A Refined Life at High Resolution: Subgrid Modelling in the Eddy-Rich Regime

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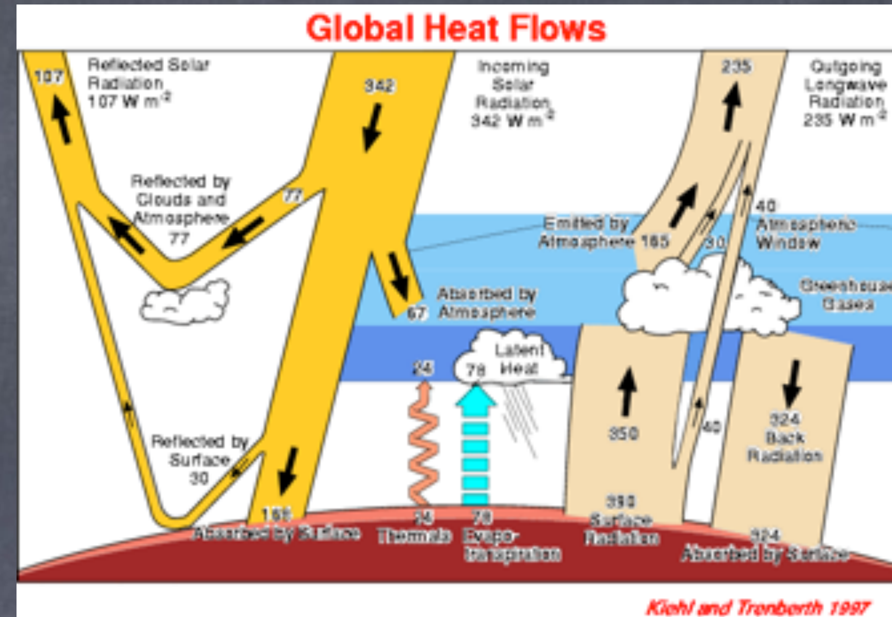
Major Contributions from Scott D. Bachman (CU-PhD, now Cambridge, UK),

Thanks for discussions with:
Stephen Griffies (GFDL), Frank Bryan, Peter Gent & John Dennis (NCAR),
Keith Julien (CU), Jim McWilliams (UCLA), Traian Iliescu (VA Tech)

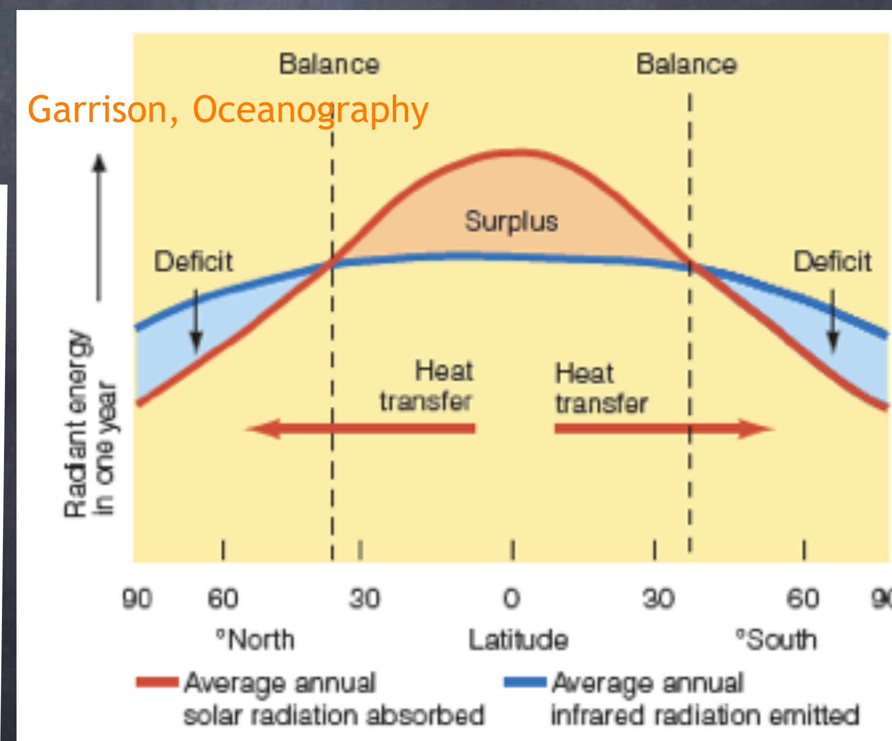
WHOI Walsh Cottage
July 16, 10:30-11:30

Sponsors: NSF 0855010, 0825614, Brown U.

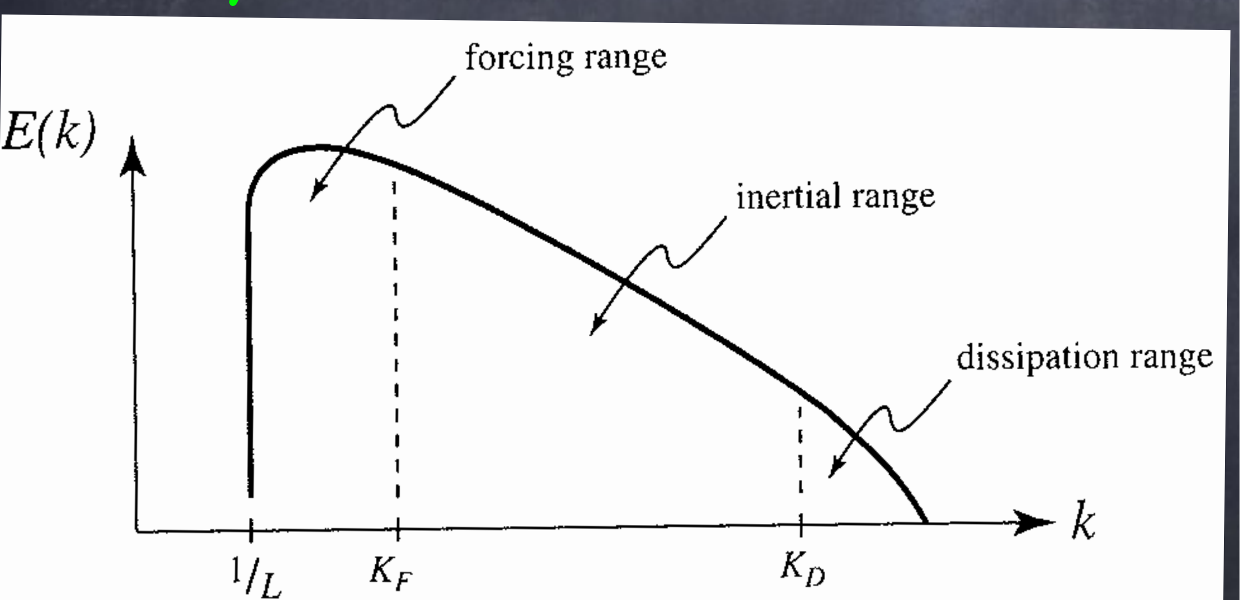
The Earth's Climate System is driven by the Sun's light (minus outgoing infrared) on a global scale



Dissipation concludes turbulence cascades on scales nearly a trillion times smaller



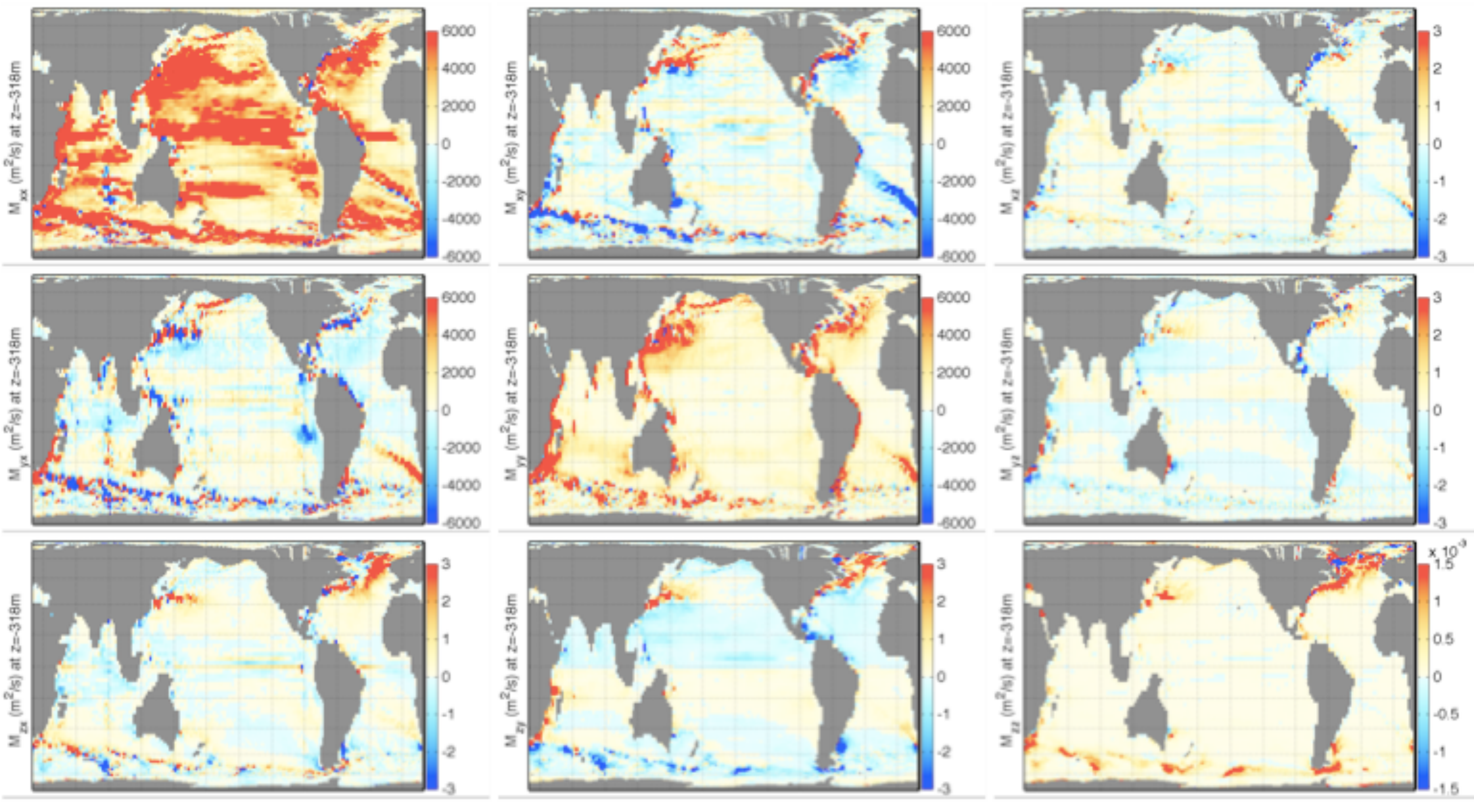
Garrison, Oceanography



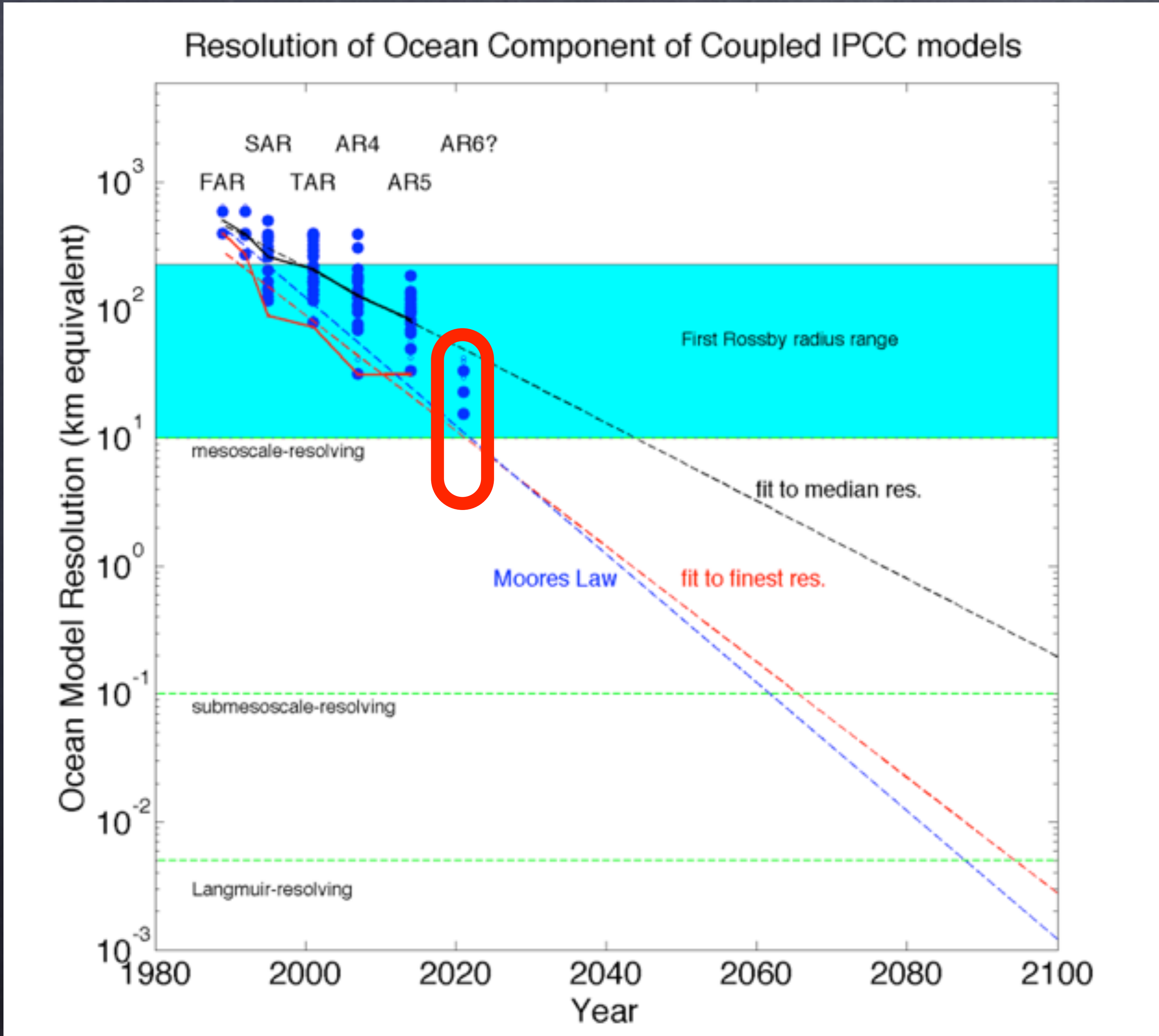
What does it take to parameterize eddies in an eddy-free model? A lot of scaling laws...

$$\overline{\mathbf{u}'\tau'} = -\mathbf{M}\nabla\overline{\tau}$$

B. Fox-Kemper, R. Lumpkin, and F. O. Bryan. Lateral transport in the ocean interior. In G. Siedler, J. Church, J. Gould, and S. Griffies, editors, Ocean Circulation and Climate - Observing and Modelling the Global Ocean. Elsevier, 2013. In press.



Resolution will be an issue for centuries to come!



