

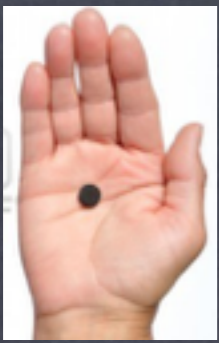
Scale-aware subgrid closures for models that partly resolve the mesoscale and submesoscale

Baylor Fox-Kemper (Brown University)

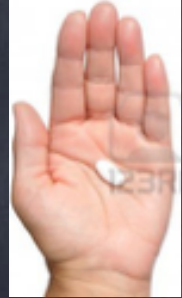
with Scott Bachman (DAMTP)

ECCO CAOS Colloquium

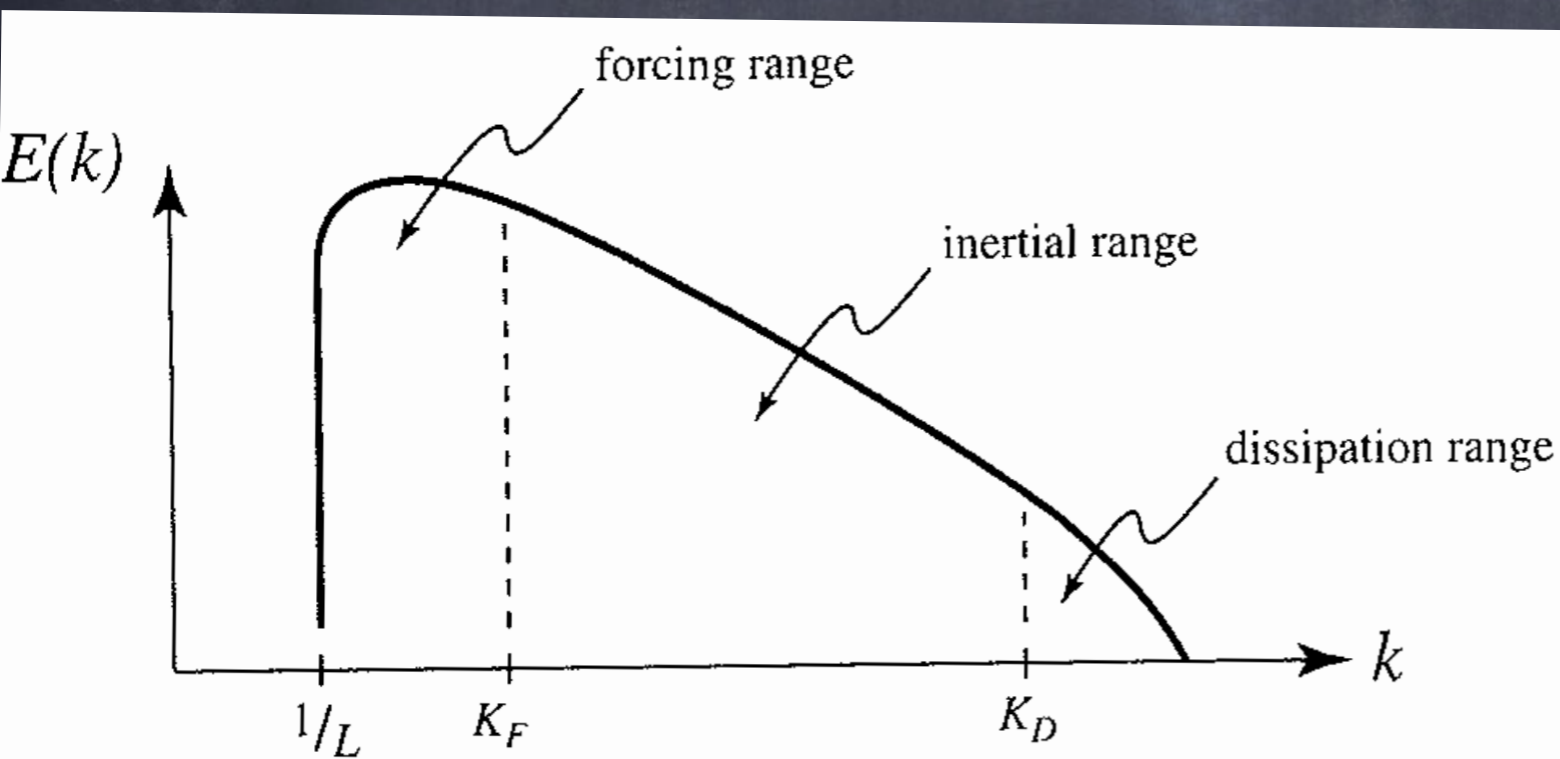
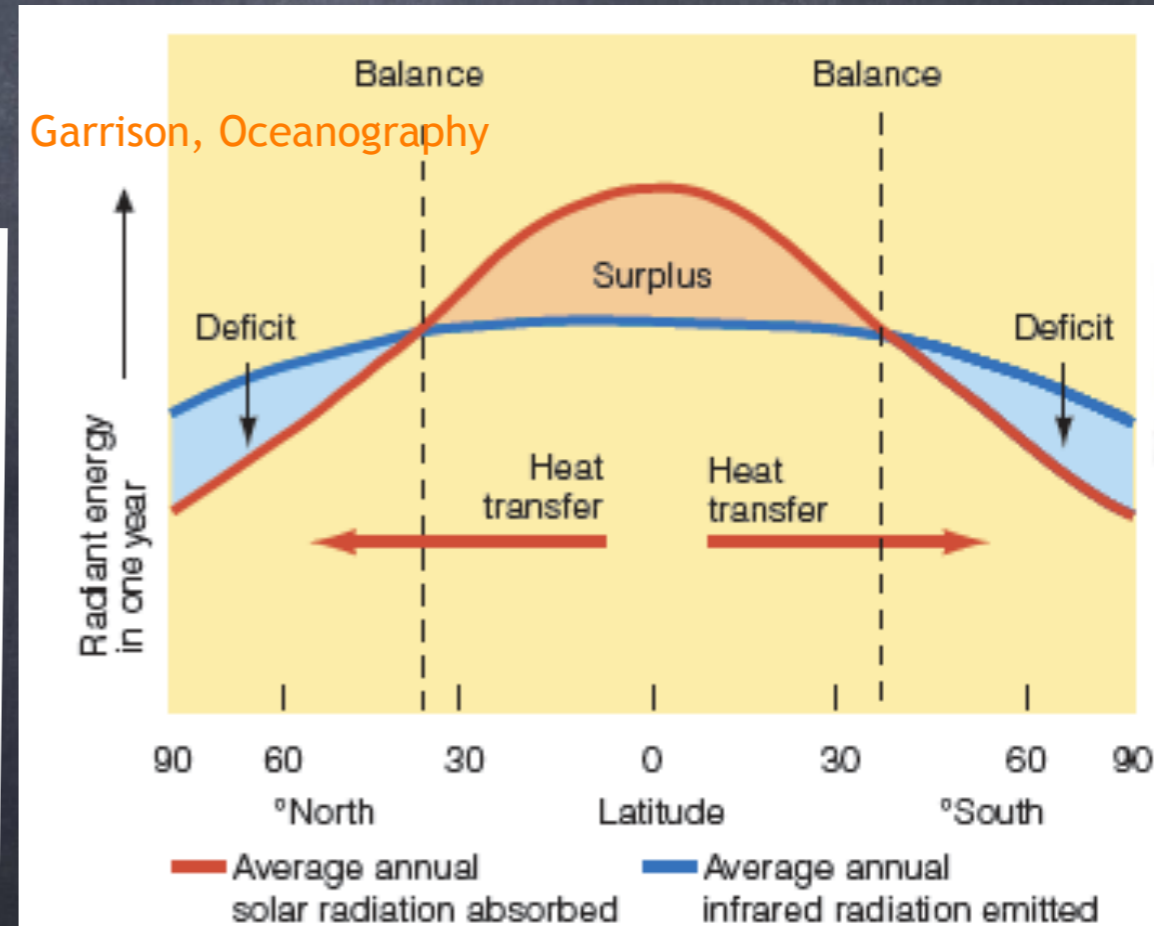
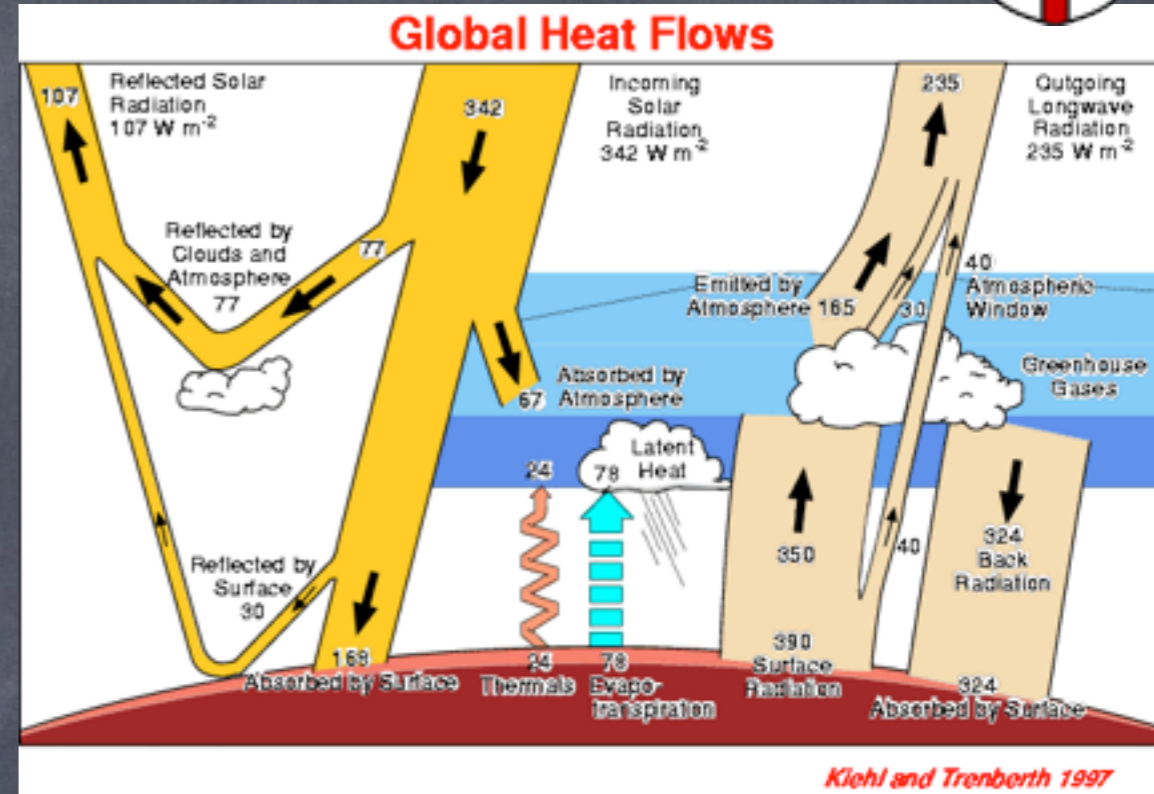
Sponsors: NSF 1258907, 1245944, 0934737, 0825614, NASA NNX09AF38G



The Earth's Climate System is driven by the Sun's light (minus outgoing infrared) on a global scale



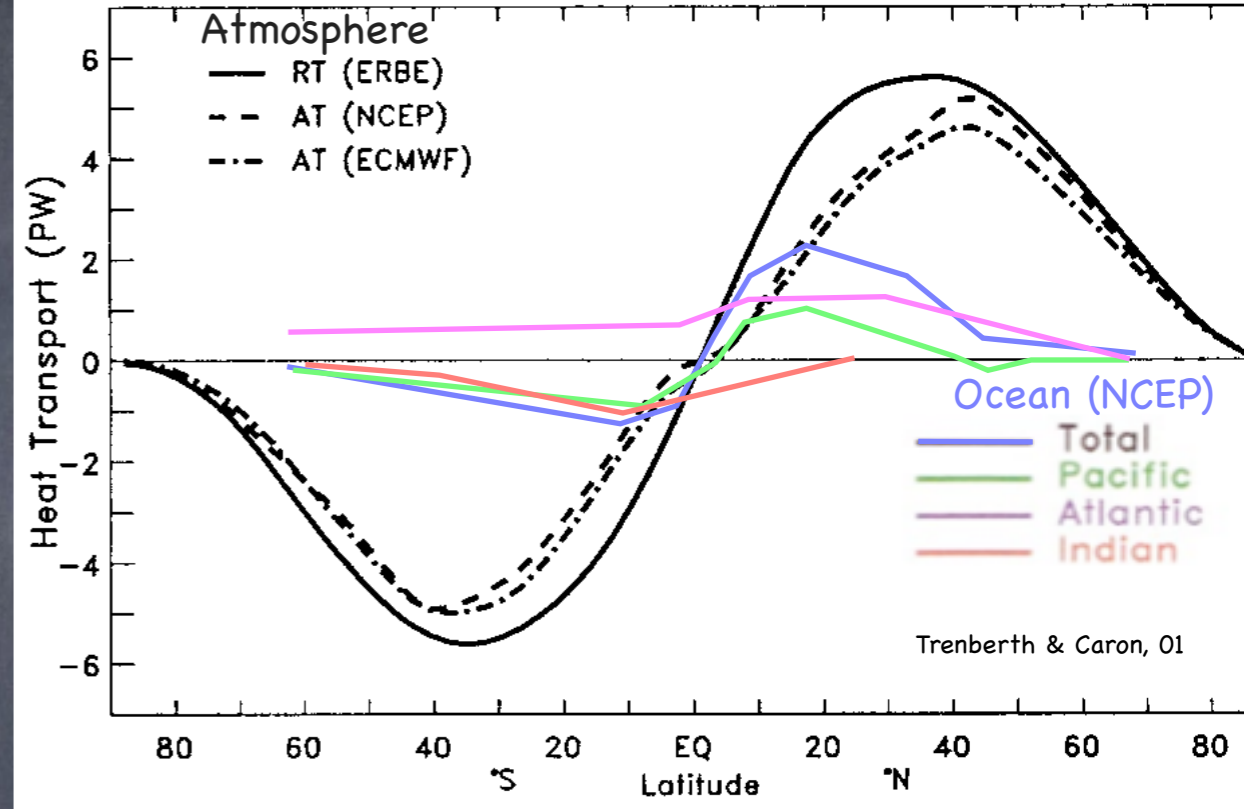
Dissipation concludes turbulence cascades to scales about a billion times smaller



Air-Sea Flux Errors vs. Data

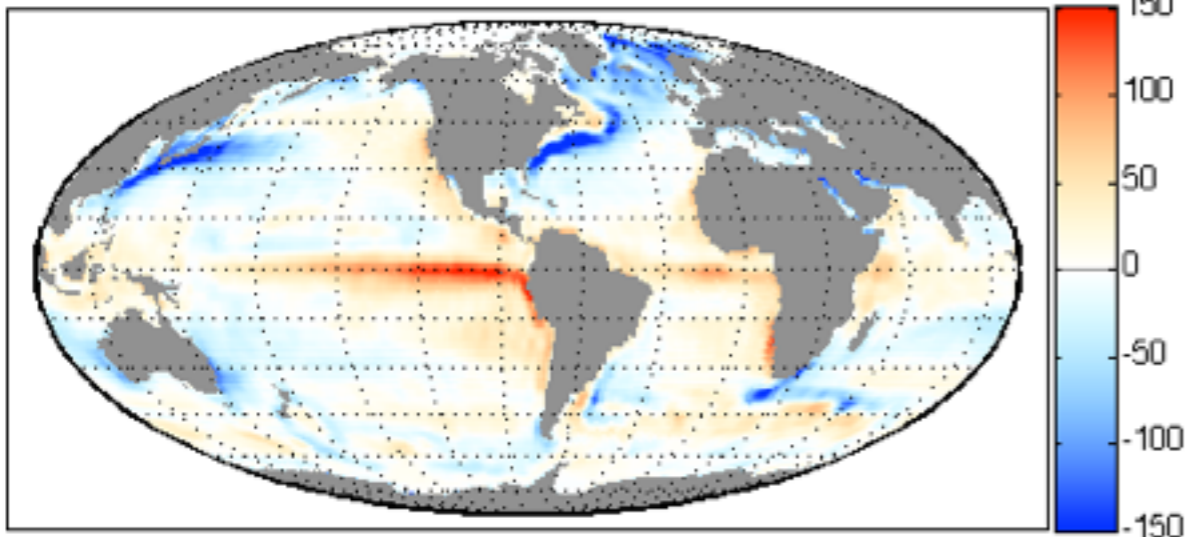
Heat capacity & mode of transport is different in A vs. O

S. C. Bates, B. Fox-Kemper, S. R. Jayne, W. G. Large, S. Stevenson, and S. G. Yeager. Mean biases, variability, and trends in air-sea fluxes and SST in the CCSM4. *Journal of Climate*, 25(22):7781-7801, 2012.

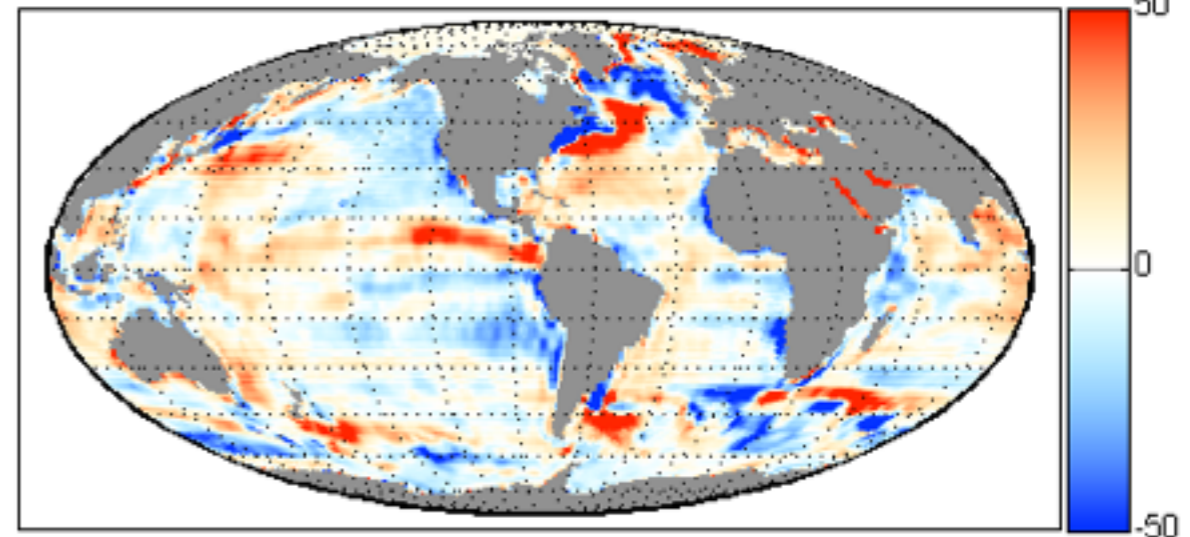


Trenberth & Caron, 01

Mean of 1986-2005 CORE Q_{as} (W/m^2)

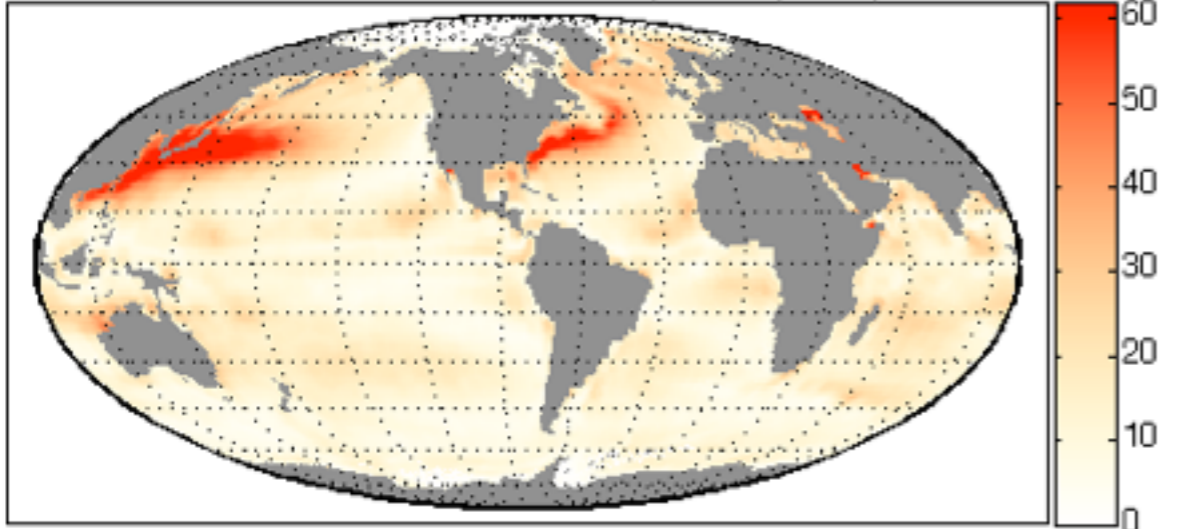


1986-2005 CCSM4-CORE Q_{as} bias, mean:1.5, rms:23 (W/m^2)

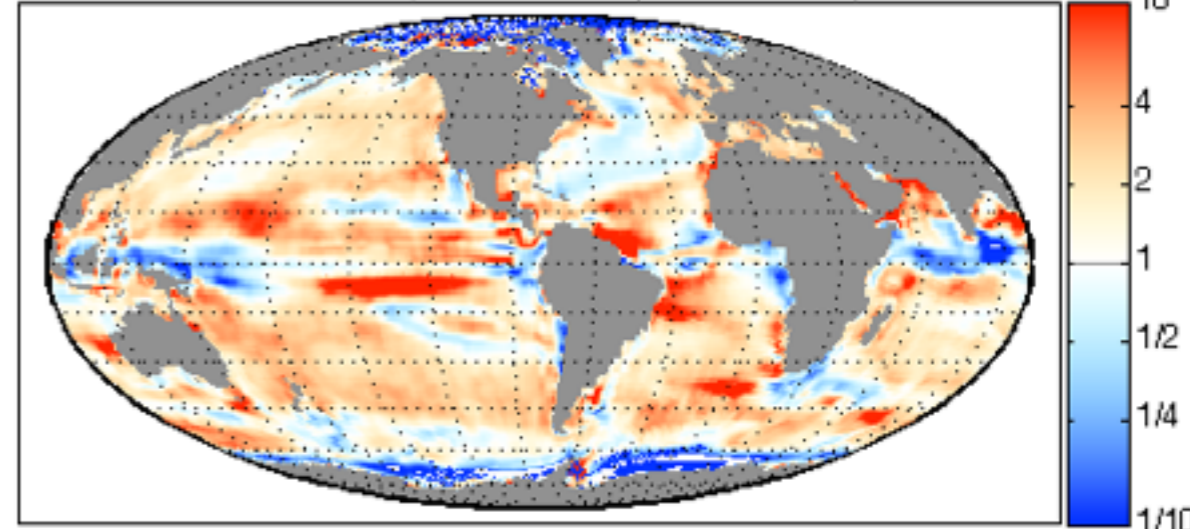


Mean

St. Dev. of CORE annual evaporation (W/m^2)

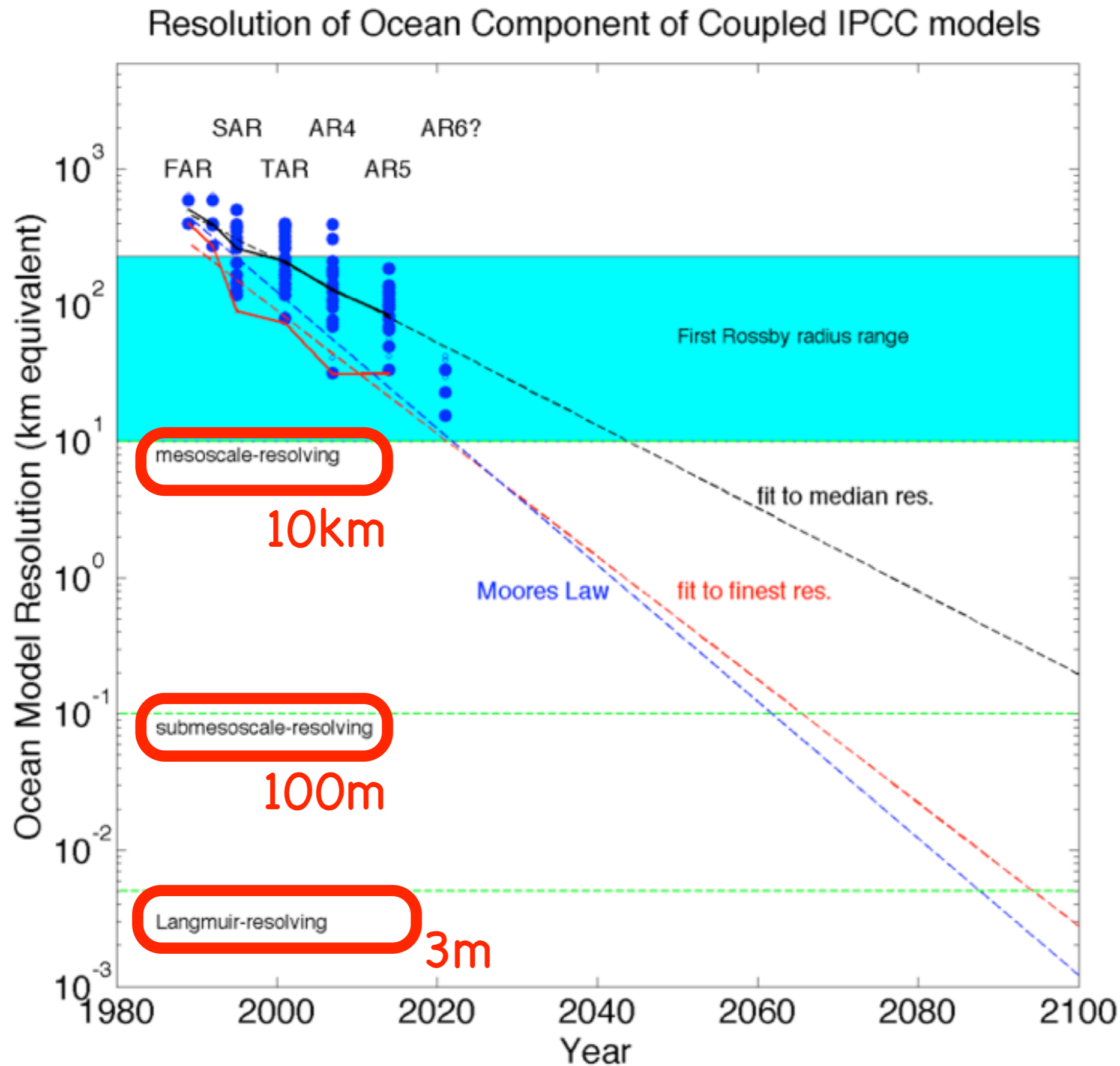


Variance ratio (CCSM4/CORE) of annual evaporation



Annual
9-15mo

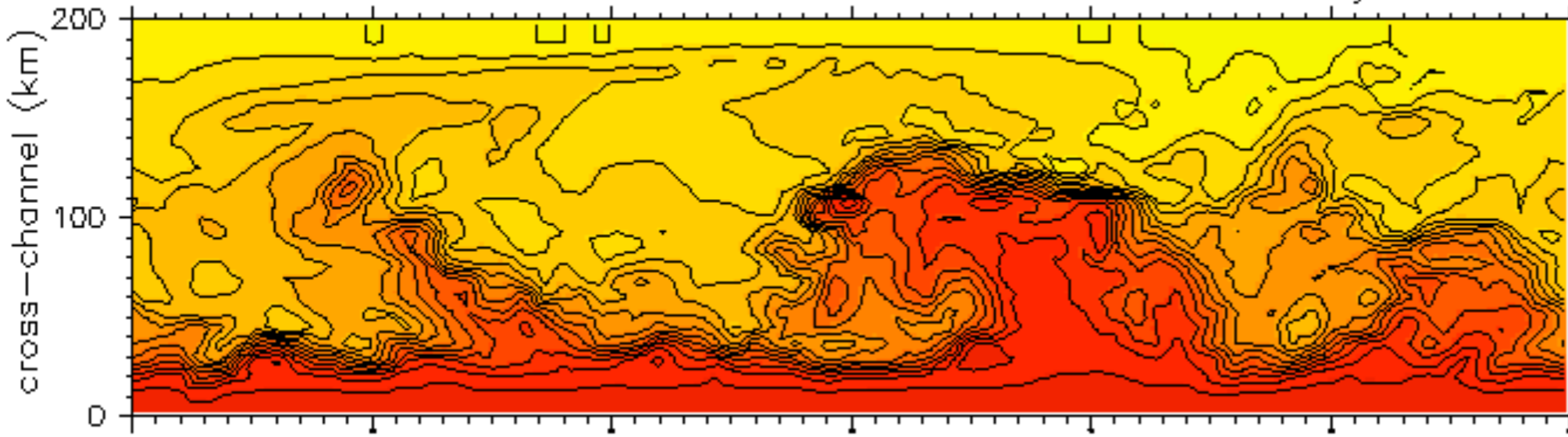
Resolution will be an issue for centuries to come!



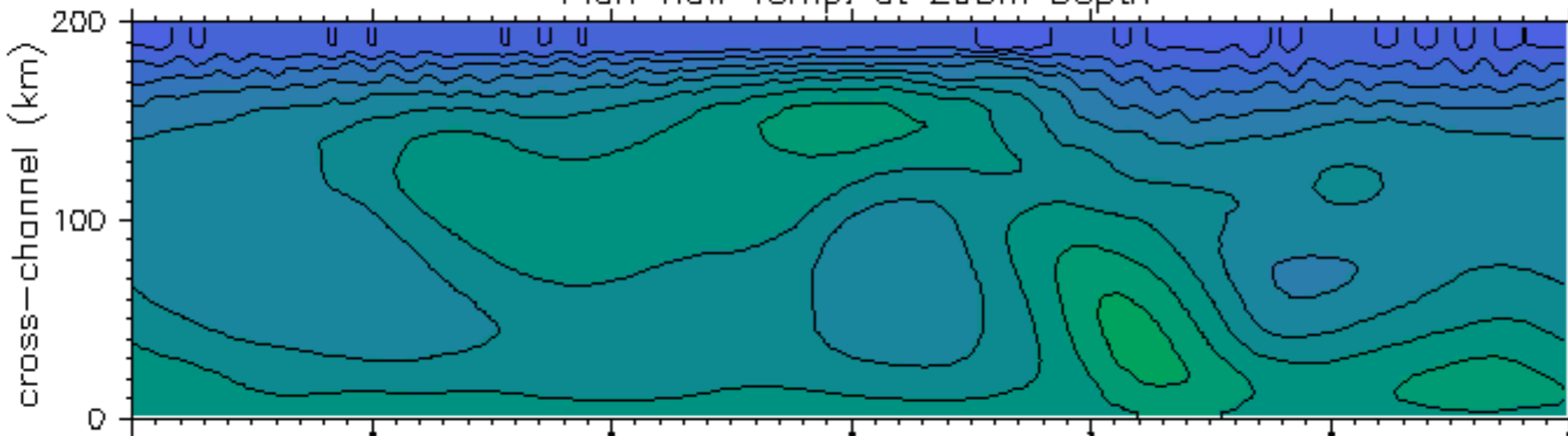
If we can't resolve a process, we need to develop a parameterization or subgrid model of its effect

Plan View Temp. at Surface

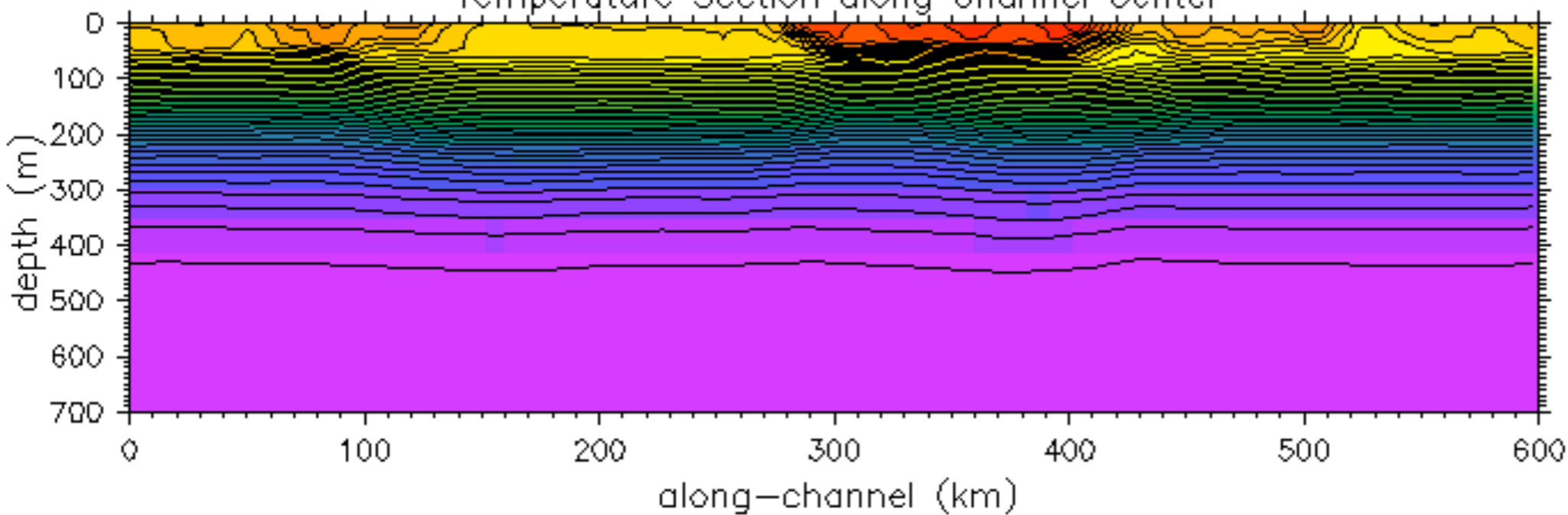
Day: 900



Plan View Temp. at 205m Depth



Temperature Section along Channel Center



Big, Deep
(mesoscale)

interact
with

Little,
Shallow
(submeso)

B. Fox-Kemper, R. Ferrari,
and R. W. Hallberg.
Parameterization of mixed
layer eddies. Part I: Theory
and diagnosis. *Journal of
Physical Oceanography*,
38(6):1145-1165, 2008.

