

A Refined Life at High Resolution: Subgrid Modelling in the Eddy-Rich Regime

Baylor Fox-Kemper (Brown University, USA)

Contributions from Scott D. Bachman (CU-PhD, now Cambridge, UK),

Thanks for discussions with:

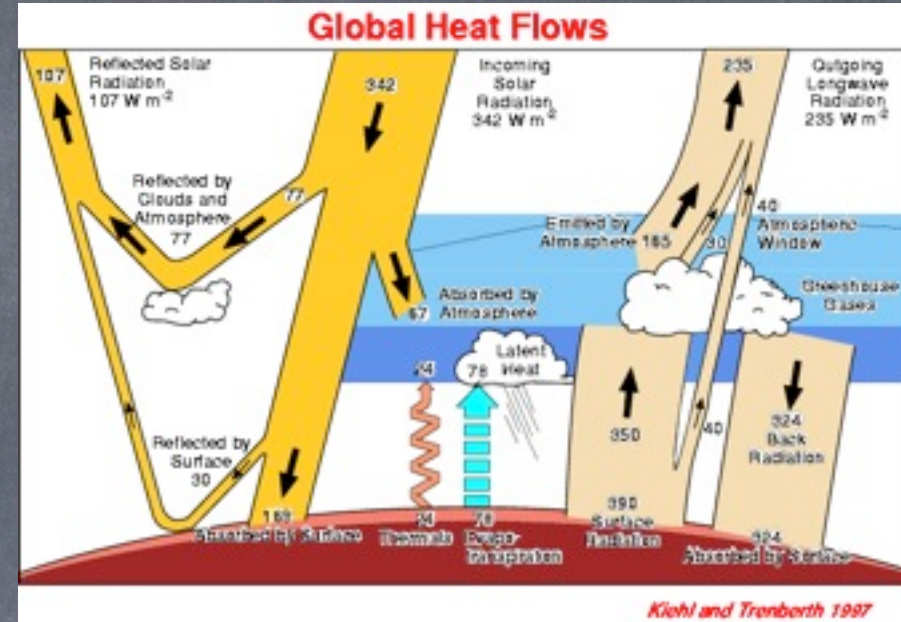
Stephen Griffies (GFDL), Frank Bryan, Peter Gent & John Dennis (NCAR),
Keith Julien (CU), Jim McWilliams (UCLA), Traian Iliescu (VA Tech)

European Geophysical Union OS4.4, Room Y2

10 Apr 2013, 14:30–15:00

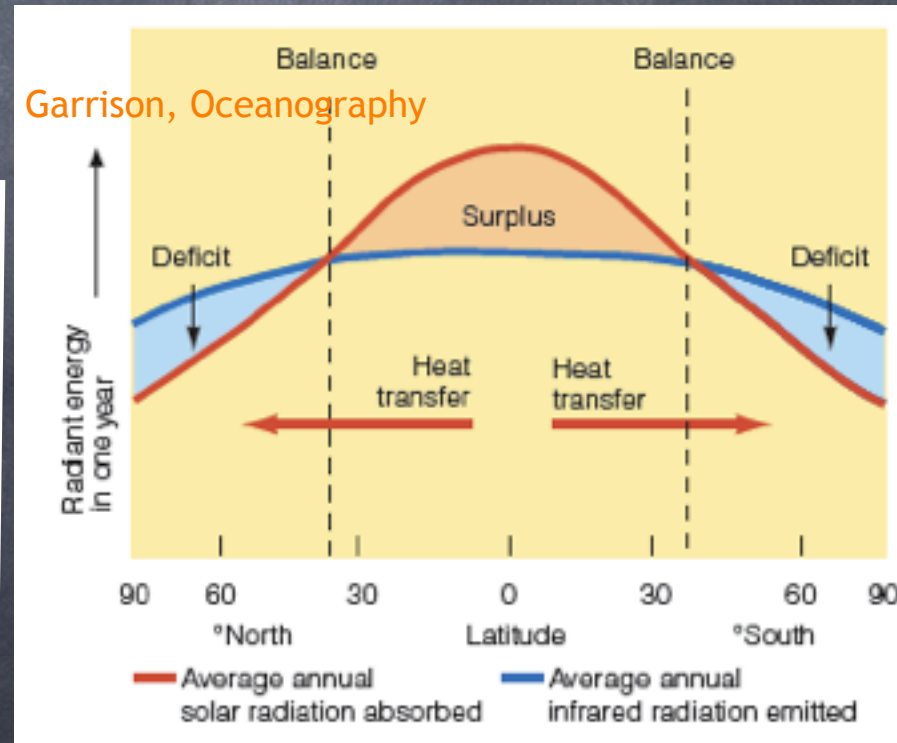
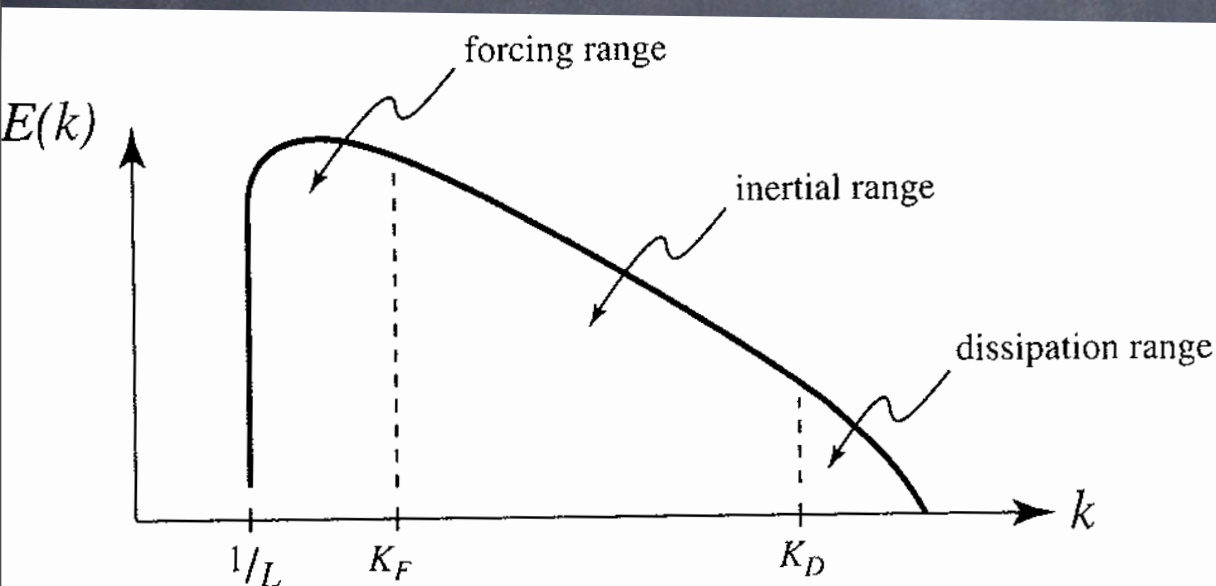
Sponsors: NSF 0855010, 0825614, Brown U.

The Earth's Climate System is driven by the Sun's light (minus outgoing infrared) on a global scale



Dissipation concludes

turbulence cascades on scales nearly a trillion times smaller



What does it take to parameterize eddies in an eddy-free model? A lot of scaling laws...

Map shows elements for anisotropic GM/Redi, also air-sea fluxes?

B. Fox-Kemper, R. Lumpkin, and F. O. Bryan. Lateral transport in the ocean interior. In G. Siedler, J. Church, J. Gould, and S. Griffies, editors, *Ocean Circulation and Climate - Observing and Modelling the Global Ocean*. Elsevier, 2013. In press.

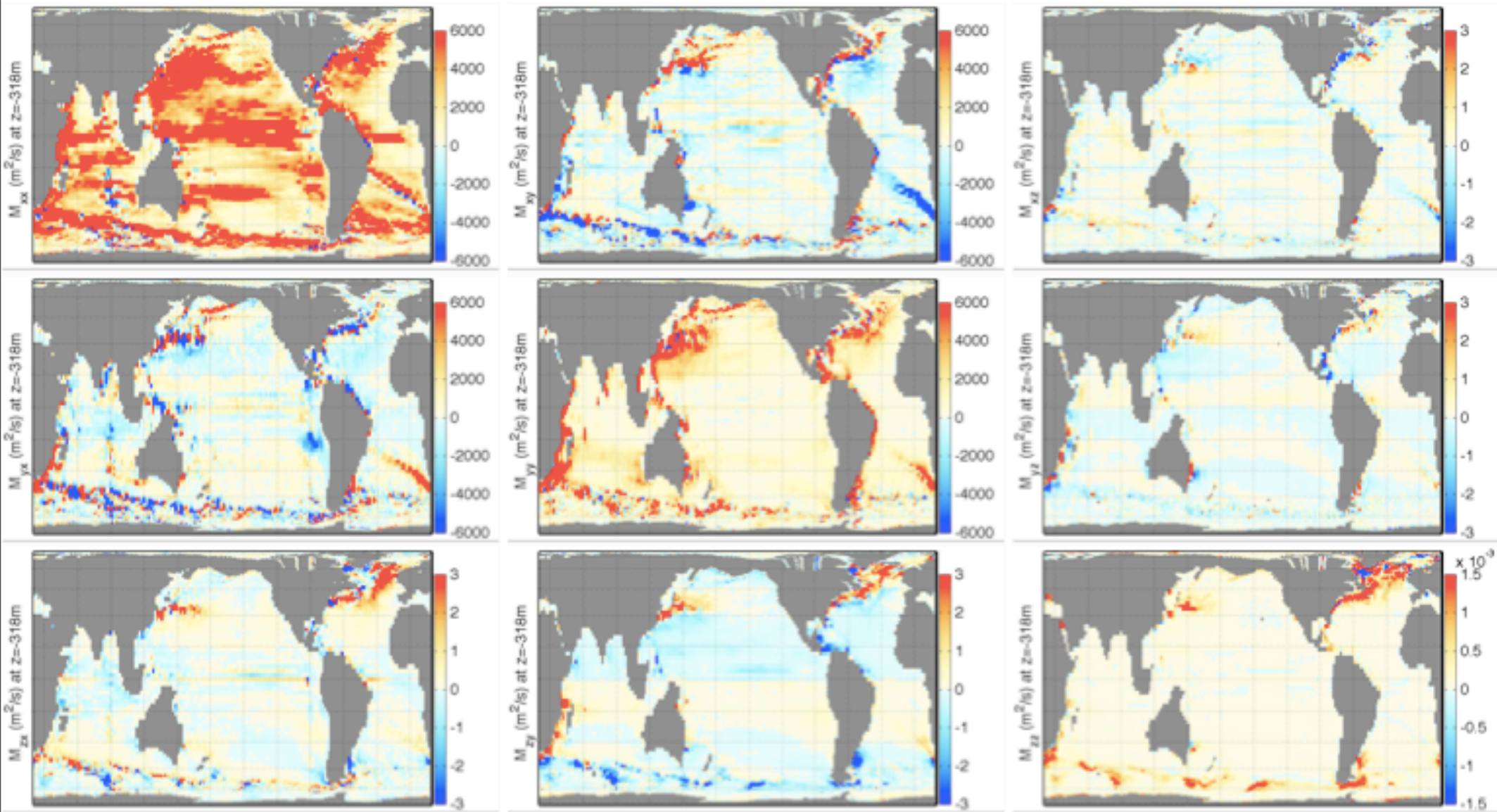
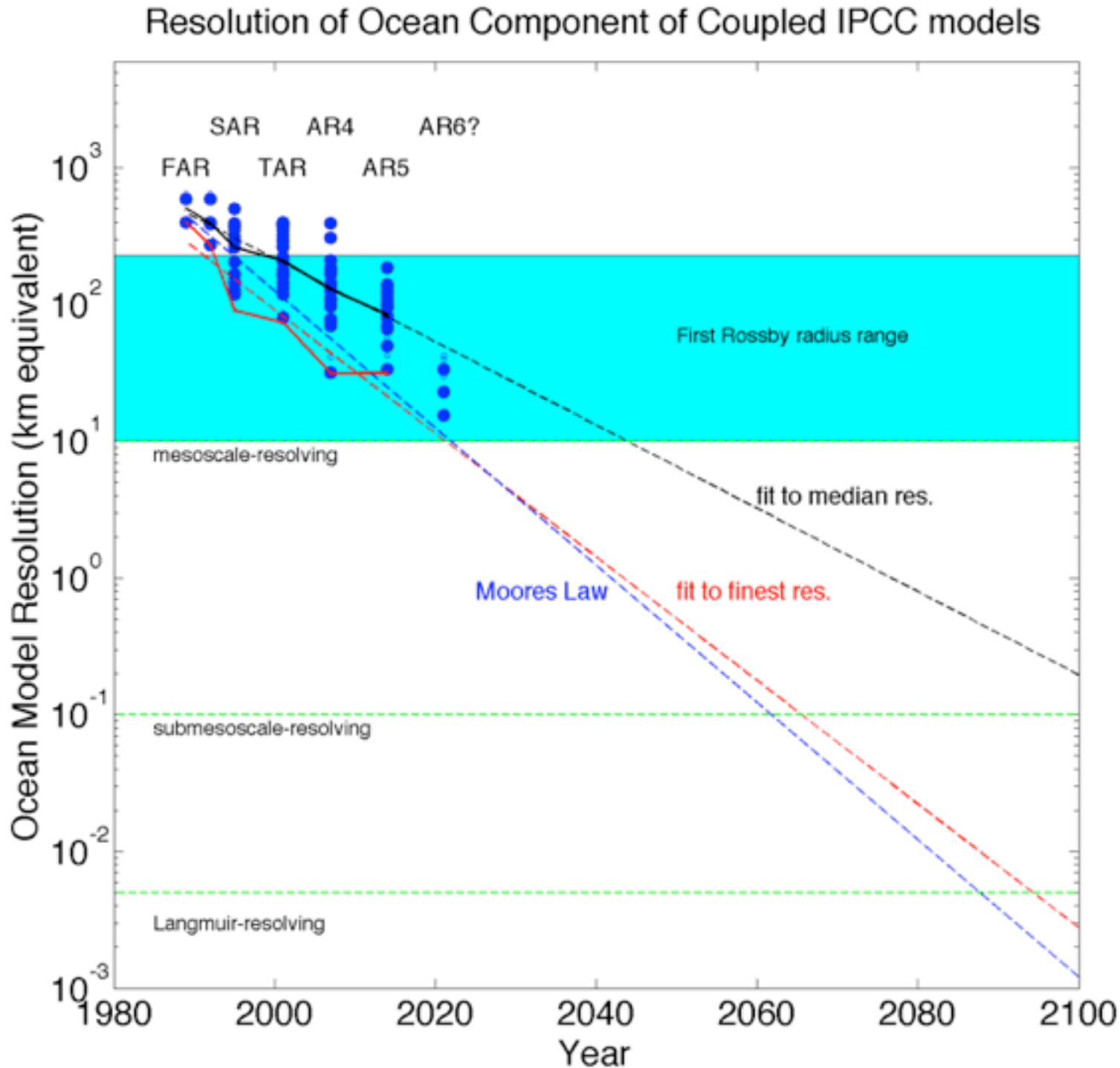
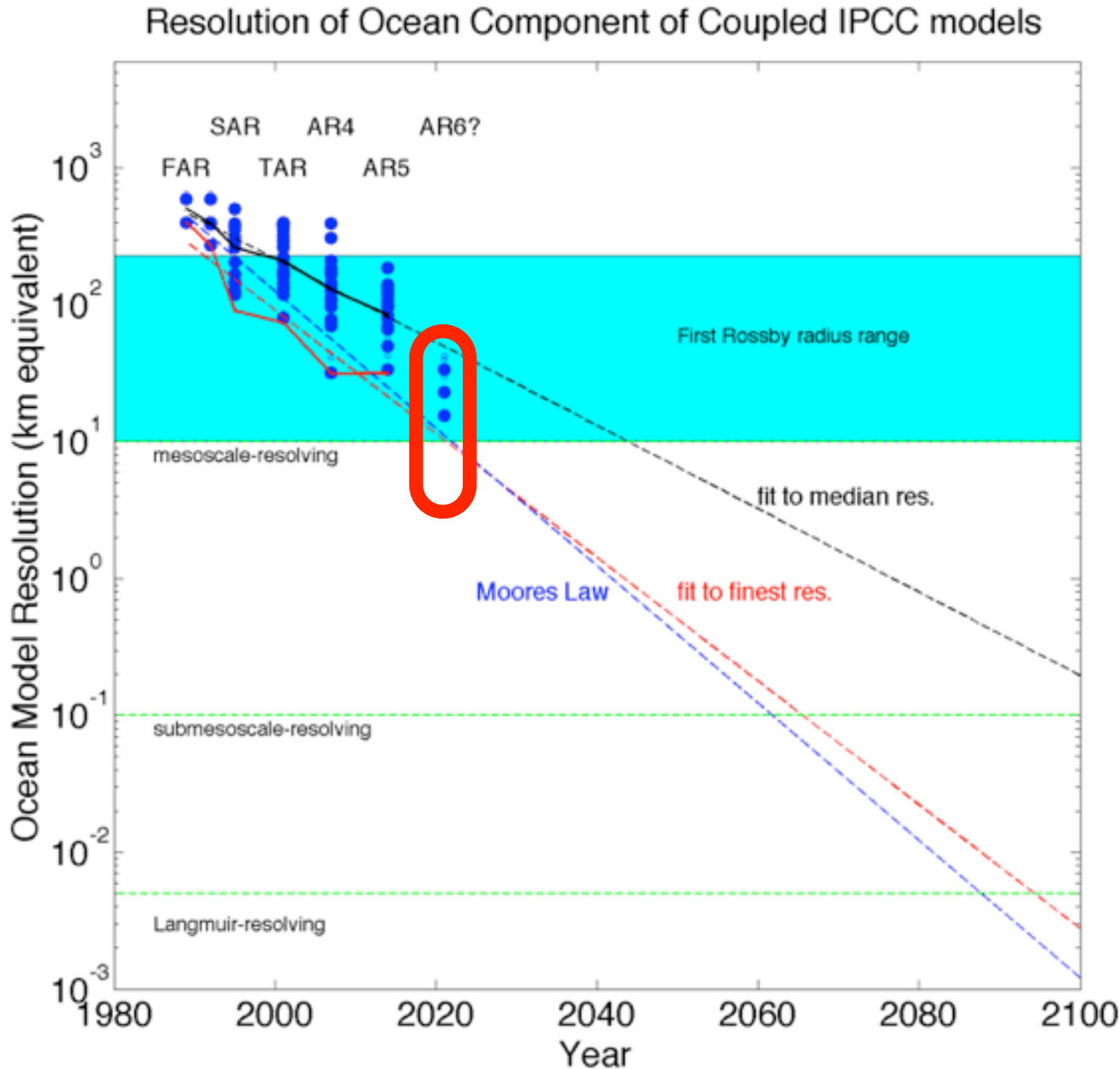


