v}\bar{\tau}_N \\ \overline{w\tau_1} - \bar{w}\bar{\tau}_1 & & \overline{w\tau_N} - \bar{w}\bar{\tau}_N \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_{1,x} & & \bar{\tau}_{N,x} \\ \bar{\tau}_{1,y} & \dots & \bar{\tau}_{N,y} \\ \bar{\tau}_{1,z} & & \bar{\tau}_{N,z} \end{bmatrix}$$

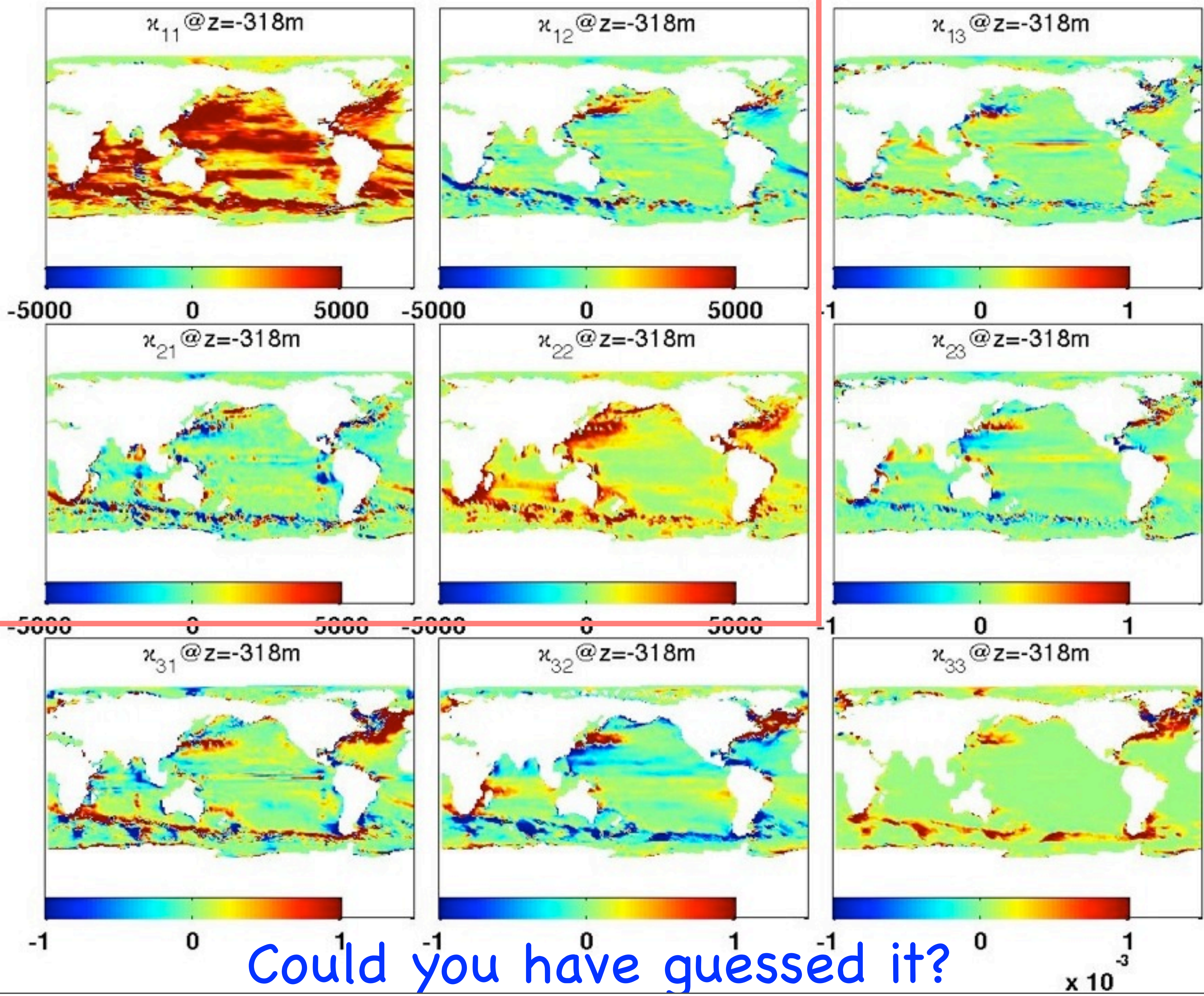
Is Lagrangian Transport Unique?