The Character of the Submesoscale

(Capet et al., 2008)

10 km
←

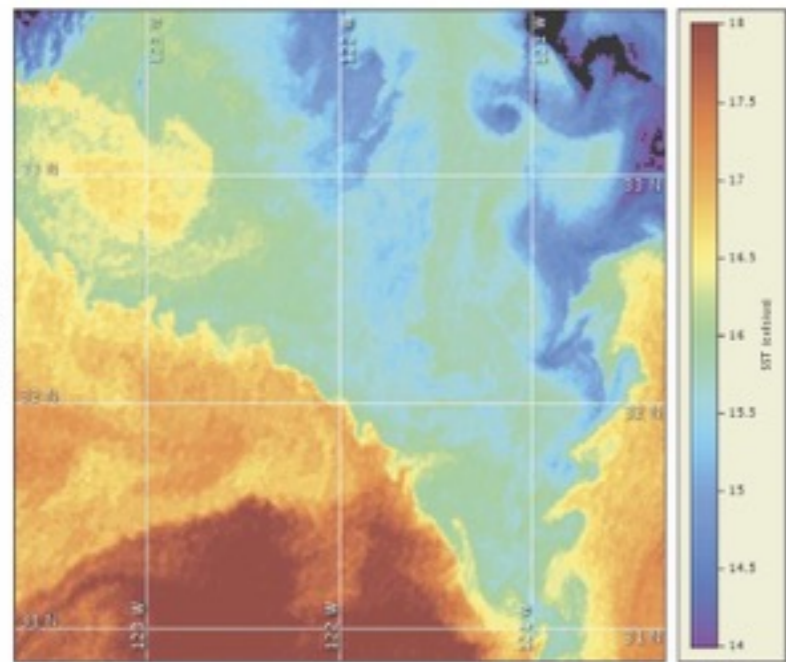
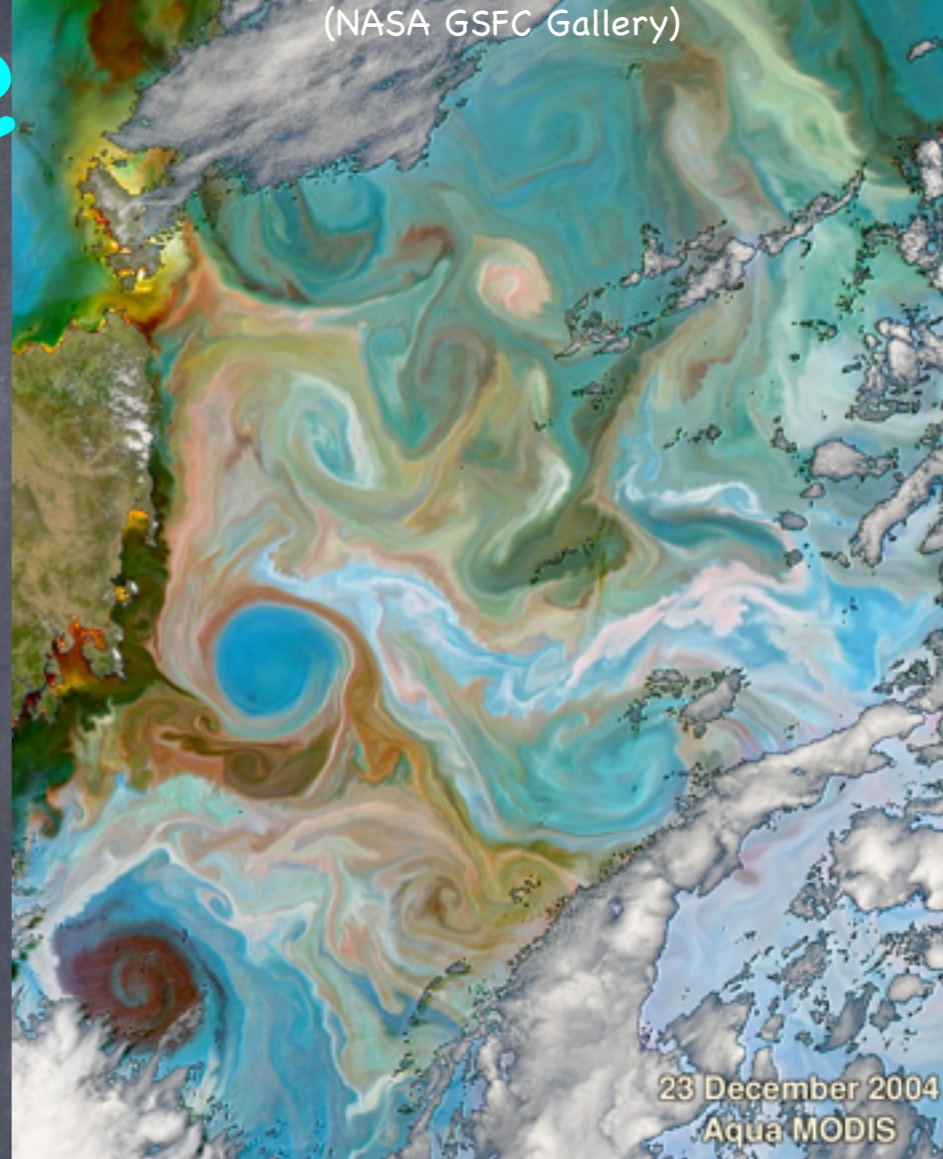
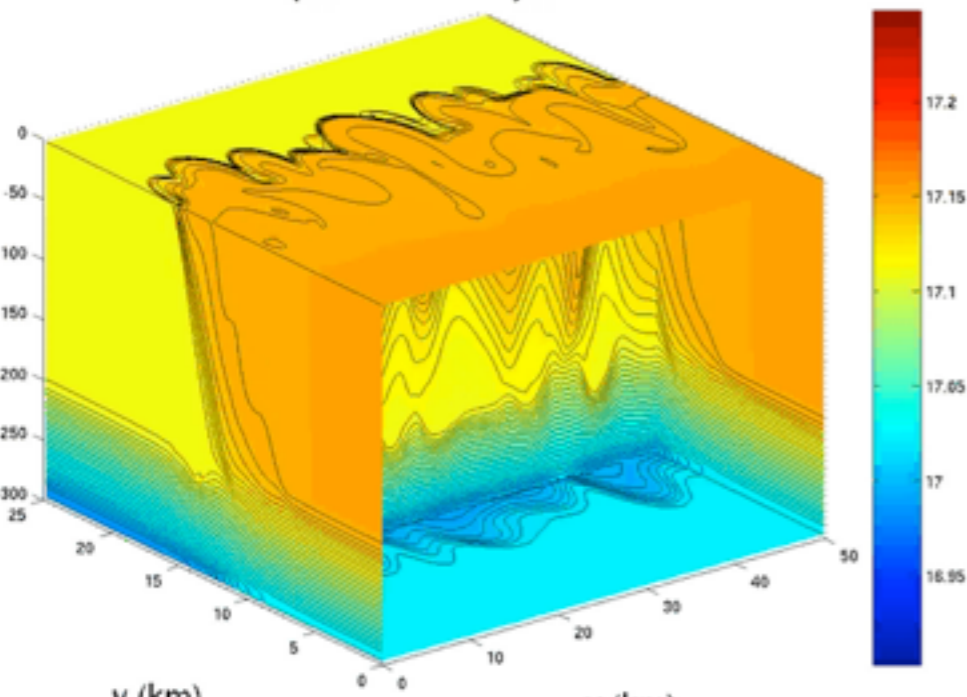


FIG. 16. Sea surface temperature measured at 1832 UTC 3 Jan 2006 off Point Conception in the California Current from CoastWatch (<http://coastwatch.pfeg.noaa.gov>). The fronts between recently

- Fronts
- Eddies
- $Ro=O(1)$
- $Ri=O(1)$
- near-surface
- 1-10km, days

Temperature on day:17.375



Eddy processes often
baroclinic instability

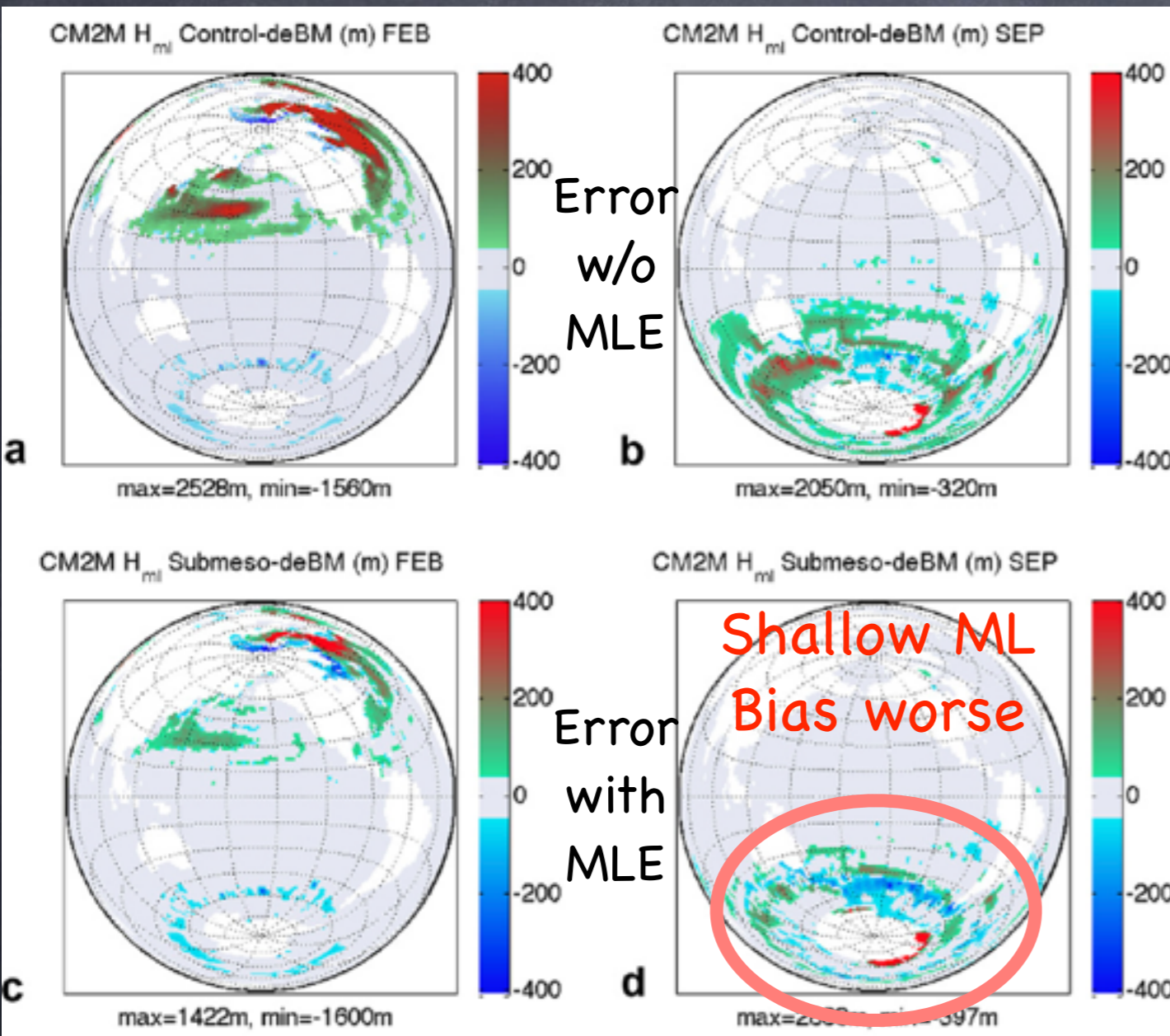
Parameterizations of
submesoscale baroclinic
instability?

B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. *Journal of Physical Oceanography*, 38(6):1145-1165, 2008

S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. *Ocean Modelling*, 64:12-28, 2013

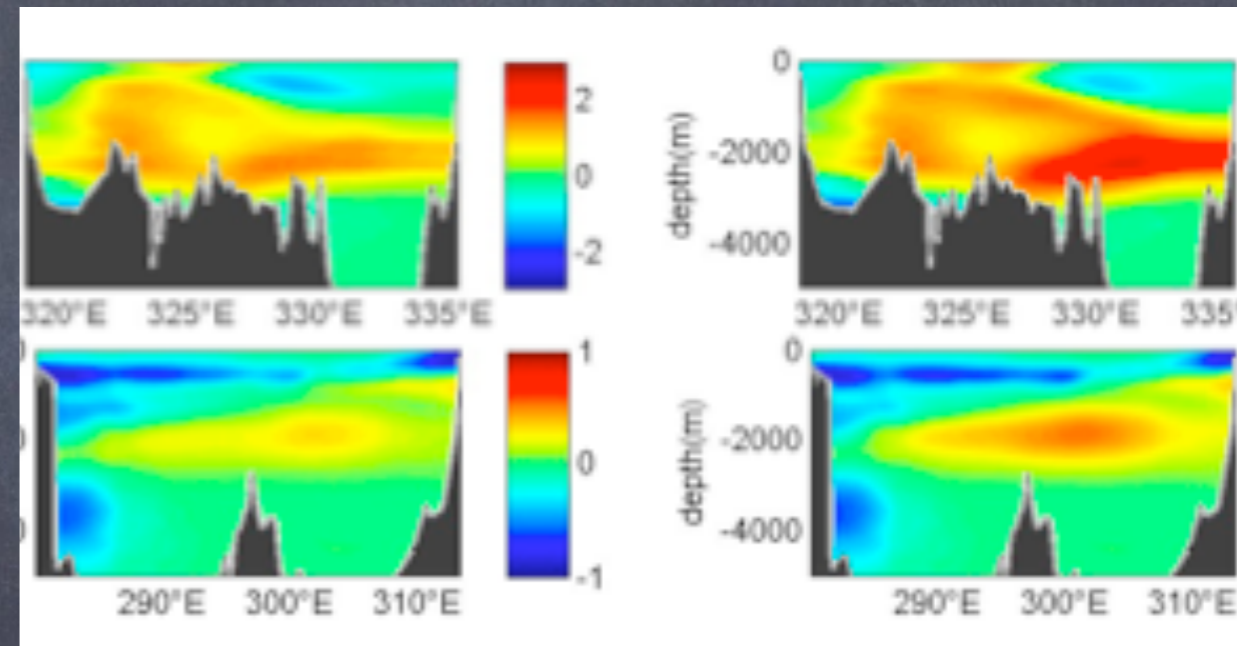
Physical Sensitivity of Ocean Climate to MLE: (submeso) Mixed Layer Eddy Restratification

Improves CFCs
(water masses)



Bias with MLE

Bias w/o MLE



A consistently restratifying,

$$\overline{w'b'} \propto \frac{H^2}{|f|} |\nabla_H \bar{b}|^2$$

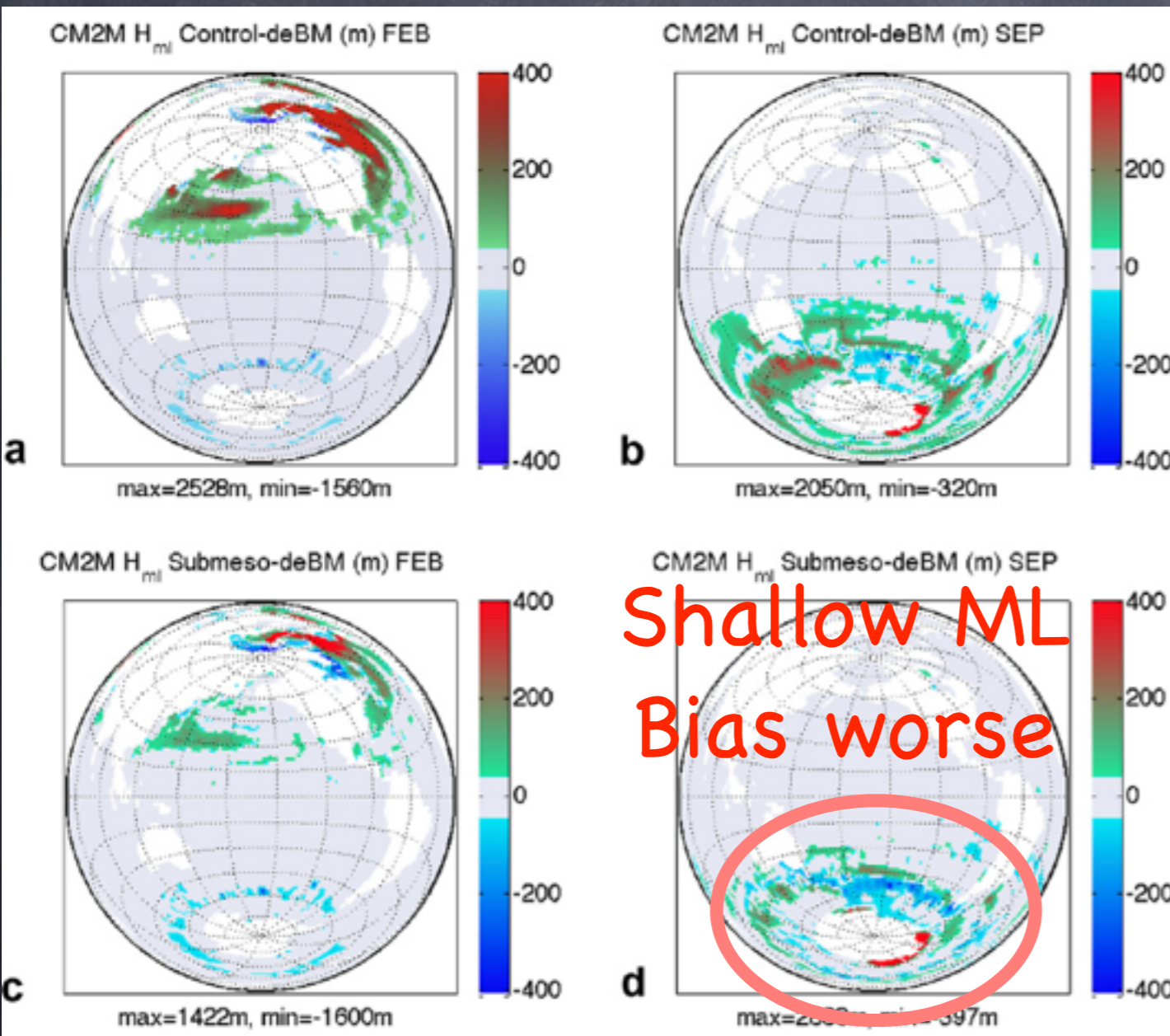
and horizontally downgradient flux.

$$\overline{\mathbf{u}'_H b'} \propto \frac{-H^2 \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla_H \bar{b}$$

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels.
Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

Mixed Layer Problem--Southern Ocean too shallow!

What's missing?



Bias w/o MLE have shown that a too shallow S. Ocean MLD is true of most* present climate models

salinity forcing or ocean physics?

*true for CMIP5 multi-model ensemble

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

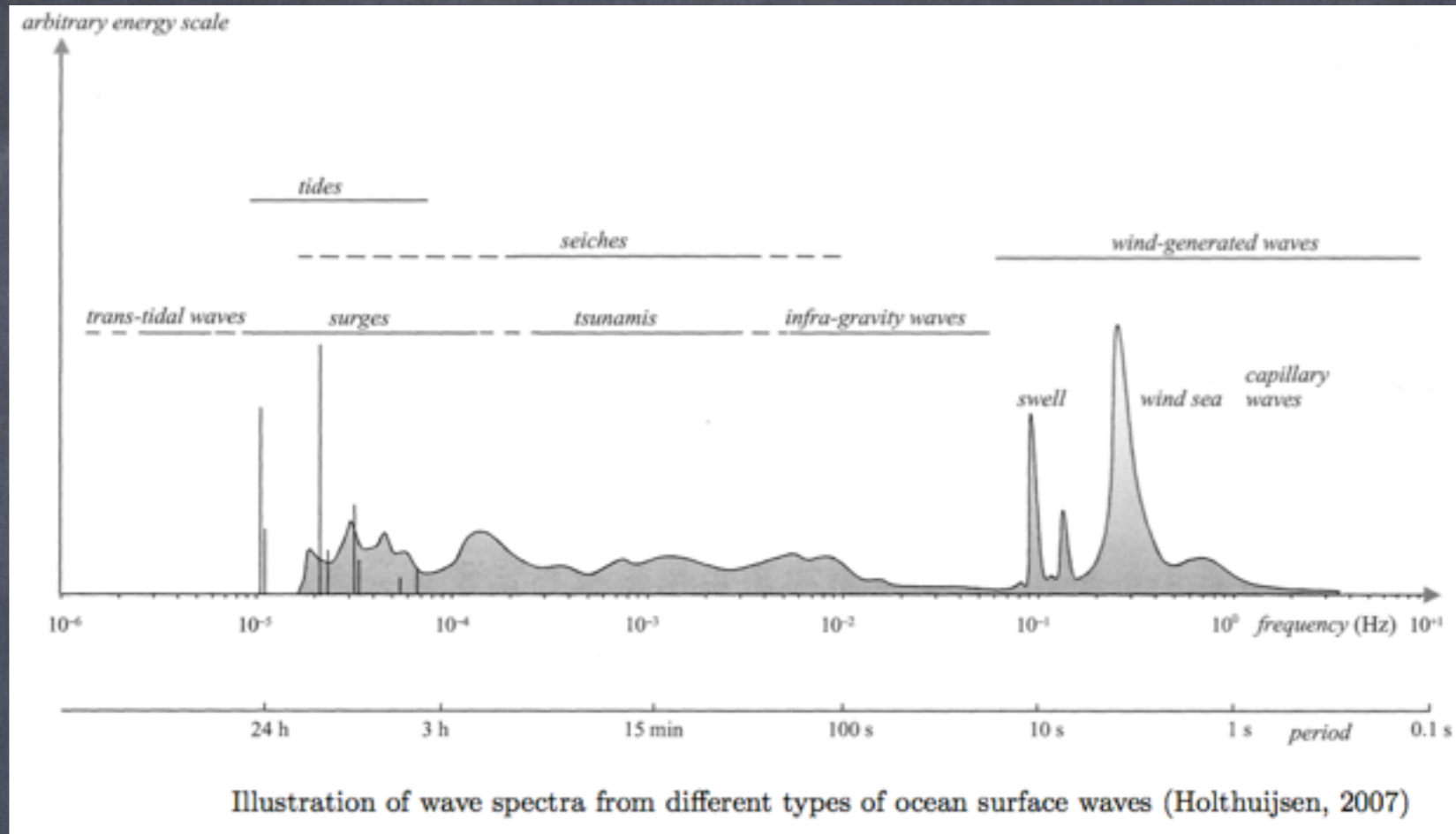
- Lesson: We can study a small-scale system (1-10km submeso mixed layer eddies), derive parameterizations, and then use them to improve climate models & assess impact globally
 - This particular process relied heavily on **thermal wind (geostrophic & hydrostatic)** scaling relationships
- Corollary: But, what about things we haven't thought of yet? e.g., things that aren't geostrophic & hydrostatic?
 - For example, waves and near-surface 3d turbulence

Waves, waves, waves

- I will discuss surface wave effects on upper ocean physics on larger & slower scales.
 - On Langmuir Turbulence Scales
 - (10–100m, 10–100min)
 - Submesoscales
 - (1–10km, 0.1 to 10 days)
- One test involving Langmuir–Submesoscale coupling (10m–10km, 30 days)

Surface Wave Primer

Look for fast, small solutions of the Boussinesq Equations:



The irrotational, incompressible flow obeys

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

$$u \equiv \frac{\partial \phi}{\partial x} \quad w \equiv \frac{\partial \phi}{\partial z}$$

The boundary conditions are:

Solid
Bottom

$$w = \frac{\partial \phi}{\partial z} = 0 \quad \text{at} \quad z = -H$$

Pressure
Matching
(dynamic)

$$p = 0 \quad \text{at} \quad z = \eta$$

Velocity
Matching
(kinematic)

$$\frac{D\eta}{Dt} = w_\eta \quad \text{at} \quad z = \eta$$



Surface Wave Primer

Look for fast, small solutions of the Boussinesq Equations:

Linearized for not steep waves

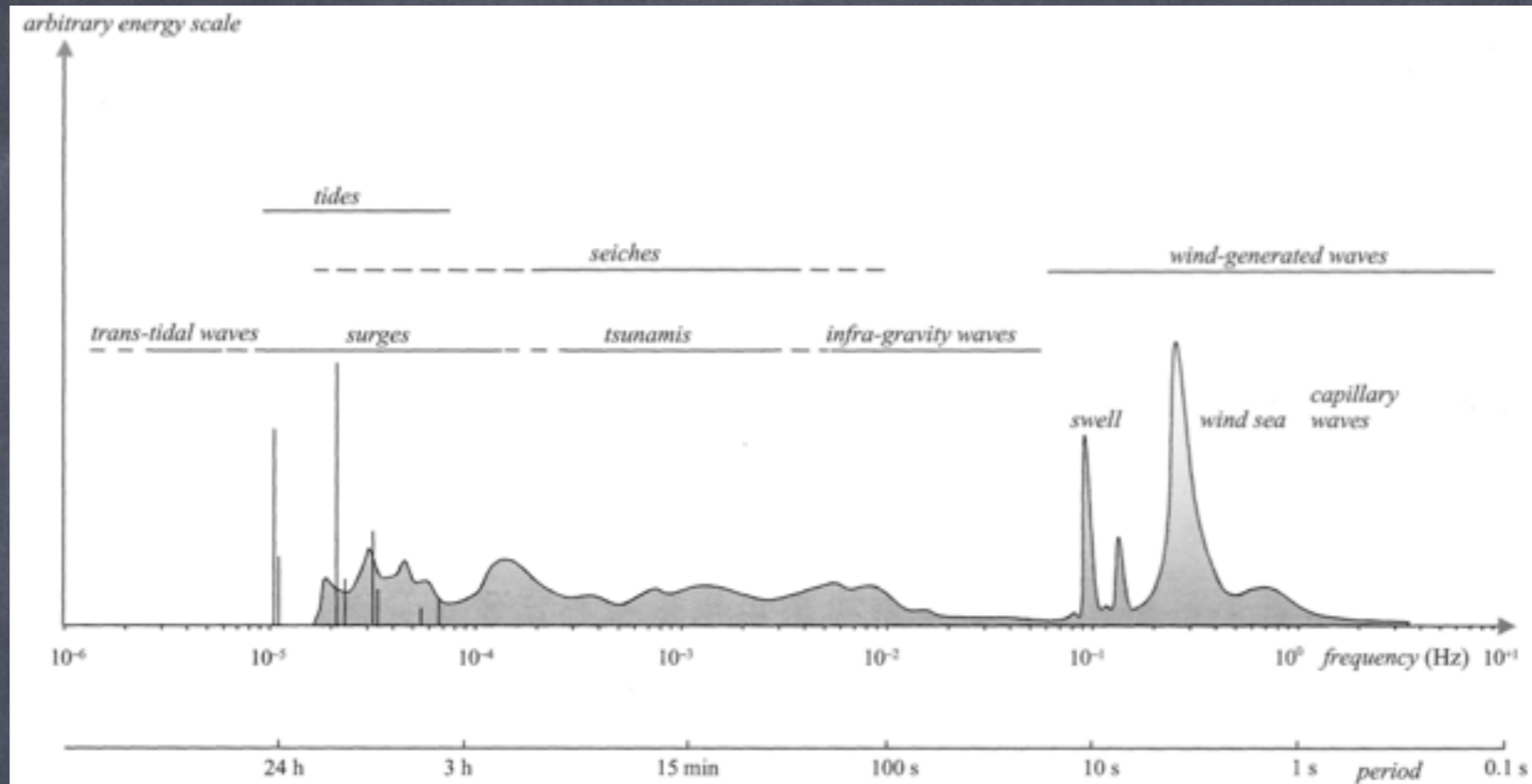


Illustration of wave spectra from different types of ocean surface waves (Holthuijsen, 2007)

The irrotational, incompressible flow obeys

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

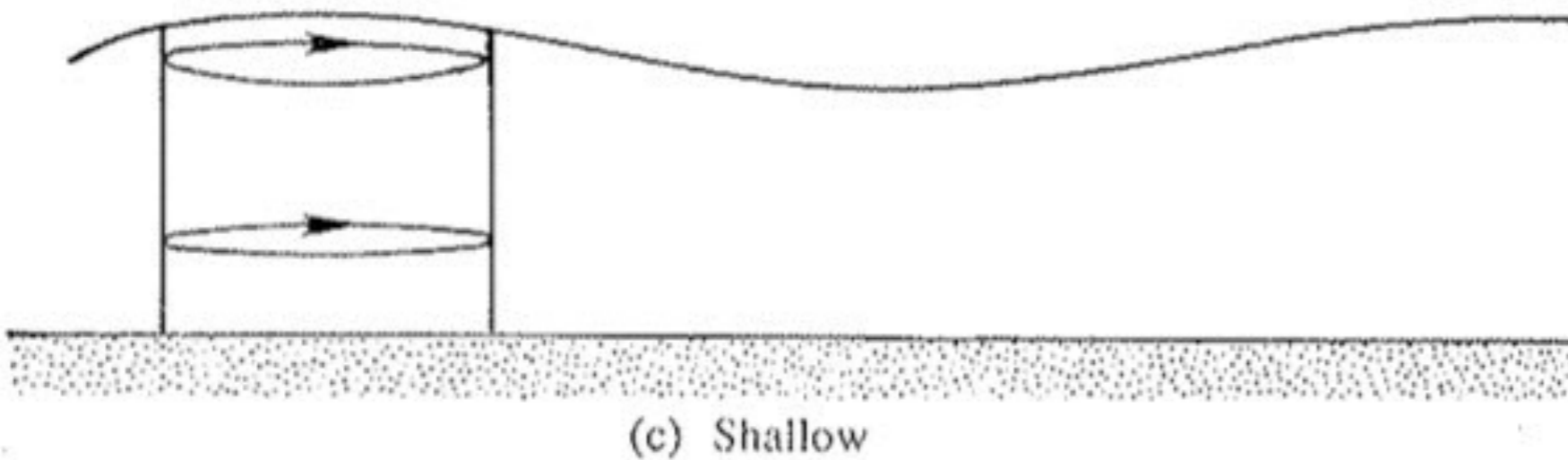
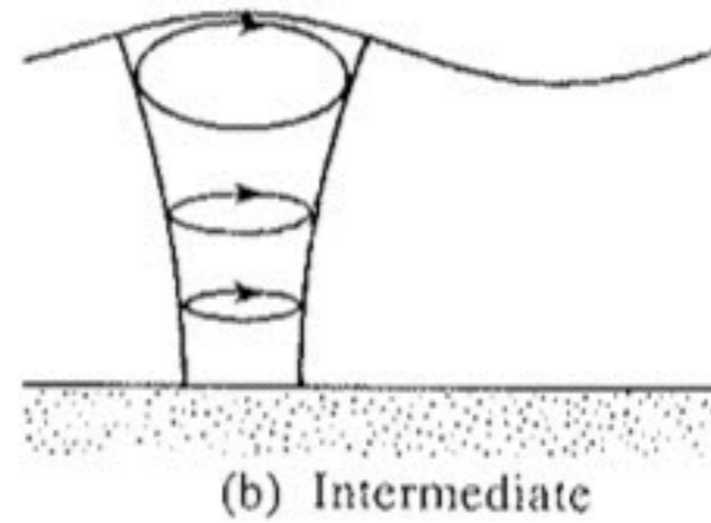
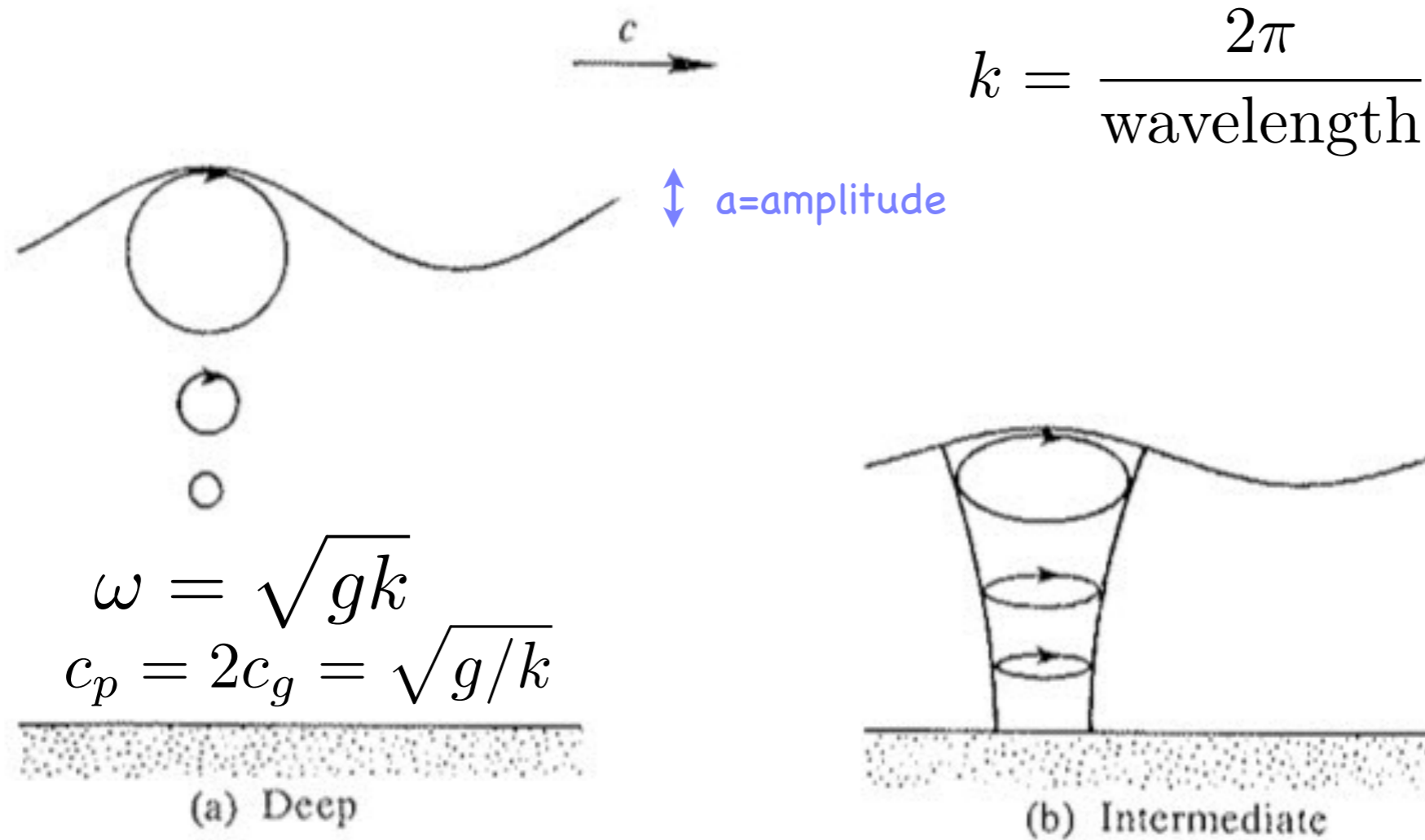
$$u \equiv \frac{\partial \phi}{\partial x} \quad w \equiv \frac{\partial \phi}{\partial z}$$

The boundary conditions are (small steepness):

Solid Bottom	$w = \frac{\partial \phi}{\partial z} = 0$	at $z = -H$
Pressure Matching (dynamic)	$\frac{\partial \phi}{\partial t} = -g\eta$	at $z = 0$
Velocity Matching (kinematic)	$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z}$	at $z = 0$



Particle motions



The u, v , decay exponentially toward the bottom with decay scale proportional to the wavelength.