Figure 4: Components of the R tensor at 318m depth, with the $K_{\alpha\beta}$ part in the upper left 4 panels.

Resolution will be an issue for centuries to come!



Resolution will be an issue for centuries to come!



Today's
Focus:

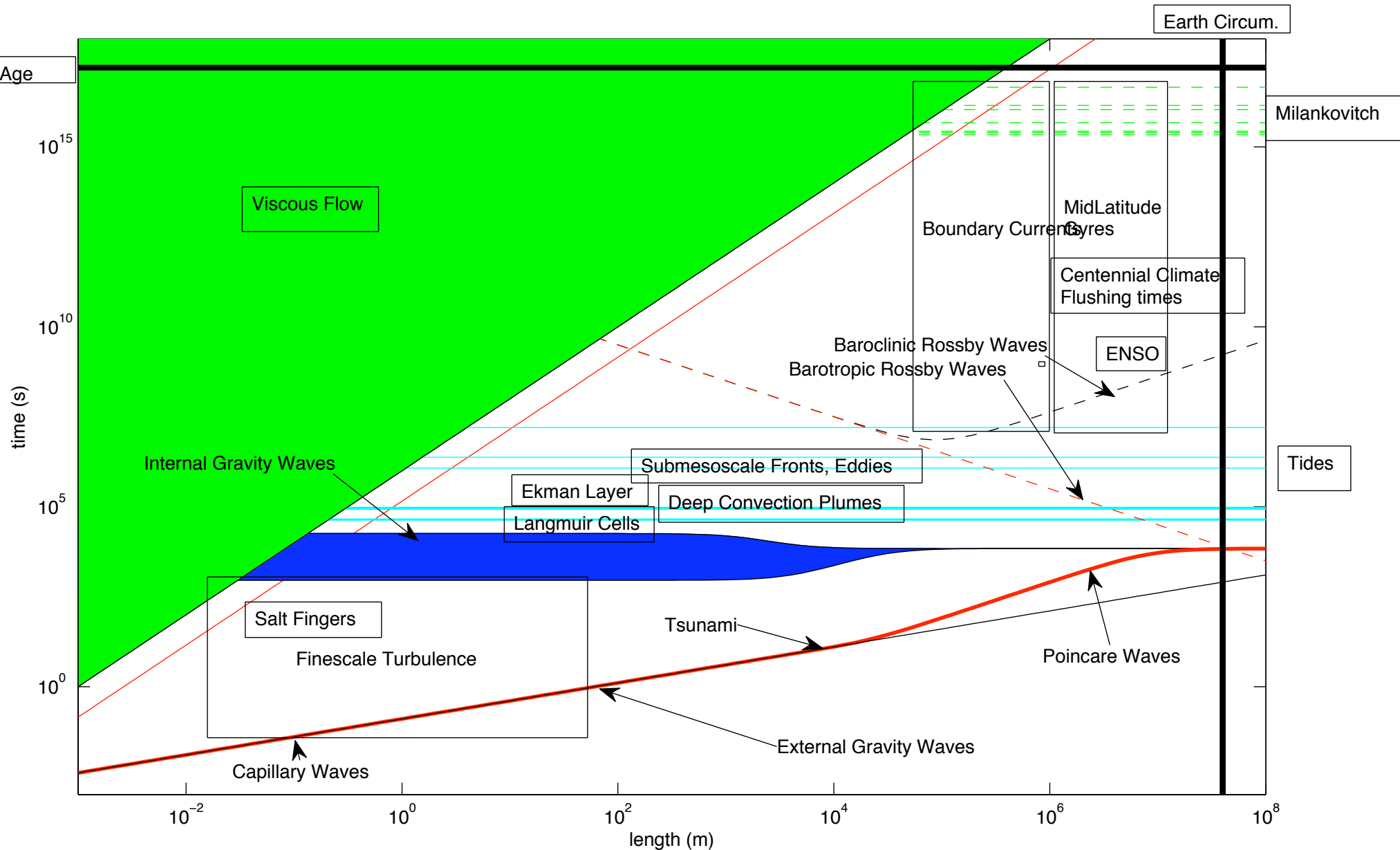
MOLES

Mesoscale
Ocean
Large Eddy
Simulations
with
 $O(2-50\text{km})$
horiz.
resolution

The Ocean is Vast and Diverse

CM2.5=GFDL Hi-Res Earth System Model

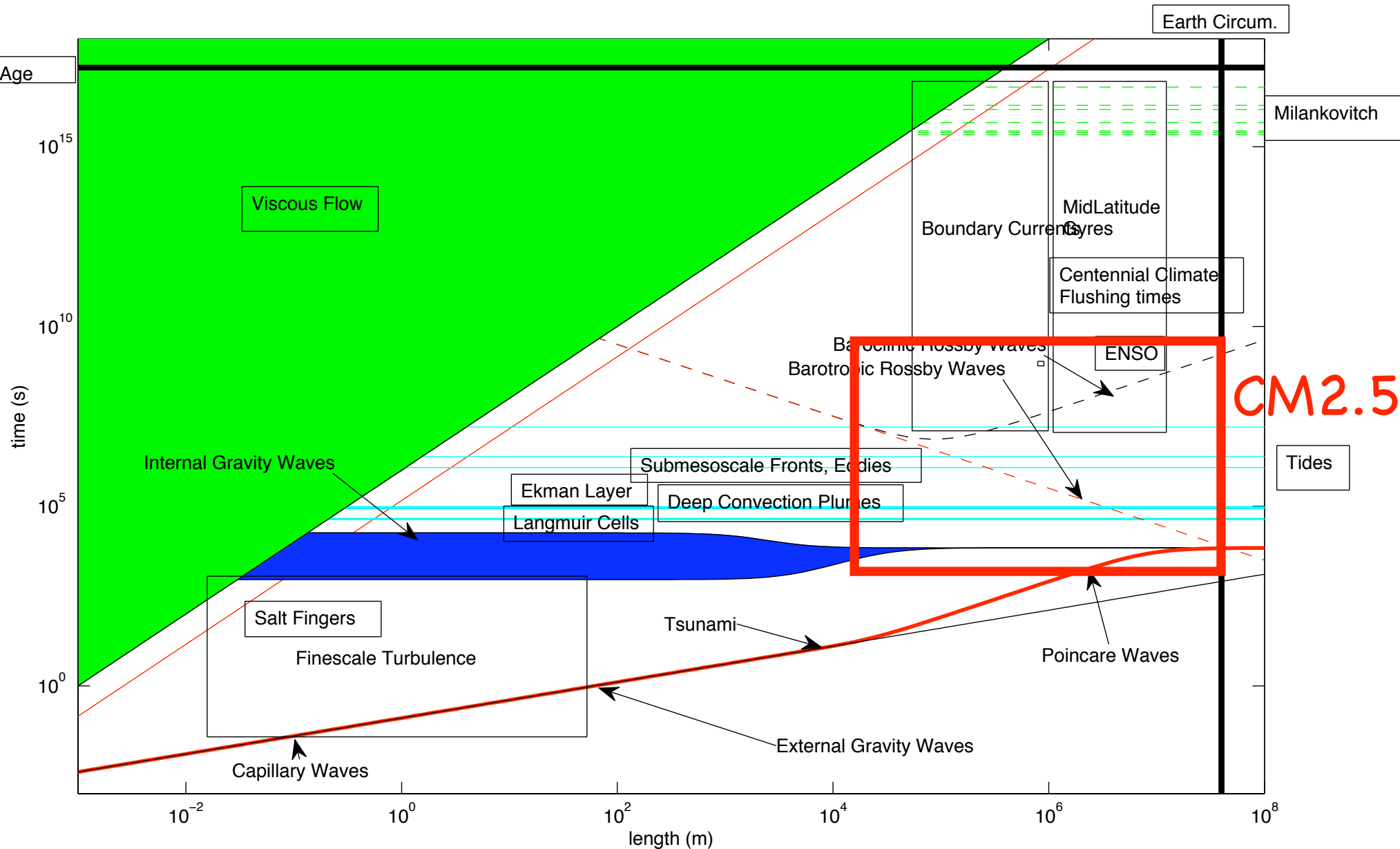
Delworth et al. 2012



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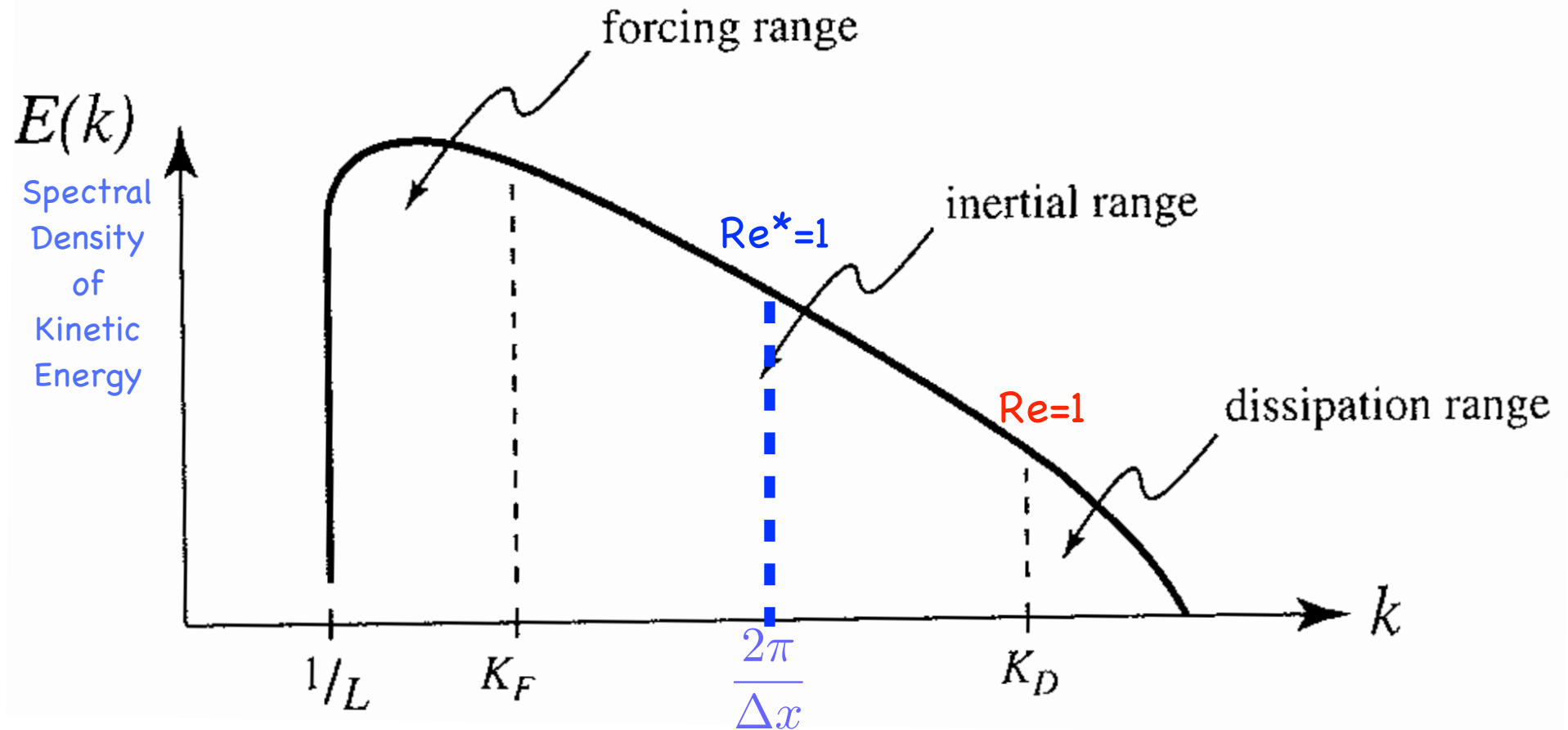
Delworth et al. 2012



So, even as we begin to resolve the mesoscale...