- Taylor 1921/1953 says yes, in a decorrelation timescale sense
- If non-conservative tracers, then no
- Nearly conservative tracers--probably something that becomes identical in limit
- Active vs. passive tracers?

What does mean mean?

- Temporal averaging -> 8yr average by season or overall
  - Must preserve covariances...
  - 20x increase in variables
- Spatial coarse-graining 10km->200km
  - 20x20 gridpoints per coarse-gridpoint
  - 400x reduction of variables
- Collect
- slow,coarse; slow,fine; fast,coarse; fast,fine

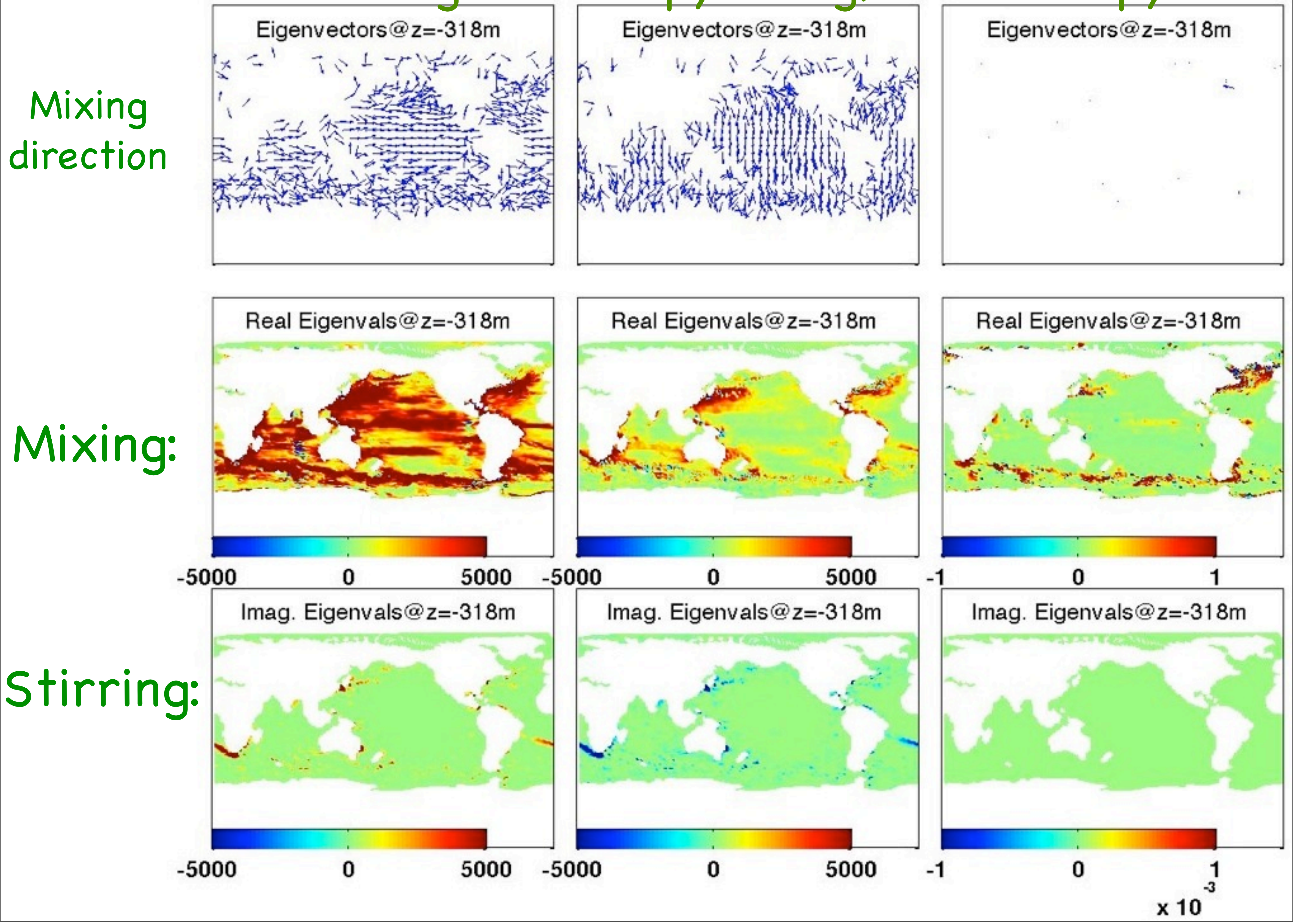
K



M

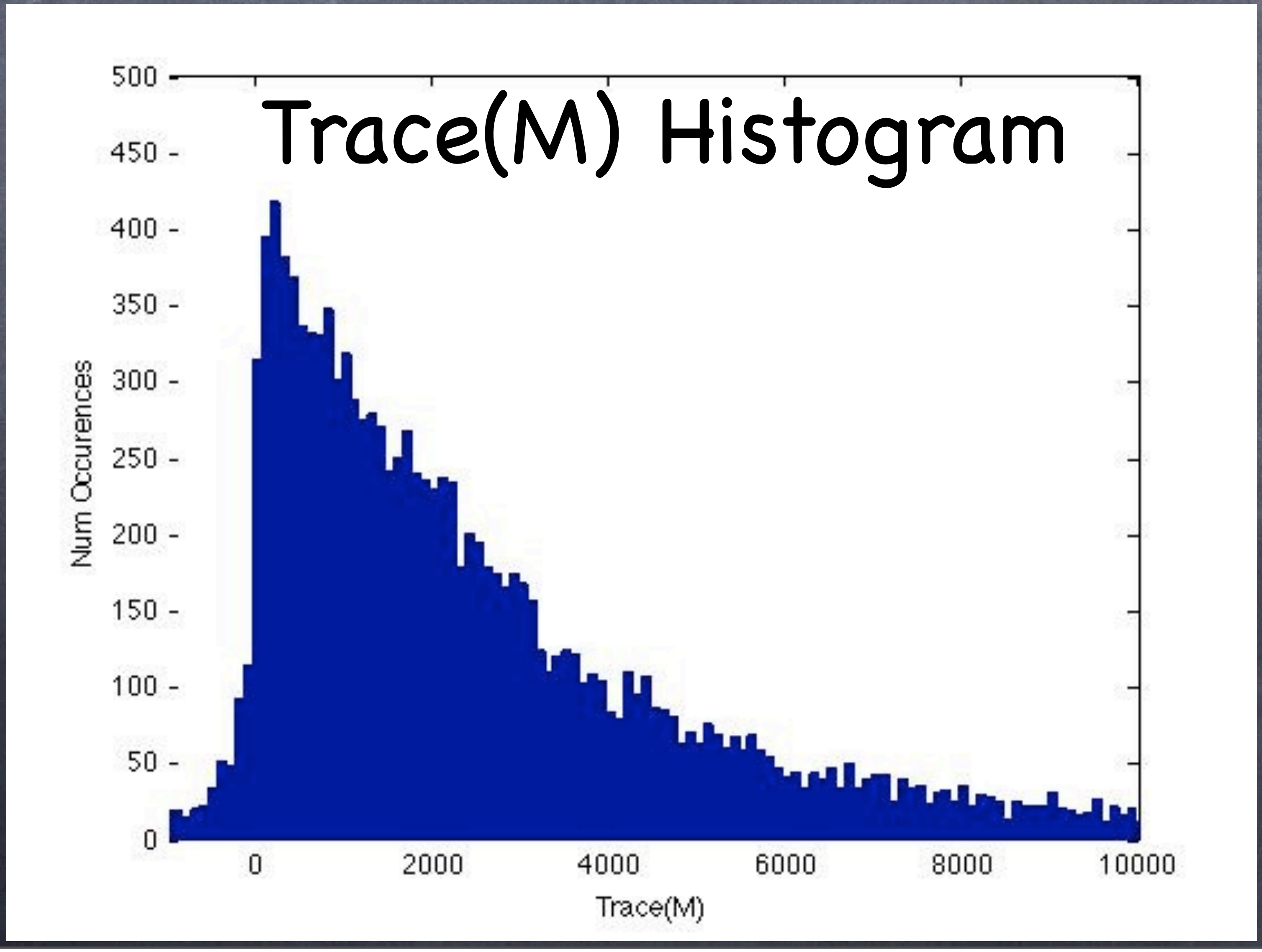
Could you have guessed it?