Thus, kH is a measure of depth

ka is a measure of steepness

Deep water waves don't "feel" the bottom. Implies nonhydrostatic () & fast timescale ($Ro \gg 1$)

Craik–Leibovich Boussinesq

- Formally a multiscale asymptotic equation set:
 - 3 classes: Small, Fast; Large, Fast; Large, Slow
 - Solve first 2 types of motion in the case of limited slope (ka), irrotational \rightarrow Deep Water Waves!
 - Must also assume slowly-varying wave packets
 - Average over deep water waves in space & time,
 - Arrive at Large, Slow equation set:

$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times (\mathbf{v} + \mathbf{v}_s) = -\nabla \pi^\dagger + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$

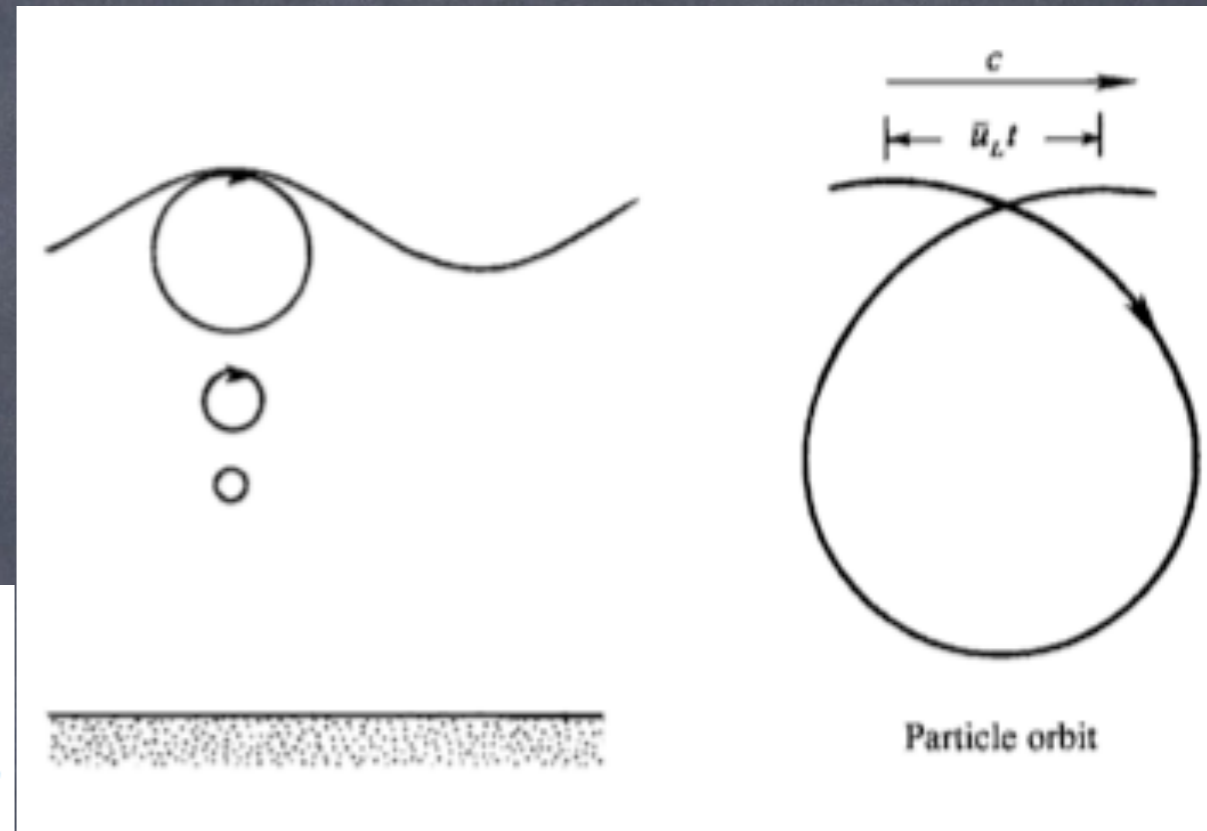
$$\frac{\partial b}{\partial t} + (\mathbf{v} + \mathbf{v}_s) \cdot \nabla b = 0 \quad \nabla \cdot \mathbf{v} = 0$$

$$\mathbf{v}_s = \text{Stokes Drift}$$

What is Stokes Drift?

Take wave solns, compare the velocity of trajectories vs. Eulerian velocity, Taylor Expand, calculate:

$$\begin{aligned} \mathbf{u}^L(\mathbf{x}_p(t_0), t) - \mathbf{u}^E(\mathbf{x}_p(t_0), t) &\approx [\mathbf{x}_p(t) - \mathbf{x}_p(t_0)] \cdot \nabla \mathbf{u}^E(\mathbf{x}_p(t_0), t) \\ &\approx \left[\int_{t_0}^t \mathbf{u}^E(\mathbf{x}_p(t_0), s') ds' \right] \cdot \nabla \mathbf{u}^E(\mathbf{x}_p(t_0), t). \end{aligned}$$



Examples:

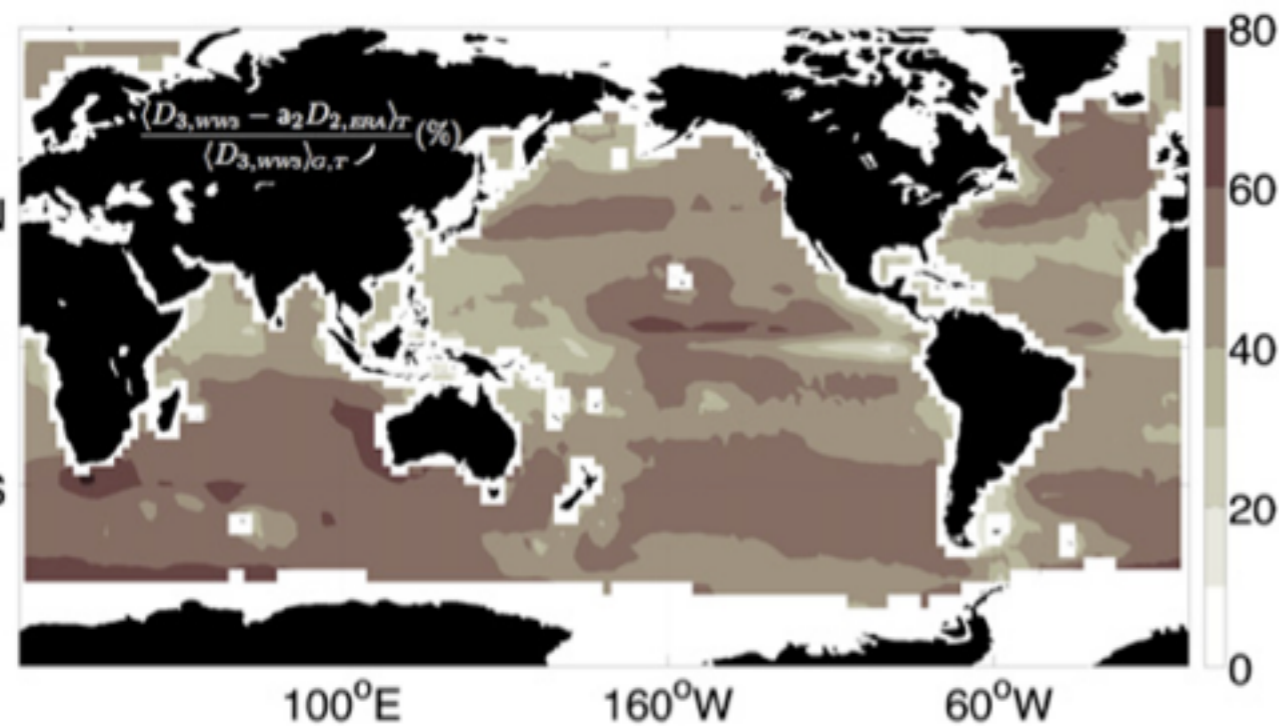
Monochromatic:

$$\mathbf{u}^S = \hat{\mathbf{e}}^w \frac{8\pi^3 a^2 f_p^3}{g} e^{\frac{8\pi^2 f_p^2}{g} z} = \hat{\mathbf{e}}^w a^2 \sqrt{gk^3} e^{2kz}.$$

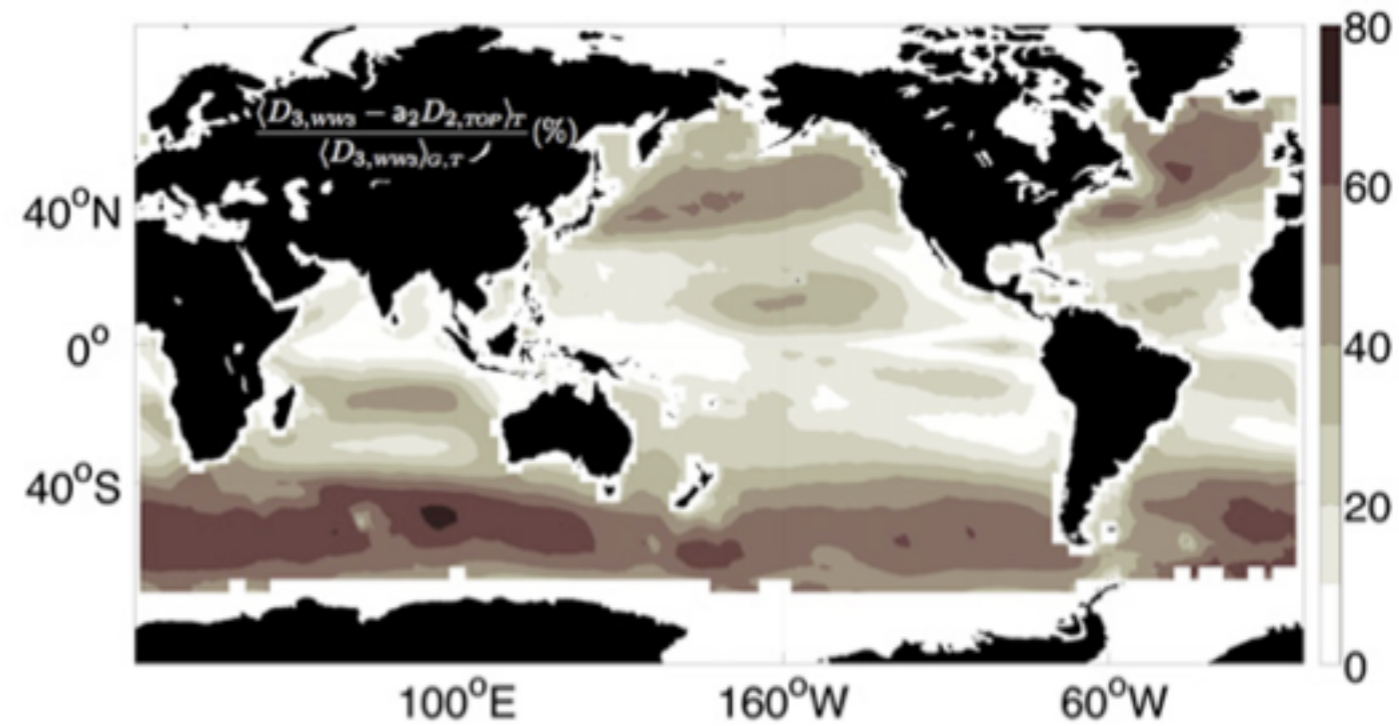
Spectrum:

$$\mathbf{u}^S = \frac{16\pi^3}{g} \int_0^\infty \int_{-\pi}^\pi (\cos \theta, \sin \theta, 0) f^3 S_{f\theta}(f, \theta) e^{\frac{8\pi^2 f^2}{g} z} d\theta df.$$

How well do we know Stokes Drift? <50% discrepancy



(e) $\langle D_{3,WW3} - a_2 D_{2,ERA} \rangle_T / \langle D_{3,WW3} \rangle_{G,T}$ (%)



(f) $\langle D_{3,WW3} - a_2 D_{2,TOPEX} \rangle_T / \langle D_{3,WW3} \rangle_{G,T}$ (%)

RMS error in measures of surface Stokes drift, 2 wave models (left), model vs. altimeter (right)

Year 2000 data & models

The Character of the Langmuir Scale

- Near-surface
- Langmuir Cells & Langmuir Turb.
- $Ro \gg 1$
- Aspect $O(1)$: Nonhydro
- 1-10m
- 10s to mins
- $w, u = O(10\text{cm/s})$
- Stokes drift
- Eqtns: Craik-Leibovich
- Params: McWilliams & Sullivan, 2000, etc.

image:
Thorpe, 04

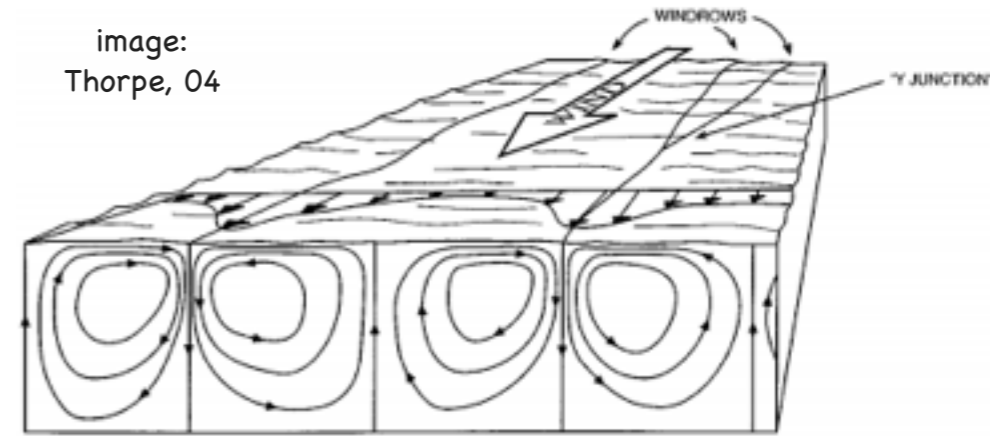
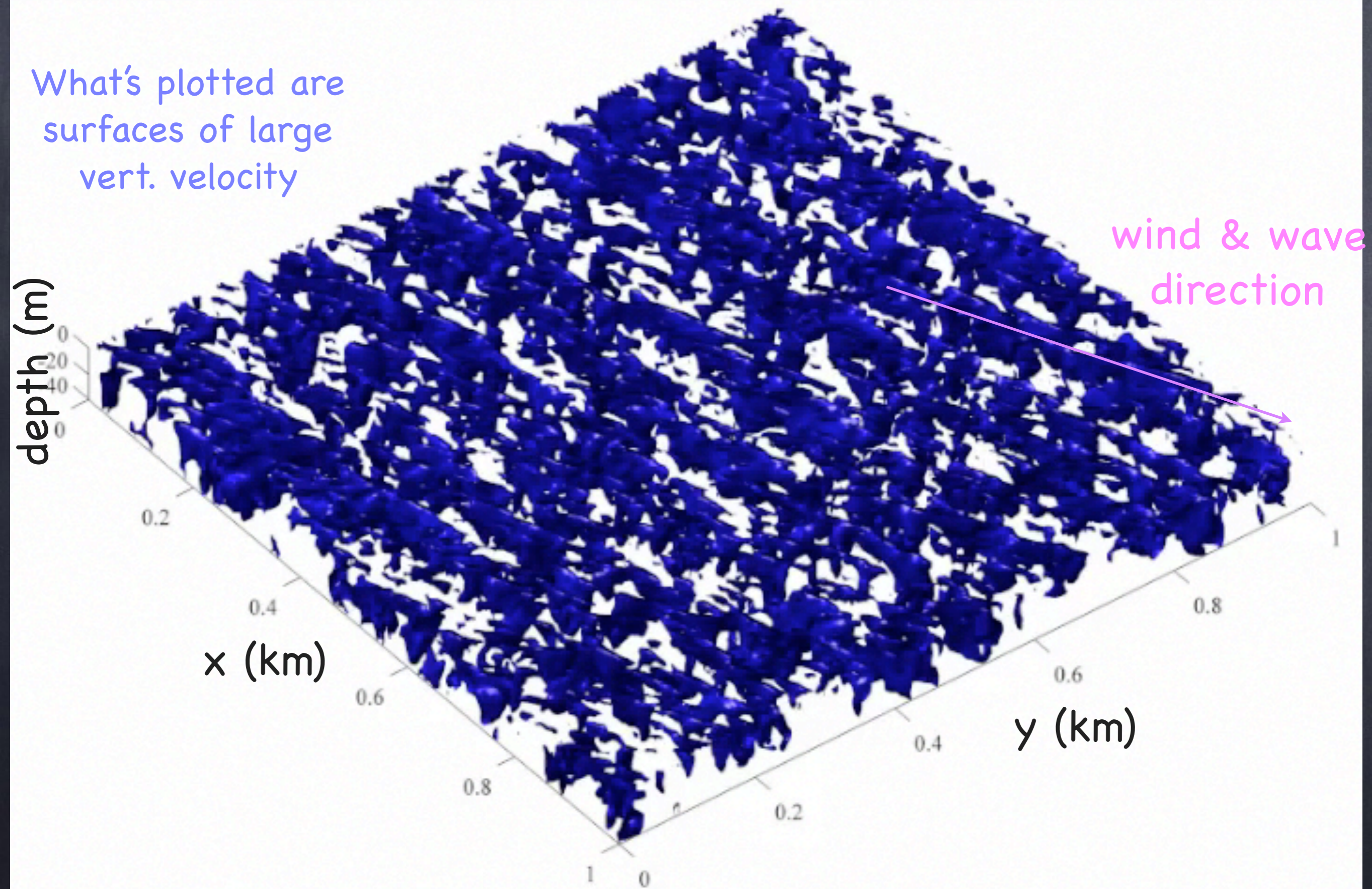


Figure 1 Sketch showing the pattern of mean flow in idealized Langmuir circulation. The windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). In practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2) amalgamate and meander in space and time. Bands of bubbles or buoyant algae may form within the downward-going (or downwelling) flow (see Figure 3).

Image: NPR.org,
Deep Water
Horizon Spill

What's plotted are
surfaces of large
vert. velocity



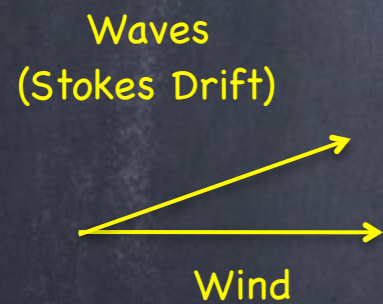
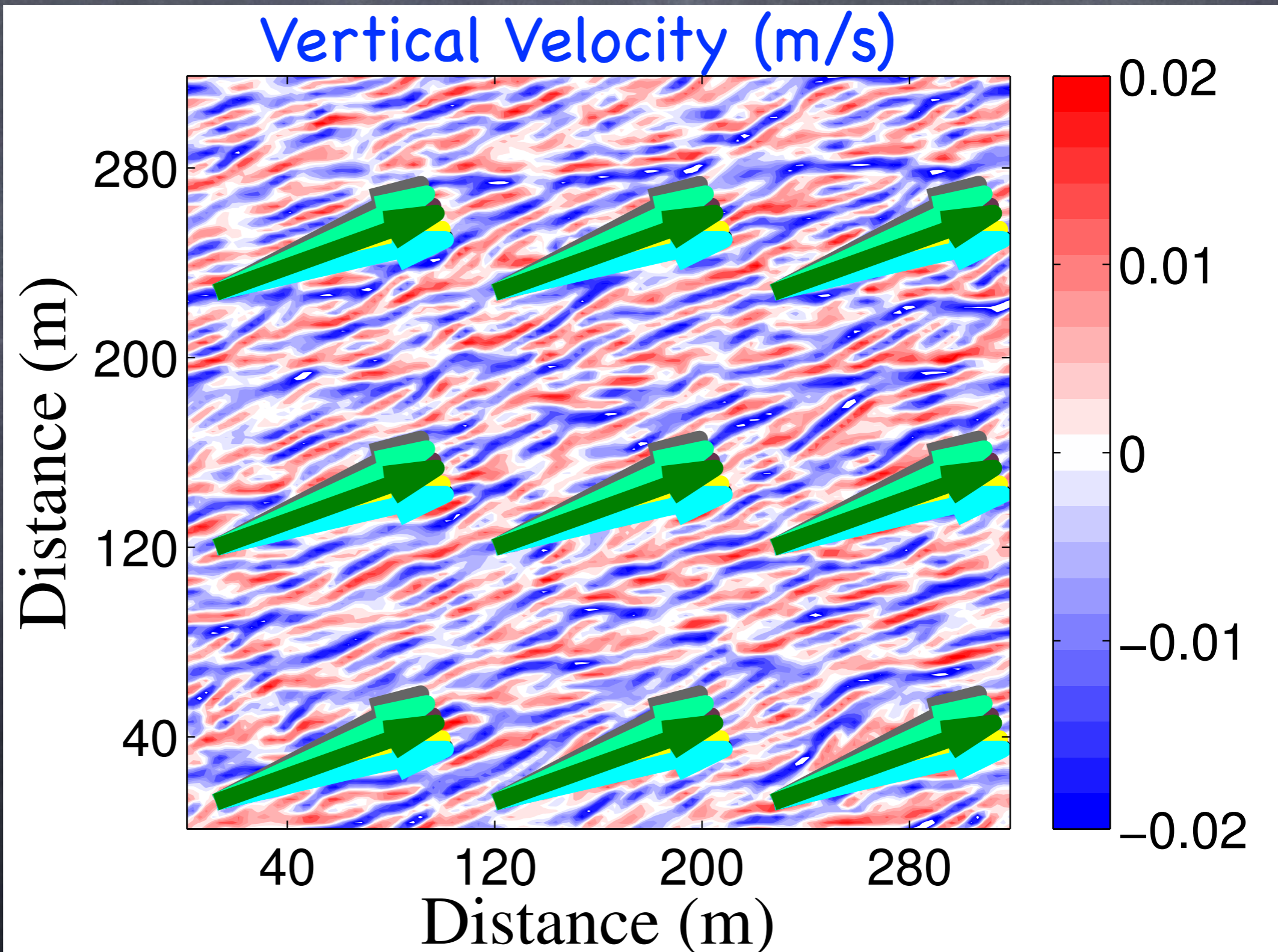
wind & wave
direction

depth (m)

x (km)

y (km)

CLB as equations for Large Eddy Simulations: Tricky: Misaligned Wind & Waves



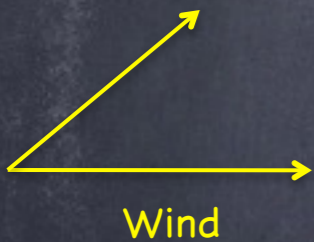
L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

Tricky: Misaligned Wind & Waves

Vertical Velocity (m/s)



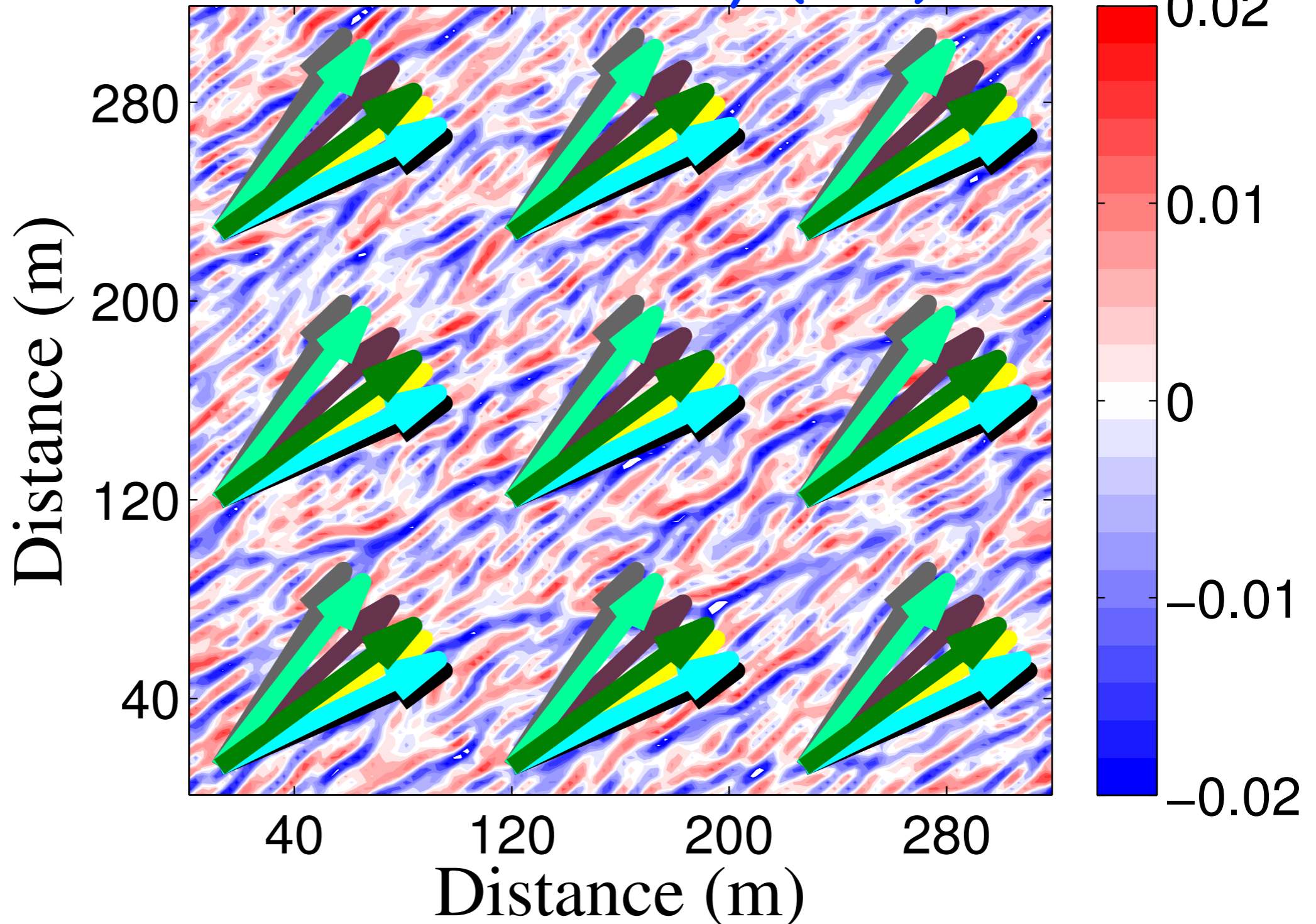
Waves
(Stokes Drift)



L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

Tricky: Misaligned Wind & Waves

Vertical Velocity (m/s)



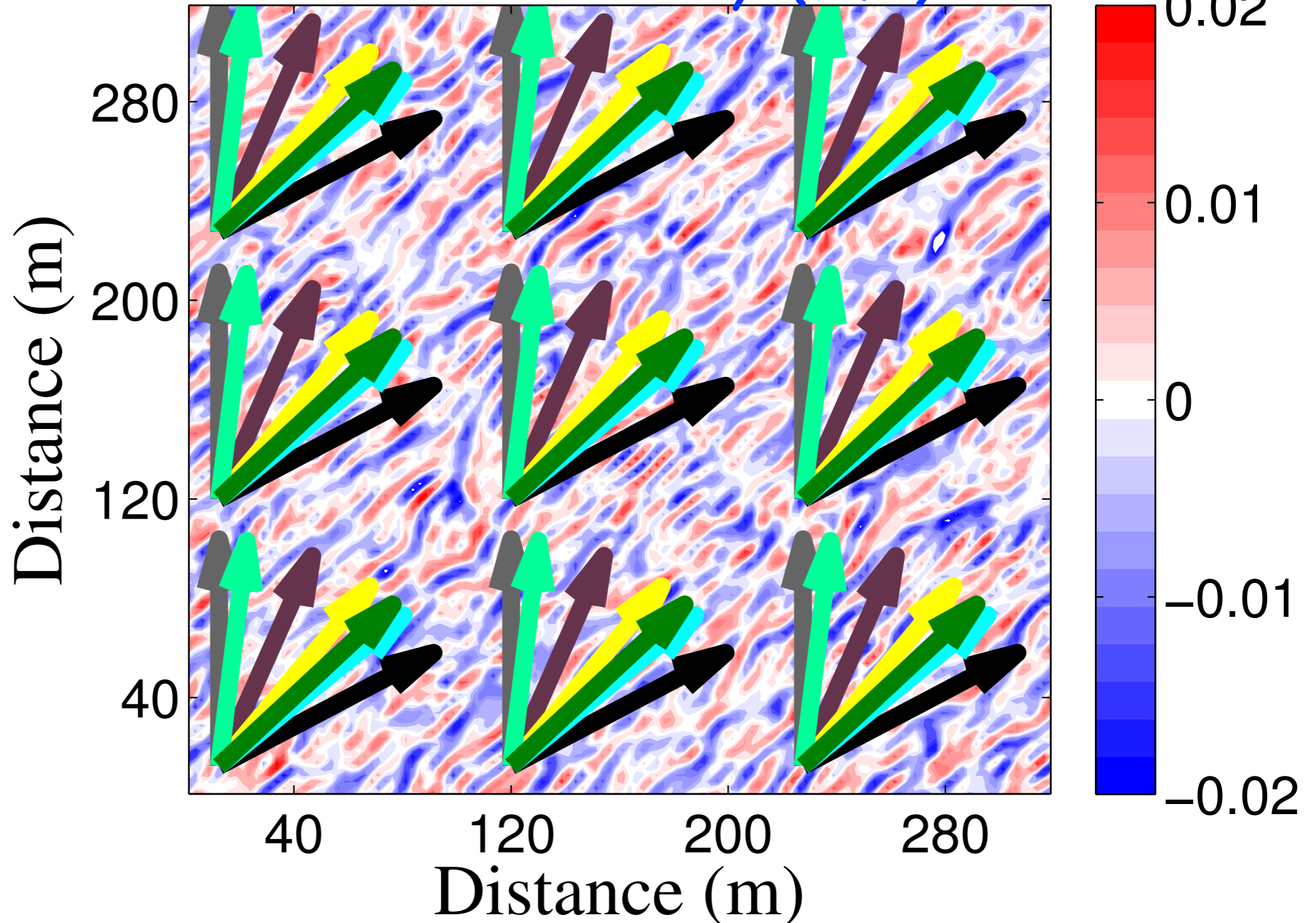
Waves
(Stokes Drift)



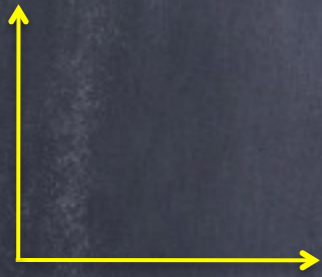
L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

Tricky: Misaligned Wind & Waves

Vertical Velocity (m/s)