Today's Focus:

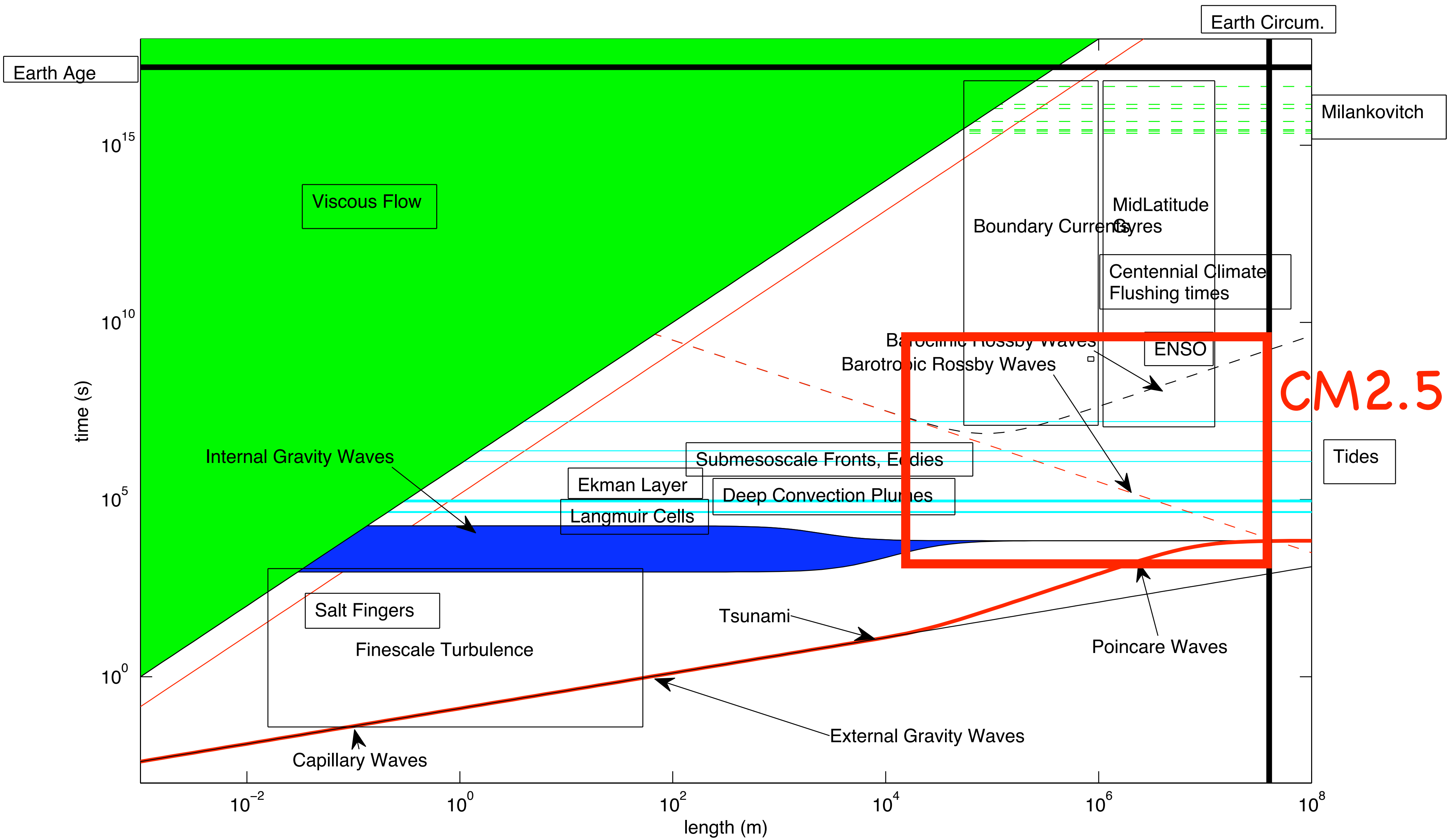
MOLES

Mesoscale Ocean Large Eddy Simulations with O(2-50km) horiz. resolution

The Ocean is Vast and Diverse

CM2.5=GFDL Hi-Res Earth System Model

Delworth et al. 2012



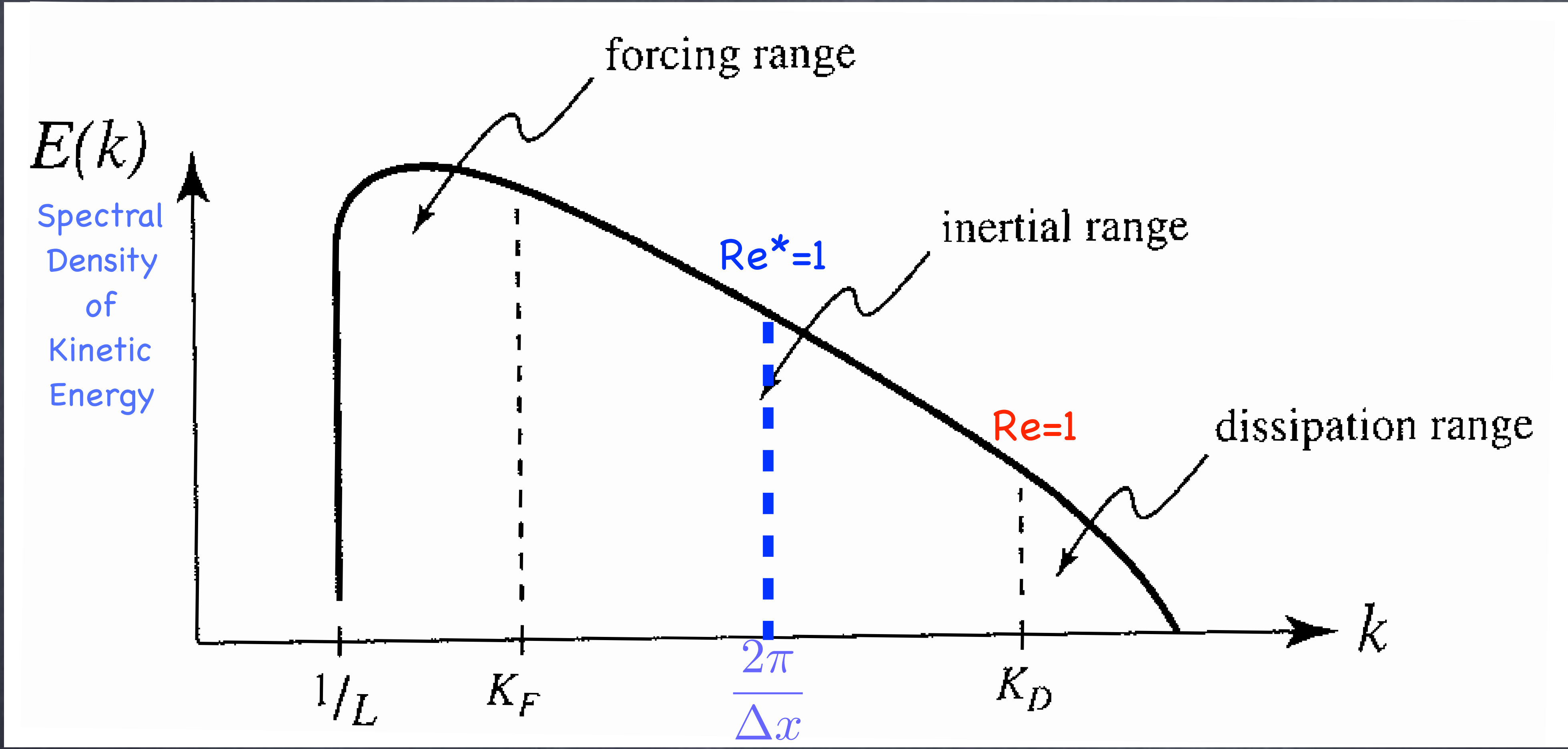
So, even as we begin to resolve the mesoscale...

- There are many, many processes left unresolved or partially resolved
- Eddy Less: For the unresolved (no eddies), need Reynolds-Average Closures (e.g., KPP, Gent-McWilliams/Redi)
- Eddy Rich: eddy-permitting to resolving, need Scale-Aware Large-Eddy-Simulation Closures (e.g., Smagorinsky)
- Some scale-aware hybrids, e.g.,

One set of possibilities: Cascade-based...

- Kolmogorov Cascades...
- Are they real?
- Does it matter?

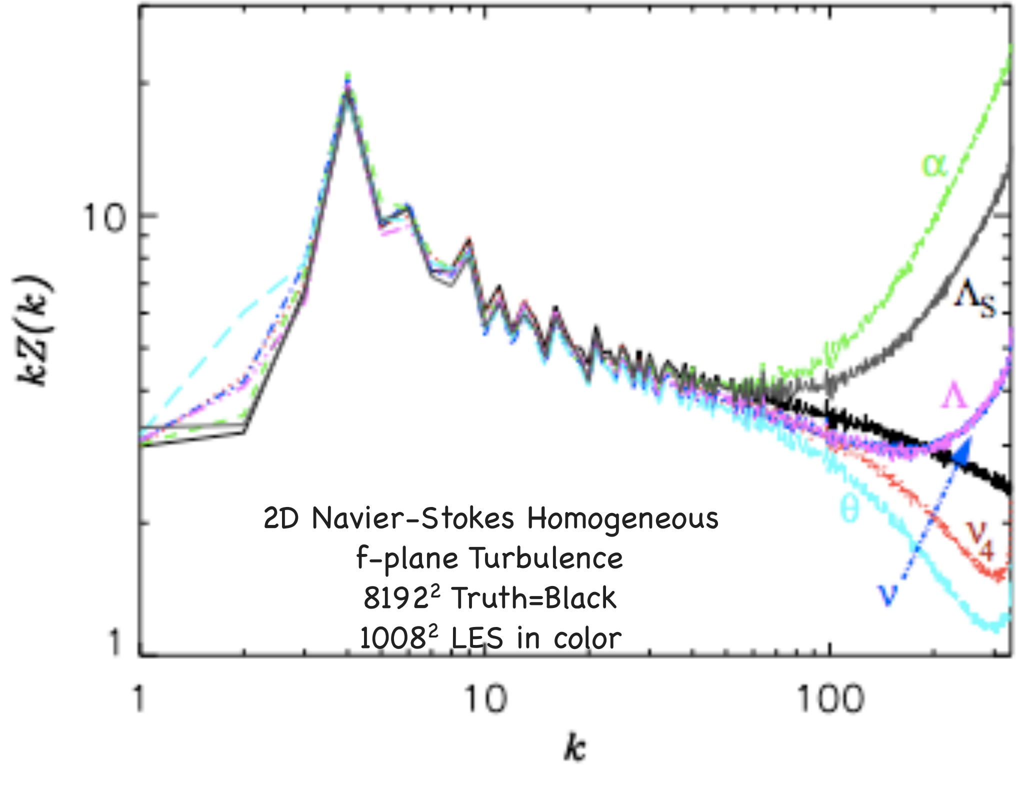
Truncation of Cascades



1963: Smagorinsky Scale & Flow Aware Viscosity Scaling,
So the Energy Cascade is Preserved,
but order-1 gridscale Reynolds #: $Re^* = UL/\nu_*$

$$\nu_{*h} = \left(\frac{\Upsilon_h \Delta x}{\pi}\right)^2 \sqrt{\left(\frac{\partial u_*}{\partial x} - \frac{\partial v_*}{\partial y}\right)^2 + \left(\frac{\partial u_*}{\partial y} + \frac{\partial v_*}{\partial x}\right)^2}$$

Some MOLES Truncation Methods In Use



Graham & Ringler, 2013 Ocean Modelling

- Harmonic/Biharmonic/Numerical
  - Many. Often not scale- or flow-aware
  - Griffies & Hallberg, 2000, is one aware example
- Fox-Kemper & Menemenlis, 2008. ECCO2.
  - Leith Viscosity (2d Enstrophy Scaling)
- Chen, Q., Gunzburger, M., Ringler, T., 2011
  - Anticipated Potential Vorticity of Sadourny
- San, Staples, Iliescu (2011, 2013)
  - Approximate Deconvolution Method
- Stochastic & Statistical Parameterizations
  - Other session going on now in Y10

See also Ramachandran et al, 2013 Ocean Modelling for SMOLES

The Character of the Mesoscale

100 km

(Capet et al., 2008)

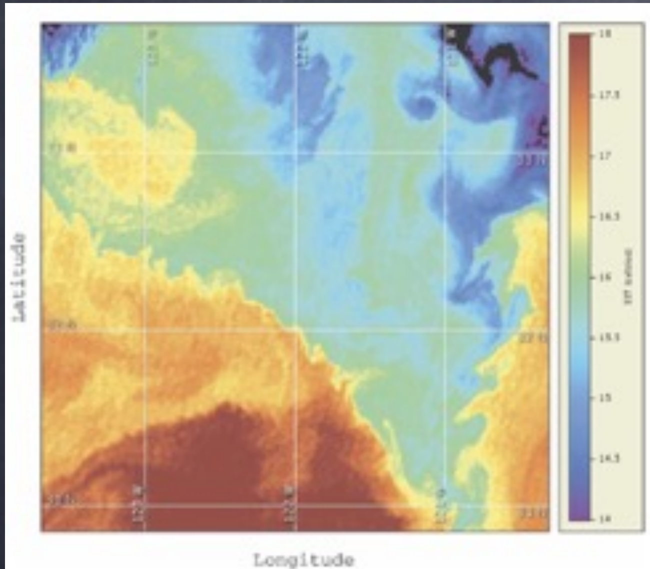
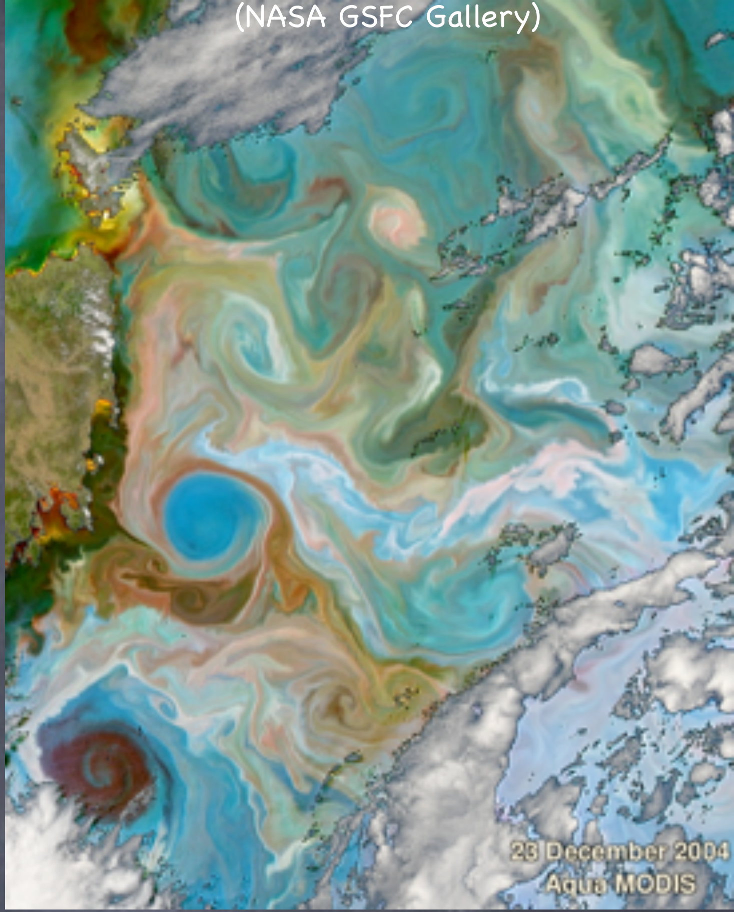


FIG. 16. Sea surface temperature measured at 1832 UTC 3 Jun 2006 off Point Conception in the California Current from CoastWatch (http://coastwatch.pfeg.noaa.gov). The fronts between recently upwelled water (i.e., 15°–16°C) and offshore water (>17°C) show submesoscale instabilities with wavelengths around 30 km (right front) or 15 km (left front). Images for 1 day earlier and 4 days later show persistence of the instability events.

