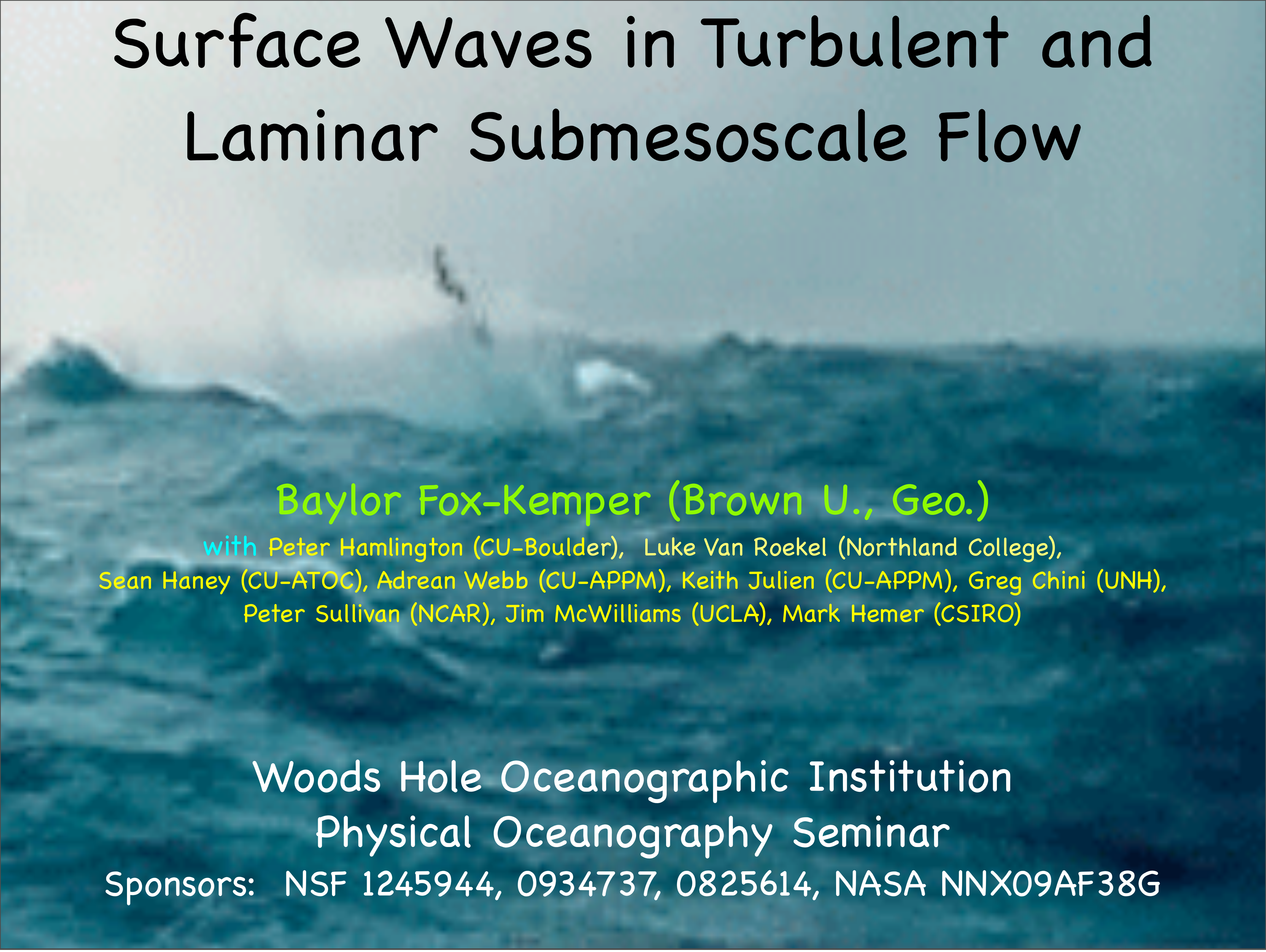


Surface Waves in Turbulent and Laminar Submesoscale Flow

Baylor Fox-Kemper (Brown U., Geo.)

with Peter Hamlington (CU-Boulder), Luke Van Roekel (Northland College),
Sean Haney (CU-ATOC), Adrean Webb (CU-APPM), Keith Julien (CU-APPM), Greg Chini (UNH),
Peter Sullivan (NCAR), Jim McWilliams (UCLA), Mark Hemer (CSIRO)

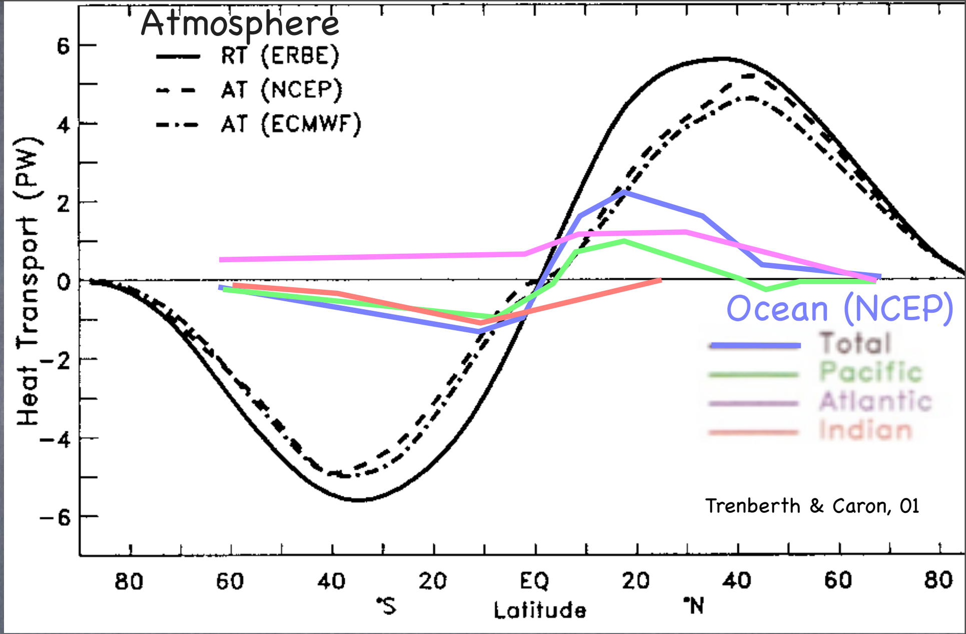
Woods Hole Oceanographic Institution
Physical Oceanography Seminar