Waves
(Stokes Drift)



Wind

L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

Why? Vortex Tilting Mechanism

In CLB: Tilting occurs in

direction of $\mathbf{u}_L = \mathbf{v} + \mathbf{v}_s$

$$\frac{\partial \xi}{\partial t} + \underbrace{(\mathbf{u}_L \cdot \nabla) \xi}_{AD} = \underbrace{(\boldsymbol{\omega}_a \cdot \nabla)(\mathbf{u}_L \cdot \hat{\mathbf{x}}')}_{TS} + \underbrace{(\nabla b \times \hat{\mathbf{z}}) \cdot \hat{\mathbf{x}}'}_{BV} + \text{SGS},$$

image:
Thorpe, 04

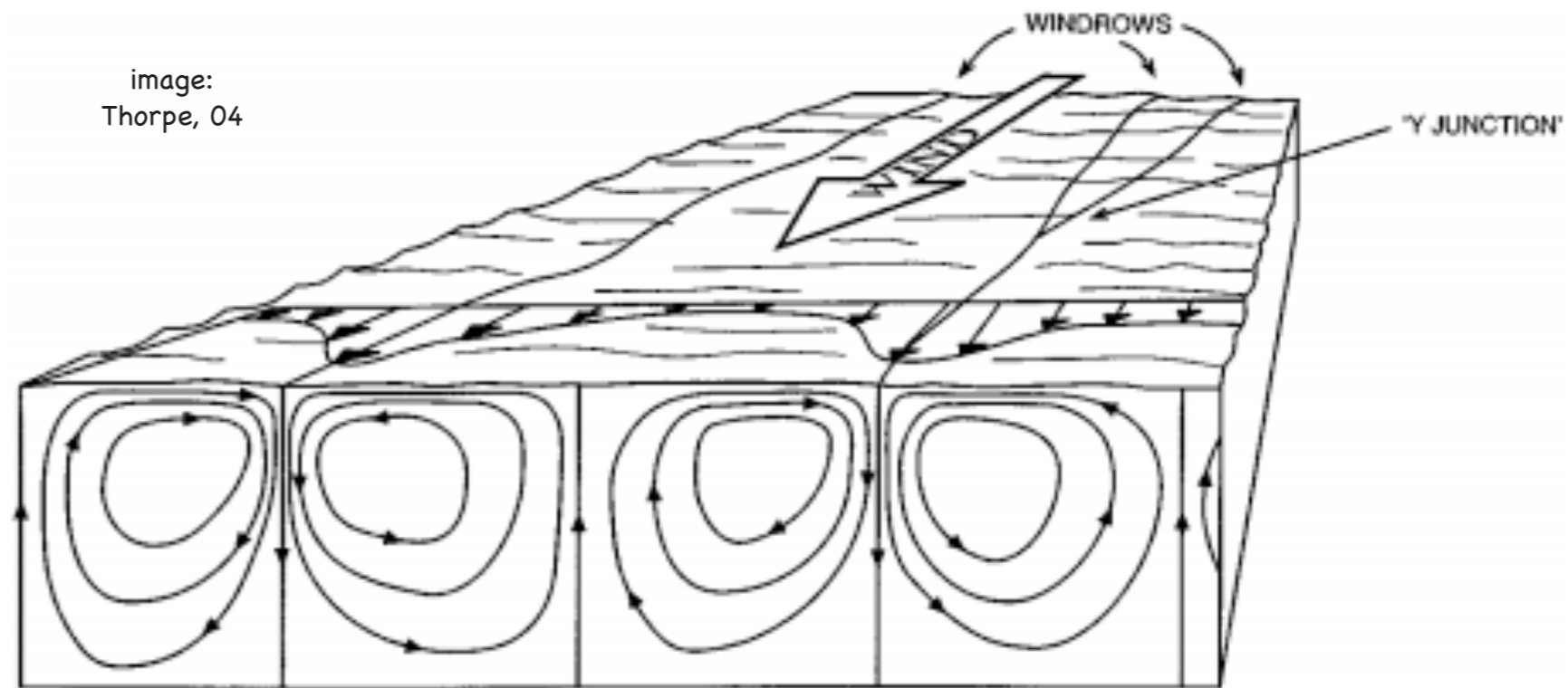
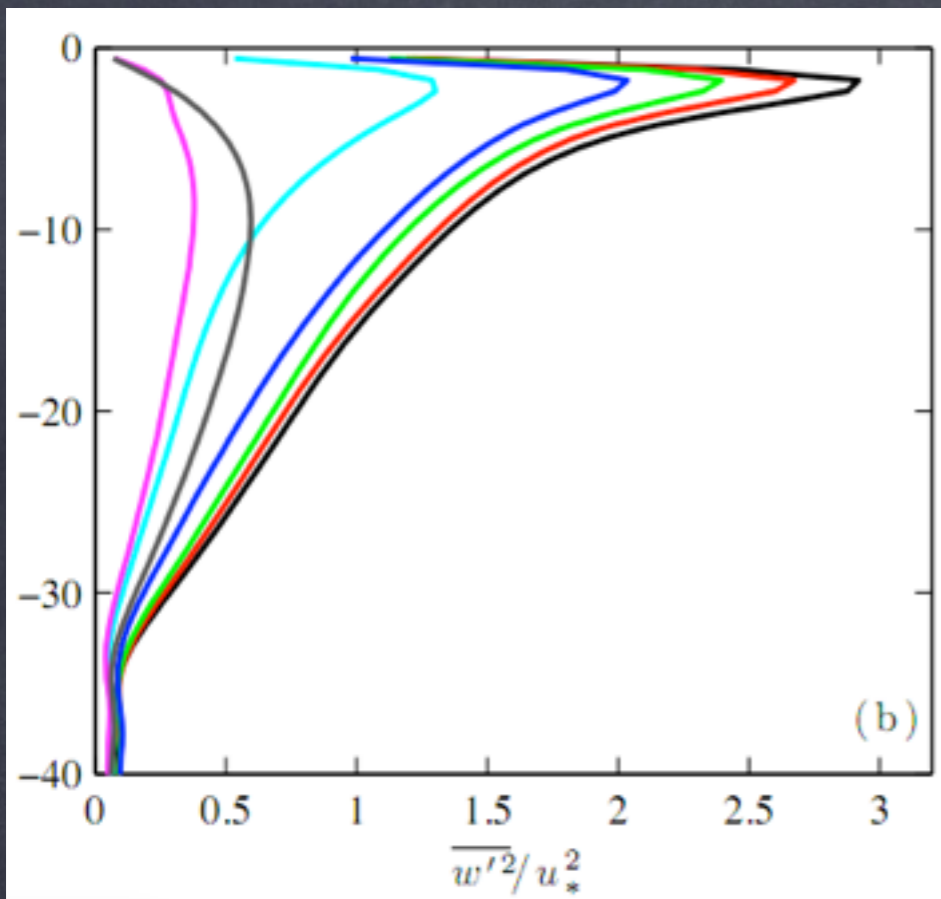


Figure 1 Sketch showing the pattern of mean flow in idealized Langmuir circulation. The windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). In practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2) amalgamate and meander in space and time. Bands of bubbles or buoyant algae may form within the downward-going (or downwelling) flow (see Figure 3).

depth

$\langle w'^2 \rangle$



Generalized Turbulent Langmuir No.,
Projection of u^* , u_s into Langmuir Direction

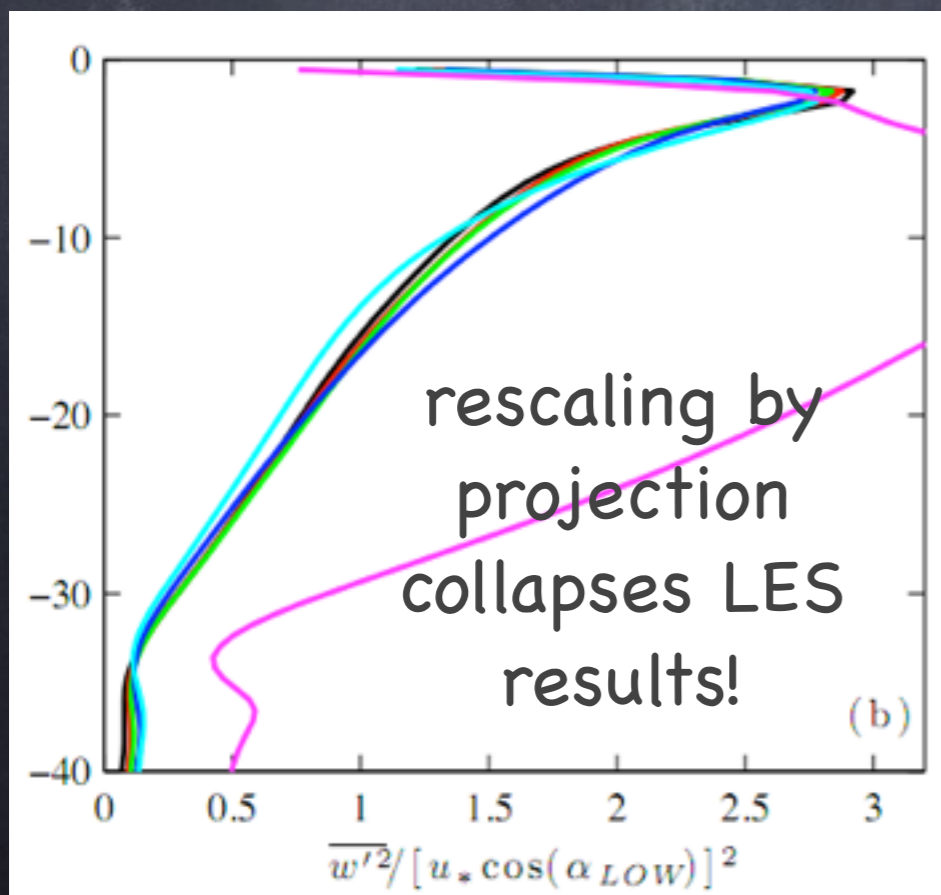
$$\frac{\langle \overline{w'^2} \rangle_{ML}}{u_*^2} = 0.6 \cos^2(\alpha_{LOW}) [1.0 + (3.1 La_{proj})^{-2} + (5.4 La_{proj})^{-4}],$$

$$La_{proj}^2 = \frac{|u_*| \cos(\alpha_{LOW})}{|u_s| \cos(\theta_{ww} - \alpha_{LOW})},$$

$$\alpha_{LOW} \approx \tan^{-1} \left(\frac{\sin(\theta_{ww})}{\frac{u_*}{u_s(0)\kappa} \ln \left(\left| \frac{H_{ML}}{z_1} \right| \right) + \cos(\theta_{ww})} \right)$$

depth

rescaled $\langle w'^2 \rangle$



A scaling for LC
strength & direction!

L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. Journal of Geophysical Research-Oceans, 117:C05001, 22pp, 2012.

Why? Vortex Tilting Mechanism

Misalignment
enhances degree
of wave-driven LT

In CLB: Tilting occurs in
direction of $\mathbf{u}_L = \mathbf{v} + \mathbf{v}_s$

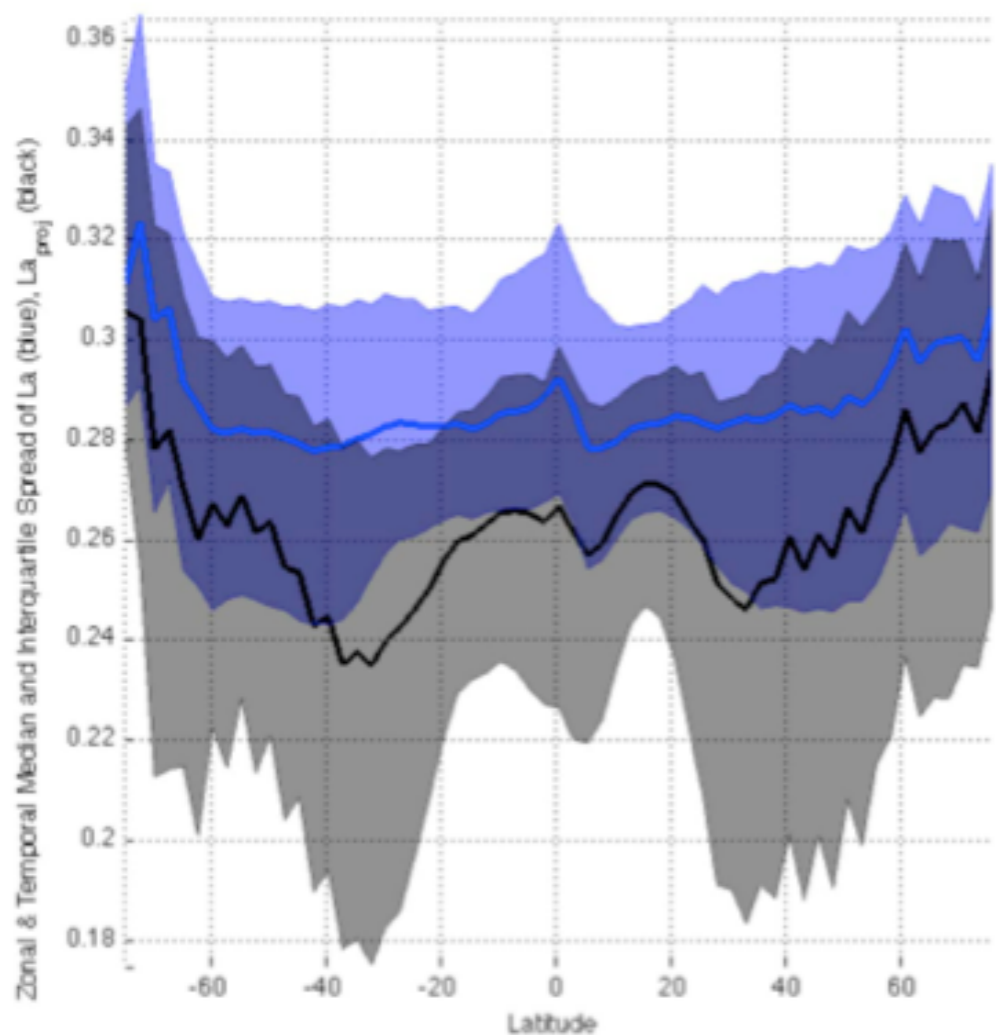
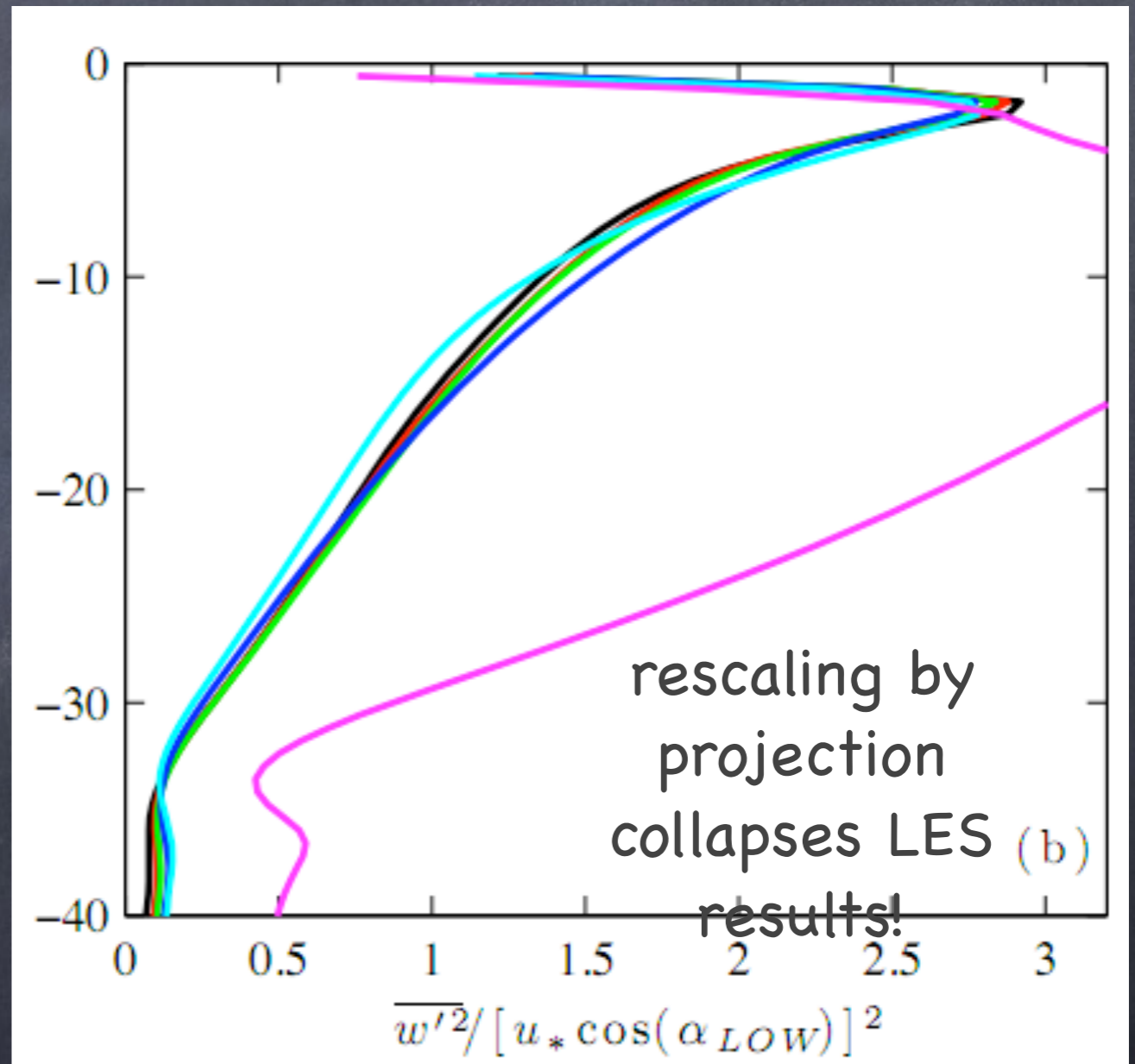


Figure 17. Temporal and zonal median and interquartile range of La_t and La_{proj} for a realistic simulation of 1994–2002 using Wave Watch III.



Physical Model by N. Suzuki (Brown)

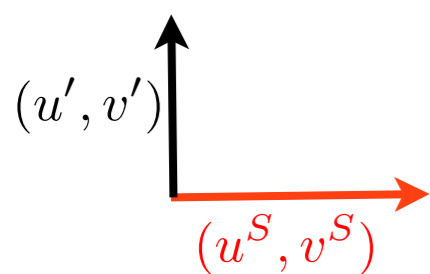
$$\partial_t u + (\vec{u}^L \cdot \nabla)u = -\partial_x \tilde{p} + f v^L$$

$$\partial_t v + (\vec{u}^L \cdot \nabla)v = -\partial_y \tilde{p} - f u^L$$

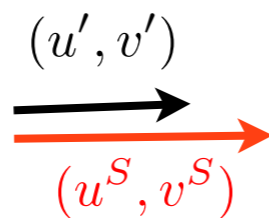
$$\partial_t w + (\vec{u}^L \cdot \nabla)w = -\partial_z \tilde{p} + \tilde{b} - (u', v') \cdot \partial_z (u^S, v^S)$$

Stokes-shear force

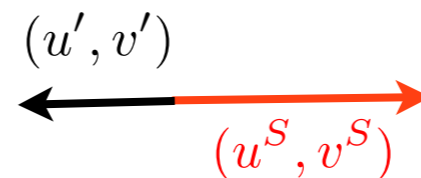
- solely responsible for the CL2 instability $-\langle (u'w', v'w') \rangle \cdot \partial_z (u^S, v^S)$
- conditional: acts only on (u', v')
- directional:



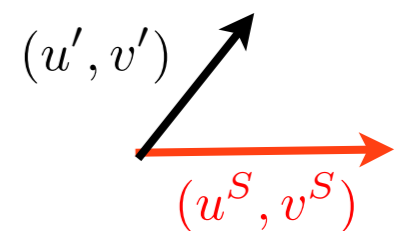
0



pushed down

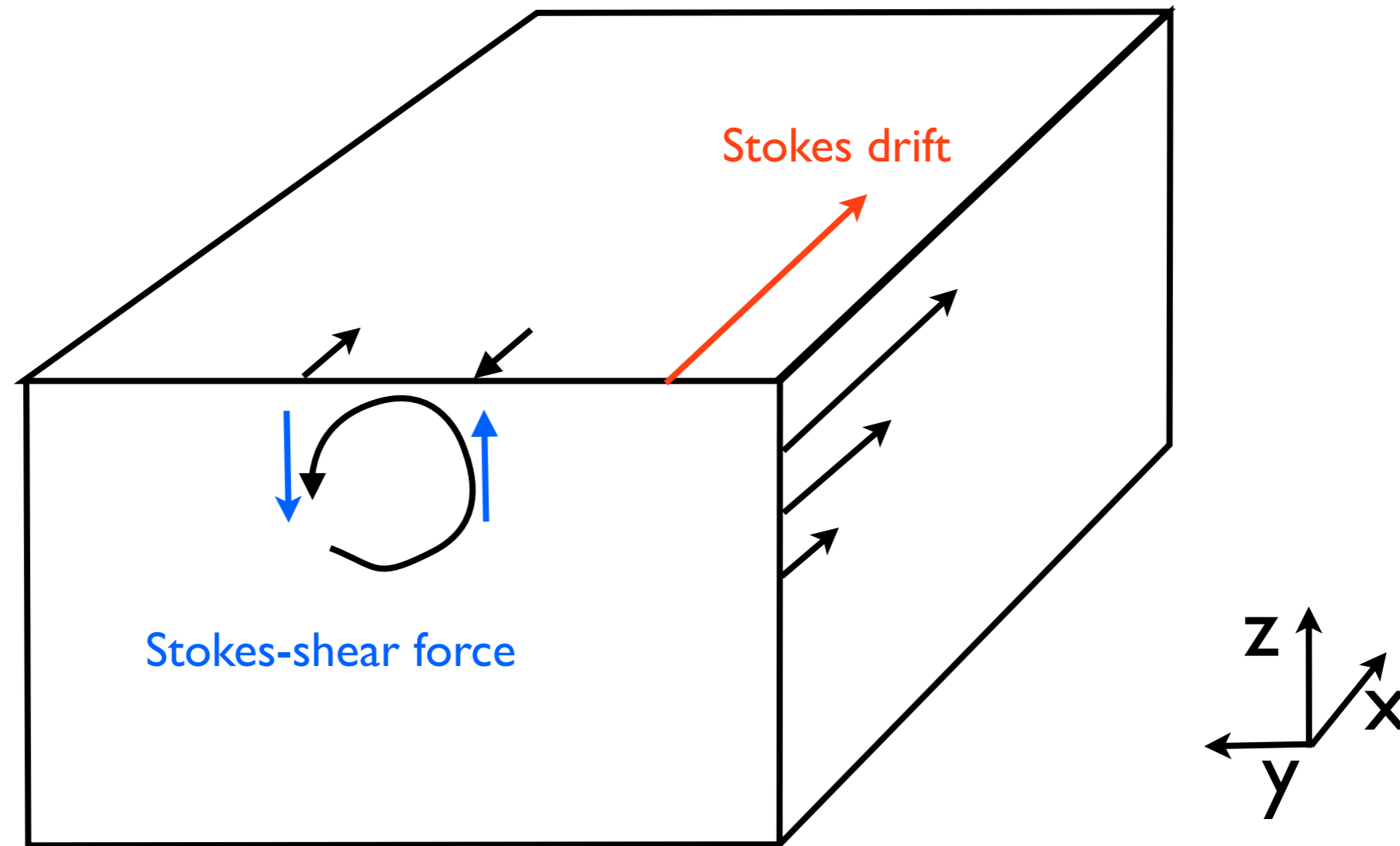


pushed up



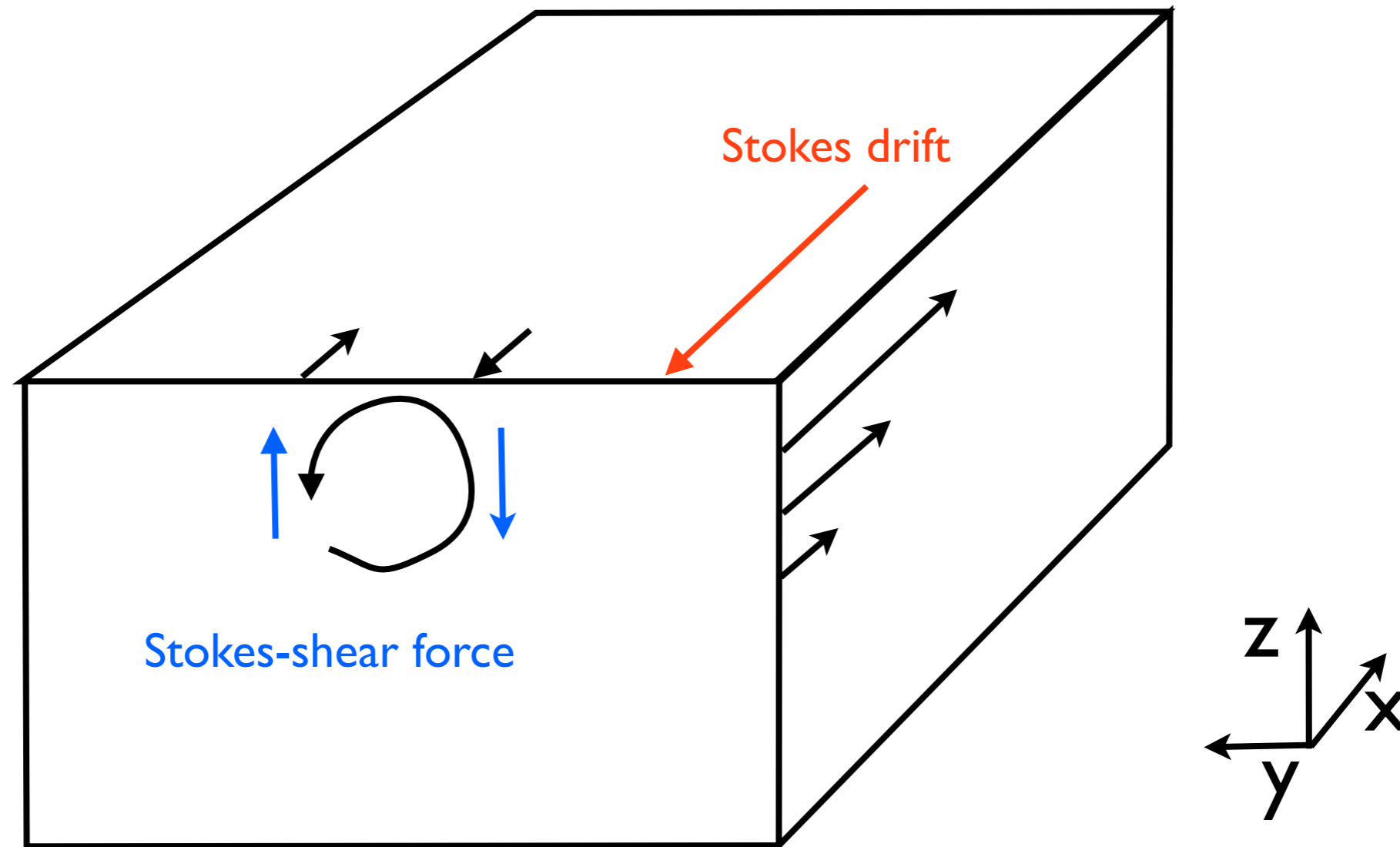
weakly
pushed down

Direct influence on shear turbulence



Enhance the shear turbulence

Direct influence on shear turbulence



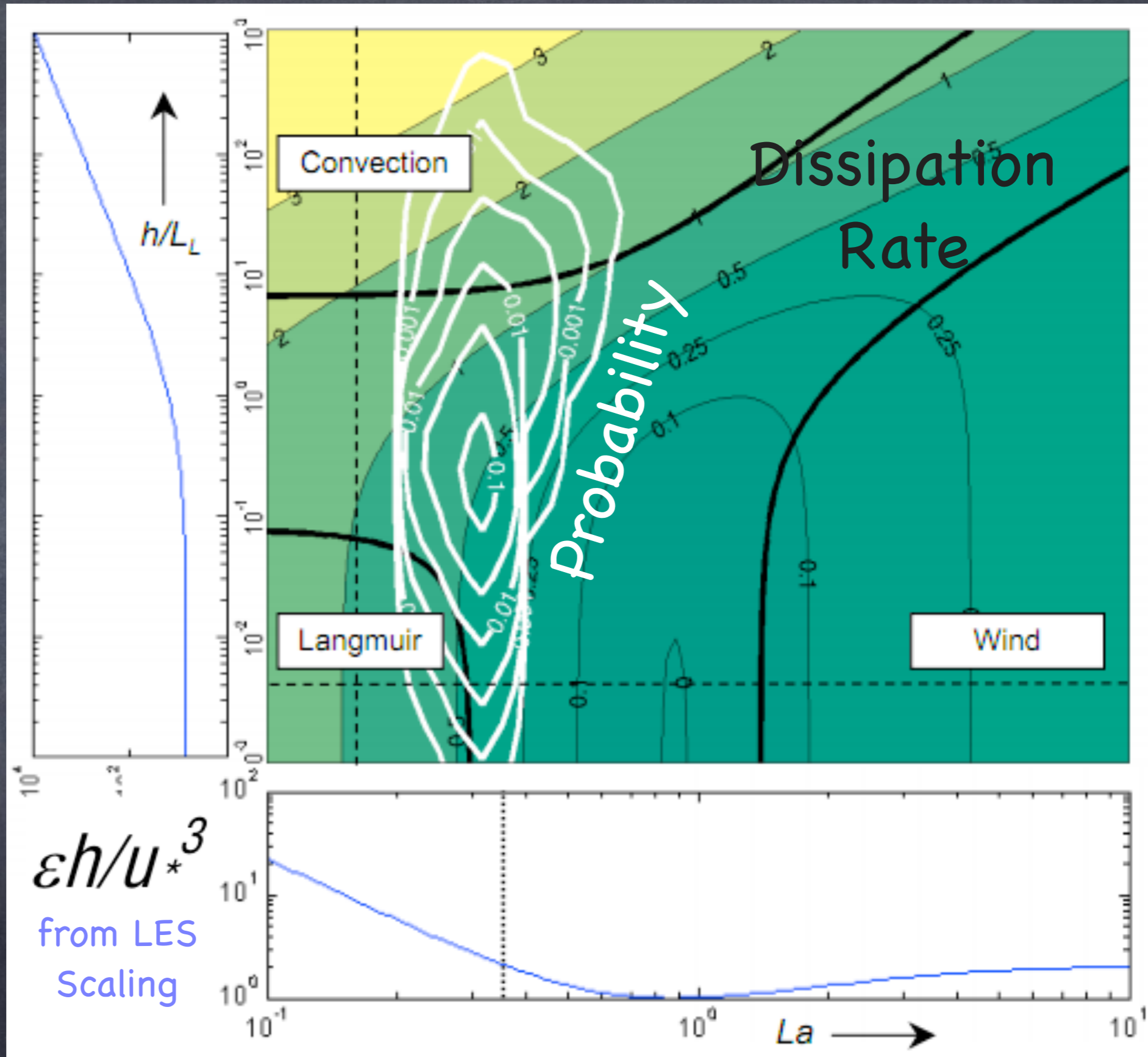
Kills the shear turbulence

But, does Langmuir Turbulence Matter?

- Langmuir turbulence can only matter, in climate modeling practice, when winds and waves are not in equilibrium.
- In this case, just knowing the winds is **insufficient** to predict the rate of Boundary Layer Mixing
- Thus, to do Langmuir mixing right, we need a wave model in addition to Atmosphere & Ocean
- But, in the meantime, we can use offline estimates using data...