Result: Strong Anisotropy Along/Across Isopycnals



Are Diffusivity Values Resonable?

$$\begin{bmatrix} \overline{u'\tau'} \\ \overline{v'\tau'} \\ \overline{w'\tau'} \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} & \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ K_{yx} & K_{yy} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \\ \hat{\mathbf{x}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \hat{\mathbf{y}}\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} & \tilde{\nabla}z\cdot\mathbf{K}\cdot\tilde{\nabla}\mathbf{z} \end{bmatrix} \begin{bmatrix} \overline{\tau}_x \\ \overline{\tau}_y \\ \overline{\tau}_z \end{bmatrix}$$



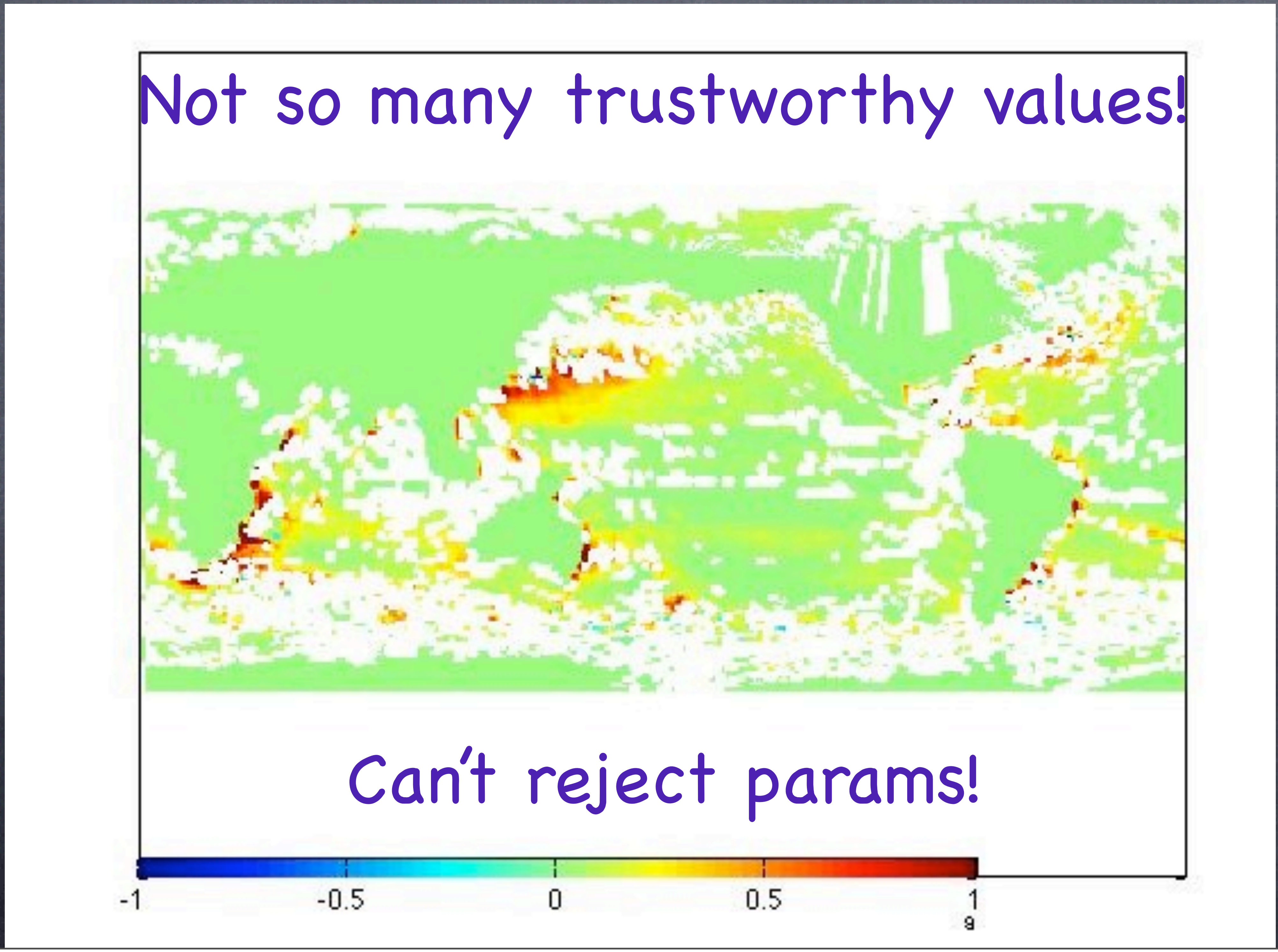
Hor. Diffusivity is roughly Trace(M)/2

Peak of Diffusivity near 250 m^2/s

Median Diffusivity near $1000 m^2/s$

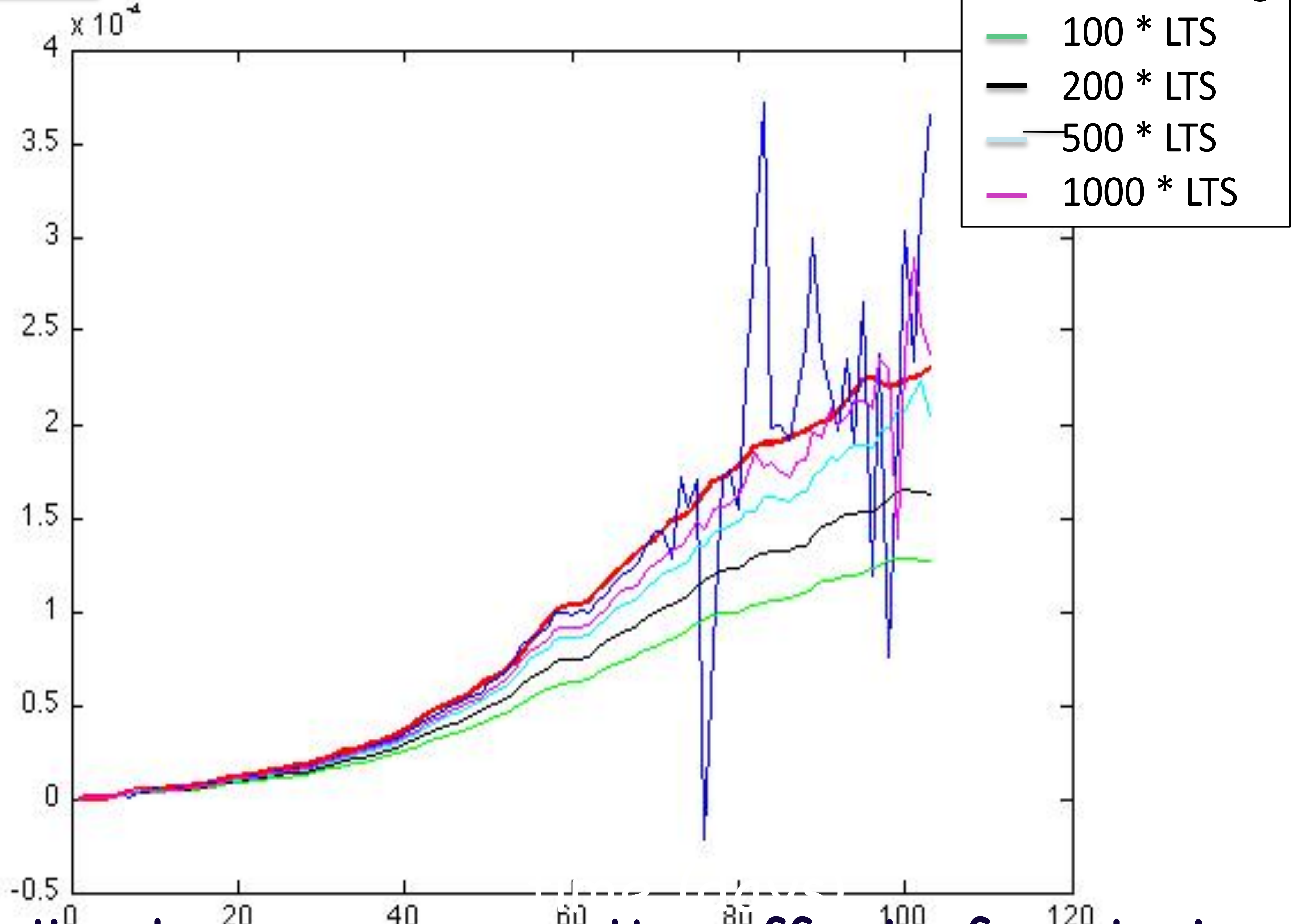
<6% negative

But, how well does it work? Suppose we only plot values where different tracer sets agree...