Sponsors: NSF 1245944, 0934737, 0825614, NASA NNX09AF38G

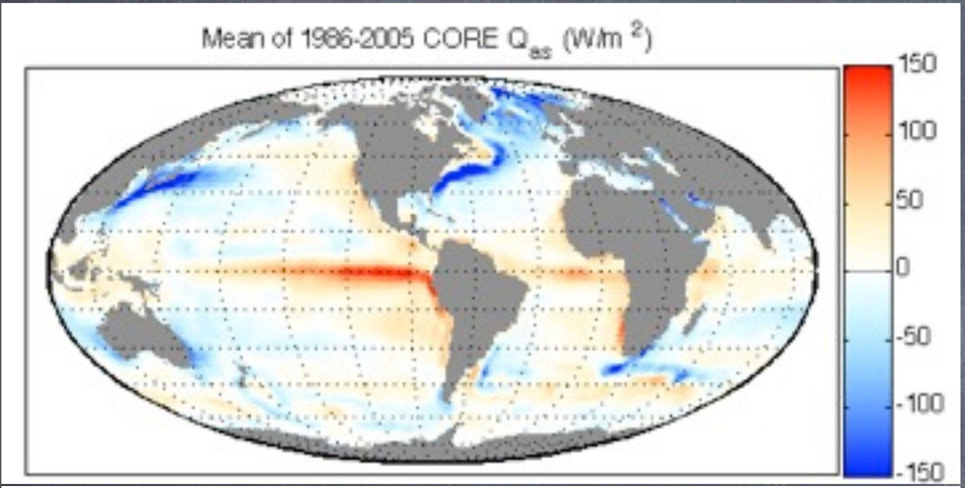
Air-Sea Flux Errors vs. Data

Heat capacity & mode of transport is different in A vs. O

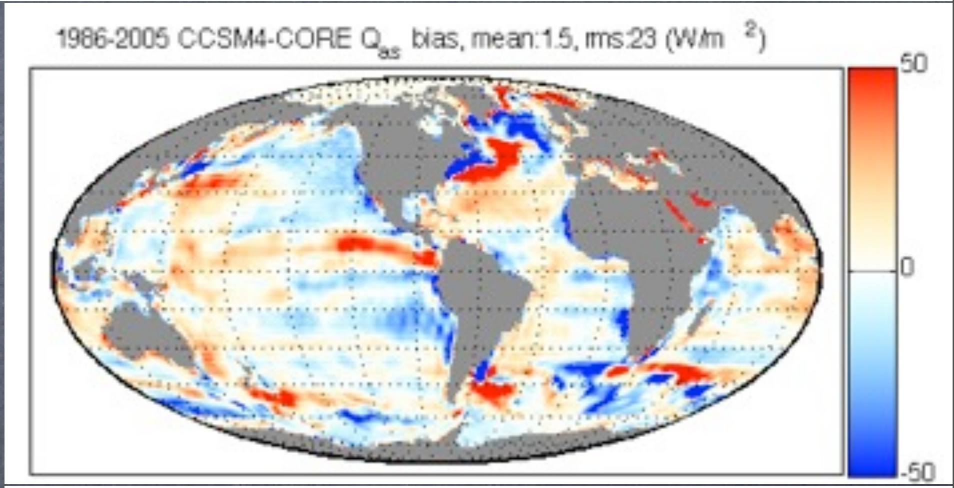
S. C. Bates, B. Fox-Kemper, S. R. Jayne, W. G. Large, S. Stevenson, and S. G. Yeager. Mean biases, variability, and trends in air-sea fluxes and SST in the CCSM4.Journal of Climate, 25(22):7781-7801, 2012.



Trenberth & Caron, 01



Mean of 1986-2005 CORE Q_{as} (W/m ^{2})

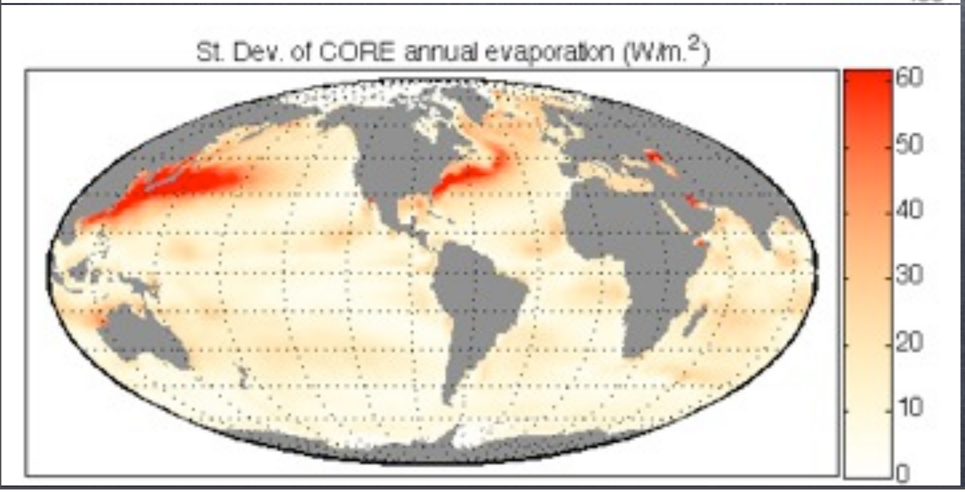


1986-2005 CCSM4-CORE Q_{as} bias, mean:1.5, rms:23 (W/m ^{2})