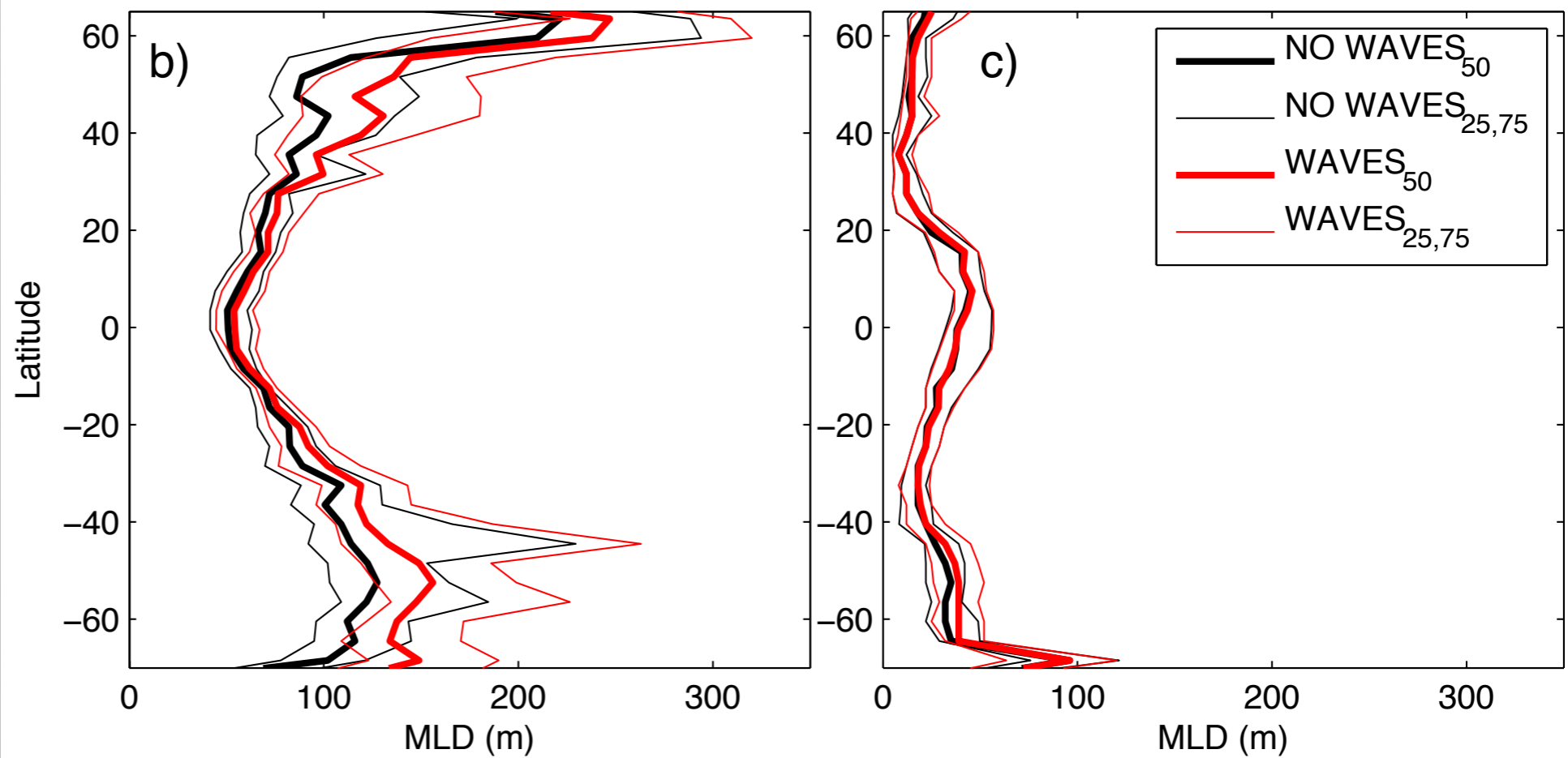
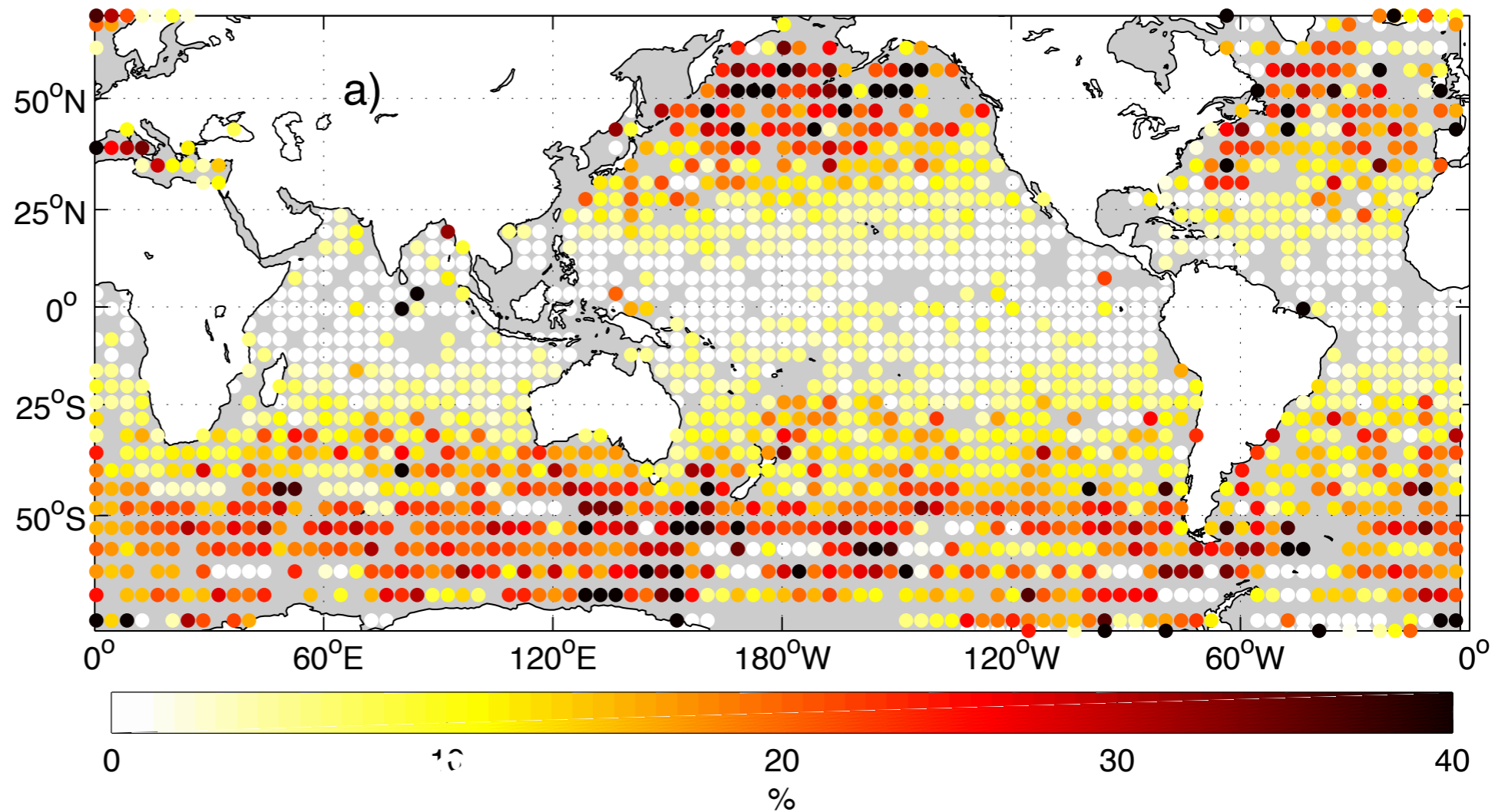
Data + LES,
Southern Ocean
mixing energy:
Langmuir (Stokes-
drift-driven) and
Convective

So, waves
can drive
mixing via
Stokes drift
(combines
with cooling
& winds)



S. E. Belcher, A. A. L. M. Grant, K. E. Hanley, B. Fox-Kemper, L. Van Roekel, P. P. Sullivan, W. G. Large, A. Brown, A. Hines, D. Calvert, A. Rutgersson, H. Petterson, J. Bidlot, P. A. E. M. Janssen, and J. A. Polton. A global perspective on Langmuir turbulence in the ocean surface boundary layer. *Geophysical Research Letters*, 39(18):L18605, 9pp, 2012.

Including Wave-driven Mixing (Harcourt 2013 parameterization) Deepens the Mixed Layer!



E. A. D'Asaro, J. Thomson, A. Y. Shcherbina, R. R. Harcourt, M. F. Cronin, M. A. Hemer, and B. Fox-Kemper. Quantifying Upper Ocean Turbulence Driven by Surface Waves. Submitted 2013.

Conclusions on wave effects on Langmuir Scale

- Wave forced turbulence is an important contributor to boundary layer mixing
- Wave effects are particularly needed in climate models to have scenarios where waves and winds are not in equilibrium, but this may require a prognostic wave model as a climate model component
- Reducing the Southern Ocean mixed layer bias is a key deliverable of this effort

The Character of the Submesoscale

(NASA GSFC Gallery)

(Capet et al., 2008)

10
km
←

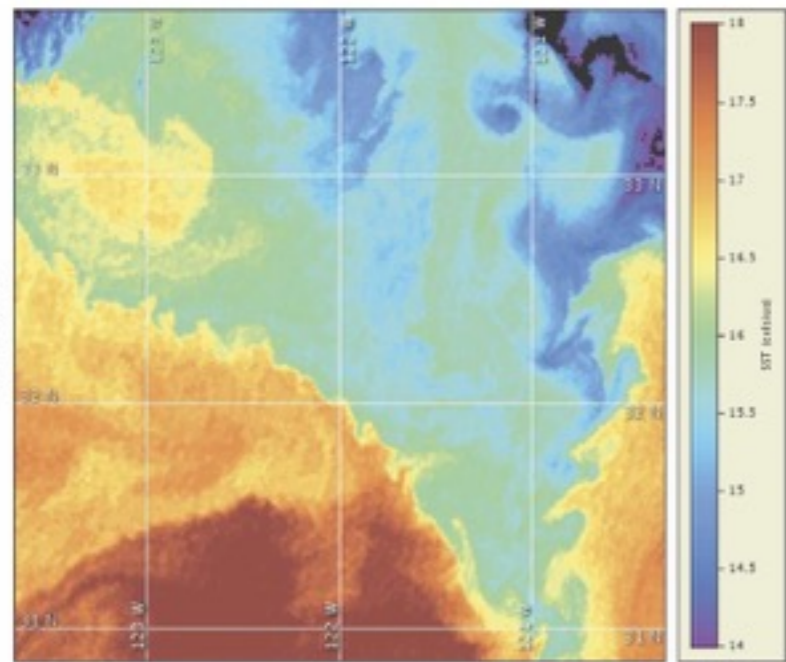
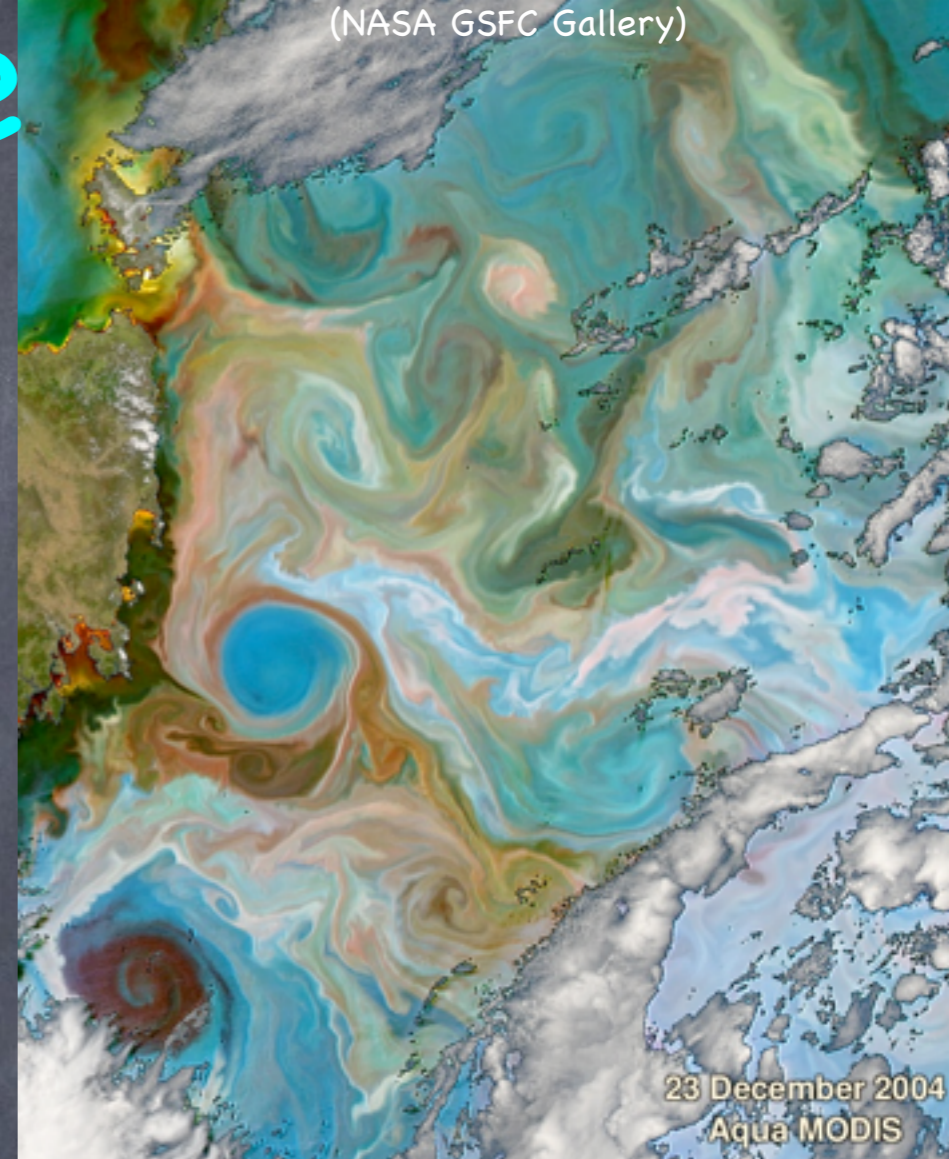
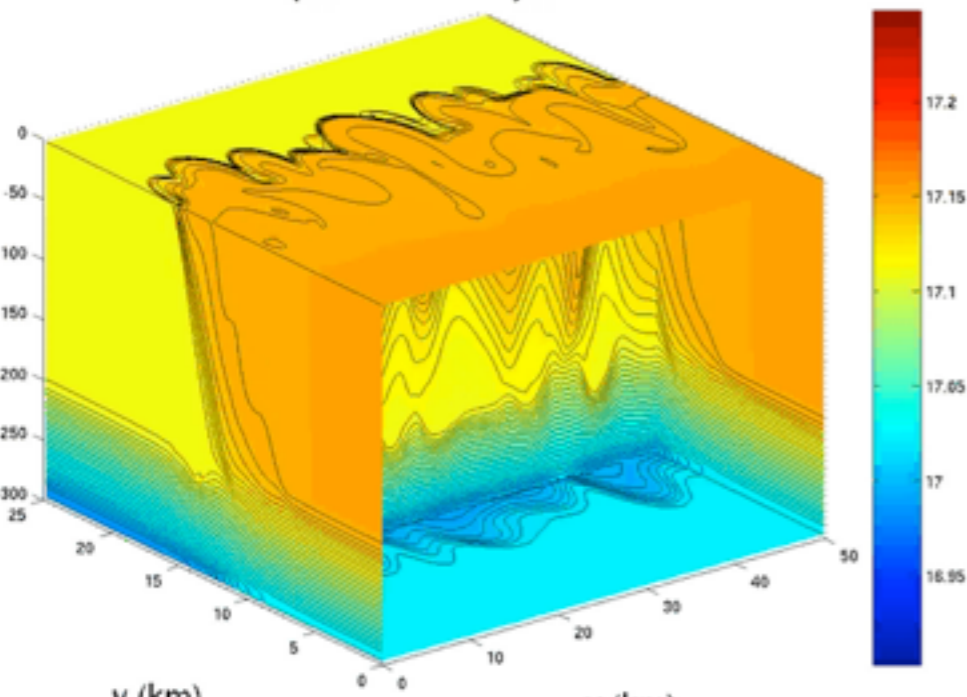


FIG. 16. Sea surface temperature measured at 1832 UTC 3 Jan 2006 off Point Conception in the California Current from CoastWatch (<http://coastwatch.pfeg.noaa.gov>). The fronts between recently

Temperature on day:17.375



- Fronts
- Eddies
- $Ro=O(1)$
- $Ri=O(1)$
- near-surface
- 1-10km, days

Eddy processes often
baroclinic instability

Parameterizations of
submesoscale baroclinic
instability?

B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. *Journal of Physical Oceanography*, 38(6):1145-1165, 2008

S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. *Ocean Modelling*, 64:12-28, 2013

Geostrophy, Hydrostasy, & Thermal Wind

Traditional Mesoscale & Weak Submesoscale Oceanography
inhabits a special distinguished limit:

Inviscid ($Re \gg 1$), rapidly rotating ($Ro < 1$), and thin* ($L \gg H$)

Full Momentum

$$\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla\phi + b\mathbf{k} + \nu\nabla^2\mathbf{v}$$

$$Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri \equiv \frac{\frac{\partial b}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \quad \alpha = H/L$$

*closely related to strong stratification & ocean dimensions

Geostrophy, Hydrostasy, & Thermal Wind

Traditional Mesoscale & Weak Submesoscale Oceanography
inhabits a special distinguished limit:

Inviscid ($Re \gg 1$), rapidly rotating ($Ro < 1$), and thin* ($L \gg H$)

(Horizontal) Geostrophic Balance

$$\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla\phi + b\mathbf{k} + \nu\nabla^2\mathbf{v}$$

$$Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri \equiv \frac{\frac{\partial b}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \quad \alpha = H/L$$

*closely related to strong stratification & ocean dimensions

Geostrophy, Hydrostasy, & Thermal Wind

Traditional Mesoscale & Weak Submesoscale Oceanography
inhabits a special distinguished limit:

Inviscid ($Re \gg 1$), rapidly rotating ($Ro < 1$), and thin* ($L \gg H$)

(Vertical) Hydrostatic Balance

$$\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla\phi + b\mathbf{k} + \nu\nabla^2\mathbf{v}$$

$$Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri \equiv \frac{\frac{\partial b}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \quad \alpha = H/L$$

*closely related to strong stratification & ocean dimensions

Geostrophy, Hydrostasy, & Thermal Wind

Traditional Mesoscale & Weak Submesoscale Oceanography
inhabits a special distinguished limit:

Inviscid ($Re \gg 1$), rapidly rotating ($Ro < 1$), and thin* ($L \gg H$)

(Combined) Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b$$

Taken together with the forcing (air-sea) of buoyancy
and the advection of buoyancy by this flow--you have
the tools to study large-scale ocean physics!

Craik–Leibovich Boussinesq

Do waves affect the (sub)mesoscale?

Yes!!

J. C. McWilliams and B. Fox–Kemper. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 730:464–490, Sept 2013.

$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times (\mathbf{v} + \mathbf{v}_s) = -\nabla \pi^\dagger + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$

$$\frac{\partial b}{\partial t} + (\mathbf{v} + \mathbf{v}_s) \cdot \nabla b = 0 \quad \nabla \cdot \mathbf{v} = 0$$

$\mathbf{v}_s =$ Stokes Drift

Now, Craik–Leibovich Boussinesq Equivalent:
(Combined) Lagrangian Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial}{\partial z} (\mathbf{v} + \mathbf{v}_s) = \mathbf{f} \times \frac{\partial \mathbf{v}_L}{\partial z} = -\nabla b$$

Now the temperature gradients govern the Lagrangian flow, not the Eulerian!

Leading order consequence for small Rossby:
Anti–Stokes Effect:

Any Stokes drift that is unbalanced will provoke an Eulerian current to cancel it out!

So, can we just forget the whole thing and interpret large scales as Lagrangian velocities?

$$[\mathbf{f} + \nabla \times \mathbf{v}] \times \frac{\partial}{\partial z} (\mathbf{v} + \mathbf{v}_s) = -\nabla b$$

Not quite, because $Ro > 0$ corrections are different!

The "Ro" for waves, is big *more often* than Ro is, especially for wide, shallow currents in a mixed layer

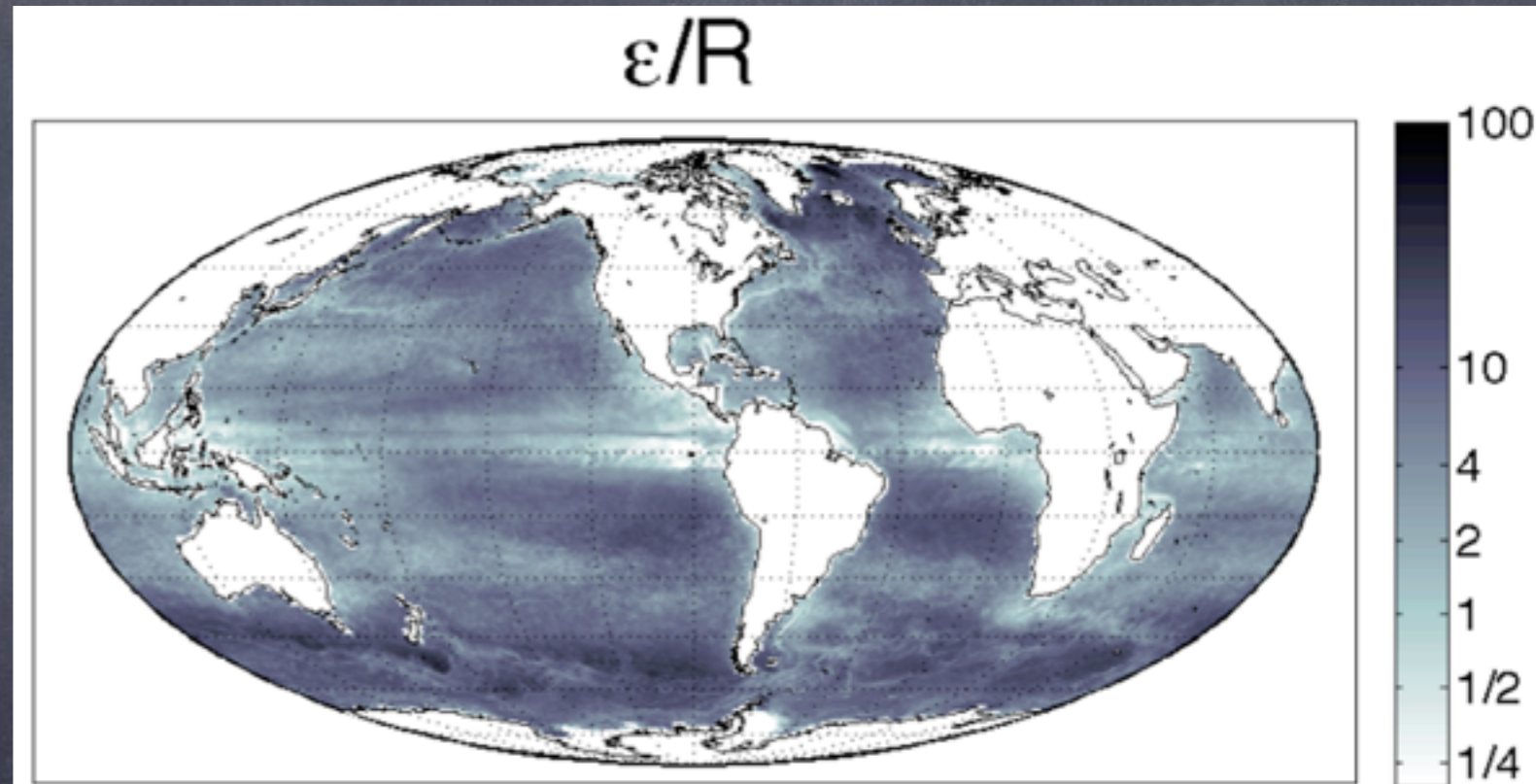
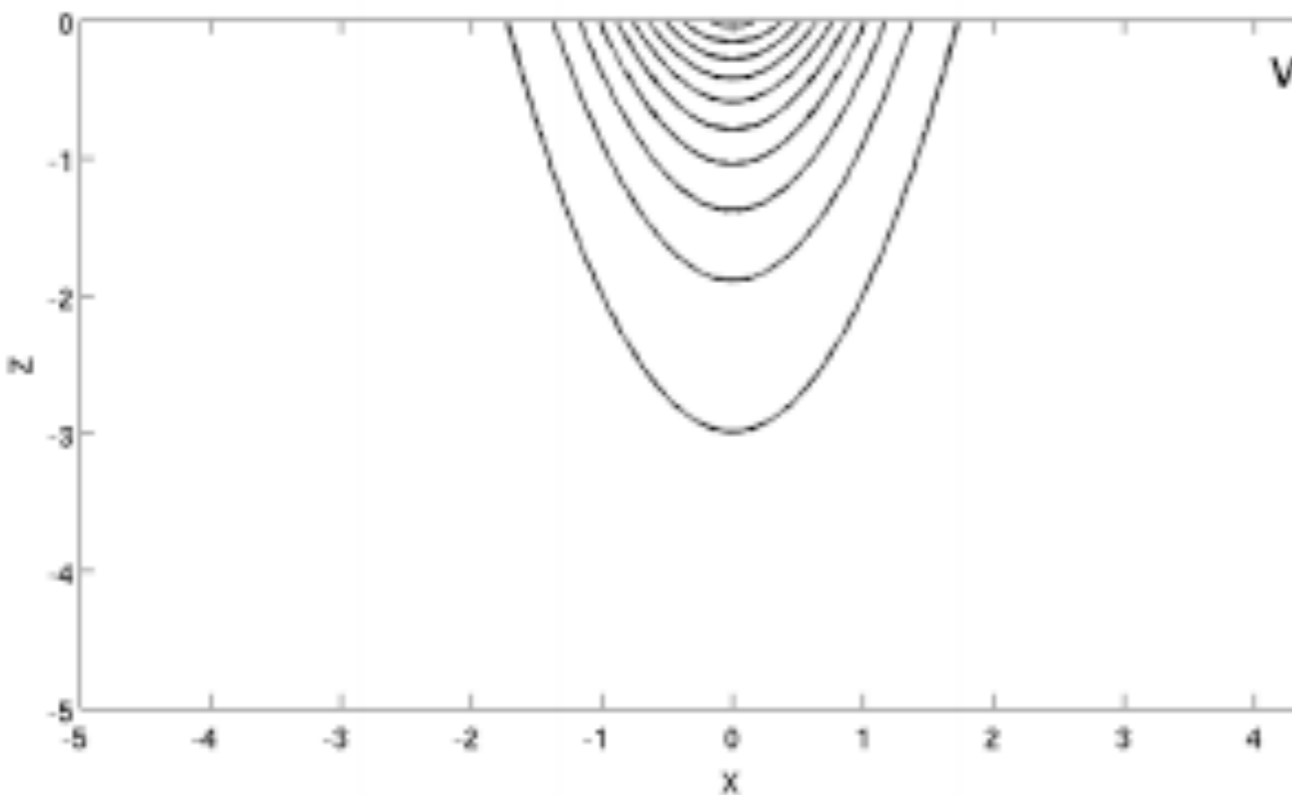


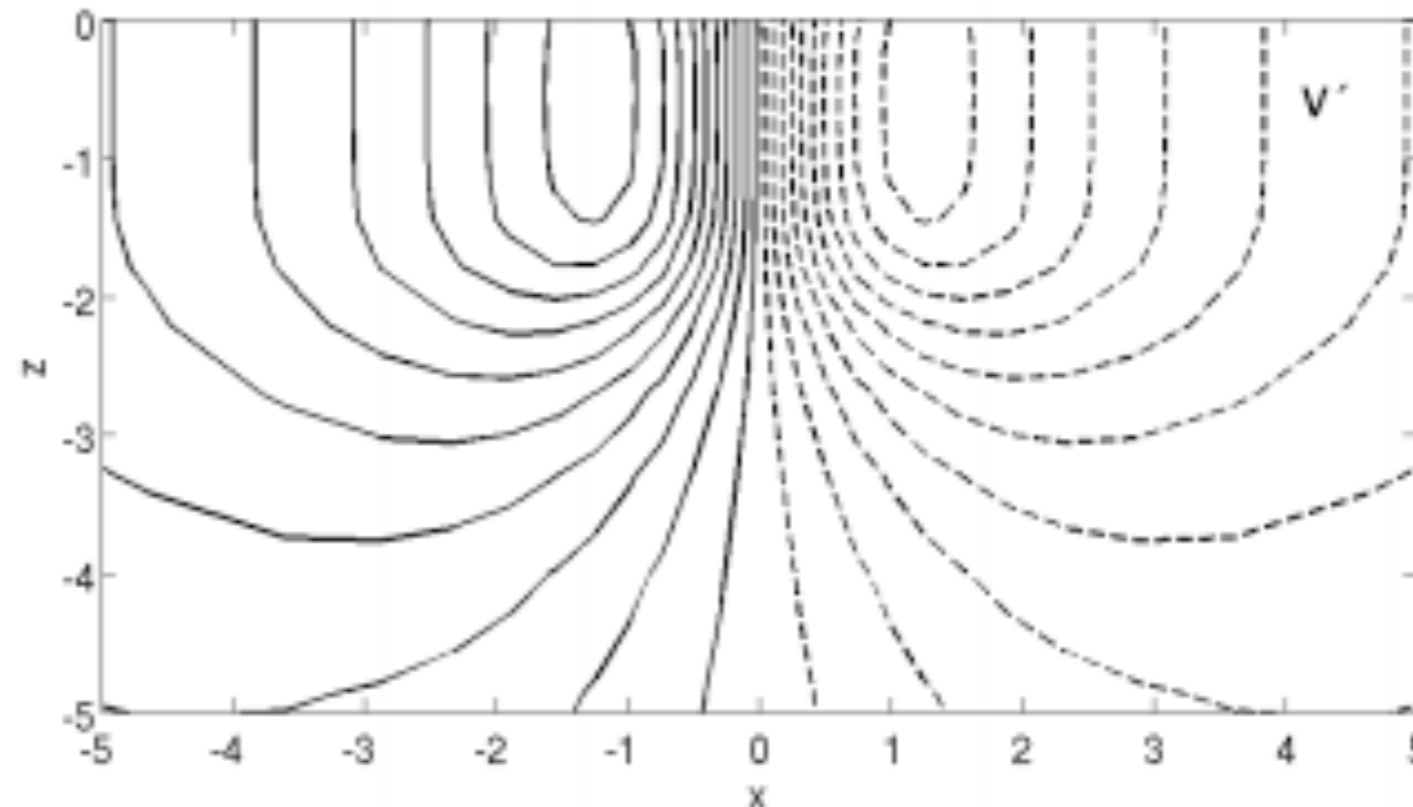
FIGURE 1. Estimated ratio $\epsilon/\mathcal{R} \approx (|\mathbf{u}_s \cdot \mathbf{u}|h)/(|\mathbf{u}|^2 h_s)$ governing the relative importance of Stokes effects versus nonlinearity. Eulerian velocity (\mathbf{u}) is taken as the AVISO weekly satellite geostrophic velocity or $-\mathbf{u}_s$ (for anti-Stokes flow) if $|\mathbf{u}_s| > |\mathbf{u}|$. The front/filament depth (h) is estimated as the mixed layer depth from the de Boyer Montégut *et al.* (2004) climatology. An exponential fit to the Stokes drift of the upper 9m projected onto the AVISO geostrophic velocity provides $\mathbf{u}_s \cdot \mathbf{u}$ and h_s . Stokes drift is taken from the WaveWatch-3 simulation described in Webb & Fox-Kemper (2011). \mathbf{u} , \mathbf{u}_s , and h_s are all for the year 2000, while h is from a climatology of observations over 1961-2008. The year 2000 average of ϵ/\mathcal{R} is shown.