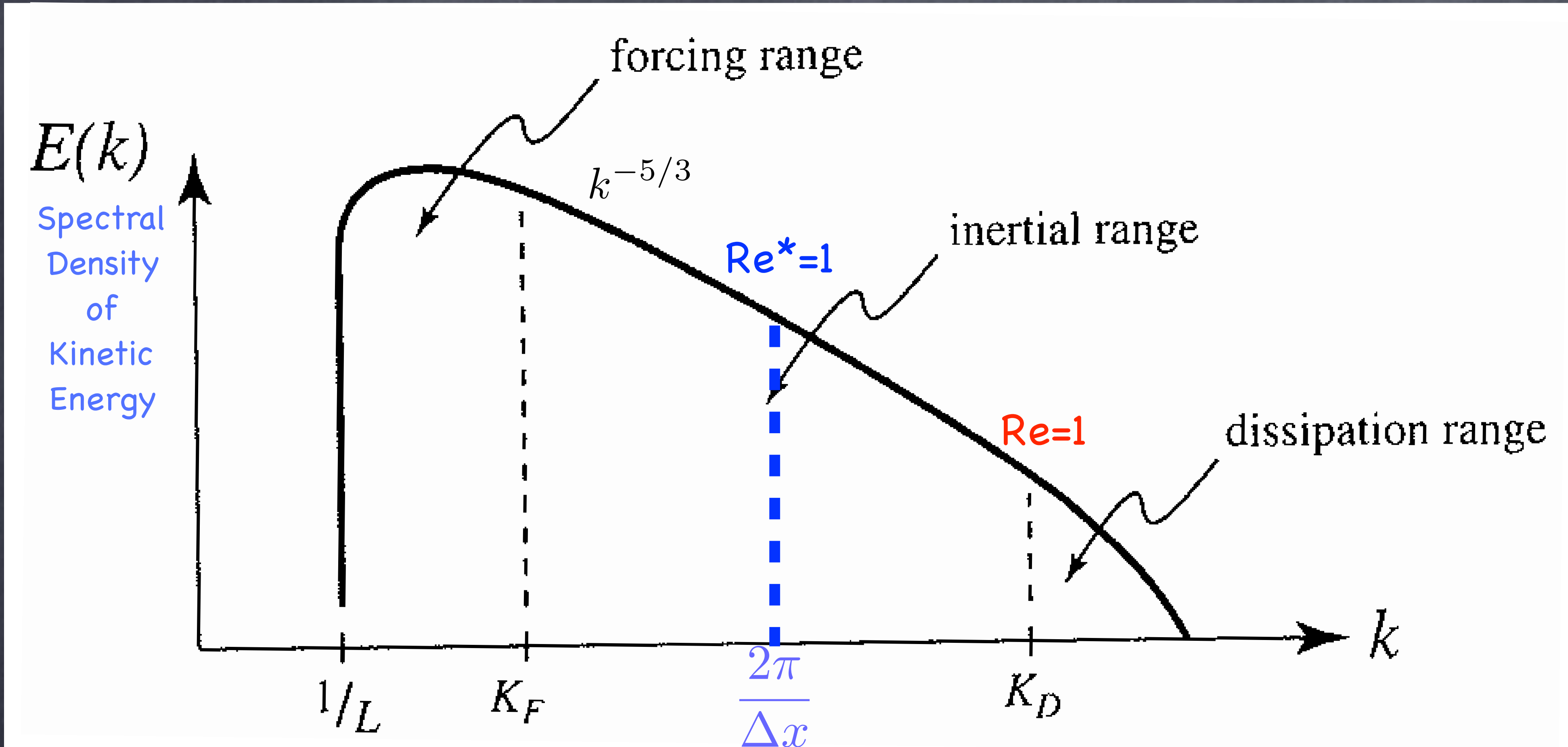
- Boundary Currents
- Eddies
- Ro=O(0.1)
- Ri=O(1000)
- Full Depth
- Quasi-2d
- Eddies strain to produce Fronts
- 100km, months

(NASA GSFC Gallery)



Eddy processes mainly baroclinic & barotropic instability.
Quasigeostrophy is likely to be very accurate.

3D Turbulence Cascade



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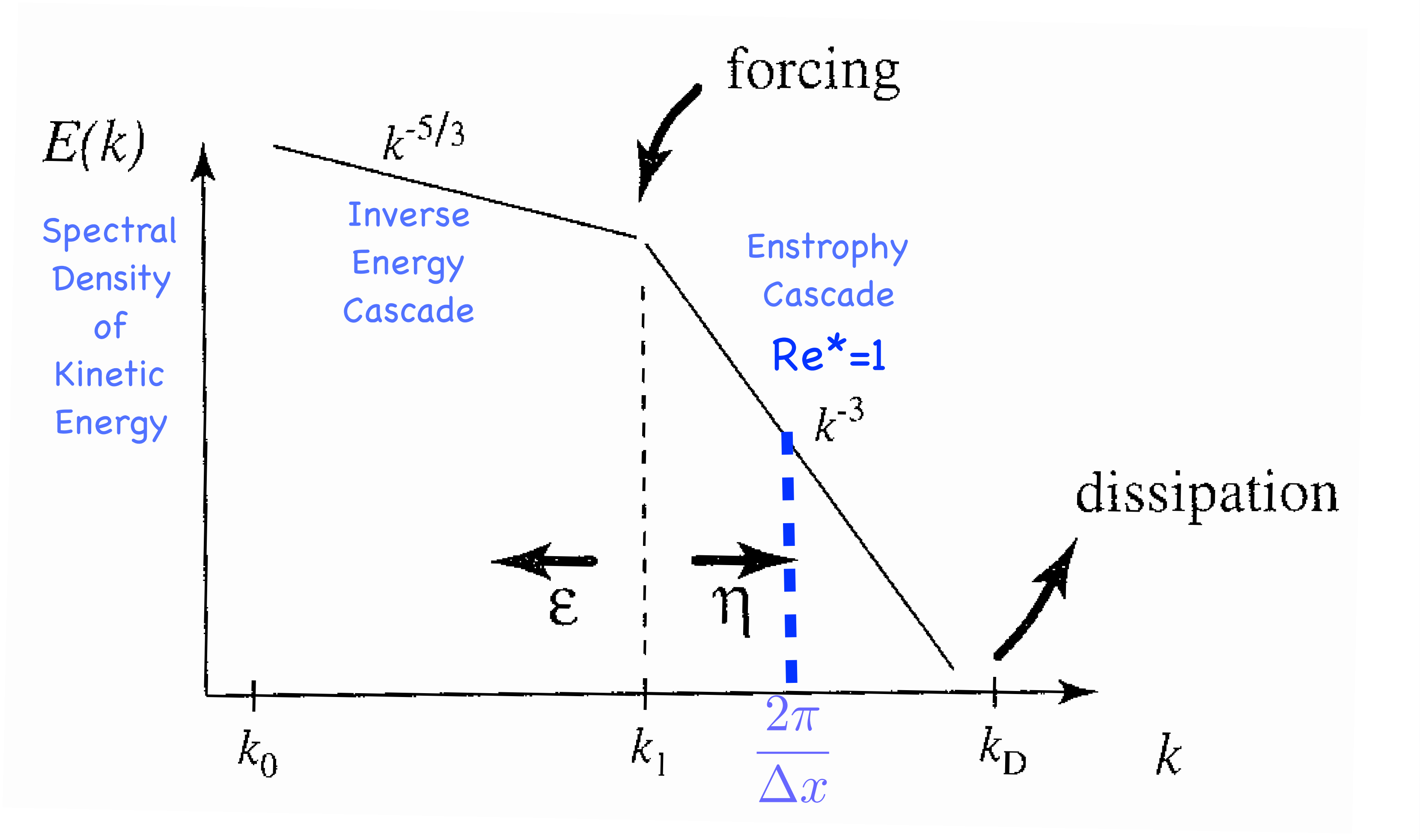
2D Turbulence Differs

R. Kraichnan, 1967 JFM

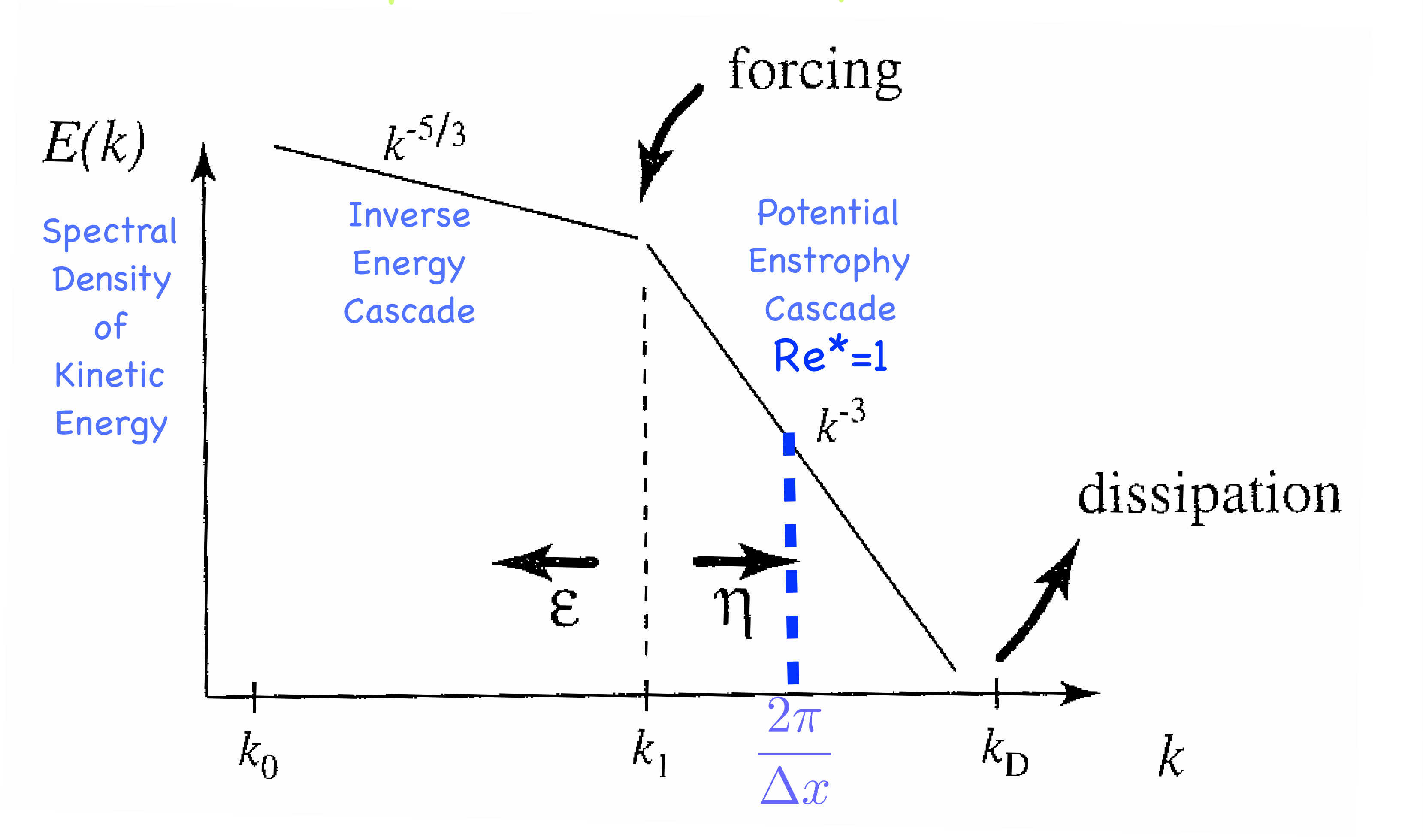


1996: Leith Devises Viscosity Scaling,
So that the Enstrophy (vorticity²) Cascade is Preserved

$$\nu_* = \left(\frac{\Lambda \Delta x}{\pi}\right)^3 \left|\nabla_h \left(\frac{\partial u_*}{\partial y} - \frac{\partial v_*}{\partial x}\right)\right|$$

QG Turbulence: Pot'l Enstrophy cascade (potential vorticity^2)

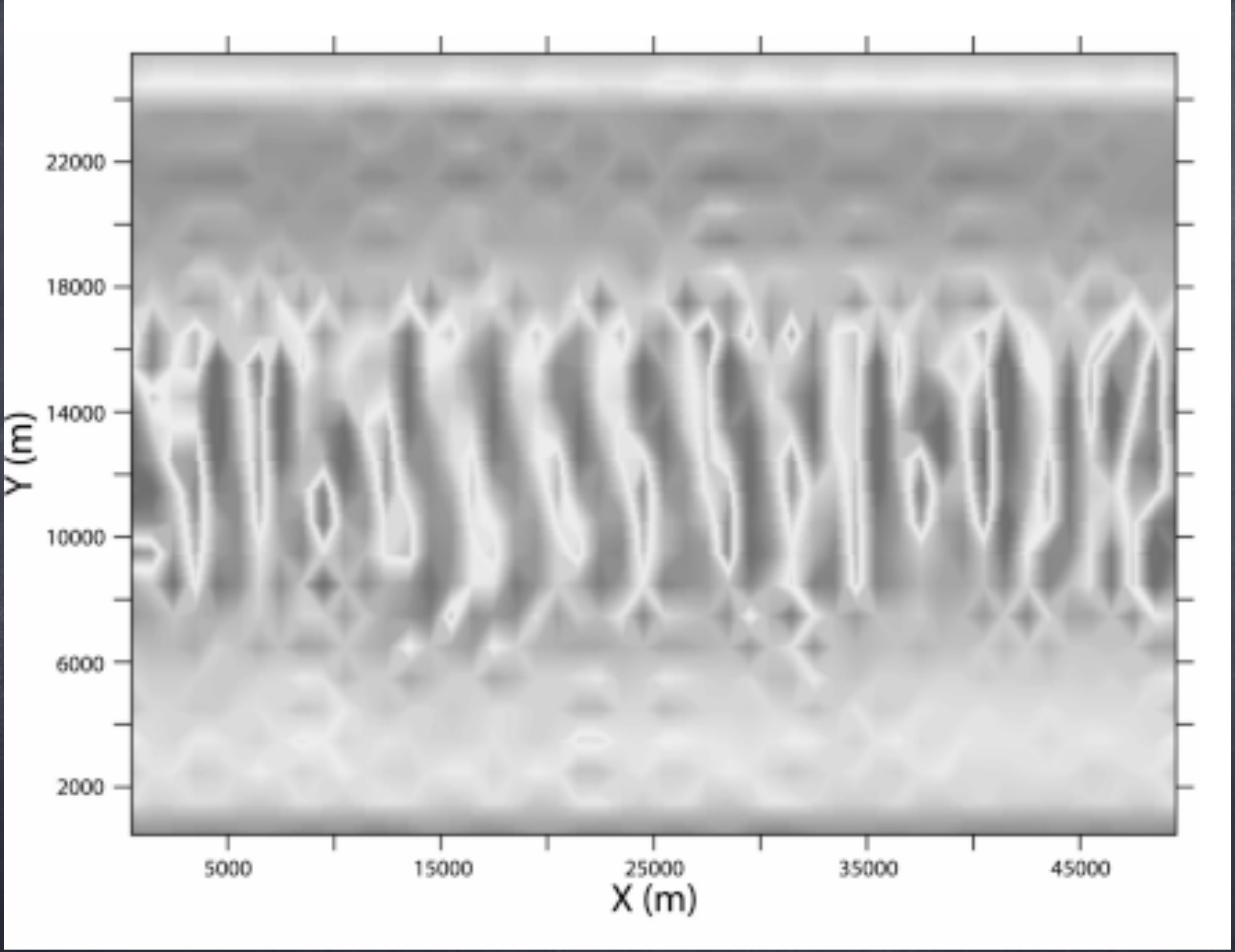
J. Charney, 1971 JAS



F-K & Menemenlis '08: Revise Leith Viscosity Scaling, So that diverging, vorticity-free, modes are also damped

