Surface Waves in Turbulent and Laminar Submesoscale Flow

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Joint Scientific Computing/LCDS seminar

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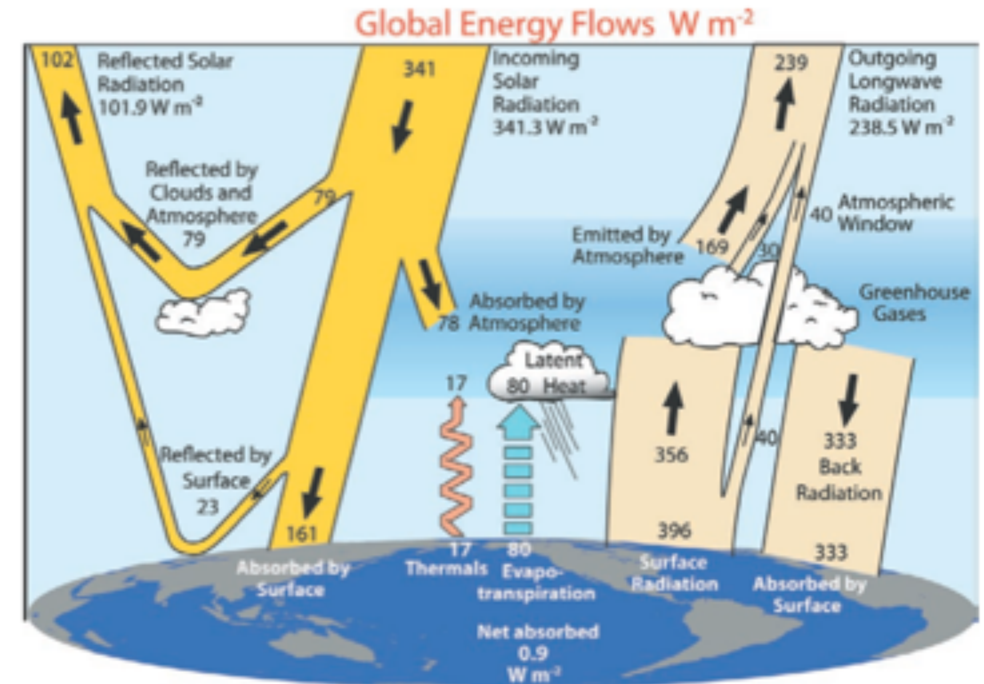
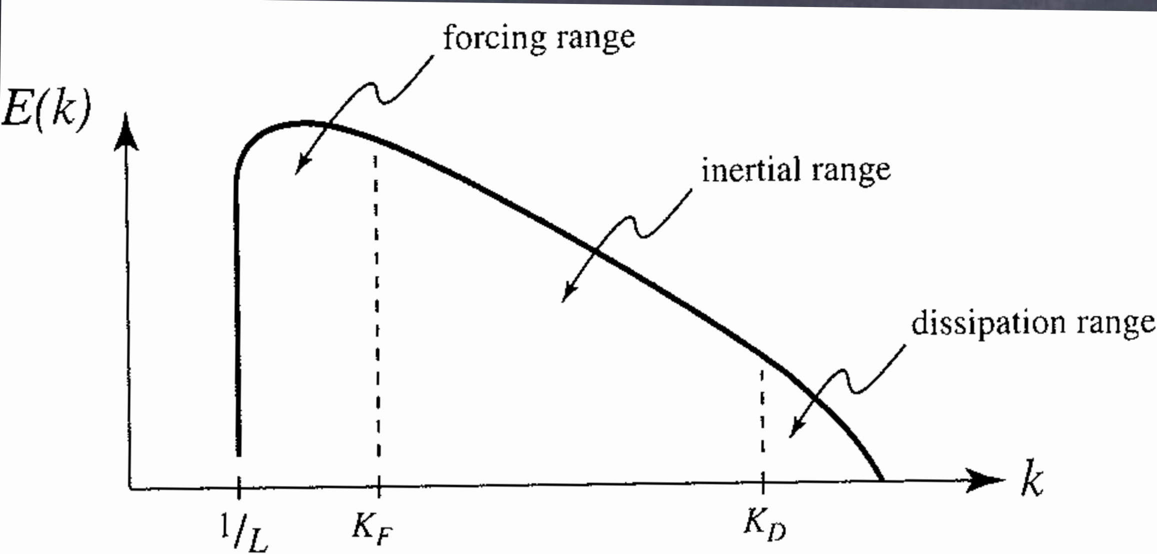
The Earth's Climate System is driven by the Sun's light (minus outgoing infrared) on a global scale



Trenberth & Fasullo, 09

FIG. 1. The global annual mean Earth's energy budget for the Mar 2000 to May 2004 period (W m⁻²). The broad arrows indicate the schematic flow of energy in proportion to their importance.

Dissipation concludes turbulent cascades on scales about a trillion times smaller



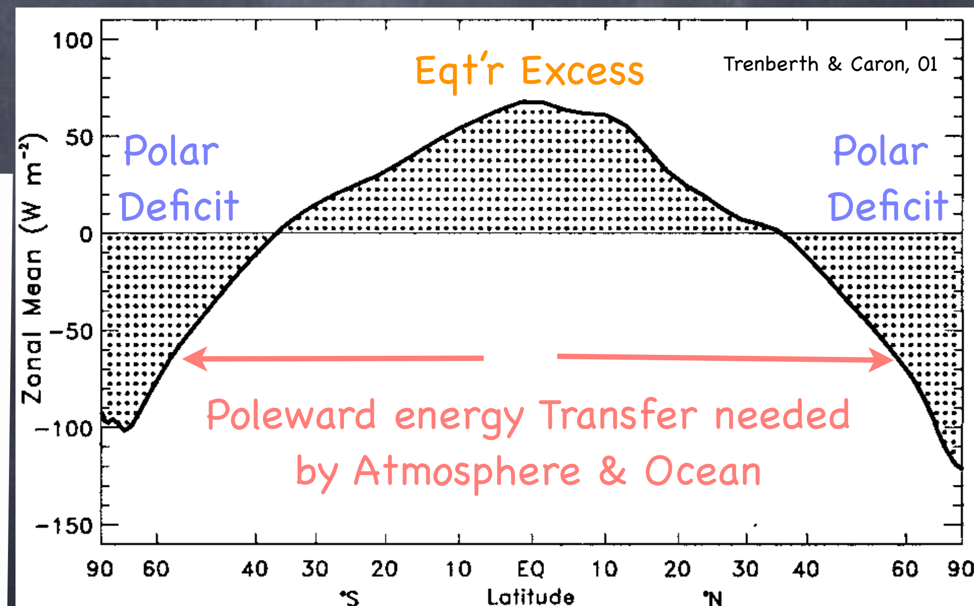
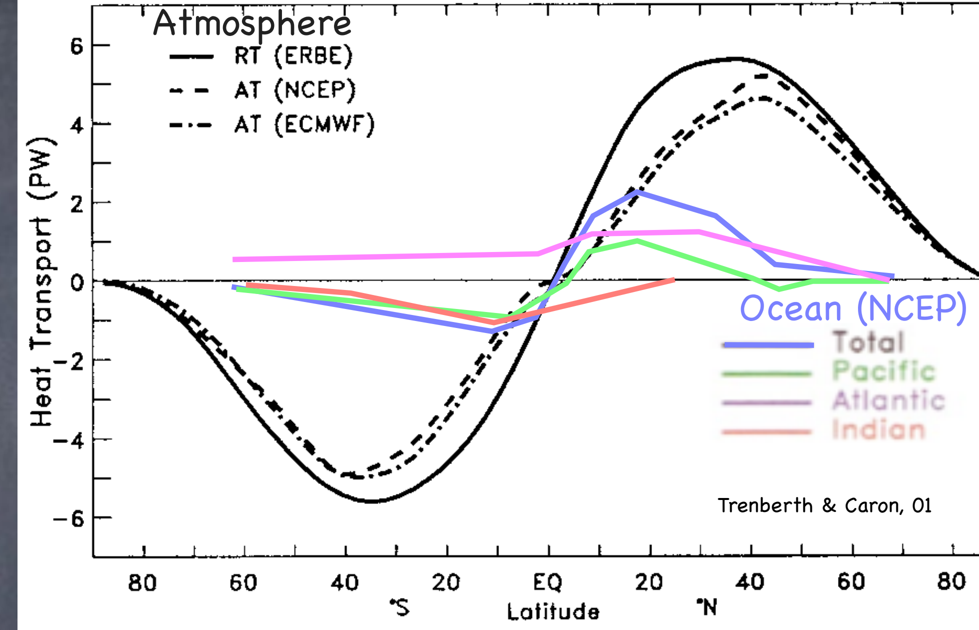


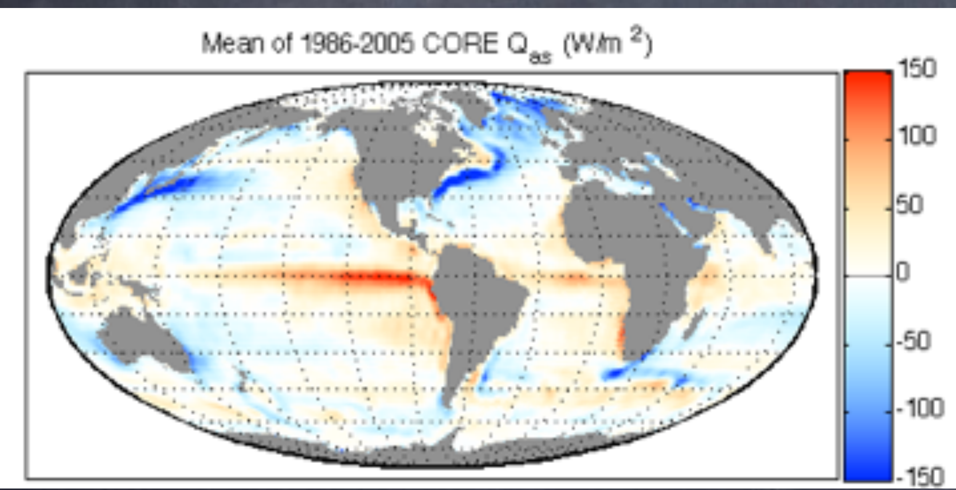
FIG. 1. TOA annualized ERBE zonal mean net radiation (W m⁻²) for Feb 1985–Apr 1989.

Air-Sea Flux Errors vs. Data

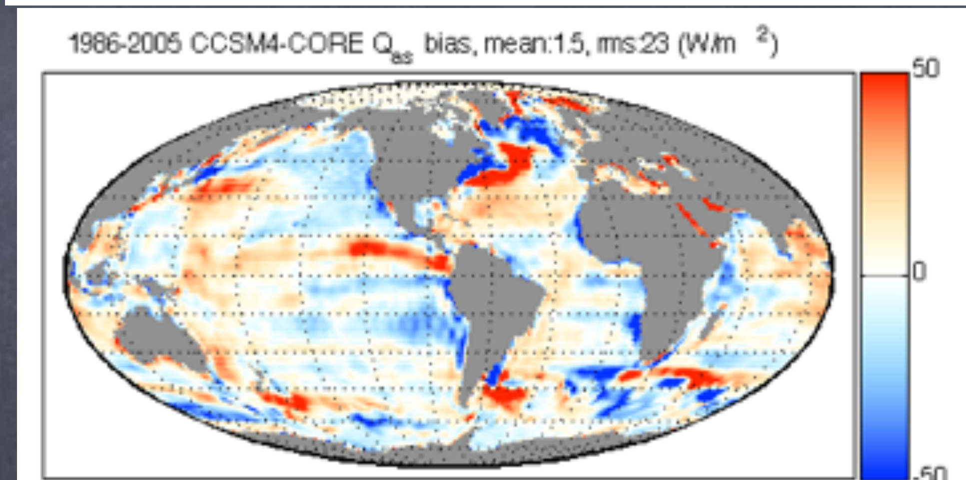
Heat capacity & mode of transport is different in A vs. O

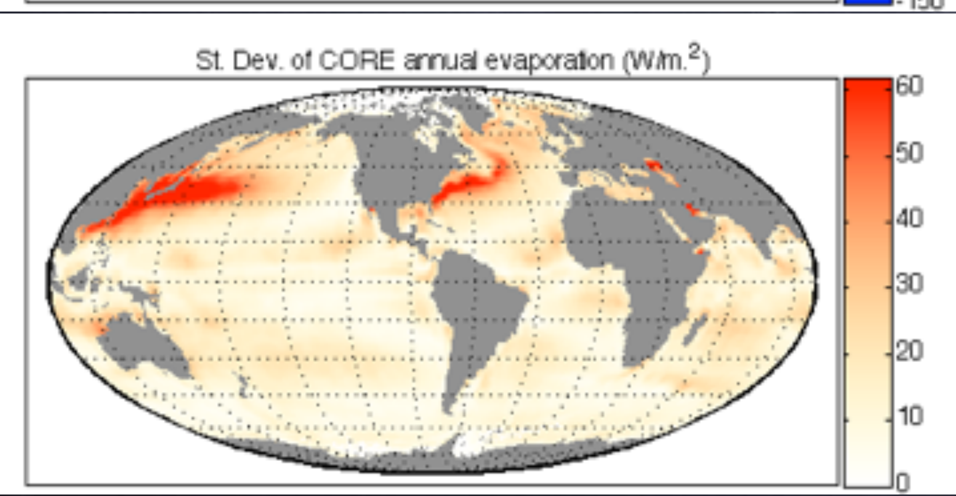
S. C. Bates, B. Fox-Kemper, S. R. Jayne, W. G. Large, S. Stevenson, and S. G. Yeager. Mean biases, variability, and trends in air-sea fluxes and SST in the CCSM4.Journal of Climate, 25(22):7781-7801, 2012.





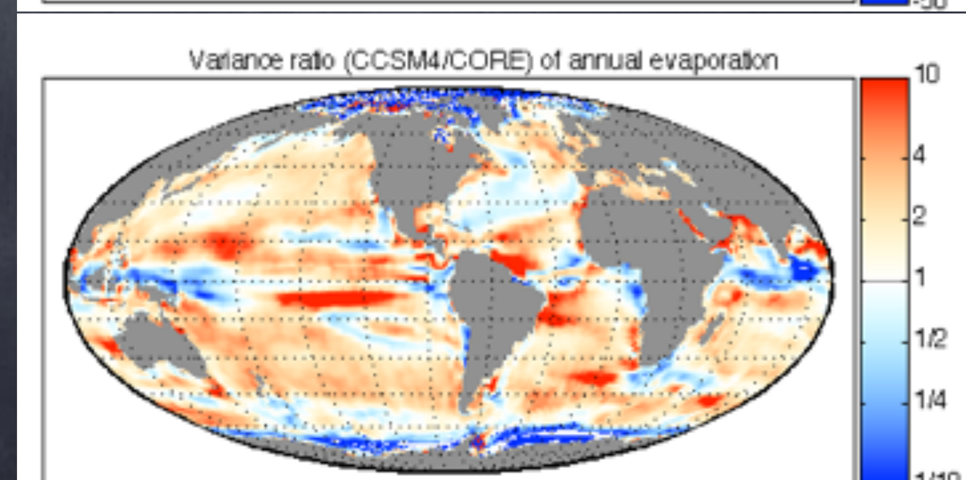
Mean



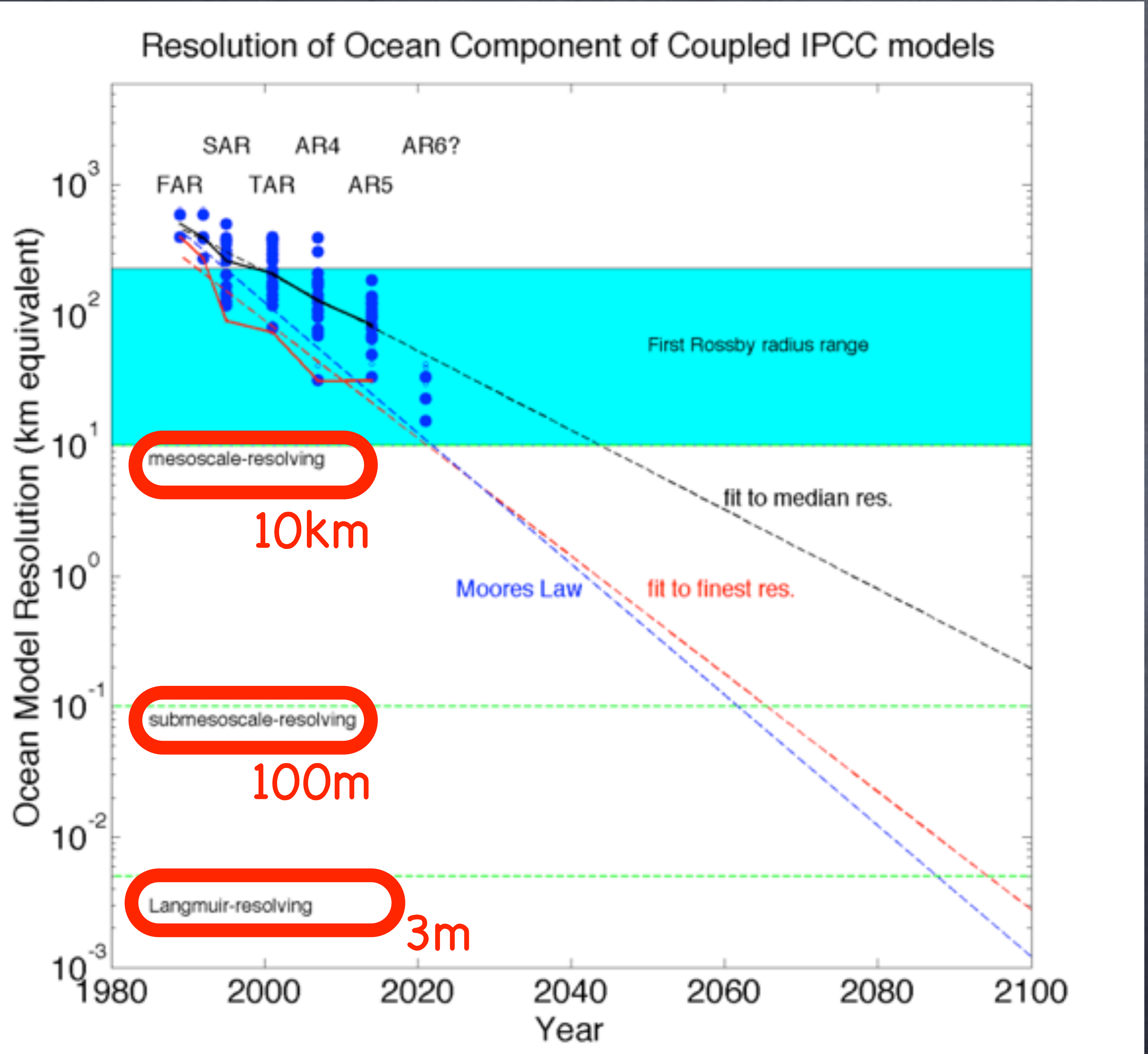


Annual

9-15mo



Resolution will be an issue for centuries to come!