- There are many, many processes left unresolved or partially resolved
- **Eddy Less:** For the unresolved (no eddies), need Reynolds-Average Closures (e.g., KPP, Gent-McWilliams/Redi)
- **Eddy Rich:** eddy-permitting to resolving, need Scale-Aware Large-Eddy-Simulation Closures (e.g., Smagorinsky)
- **Some scale-aware hybrids, e.g., Mixed Layer Eddies: Fox-Kemper et al. 2011**

Truncation of Cascades

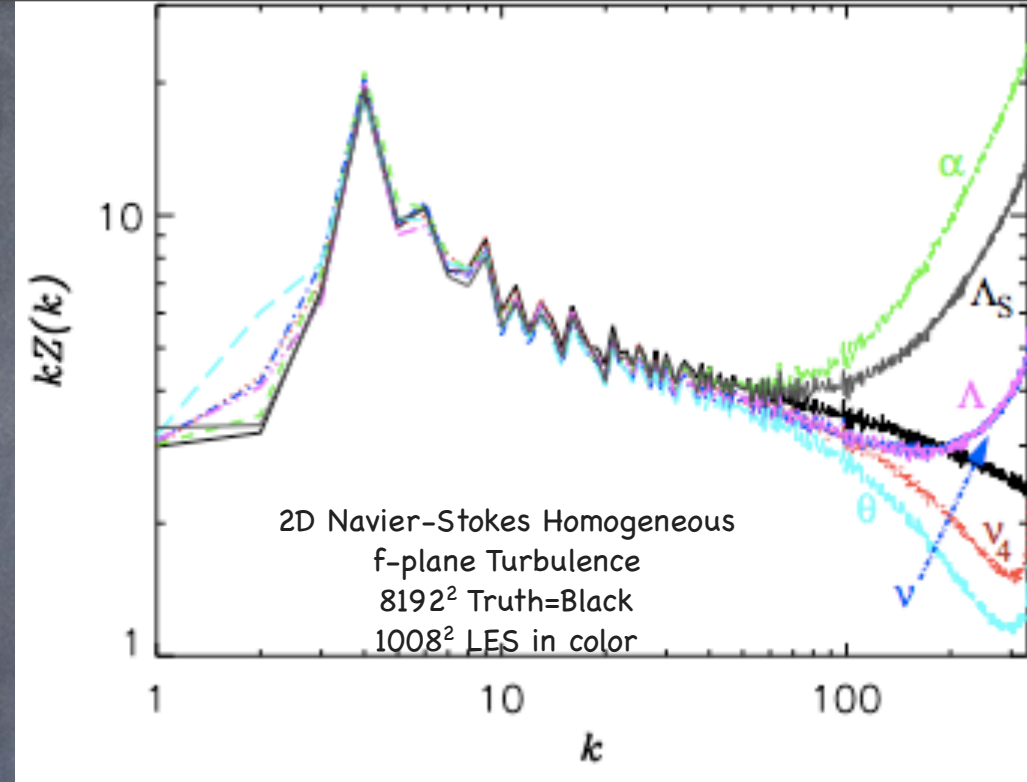


1963: Smagorinsky Scale & Flow Aware Viscosity Scaling,
 So the Energy Cascade is Preserved,
 but order-1 gridscale Reynolds #: $Re^* = UL/\nu_*$

$$\nu_{*h} = \left(\frac{\Upsilon_h \Delta x}{\pi} \right)^2 \sqrt{\left(\frac{\partial u_*}{\partial x} - \frac{\partial v_*}{\partial y} \right)^2 + \left(\frac{\partial u_*}{\partial y} + \frac{\partial v_*}{\partial x} \right)^2}$$

Some MOLES Truncation Methods In Use

- Harmonic/Biharmonic/Numerical
 - Many. Often not scale- or flow-aware
 - Griffies & Hallberg, 2000, is one aware example
- Fox-Kemper & Menemenlis, 2008. ECCO2.
 - Leith Viscosity (2d Enstrophy Scaling)
- Chen, Q., Gunzburger, M., Ringler, T., 2011
 - Anticipated Potential Vorticity of Sadourny
- San, Staples, Iliescu (2011, 2013)
 - Approximate Deconvolution Method
- Stochastic & Statistical Parameterizations
 - Other session going on now in Y10

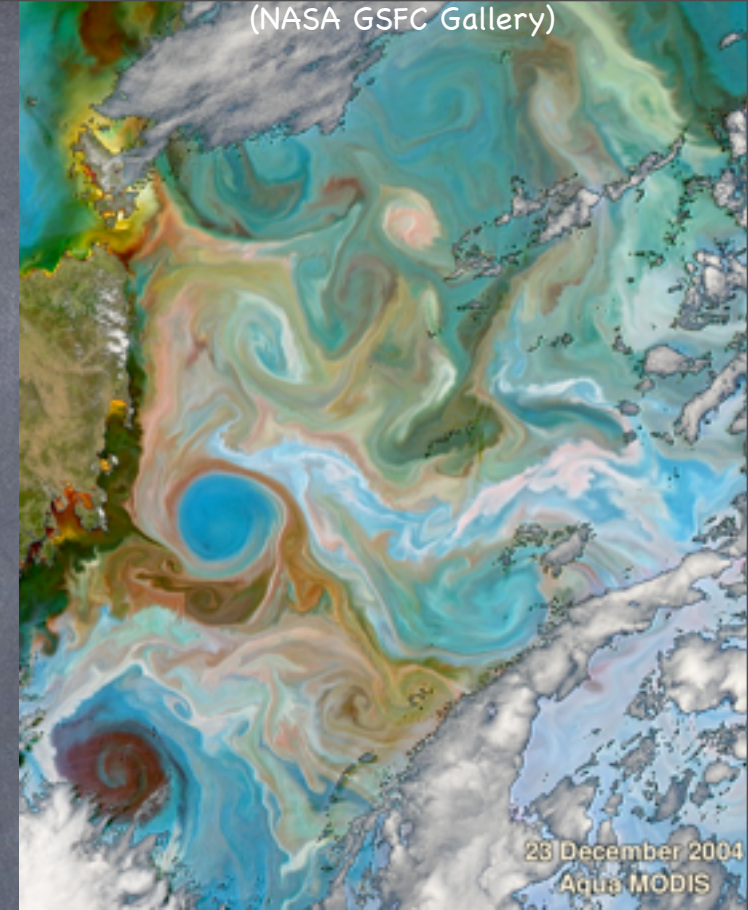


Graham & Ringler, 2013 Ocean Modelling

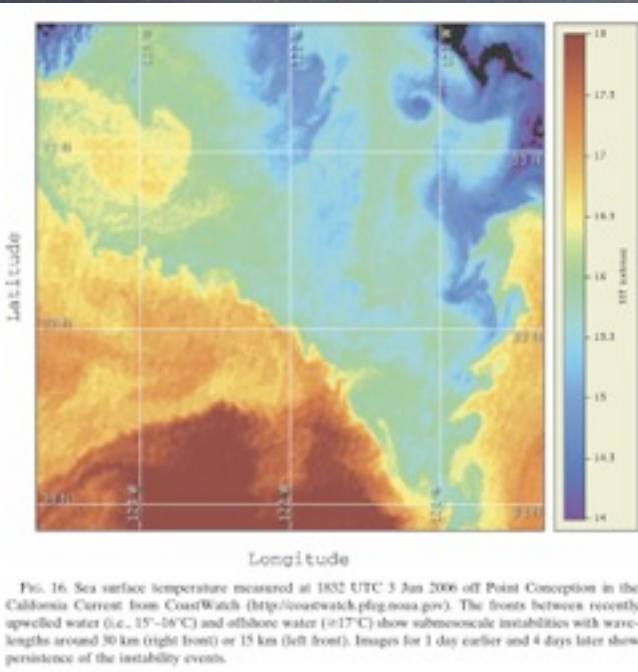
See also Ramachandran et al, 2013
Ocean Modelling for SMOLES

The Character of the Mesoscale

← 100 km



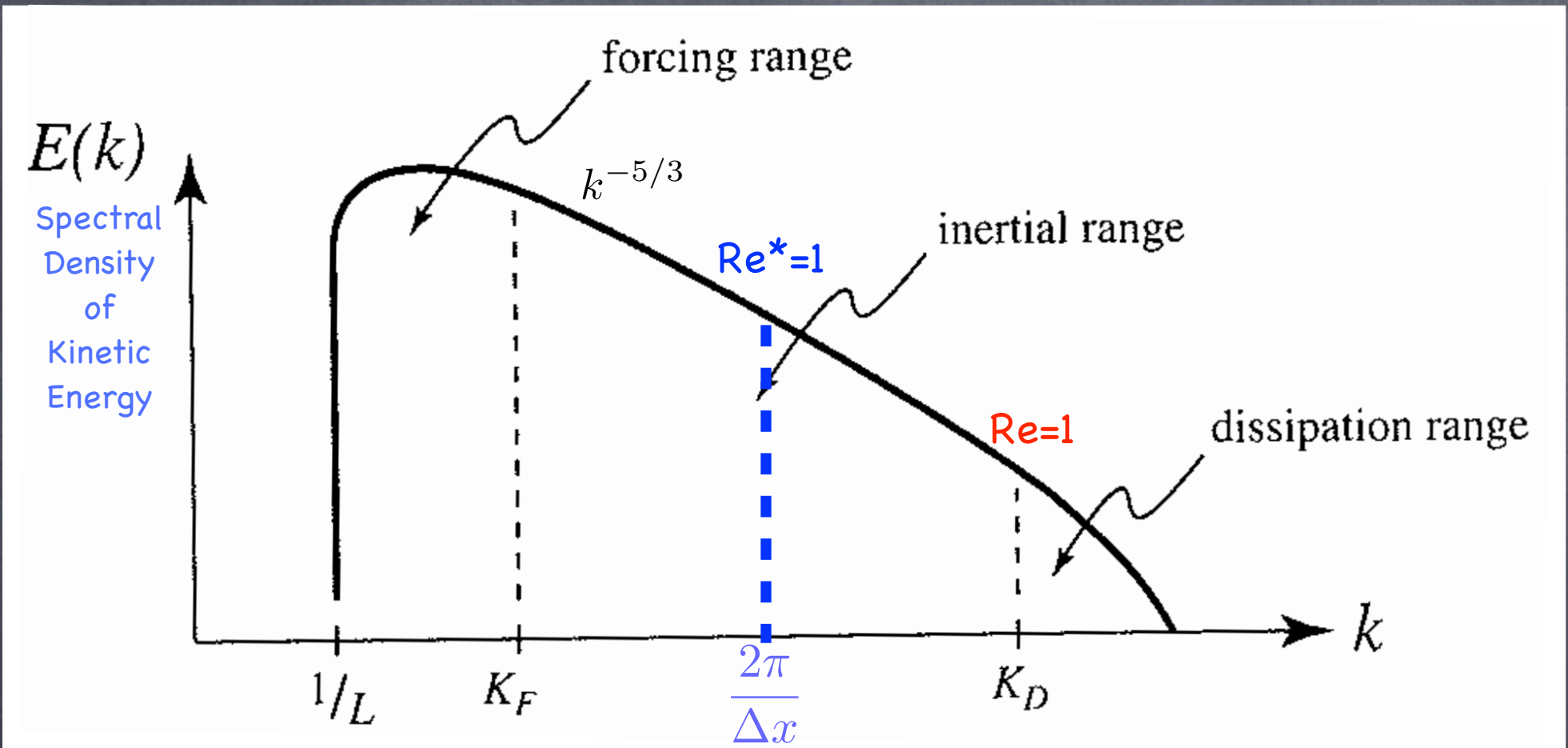
(Capet et al., 2008)



- Boundary Currents
- Eddies
- $Ro=O(0.1)$
- $Ri=O(1000)$
- Full Depth
- Quasi-2d
- Eddies strain to produce Fronts
- 100km, months

Eddy processes mainly **baroclinic & barotropic instability**.
Quasigeostrophy is likely to be very accurate.