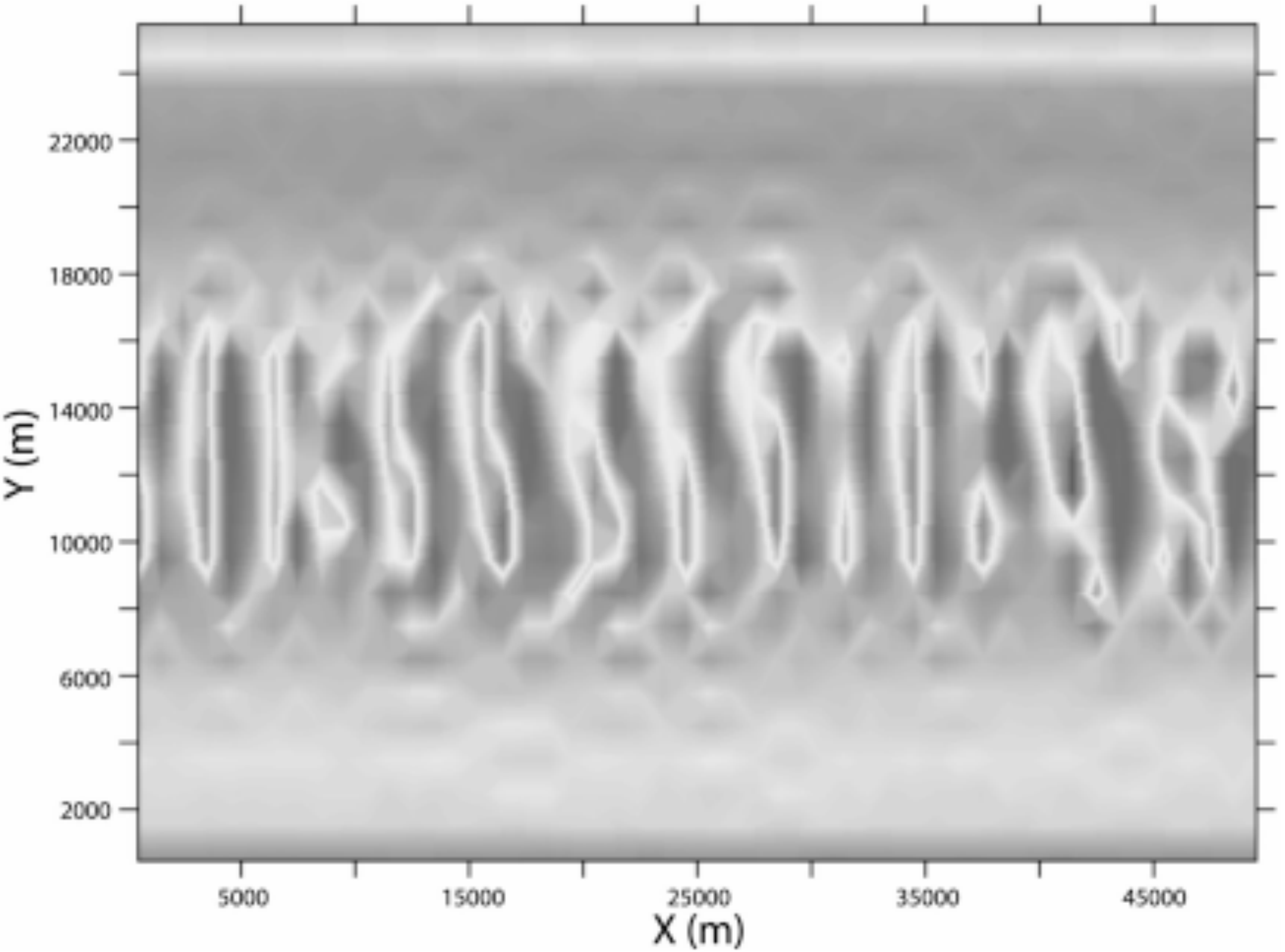
$$\nu_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2}$$

B. Fox-Kemper and D. Menemenlis. Can large eddy simulation techniques improve mesoscale-rich ocean models? In M. Hecht and H. Hasumi, editors, Ocean Modeling in an Eddying Regime, volume 177, pages 319-338. AGU Geophysical Monograph Series, 2008.



Fox-Kemper & Menemenlis, 2008

Vertical Velocity with Leith Viscosity

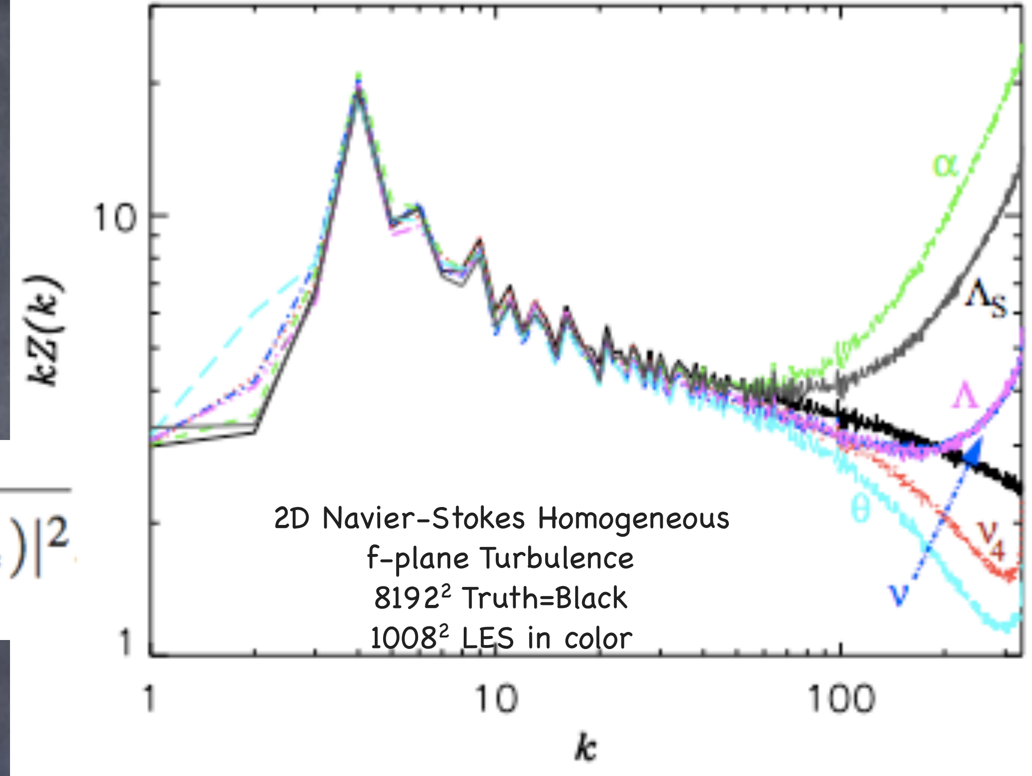


Fox-Kemper & Menemenlis, 2008

Vertical Velocity with LeithPlus Viscosity

Key Advantages of LeithPlus

$$\nu_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2}$$



Graham & Ringler, 2013 Ocean Modelling

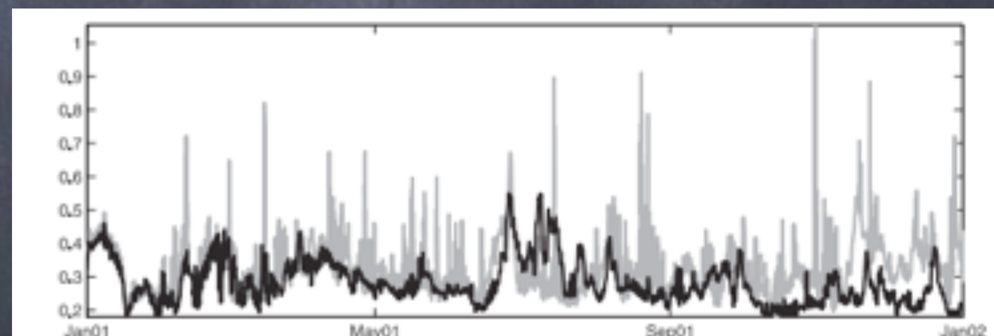
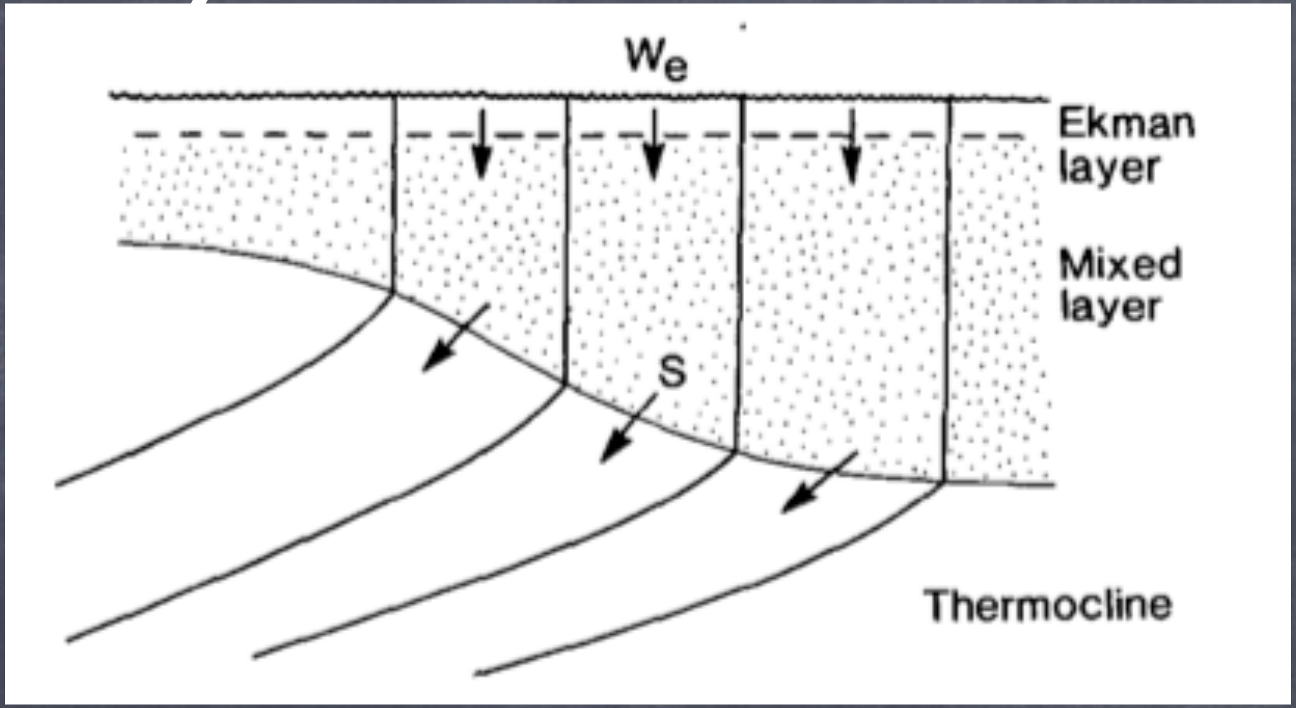


Figure 4. Maximum Courant number, $w\Delta t/\Delta z$, for vertical advection. Gray line is from the *LeithOnly* integration, and black line from the *LeithPlus* integration.

Fox-Kemper & Menemenlis, 2008

- Fox-Kemper & Menemenlis, 2008. ECCO2.
  - LeithPlus Viscosity (2d Enstrophy Scaling)
- At low Rossby number, becomes same as Leith, which is a good scaling for low Rossby
- LeithPlus Allows longer timesteps than Leith (by damping converging motions-->smaller w).
- Excellent numerical stability/damping properties
- Leith does a good job for 2-d flows
- More scale-selective than Smagorinsky or Biharmonic Smagorinsky

Is 2D Turbulence a good proxy for neutral flow?



Nurser & Marshall, 1991 JPO

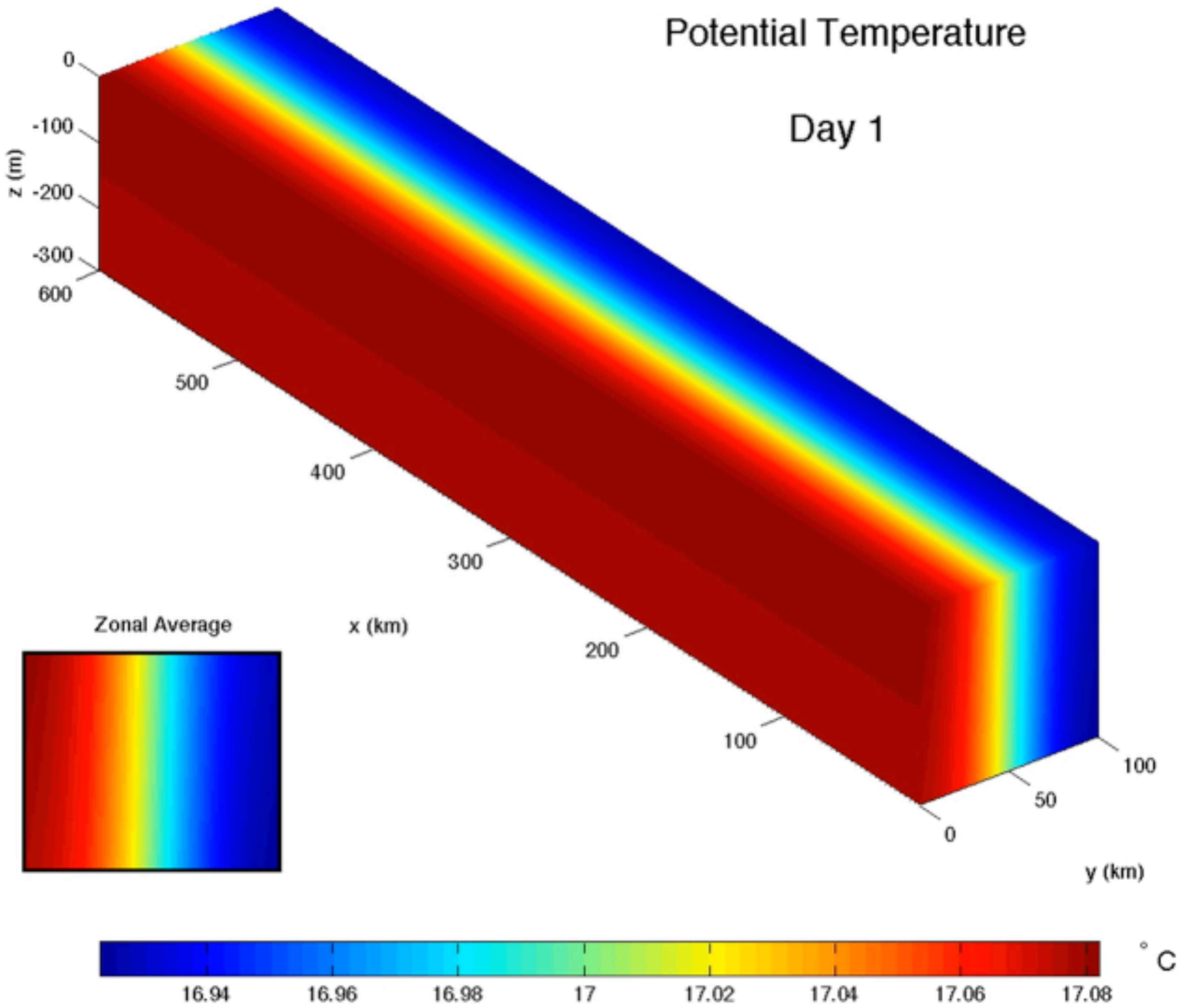
Yes:

- For a few eddy time-scales QG & 2D AGREE (Bracco et al. '04)
- Barotropic Flow--Obvious 2d analogue

No:

- Bolus Fluxes--Divergent 2d flow
- Sloped, not horiz.
- Surface Effects?