IPCC: Intergovernmental Panel on Climate Change

They won the Nobel (Peace) Prize with Al Gore

Here are the collection of IPCC models...

If we can't resolve a process, we need to develop a parameterization or subgrid model of its effect

What is a parameterization/subgrid model?

Fluid equations for A&O are PDEs (Rotating, Stratified Navier-Stokes), but we cannot resolve to dissipation, so we use statistical or bulk subgrid models to capture multiscale interactions:

- Express the coarse-grain averages of quantities (including the subgrid effects), e.g.:

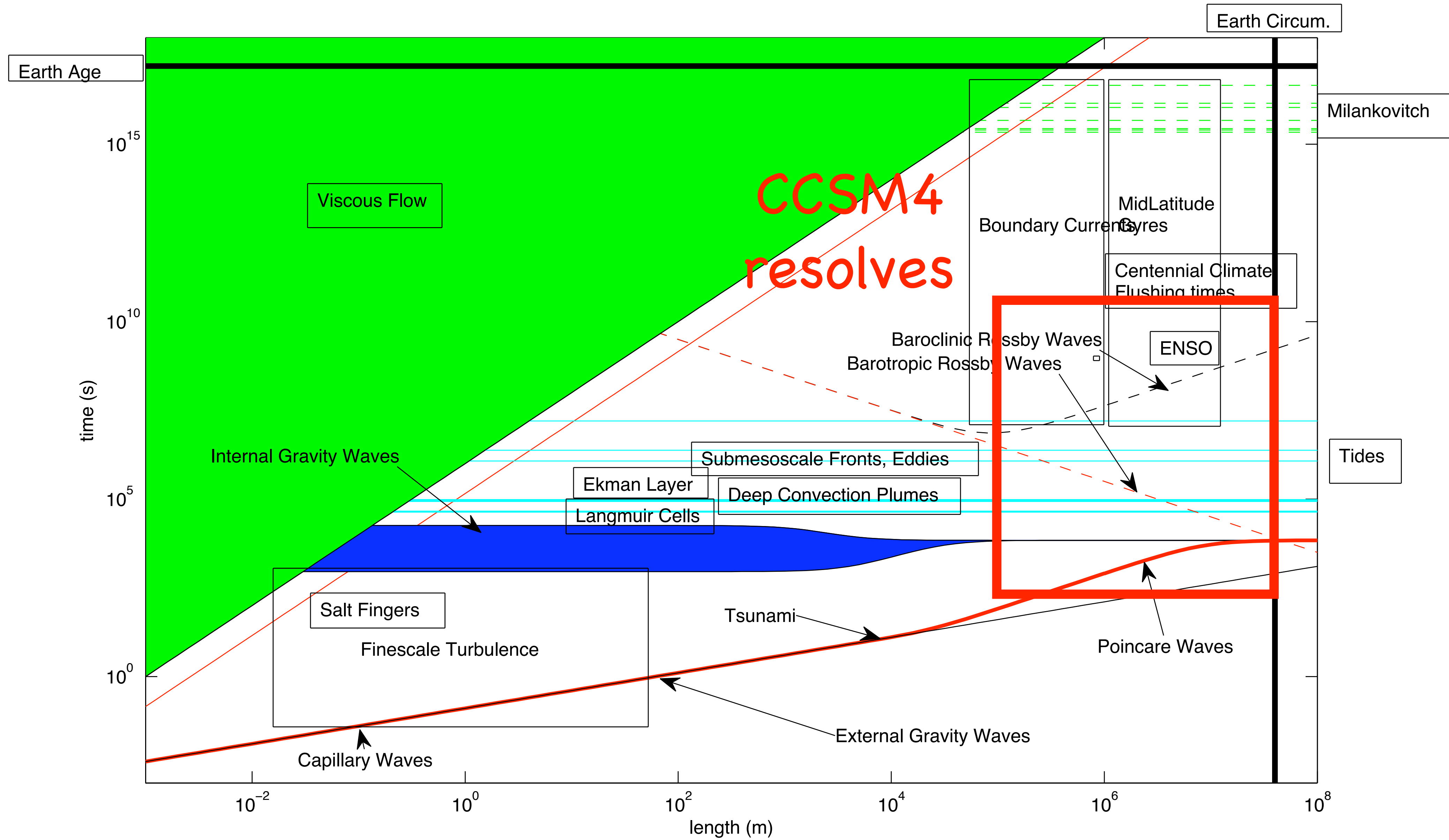
$$\overline{\frac{\partial \tau}{\partial t}} \qquad \overline{\frac{\partial u}{\partial x}} \qquad \overline{\frac{\partial u\tau}{\partial x}}$$

- As a function of the resolved coarse-grain fields

$$\overline{\frac{\partial \tau}{\partial t}} = \frac{\partial \overline{\tau}}{\partial t} \qquad \overline{\frac{\partial u}{\partial x}} = \frac{\partial \overline{u}}{\partial x} \qquad \overline{\frac{\partial u\tau}{\partial x}} = \frac{\partial \overline{u}\ \overline{\tau}}{\partial x} + \frac{\partial \overline{u'\tau'}}{\partial x}$$

- Note that nonlinear terms require special treatment
- These couple different scales, small talks to large

The Ocean is Vast & Diverse:
Q: What processes to parameterize?
Today's A: Unresolved Upper Ocean with Air-Sea Impact



Fundamental Equations of Motion of a Fluid

The following constitutes, in principle, a complete set of equations for an inviscid fluid heated at a rate $\dot{Q}$ and whose composition, S, changes at a rate $\dot{S}$.

$$\frac{D?}{Dt} \equiv \frac{\partial ?}{\partial t} + \mathbf{v} \cdot \nabla ?$$

Evolution equations for velocity, density and composition:

$$\frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} = -\frac{\nabla p}{\rho} + \boldsymbol{F}', \qquad \frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho \nabla \cdot \boldsymbol{v} = 0, \qquad \frac{\mathrm{D}S}{\mathrm{D}t} = \dot{S}. \tag{F.1}$$

Internal energy equation or entropy equation:

$$\frac{\mathrm{D}I}{\mathrm{D}t} - \frac{p}{\rho}\nabla \cdot \boldsymbol{v} = \dot{Q}_T, \qquad \frac{\mathrm{D}\eta}{\mathrm{D}t} = \frac{1}{T}\dot{Q}. \tag{F.2}$$

Text

where $\dot{Q}_T = \dot{Q} + \mu\dot{S}$ is the total rate of energy input.

Fundamental equation of state:

$$I = I(\rho, S, \eta). \tag{F.3}$$

Diagnostic equations for temperature and pressure:

$$T = \left(\frac{\partial I}{\partial \eta}\right)_{\alpha,S}, \qquad p = -\left(\frac{\partial I}{\partial \alpha}\right)_{\eta,S}. \tag{F.4}$$

Vallis, 06

9 Variables 9 Equations. Brutal but complete.

With nearly incompressible (small density variations) approximation & approximated rotating Earth:

A simpler set of 5 vars

Summary of Boussinesq Equations

$$\frac{D?}{Dt} \equiv \frac{\partial ?}{\partial t} + \mathbf{v} \cdot \nabla ?$$

The simple Boussinesq equations are, for an inviscid fluid:

momentum equations: $$\frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} + \boldsymbol{f} \times \boldsymbol{v} = -\nabla\phi + b\mathbf{k}, \qquad \text{(B.1)}$$

mass conservation: $$\nabla \cdot \boldsymbol{v} = 0, \qquad \text{(B.2)}$$

buoyancy equation: $$\frac{\mathrm{D}b}{\mathrm{D}t} = \dot{b}. \qquad \text{(B.3)}$$

Vallis, 06

If you want, it's easy to distinguish buoyancy into contributions from Temperature and from Salinity

Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished limit:

Inviscid (Re>>1), rapidly rotating (Ro<<1), and thin* (L>>H)

Full Momentum

$$\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla\phi + b\mathbf{k} + \nu\nabla^2\mathbf{v}$$

$$Re = \frac{UL}{\nu} \qquad Ro = \frac{U}{fL} \qquad Ri \equiv \frac{\frac{\partial b}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \qquad \alpha = H/L$$

*closely related to strong statification & ocean dimensions

Geostrophy, Hydrostasy, & Thermal Wind

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inhabits a special distinguished limit:
Inviscid (Re>>1), rapidly rotating (Ro<<1), and thin* (L>>H)

(Horizontal) Geostrophic Balance

$$\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla\phi + b\mathbf{k} + \nu\nabla^2\mathbf{v}$$

$$Re = \frac{UL}{\nu} \qquad Ro = \frac{U}{fL} \qquad Ri \equiv \frac{\frac{\partial b}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \qquad \alpha = H/L$$

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Geostrophy, Hydrostasy, & Thermal Wind

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(Vertical) Hydrostatic Balance

$$\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla\phi + b\mathbf{k} + \nu\nabla^2\mathbf{v}$$

$$Re = \frac{UL}{\nu} \qquad Ro = \frac{U}{fL} \qquad Ri \equiv \frac{\frac{\partial b}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \qquad \alpha = H/L$$

*closely related to strong statification & ocean dimensions

Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished limit:

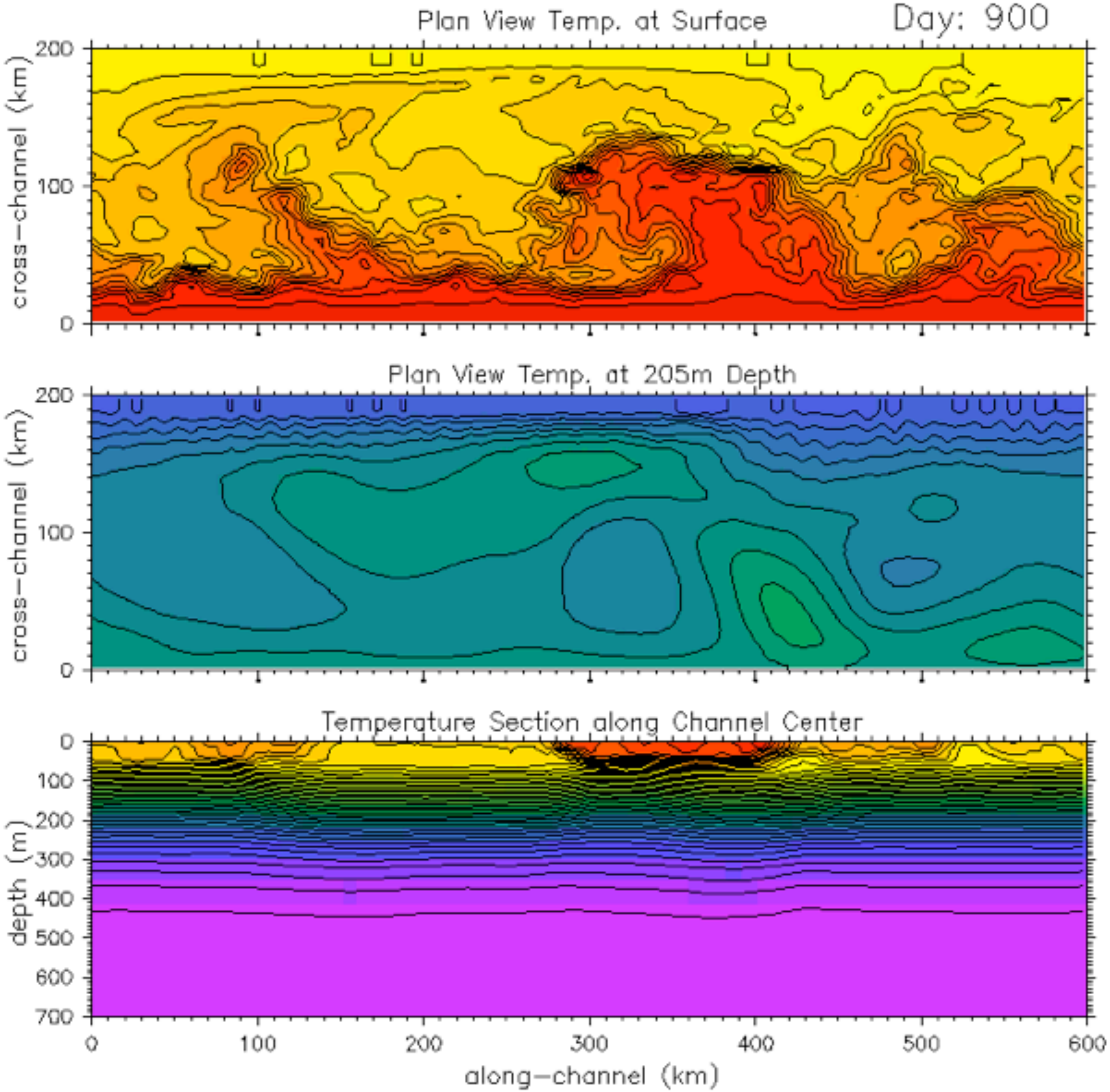
Inviscid (Re>>1), rapidly rotating (Ro<<1), and thin* (L>>H)

(Combined) Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b$$

Taken together with the forcing (air-sea) of buoyancy and the advection of buoyancy by this flow--you have the tools to study large-scale ocean physics!

Let’s see some examples of
Bousinesq, Hydrostatic Models
at work in the
mesoscale (10-100km) &
submesoscale (100m-10km)



Big, Deep (mesoscale)

interact with

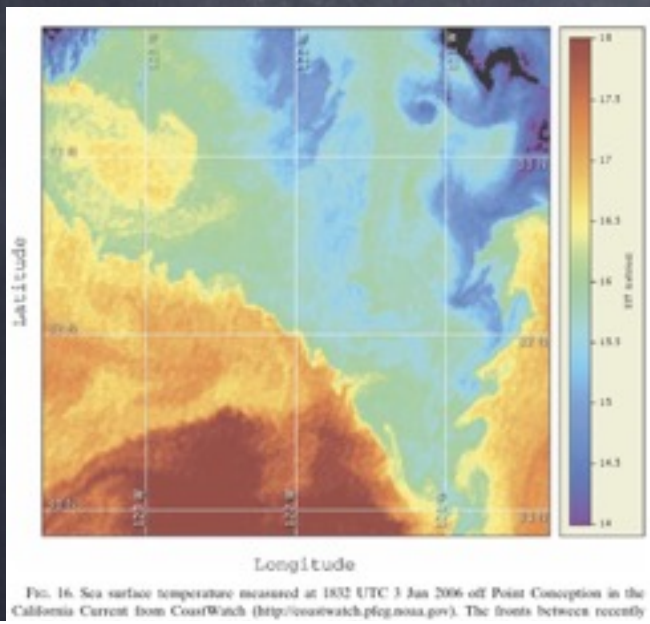
Little, Shallow (submeso)

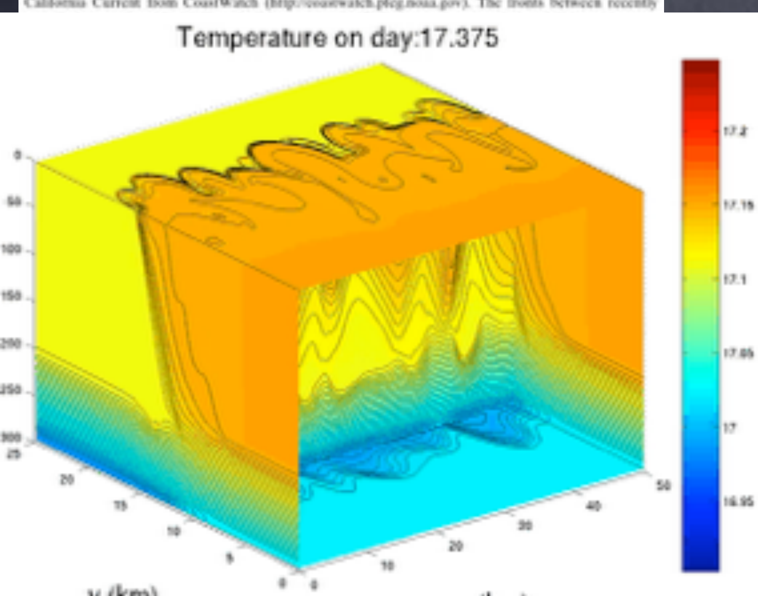
B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. Journal of Physical Oceanography, 38(6):1145-1165, 2008.

The Character of the Submesoscale

← 10 km

(Capet et al., 2008)



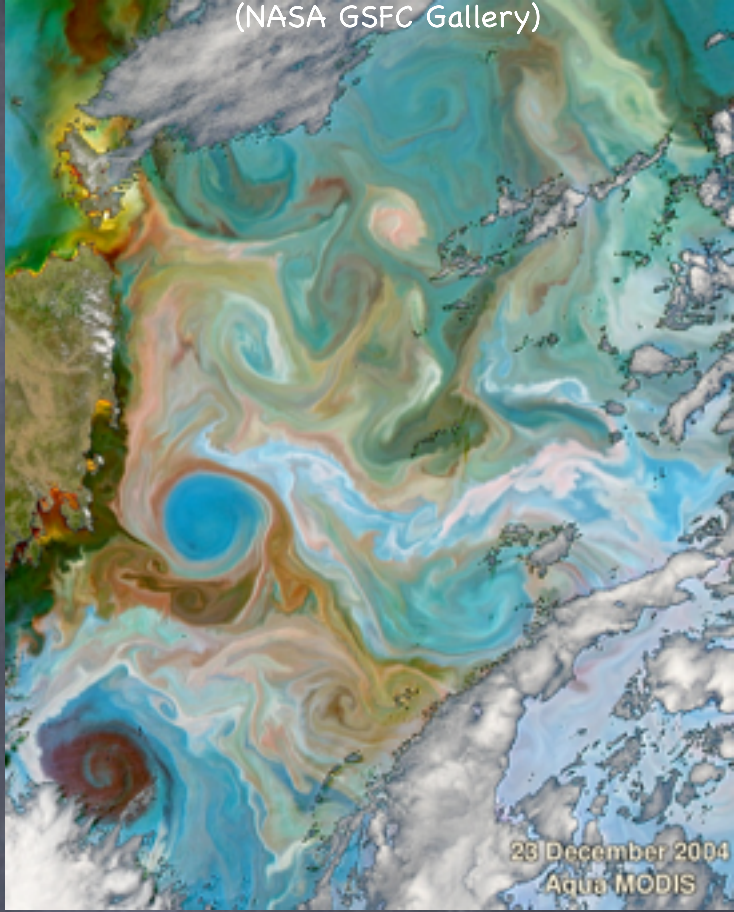


- Fronts
- Eddies
- Ro=O(1)
- Ri=O(1)
- near-surface
- 1-10km, days

Eddy processes often baroclinic instability

Parameterizations of submesoscale baroclinic instability?