3D Turbulence Cascade

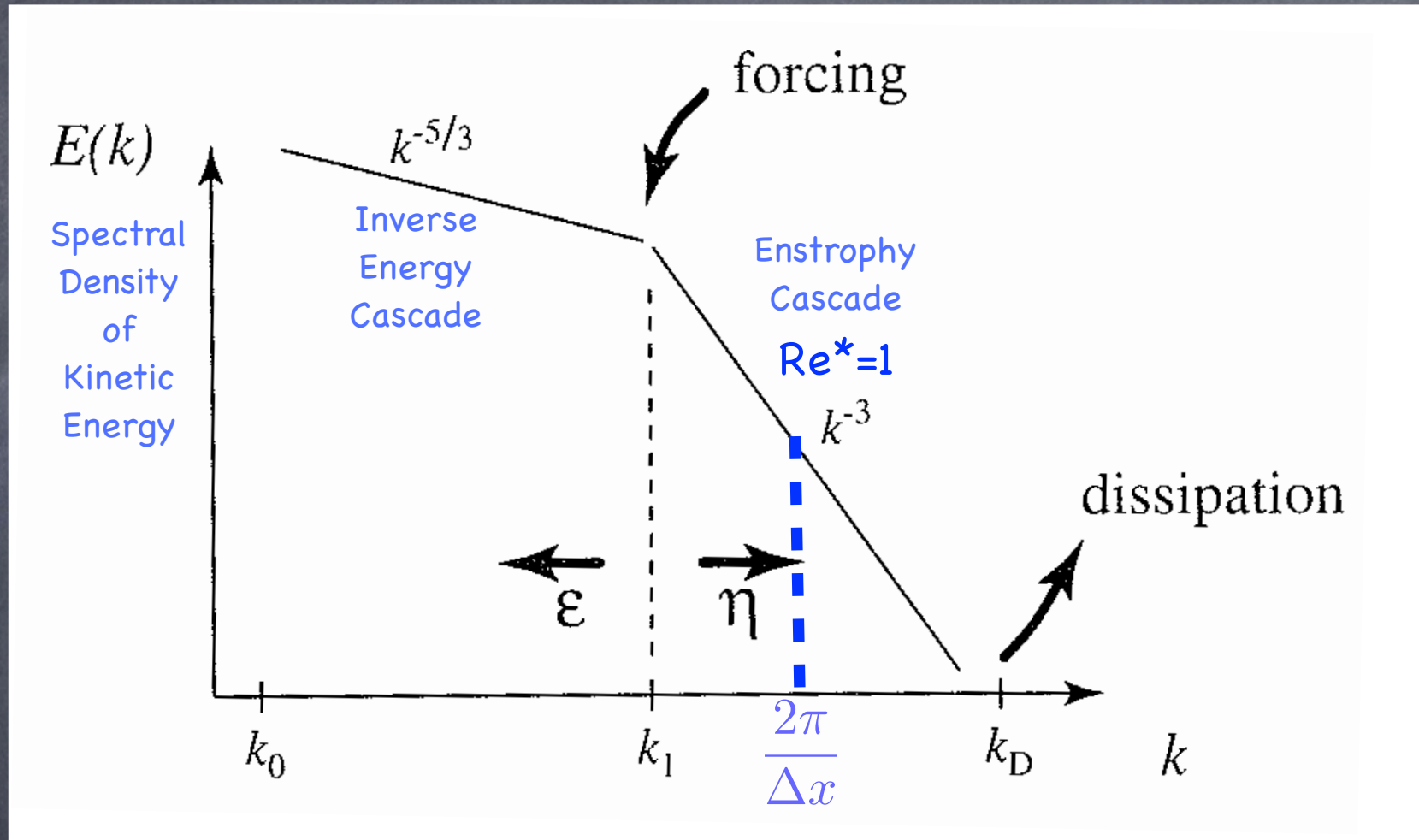


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2D Turbulence Differs

R. Kraichnan, 1967 JFM

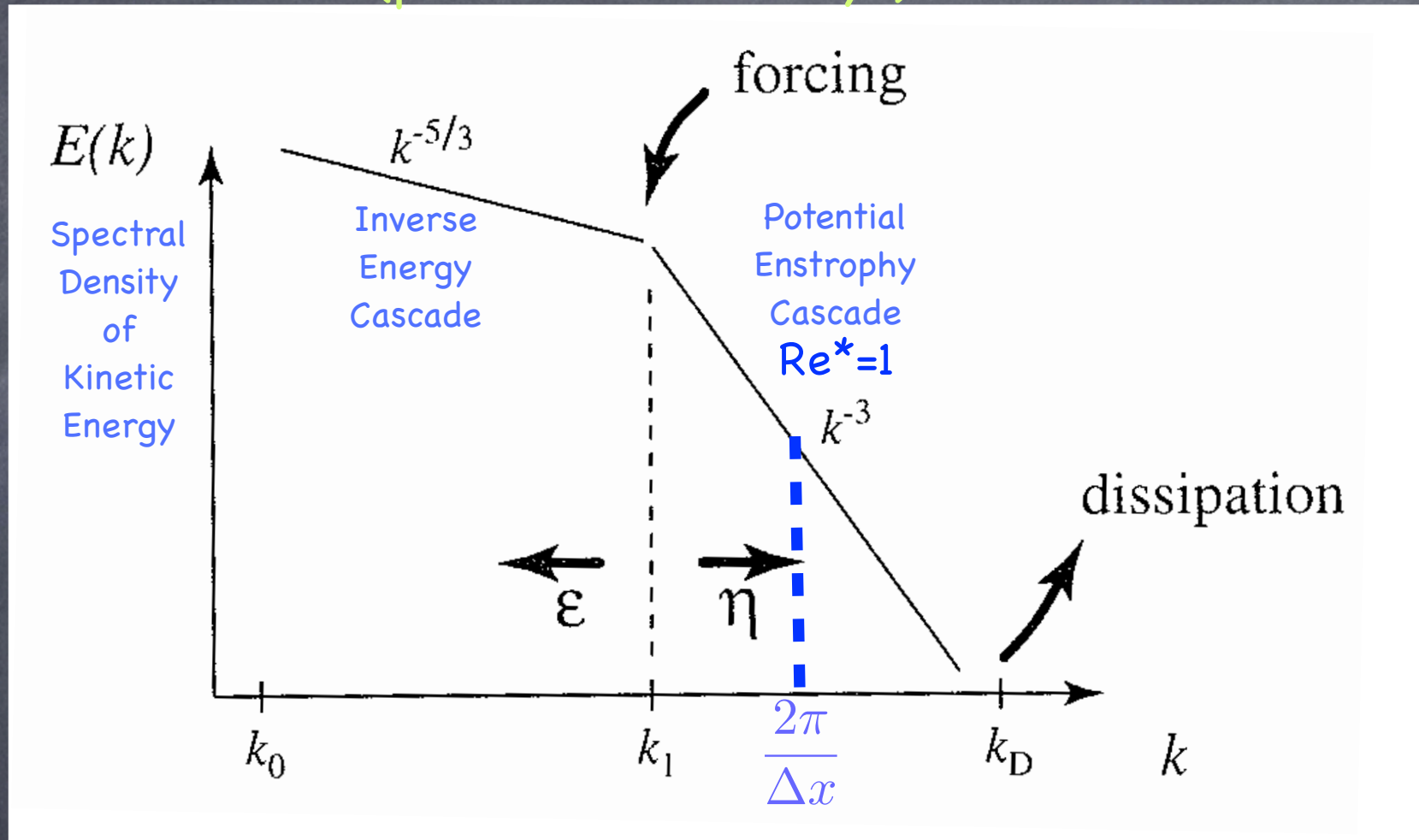


1996: Leith Devises Viscosity Scaling,
So that the Enstrophy (vorticity²) Cascade is Preserved

$$\mathbf{v}_* = \left(\frac{\Lambda \Delta x}{\pi} \right)^3 \left| \nabla_h \left(\frac{\partial u_*}{\partial y} - \frac{\partial v_*}{\partial x} \right) \right|$$

QG Turbulence: Pot'l Enstrophy cascade (potential vorticity²)

J. Charney, 1971 JAS



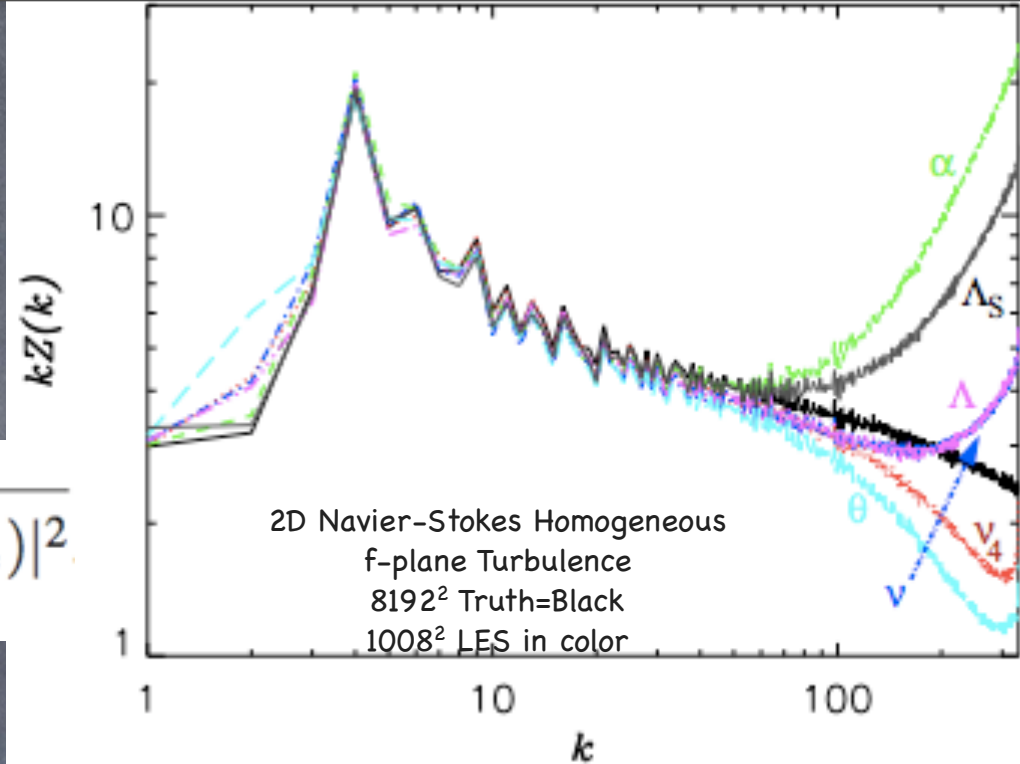
**F-K & Menemenlis '08: Revise Leith Viscosity Scaling,
So that diverging, vorticity-free, modes are also damped**

$$\mathbf{v}_* = \left(\frac{\Delta x}{\pi} \right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2}$$

B. Fox-Kemper and D. Menemenlis. Can large eddy simulation techniques improve mesoscale-rich ocean models? In M. Hecht and H. Hasumi, editors, Ocean Modeling in an Eddy Regime, volume 177, pages 319-338. AGU Geophysical Monograph Series, 2008.

Key Advantages of LeithPlus

$$\mathbf{v}_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2}$$



Graham & Ringler, 2013 Ocean Modelling

Fox-Kemper & Menemenlis, 2008. ECCO2.

- LeithPlus Viscosity (2d Enstrophy Scaling)

- At low Rossby number, becomes same as Leith, which is a good scaling for low Rossby

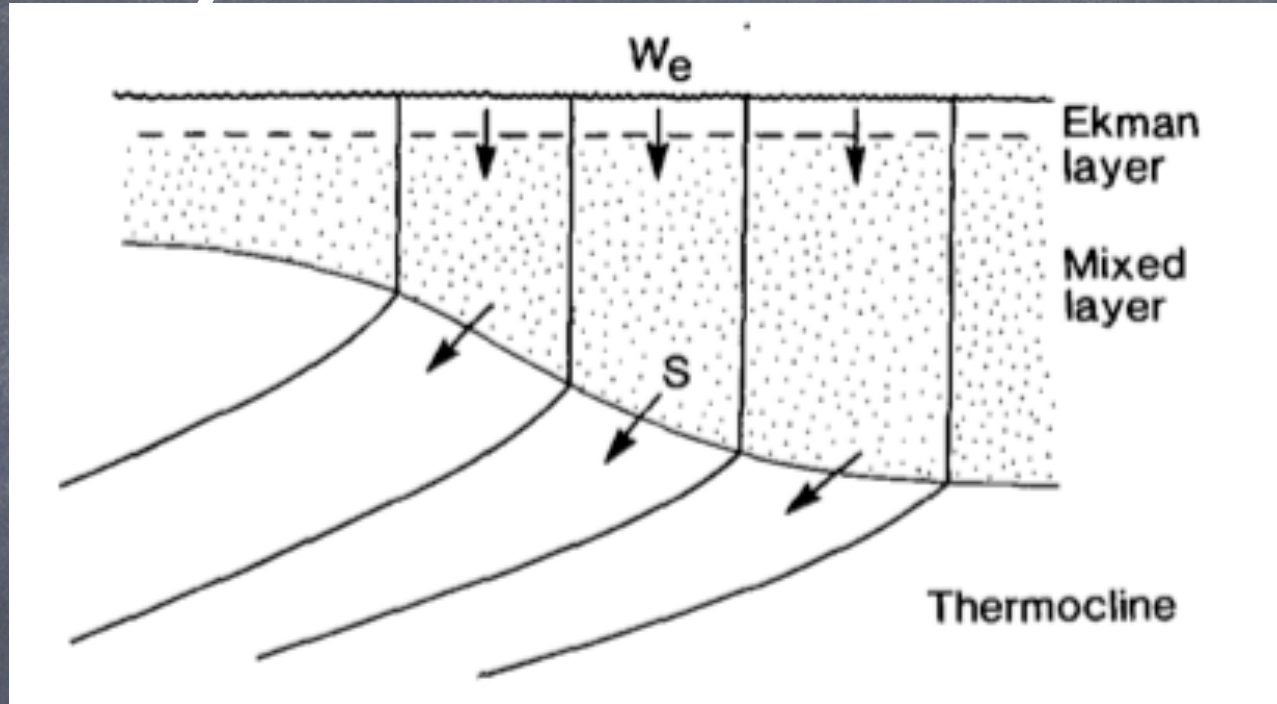
- LeithPlus Allows longer timesteps than Leith (by damping converging motions-->smaller w).

- Excellent numerical stability/damping properties

- Leith does a good job for 2-d flows

- More scale-selective than Smagorinsky or Biharmonic Smagorinsky

Is 2D Turbulence a good proxy for neutral flow?



Yes:

- For a few eddy time-scales QG & 2D AGREE (Bracco et al. '04)
- Barotropic Flow-- Obvious 2d analogue

No:

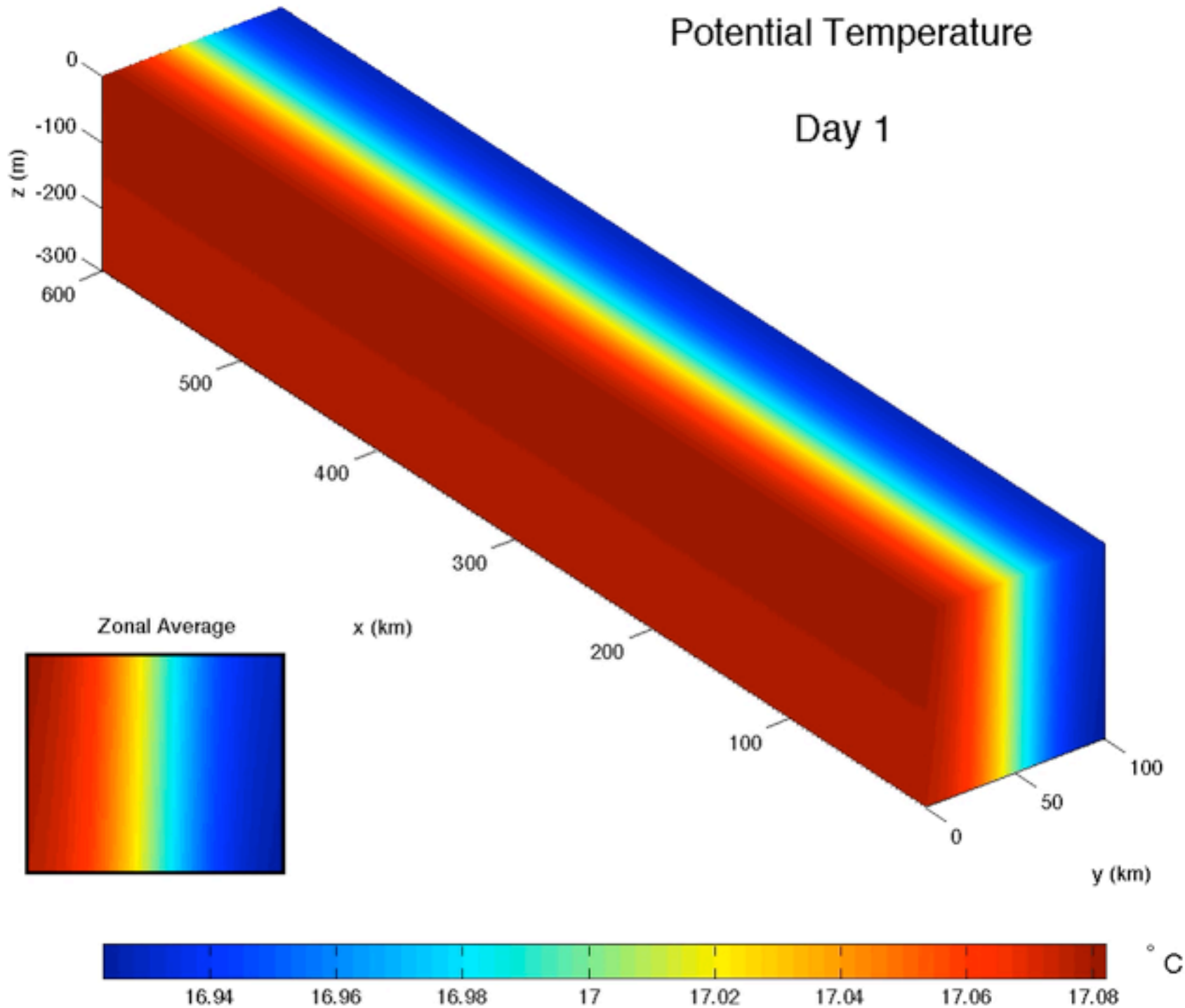
Nurser & Marshall, 1991 JPO

- Bolus Fluxes-- Divergent 2d flow
- Sloped, not horiz.
- Surface Effects?