$$\frac{d\tau}{dt} = -\lambda(\tau - \tau_0)$$

$$\overline{v'b'}_{\rm rec} = -\mathbf{M}\nabla\overline{b}$$



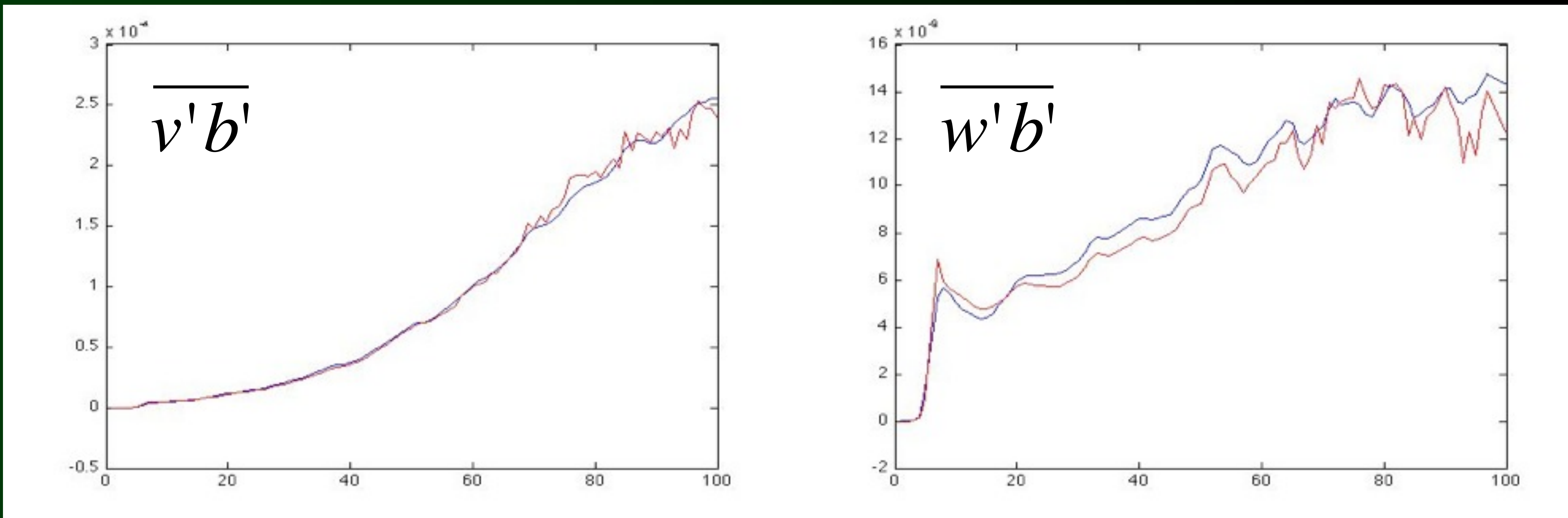
In idealized runs, can see the effect of restoring. Whatever we do, we need to get buoyancy right!

In idealized setting, can do better

Reconstruction of eddy buoyancy fluxes

Original fluxes

Reconstructed fluxes



Using specially-tailored non-restored tracers improves estimate (error is now < 10%)... but not feasible in realistic diagnosis.

In realistic diagnosis, we can improve the estimate a bit by approximating restoring effect

Uncertainty... How many tracers needed?

- Distribution?