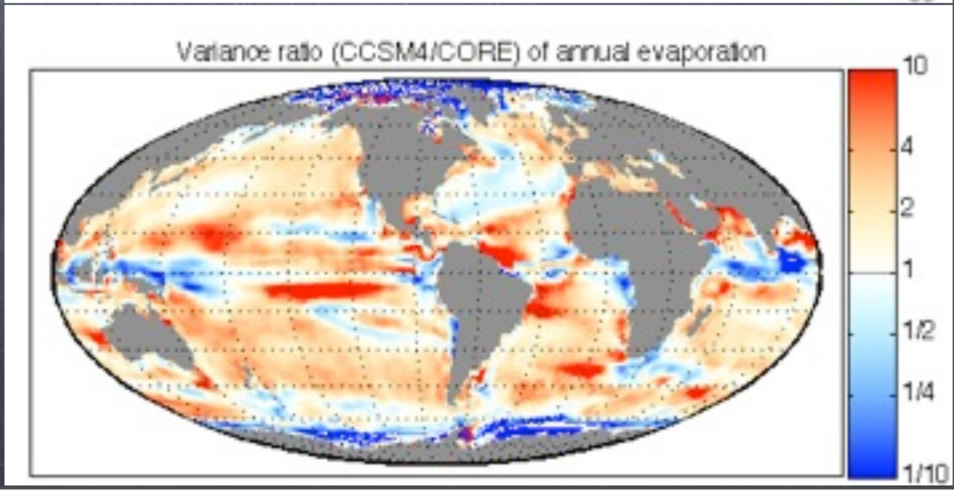
Mean

Annual

9-15mo

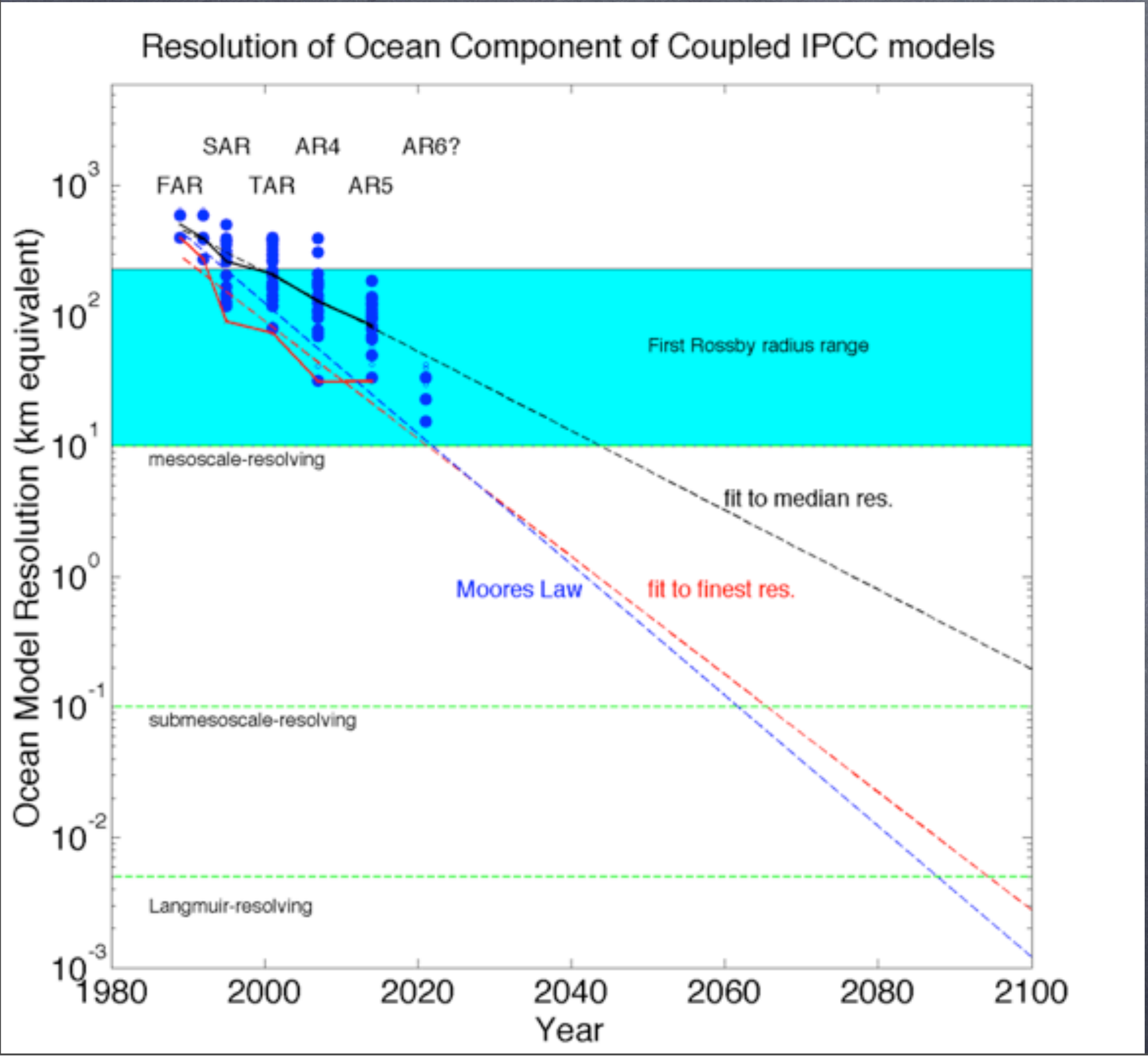


St. Dev. of CORE annual evaporation (W/m. ^{2})



Variance ratio (CCSM4/CORE) of annual evaporation

Resolution will be an issue for centuries to come!