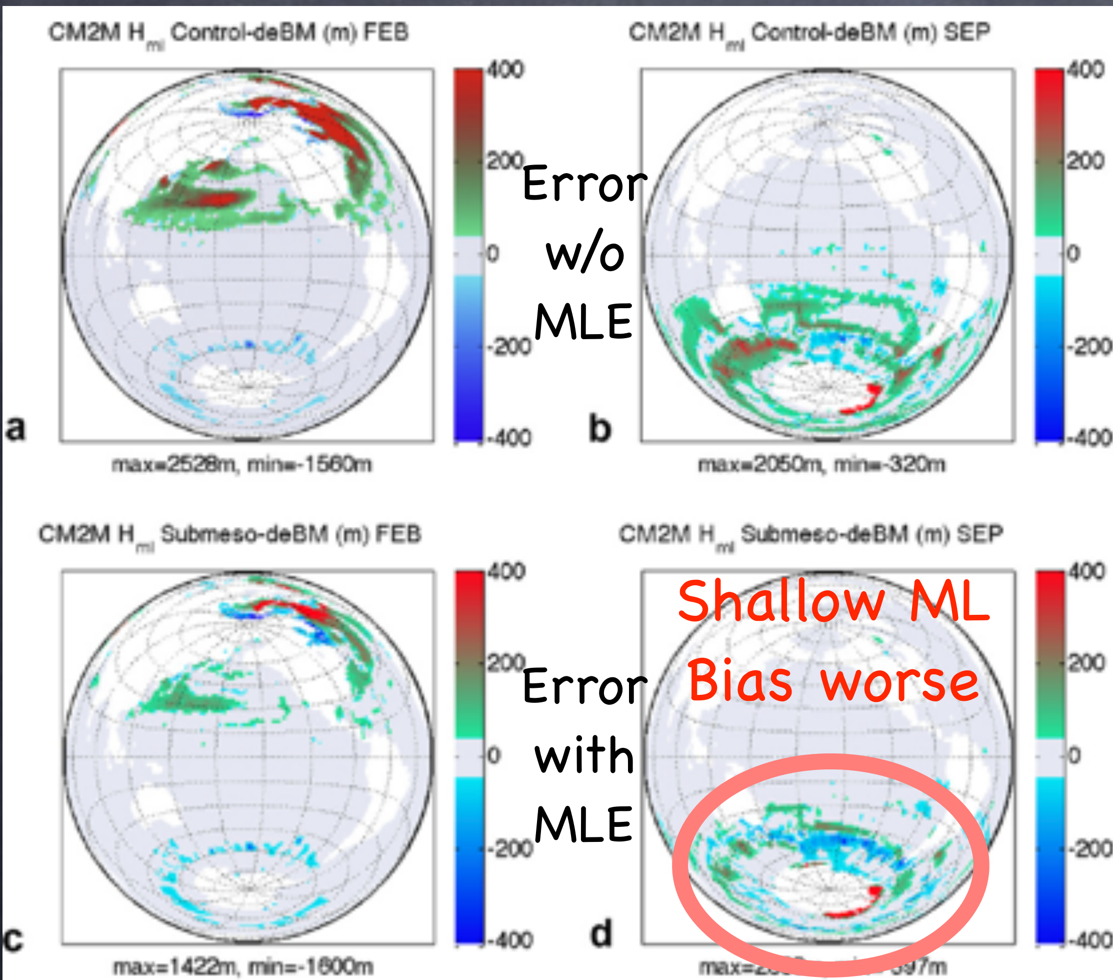
(NASA GSFC Gallery)



B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. Journal of Physical Oceanography, 38(6):1145-1165, 2008

S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013

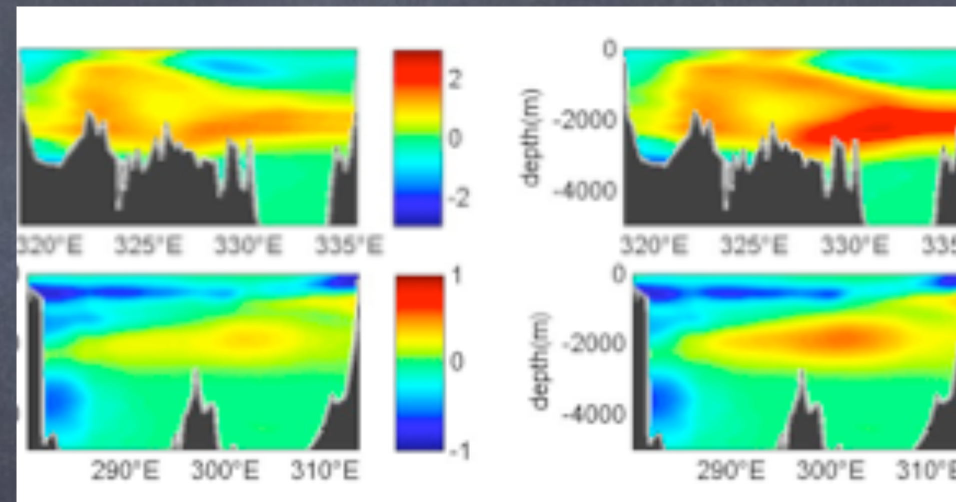
Physical Sensitivity of Ocean Climate to MLE:
(submeso) Mixed Layer Eddy Restratification



Improves CFCs (water masses)

Bias with MLE

Bias w/o MLE



A consistently restratifying,

$$\overline{w'b'} \propto \frac{H^2}{|f|} \left|\nabla_H \bar{b}\right|^2$$

and horizontally downgradient flux.

$$\overline{\mathbf{u}'_H b'} \propto \frac{-H^2 \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla_H \bar{b}$$

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

- So, we’ve seen that we can study a small-scale system (1-10km submeso mixed layer eddies), derive parameterizations, and then use them to improve climate models & assess impact globally
  - This particular process relied heavily on thermal wind scaling relationships
- But, what about the effects of things that aren’t geostrophic & hydrostatic?
  - For example, waves and near-surface 3d turbulence

Surface Wave Primer

Look for fast, small solutions of the Boussinesq Equations:

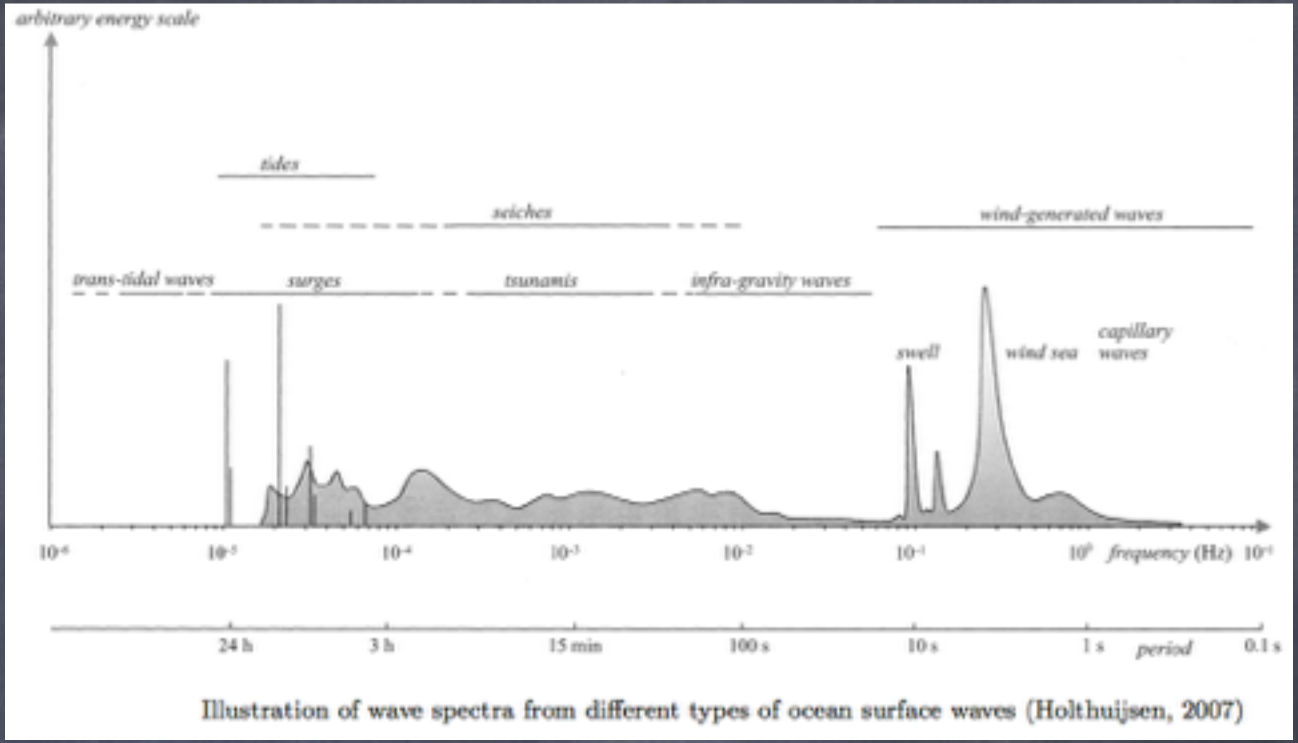


Illustration of wave spectra from different types of ocean surface waves (Holthuijsen, 2007)

The irrotational, incompressible flow obeys

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

$$u = \frac{\partial \phi}{\partial x} \qquad w = \frac{\partial \phi}{\partial z}$$

The boundary conditions are:

Solid Bottom: $w = \frac{\partial \phi}{\partial z} = 0 \quad \text{at} \quad z = -H$

Pressure Matching (dynamic): $p = 0 \quad \text{at} \quad z = \eta$

Velocity Matching (kinematic): $\frac{D\eta}{Dt} = w_\eta \quad \text{at} \quad z = \eta$

Surface Wave Primer

Look for fast, small solutions of the Boussinesq Equations:

Linearized for not steep waves

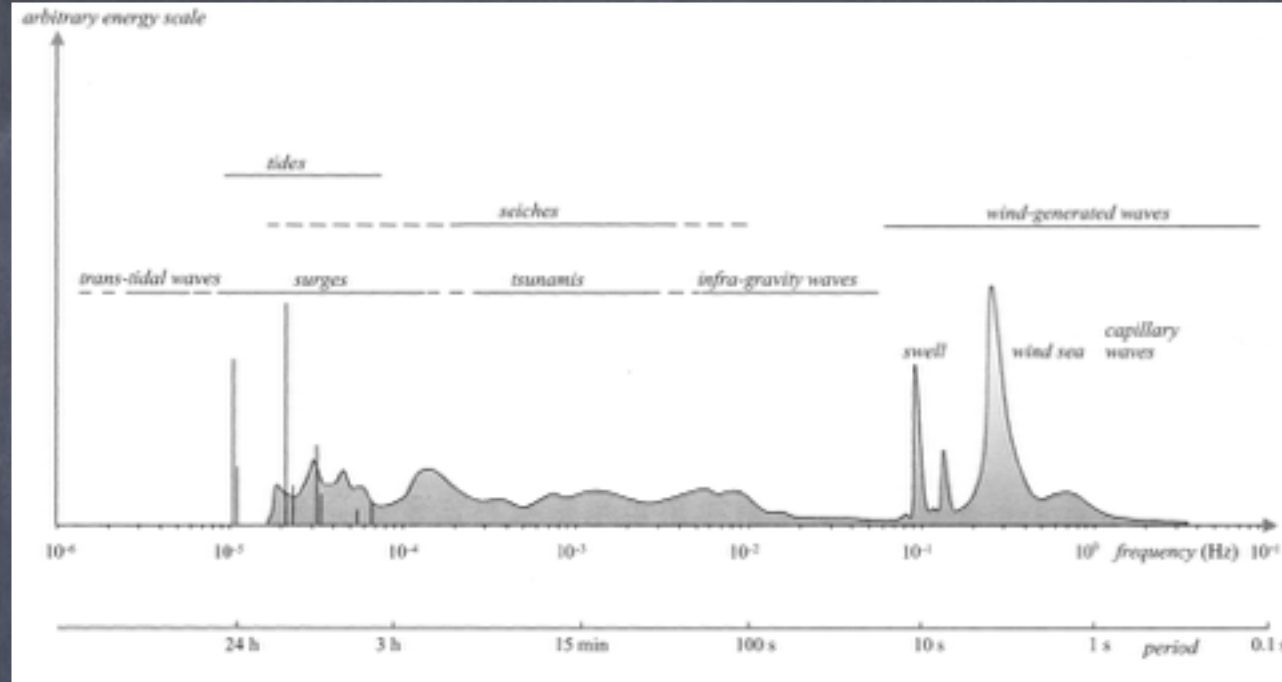


Illustration of wave spectra from different types of ocean surface waves (Holthuijsen, 2007)

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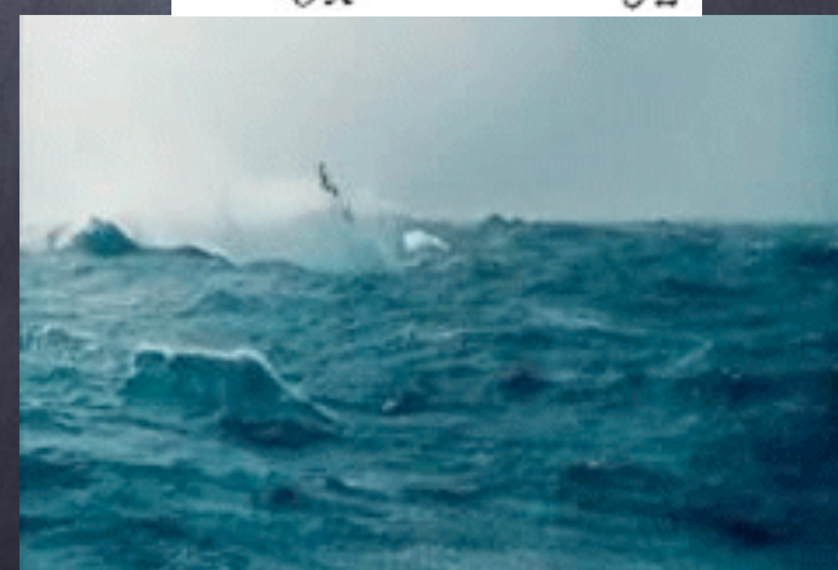
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

$$u = \frac{\partial \phi}{\partial x} \qquad w = \frac{\partial \phi}{\partial z}$$

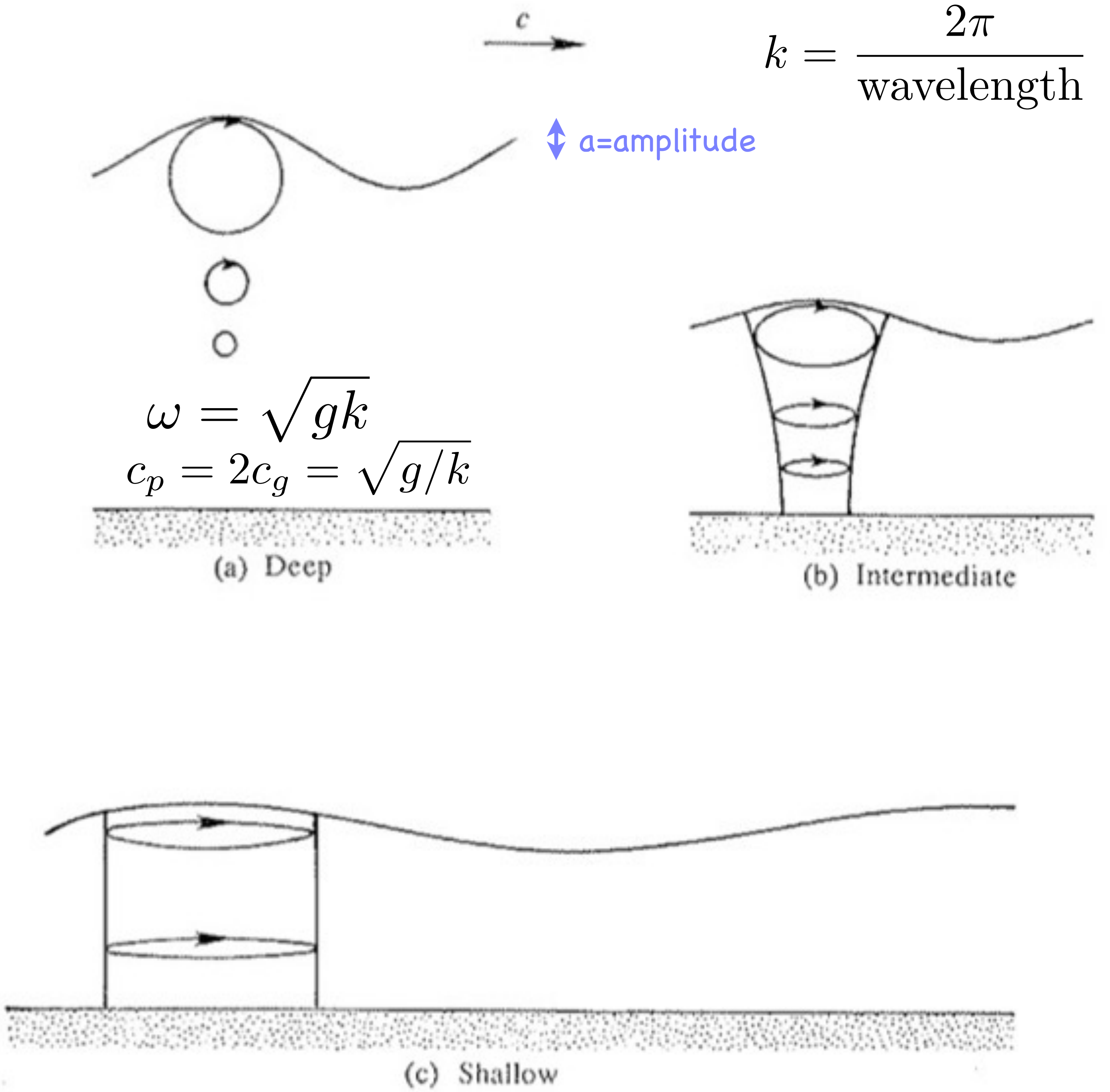
The boundary conditions are (small steepness):

Solid Bottom	$w = \frac{\partial \phi}{\partial z} = 0$ at $z = -H$
Pressure Matching (dynamic)	$\frac{\partial \phi}{\partial t} = -g\eta$ at $z = 0$
Velocity Matching (kinematic)	$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z}$ at $z = 0$



