### Surface Waves in Turbulent and Laminar Submesoscale Flow

#### Baylor Fox-Kemper (Brown U., Geo.)

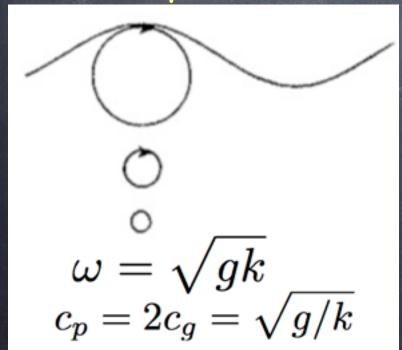
with Peter Hamlington (CU-Boulder), Luke Van Roekel (Northland College), Sean Haney (CU-ATOC), Adrean Webb (CU-APPM), Keith Julien (CU-APPM), Greg Chini (UNH), Peter Sullivan (NCAR), Jim McWilliams (UCLA), Mark Hemer (CSIRO)

> AMS's 19th AOFD Meeting June 20, 2013; 14:00-14:15

Sponsors: NSF 1245944, 0934737, 0825614, NASA NNX09AF38G

Look for fast, small solutions of the freesurface Boussinesq Equations

Linearize for not steep waves



### Ageostrophic, Nonhydrostatic Waves

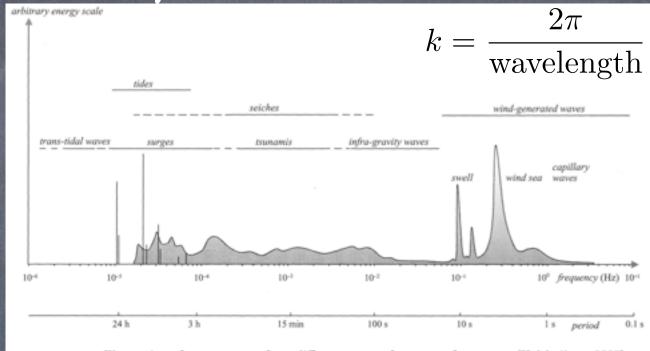


Illustration of wave spectra from different types of ocean surface waves (Holthuijsen, 2007)



# vs. Geostrophic, Hydrostatic, (Thermal Wind)

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished limit:

Inviscid (Re>>1), rapidly rotating (Ro<<1), and thin (L>>H)

(Combined) Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b$$

Adding forcing (air-sea) and advection of buoyancy by this flow--you have (nearly) all large-scale ocean physics!

## Craik-Leibovich Boussinesq

- Formally a multiscale asymptotic equation set:
  - 3 classes: Small, Fast; Large, Fast; Large, Slow
  - Solve first 2 types of motion in the case of limited slope (ka), irrotational --> Deep Water Waves!
  - Must also assume slowly-varying wave packets
  - Average over deep water waves in space & time,
  - Arrive at Large, Slow equation set:

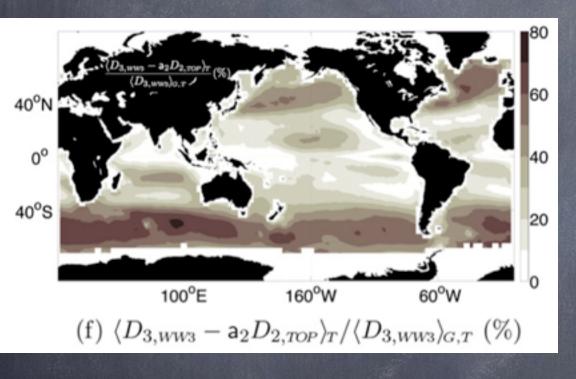
$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times (\mathbf{v} + \mathbf{v}_s) = -\nabla \pi^{\dagger} + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$

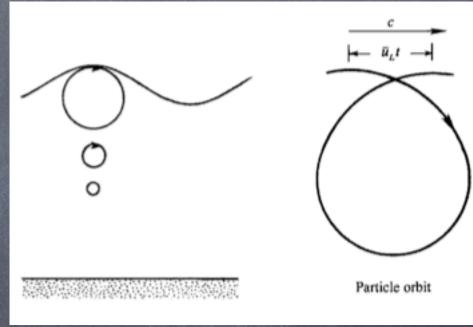
$$\frac{\partial b}{\partial t} + (\mathbf{v} + \mathbf{v}_s) \cdot \nabla b = 0 \qquad \nabla \cdot \mathbf{v} = 0$$

 $\mathbf{v}_s = \text{Stokes Drift}$ 

Craik & Leibovich 1976; Gjaja & Holm 1996; McWilliams et al. 2004

# How well do we know Stokes Drift? <50% discrepancy





RMS error in measures of surface Stokes drift, between wave models (not shown) or model vs. altimeter (shown)

# Now, we've got the CLB equations & estimated global Stokes, what to do?

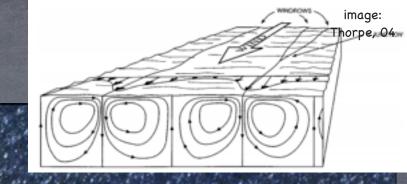
- 1) Stokes (wave)-driven small-scale turbulence (Large Eddy Simulations of CLB)
- 2) Laminar submesoscale flow with Stokes Coriolis & Stokes Vortex forces (Analytic Solns of CLB)
- 3) Wave-driven turbulence interacting with submesoscale flow (Multiscale LES of CLB)

### The Character of the

#### Langmuir Turbulence

- Near-surface
- wave & wind driven
- Langmuir Cells & Langmuir Turb.
- Ro>>1
- Ri<1: Nonhydro
- 1–10m
- 10s to mins
- w, u=O(10cm/s)
- Stokes drift
- Eqtns:Craik-Leibovich
- Params: McWilliams & Sullivan, 2000, etc.

lmage: NPR.org, Digitalglobe Seabird Deep Water Horizon Spil





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#### Call for Papers: Gulf of Mexico Modelling: Lessons learned from the spill

The Gulf of Mexico (GoM) is a complex, semi-enclosed basin of great environmental and economic importance. On 20 April 2010, the Deepwater Horizon drilling rig experienced a catastrophic failure, which claimed 11 lives and set off an 87 day oil spill in the GoM. Academic, governmental and private sector research has contributed to mitigation efforts, and the GoM has received unprecedented attention over the last three years. At present, no single ocean model is capable of handling the wide range of scales and complex dynamics necessary to understand the GoM circulation and dispersion of the oil spill. Instead, different model configurations have been used to capture a subset of the GoM dynamics.

Ocean Modelling will host a Virtual Special Issue (VSI): "Gulf of Mexico Modelling: Lessons learned from the spill" to collect the last three years of intense research concerning GoM modelling. The VSI will serve as a standard and influence for future GoM modelling efforts and development. While the VSI will focus on the GoM, submissions that address the modelling advances required to understand this basin's circulation and dispersion of pollutants but also have broader applicability are encouraged.

This VSI would be open to all modelling efforts related to GoM, as well as studies of processes or observations found to be important or needed for GoM modelling. Submissions which address oil spill related science in the following areas are encouraged:

- GoM basin or shelf scale physical/biological/chemical processes
- 2. GoM open-coastal ocean connectivity and cross-topography transport
- Bubble/droplet scale dynamics including biological and chemical degradation and dispersant application effects
- 4. Air-sea and boundary layer processes
- 5. Surfactant or emulsion dispersion processes

Contributions should address: Why does the particular method of investigation appropriately model the physical process of interest? How does the particular method advance GoM modelling? What are the future implications of the work to GoM modelling and related modelling worldwide?