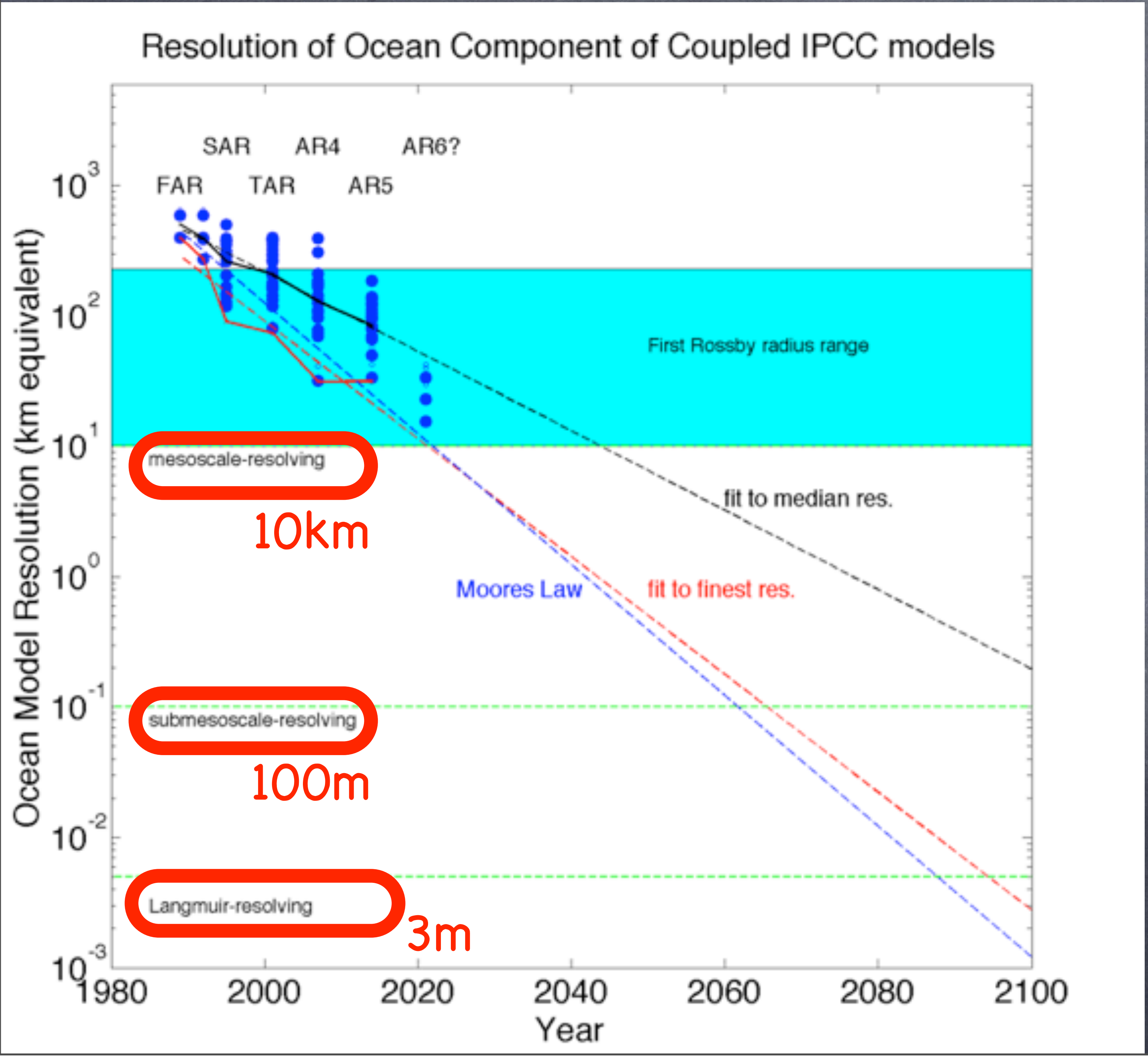
IPCC: Intergovernmental Panel on Climate Change

They won the Nobel (Peace) Prize with Al Gore

Here are the collection of IPCC models...

If we can't resolve a process, we need to develop a parameterization or subgrid model of its effect

Resolution will be an issue for centuries to come!



IPCC: Intergovernmental Panel on Climate Change

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Here are the collection of IPCC models...

If we can't resolve a process, we need to develop a parameterization or subgrid model of its effect

What is a parameterization/subgrid model?

Fluid equations for A&O are PDEs (Rotating, Stratified Navier-Stokes), but we cannot resolve to dissipation, so we use statistical or bulk subgrid models to capture multiscale interactions:

- Express the coarse-grain averages of quantities (including the subgrid effects), e.g.:

$$\overline{\frac{\partial \tau}{\partial t}} \qquad \overline{\frac{\partial u}{\partial x}} \qquad \overline{\frac{\partial u\tau}{\partial x}}$$

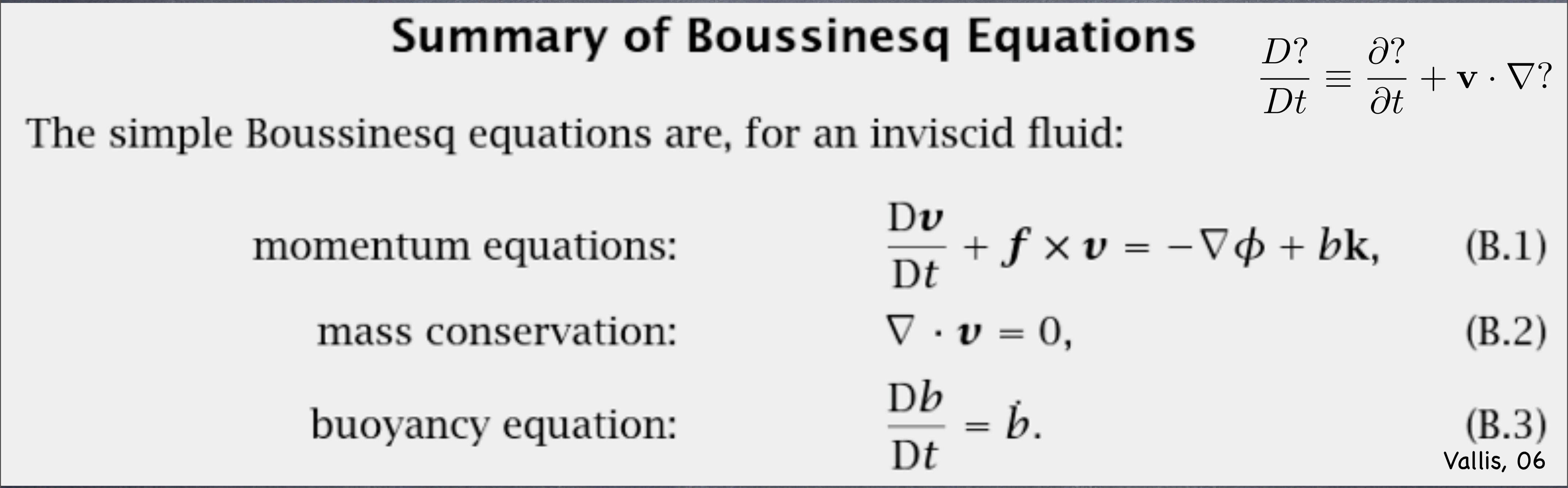
- As a function of the resolved coarse-grain fields

$$\overline{\frac{\partial \tau}{\partial t}} = \frac{\partial \overline{\tau}}{\partial t} \qquad \overline{\frac{\partial u}{\partial x}} = \frac{\partial \overline{u}}{\partial x} \qquad \overline{\frac{\partial u\tau}{\partial x}} = \frac{\partial \overline{u}\;\overline{\tau}}{\partial x} + \frac{\partial \overline{u'\tau'}}{\partial x}$$