Underdetermined

$$\begin{bmatrix} \overline{u\tau_1} - \bar{u}\bar{\tau}_1 \\ \overline{v\tau_1} - \bar{v}\bar{\tau}_1 \\ \overline{w\tau_1} - \bar{w}\bar{\tau}_1 \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_{1,x} \\ \bar{\tau}_{1,y} \\ \bar{\tau}_{1,z} \end{bmatrix}$$

Just-determined

$$\begin{bmatrix} \overline{u\tau_1} - \bar{u}\bar{\tau}_1 & \overline{u\tau_2} - \bar{u}\bar{\tau}_2 & \overline{u\tau_3} - \bar{u}\bar{\tau}_3 \\ \overline{v\tau_1} - \bar{v}\bar{\tau}_1 & \overline{v\tau_2} - \bar{v}\bar{\tau}_2 & \overline{v\tau_3} - \bar{v}\bar{\tau}_3 \\ \overline{w\tau_1} - \bar{w}\bar{\tau}_1 & \overline{w\tau_2} - \bar{w}\bar{\tau}_2 & \overline{w\tau_3} - \bar{w}\bar{\tau}_3 \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_{1,x} & \bar{\tau}_{2,x} & \bar{\tau}_{3,x} \\ \bar{\tau}_{1,y} & \bar{\tau}_{2,y} & \bar{\tau}_{3,y} \\ \bar{\tau}_{1,z} & \bar{\tau}_{2,z} & \bar{\tau}_{3,z} \end{bmatrix}$$

Overdetermined

$$\begin{bmatrix} \overline{u\tau_1} - \bar{u}\bar{\tau}_1 & & \overline{u\tau_N} - \bar{u}\bar{\tau}_N \\ \overline{v\tau_1} - \bar{v}\bar{\tau}_1 & \ldots & \overline{v\tau_N} - \bar{v}\bar{\tau}_N \\ \overline{w\tau_1} - \bar{w}\bar{\tau}_1 & & \overline{w\tau_N} - \bar{w}\bar{\tau}_N \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_{1,x} & & \bar{\tau}_{N,x} \\ \bar{\tau}_{1,y} & \ldots & \bar{\tau}_{N,y} \\ \bar{\tau}_{1,z} & & \bar{\tau}_{N,z} \end{bmatrix}$$

Uncertainty... Coarse-grain? What is the mean & eddy?

- Flierl & McWilliams '77: ~30 yrs of Eulerian obs. needed for variances & co-variances
- Single snapshots are huge at Global 10km
  - (>2.10^8 gridpoints, 5 state variables, 9 tensor elements, 3N fluxes, N tracers, N variances, etc)
- Spatial coarse-graining desirable to improve averaging and condense dataset

Underdetermined

$$\begin{bmatrix} \overline{u\tau_1} - \bar{u}\bar{\tau}_1 \\ \overline{v\tau_1} - \bar{v}\bar{\tau}_1 \\ \overline{w\tau_1} - \bar{w}\bar{\tau}_1 \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_{1,x} \\ \bar{\tau}_{1,y} \\ \bar{\tau}_{1,z} \end{bmatrix}$$

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Overdetermined

$$\begin{bmatrix} \overline{u\tau_1} - \bar{u}\bar{\tau}_1 & & \overline{u\tau_N} - \bar{u}\bar{\tau}_N \\ \overline{v\tau_1} - \bar{v}\bar{\tau}_1 & \dots & \overline{v\tau_N} - \bar{v}\bar{\tau}_N \\ \overline{w\tau_1} - \bar{w}\bar{\tau}_1 & & \overline{w\tau_N} - \bar{w}\bar{\tau}_N \end{bmatrix} = - \begin{bmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{bmatrix} \begin{bmatrix} \bar{\tau}_{1,x} & & \bar{\tau}_{N,x} \\ \bar{\tau}_{1,y} & \dots & \bar{\tau}_{N,y} \\ \bar{\tau}_{1,z} & & \bar{\tau}_{N,z} \end{bmatrix}$$

Could INCLUDE AVERAGING in linear model:

Then get distribution pre-coarse graining & time avg

Conclusions: None

Prospects:

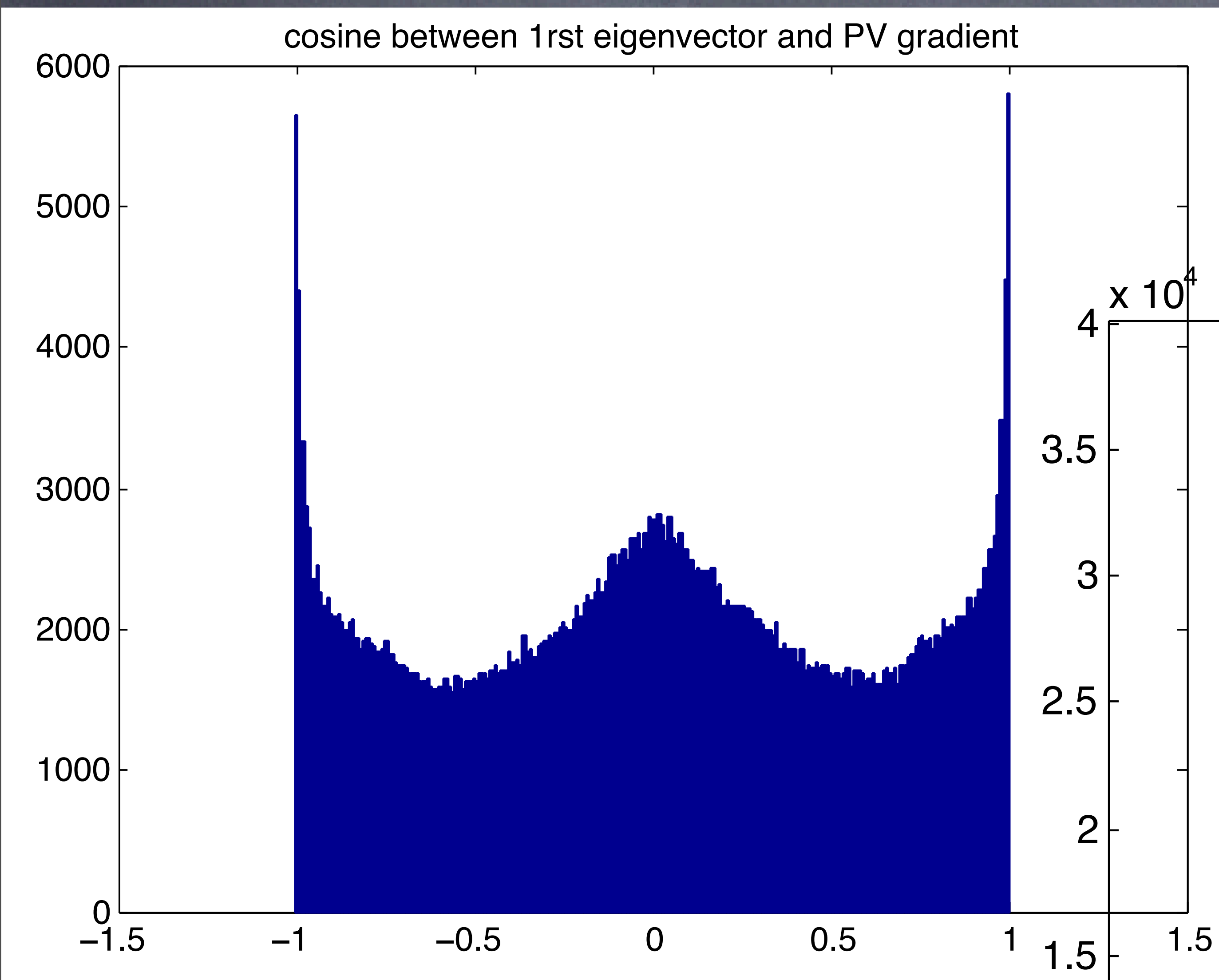
- Can we make a better measure of the uncertainty profile?
- Can we use the distribution in stochastic, rather than deterministic, parameterizations?
- Can we detect times when the flux-gradient relationship itself fails?
- Can we specify better averaging kernels?

$$\overline{\mathbf{u}\tau} - \overline{\mathbf{u}}\,\overline{\tau} = -\mathbf{M}\nabla\overline{\tau} + \overline{\epsilon}$$

Result: Strong Anisotropy Along/Across PV Grads.

Mixing direction

Either along PV contours or across



1rst Eigenvector

2nd Eigenvector Across PV contours