As a Virtual Special Issue, accepted papers will appear in *Ocean Modelling* as per a normal submission, but designated as part of the "Gulf of Mexico Ocean Modelling: Lessons learned from the spill" Special Issue. All papers will be linked online to other "Gulf of Mexico Modelling: Lessons learned from the spill." The first papers are expected to appear late in 2013 or early 2014.

#### Special Issue Editor(s):

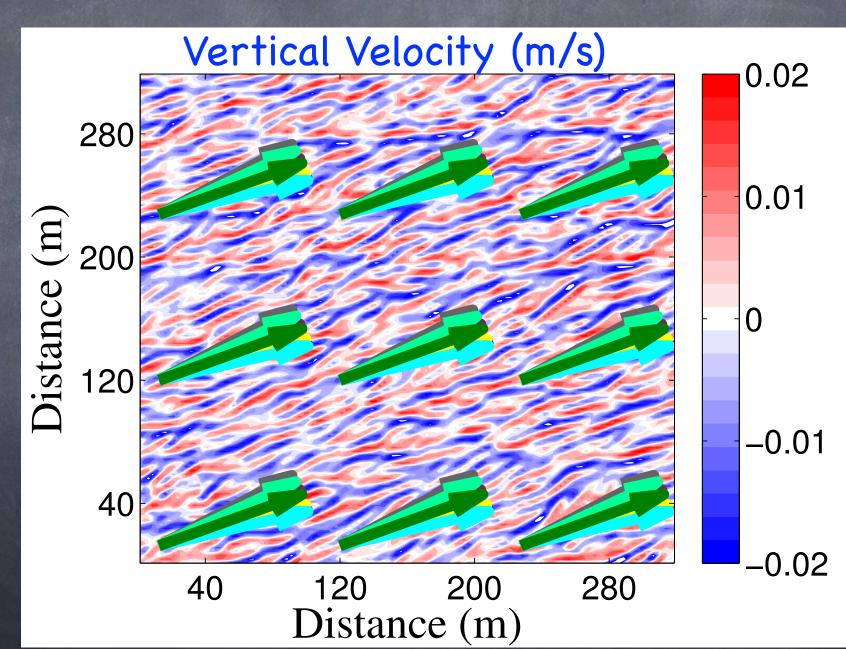
Dr. Baylor Fox-Kemper

Dr. Joseph Kuehl (assistant)

Blue stripes here are windrows-convergences

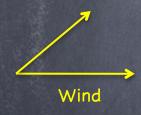
Waves
(Stokes Drift)

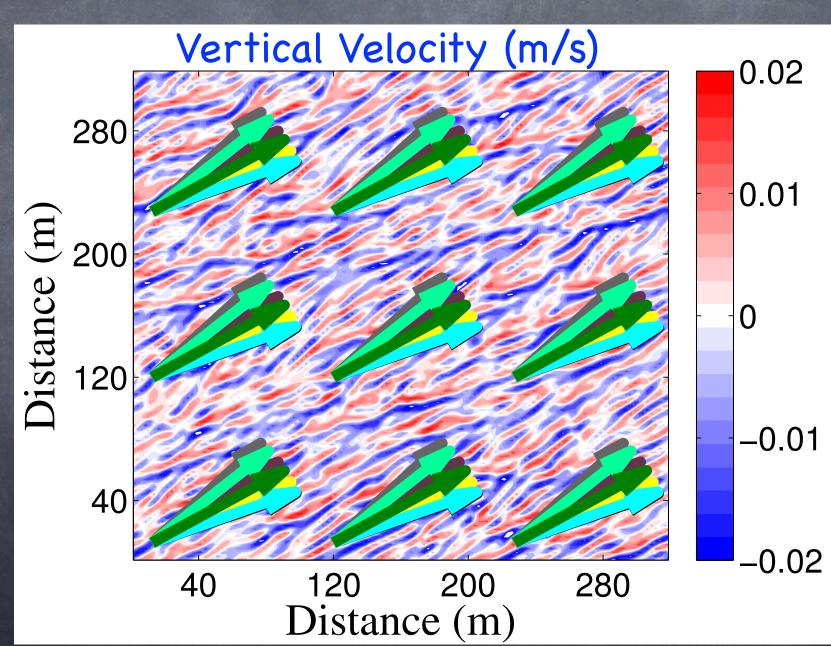
Wind



Blue stripes here are windrows-convergences

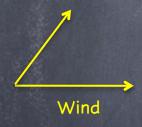
Waves (Stokes Drift)

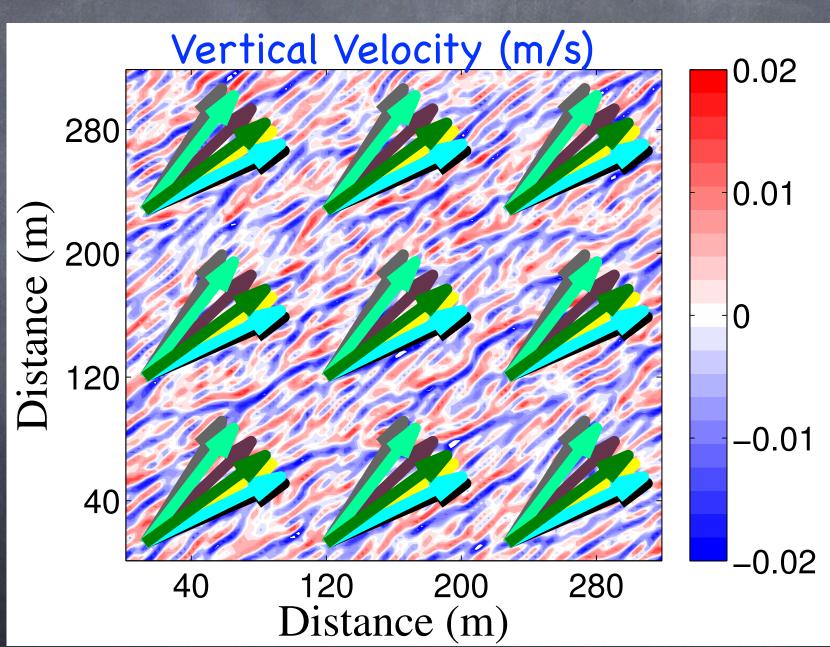




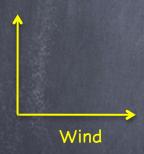
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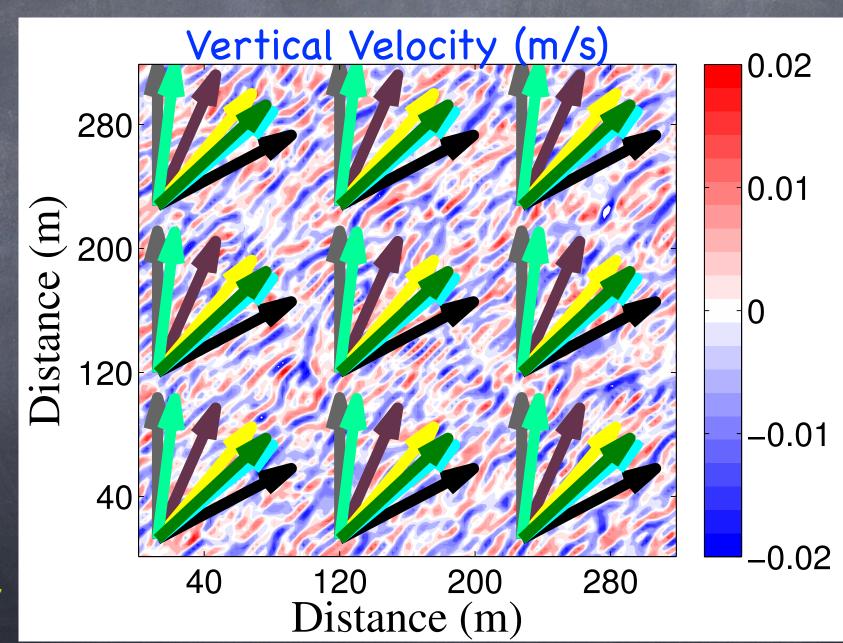
Waves (Stokes Drift)