- Note that nonlinear terms require special treatment
- These couple different scales, small talks to large

With nearly incompressible (small density variations) approximation & approximated rotating Earth:

A simpler set of 5 vars



Summary of Boussinesq Equations

$$\frac{D?}{Dt} \equiv \frac{\partial ?}{\partial t} + \mathbf{v}\cdot\nabla ?$$

The simple Boussinesq equations are, for an inviscid fluid:

momentum equations: $$\frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} + \boldsymbol{f}\times\boldsymbol{v} = -\nabla\phi + b\mathbf{k}, \qquad \text{(B.1)}$$

mass conservation: $$\nabla\cdot\boldsymbol{v} = 0, \qquad \text{(B.2)}$$

buoyancy equation: $$\frac{\mathrm{D}b}{\mathrm{D}t} = \dot{b}. \qquad \text{(B.3)}$$

Vallis, 06

If you want, it's easy to distinguish buoyancy into contributions from Temperature and from Salinity (since we are near surface--linear EOS is OK)

Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished subinertial limit:

Inviscid (Re>>1), rapidly rotating (Ro<<1), and thin* (L>>H)

Full Momentum

$$\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla\phi + b\mathbf{k} + \nu\nabla^2\mathbf{v}$$

$$Re = \frac{UL}{\nu} \qquad Ro = \frac{U}{fL} \qquad Ri \equiv \frac{\frac{\partial b}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \qquad \alpha = H/L$$

*closely related to strong statification & ocean dimensions

Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished subinertial limit:

Inviscid (Re>>1), rapidly rotating (Ro<<1), and thin* (L>>H)

(Horizontal) Geostrophic Balance

$$\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla\phi + b\mathbf{k} + \nu\nabla^2\mathbf{v}$$

$$Re = \frac{UL}{\nu} \qquad Ro = \frac{U}{fL} \qquad Ri \equiv \frac{\frac{\partial b}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \qquad \alpha = H/L$$

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Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished subinertial limit:

Inviscid (Re>>1), rapidly rotating (Ro<<1), and thin* (L>>H)

(Vertical) Hydrostatic Balance

$$\frac{D\mathbf{v}}{Dt} + \mathbf{f} \times \mathbf{v} = -\nabla\phi + b\mathbf{k} + \nu\nabla^2\mathbf{v}$$

$$Re = \frac{UL}{\nu} \qquad Ro = \frac{U}{fL} \qquad Ri \equiv \frac{\frac{\partial b}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \qquad \alpha = H/L$$

*closely related to strong statification & ocean dimensions

Geostrophy, Hydrostasy, & Thermal Wind

Traditional Oceanography & Resolved Flow in IPCC models inhabits a special distinguished limit:

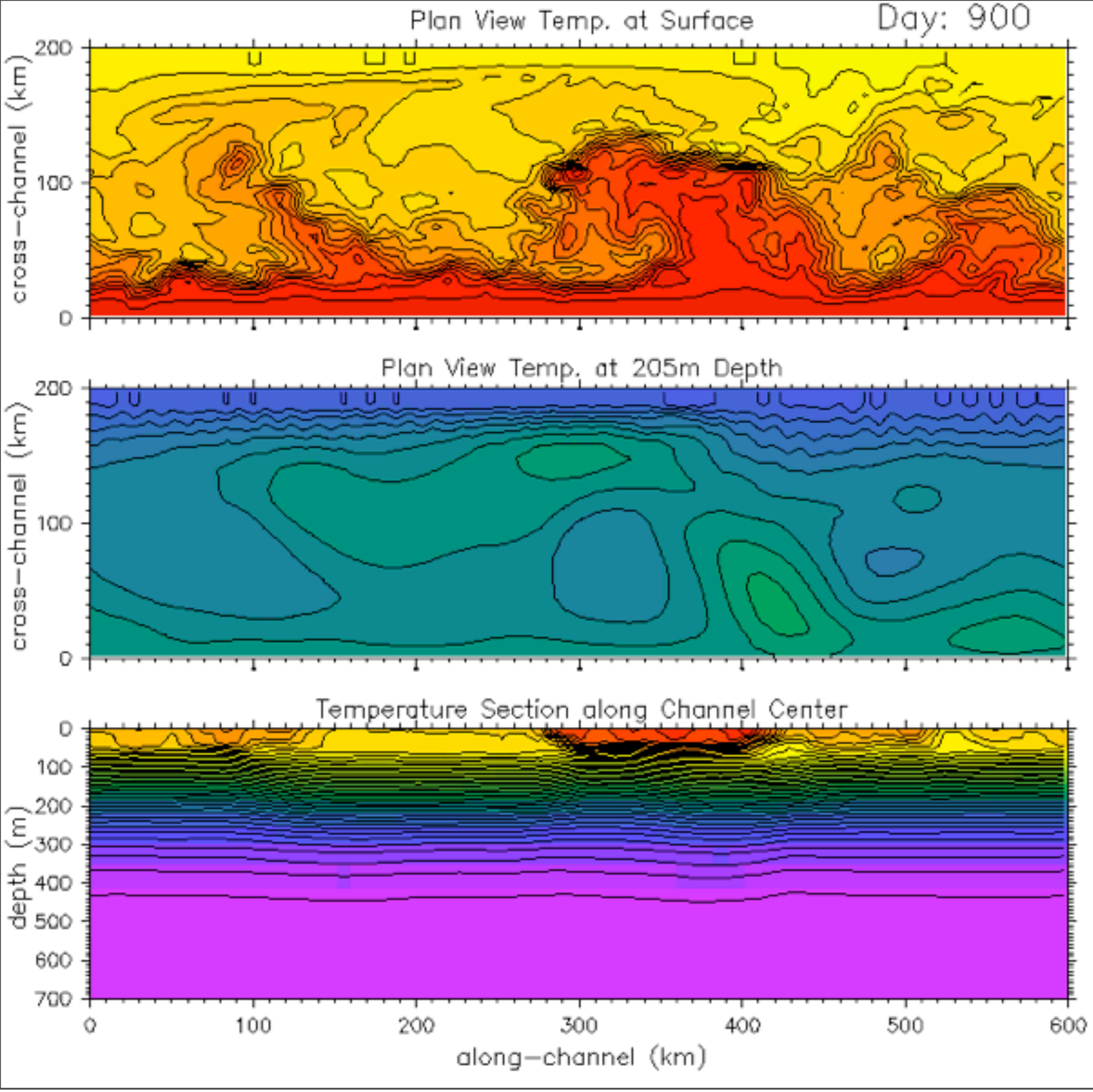
Inviscid (Re>>1), rapidly rotating (Ro<<1), and thin* (L>>H)

(Combined) Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b$$

Taken together with the forcing (air-sea) of buoyancy and the advection of buoyancy by this flow--you have the tools to study large-scale ocean physics!