Movie: S. Bachman



S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013.

The Dream Parameterization...

Pot'l Enstrophy

$$\nu_{qg} = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^3 |\nabla q_{qg}| = \left(\frac{\Lambda_L \Delta x}{\pi}\right)^3 \left|\nabla_h \left[\beta y + \nabla_h^2 \psi + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial \psi}{\partial z}\right)\right]\right|.$$

PV equation sink consistency

Horizontal=Isoneutral in QG

Bolus Equivalence--Dukowicz & Smith (97)

$$\nu_{qg} = \kappa_{Redi} = \kappa_{GM} = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^3 |\nabla q_{qg}|.$$

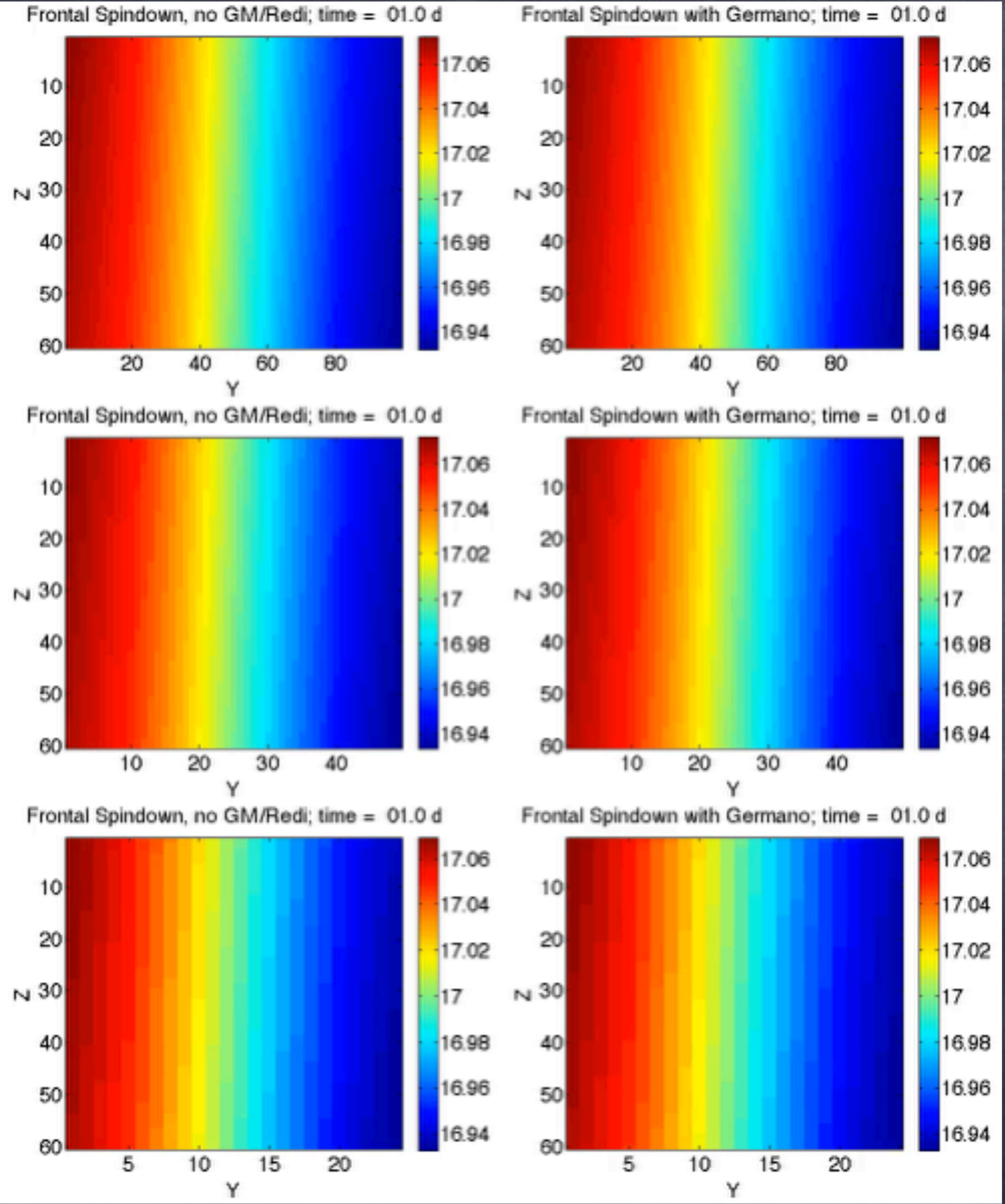
Even Better! Germano!

$$\mathcal{L} \equiv \mathbf{T} - \overline{\tau}^{F},$$

$$\mathcal{L} \equiv \overline{\overline{\mathbf{u}}^{G}\,\overline{c}^{G}}^{F} - \overline{\overline{\mathbf{u}}^{G}}^{F}\,\overline{\overline{c}^{G}}^{F}$$

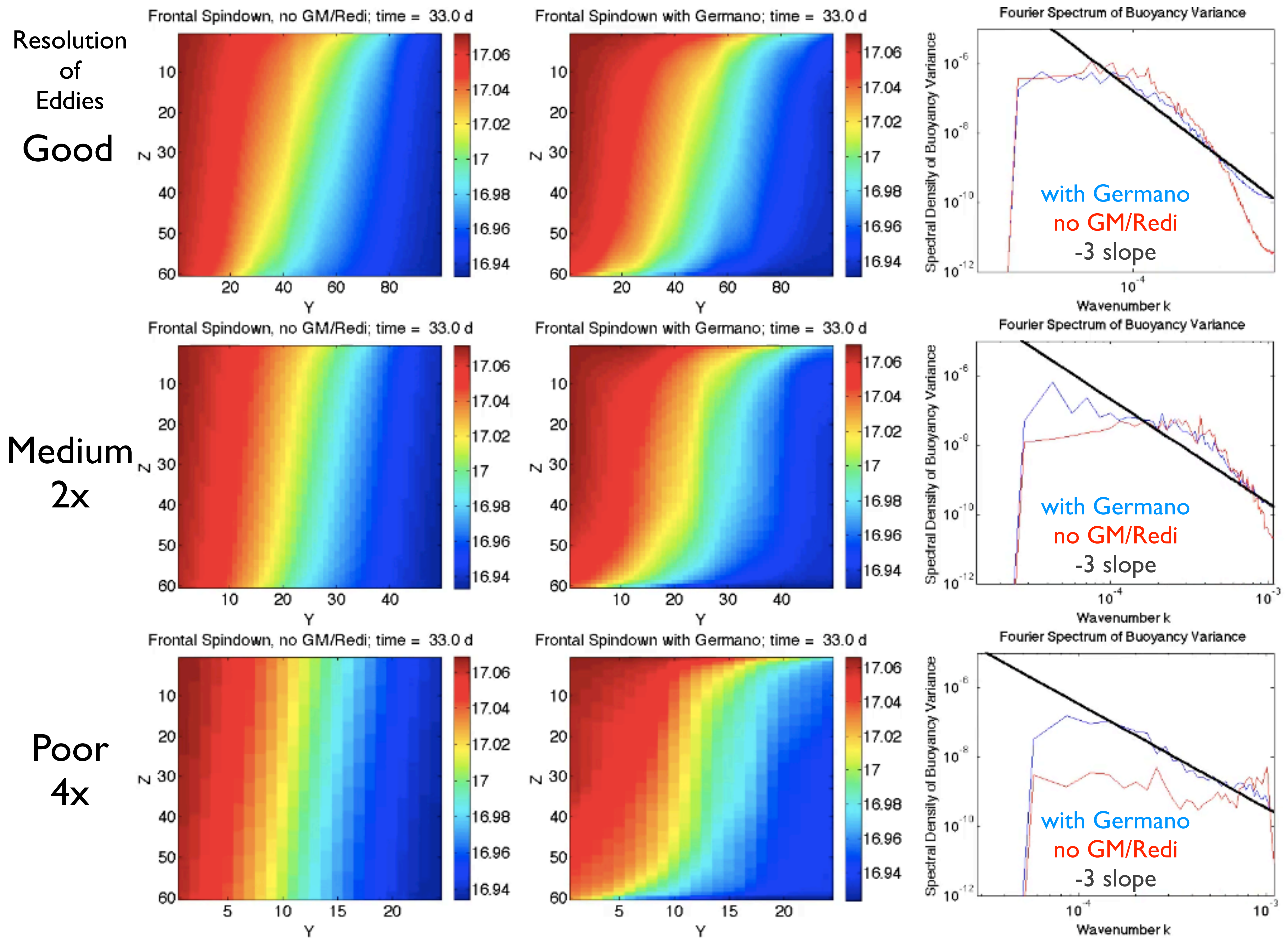
$$\mathbf{T} \equiv \overline{\overline{\mathbf{u}c}^{G}}^{F} - \overline{\overline{\mathbf{u}}^{G}}^{F}\,\overline{\overline{c}^{G}}^{F}$$

$$\overline{\tau}^{F} \equiv \overline{\overline{\mathbf{u}c}^{G} - \overline{\mathbf{u}}^{G}\,\overline{c}^{G}}^{F}.$$



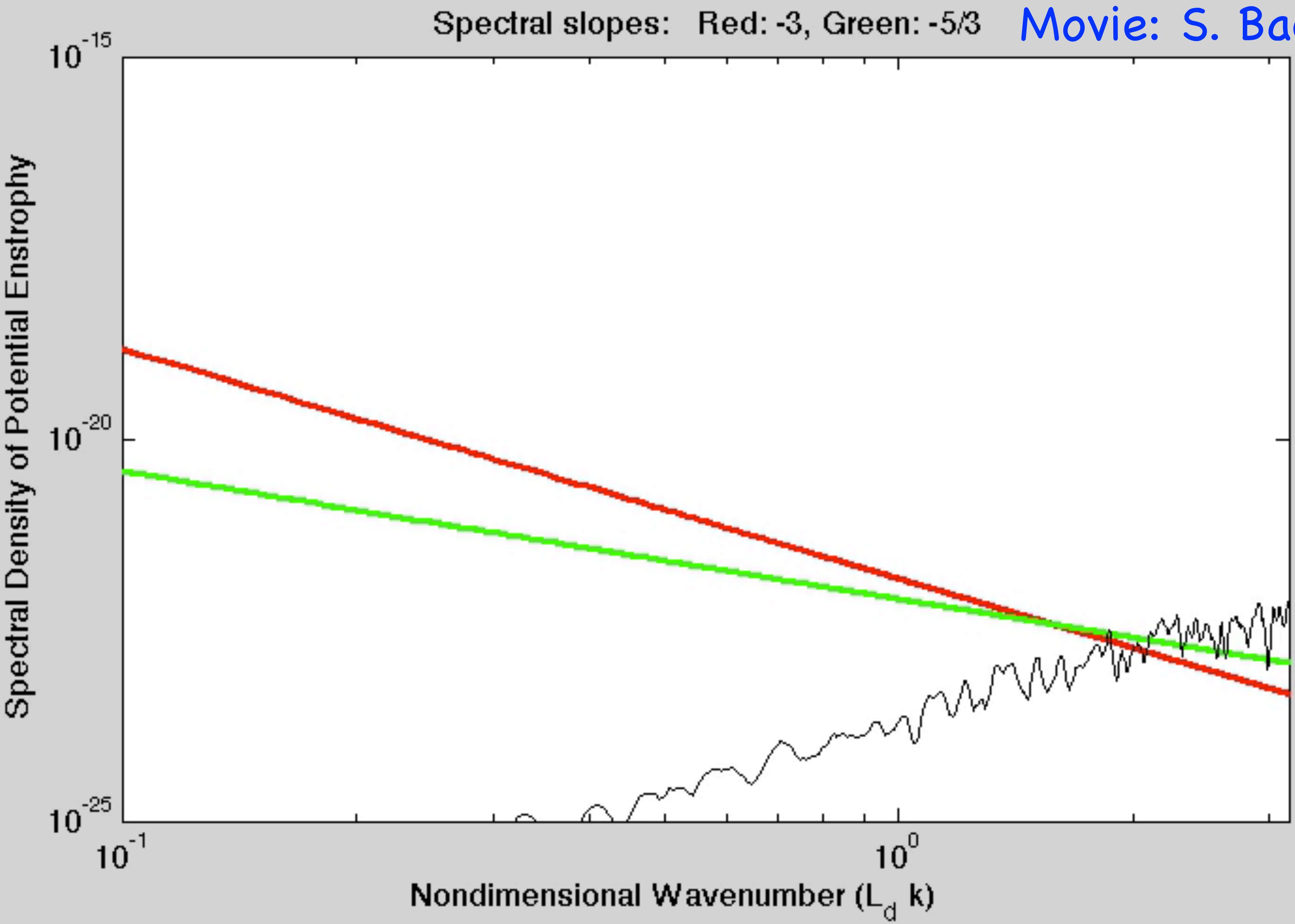
Thanks to
Scott Bachman--

former PhD,
now at DAMTP
with John Taylor


Resolution of Eddies
Good
Medium 2x
Poor 4x
Frontal Spindown, no GM/Redi; time = 33.0 d
Frontal Spindown with Germano; time = 33.0 d
Fourier Spectrum of Buoyancy Variance
Spectral Density of Buoyancy Variance
Wavenumber k
Y
Z
with Germano
no GM/Redi
-3 slope

Cascade of Pot'l Enstrophy?

Movie: S. Bachman

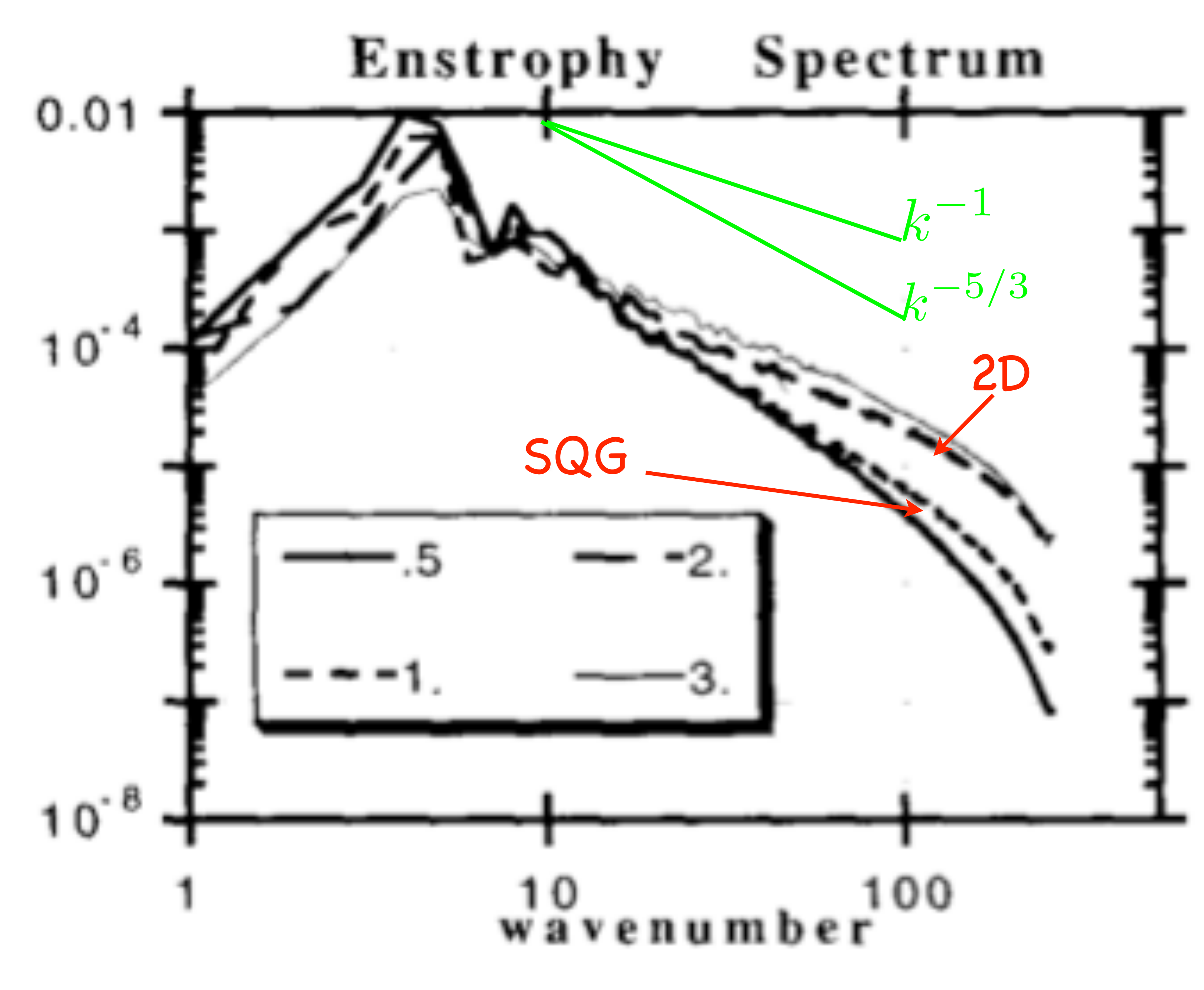


In real stratified flows, things are a bit more complex than in 2d

Even more than QG...