Blue stripes
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windrows-convergences
Waves
(Stokes Drift)





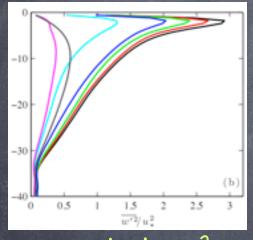
# Why? Vortex Tilting Mechanism In CLB: Tilting & Stretching occur in

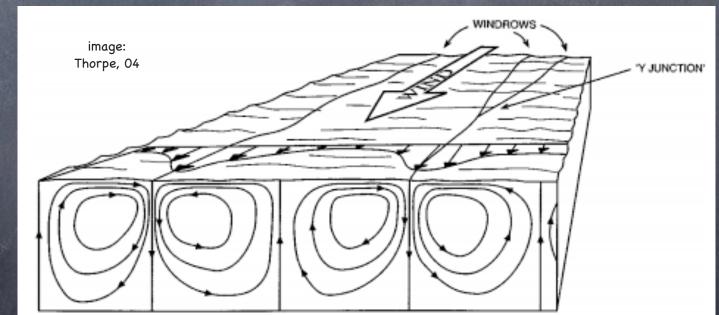
In CLB: Tilting & Stretching occur in direction of Lagrangian shear:

rescaling by projection collapses LES results!

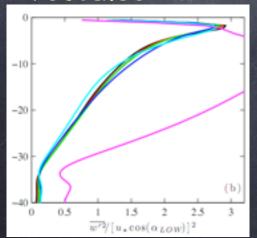
 $\frac{\partial \omega}{\partial t} + \underbrace{(\mathbf{v}_{L} \cdot \nabla) \,\omega}_{AD} = \underbrace{([\omega + f\hat{\mathbf{z}}] \cdot \nabla) \,\mathbf{v}_{L}}_{TS} + \underbrace{\nabla b \times \hat{\mathbf{z}}}_{BT}$ 

$$\mathbf{v}_L \equiv \mathbf{v} + \mathbf{v}_s, \qquad \omega \equiv \nabla \times \mathbf{v}$$





#### rescaled <w2>



L. P. Van Roekel, BFK, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. Journal of Geophysical Research-Oceans, 117:C05001, 22pp, 2012.

depth

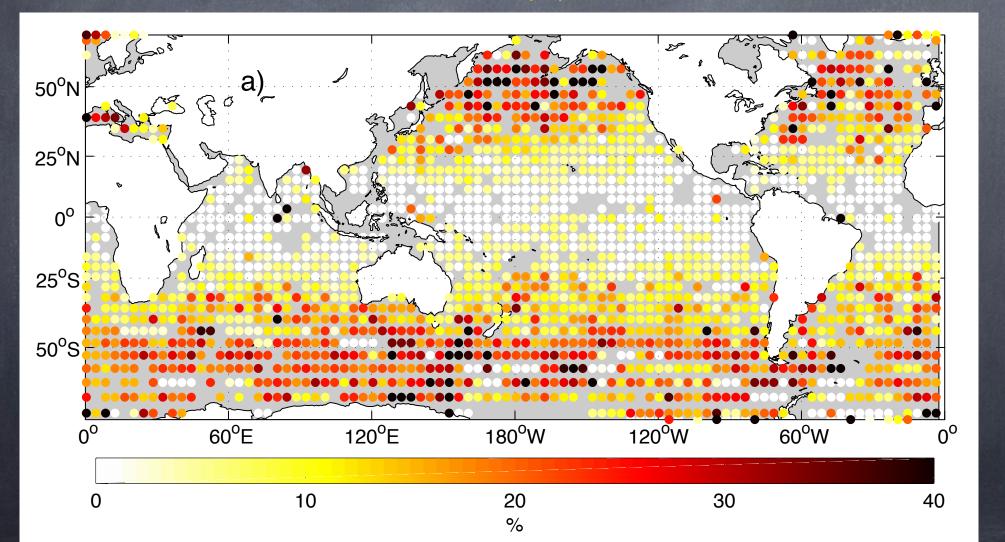
depth

# Including wave-driven mixing appreciably changes the mixed layer depth

(Harcourt 2013 parameterization)

S. E. Belcher, A. A. L. M. Grant, K. E. Hanley, BFK, L. Van Roekel, P. P. Sullivan, W. G. Large, A. Brown, A. Hines, D. Calvert, A. Rutgersson, H. Petterson, J. Bidlot, P. A. E. M. Janssen, and J. A. Polton. A global perspective on Langmuir turbulence in the ocean surface boundary layer. Geophysical Research Letters, 39(18):L18605, 9pp, 2012.

M. A. Hemer, BFK, & R. R. Harcourt. Quantifying the effects of wind waves the the coupled climate system, in prep. 2013.



So, Waves can Drive turbulence that affects large scale:

What about direct effects of waves on submeso- & meso-scales?

Stokes Coriolis & Stokes Vortex

Recall, Subinertial Boussinesq Equations Dominated by: (Combined) Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b$$

# Craik-Leibovich Boussinesq Subinertial Dominated By: (Combined) Lagrangian Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial}{\partial z} \left( \mathbf{v} + \mathbf{v}_s \right) = \mathbf{f} \times \frac{\partial \mathbf{v}_L}{\partial z} = -\nabla b$$

Now the buoyancy gradients govern the Lagrangian flow, not the not the Eulerian!

#### All GFD is for the Lagrangian Flow??

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

Why can't we just forget the whole thing and interpret large scales as Lagrangian velocities?

$$[\mathbf{f} + \nabla \times \mathbf{v}] \times \frac{\partial}{\partial z} (\mathbf{v} + \mathbf{v}_s) = -\nabla b$$

because vortex force & BCs are different!

The "Rossby No." for waves, is big \*more often\* than Ro is

See Haney's Poster for more!!!

#### ε/R

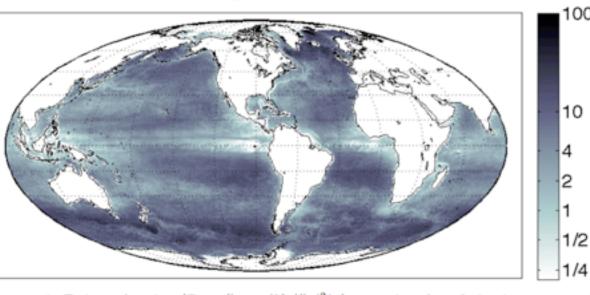
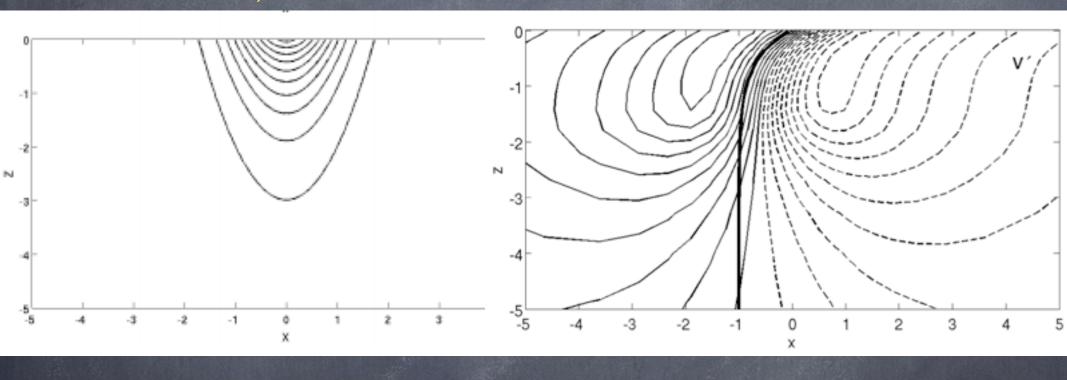