Movie: S. Bachman

Potential Temperature

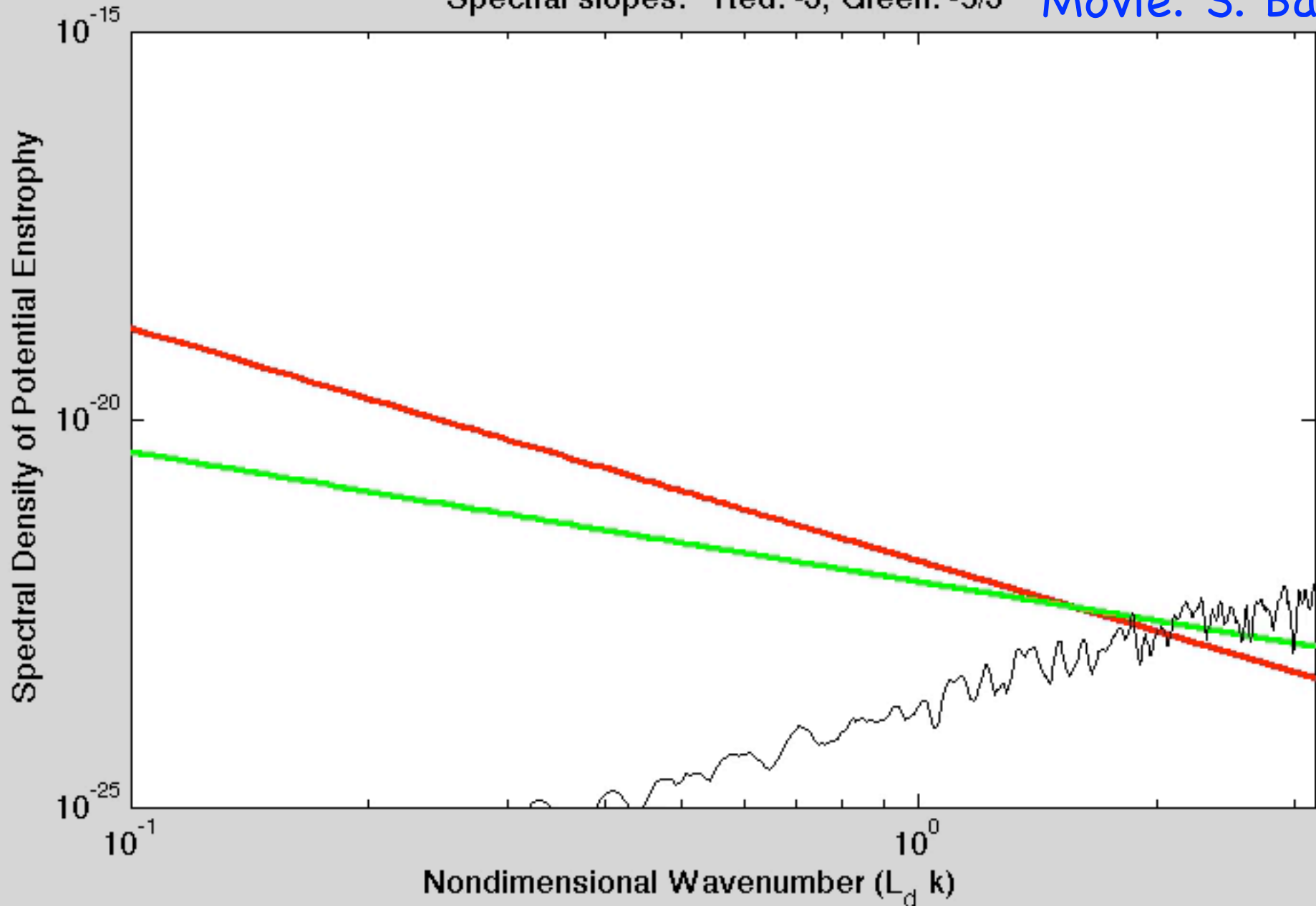
Day 1



S. Bachman and
B. Fox-Kemper.
Eddy
parameterization
challenge suite. I:
Eddy spindown.
Ocean Modelling,
64:12-28, 2013.

Cascade of Pot'l Enstrophy?

Spectral slopes: Red: -3, Green: -5/3 Movie: S. Bachman



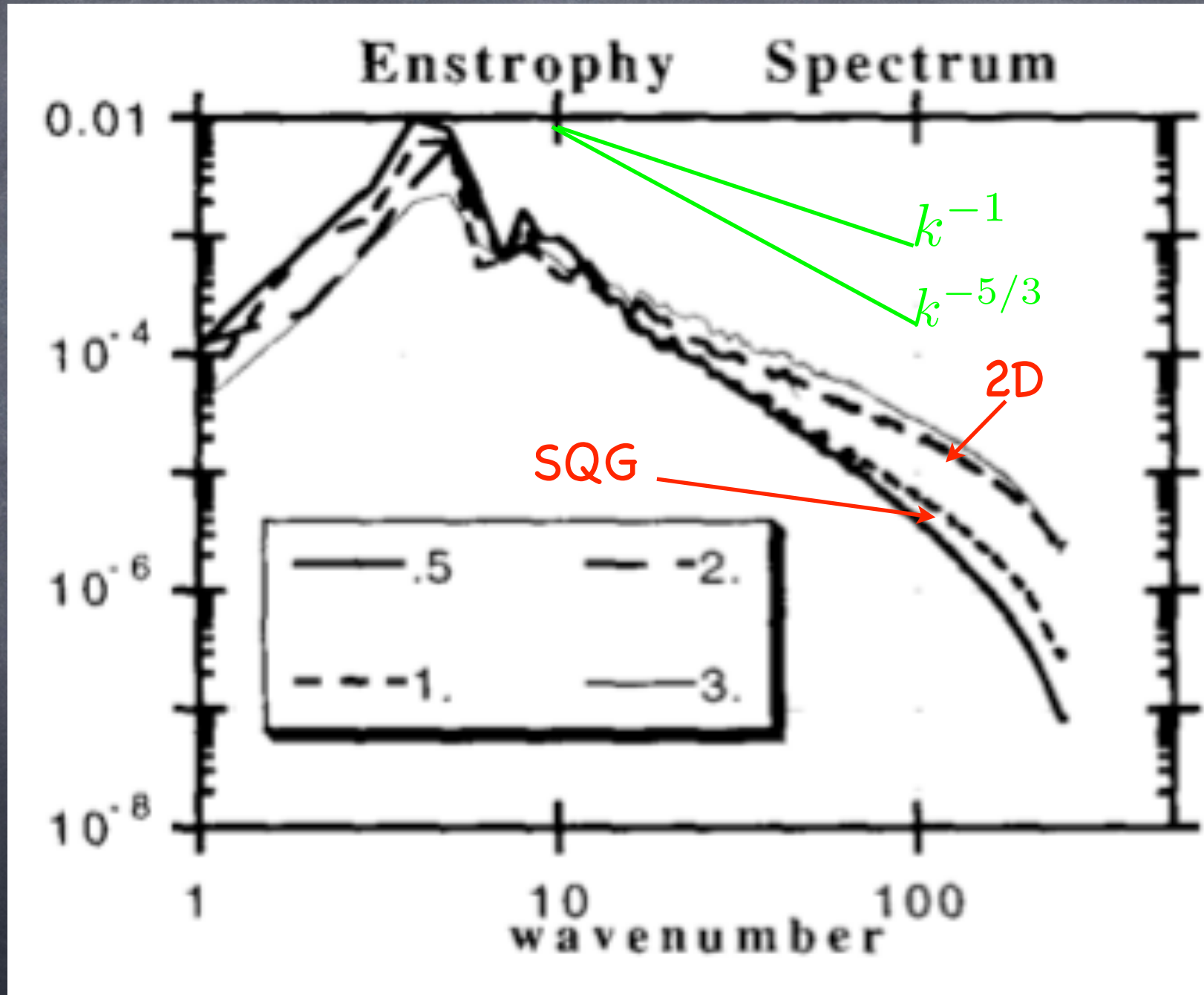
In real stratified flows,

things are a bit more complex than in 2d

Even more than QG...

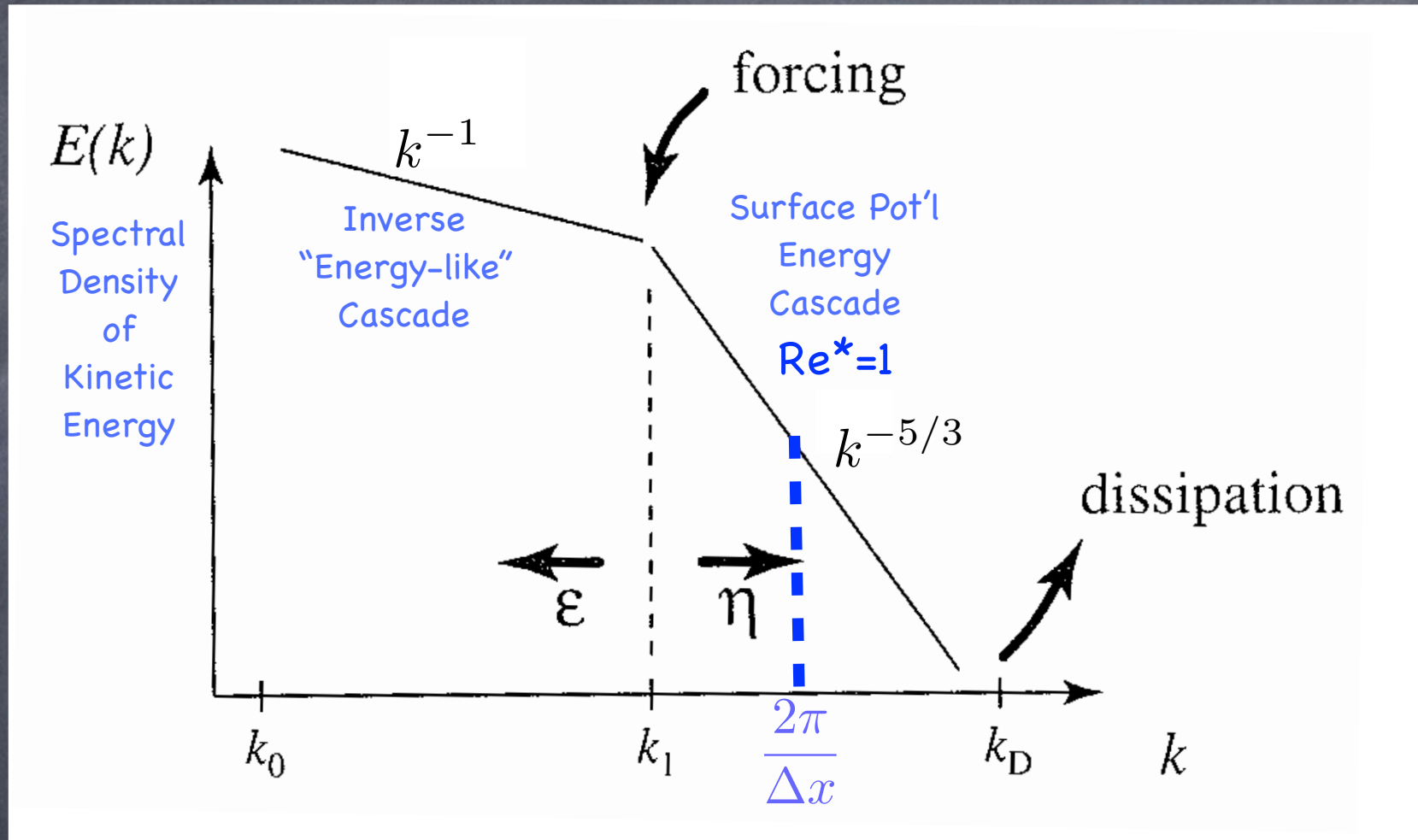
Surface Effects may dominate

Pierrehumbert, Held, Swanson, 1994 Chaos Spectra of Local and Nonlocal Two-dimensional Turbulence



SQG Turbulence: Surface Buoyancy & Velocity cascade

W. Blumen, 1978 JAS
 Held et al 1995, JFM.
 Smith et al. 2002, JFM



Smag-Like:
$$\kappa_* = \left(\frac{\Upsilon \Delta x}{\pi} \right)^{4/3} \left| \frac{1}{f} \nabla_h b \right|^{2/3}$$

Leith-Like:
$$\kappa_* = \left(\frac{\Lambda \Delta x}{2\pi} \right)^{3/2} \left[-\frac{\partial}{\partial z} |\nabla_h \psi|^2 \right]^{1/2}$$

Many observations tell us:

The spectrum of potential density and buoyancy often scales as k^{-2} , which isn't too far from $k^{-5/3}$