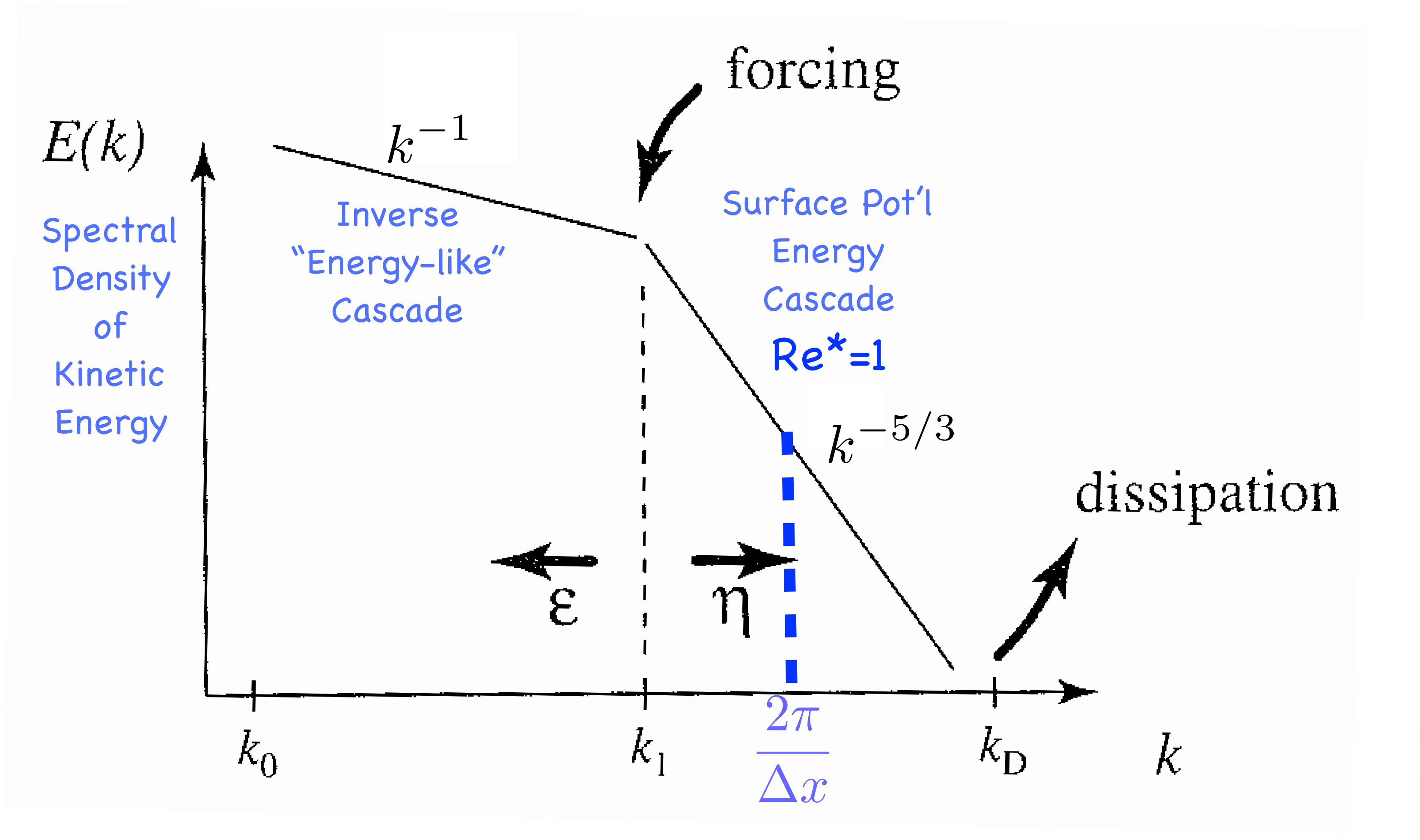
Surface Effects may dominate

Pierrehumbert, Held, Swanson, 1994 Chaos
Spectra of Local and Nonlocal Two-dimensional Turbulence



SQG Turbulence: Surface Buoyancy & Velocity cascade

W. Blumen, 1978 JAS
Held et al 1995, JFM.
Smith et al. 2002, JFM



Smag-Like: $$\kappa_* = \left(\frac{\Upsilon \Delta x}{\pi}\right)^{4/3} \left|\frac{1}{f}\nabla_h b\right|^{2/3}$$

Leith-Like: $$\kappa_* = \left(\frac{\Lambda \Delta x}{2\pi}\right)^{3/2} \left[-\frac{\partial}{\partial z}|\nabla_h \psi|^2\right]^{1/2}$$

Many observations tell us:

The spectrum of potential density and buoyancy often scales as k^{-2}, which isn't too far from $k^{-5/3}$

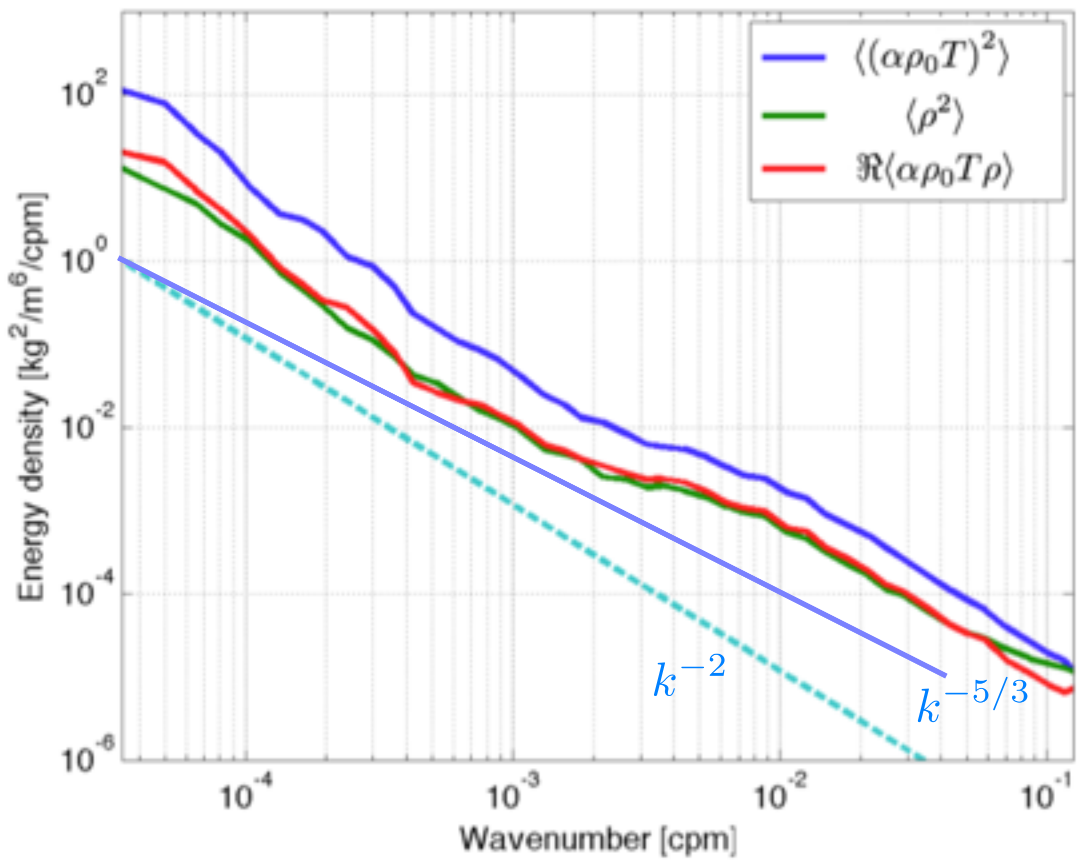
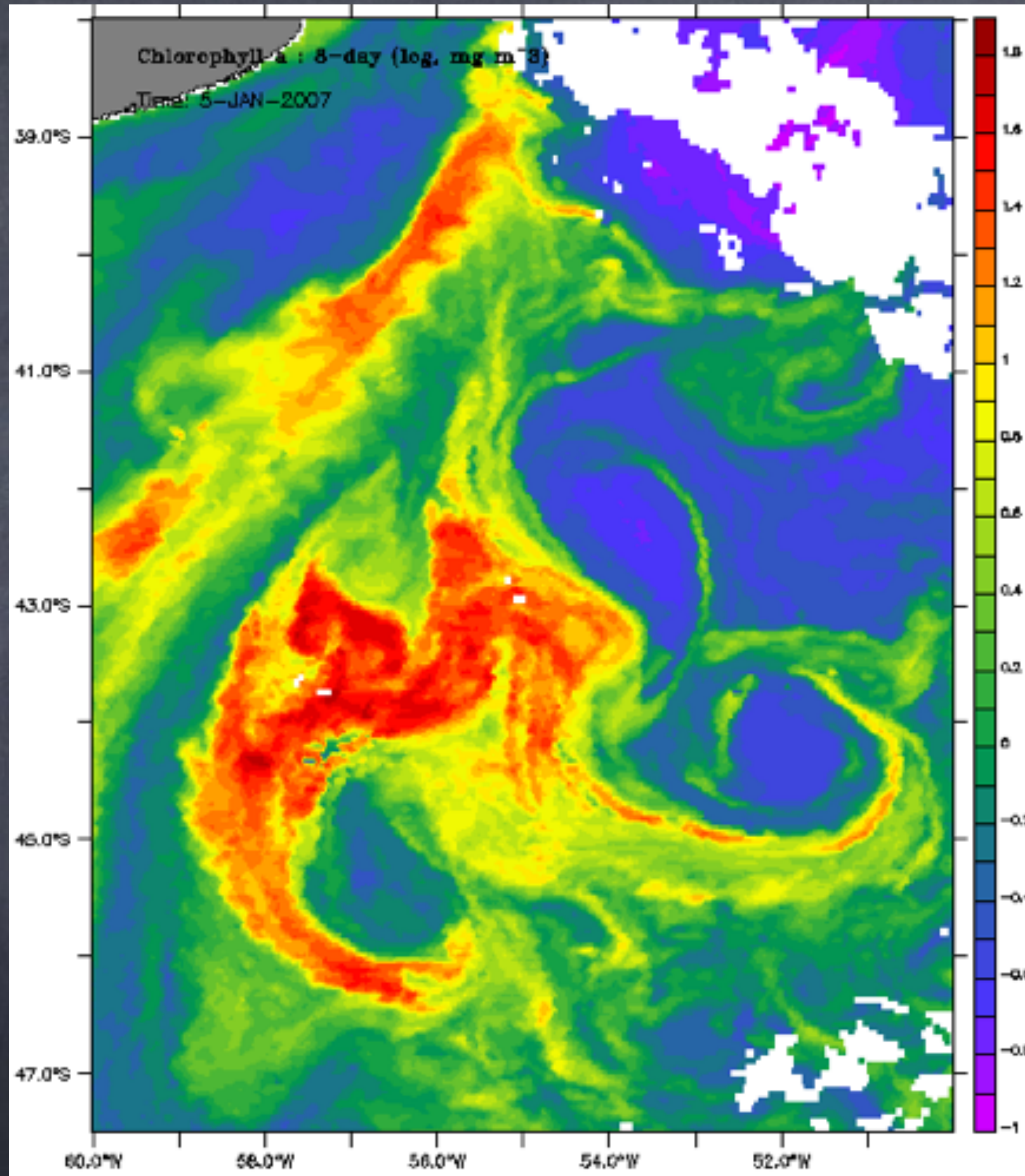


Figure 1: Observed spectra of mixed layer potential density variance (green), temperature contribution to potential density (blue), and temperature-density co-spectrum (red) from SeaSoar towed CTD and shipboard ADCP sections (data from Ferrari and Rudnick, 2000). A dashed line indicates k^{-2} scaling.

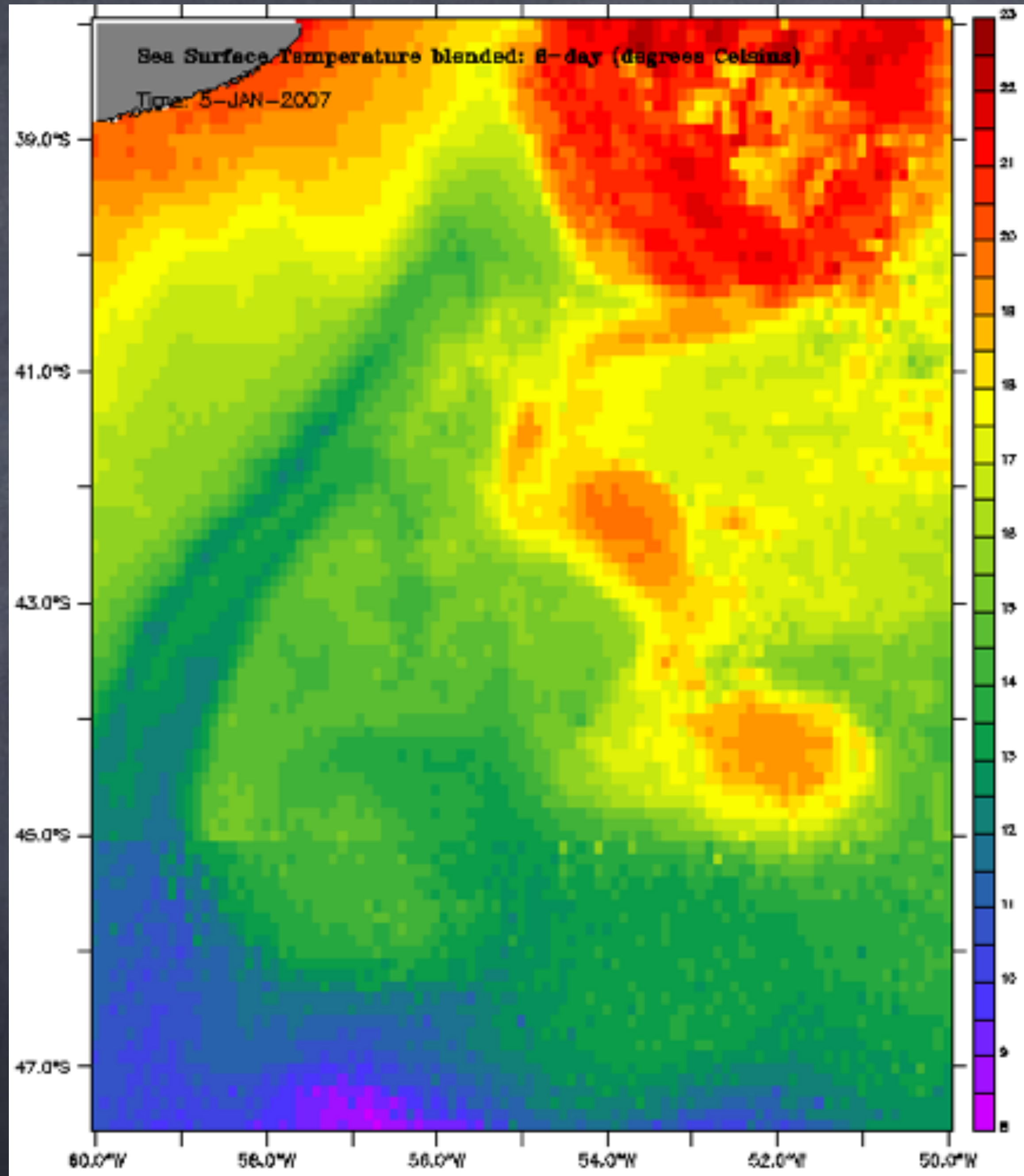
B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. Ocean Modelling, 39:61-78, 2011.