FIGURE 1. Estimated ratio  $\epsilon/\mathcal{R} \approx (|\mathbf{u}_s \cdot \mathbf{u}|h)/(|\mathbf{u}|^2h_s)$  governing the relative importance of Stokes effects versus nonlinearity. Eulerian velocity ( $\mathbf{u}$ ) is taken as the AVISO weekly satellite geostrophic velocity or  $-\mathbf{u}_s$  (for anti-Stokes flow) if  $|\mathbf{u}_s| > |\mathbf{u}|$ . The front/filament depth (h) is estimated as the mixed layer depth from the de Boyer Montégut et al. (2004) climatology. An exponential fit to the Stokes drift of the upper 9m projected onto the AVISO geostrophic velocity provides  $\mathbf{u}_s \cdot \mathbf{u}$  and  $h_s$ . Stokes drift is taken from the WaveWatch-3 simulation described in Webb & Fox-Kemper (2011).  $\mathbf{u}$ ,  $\mathbf{u}_s$ , and  $h_s$  are all for the year 2000, while h is from a climatology of observations over 1961-2008. The year 2000 average of  $\epsilon/\mathcal{R}$  is shown.

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

Waves (Stokes Vortex Force) example of wave-balanced Submeso flow  $\epsilon=2, \epsilon\gg \mathcal{R} \qquad \text{Near the "sweet spot"}$ 



Initial Submeso Front

Contours: 0.1

Perturbation on that scale due to waves

Contours: 1.4

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

### What about Langmuir-Submeso Interactions?

Movie: P. Hamlington See his poster for more!!

Perform large eddy simulations (LES) of CLB with a submesoscale temperature front with winds-with and without Stokes drift

$$\frac{\partial \rho}{\partial t} + \mathbf{u}_L \cdot \nabla \rho = SGS \qquad \nabla \cdot \mathbf{u} = 0$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\boldsymbol{\omega} + f\hat{\mathbf{z}}) \times \mathbf{u}_L = -\nabla \pi - \frac{g\rho\hat{\mathbf{z}}}{\rho_0} + SGS$$

Computational parameters:

Domain size:  $20 \text{km} \times 20 \text{km} \times -160 \text{m}$ 

Grid points: 4096 x 4096 x 128

Resolution:  $5m \times 5m \times -1.25m$ 



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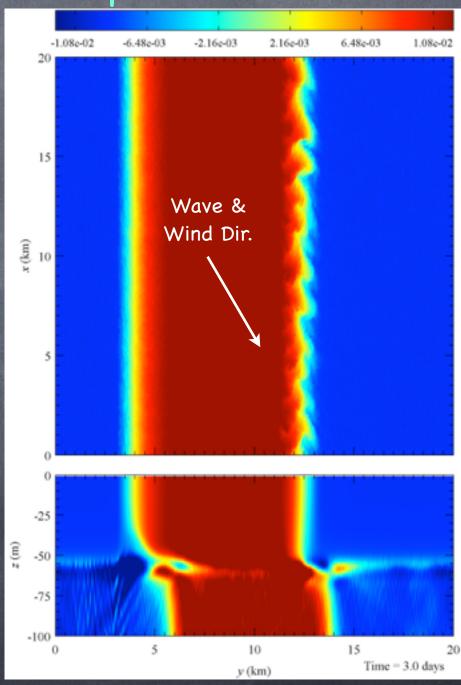
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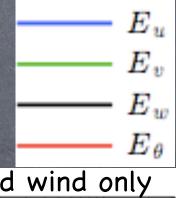
#### Movie: P. Hamlington See his poster for more!!

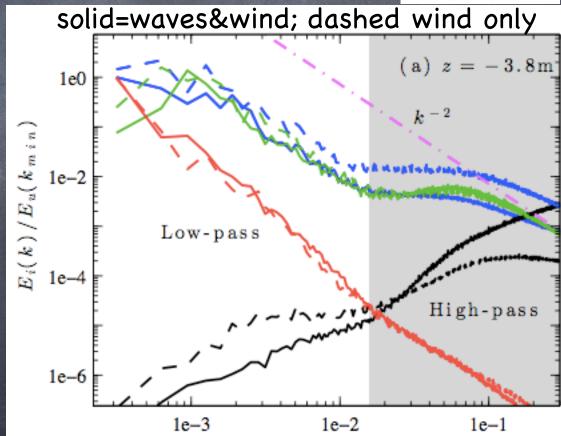


- Submesoscale flow is affected by wave-balance and enhanced <u'w'> (weaker surf. w/ Stokes)
- Strong two-way turbulent interactions are rare for this configuration
- Two turbulent cascades.
- Presence of waves greatly changes small scale from symmetric instability to gravitational
- P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, G. P. Chini. Langmuir-Submesoscale Interactions: Descriptive Analysis of Multiscale Frontal Spin-down Simulations, *JGR-Oceans*, 2013. In prep.

# Overall results from multiscale

LES
See Hamlington
poster for more!!