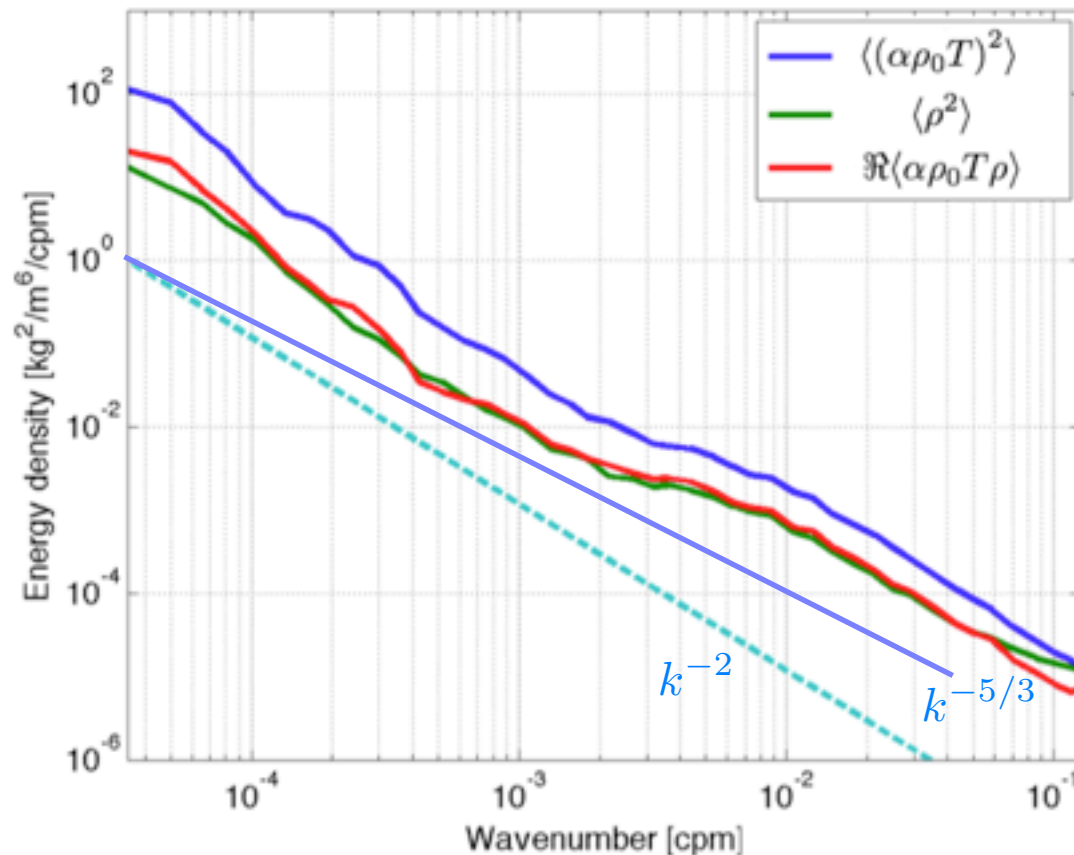
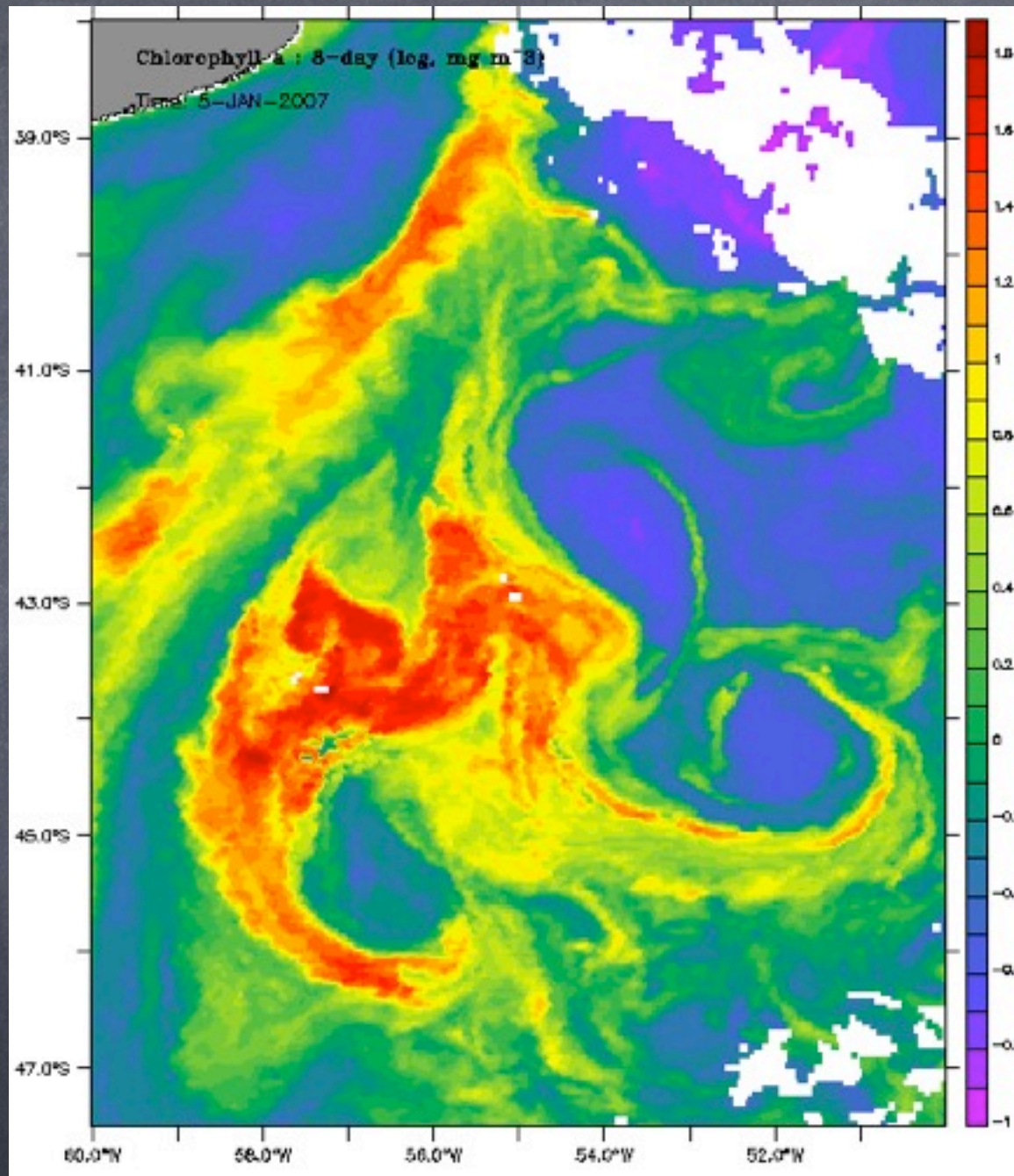


Figure 1: Observed spectra of mixed layer potential density variance (green), temperature contribution to potential density (blue), and temperature-density co-spectrum (red) from SeaSoar towed CTD and shipboard ADCP sections (data from Ferrari and Rudnick, 2000). A dashed line indicates k^{-2} scaling.

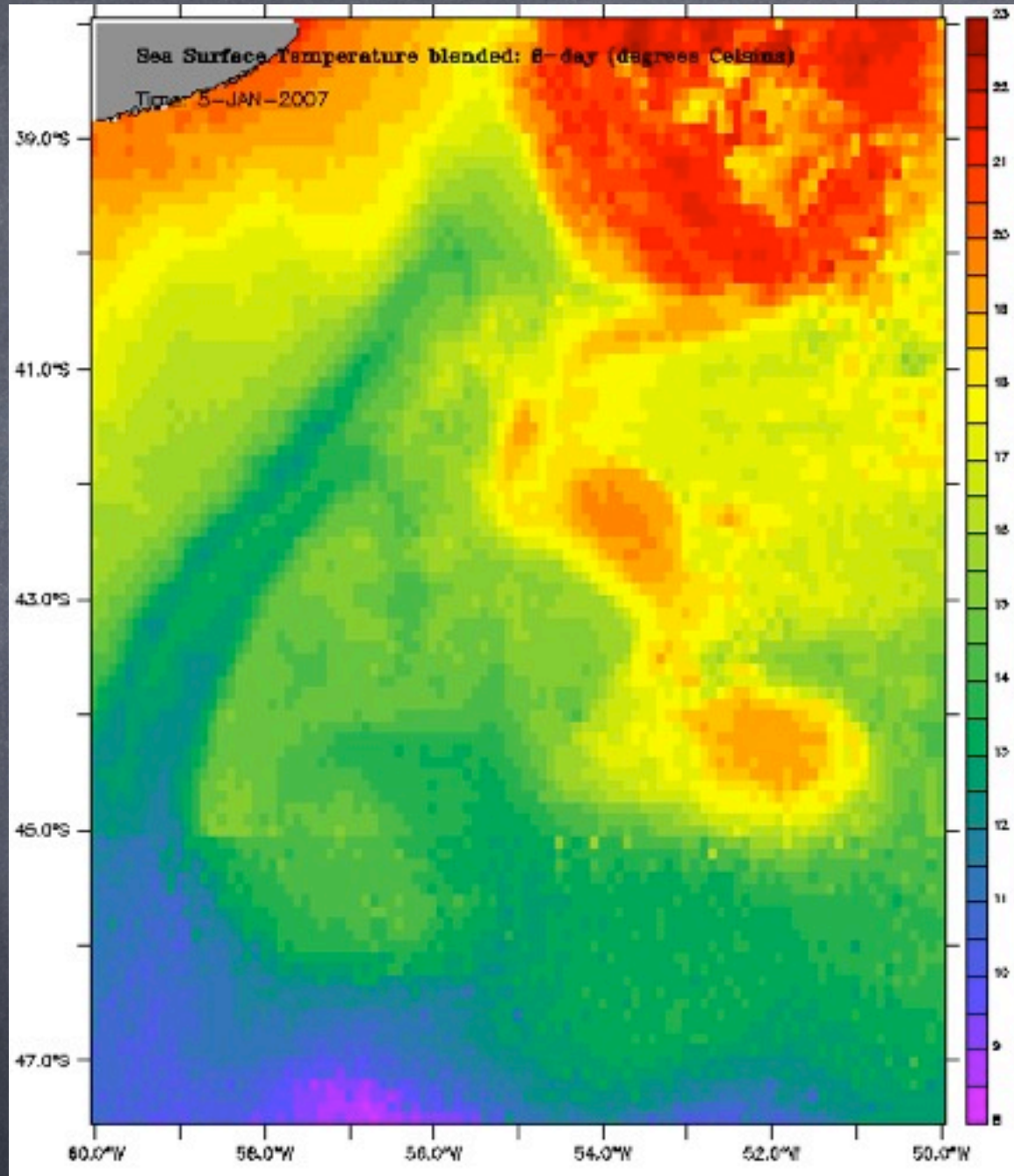
B. Fox-Kemper,
G. Danabasoglu,
R. Ferrari, S. M. Griffies,
R. W. Hallberg, M. M.
Holland, M. E. Maltrud,
S. Peacock, and B. L.
Samuels. Parameterization
of mixed layer eddies. III:
Implementation and impact
in global ocean climate
simulations. *Ocean
Modelling*, 39:61-78, 2011.

Examples: Jan 5, 07 East of Argentina



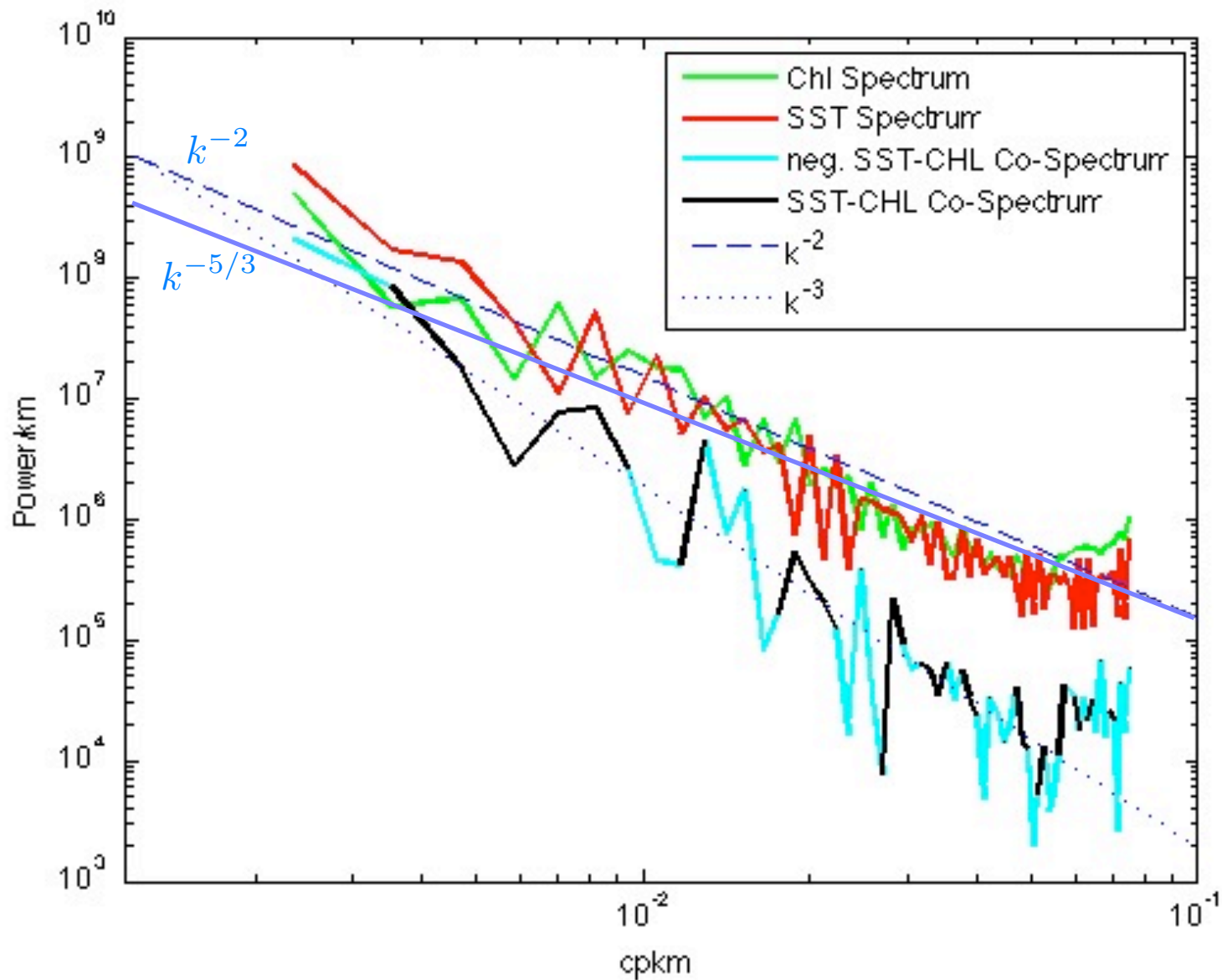
MODIS on Aqua Chl

Examples: Jan 5, 07 East of Argentina



Remote Sensing Systems Inc. (www.remss.com) Blended SST blended

Spectra: Jan 5, 07 East of Argentina



It is not clear that inertial ranges exist.