Waves (Stokes Drift Vortex Force) → Submeso, Meso: An example



Initial Submeso Front

Contours: 0.1

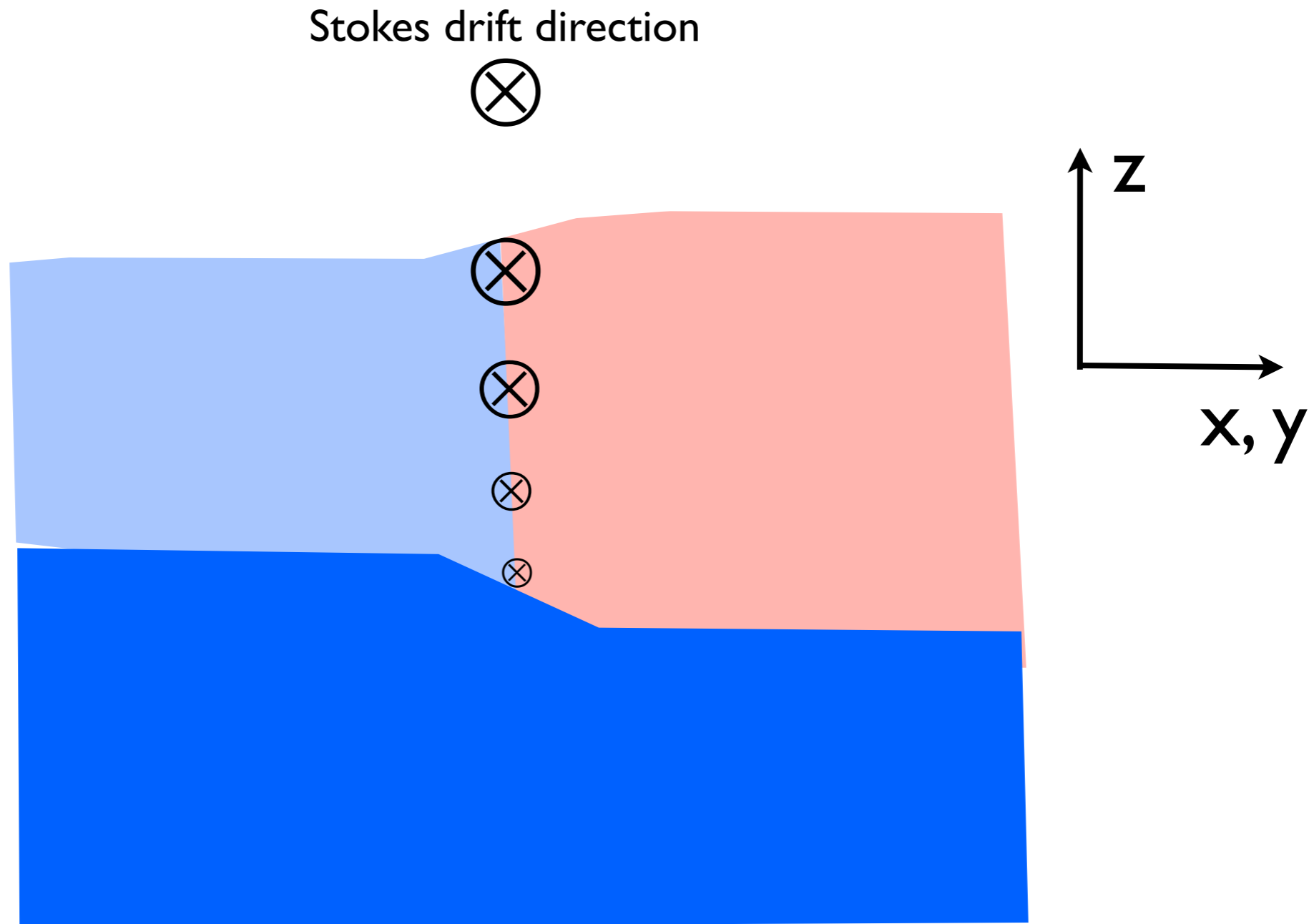


Perturbation on that scale
due to waves

Contours: 0.014

Direct influence on the thermal wind

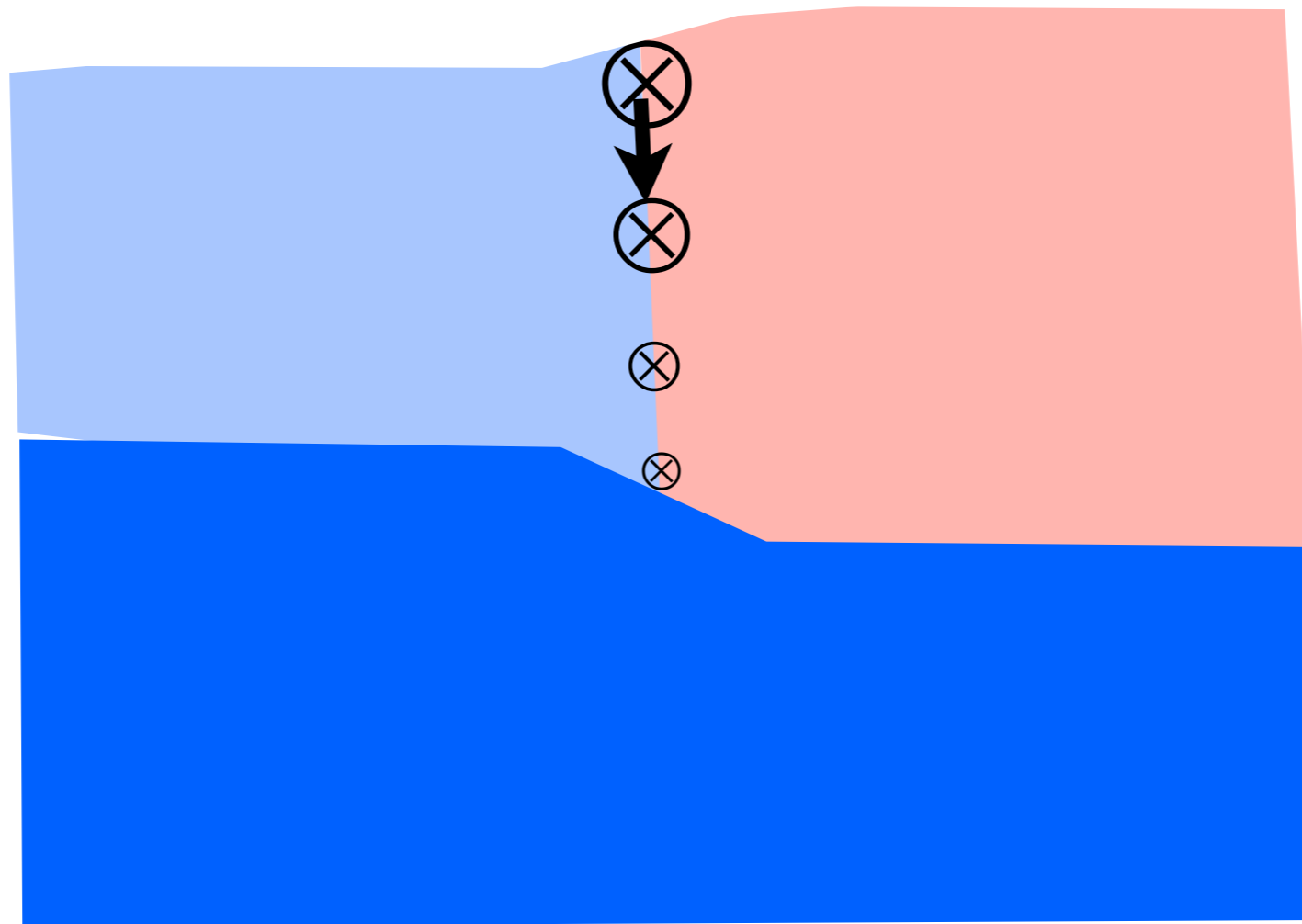
James C. McWilliams and Baylor Fox-Kemper, Oceanic wave-balanced surface fronts and filaments, J. Fluid Mech. (2013), vol. 730, pp. 464490.



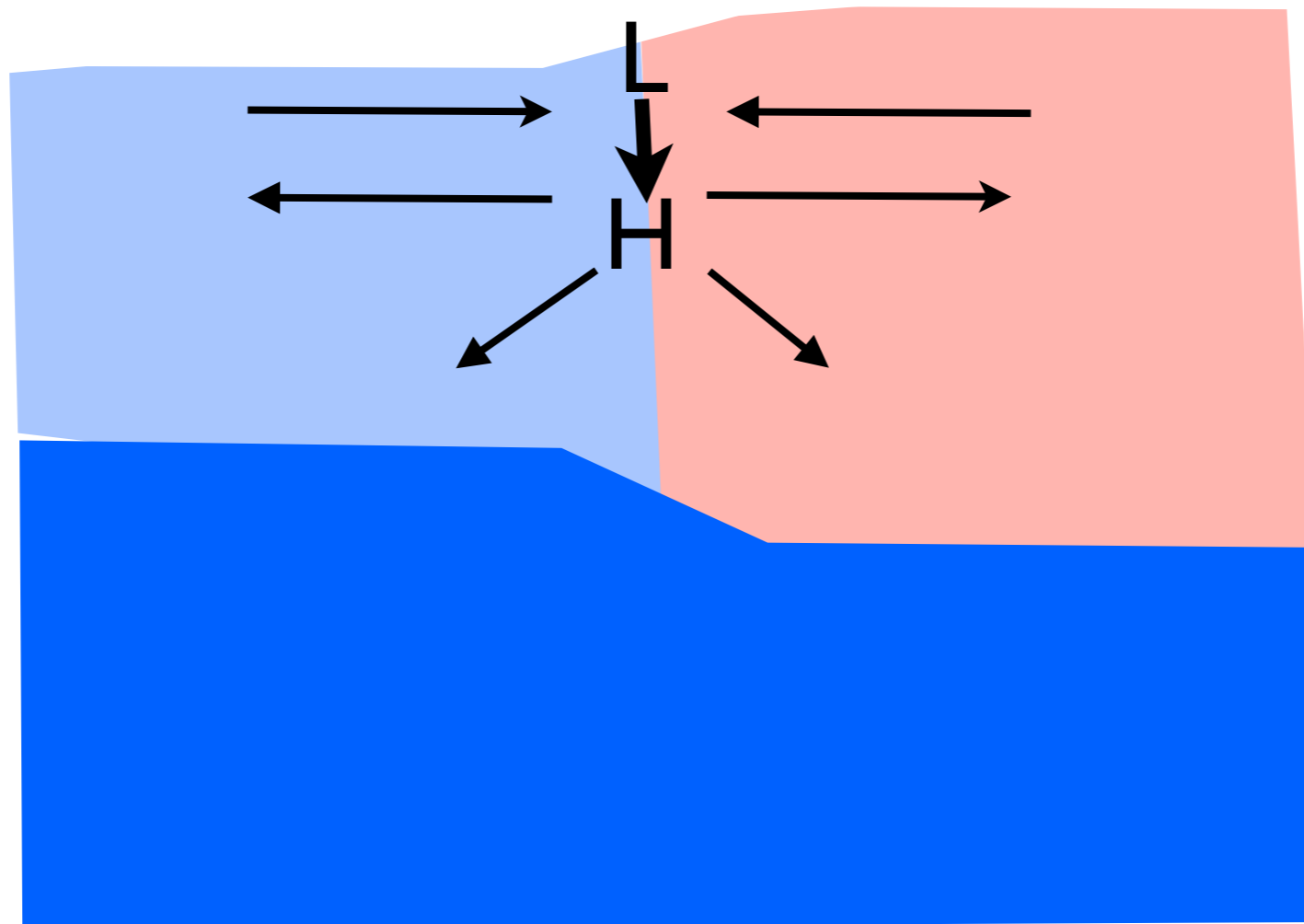
Physical Model by N. Suzuki (Brown)

Direct influence on the thermal wind

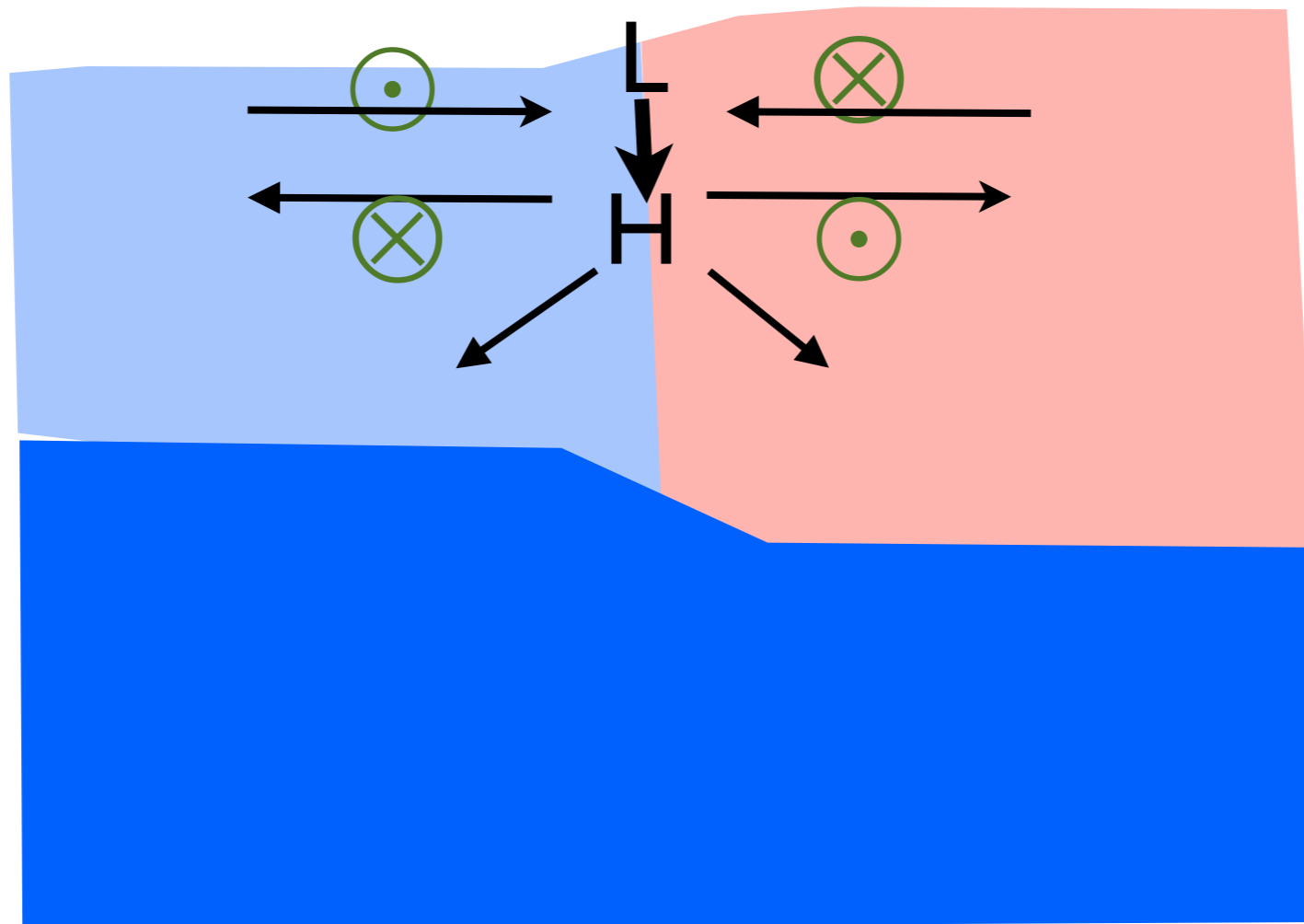
Stokes drift direction



Direct influence on the thermal wind

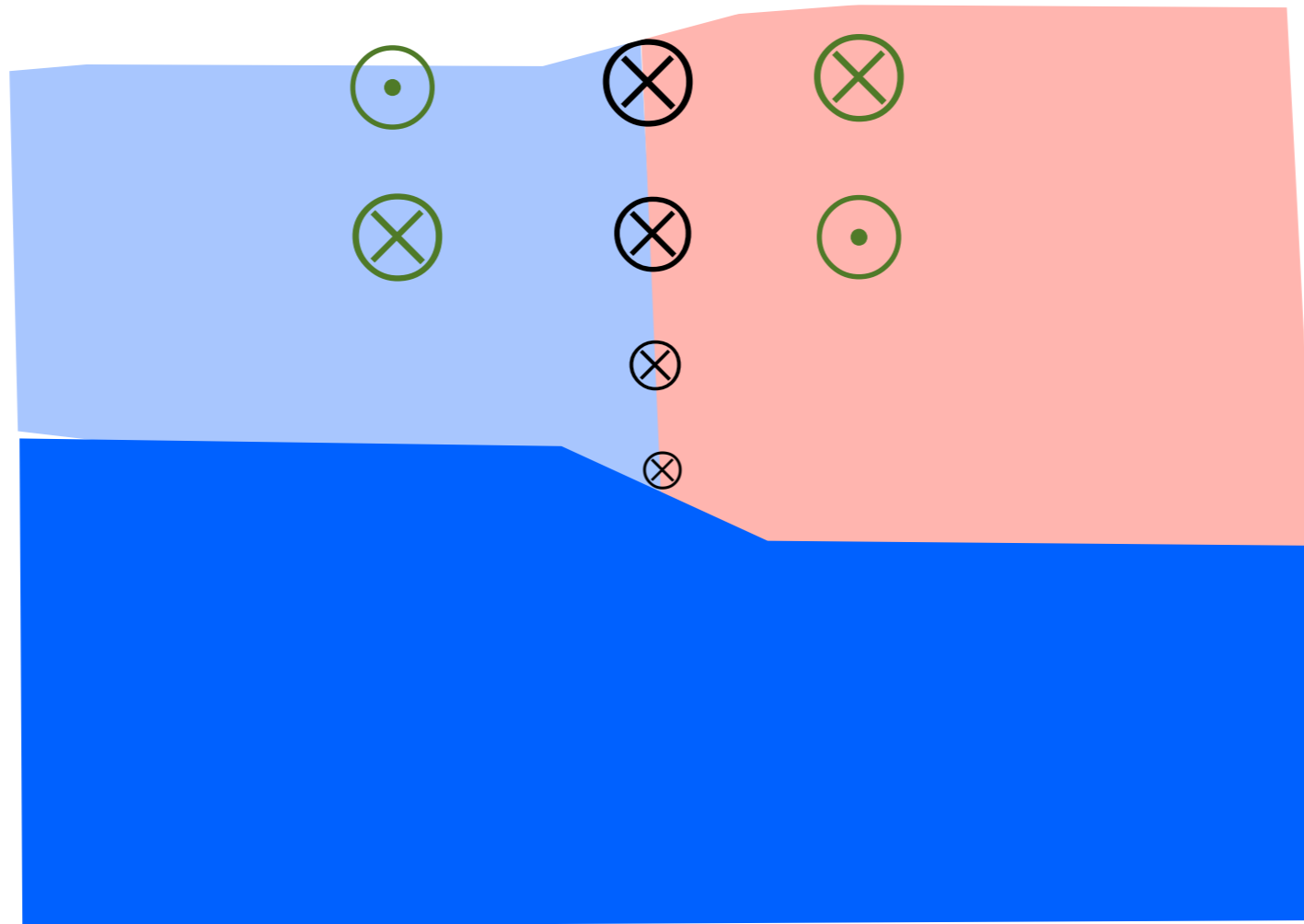


Direct influence on the thermal wind



Direct influence on the thermal wind

Stokes drift direction



Conclusions on wave effects on (sub)mesoscale

- Wave forces significantly affect the dominant (sub)mesoscale balances in many places
- The primary effect is Anti-Stokes Flow
- The secondary effect is a Stokes vortex/ Stokes shear force effect that disturbs hydrostatic & geostrophic balances

What about Langmuir-Submeso Interactions?

Movie: P. Hamlington

Perform large eddy simulations (LES) of Langmuir turbulence with a submesoscale temperature front

Use NCAR LES model to solve Craik-Leibovich equations (Moeng, 1984, McWilliams et al, 1997)

$$\frac{\partial \rho}{\partial t} + \mathbf{u}_L \cdot \nabla \rho = \text{SGS}$$

$$\nabla \cdot \mathbf{u} = 0$$

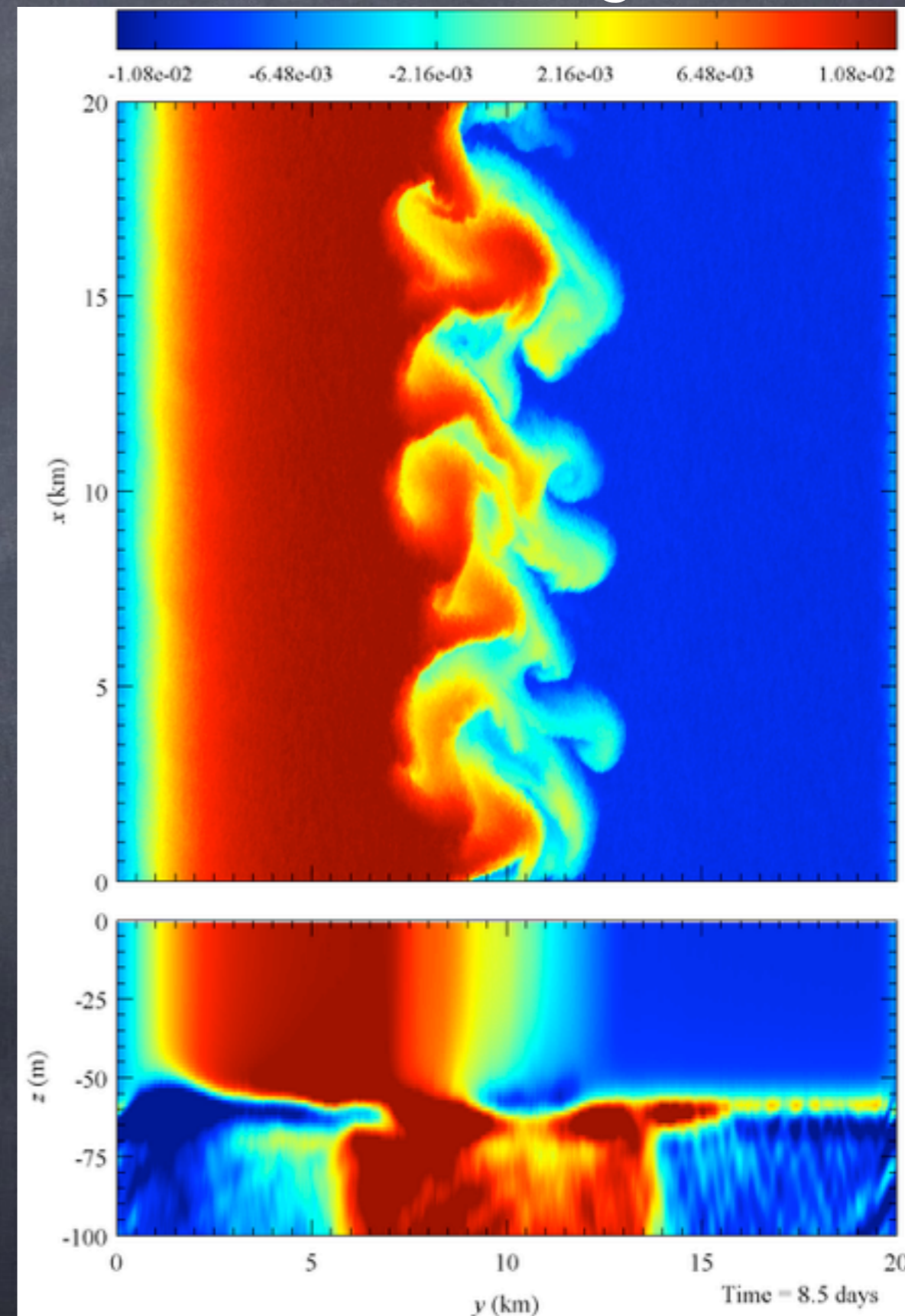
$$\frac{\partial \mathbf{u}}{\partial t} + (\boldsymbol{\omega} + f\hat{\mathbf{z}}) \times \mathbf{u}_L = -\nabla \pi - \frac{g\rho\hat{\mathbf{z}}}{\rho_0} + \text{SGS}$$

Computational parameters:

Domain size: 20km x 20km x -160m

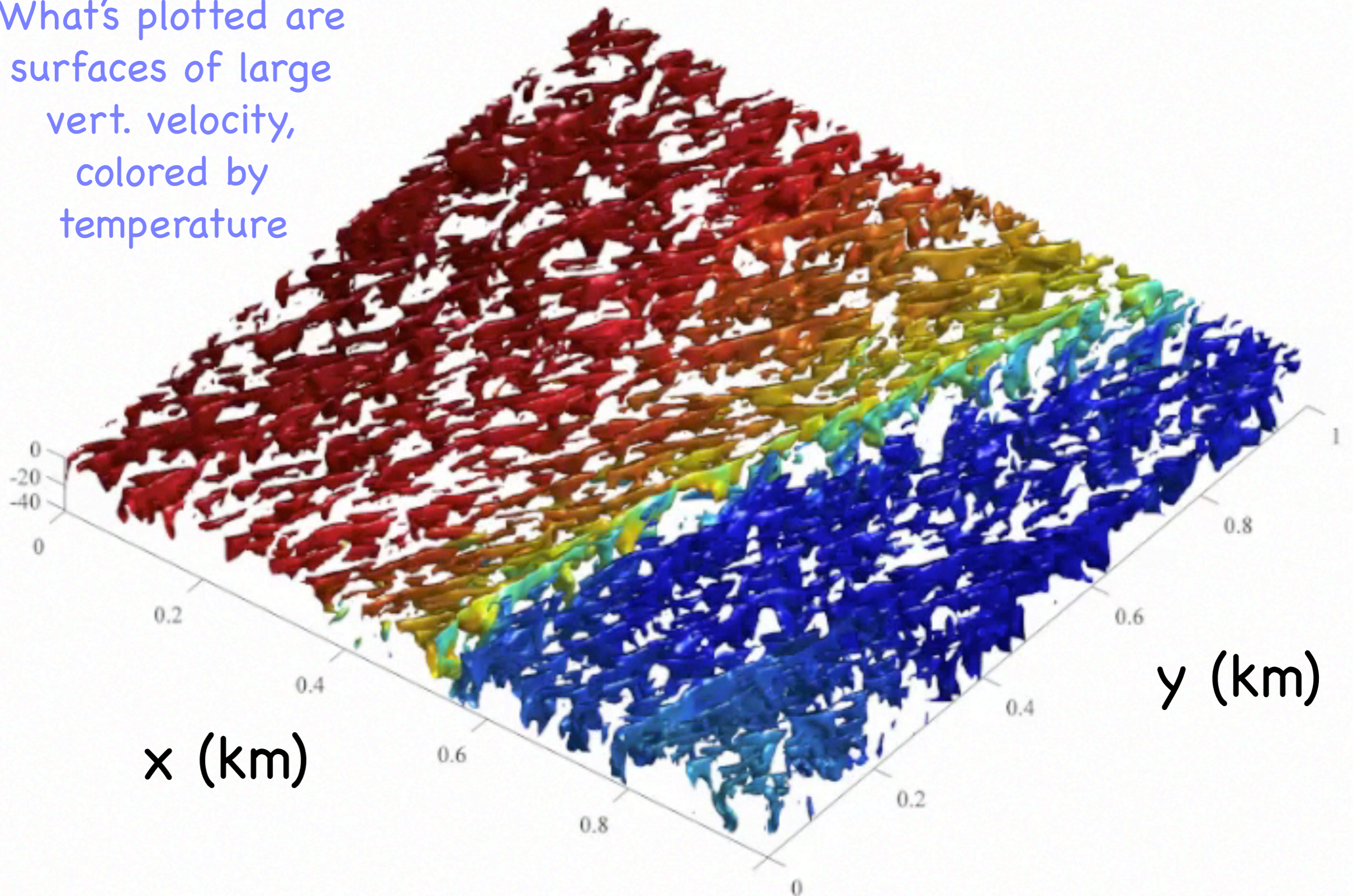
Grid points: 4096 x 4096 x 128

Resolution: 5m x 5m x -1.25m

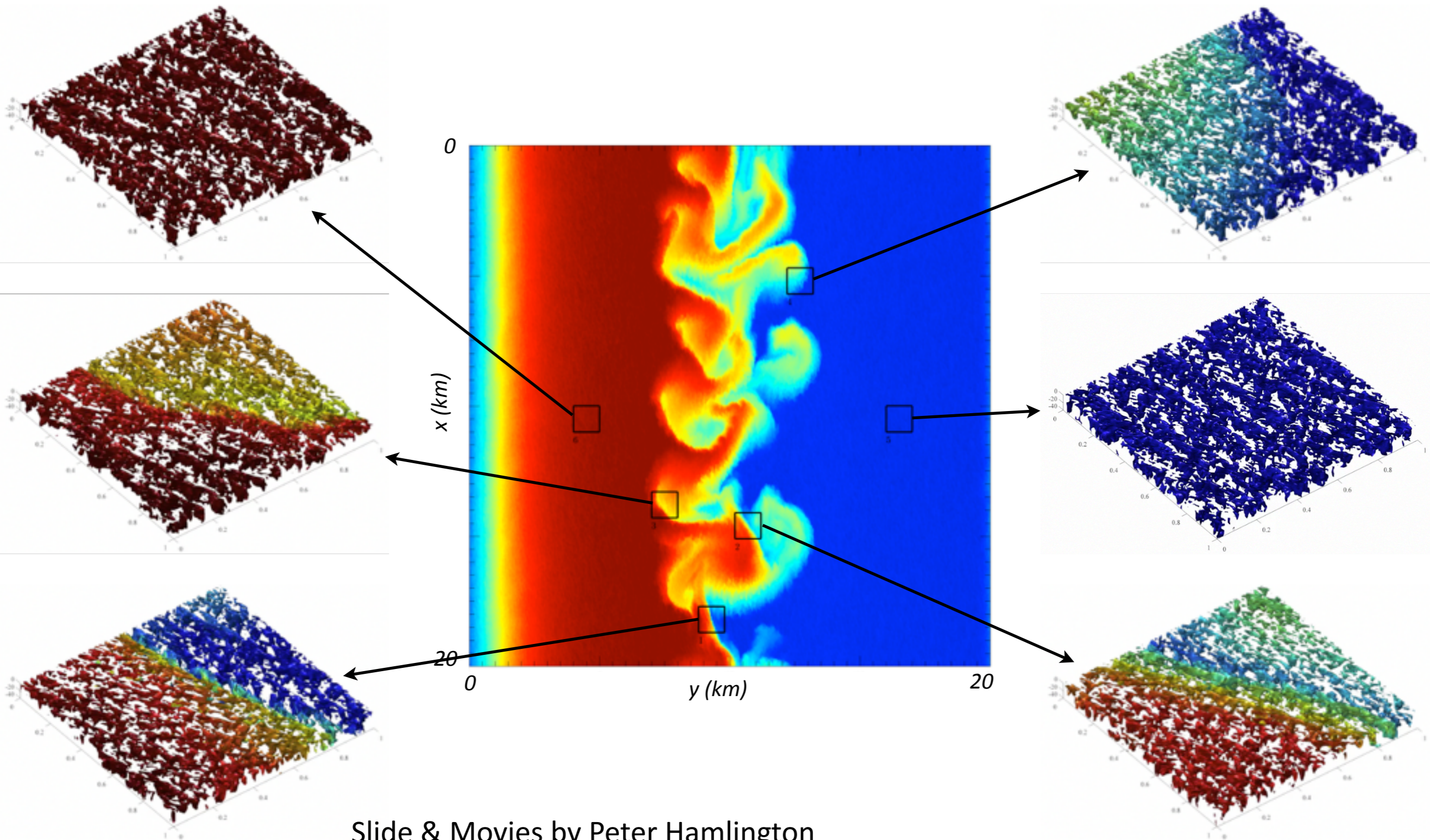


Zoom: Submeso-Langmuir Interaction!

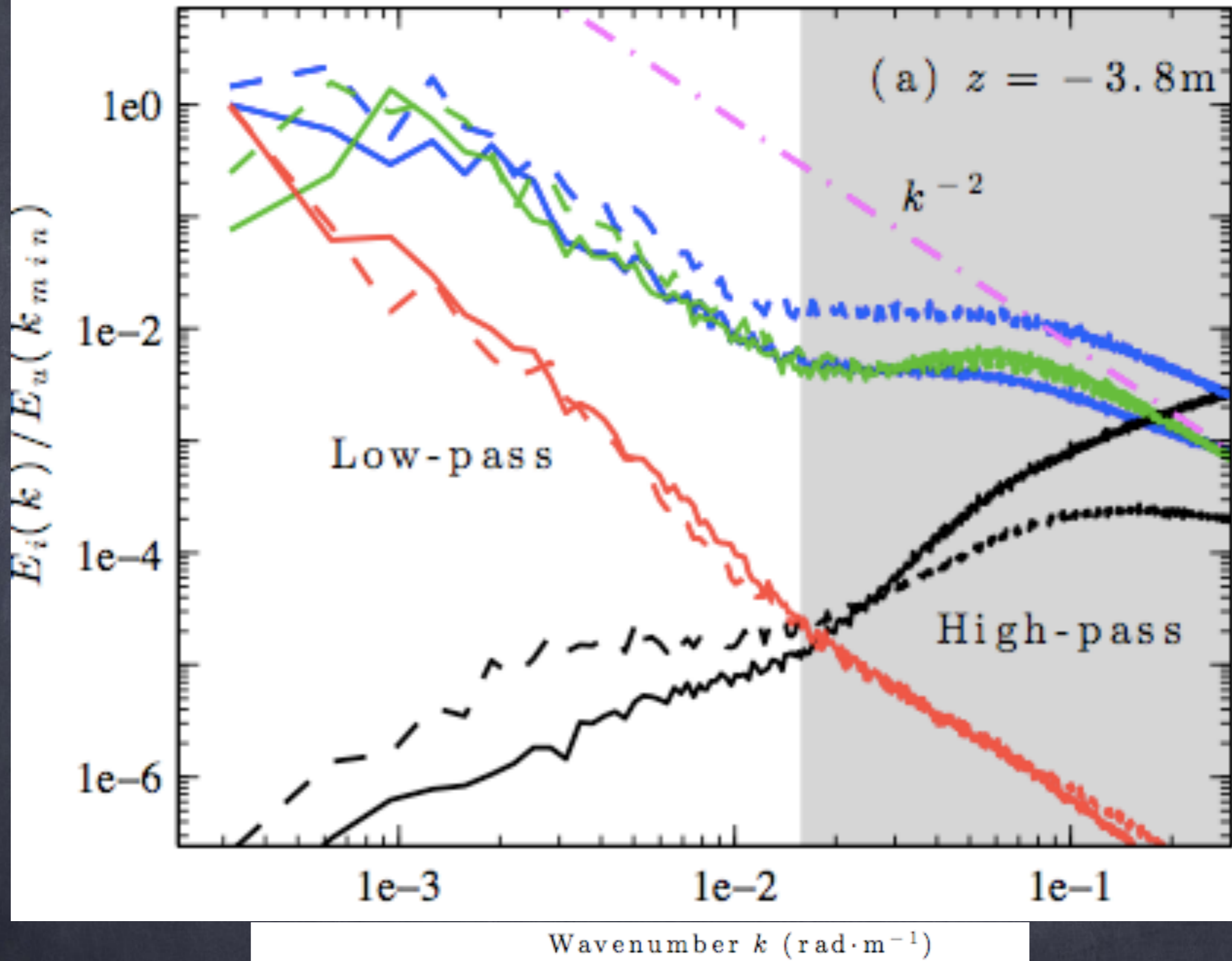
What's plotted are
surfaces of large
vert. velocity,
colored by
temperature



Diverse types of interaction



Slide & Movies by Peter Hamlington



Solid With Stokes

Dashed Without Stokes

Both Submeso & Langmuir-scale impacts of Stokes

Stokes

No
Stokes

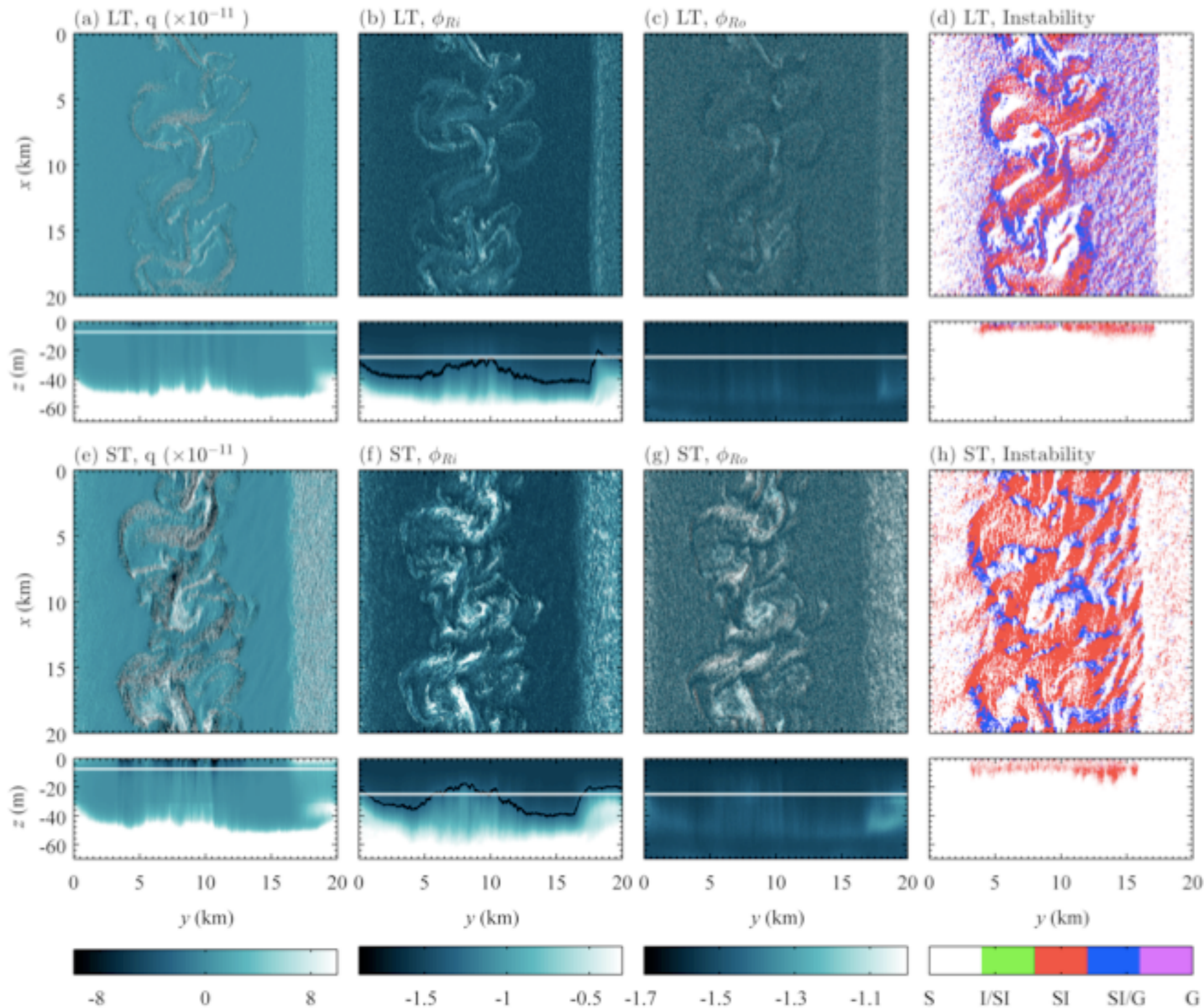


FIG. 12. Potential vorticity q (a,e), modified Richardson number ϕ_{Ri} (b,f), modified Rossby number ϕ_{Ro} (c,g), and instability maps (d,h) in $x-y$ planes (top panels) and as a function

Submeso-Langmuir results

- Strong interactions between small & large scales are rare in this configuration
- Two relatively independent turbulent spectral cascades near the surface. Only submeso at depth.
- Presence of waves greatly changes small scale instability character from symmetric instability to gravitational--Stokes shear force explains this!
- Key Asymptotic divide between Submeso and Langmuir Turbulence is aspect ratio/nonhydrostatic

Conclusions

- Climate modeling is challenging partly due to the vast and diverse scales of fluid motions
- In the upper ocean, horizontal scales as big as basins, and as small as meters contribute non-negligibly to the air-sea exchange
- Process models, especially those spanning a whole or multiple scales, are a powerful tool in studying these connections and improving subgrid models.
- Based on present rates of increase of computing power, we will need these subgrid models for at least another century!