Let's see some examples of Bousinesq, Hydrostatic Models at work in the mesoscale (10-100km) & submesoscale (100m-10km)



Big, Deep (mesoscale)

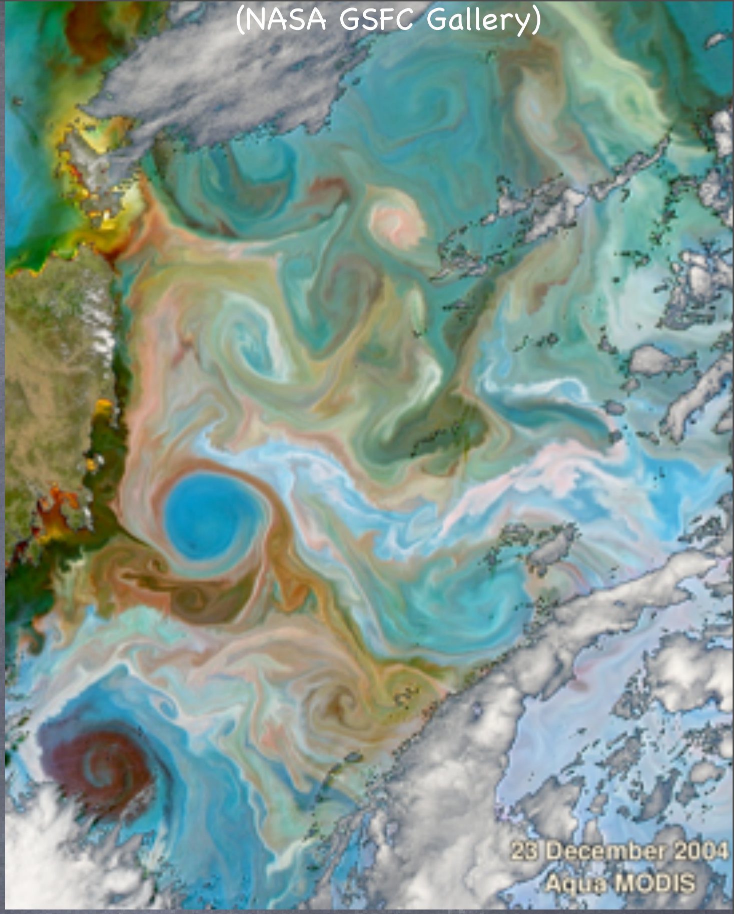
interact with

Little, Shallow (submeso)

B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. Journal of Physical Oceanography, 38(6):1145-1165, 2008.

The Character of the Submesoscale

← 10 km


(NASA GSFC Gallery)

(Capet et al., 2008)

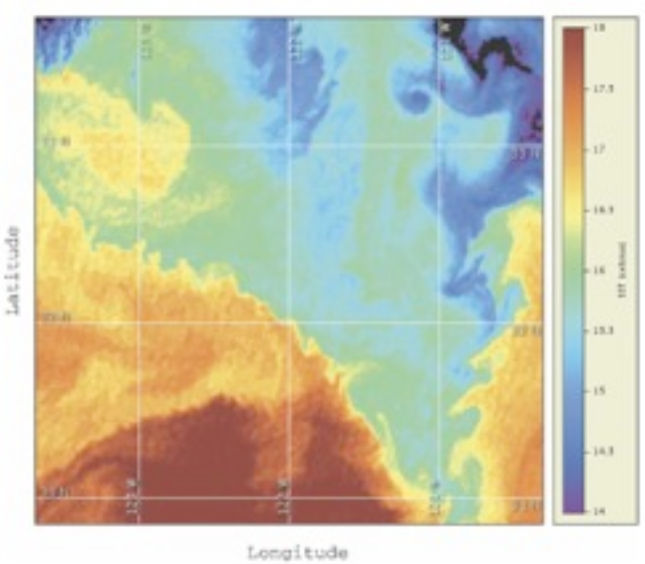
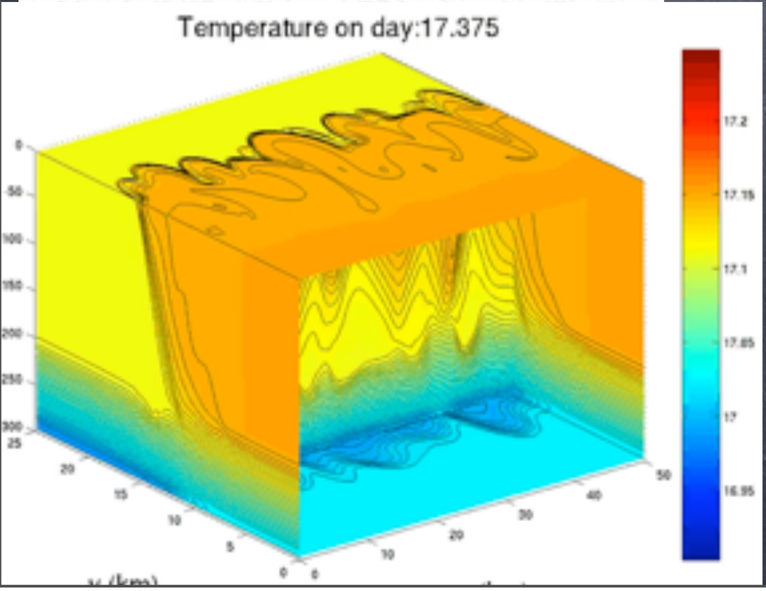