This spectrum shows that topographic interactions change the spectrum at depth dramatically

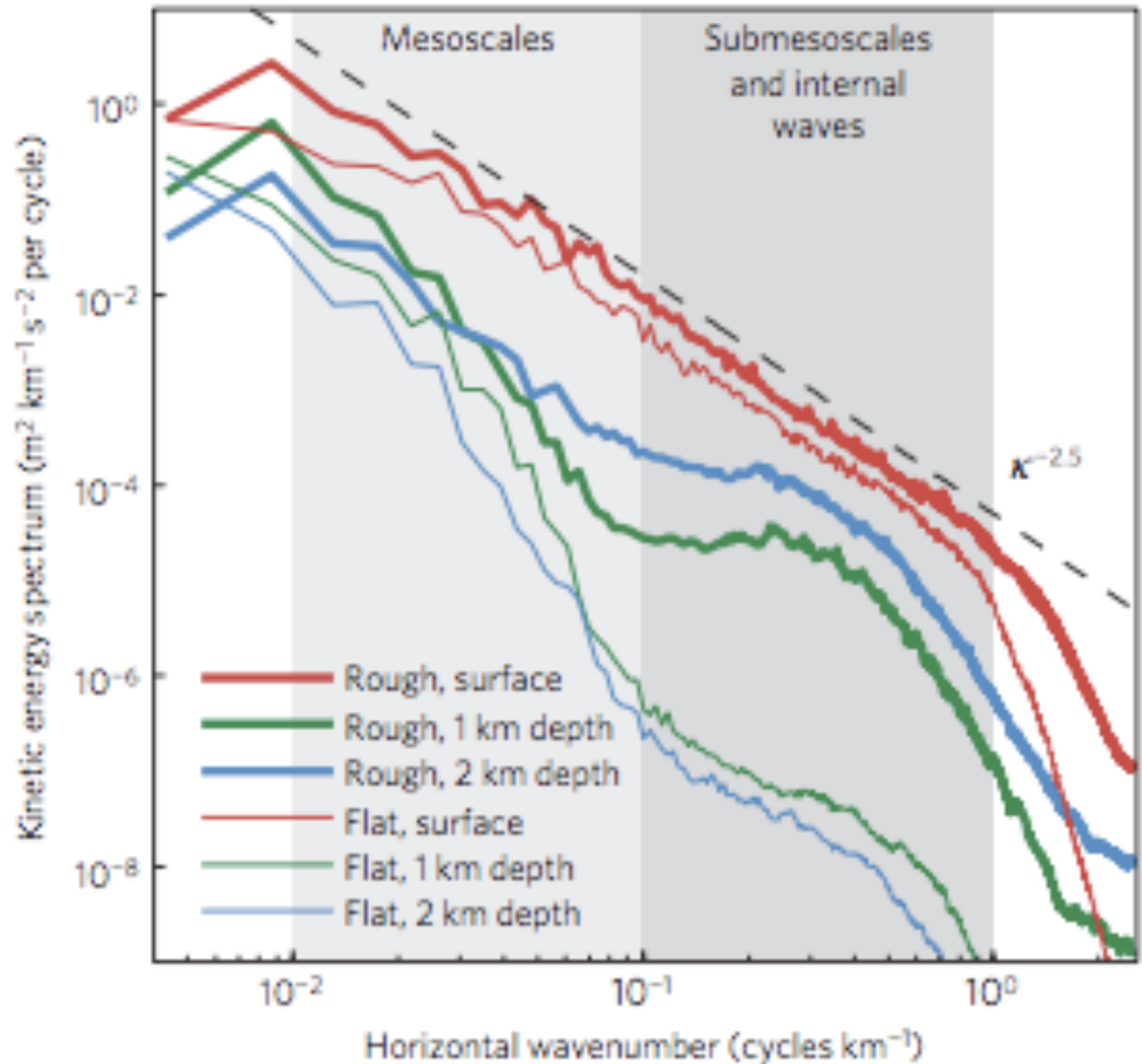


Figure 4 | Horizontal wavenumber kinetic energy spectra. Spectra

Reynolds vs. Péclet: Prandtl=1?

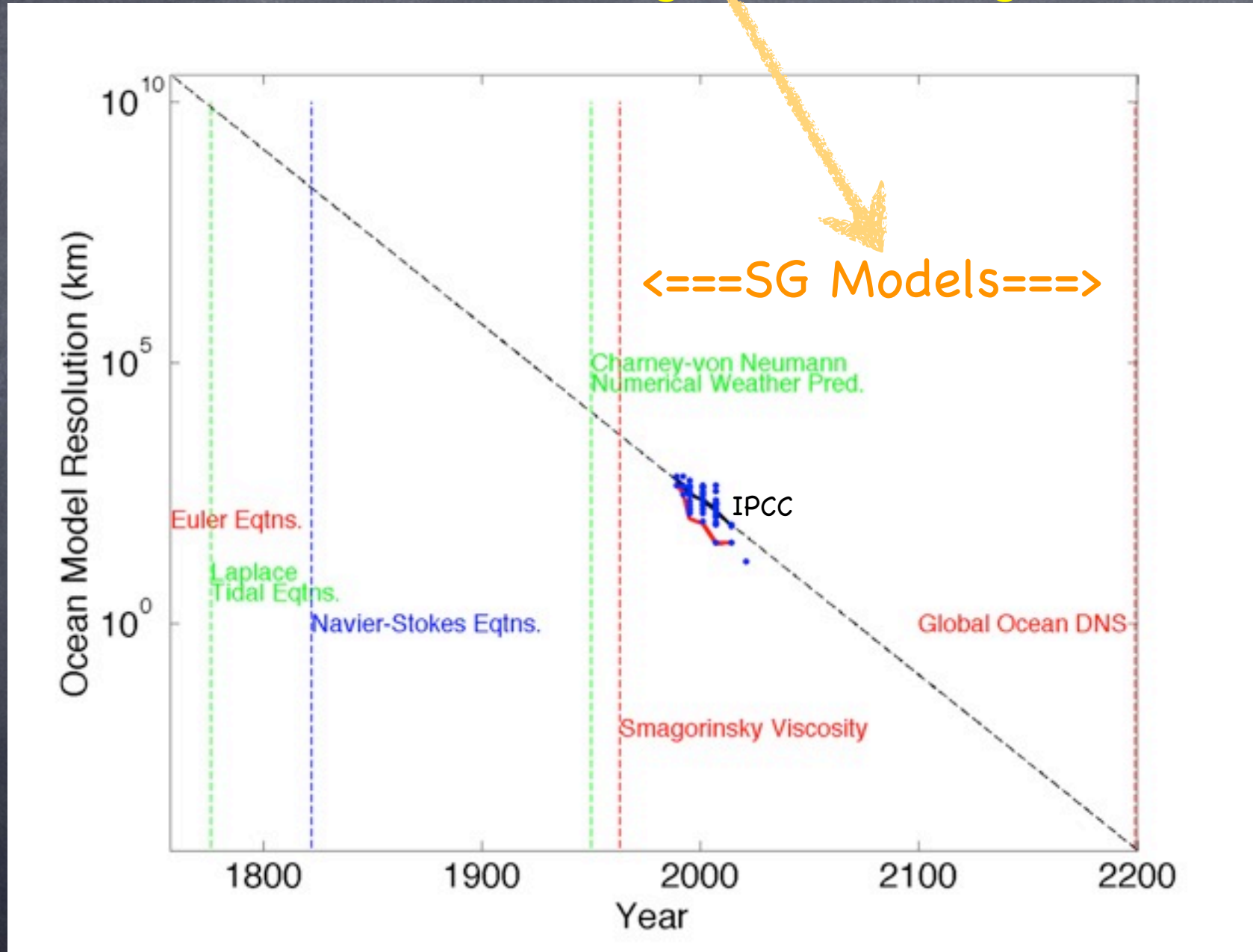
- In all cascade examples, the truncation occurs at large Reynolds and Péclet, so it is reasonable to assume diffusivity=viscosity
- In the QG framework, diffusivity *must* equal viscosity to avoid spurious generation of potential vorticity by the subgrid model
- In the SQG framework, it is surface diffusivity of buoyancy that rules-- associating viscosity is only convenient

A Prescription for Parameterization...

Accuracy TBD

- QG Leith & Potential Vorticity to generate #1 viscosity
- 2D Leith & Barotropic Vorticity to generate #2 viscosity
- SQG Leith & Surf. Buoyancy to generate #3 diffusivity
- Take $\max(\#1, \#2, \#3)$ as viscosity, Redi diffusivity, *and* as GM transfer coeff. Nearly suggested by Roberts & Marshall, 98, JPO
- Note: Unlike Eddy-Free closures, e.g., Visbeck et al (97), Eddy-Rich closures take advantage of resolved eddies & instabilities, only need a boost from eddy-permitting to eddy-resolving (and for numerical stability)

Extrapolate for historical perspective: The Golden Era of Subgrid Modeling is Now!



All papers at: fox-kemper.com/research

Conclusions

- Climate modeling is challenging partly due to the vast and diverse scales of fluid motions
- Physically-motivated subgrid models result when the gridscale is in an approximate inertial range
 - E.g., Leith Family are good for MOLES
- Viscosity, diffusivity, and streamfunction closures can be found in this case
- Improved Large Eddy Simulations will* result: boundary current separation, SSH & SST variance, spectral properties, tracer transport, fewer arbitrary parameters

*my conjecture

All papers at: fox-kemper.com/research

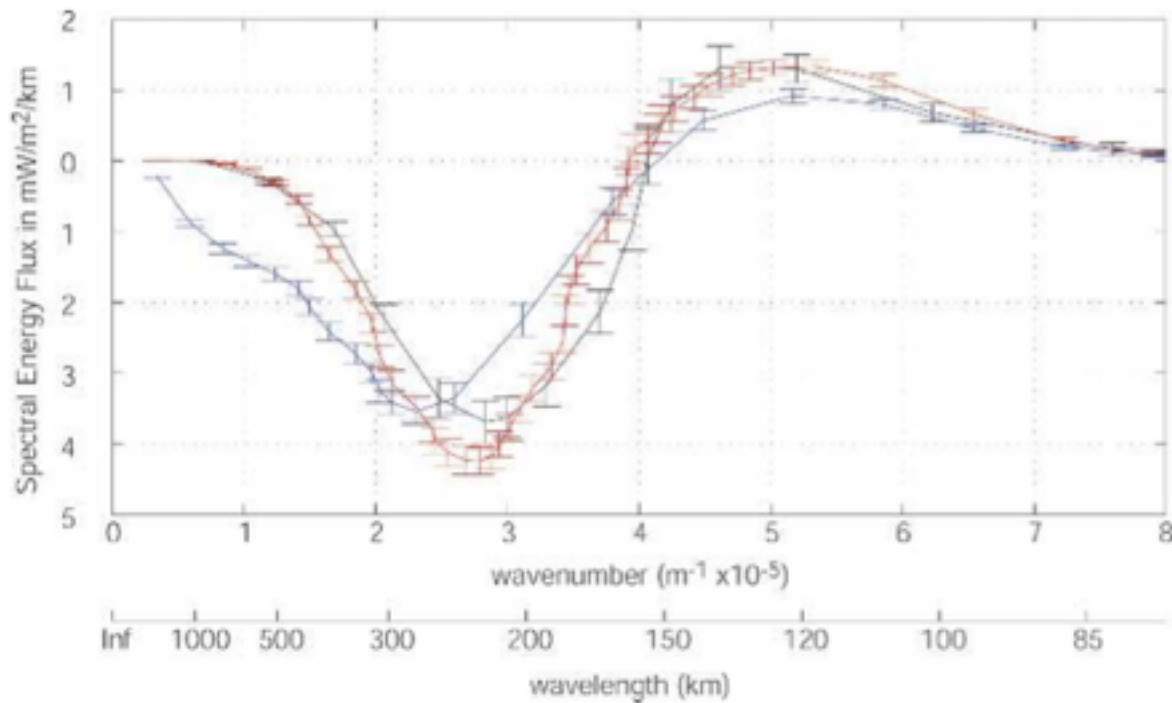
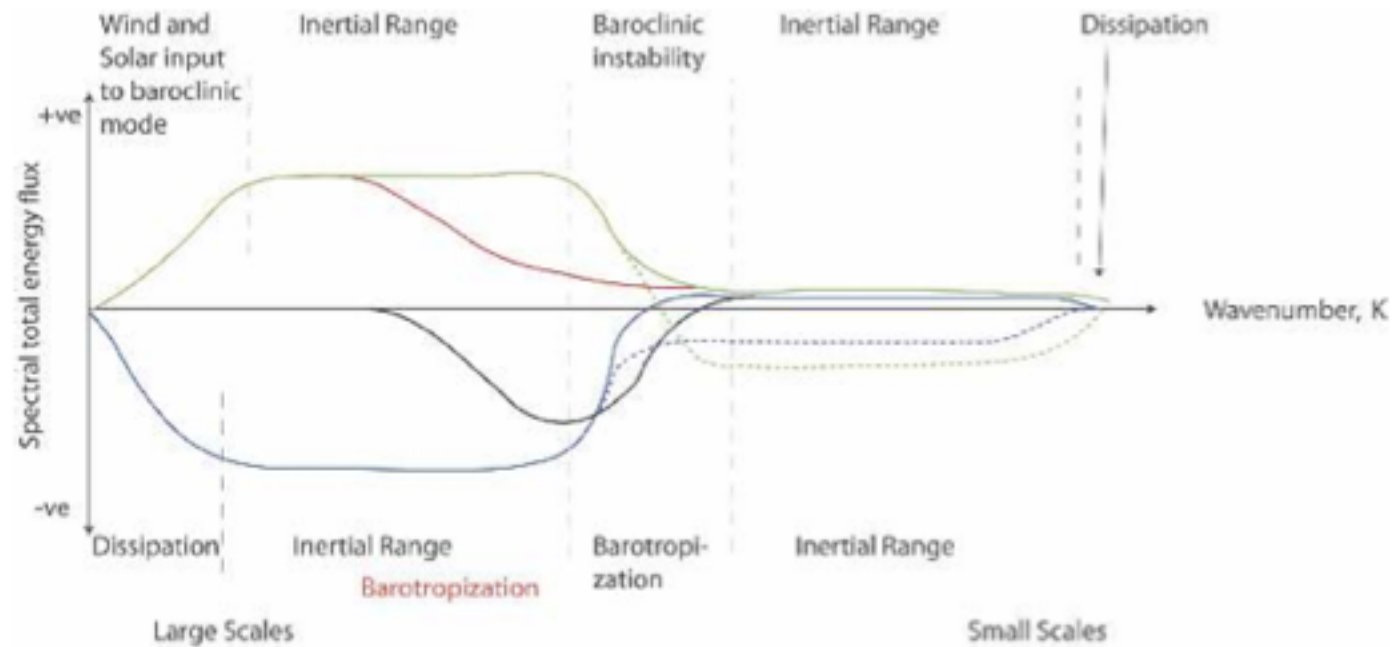
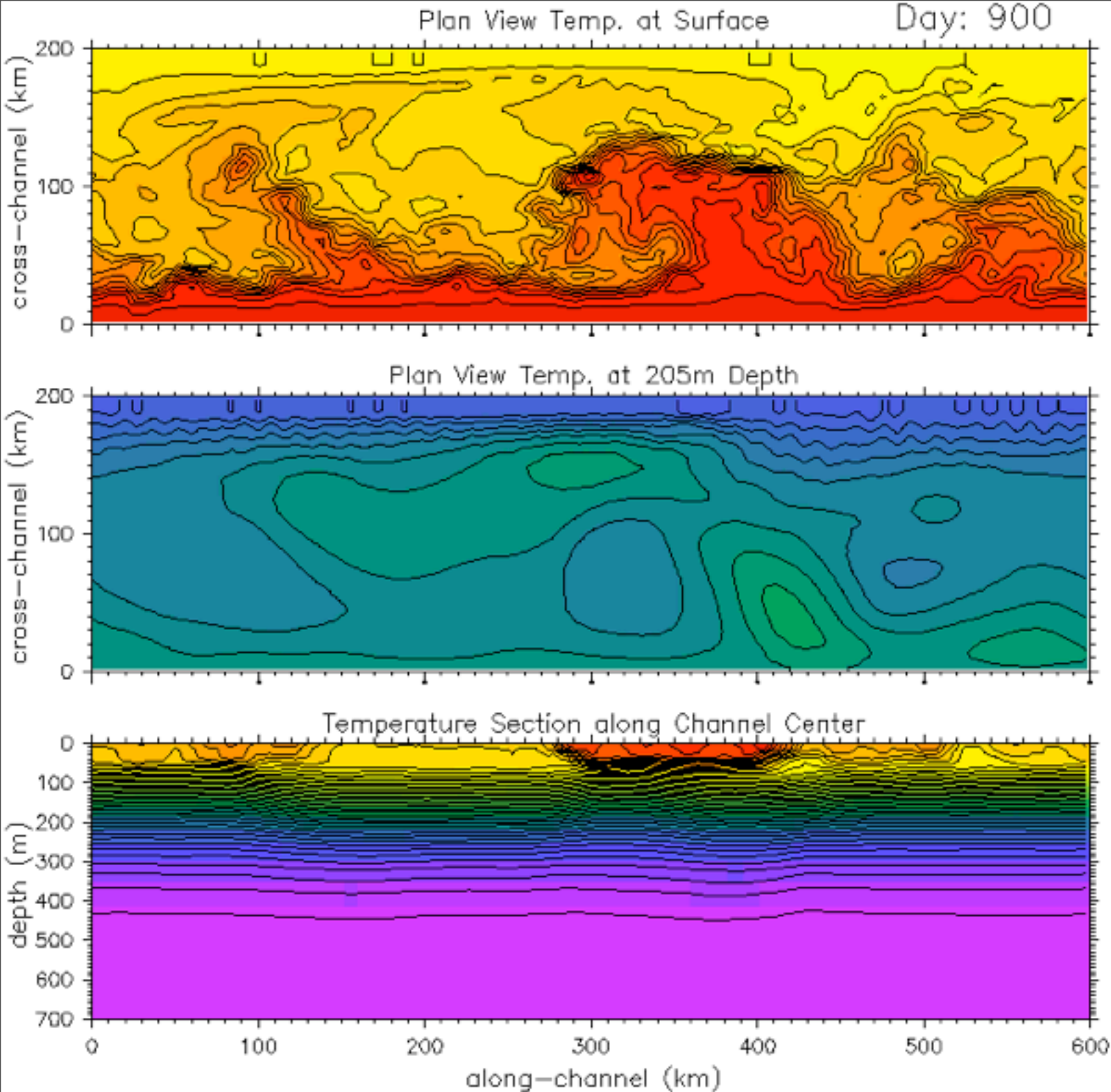