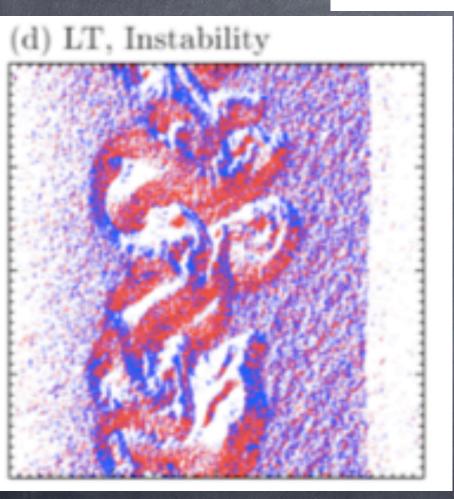




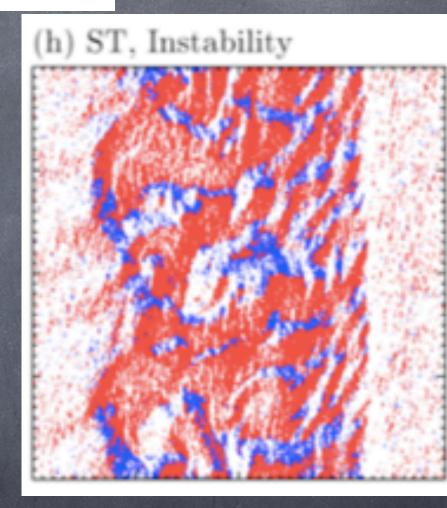
With Stokes Drift



Without Stokes Drift



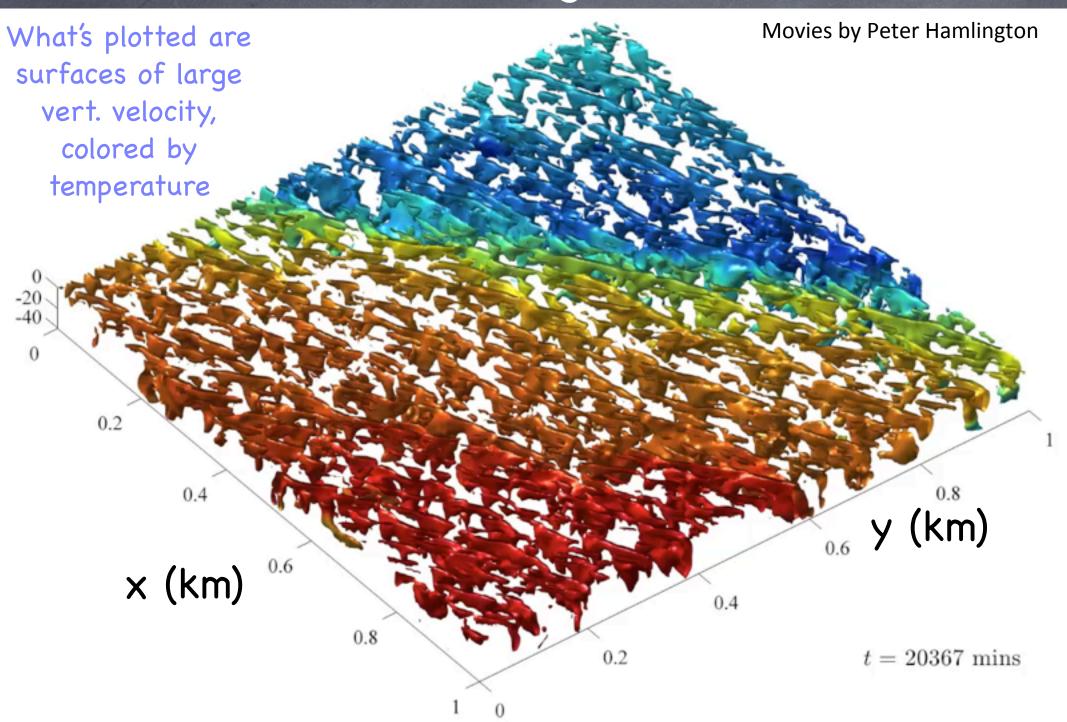
Mostly Baroclinic & Symmetric & Gravitational Instability



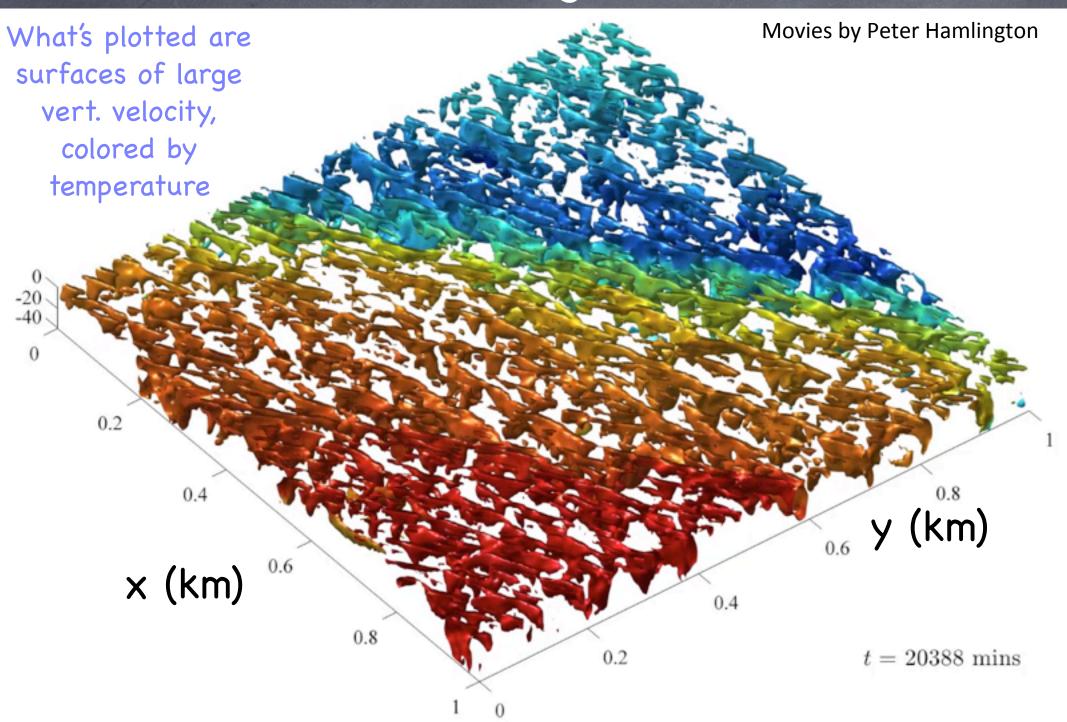
Mostly Baroclinic & Symmetric Instability

P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale simulations. J. Phys Oceanogr. submitted, 2013.

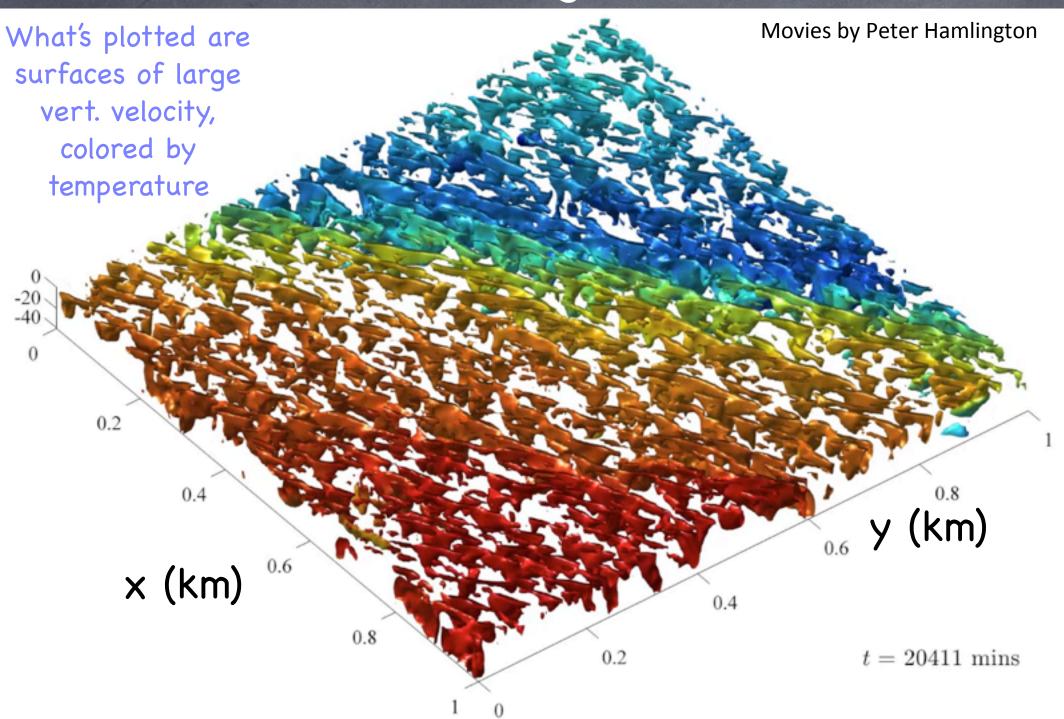
#### Zoom: Submeso-Langmuir Interaction!