Particle motions

$$k = \frac{2\pi}{\text{wavelength}}$$



$$\omega = \sqrt{gk}$$

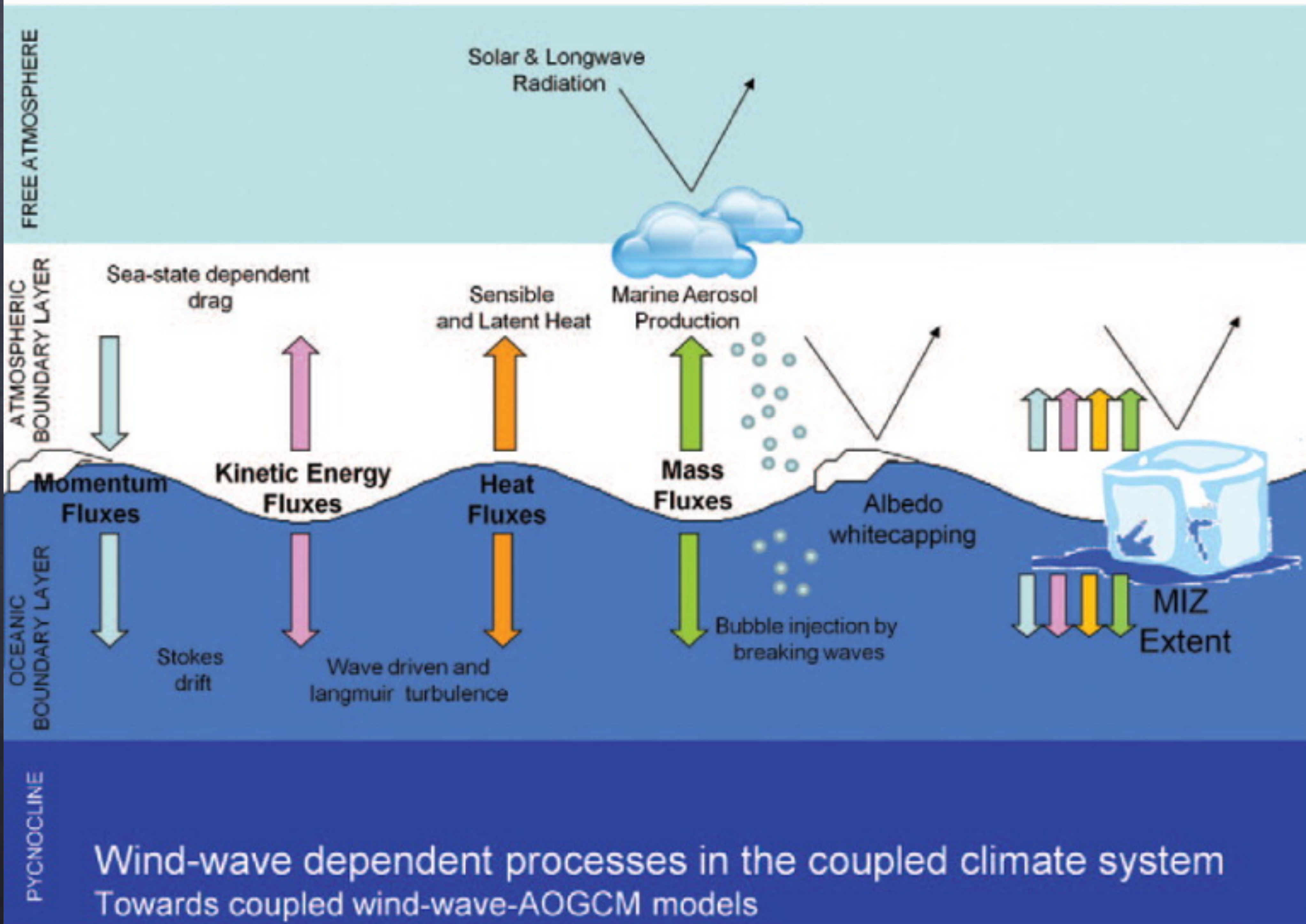
$$c_p = 2c_g = \sqrt{g/k}$$

The u, v, decay exponentially toward the bottom with decay scale proportional to the wavelength.

Thus, kH is a measure of depth

ka is a measure of steepness

Deep water waves don't "feel" the bottom. Implies nonhydrostatic ($H \approx L$) & fast timescale (Ro>>1)



L. Cavaleri, B. Fox-Kemper, and M. Hemer. Wind waves in the coupled climate system. Bulletin of the American Meteorological Society, 93(11):1651-1661, 2012.

The Character of the Langmuir Scale

- Near-surface
- Langmuir Cells & Langmuir Turb.
- Ro>>1
- Ri<1: Nonhydro
- 1-10m
- 10s to mins
- w, u=O(10cm/s)
- Stokes drift
- Eqtns:Craik-Leibovich
- Params: McWilliams & Sullivan, 2000, etc.

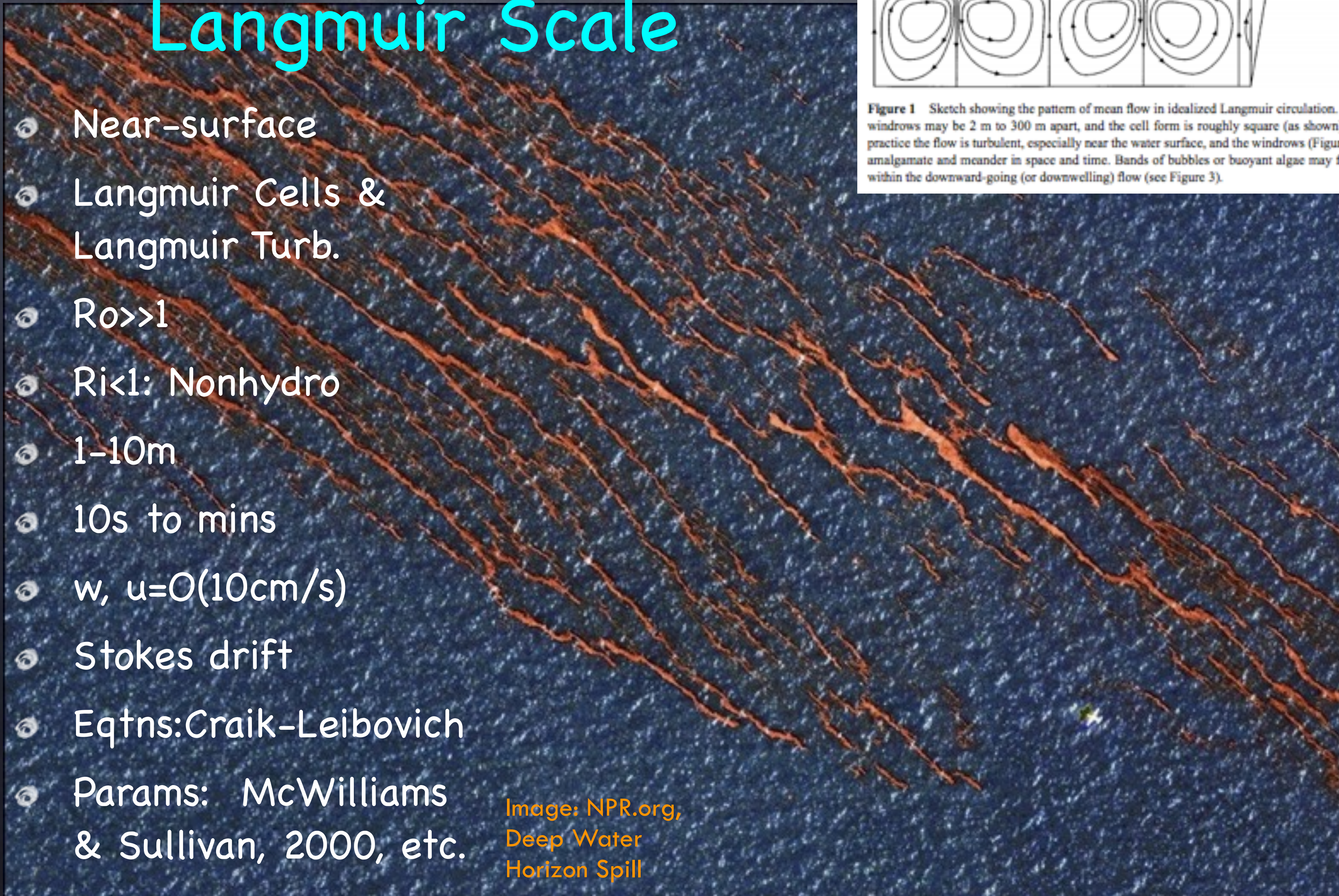


Image: NPR.org, Deep Water Horizon Spill

image: Thorpe, 04

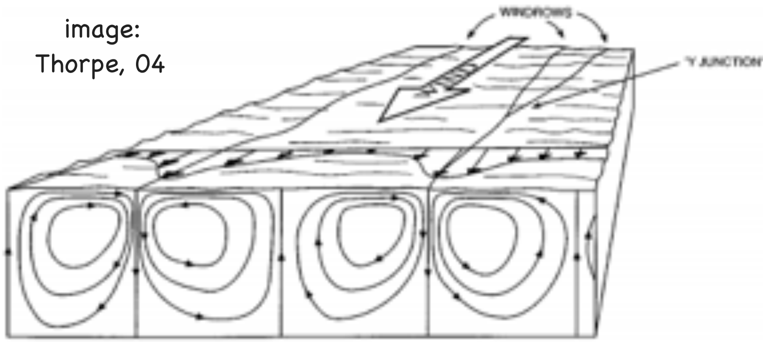


Figure 1 Sketch showing the pattern of mean flow in idealized Langmuir circulation. Th windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). In practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2 amalgamate and meander in space and time. Bands of bubbles or buoyant algae may forn within the downward-going (or downwelling) flow (see Figure 3).

Craik-Leibovich Boussinesq

Old Boussinesq (written in vortex force form)

$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times \mathbf{v} = -\nabla \pi + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$

$$\frac{\partial b}{\partial t} + \mathbf{v} \cdot \nabla b = 0 \qquad \nabla \cdot \mathbf{v} = 0$$

Craik-Leibovich Boussinesq $\quad \mathbf{v}_s = \text{Stokes Drift}$

$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times (\mathbf{v} + \mathbf{v}_s) = -\nabla \pi^\dagger + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$

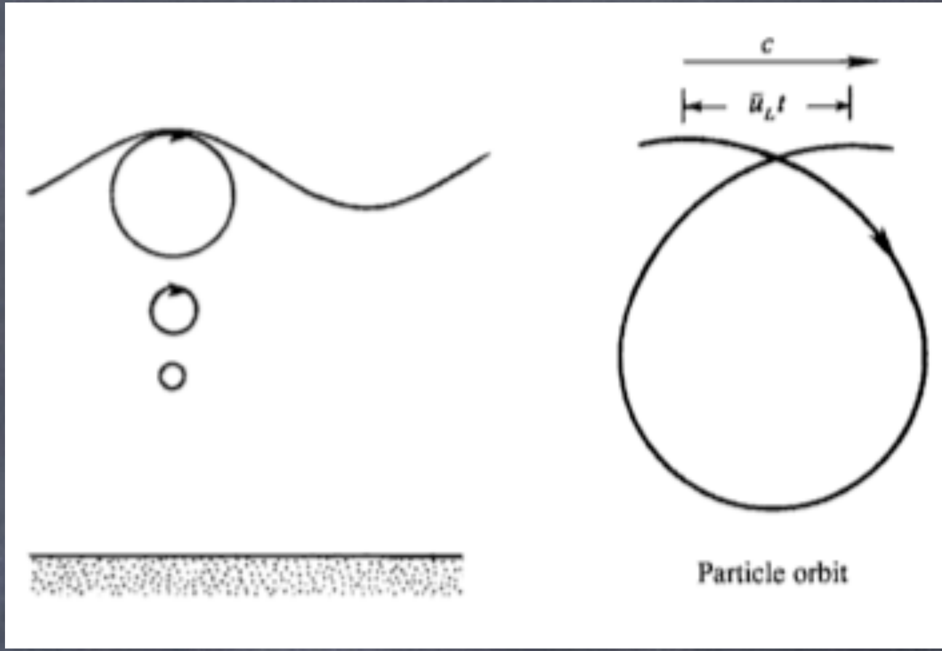
$$\frac{\partial b}{\partial t} + (\mathbf{v} + \mathbf{v}_s) \cdot \nabla b = 0$$

$$\nabla \cdot \mathbf{v} = 0$$

What is Stokes Drift?

Take wave solns, compare the velocity of trajectories vs. Eulerian velocity, Taylor Expand, calculate:

$$\boldsymbol{u}^L(\boldsymbol{x}_p(t_0), t) - \boldsymbol{u}^E(\boldsymbol{x}_p(t_0), t) \approx [\boldsymbol{x}_p(t) - \boldsymbol{x}_p(t_0)] \cdot \nabla \boldsymbol{u}^E(\boldsymbol{x}_p(t_0), t)$$
$$\approx \left[\int_{t_0}^{t} \boldsymbol{u}^E(\boldsymbol{x}_p(t_0), s')ds'\right] \cdot \nabla \boldsymbol{u}^E(\boldsymbol{x}_p(t_0), t).$$

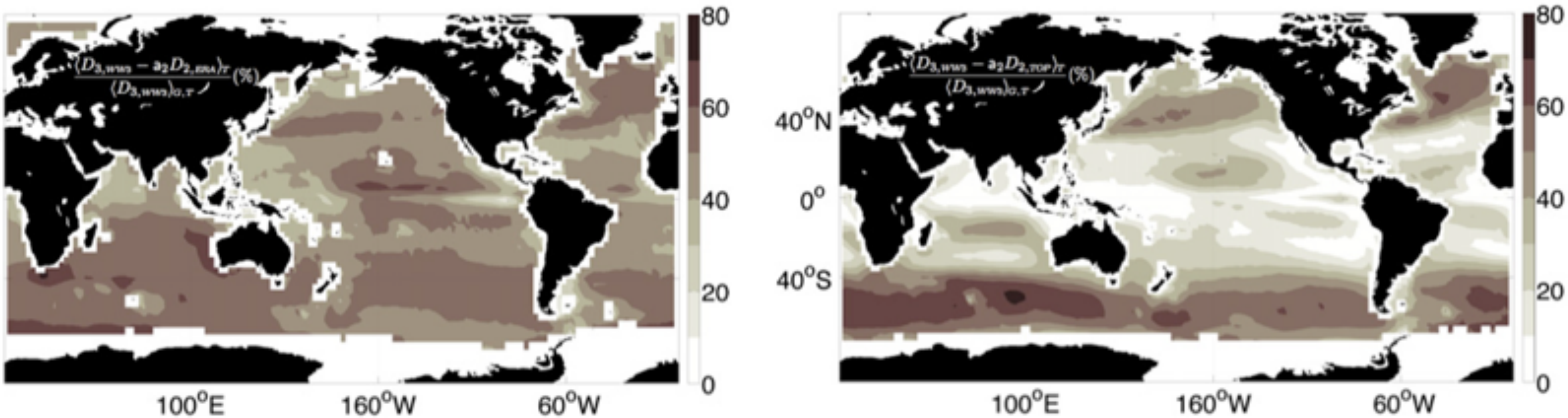


Examples:

Monochromatic: $$\boldsymbol{u}^S = \hat{\mathbf{e}}^{\mathrm{w}} \frac{8\pi^3 a^2 f_p^3}{g} e^{\frac{8\pi^2 f_p^2}{g} z} = \hat{\mathbf{e}}^{\mathrm{w}} a^2 \sqrt{gk^3} e^{2kz}.$$

Spectrum: $$\boldsymbol{u}^S = \frac{16\pi^3}{g} \int_0^{\infty} \int_{-\pi}^{\pi} (\cos\theta, \sin\theta, 0) f^3 S_{f\theta}(f,\theta) e^{\frac{8\pi^2 f^2}{g} z} d\theta df.$$

How well do we know Stokes Drift? <50% discrepancy



(e) $\langle D_{3,WW3} - a_2 D_{2,ERA}\rangle_T / \langle D_{3,WW3}\rangle_{G,T}$ (%)　(f) $\langle D_{3,WW3} - a_2 D_{2,TOP}\rangle_T / \langle D_{3,WW3}\rangle_{G,T}$ (%)

RMS error in measures of surface Stokes drift, 2 wave models (left), model vs. altimeter (right)

Year 2000 data & models

A. Webb and B. Fox-Kemper. Wave spectral moments and Stokes drift estimation. *Ocean Modelling*, 40(3-4):273-288, 2011.

Craik-Leibovich Boussinesq

- Formally a multiscale asymptotic equation set:
  - 3 classes: Small, Fast; Large, Fast; Large, Slow
  - Solve first 2 types of motion in the case of limited slope (ka), irrotational --> Deep Water Waves!
  - Average over deep water waves in space & time,

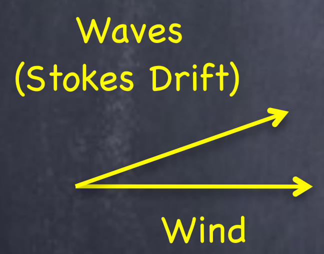
$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times (\mathbf{v} + \mathbf{v}_s) = -\nabla \pi^{\dagger} + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$

$$\frac{\partial b}{\partial t} + (\mathbf{v} + \mathbf{v}_s) \cdot \nabla b = 0 \qquad \nabla \cdot \mathbf{v} = 0$$