FIG. 2. Time mean, spectral kinetic energy flux $\bar{\Pi}(K)$ vs total wavenumber K in a homogeneous ACC region (rectangles centered at 57°S , 120°W): black curve using SSH on a 32×32 grid, red curve using SSH on a 64×64 grid, blue curve using velocity on a 64×64 grid. Positive slope reveals a source of energy. The larger negative lobe reveals a net inverse cascade to lower wavenumber. Error bars represent standard error.

Implications from
Scott & Wang (05)

Lengthscales of
Energy
Transfer



Big, Deep
(meso)

interact
with

Little,
Shallow
(submeso)

B. Fox-Kemper, R. Ferrari,
and R. W. Hallberg.
Parameterization of mixed
layer eddies. Part I: Theory
and diagnosis. *Journal of
Physical Oceanography*,
38(6):1145-1165, 2008.

A Global Parameterization of Mixed Layer Eddy Flow & Scale Aware Restratification validated against simulations

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

$$\overline{\mathbf{u}'b'} \equiv \Psi \times \nabla \bar{b}$$

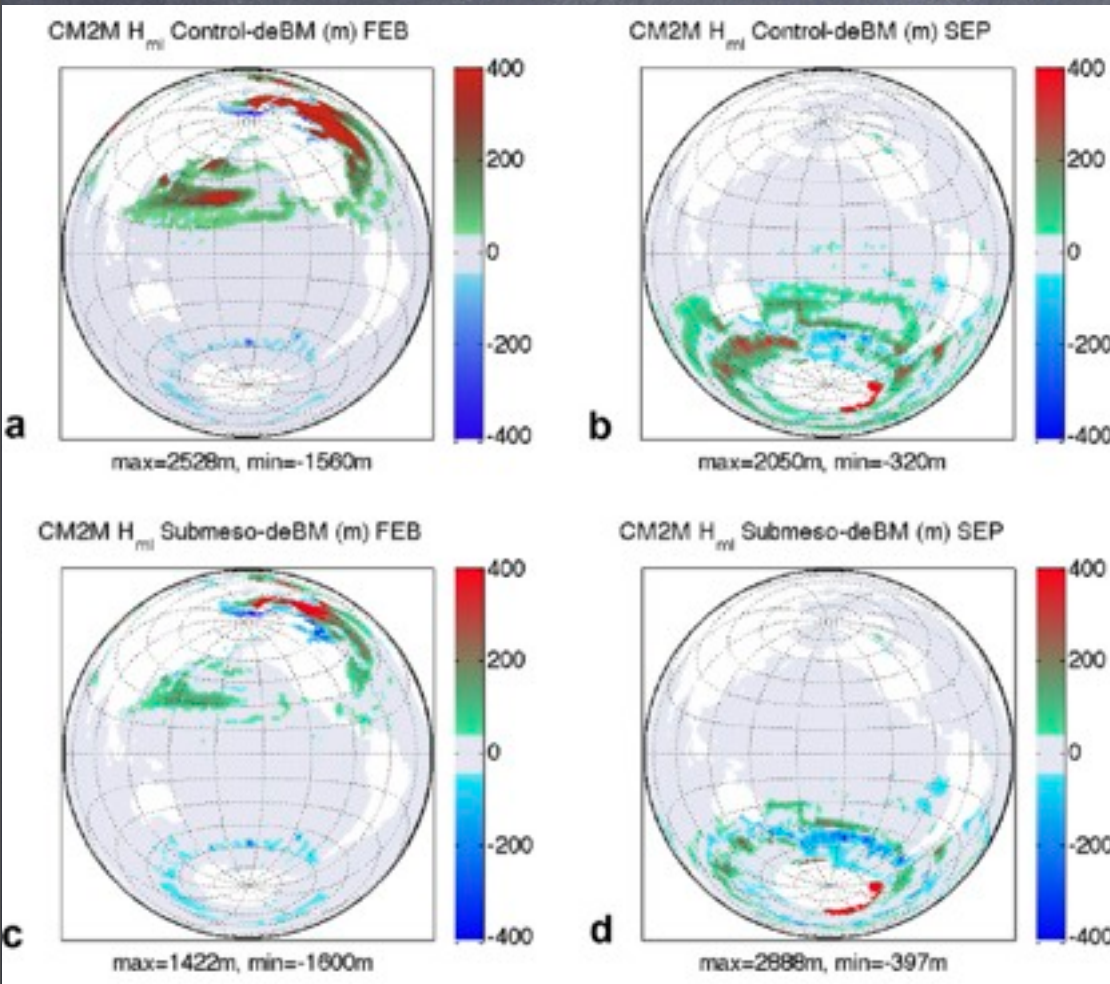
$$\Psi = \left[\frac{\Delta x}{L_f} \right] \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla \bar{b} \times \hat{\mathbf{z}}$$

Compare to the original **singular, unrescaled** version

$$\Psi = \left| \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{\mathbf{z}} \right.$$

New version **handles the equator**, and **averages over many fronts**

Physical Sensitivity of Ocean Climate to MLE: Mixed Layer Eddy Restratification

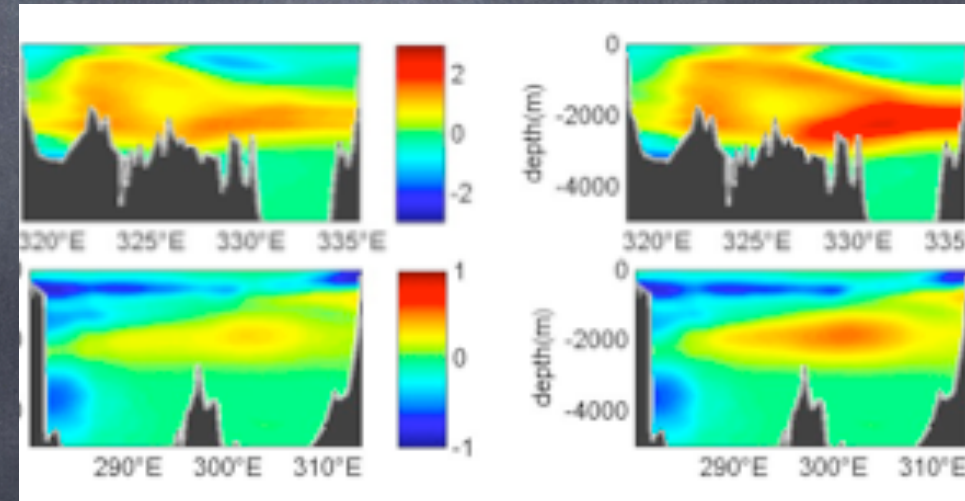


Bias
w/o
MLE

Improves CFCs
(water masses)

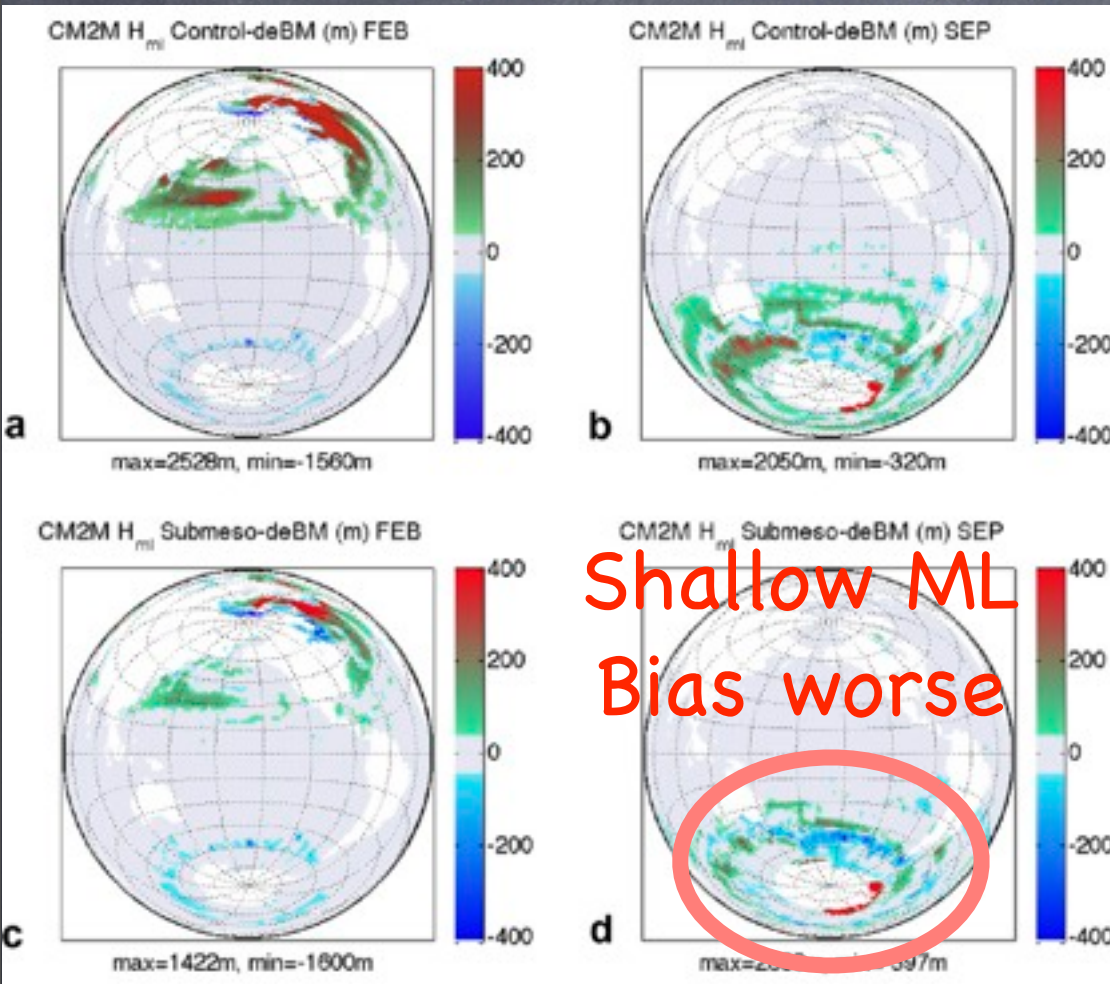
Bias with MLE

Bias w/o MLE



B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels.
Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

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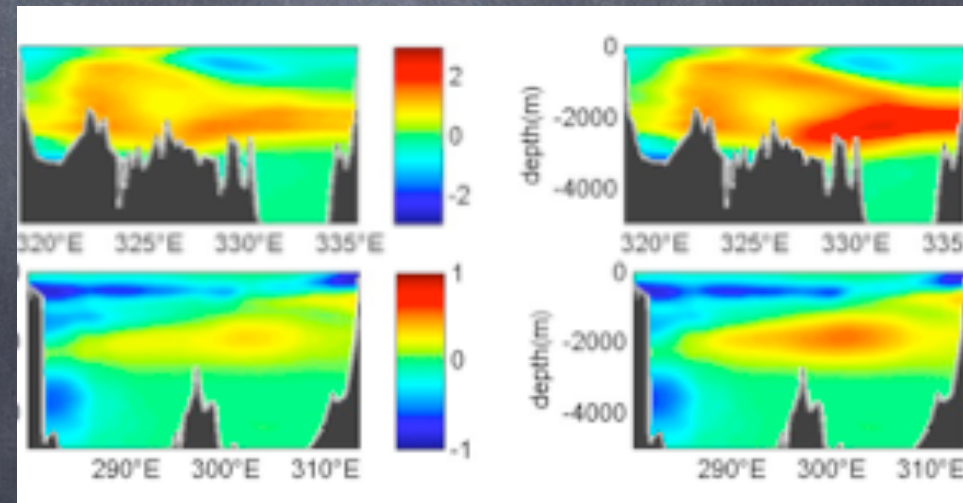


Bias
w/o
MLE

Improves CFCs
(water masses)

Bias with MLE

Bias w/o MLE



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 Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.