Many more wave-climate effects to come... stay tuned!

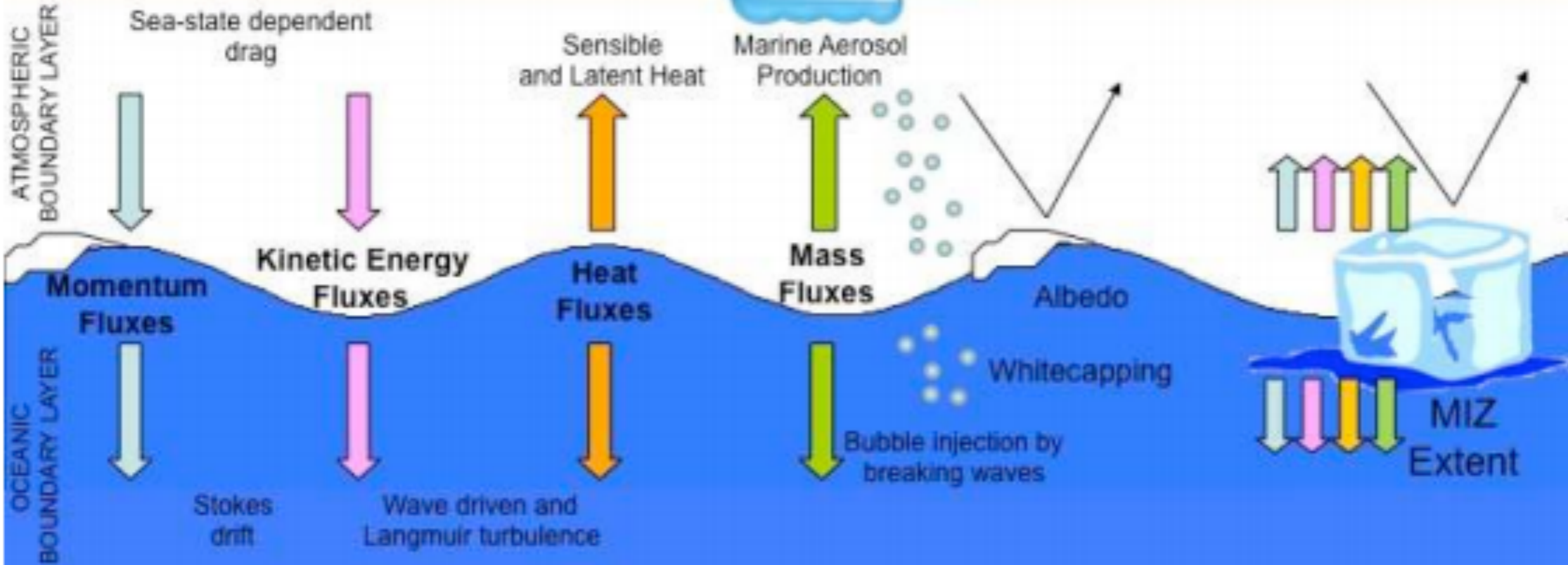
FREE ATMOSPHERE

Solar & Longwave Radiation

ATMOSPHERIC BOUNDARY LAYER

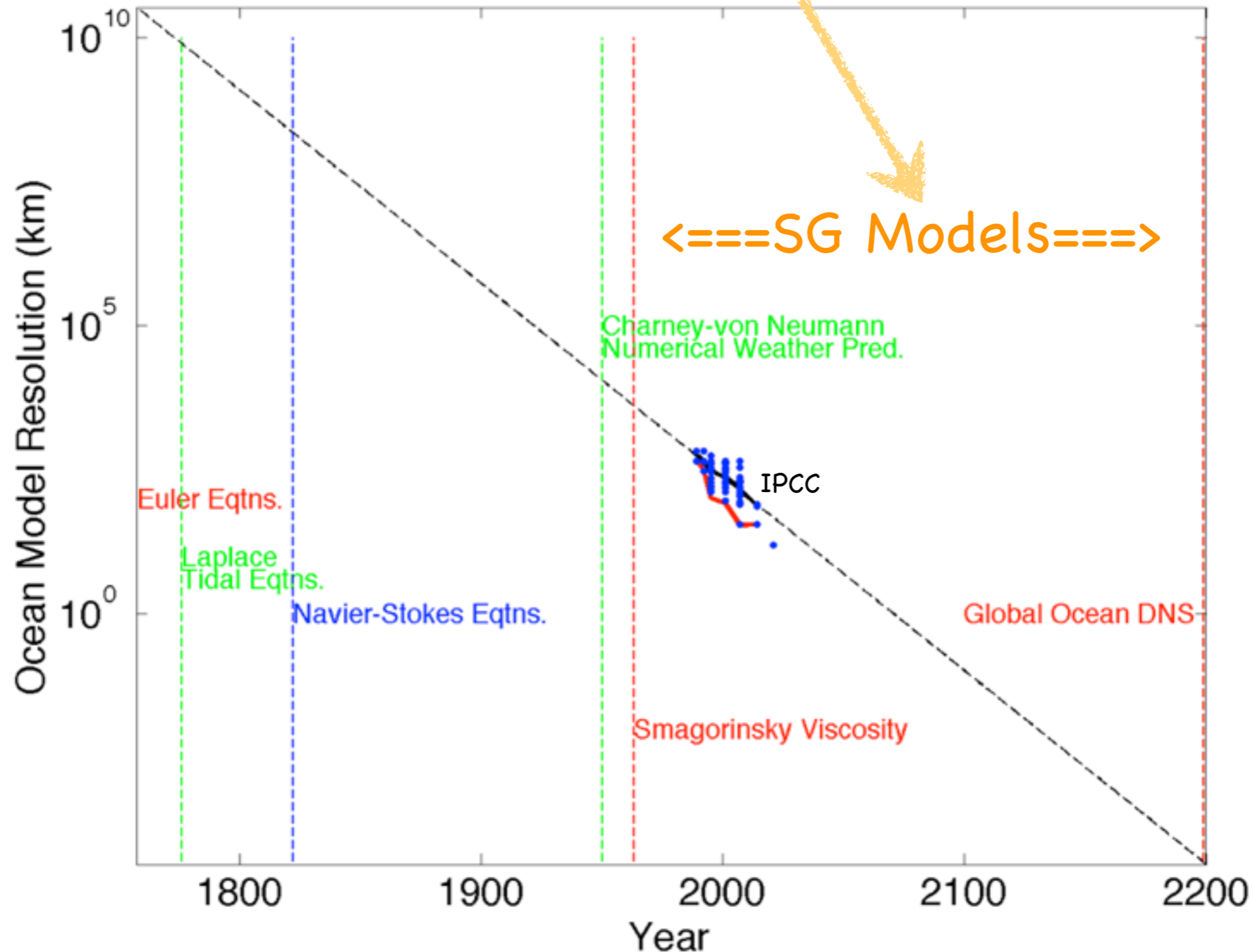
OCEANIC BOUNDARY LAYER

WIND WAVE



Wind-wave dependent processes in the coupled climate system
Towards coupled wind-wave-AOGCM models

Extrapolate for historical perspective: The Golden Era of Subgrid Modeling is Now!

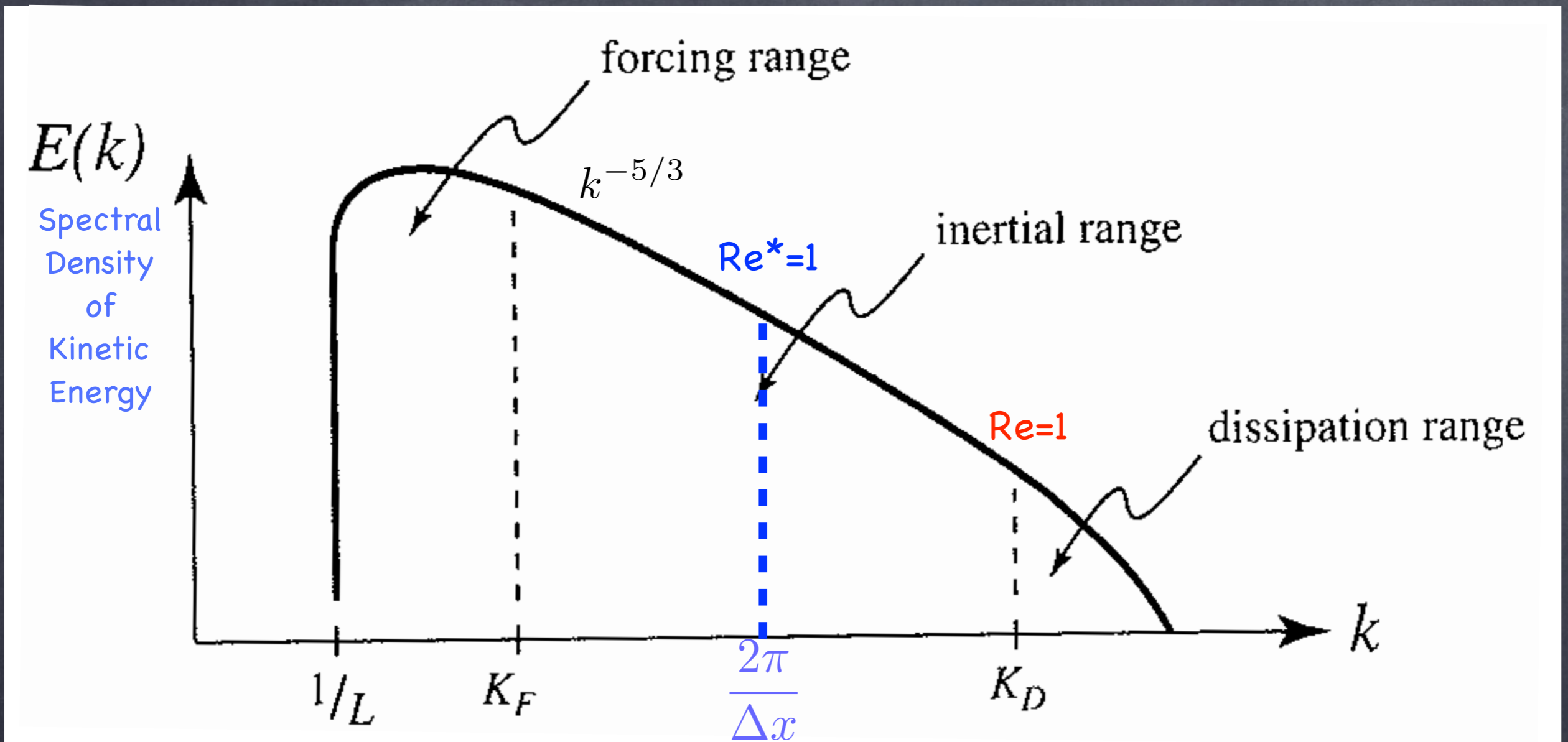


All papers at: fox-kemper.com/research

So, even as we begin to resolve the mesoscale...

- There are many, many processes left unresolved or partially resolved
- **Eddy Less:** For the unresolved (no eddies), need Reynolds-Average Closures (e.g., KPP, Gent-McWilliams, Redi)
- **Eddy Rich:** eddy-permitting to resolving, need Large-Eddy-Simulation Closures (e.g., Smagorinsky)
- Some scale-aware hybrids, e.g., Mixed Layer Eddies: Fox-Kemper et al. 2011, Hallberg 2013

3D Turbulence Cascade

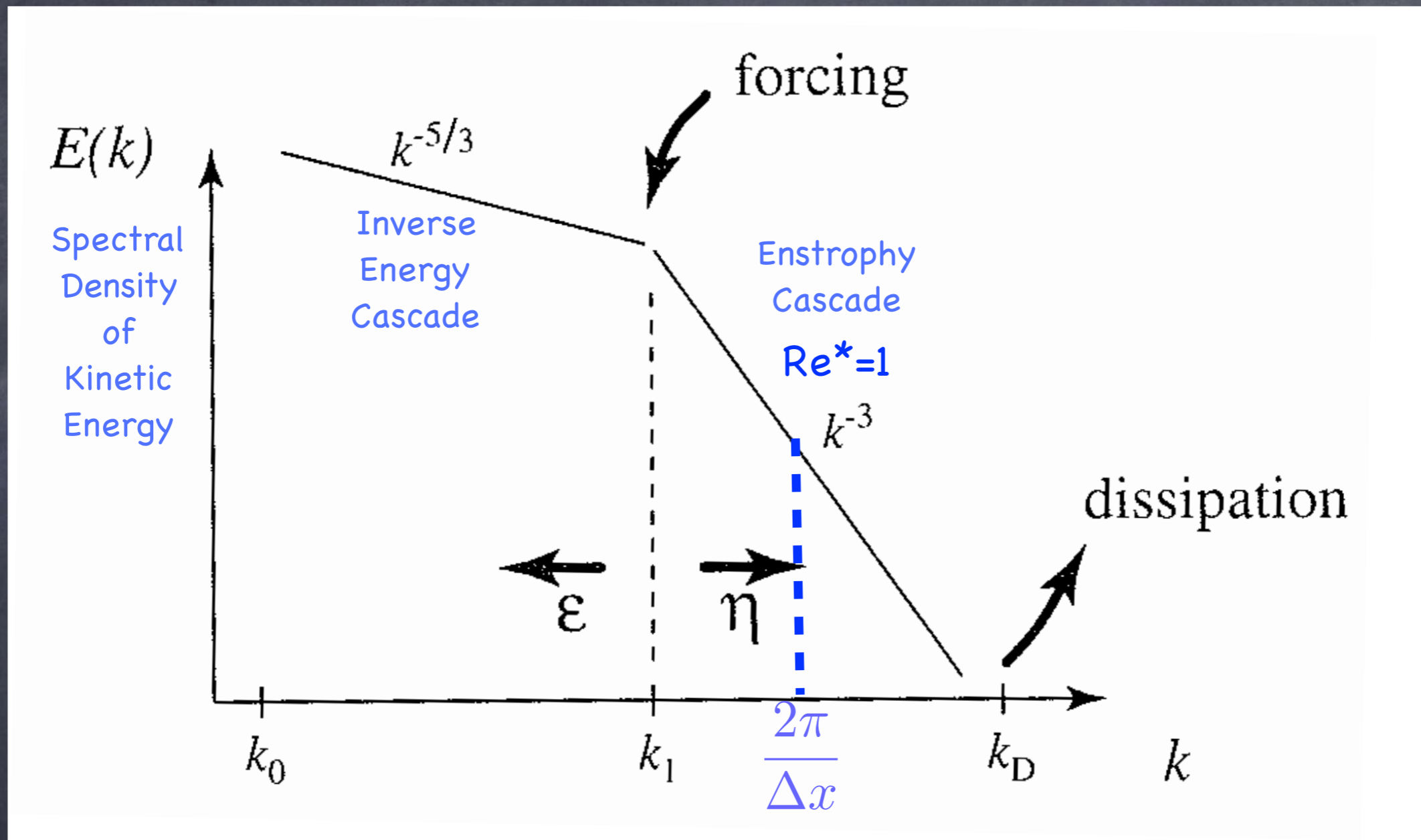


1963: Smagorinsky Scale & Flow Aware Viscosity Scaling,
 So the Energy Cascade is Preserved,
 but order-1 gridscale Reynolds #: $Re^* = UL/\nu_*$

$$\nu_{*h} = \left(\frac{\Upsilon_h \Delta x}{\pi} \right)^2 \sqrt{\left(\frac{\partial u_*}{\partial x} - \frac{\partial v_*}{\partial y} \right)^2 + \left(\frac{\partial u_*}{\partial y} + \frac{\partial v_*}{\partial x} \right)^2}$$

2D Turbulence Differs

R. Kraichnan, 1967 JFM

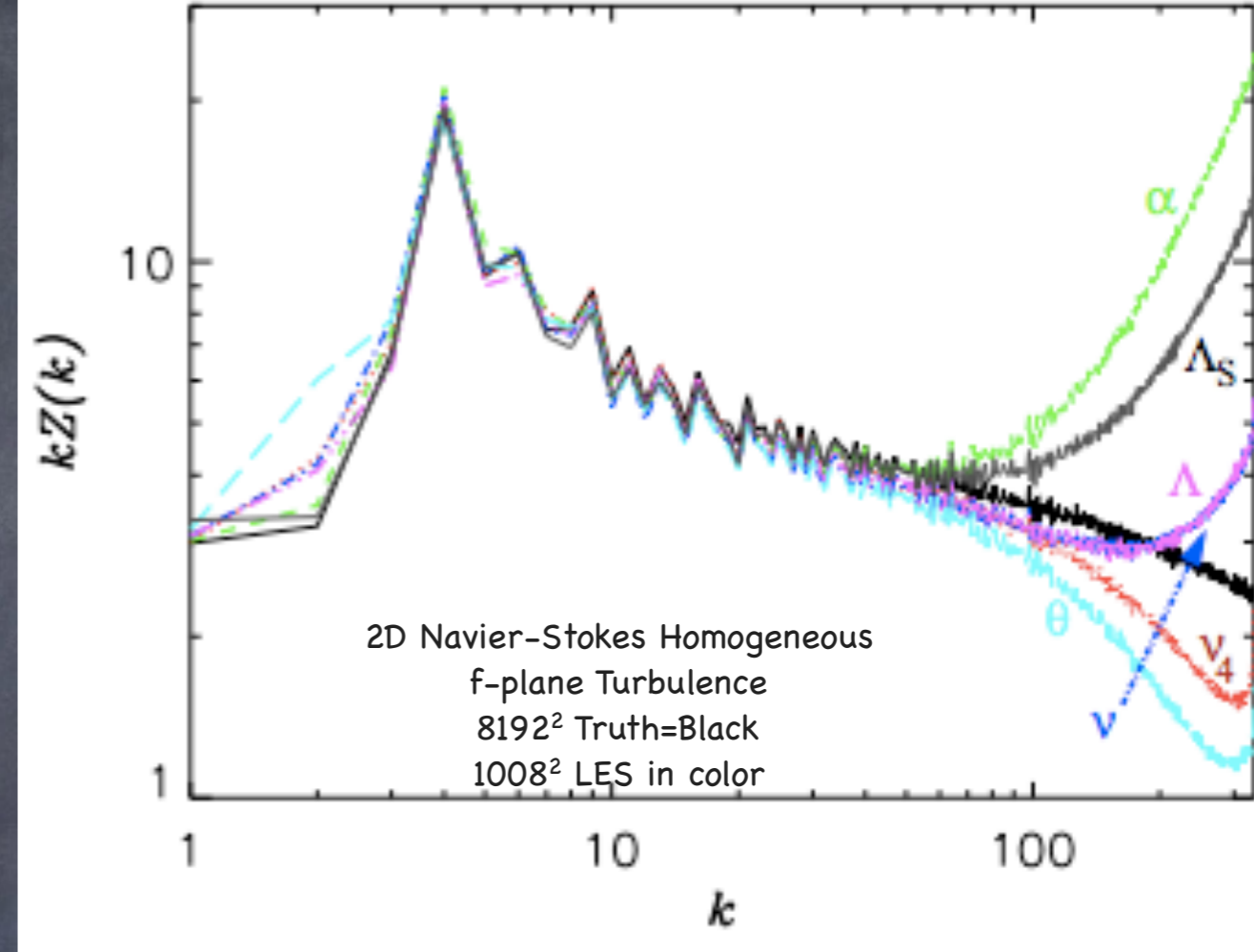


1996: Leith Devises Viscosity Scaling,
So that the Enstrophy (vorticity²) Cascade is Preserved

$$\mathbf{v}_* = \left(\frac{\Lambda \Delta x}{\pi} \right)^3 \left| \nabla_h \left(\frac{\partial u_*}{\partial y} - \frac{\partial v_*}{\partial x} \right) \right|$$

Some MOLES Truncation Methods In Use 2d (SWE) test

- Harmonic/Biharmonic/Numerical
 - Many. Often not scale- or flow-aware
 - Griffies & Hallberg, 2000, is one aware example
- Fox-Kemper & Menemenlis, 2008. ECCO2.
 - Leith Viscosity (2d Enstrophy Scaling)
- Chen, Q., Gunzburger, M., Ringler, T., 2011
 - Anticipated Potential Vorticity of Sadourny
- San, Staples, Iliescu (2011, 2013)
 - Approximate Deconvolution Method
- Stochastic & Statistical Parameterizations
 - Other session going on now in Y10

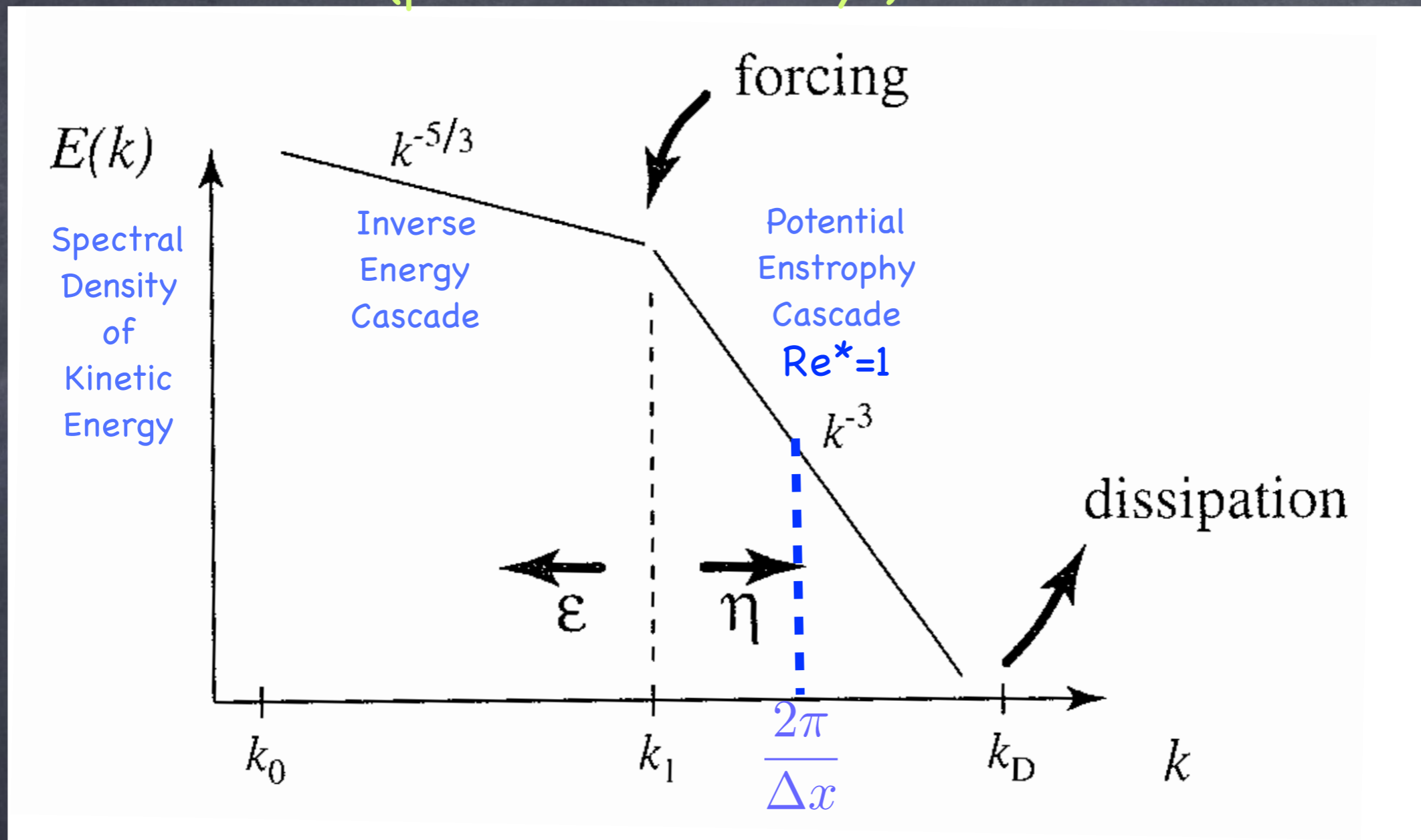


Graham & Ringler, 2013 Ocean Modelling

See also Ramachandran et al, 2013
Ocean Modelling for SMOLES

QG Turbulence: Pot'l Enstrophy cascade (potential vorticity²)

J. Charney, 1971 JAS

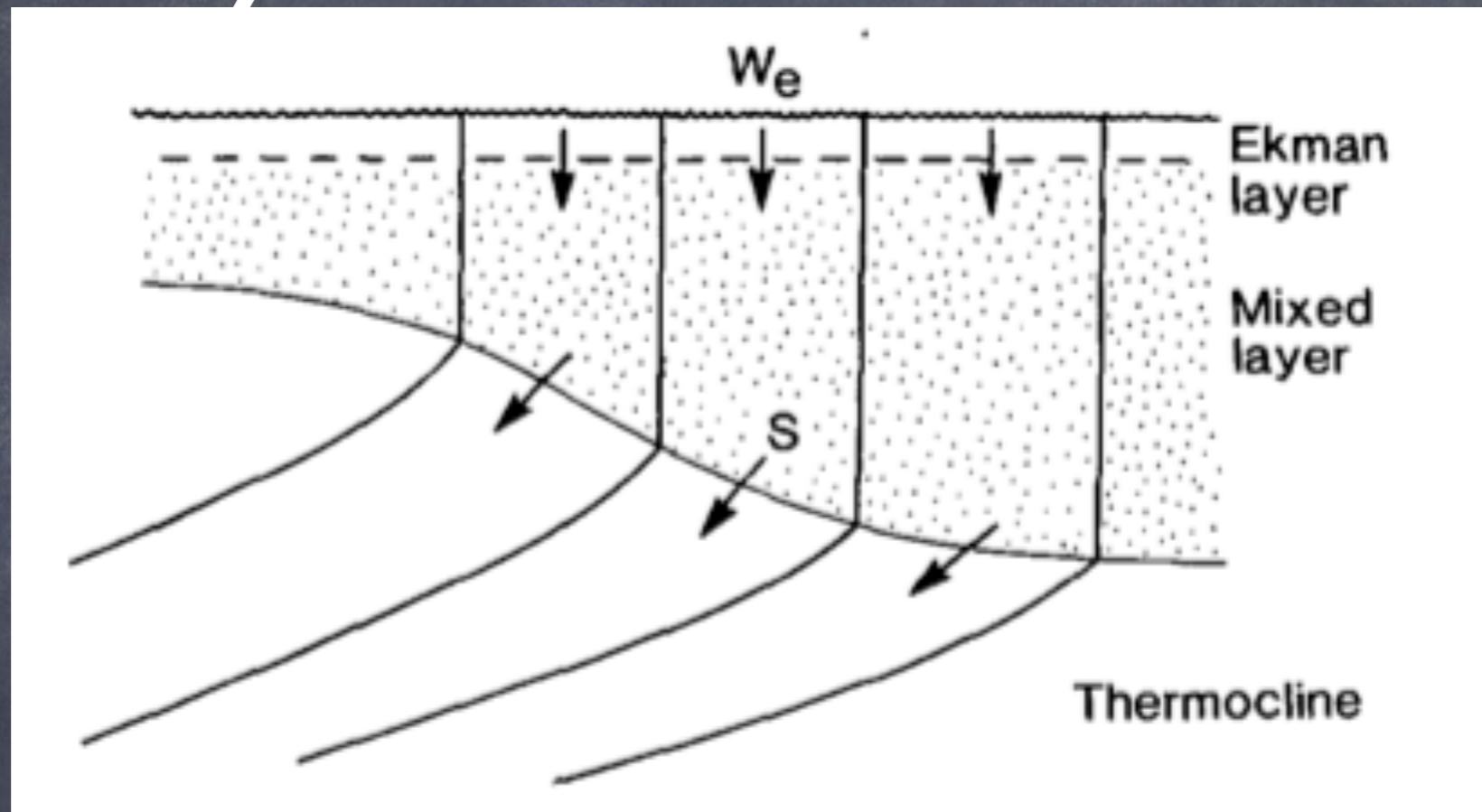


F-K & Menemenlis '08: Revise Leith Viscosity Scaling,
So that diverging, vorticity-free, modes are also damped

$$\mathbf{v}_* = \left(\frac{\Delta x}{\pi} \right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2}$$

B. Fox-Kemper and D. Menemenlis. Can large eddy simulation techniques improve mesoscale-rich ocean models? In M. Hecht and H. Hasumi, editors, Ocean Modeling in an Eddy Regime, volume 177, pages 319-338. AGU Geophysical Monograph Series, 2008.

Is 2D Turbulence a good proxy for neutral flow?