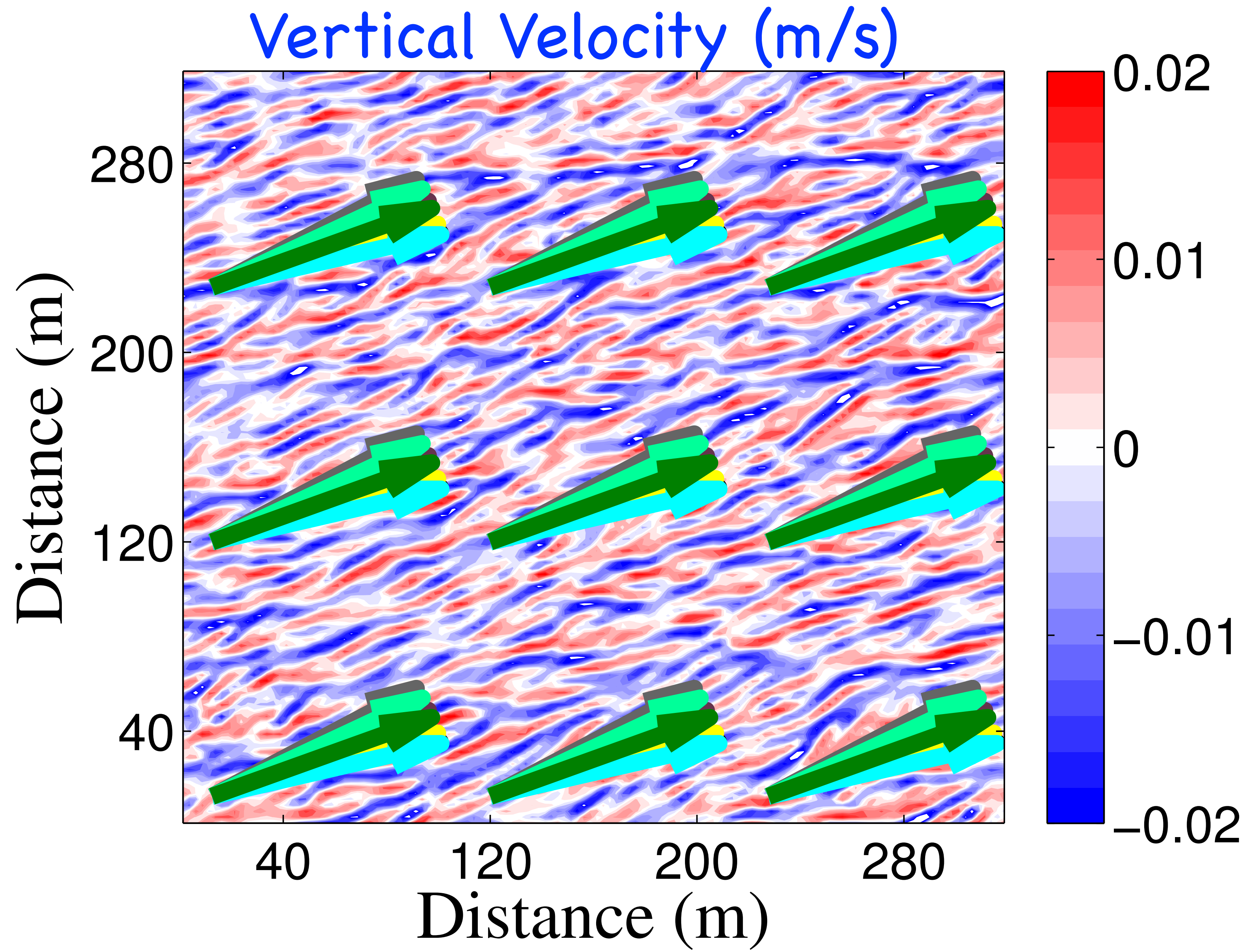
$$\mathbf{v}_s = \text{Stokes Drift}$$

Craik & Leibovich 1976; Gjaja & Holm 1996; McWilliams *et al.* 2004

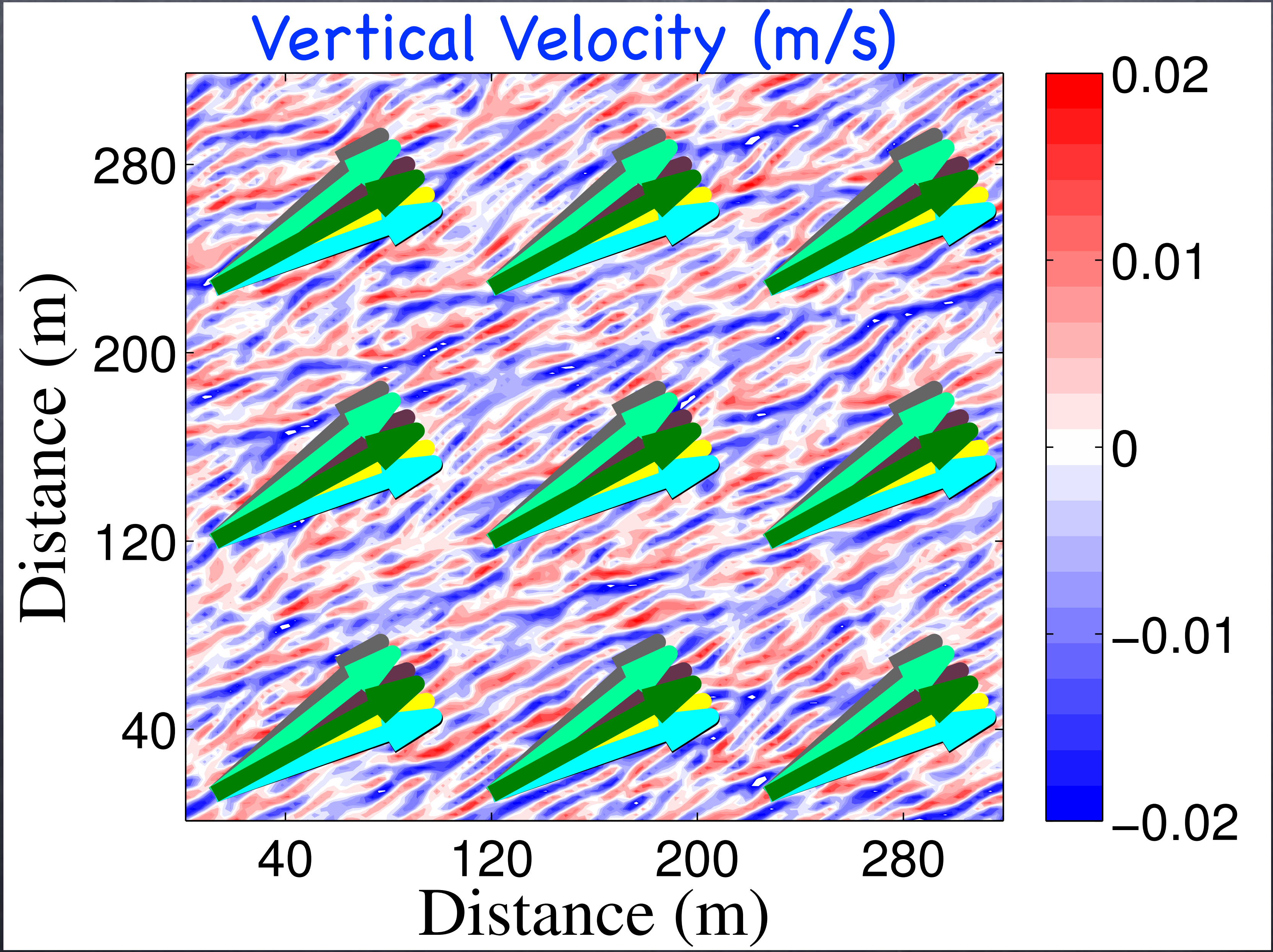
CLB as equations for Large Eddy Simulations: Tricky: Misaligned Wind & Waves



L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. Journal of Geophysical Research-Oceans, 117:C05001, 22pp, May 2012.



Tricky: Misaligned Wind & Waves



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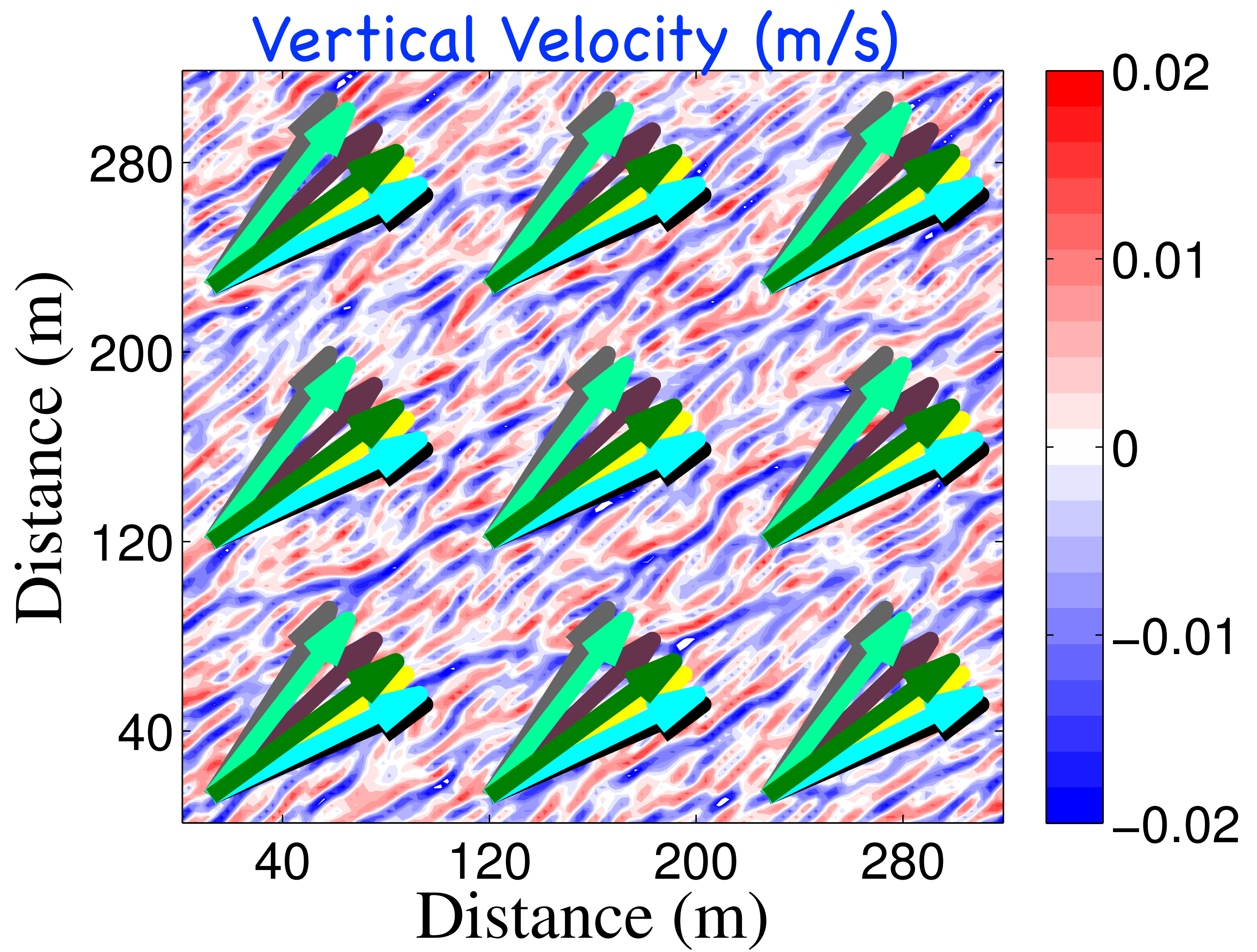

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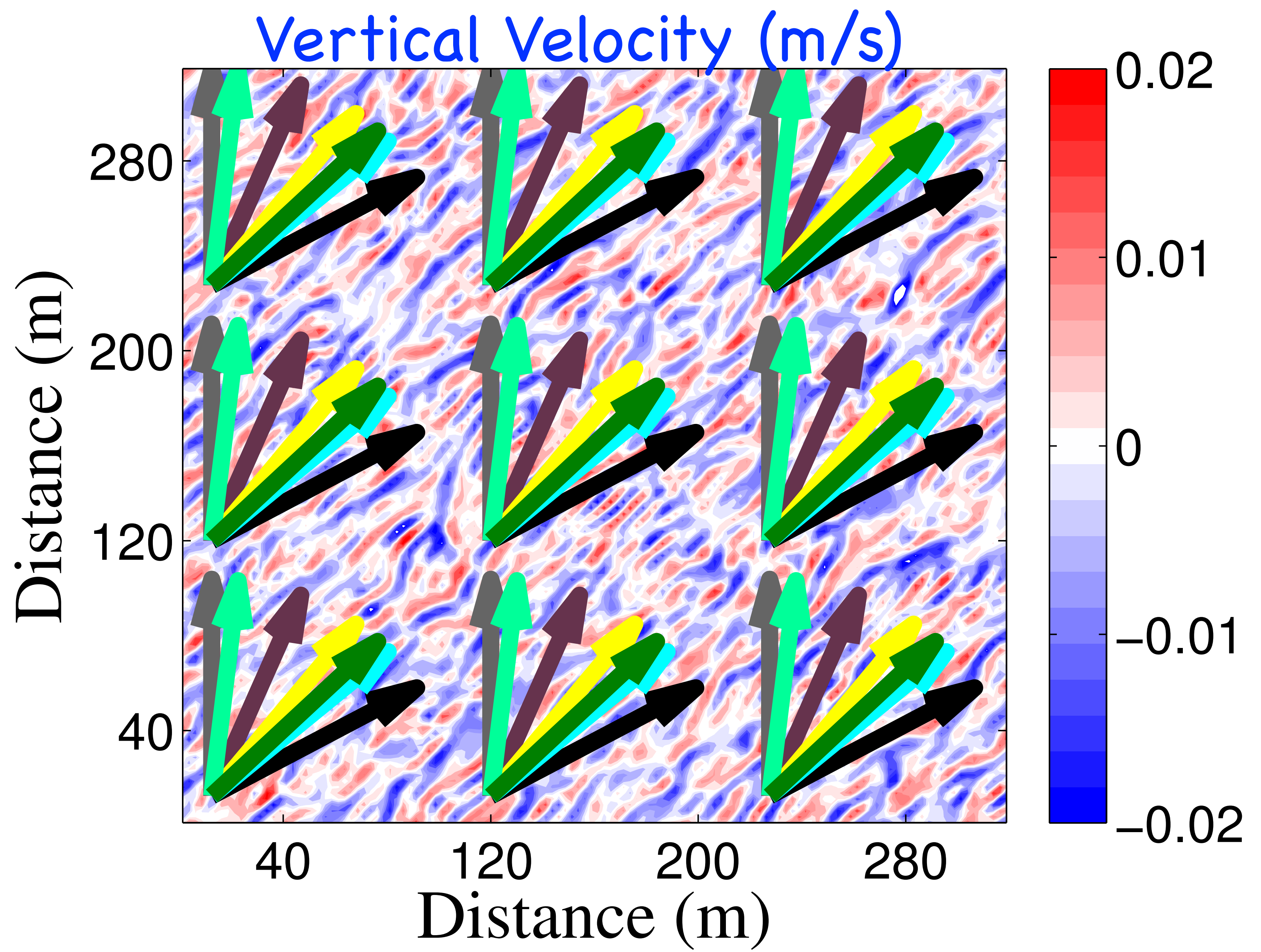
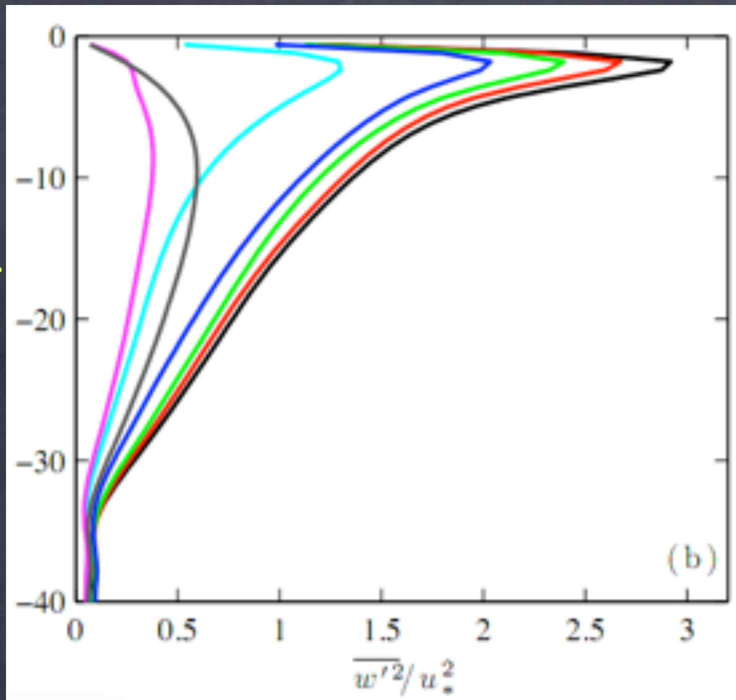
Tricky: Misaligned Wind & Waves

L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. Journal of Geophysical Research-Oceans, 117:C05001, 22pp, May 2012.

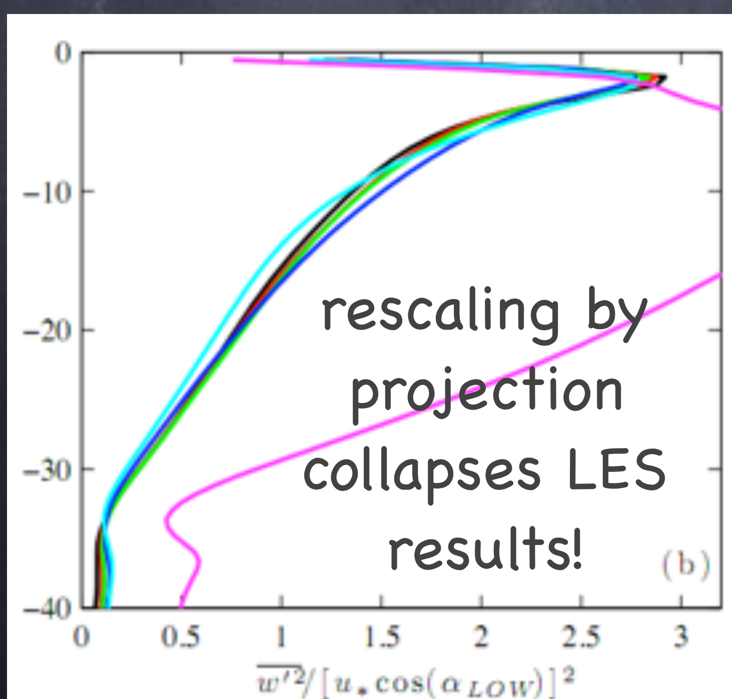


$\langle w^2 \rangle$



depth

rescaled $\langle w^2 \rangle$



depth

Generalized Turbulent Langmuir No.,

Projection of u*, u_s into Langmuir Direction

$$\frac{\left\langle \overline{w'^2} \right\rangle_{ML}}{u_*^2} = 0.6\cos^2(\alpha_{LOW})\left[1.0 + (3.1 La_{proj})^{-2} + (5.4 La_{proj})^{-4}\right],$$

$$La^2_{proj} = \frac{|u_*|\cos(\alpha_{LOW})}{|u_s|\cos(\theta_{ww}-\alpha_{LOW})},$$

$$\alpha_{LOW} \approx \tan^{-1}\left(\frac{\sin(\theta_{ww})}{\frac{u_*}{u_s(0)\kappa}\ln\left(\left|\frac{H_{ML}}{z_1}\right|\right)+\cos(\theta_{ww})}\right)$$

A scaling for LC strength & direction!

L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. Journal of Geophysical Research-Oceans, 117:C05001, 22pp, 2012.