Examples: Jan 5, 07 East of Argentina



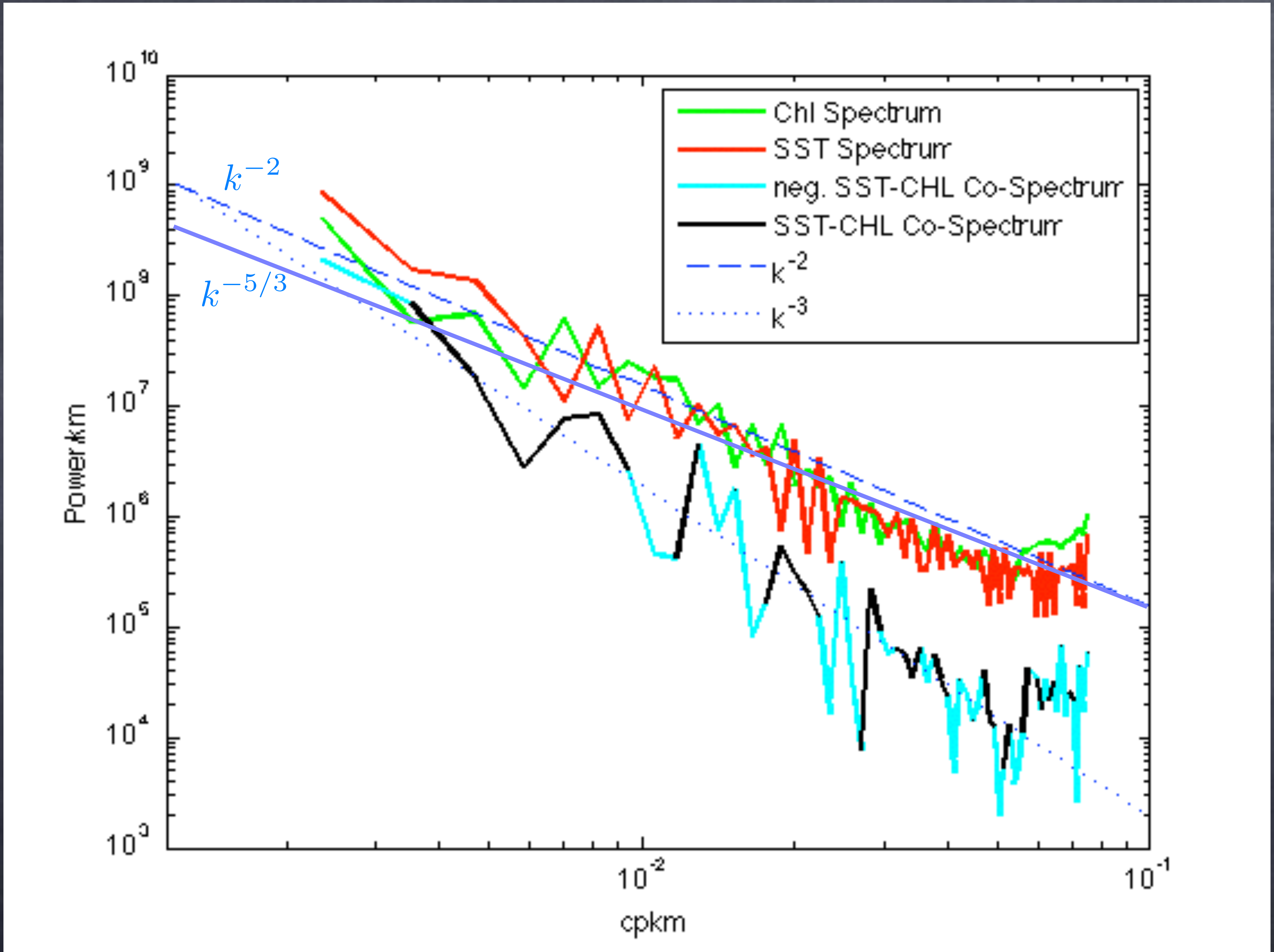
MODIS on Aqua Chl

Examples: Jan 5, 07 East of Argentina



Remote Sensing Systems Inc. (www.remss.com) Blended SST blended

Spectra: Jan 5, 07 East of Argentina



Nikurashin, Vallis, Adcroft, 2013 Nature Geoscience
Routes to energy dissipation for geostrophic flows in the Southern Ocean

It is not clear that inertial ranges exist.

This spectrum shows that topographic interactions change the spectrum at depth dramatically

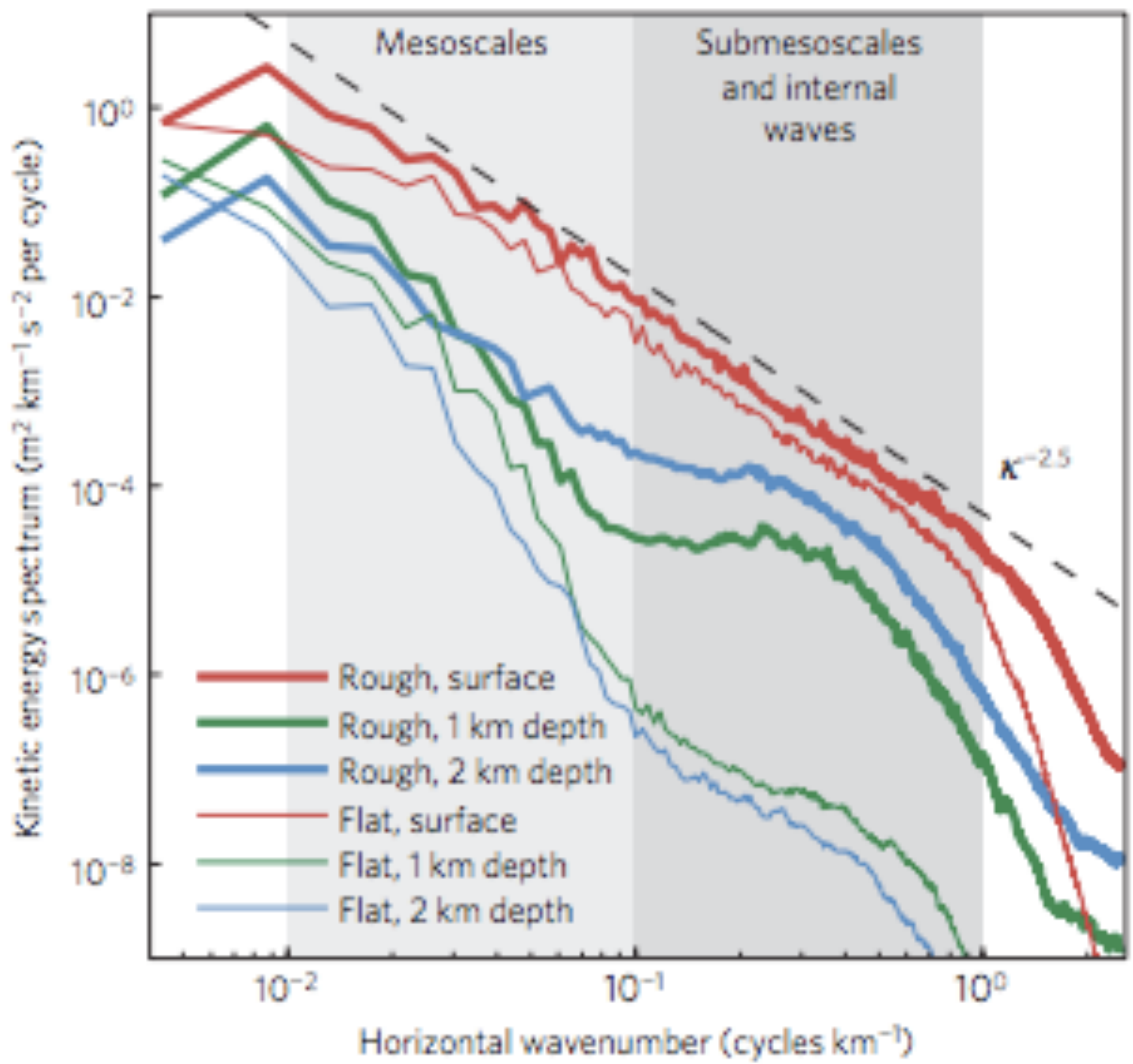


Figure 4 | Horizontal wavenumber kinetic energy spectra. Spectra

Reynolds vs. Péclet: Prandtl=1?

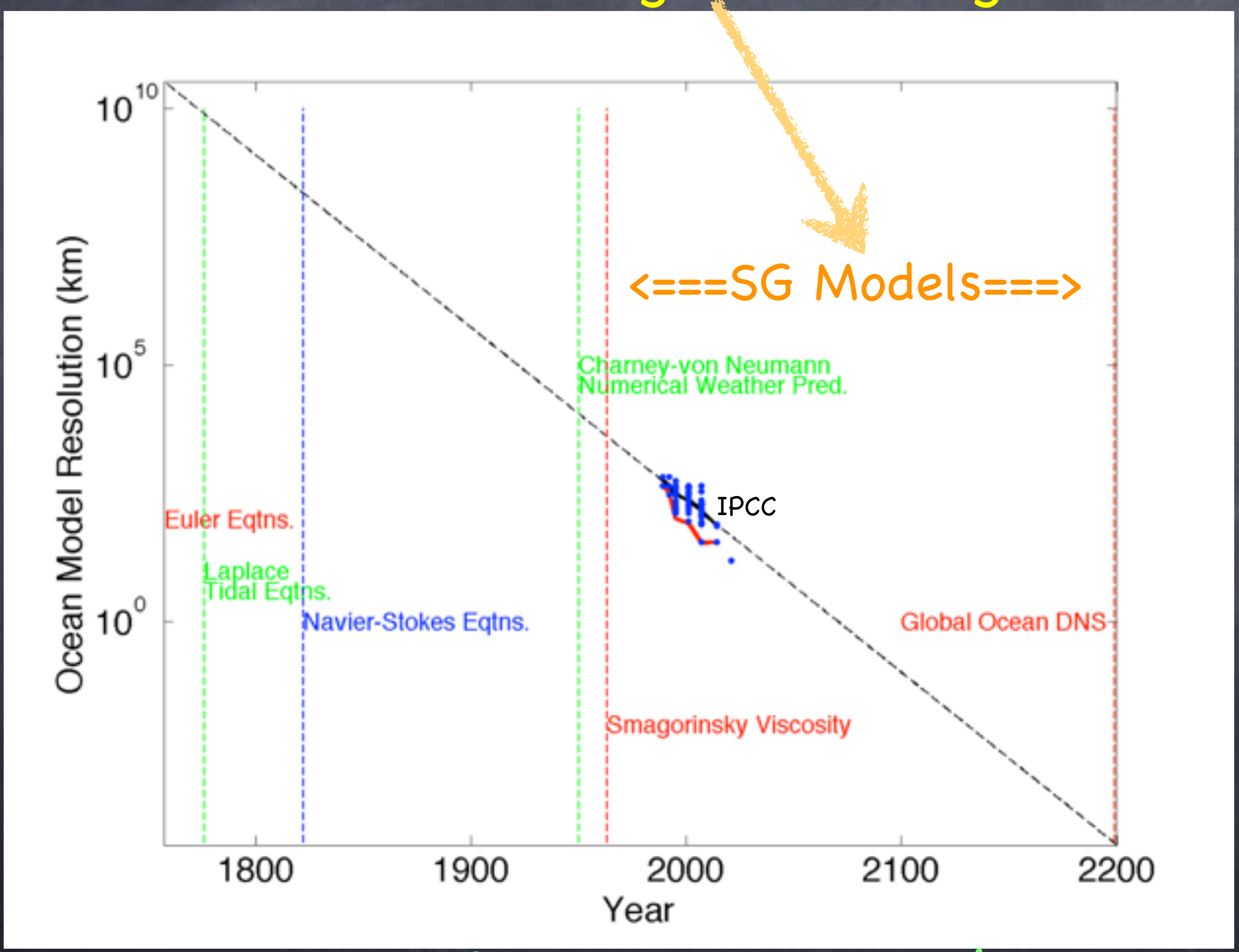
- In all cascade examples, the truncation occurs at large Reynolds and Péclet, so it is reasonable to assume diffusivity=viscosity
- In the QG framework, diffusivity *must* equal viscosity to avoid spurious generation of potential vorticity by the subgrid model
- In the SQG framework, it is surface

A Prescription for Parameterization… Accuracy TBD

- QG Leith & Potential Vorticity to generate #1 viscosity
- 2D Leith & Barotropic Vorticity to generate #2 viscosity
- SQG Leith & Surf. Buoyancy to generate #3 diffusivity

  Nearly suggested by Roberts & Marshall, 98, JPO
- Take max(#1, #2, #3) as viscosity, Redi diffusivity, *and* as GM transfer coeff.
- Note: Unlike Eddy-Free closures, e.g., Visbeck et al (97), Eddy-Rich closures take advantage of resolved

Extrapolate for historical perspective:
The Golden Era of Subgrid Modeling is Now!