Yes:

- For a few eddy time-scales QG & 2D AGREE (Bracco et al. '04)
- Barotropic Flow--Obvious 2d analogue

No:

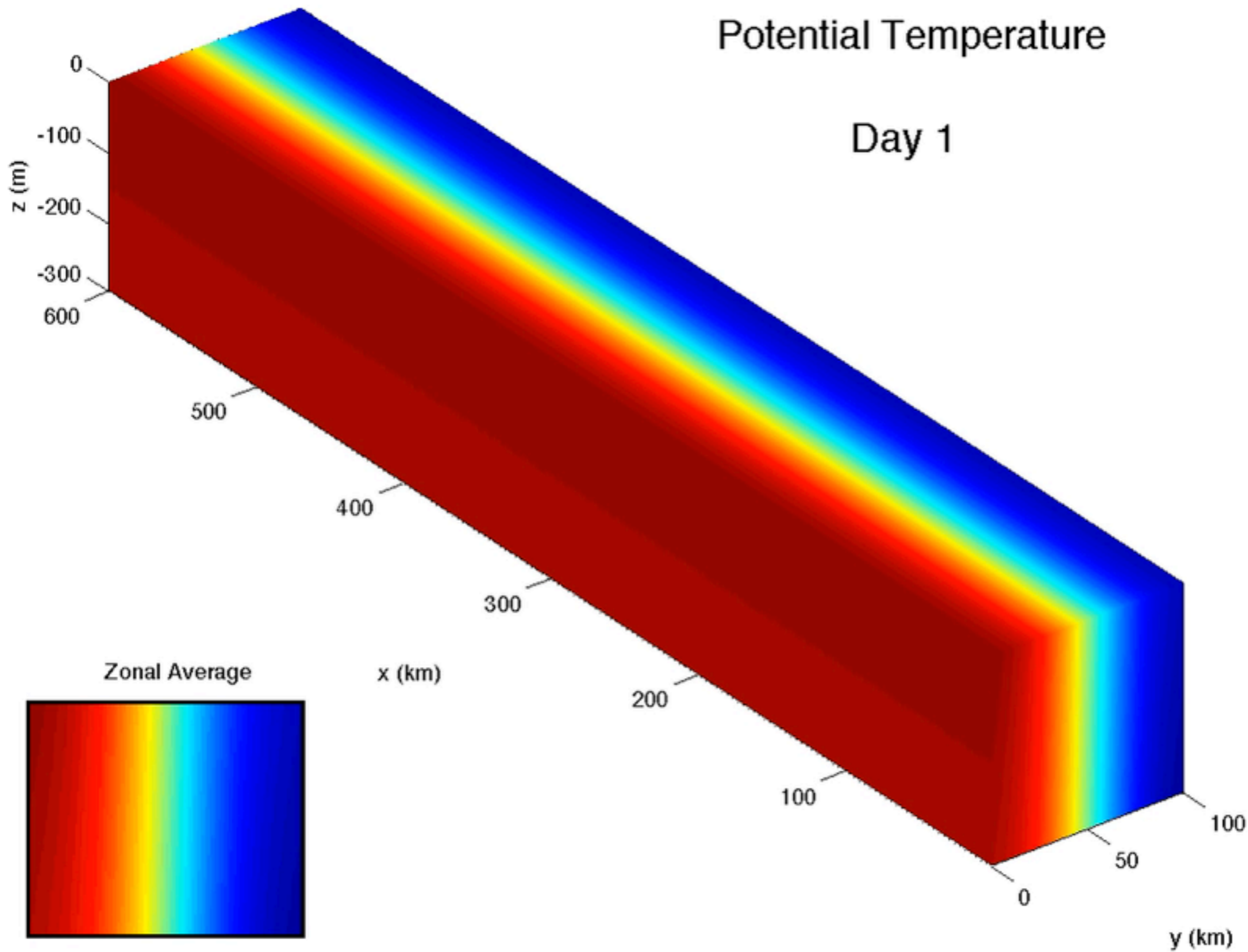
- Bolus Fluxes-- Divergent 2d flow
- Sloped, not horiz.
- Surface Effects?

Nurser & Marshall, 1991 JPO

Movie: S. Bachman

Potential Temperature

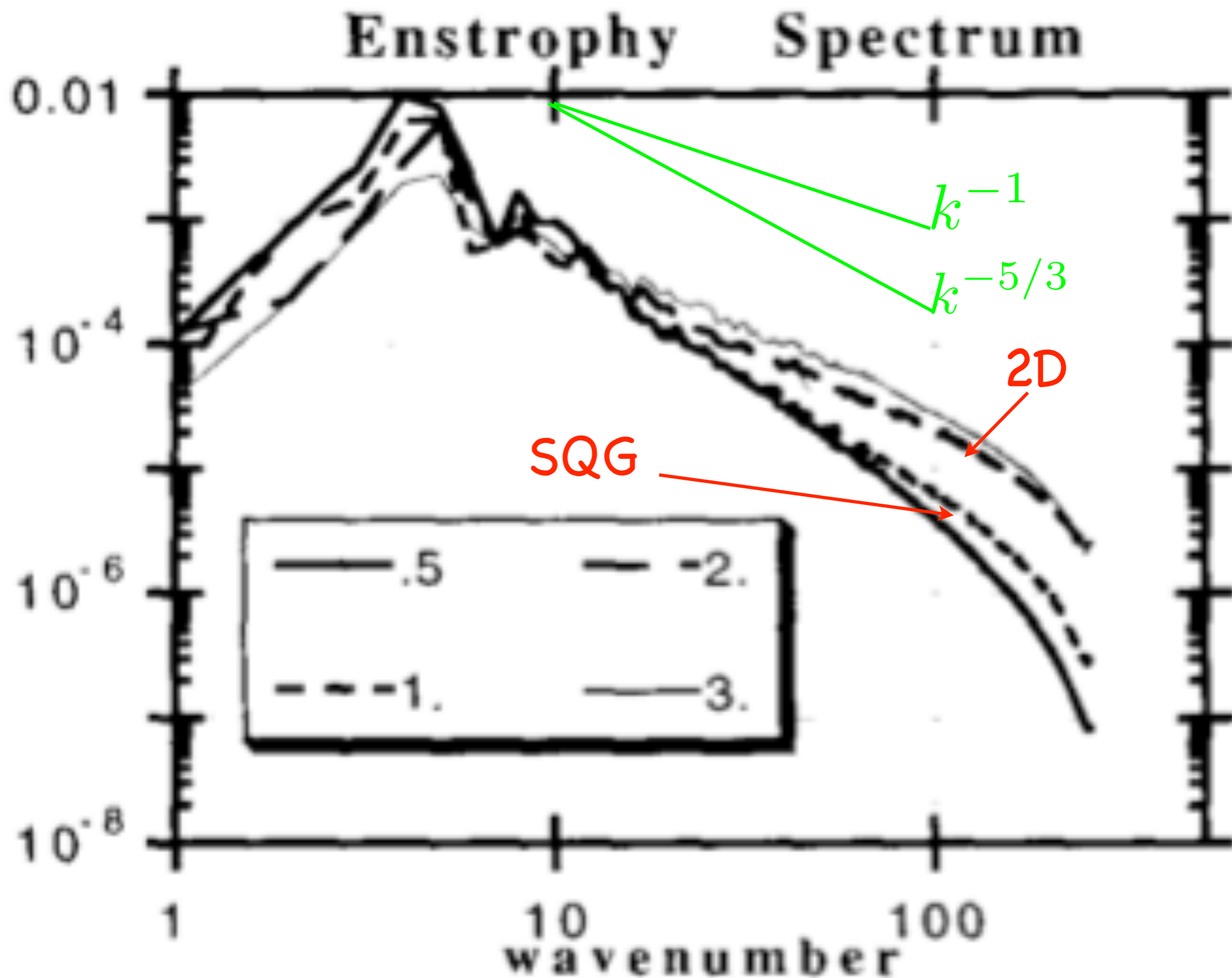
Day 1



S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013.

In real stratified flows, things are a bit more complex than in 2d. Even more than QG... Surface Effects may dominate

Pierrehumbert, Held, Swanson, 1994 Chaos Spectra of Local and Nonlocal Two-dimensional Turbulence



Many observations tell us:

The spectrum of potential density and buoyancy often scales as k^{-2} , which isn't too far from $k^{-5/3}$

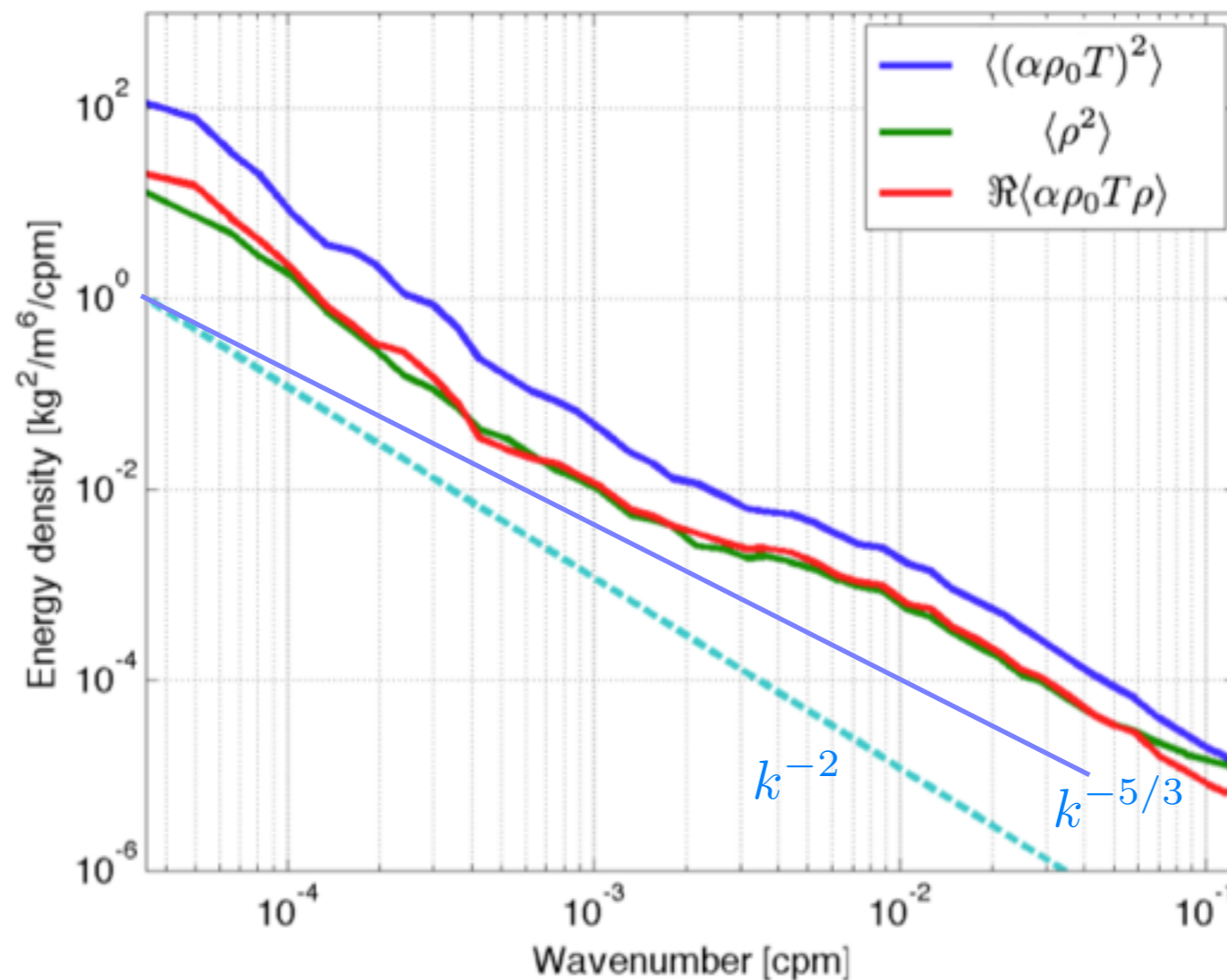
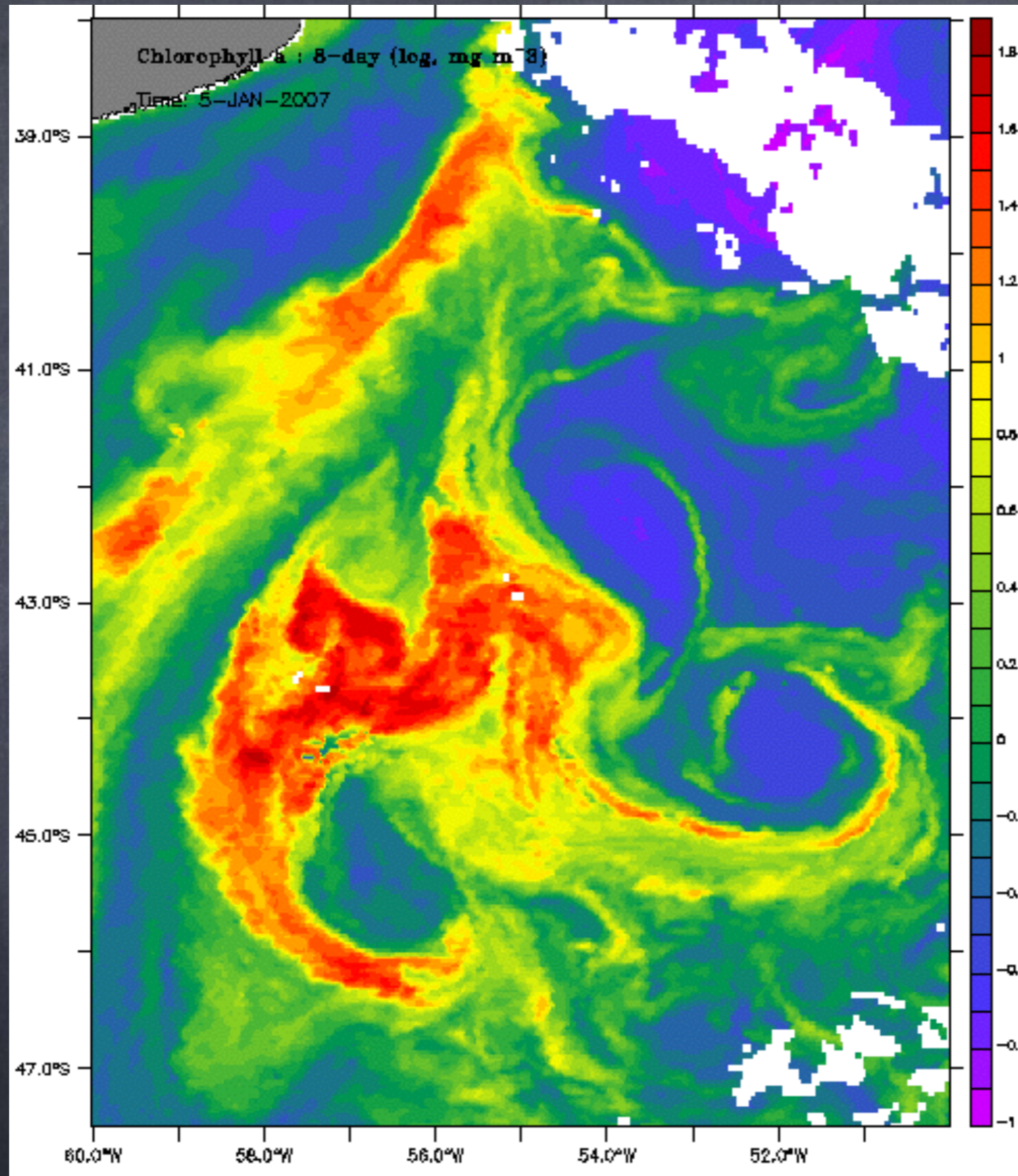


Figure 1: Observed spectra of mixed layer potential density variance (green), temperature contribution to potential density (blue), and temperature-density co-spectrum (red) from SeaSoar towed CTD and shipboard ADCP sections (data from Ferrari and Rudnick, 2000). A dashed line indicates k^{-2} scaling.

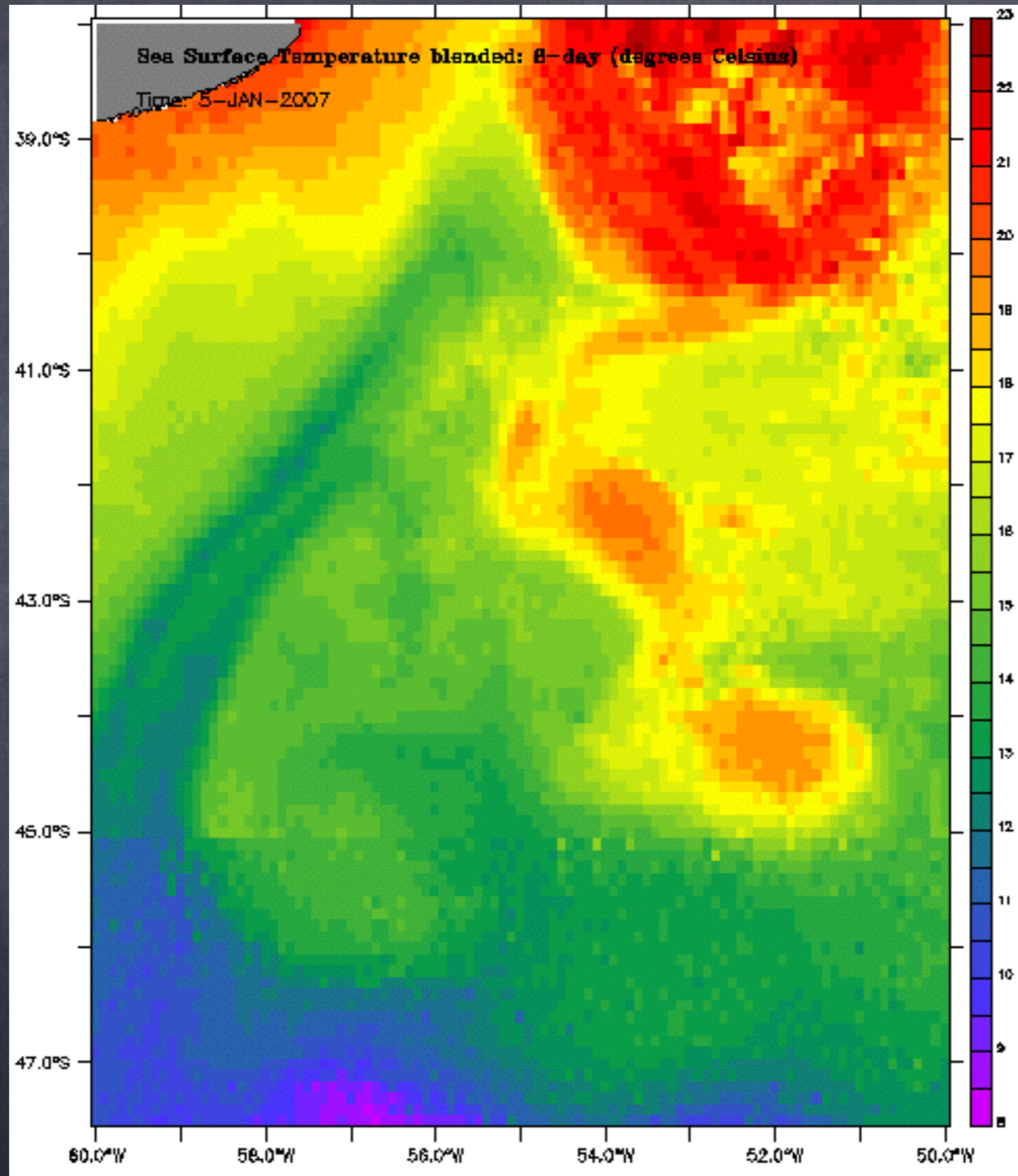
B. Fox-Kemper,
G. Danabasoglu, R. Ferrari,
S. M. Griffies, R. W. Hallberg,
M. M. Holland, M. E. Maltrud,
S. Peacock, and B. L.
Samuels. Parameterization of
mixed layer eddies. III:
Implementation and impact in
global ocean climate
simulations. *Ocean Modelling*,
39:61-78, 2011.

Examples: Jan 5, 07 East of Argentina



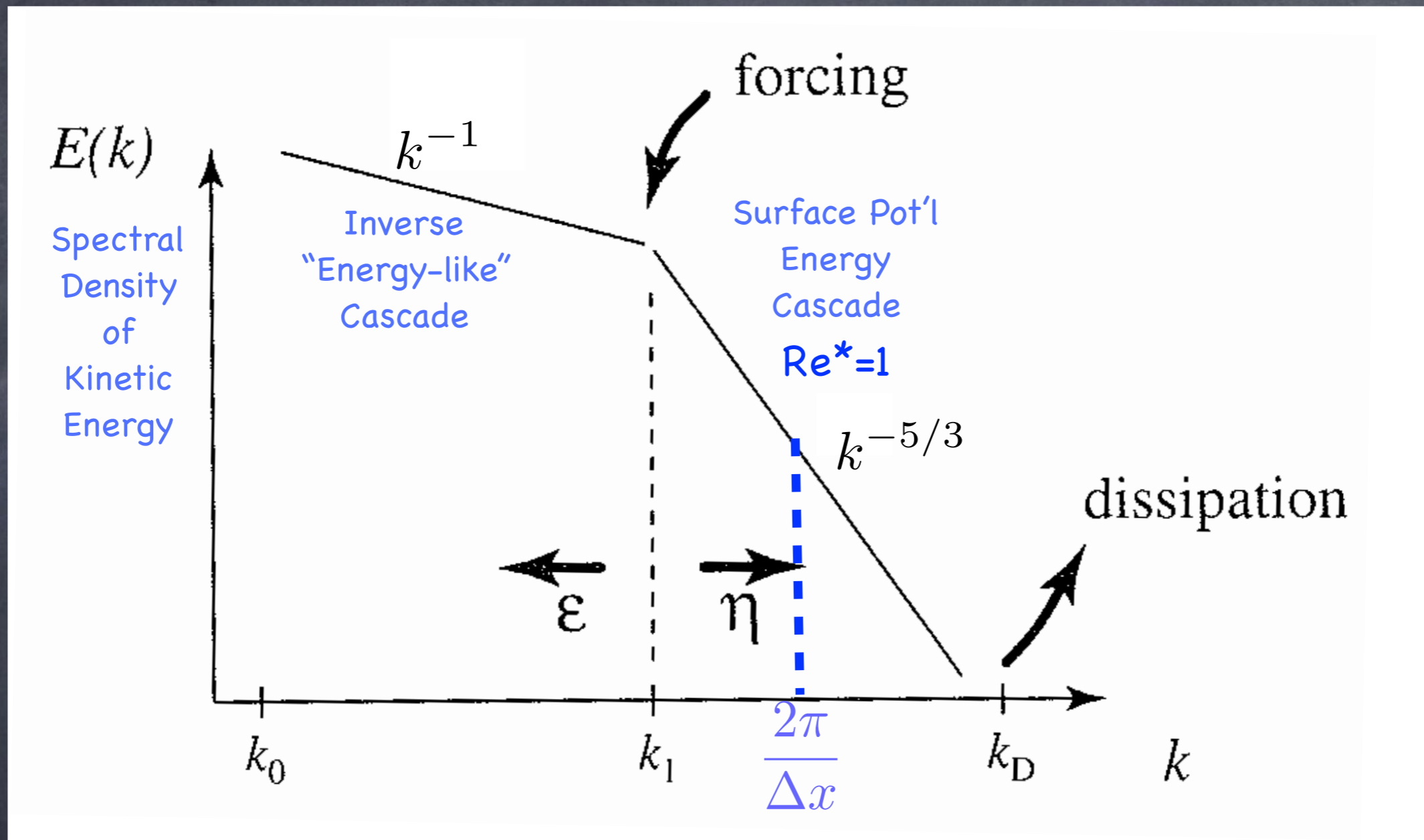
MODIS on Aqua Chl

Examples: Jan 5, 07 East of Argentina



SQG Turbulence: Surface Buoyancy & Velocity cascade

W. Blumen, 1978 JAS
 Held et al 1995, JFM.
 Smith et al. 2002, JFM

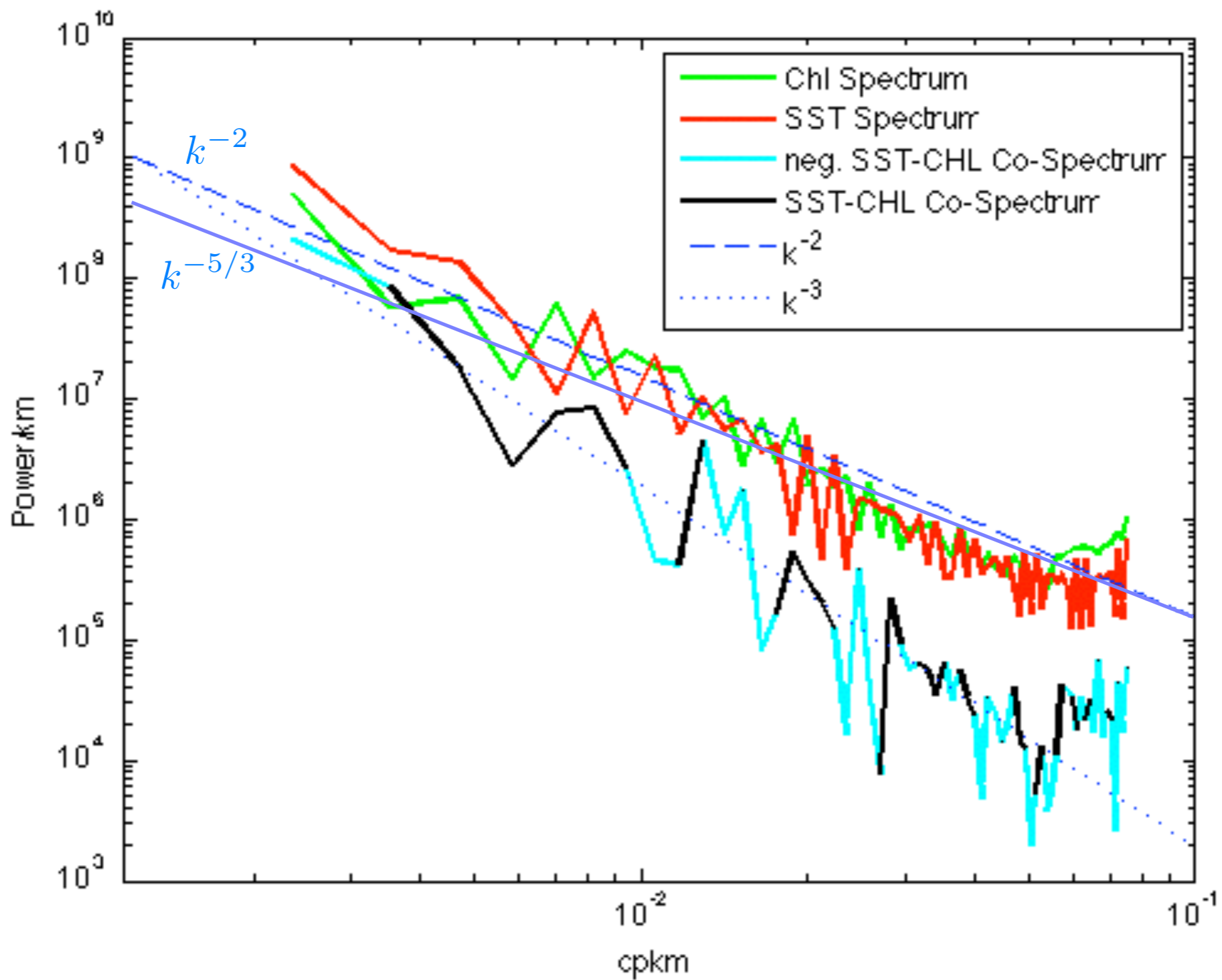


Smag-Like
 (Inverse):
 Leith-Like
 (Direct):

$$\kappa_* = \left(\frac{\Upsilon \Delta x}{\pi} \right)^{4/3} \left| \frac{1}{f} \nabla_h b \right|^{2/3}$$

$$\kappa_* = \left(\frac{\Lambda \Delta x}{2\pi} \right)^{3/2} \left[-\frac{\partial}{\partial z} |\nabla_h \psi|^2 \right]^{1/2}$$

Spectra: Jan 5, 07 East of Argentina



It is not clear that inertial ranges exist.

This spectrum shows that topographic interactions change the spectrum at depth dramatically

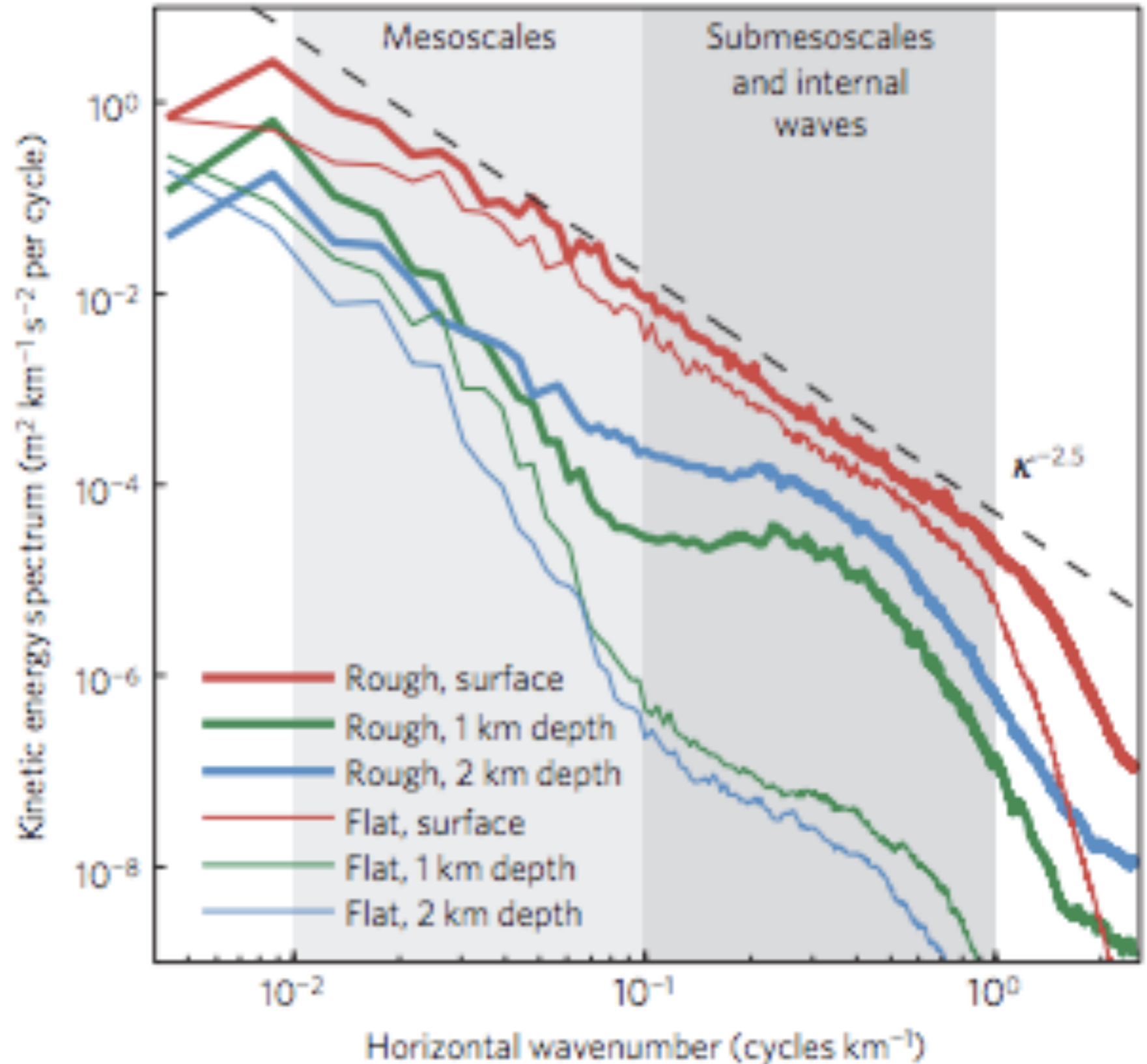


Figure 4 | Horizontal wavenumber kinetic energy spectra. Spectra

Reynolds vs. Péclet: Prandtl=1?

- In all cascade examples, the truncation occurs at large Reynolds and Péclet, so it is reasonable to assume diffusivity=viscosity
- In the QG framework, diffusivity *must* equal viscosity to avoid spurious generation of potential vorticity by the subgrid model
- For Baroclinic QG eddies, Dukowicz & Smith (97) showed that GM coefficient should equal Redi diffusivity.
- Thus, viscosity=diffusivity=GM coefficient

And it is ... ongoing

- Scott Bachman (DAMTP) has implemented this QG Leith closure in the MITgcm
 - Both Germano Dynamic and Fixed Coefficient
- Sets viscosity=diffusivity=GM coefficient
- Both are stable and robust
- Both work better than Smagorinsky, smoother spectrum to grid scale.
- But, we don't yet understand the spectral behavior of all test cases. 2d barotropic,

A Prescription for Parameterization...

Accuracy TBD

- QG Leith & Potential Vorticity to generate #1 viscosity
- 2D Leith & Barotropic Vorticity to generate #2 viscosity
- SQG Leith & Surf. Buoyancy to generate #3 diffusivity
- Take $\max(\#1, \#2, \#3)$ as viscosity, Redi diffusivity, *and* as GM transfer coeff.

Nearly suggested by Roberts & Marshall, 98, JPO

- Note: Unlike Eddy-Free closures, e.g., Visbeck et al (97), Eddy-Rich closures take advantage of resolved eddies & instabilities, only need a boost from eddy-permitting to eddy-resolving (and for numerical stability)

So, no problems? Just crunch away with CLB?

- Let's revisit our assumptions for scale separation:
 - CLB wave equations require limited *wave steepness* and irrotational flow
 - Real wind-waves are not monochromatic, but incorporate a spectrum of waves, and...



Power Spectrum of wave height

$$\langle \eta^2 \rangle = \int_0^{\infty} E(k) dk = C_0 + \int_{k_h}^{\infty} C_1 k^{-2} dk$$

Power Spectrum of wave steepness:
INFINITE!

$$\langle k^2 \eta^2 \rangle = \int_0^{\infty} k^2 E(k) dk = D_0 + \int_{k_h}^{\infty} D_1 dk$$

Steep waves break → vortex motion & small scale turbulence!

A Global Parameterization of Mixed Layer Eddy Flow & Scale Aware Restratification validated against simulations

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

$$\overline{\mathbf{u}'b'} \equiv \Psi \times \nabla \bar{b}$$

$$\Psi = \left[\frac{\Delta x}{L_f} \right] \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla \bar{b} \times \hat{\mathbf{z}}$$

Compare to the original **singular, unrescaled** version

$$\Psi = \left| \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{\mathbf{z}} \right.$$

New version **handles the equator**, and **averages over many fronts**