Why? Vortex Tilting Mechanism

In CLB: Tilting occurs in direction of $\mathbf{u}_L = \mathbf{v} + \mathbf{v}_s$

Misalignment enhances degree of wave-driven LT

$$\frac{\partial \xi}{\partial t} + \underbrace{(\mathbf{u}_L \cdot \nabla)\xi}_{\text{AD}} = \underbrace{(\omega_a \cdot \nabla)(\mathbf{u}_L \cdot \hat{\mathbf{x}}')}_{\text{TS}} + \underbrace{(\nabla b \times \hat{\mathbf{z}}) \cdot \hat{\mathbf{x}}'}_{\text{BV}} + \text{SGS},$$

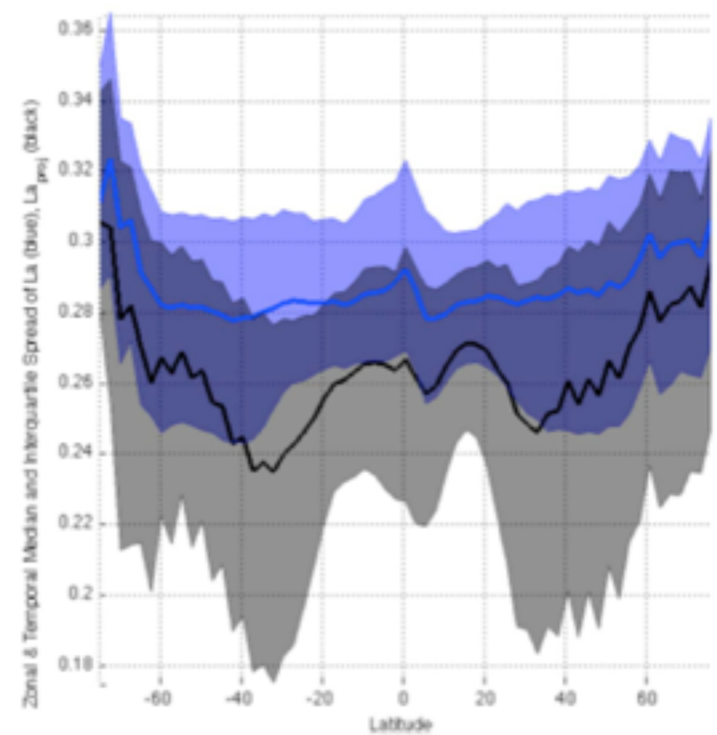


Figure 17. Temporal and zonal median and interquartile range of La_t and La_{proj} for a realistic simulation of 1994–2002 using Wave Watch III.

image: Thorpe, 04

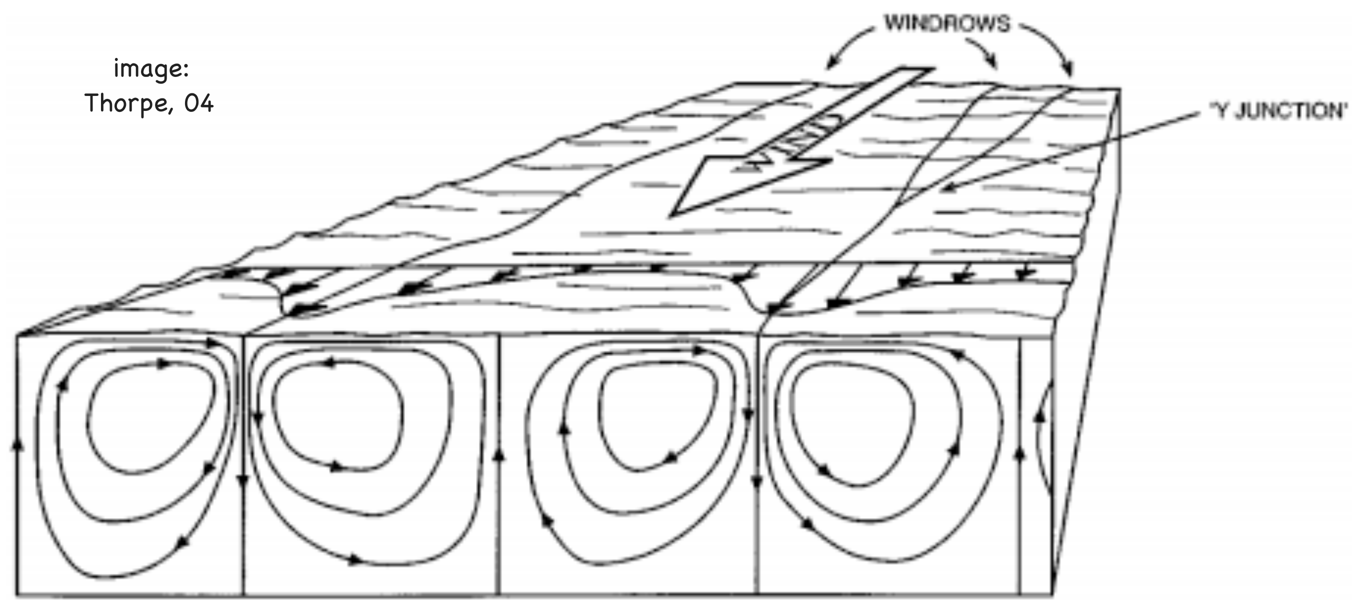
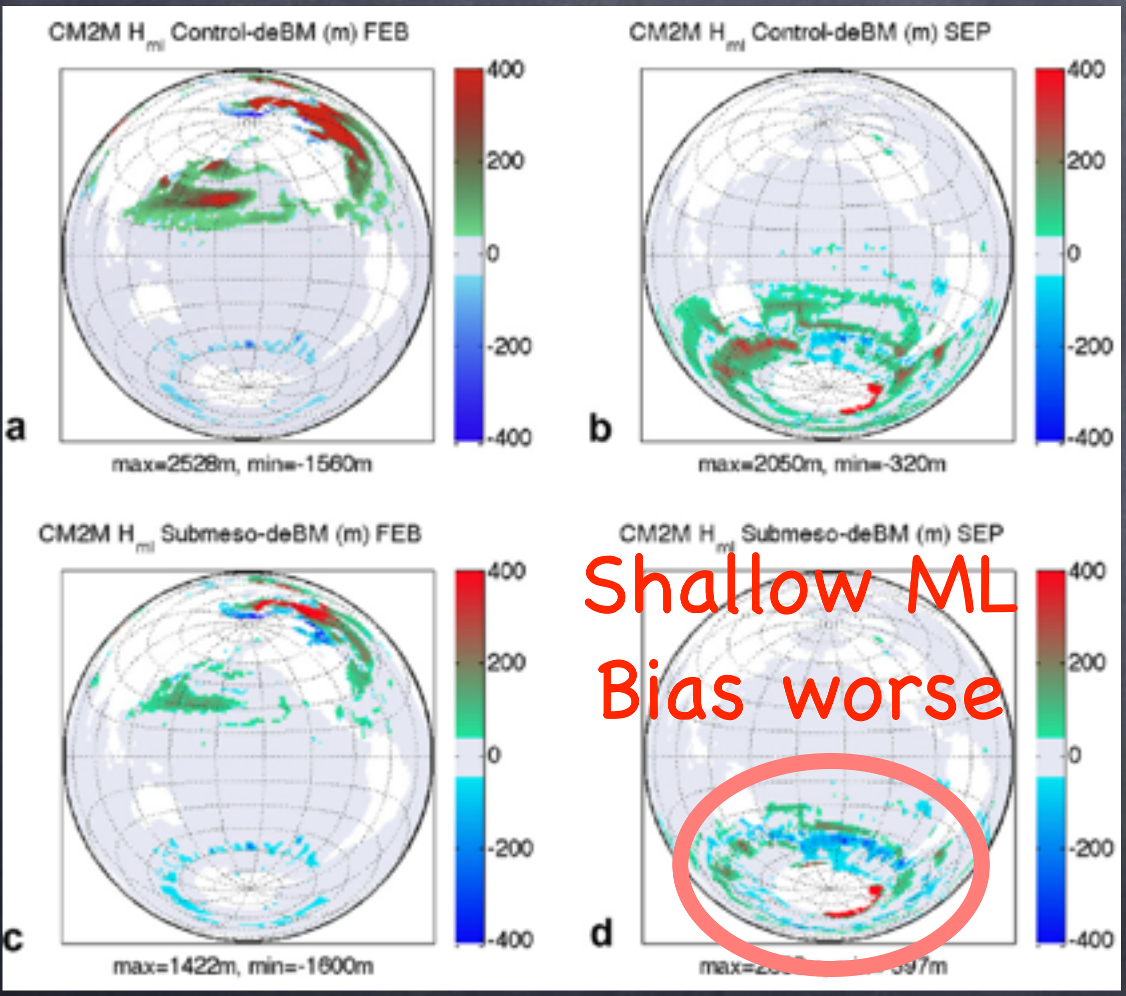


Figure 1 Sketch showing the pattern of mean flow in idealized Langmuir circulation. The windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). In practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2) amalgamate and meander in space and time. Bands of bubbles or buoyant algae may form within the downward-going (or downwelling) flow (see Figure 3).

Recall our problem with the (submeso) Mixed Layer Eddy Restratification--Southern Ocean too shallow!



Bias w/o MLE

Sallee et al. (2013) have shown that a too shallow S. Ocean MLD is true of most* present climate models

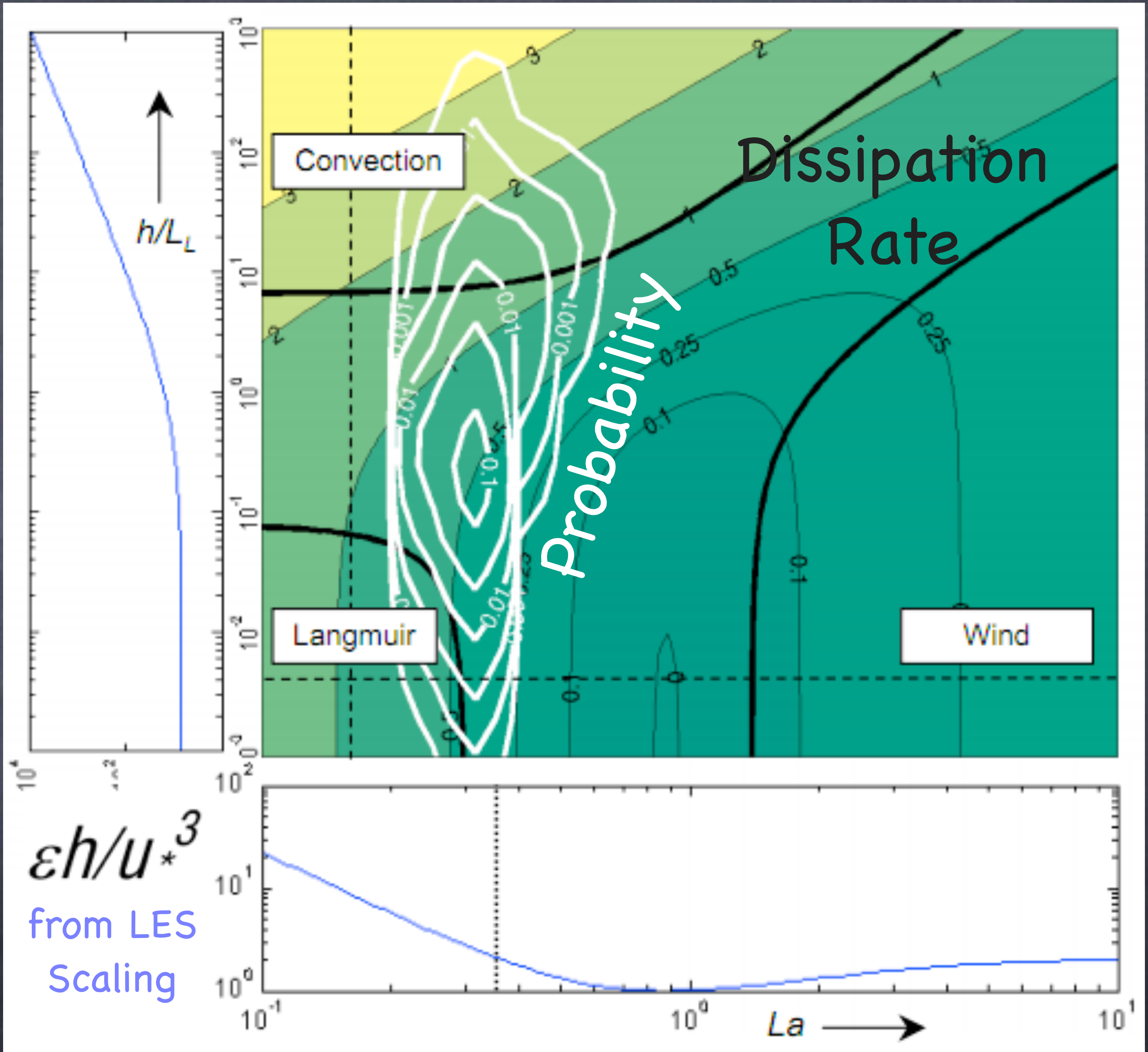
salinity forcing or ocean physics?

*true for CMIP5 multi-model ensemble

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

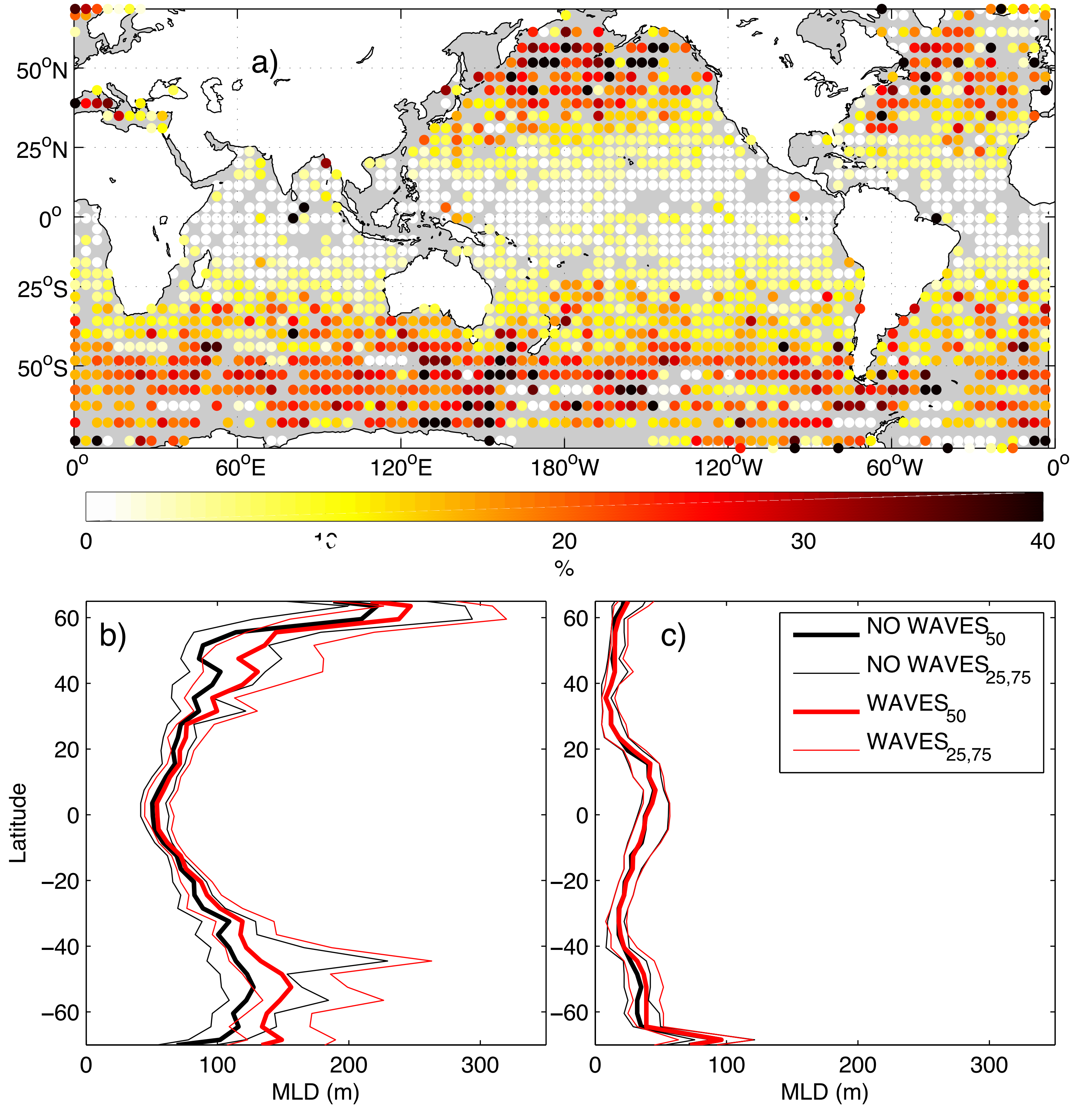
Data + LES, Southern Ocean mixing energy: Langmuir (Stokes-drift-driven) and Convective

So, waves can drive mixing via Stokes drift (combines with cooling & winds)



S. E. Belcher, A. A. L. M. Grant, K. E. Hanley, B. Fox-Kemper, L. Van Roekel, P. P. Sullivan, W. G. Large, A. Brown, A. Hines, D. Calvert, A. Rutgersson, H. Petterson, J. Bidlot, P. A. E. M. Janssen, and J. A. Polton. A global perspective on Langmuir turbulence in the ocean surface boundary layer. Geophysical Research Letters, 39(18):L18605, 9pp, 2012.

Including Wave-driven Mixing (Harcourt 2013 parameterization) Deepens the Mixed Layer!



M. A. Hemer, B. Fox-Kemper, & R. R. Harcourt. Quantifying the effects of wind waves the the coupled climate system, in prep. 2013.

What about Langmuir-Submeso Interactions?