FIG. 16. Sea surface temperature measured at 1832 UTC 3 Jun 2006 off Point Conception in the California Current from CoastWatch (http://coastwatch.pfeg.noaa.gov). The fronts between recently



- Fronts
- Eddies
- Ro=O(1)
- Ri=O(1)
- near-surface
- 1-10km, days

Eddy processes often baroclinic instability

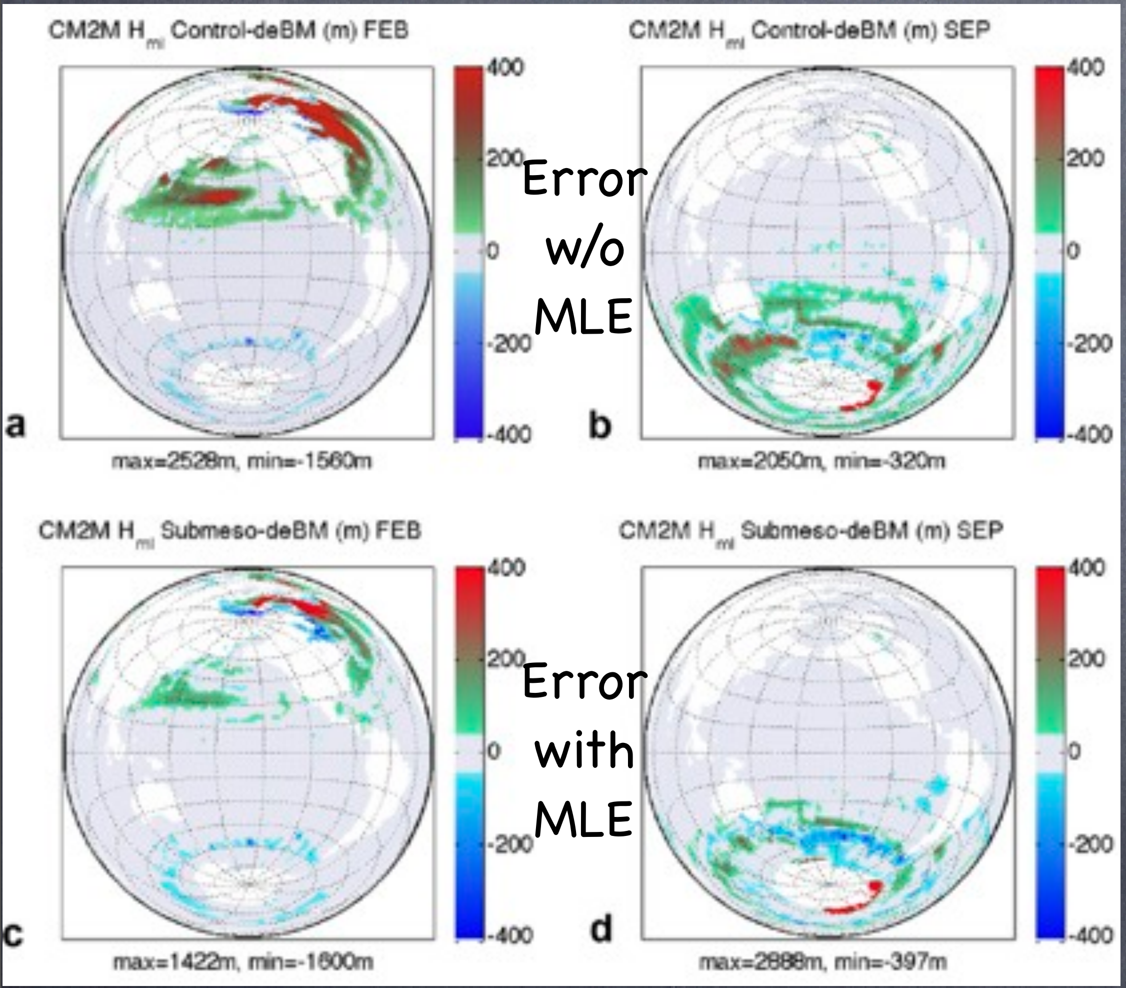
Parameterizations of submesoscale baroclinic instability?

B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. Journal of Physical Oceanography, 38(6):1145-1165, 2008

S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013

Climate affected by (Submeso) Mixed Layer Eddy Restratification

Affects AMOC, Sea Ice, SST, SSS, Air-Sea, etc.



Error w/o MLE

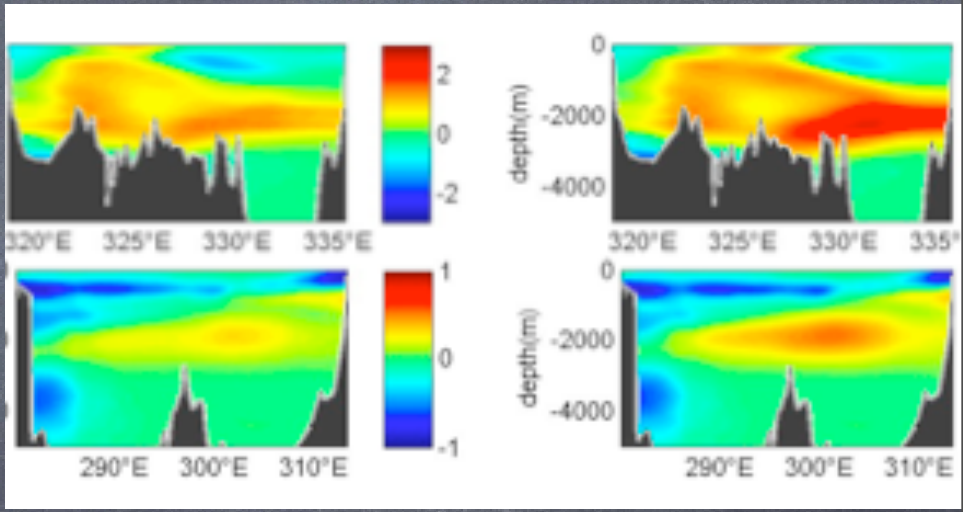
Error with MLE

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

Improves CFCs (water masses)