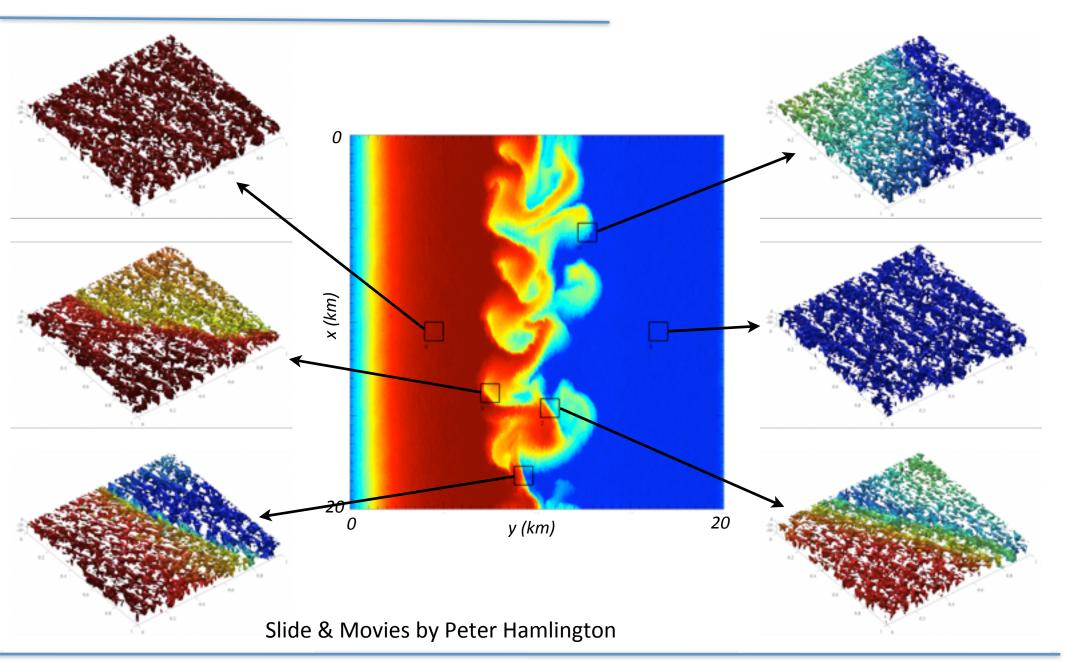
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#### Zoom: Submeso-Langmuir Interaction!



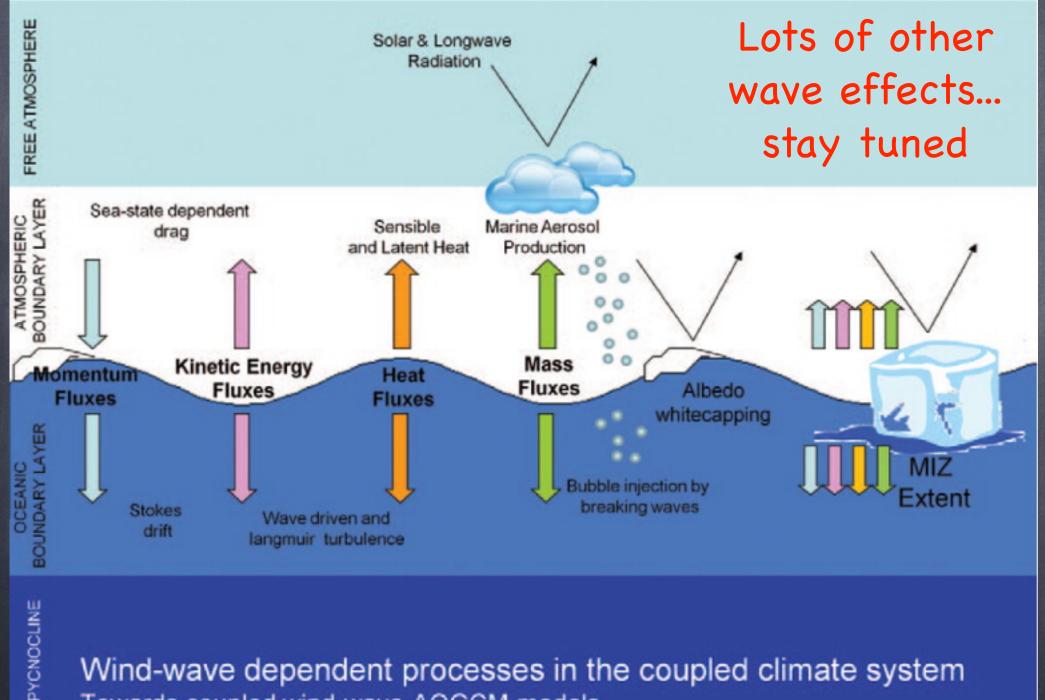
#### Diverse types of interaction



P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale simulations. J. Phys. Oceangr., submitted, 2013.

#### Conclusions

- Waves are a dominant feature of the upper ocean on short timescales
- On longer timescales, rectified effects of waves the Stokes drift—changes boundary layer turbulence and submesoscale dynamics
- Critical concept: Lagrangian shear takes over for Eulerian--except for a different \*vortex force\*
- Wave, convective, & wind effects are particularly important when transient
  - e.g., waves \*not fully developed\* which is most of the time for long fetch (i.e., open ocean)



Wind-wave dependent processes in the coupled climate system Towards coupled wind-wave-AOGCM models

L. Cavaleri, B. Fox-Kemper, and M. Hemer. Wind waves in the coupled climate system. Bulletin of the American Meteorological Society, 93(11):1651-1661, 2012.

# So, no problems? Just crunch away with CLB?

- Let's revisit our assumptions for scale separation:
  - CLB wave equations require limited \*wave steepness\* and irrotational flow
  - Real wind-waves are not monochromatic, but incorporate a spectrum of waves, and...



Power Spectrum of wave height

$$\langle \eta^2 \rangle = \int_0^\infty E(k)dk = C_0 + \int_{k_h}^\infty C_1 k^{-2} dk$$

Power Spectrum of wave steepness: INFINITE!

$$\langle k^2 \eta^2 \rangle = \int_0^\infty k^2 E(k) dk = D_0 + \int_{k_h}^\infty D_1 dk$$

Steep waves break->vortex motion & small scale turbulence!

# So, no problems? Just crunch away with CLB?

- Let's revisit our assumptions for scale separation:
  - Also, what about finite wave packets?
  - What about co-evolution of the submesoscale flow and wave packets?
  - What about steep wave effects? Breaking?

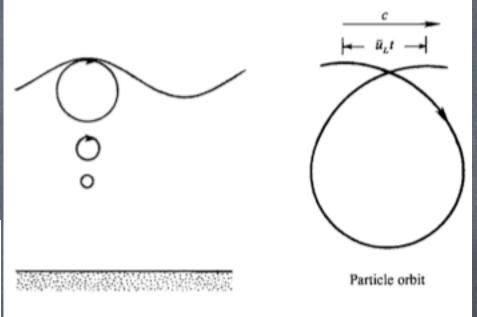
Are there other ways for waves to drive turbulence?

Steep waves break->vortex motion & small scale turbulence!

#### What is Stokes Drift?

Take wave solns, compare the velocity of trajectories vs. Eulerian velocity, Taylor Expand, calculate:

$$egin{aligned} oldsymbol{u}^L(oldsymbol{x}_p(t_0),t) & - oldsymbol{u}^E(oldsymbol{x}_p(t_0),t) & \sim \left[oldsymbol{x}_p(t_0), x'(t_0)\right] \cdot 
abla oldsymbol{u}^E(oldsymbol{x}_p(t_0),t) \\ & \sim \left[\int_{t_0}^t oldsymbol{u}^E(oldsymbol{x}_p(t_0),s')ds'\right] \cdot 
abla oldsymbol{u}^E(oldsymbol{x}_p(t_0),t) \,. \end{aligned}$$



#### Examples:

Monochromatic: 
$$u^{S} = \hat{e}^{w} \frac{8\pi^{3}a^{2}f_{p}^{3}}{g} e^{\frac{8\pi^{2}f_{p}^{2}}{g}z} = \hat{e}^{w}a^{2}\sqrt{gk^{3}}e^{2kz}$$
.