Movie: P. Hamlington

Perform large eddy simulations (LES) of Langmuir turbulence with a submesoscale temperature front

Use NCAR LES model to solve Craik-Leibovich equations (Moeng, 1984, McWilliams et al, 1997)

$$\frac{\partial \rho}{\partial t} + \mathbf{u}_L \cdot \nabla \rho = \mathrm{SGS}$$

$$\nabla \cdot \mathbf{u} = 0$$

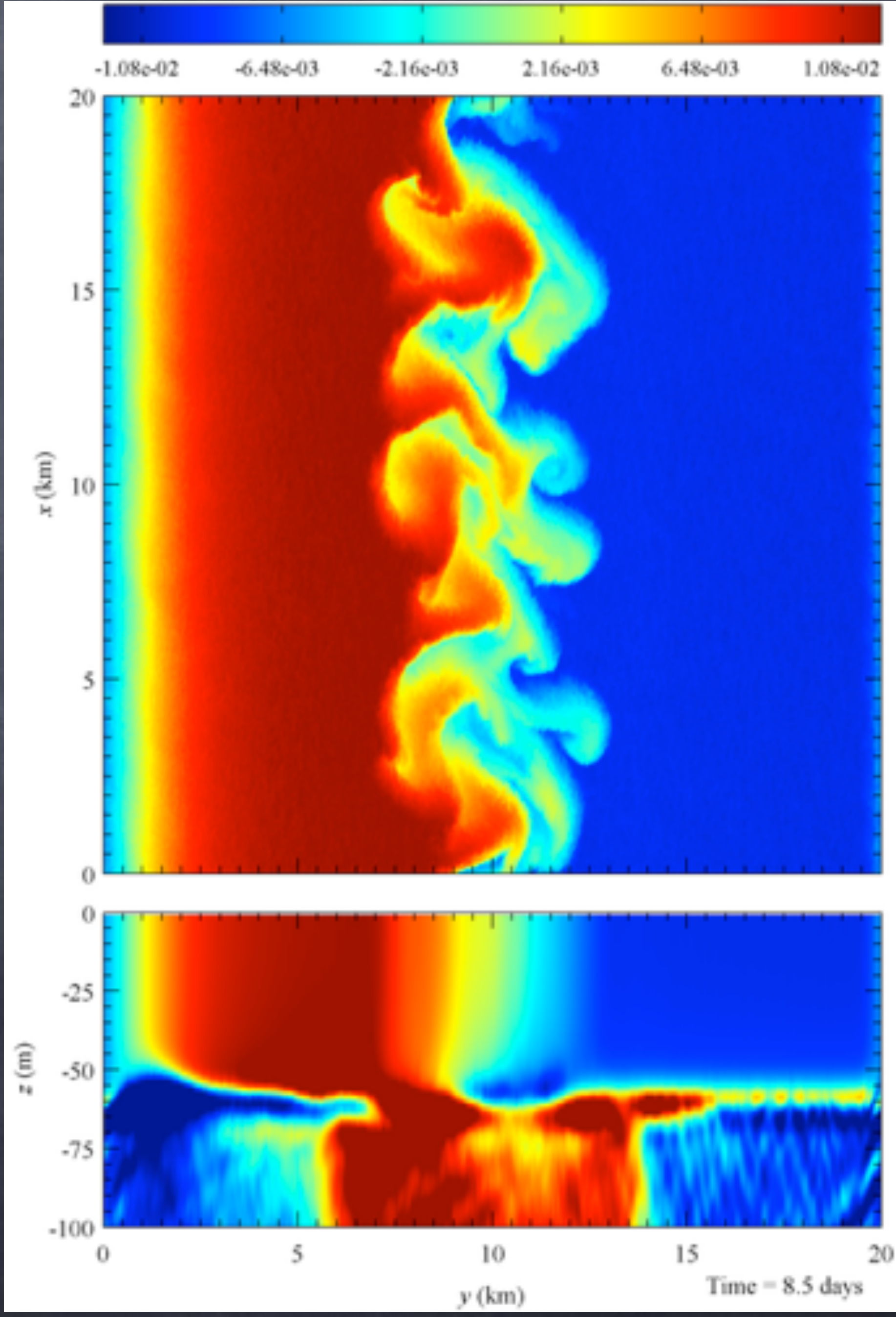
$$\frac{\partial \mathbf{u}}{\partial t} + (\boldsymbol{\omega} + f\hat{\mathbf{z}}) \times \mathbf{u}_L = -\nabla \pi - \frac{g\rho\hat{\mathbf{z}}}{\rho_0} + \mathrm{SGS}$$

Computational parameters:

Domain size: 20km x 20km x -160m

Grid points: 4096 x 4096 x 128

Resolution: 5m x 5m x -1.25m

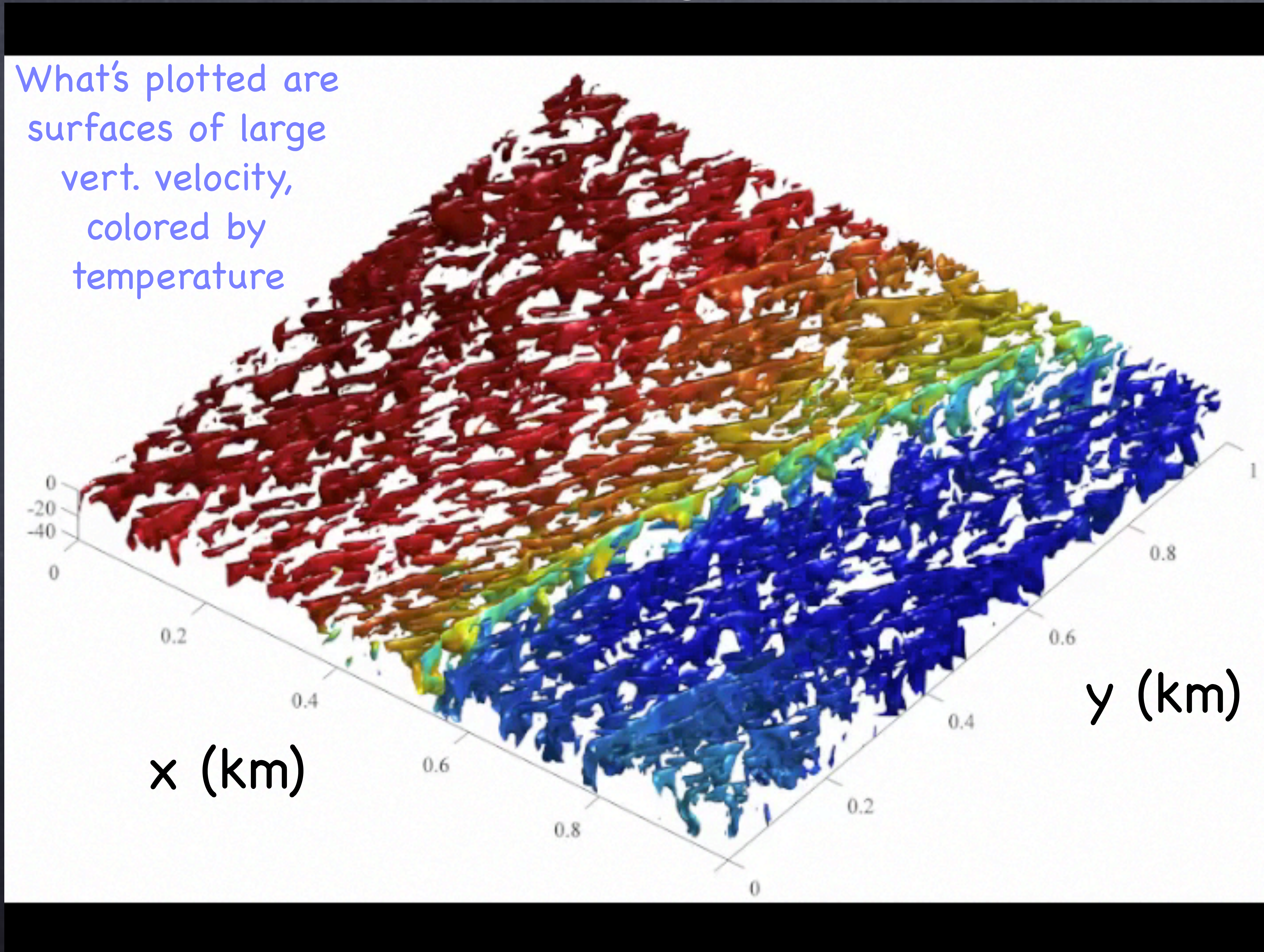


Overall results

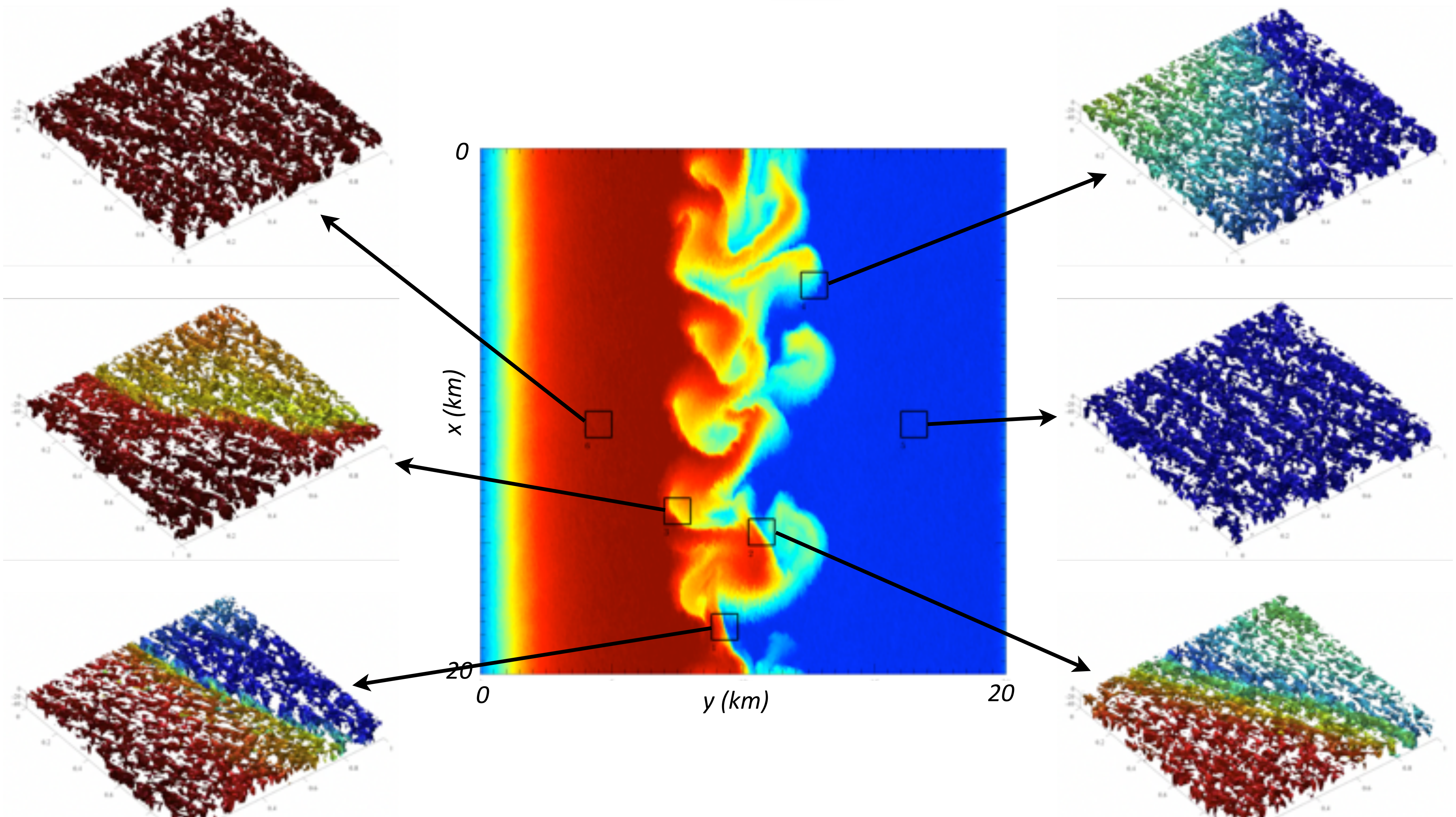
- Strong interactions between small & large scales are rare in this configuration
- Two relatively independent turbulent spectral cascades near the surface. Only one (submeso) at depth.
- Presence of waves greatly changes small scale instability character from symmetric instability to gravitational--this will matter!

P. E. Hamlington, L. P. Van Roekel, B. Fox-Kemper, K. Julien, G. P. Chini. Langmuir-Submesoscale Interactions: Descriptive Analysis of Multiscale Frontal Spin-down Simulations, *JGR-Oceans*, 2013. In prep.

Zoom: Submeso-Langmuir Interaction!



Diverse types of interaction



Slide & Movies by Peter Hamlington

P. E. Hamlington, L. P. Van Roekel, B. Fox-Kemper, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale simulations. In preparation, 2012.

So, Waves can Drive turbulence that affect larger scales indirectly:

What about direct effects of waves on larger scales?

Recall, from regular Boussinesq Equations:

(Combined) Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b$$

So, Waves can Drive turbulence that affect larger scales indirectly:

What about direct effects of waves on larger scales?

Now, Craik-Leibovich Boussinesq Equivalent:

(Combined) Lagrangian Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial}{\partial z}(\mathbf{v} + \mathbf{v}_s) = \mathbf{f} \times \frac{\partial \mathbf{v}_L}{\partial z} = -\nabla b$$

Now the temperature gradients govern the Lagrangian flow, not the not the Eulerian!

J. C. McWilliams and B. Fox-Kemper. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

So, can we just forget the whole thing and interpret large scales as Lagrangian velocities?

$$[\mathbf{f} + \nabla \times \mathbf{v}] \times \frac{\partial}{\partial z}(\mathbf{v} + \mathbf{v}_s) = -\nabla b$$

Not quite, because Ro>0 corrections are different!

The “Ro” for waves, is big *more often* than Ro is, especially for wide, shallow currents in a mixed layer

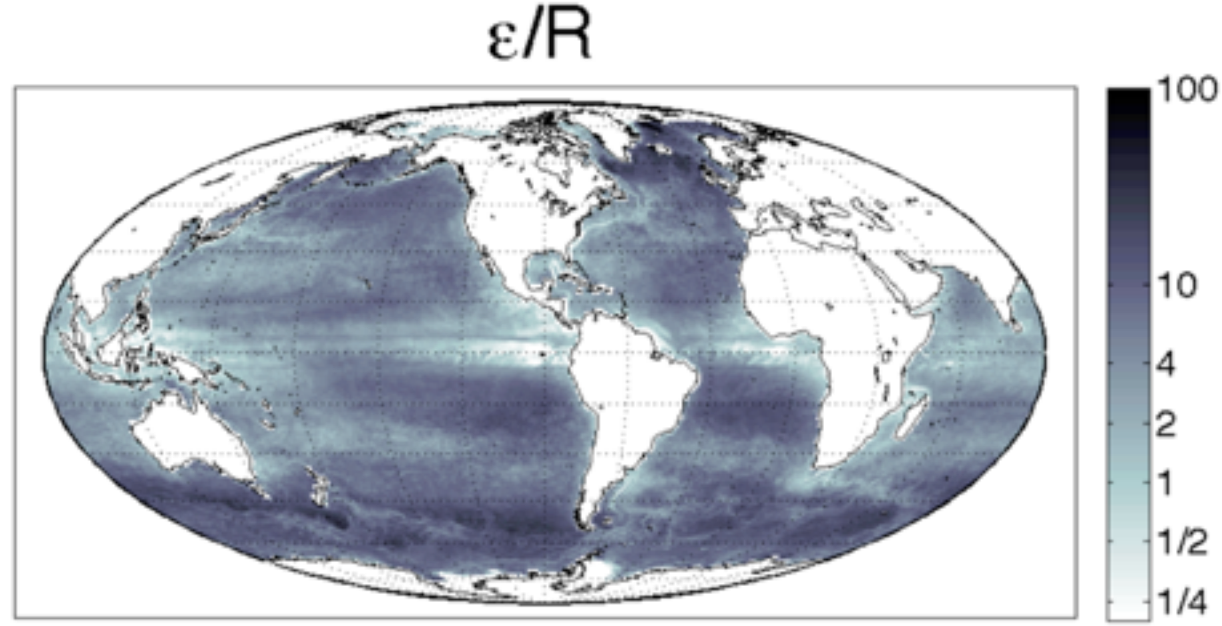
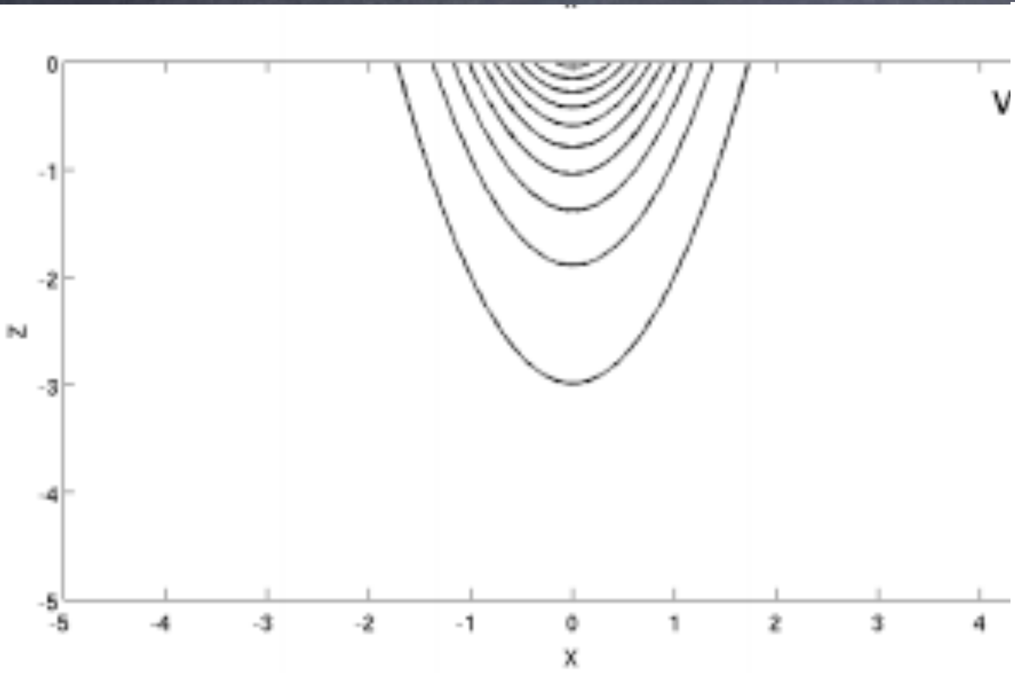
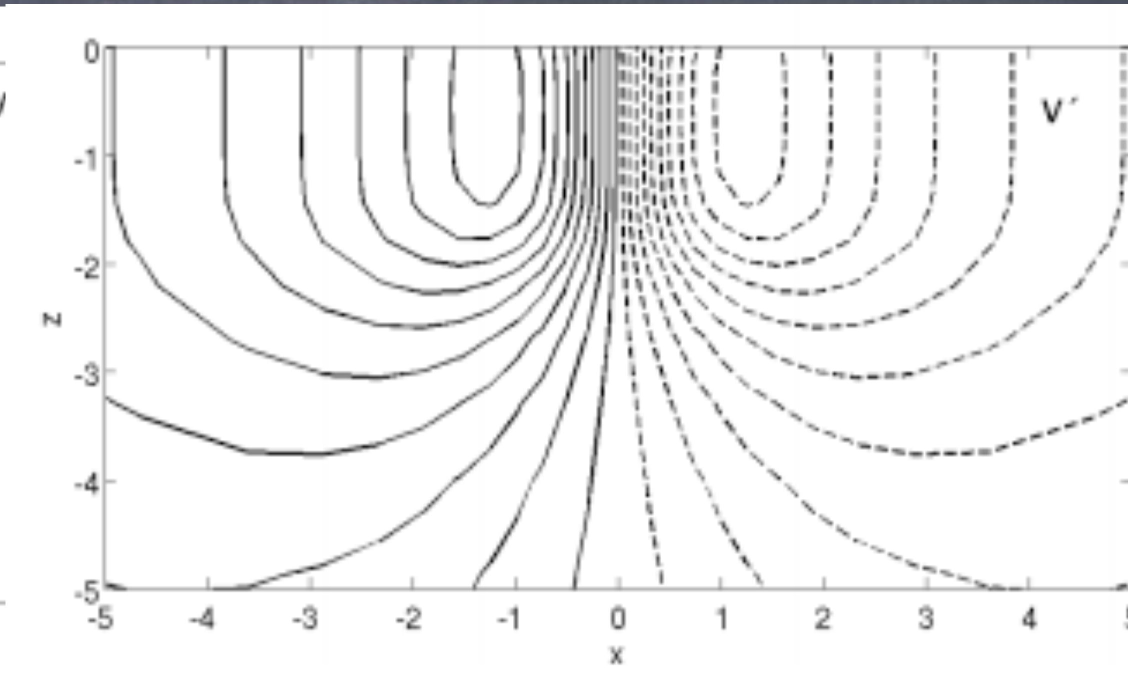


FIGURE 1. Estimated ratio $\epsilon/\mathcal{R} \approx (|\mathbf{u}_s \cdot \mathbf{u}|h)/(|\mathbf{u}|^2 h_s)$ governing the relative importance of Stokes effects versus nonlinearity. Eulerian velocity ($\mathbf{u}$) is taken as the AVISO weekly satellite geostrophic velocity or $-\mathbf{u}_s$ (for anti-Stokes flow) if $|\mathbf{u}_s| > |\mathbf{u}|$. The front/filament depth (h) is estimated as the mixed layer depth from the de Boyer Montégut *et al.* (2004) climatology. An exponential fit to the Stokes drift of the upper 9m projected onto the AVISO geostrophic velocity provides $\mathbf{u}_s \cdot \mathbf{u}$ and h_s. Stokes drift is taken from the WaveWatch-3 simulation described in Webb & Fox-Kemper (2011). $\mathbf{u}$, $\mathbf{u}_s$, and h_s are all for the year 2000, while h is from a climatology of observations over 1961-2008. The year 2000 average of $\epsilon/\mathcal{R}$ is shown.