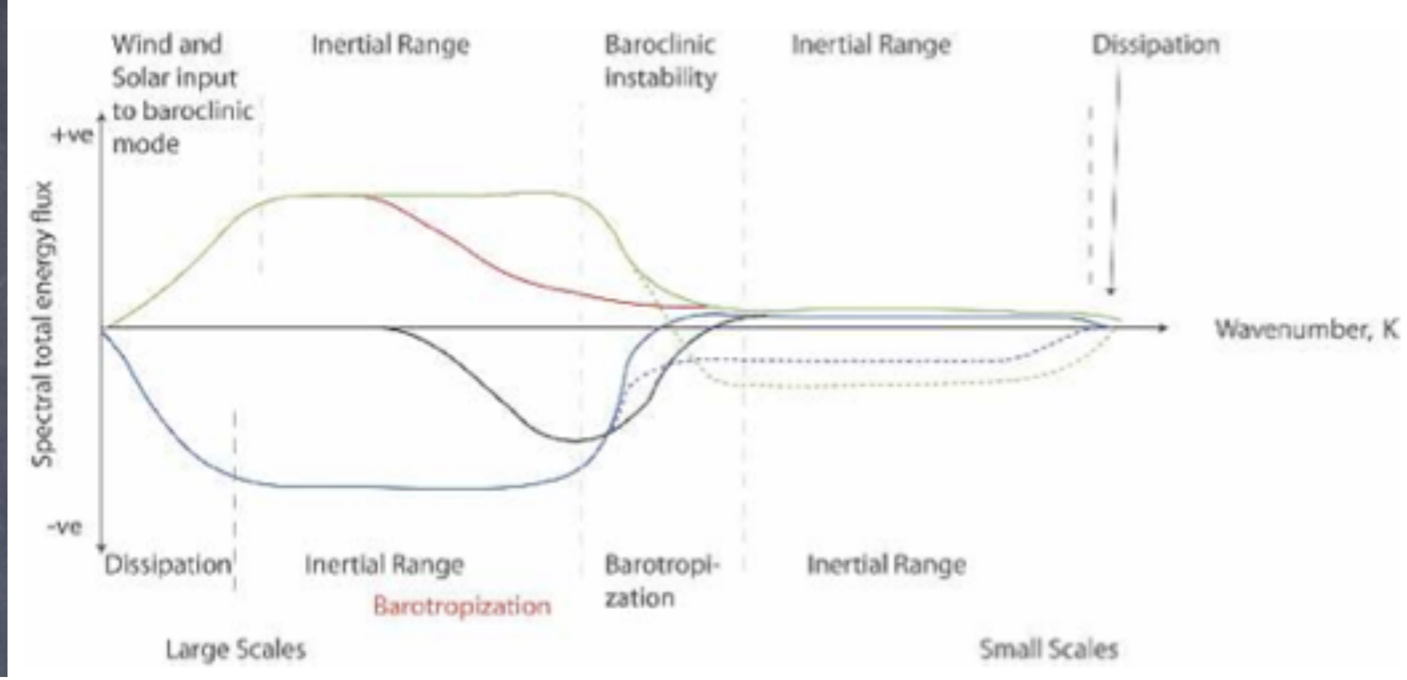
All papers at: fox-kemper.com/research

Conclusions

- Climate modeling is challenging partly due to the vast and diverse scales of fluid motions
- Physically-motivated subgrid models result when the gridscale is in an approximate inertial range
  - E.g., Leith Family are good for MOLES
- Viscosity, diffusivity, and streamfunction closures can be found in this case
- Improved Large Eddy Simulations will* result: boundary current separation, SSH & SST variance, spectral properties, tracer transport, fewer arbitrary parameters

*my conjecture

All papers at: fox-kemper.com/research



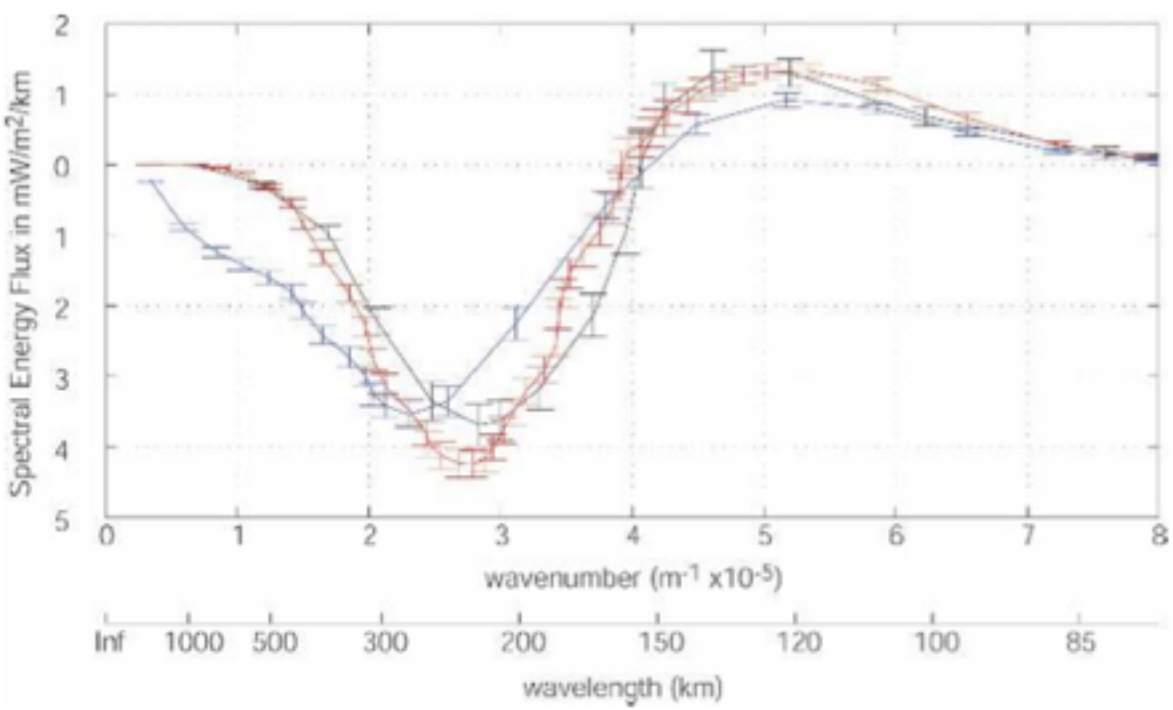
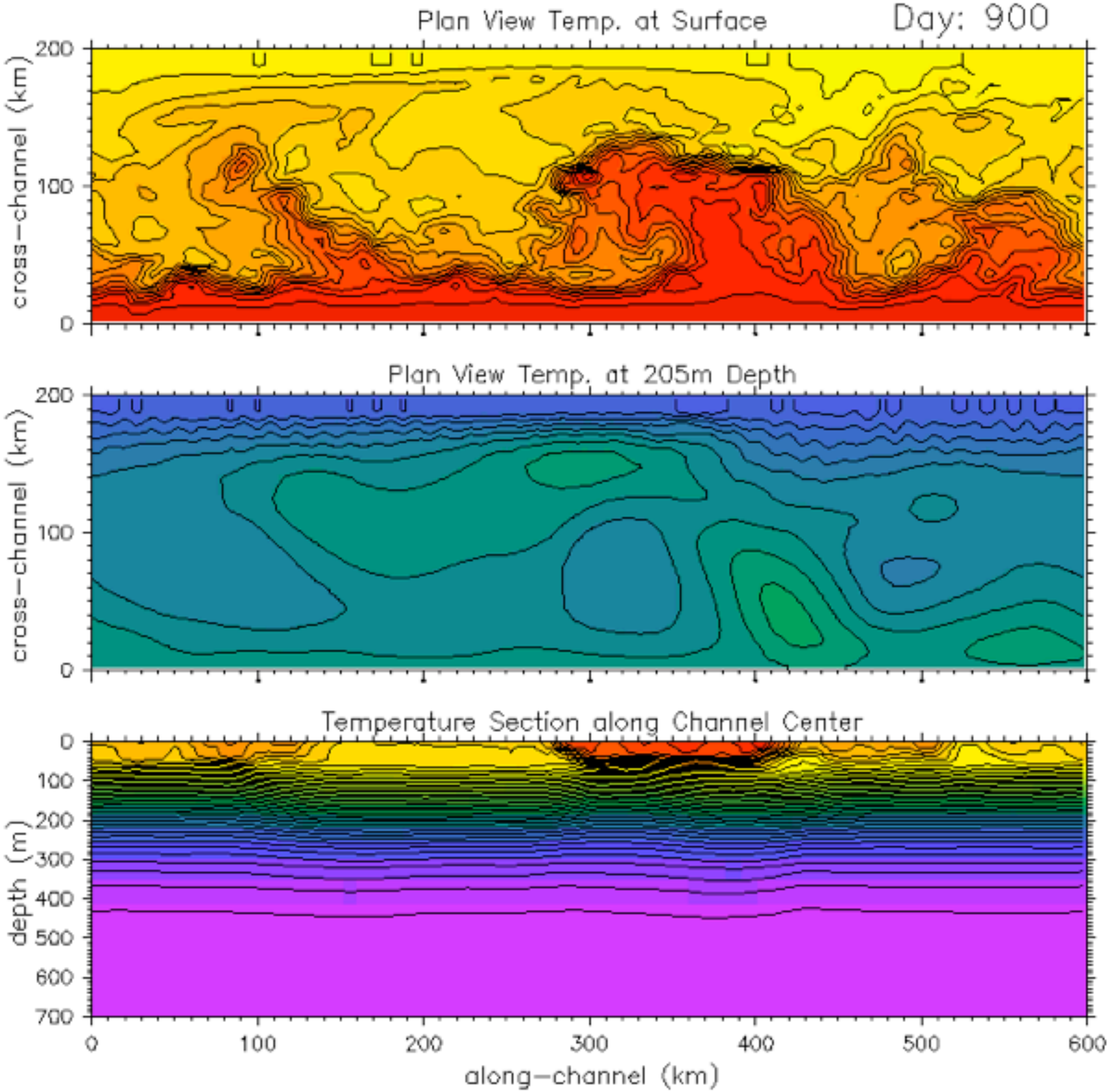


FIG. 2. Time mean, spectral kinetic energy flux $\overline{\Pi}(K)$ vs total wavenumber K in a homogeneous ACC region (rectangles centered at 57°S, 120°W): black curve using SSH on a 32 × 32 grid, red curve using SSH on a 64 × 64 grid, blue curve using velocity on a 64 × 64 grid. Positive slope reveals a source of energy. The larger negative lobe reveals a net inverse cascade to lower wavenumber. Error bars represent standard error.

Implications from Scott & Wang (05)

Lengthscales of Energy Transfer



Big, Deep (meso)

interact with

Little, Shallow (submeso)

B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. Journal of Physical Oceanography, 38(6):1145-1165, 2008.

A Global Parameterization of Mixed Layer Eddy Flow & Scale Aware Restratification validated against simulations

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. Ocean Modelling, 39:61-78, 2011.

$$\overline{\mathbf{u}'b'} \equiv \mathbf{\Psi} \times \nabla \bar{b}$$

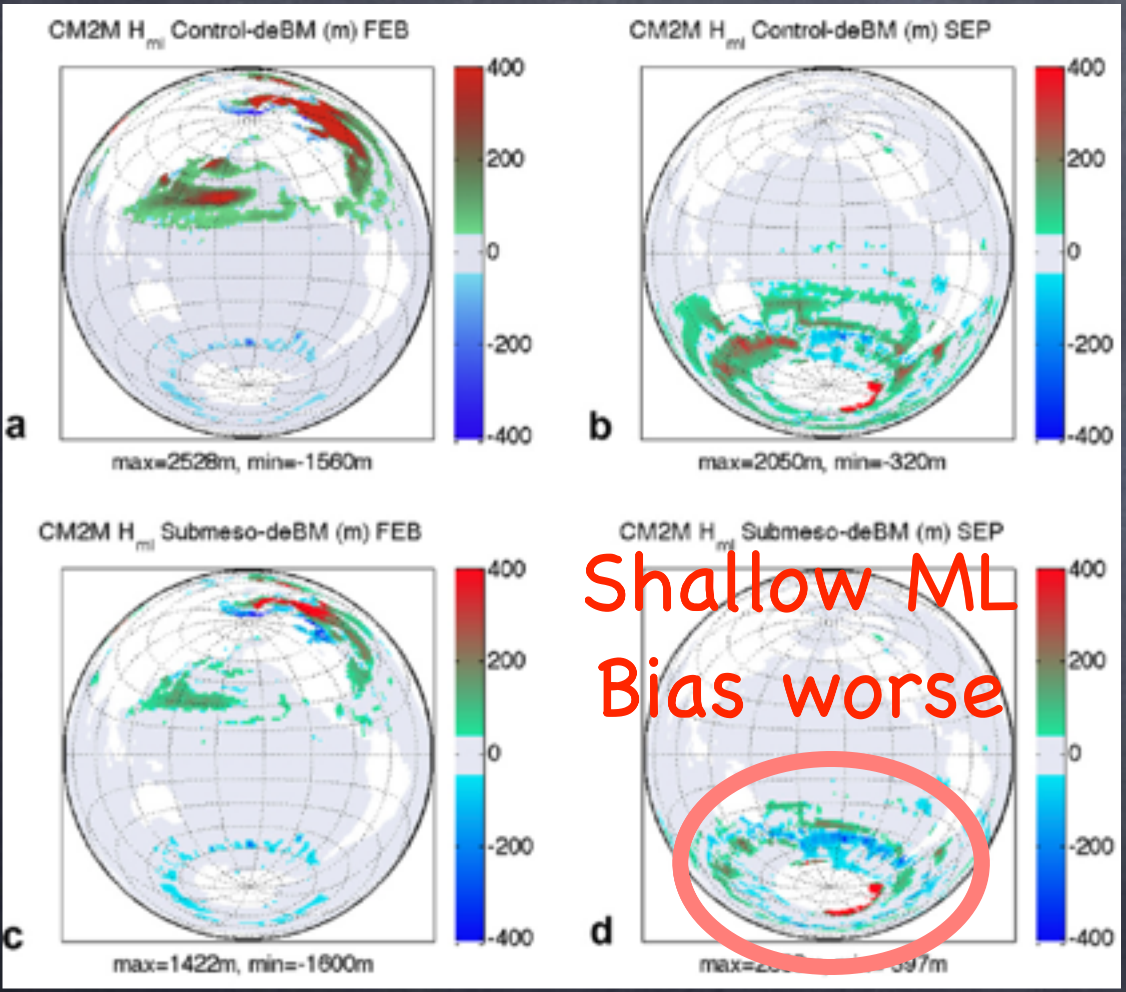
$$\mathbf{\Psi} = \boxed{\left[\frac{\Delta x}{L_f}\right]} \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla \bar{b} \times \hat{\mathbf{z}}$$

Compare to the original singular, unrescaled version

$$\mathbf{\Psi} = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{\mathbf{z}}$$

New version handles the equator, and averages over many fronts

Physical Sensitivity of Ocean Climate to MLE: Mixed Layer Eddy Restratification

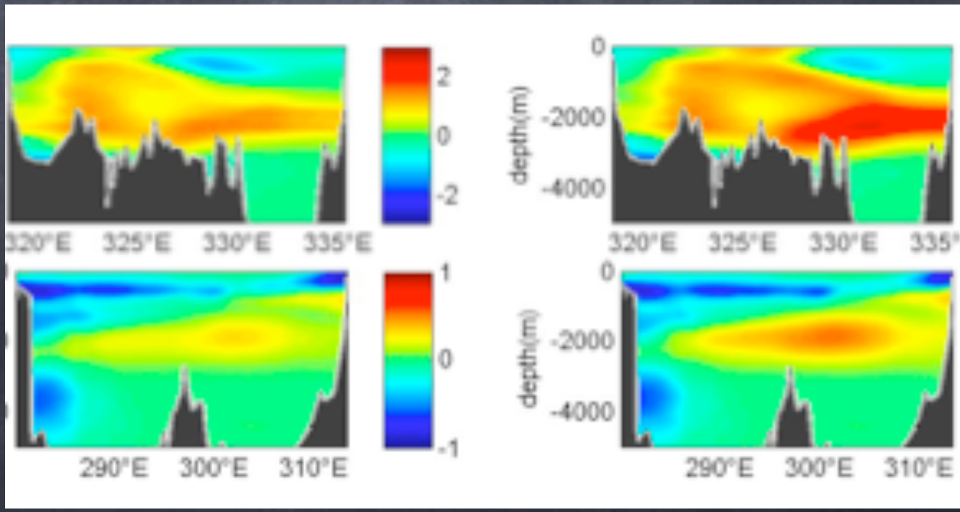


Bias w/o MLE

Improves CFCs (water masses)

Bias with MLE

Bias w/o MLE



B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.