Spectrum: 
$$\mathbf{u}^{S} = \frac{16\pi^{3}}{g} \int_{0}^{\infty} \int_{-\pi}^{\pi} (\cos \theta, \sin \theta, \mathbf{0}) f^{3} \mathcal{S}_{f\theta}(f, \theta) e^{\frac{8\pi^{2}f^{2}}{g}z} d\theta df.$$

## Craik-Leibovich Boussinesq

Old Boussinesq (written in vortex force form)

$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times \mathbf{v} = -\nabla \pi + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$
$$\frac{\partial b}{\partial t} + \mathbf{v} \cdot \nabla b = 0 \qquad \nabla \cdot \mathbf{v} = 0$$

Craik-Leibovich Boussinesq

 $\mathbf{v}_s = \text{Stokes Drift}$ 

$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times (\mathbf{v} + \mathbf{v}_s) = -\nabla \pi^{\dagger} + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$

$$\frac{\partial b}{\partial t} + (\mathbf{v} + \mathbf{v}_s) \cdot \nabla b = 0$$

$$\nabla \cdot \mathbf{v} = 0$$

Global Picture: Misalignment enhances degree to which we expect wave-driven turbulence in Boundary layer

Wind-Driven

Wave-Driven

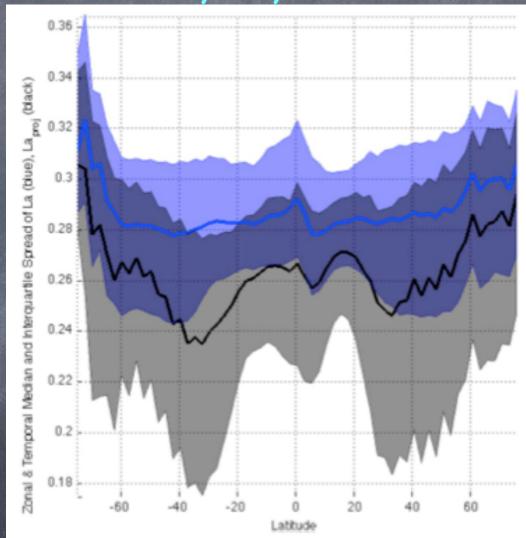


Figure 17. Temporal and zonal median and interquartile range of  $La_t$  and  $La_{proj}$  for a realistic simulation of 1994–2002 using Wave Watch III.

## Surface Waves

Look for fast, small solutions of the Boussinesq Equations:

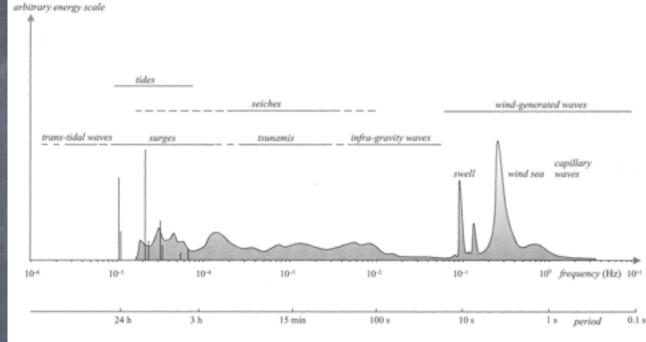


Illustration of wave spectra from different types of ocean surface waves (Holthuijsen, 2007)

#### The irrotational, incompressible flow obeys

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

#### The boundary conditions are:

$$w = \frac{\partial \phi}{\partial z} = 0$$

at 
$$z = -H$$

Pressure Matching (dynamic)

$$p = 0$$

at 
$$z = \eta$$

Velocity Matching (kinematic)

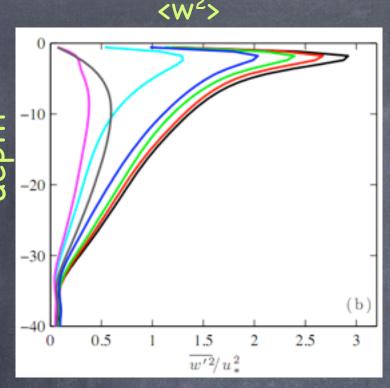
$$\frac{D\eta}{Dt} = w_{\eta}$$

$$z = \eta$$

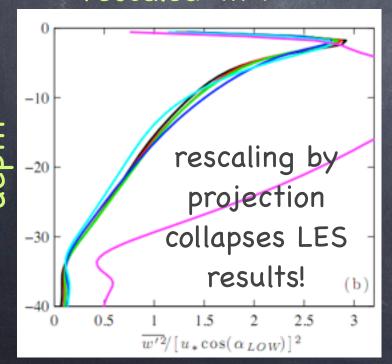
$$u = \frac{\partial q}{\partial x}$$

$$w \equiv \frac{\partial \phi}{\partial z}$$





#### rescaled <w2>

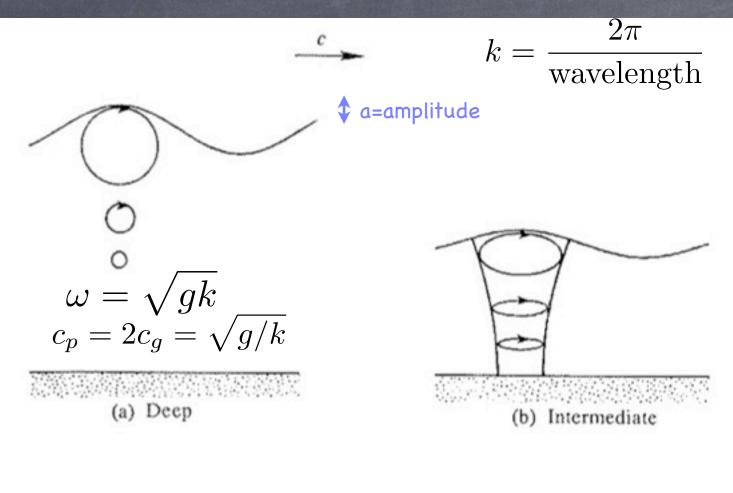


# Generalized Parameters: Predict & Project into Lagrangian Shear Direction

$$\frac{\left\langle \overline{w'^2} \right\rangle_{ML}}{u_*^2} = 0.6 \cos^2 \left( \alpha_{LOW} \right) \left[ 1.0 + (3.1 L a_{proj})^{-2} \right] 
+ (5.4 L a_{proj})^{-4} \right], 
L a_{proj}^2 = \frac{\left| u_* \right| \cos(\alpha_{LOW})}{\left| u_s \right| \cos(\theta_{ww} - \alpha_{LOW})}, 
\alpha_{LOW} \approx \tan^{-1} \left( \frac{\sin(\theta_{ww})}{\frac{u_*}{u_s(0)\kappa} \ln\left(\left| \frac{H_{ML}}{z_1} \right|\right) + \cos(\theta_{ww})} \right)$$

## = parameterization for LC strength!

#### Particle motions

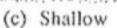


The u, v, decay
exponentially
toward the bottom
with decay scale
proportional to the
wavelength.