J. C. McWilliams and B. Fox-Kemper. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

Waves (Stokes Drift Vortex Force) -> Submeso, Meso: An example





Initial Submeso Front

Contours: 0.1

Perturbation on that scale due to waves

Contours: 0.014

J. C. McWilliams and B. Fox-Kemper. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

So, no problems? Just crunch away with CLB?

- Let's revisit our assumptions for scale separation:
  - CLB wave equations require limited *wave steepness* and irrotational flow
  - Real wind-waves are not monochromatic, but incorporate a spectrum of waves, and

Power Spectrum of wave height

$$\langle \eta^2 \rangle = \int_0^\infty E(k)dk = C_0 + \int_{k_h}^\infty C_1 k^{-2} dk$$

Power Spectrum of wave steepness: INFINITE!

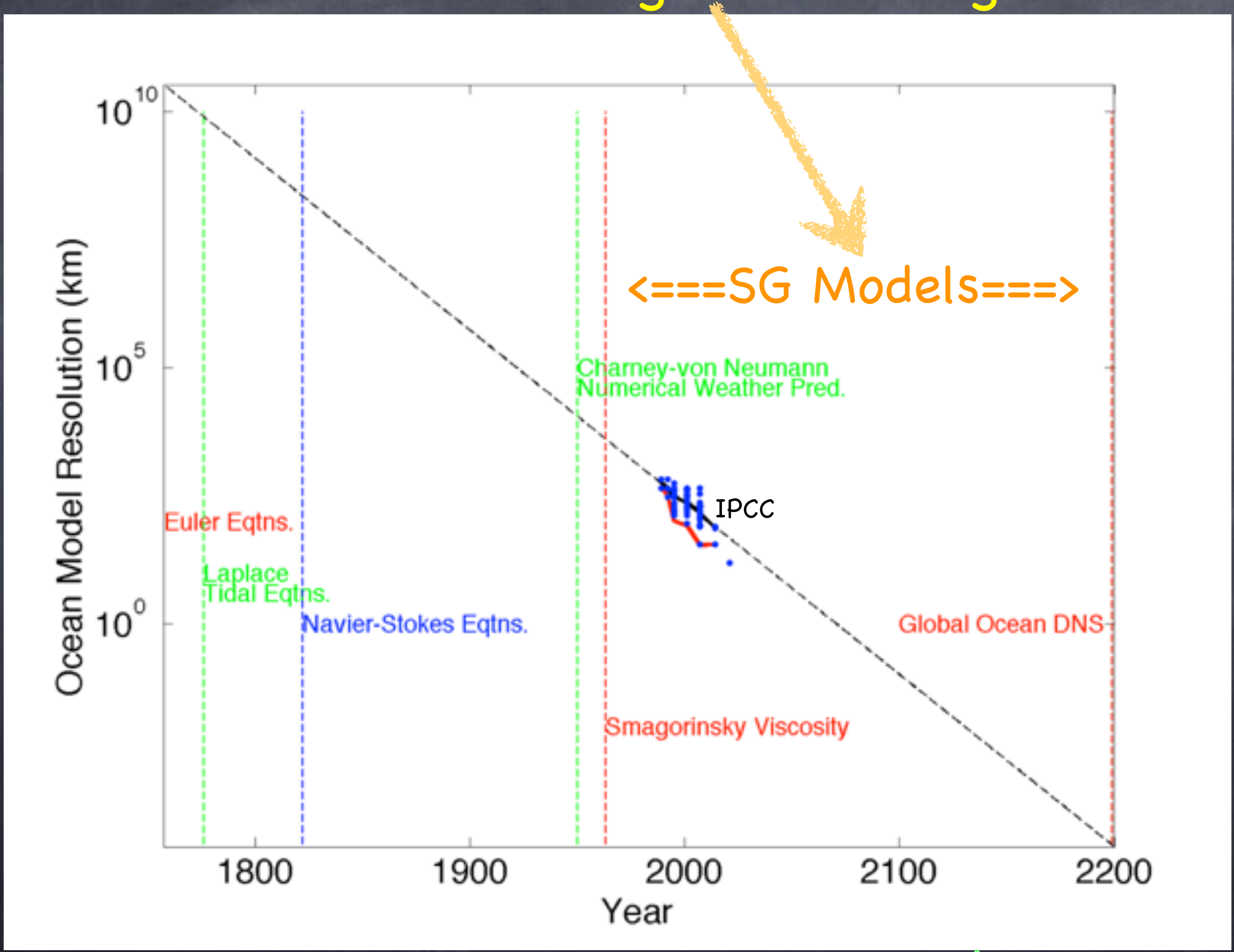
$$\langle k^2 \eta^2 \rangle = \int_0^\infty k^2 E(k)dk = D_0 + \int_{k_h}^\infty D_1 dk$$

Steep waves break->vortex motion & small scale turbulence!

Conclusions

- Climate modeling is challenging partly due to the vast and diverse scales of fluid motions
- In the upper ocean, horizontal scales as big as basins, and as small as meters contribute non-negligibly to the air-sea exchange
- Process models, especially those spanning a whole or multiple scales, are needed to study these connections and improve subgrid models.
- Interesting are the submeso to Langmuir scales, as nonhydro. & ageostrophic effects begin to dominate
- The CLB are good for LES & analysis in this range, but cannot capture some effects of small, steep waves (breaking, spray, nearshore, etc.)

Extrapolate for historical perspective:
The Golden Era of Subgrid Modeling is Now!



All papers at: fox-kemper.com/research

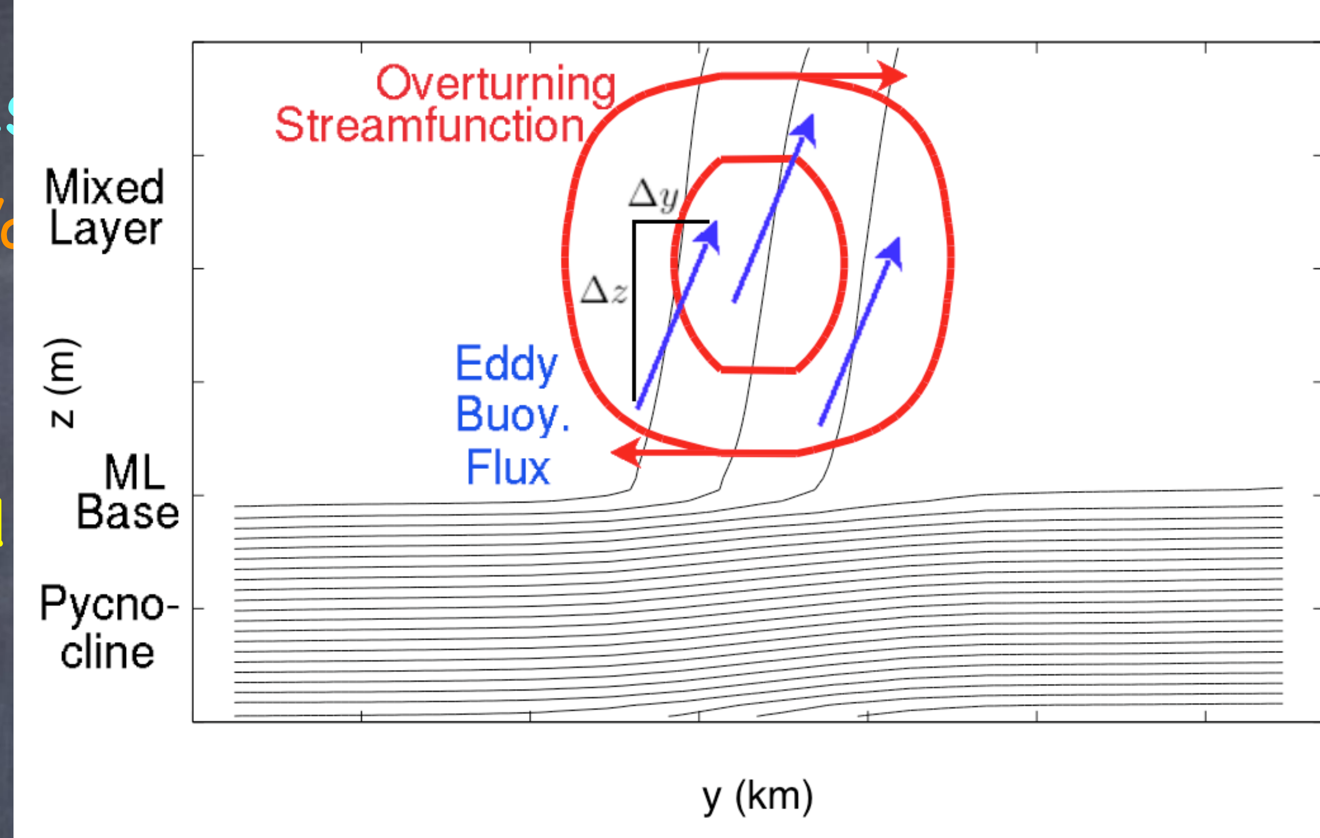
Mixed Layer Eddy Res

Estimating eddy buoyancy/c

$$\overline{\mathbf{u}'b'} \equiv \mathbf{\Psi} \times \nabla \bar{b}$$

A submeso eddy-induced

$$\mathbf{\Psi} = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times$$



in ML only:

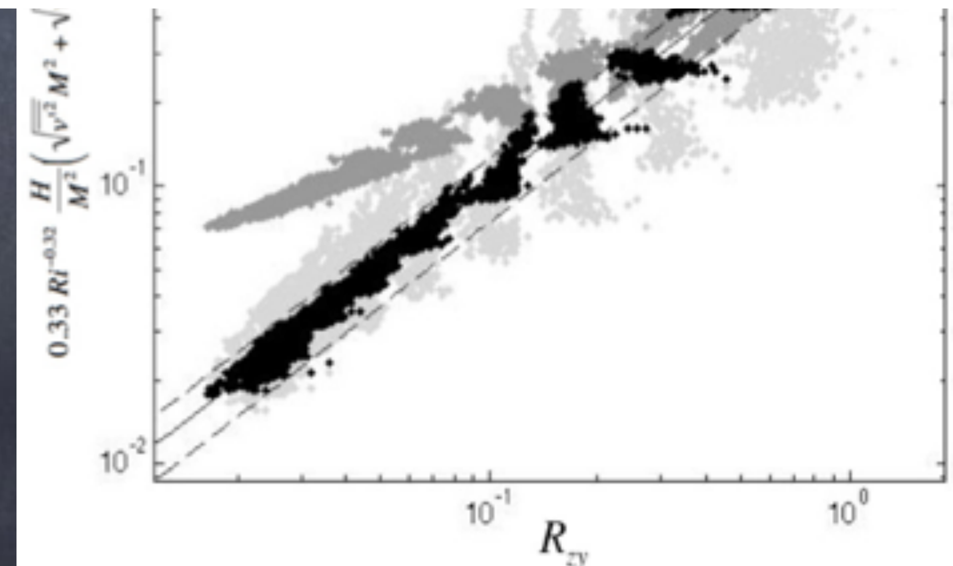
$$\mu(z) = 0 \text{ if } z < -H$$

For a consistently restratifying,

$$\overline{w'b'} \propto \frac{H^2}{|f|} \left|\nabla_H \bar{b}\right|^2$$

and horizontally downgradient flux.

$$\overline{\mathbf{u}'_H b'} \propto \frac{-H^2 \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla_H \bar{b}$$



S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013

Sensitivity of Climate to Submeso: AMOC & Cryosphere Impacts

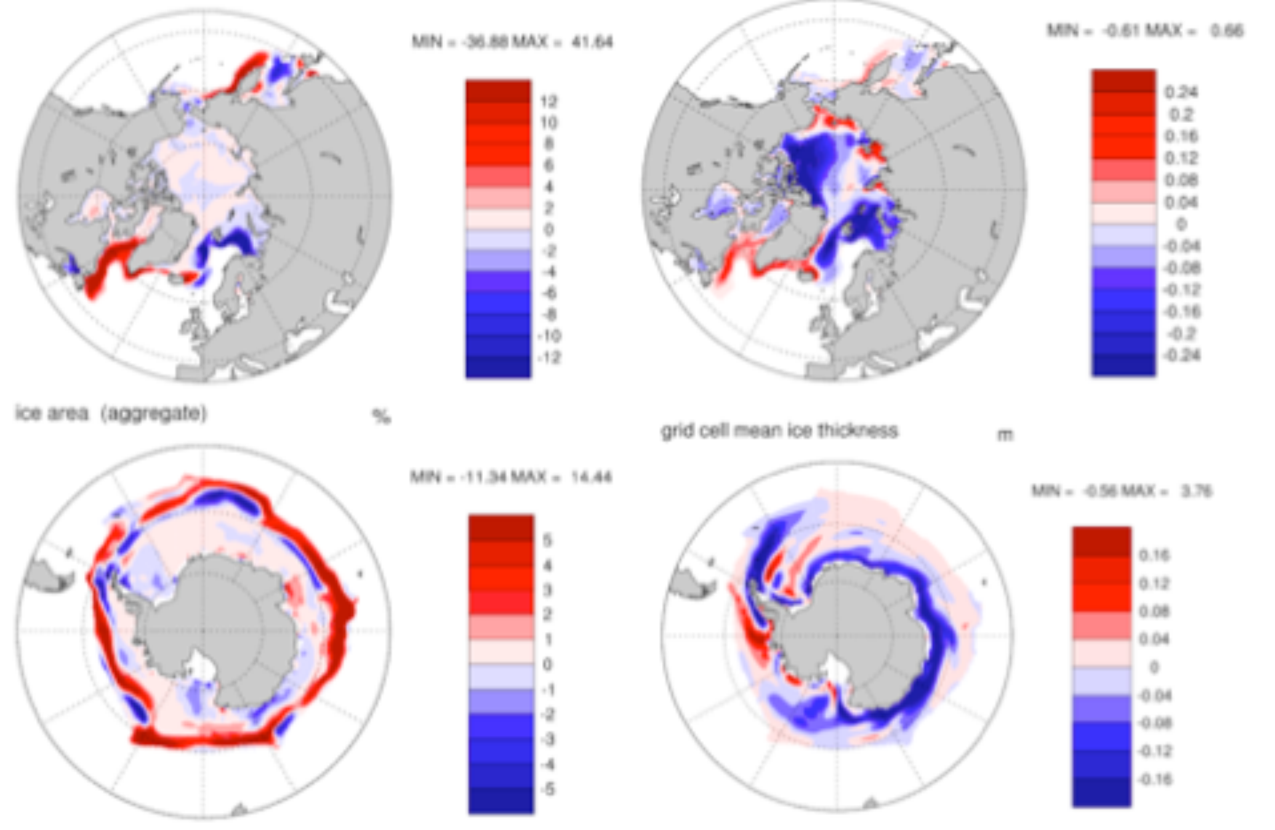
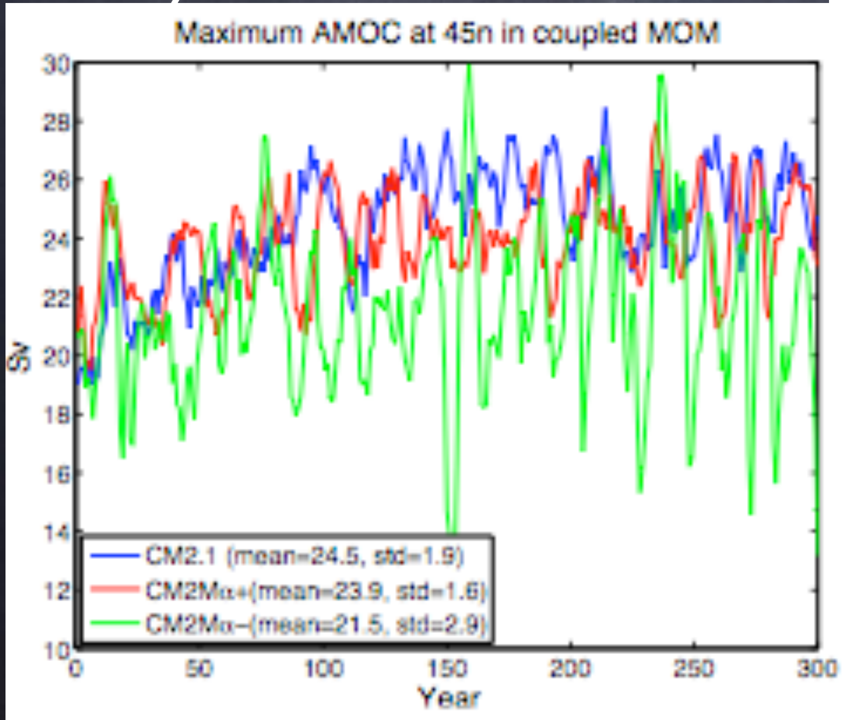


Figure 10: Wintertime sea ice sensitivity to introduction of MLE parameterization ($CCSM^+$ minus $CCSM^-$): January to March Northern Hemisphere a) ice area and b) thickness and July to September Southern Hemisphere c) ice area and d) thickness.

May Stabilize AMOC



Affects sea ice

NO RETUNING NEEDED!!!

These are impacts: bias change unknown