Thus, kH is a measure of depth

ka is a measure of steepness

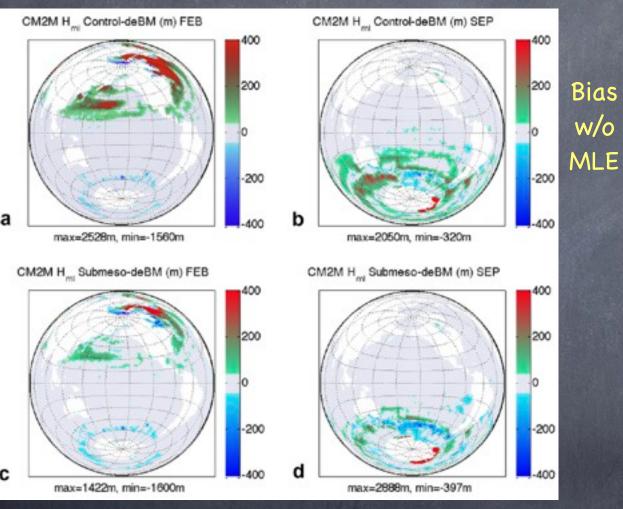
Deep water waves don't "feel" the bottom. Implies nonhydrostatic  $(H \approx L)$  & fast timescale (Ro>>1)



#### Recall our problem with the (submeso) Mixed Layer Eddy Restratification--Southern Ocean too shallow!

Bias

w/o



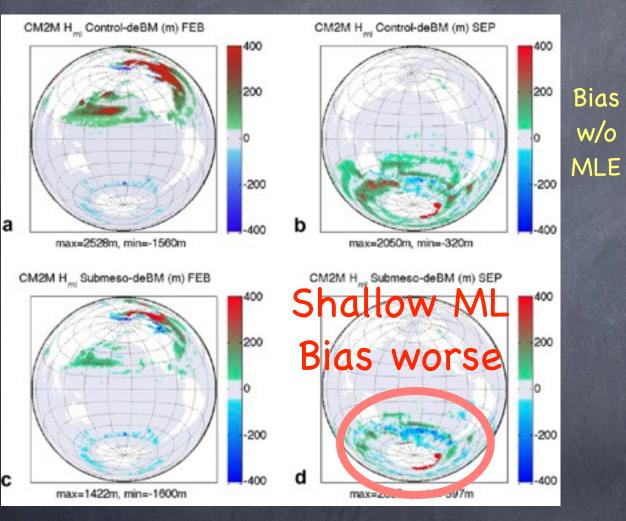
BFK, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. Ocean Modelling, 39:61-78, 2011.

Sallee et al. (2013) have shown that a too shallow S. Ocean MLD is true of most\* present climate models

salinity forcing or ocean physics?

\*true for CMIP5 multi-model ensemble

# Recall our problem with the (submeso) Mixed Layer Eddy Restratification—Southern Ocean too shallow!



Sallee et al. (2013)
have shown that a
too shallow S. Ocean
MLD is true of most\*
present climate
models

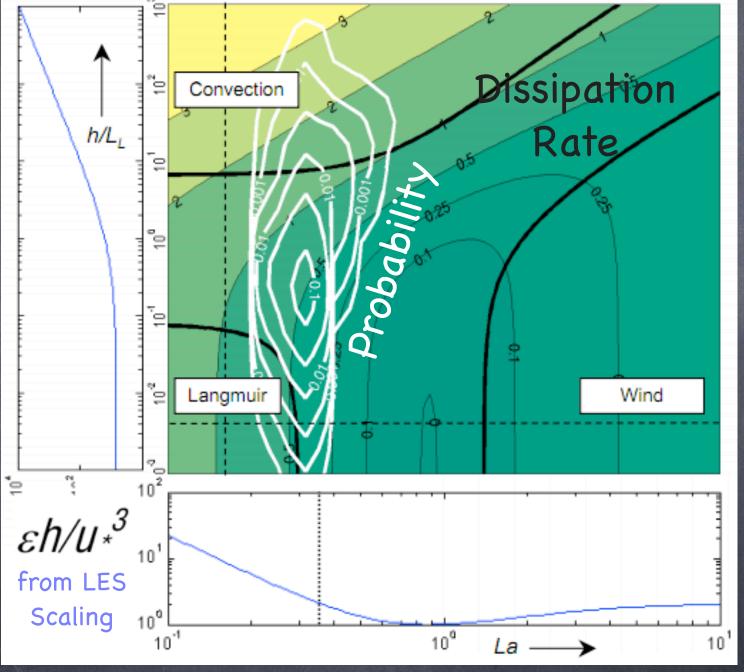
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Data + LES,
Southern Ocean
mixing energy:
Langmuir (Stokesdrift-driven) and
Convective

So, waves
can drive
mixing via
Stokes drift
(combines
with cooling
& winds)

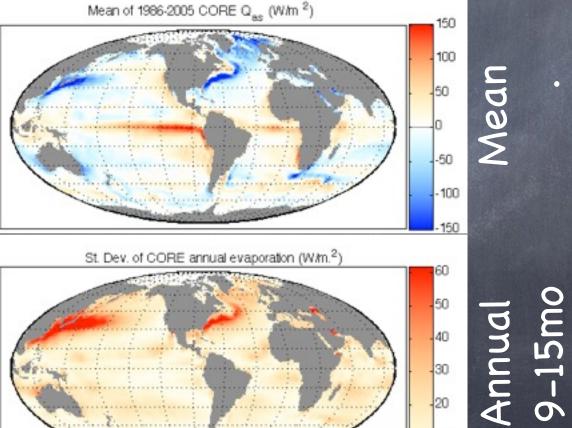


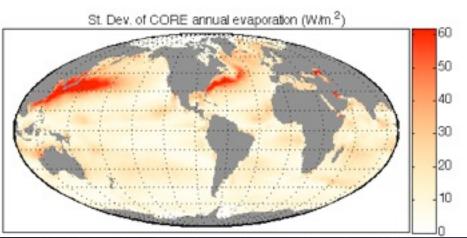
S. E. Belcher, A. A. L. M. Grant, K. E. Hanley, B. Fox-Kemper, L. Van Roekel, P. P. Sullivan, W. G. Large, A. Brown, A. Hines, D. Calvert, A. Rutgersson, H. Petterson, J. Bidlot, P. A. E. M. Janssen, and J. A. Polton. A global perspective on Langmuir turbulence in the ocean surface boundary layer. Geophysical Research Letters, 39(18):L18605, 9pp, 2012.

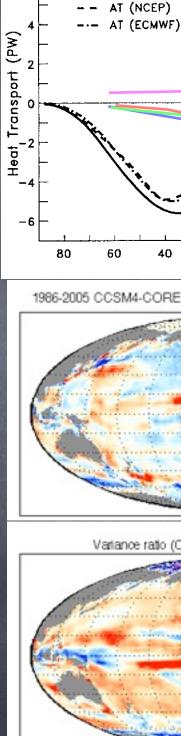
Air-Sea Flux Errors vs. Data

Heat capacity & mode of transport is different in A vs. O

S. C. Bates, BFK, S. R. Jayne, W. G. Large, S. Stevenson, and S. G. Yeager. Mean biases, variability, and trends in air-sea fluxes and SST in the CCSM4. Journal of Climate, 25(22):7781-7801, 2012.

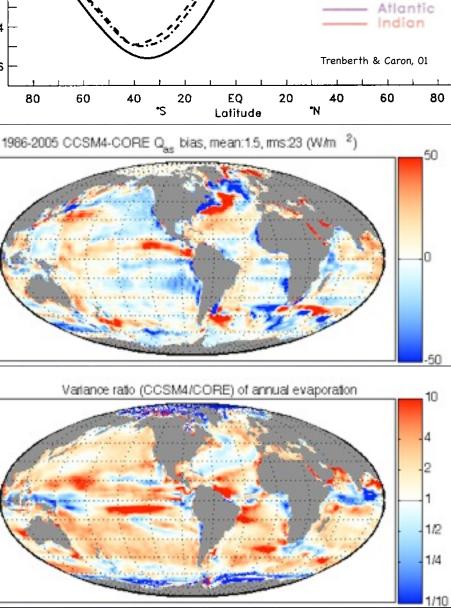






Atmosphere

RT (ERBE)



Ocean (NCEP)