Bias with MLE

Bias w/o MLE



A consistently restratifying,

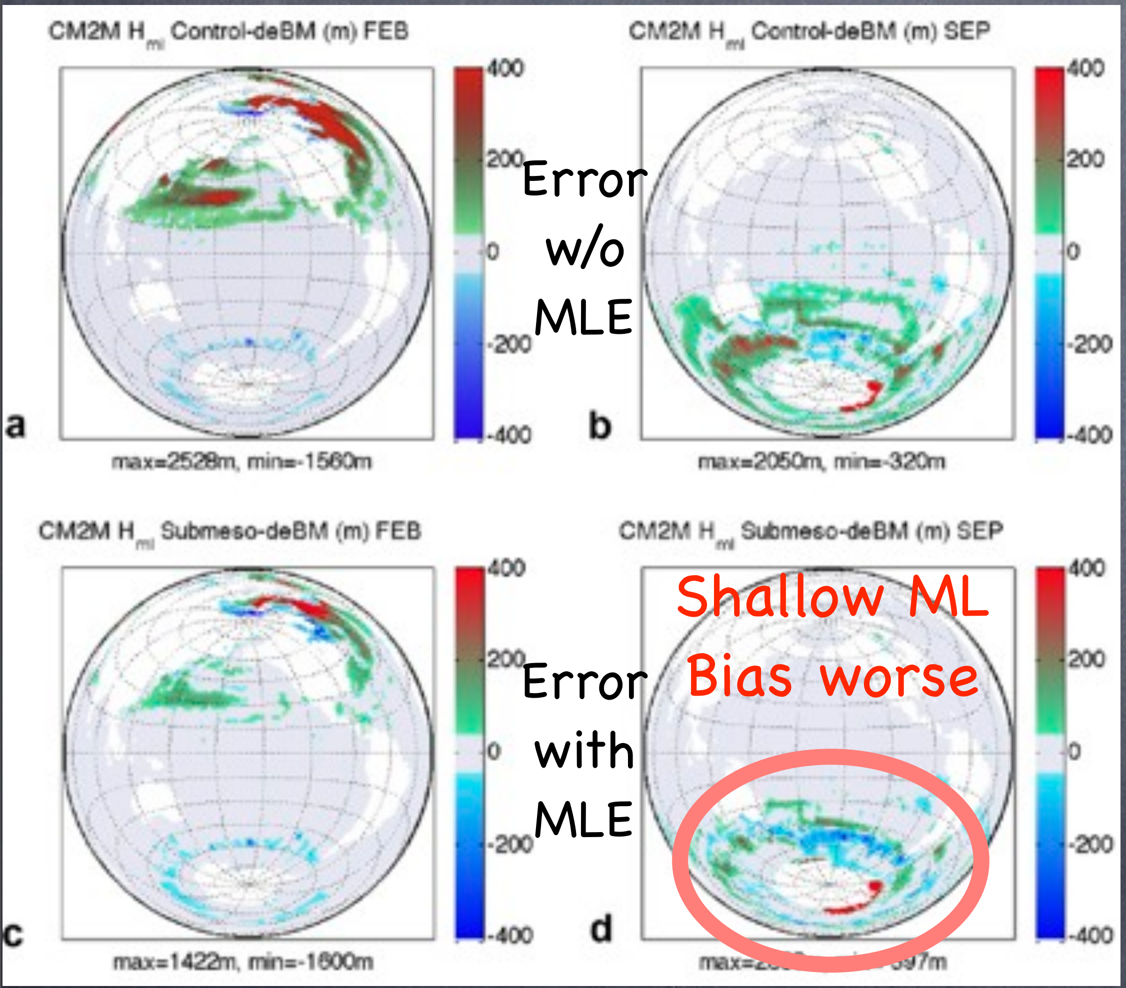
$$\overline{w'b'} \propto \frac{H^2}{|f|}\left|\nabla_H \bar{b}\right|^2$$

and horizontally downgradient flux.

$$\overline{\mathbf{u}'_H b'} \propto \frac{-H^2 \frac{\partial \bar{b}}{\partial z}}{|f|}\nabla_H \bar{b}$$

Climate affected by (Submeso) Mixed Layer Eddy Restratification

Affects AMOC, Sea Ice, SST, SSS, Air-Sea, etc.



Error w/o MLE

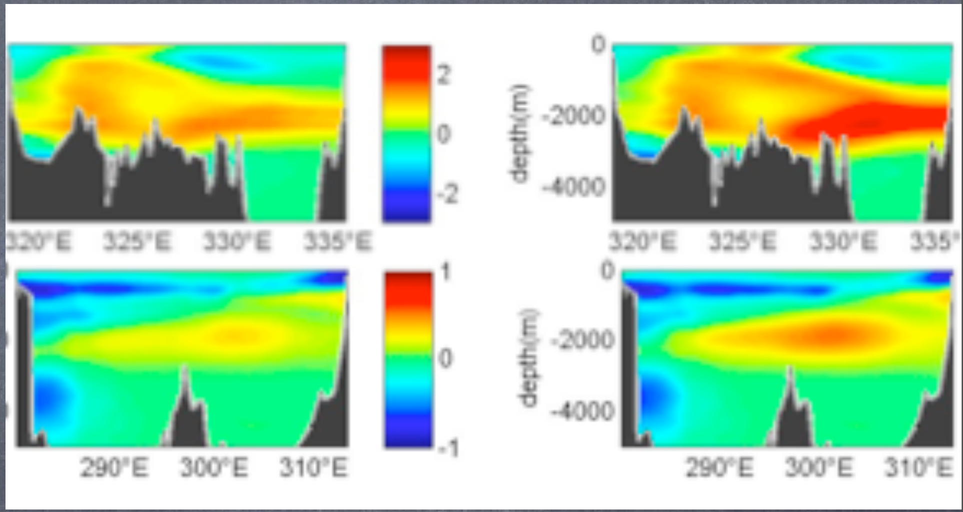
Error with MLE

Shallow ML Bias worse

Improves CFCs (water masses)

Bias with MLE

Bias w/o MLE



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B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

- Method: Study a small-scale phenomenon (100m-10km submeso mixed layer fronts & eddies), parameterize, assess impact globally, and improve climate models
    - In this case, we relied heavily on thermal wind
- But, what about the effects of things that aren't geostrophic & hydrostatic?
    - For example, waves and near-surface 3d turbulence

Surface Waves

Look for fast, small solutions of the Boussinesq Equations:

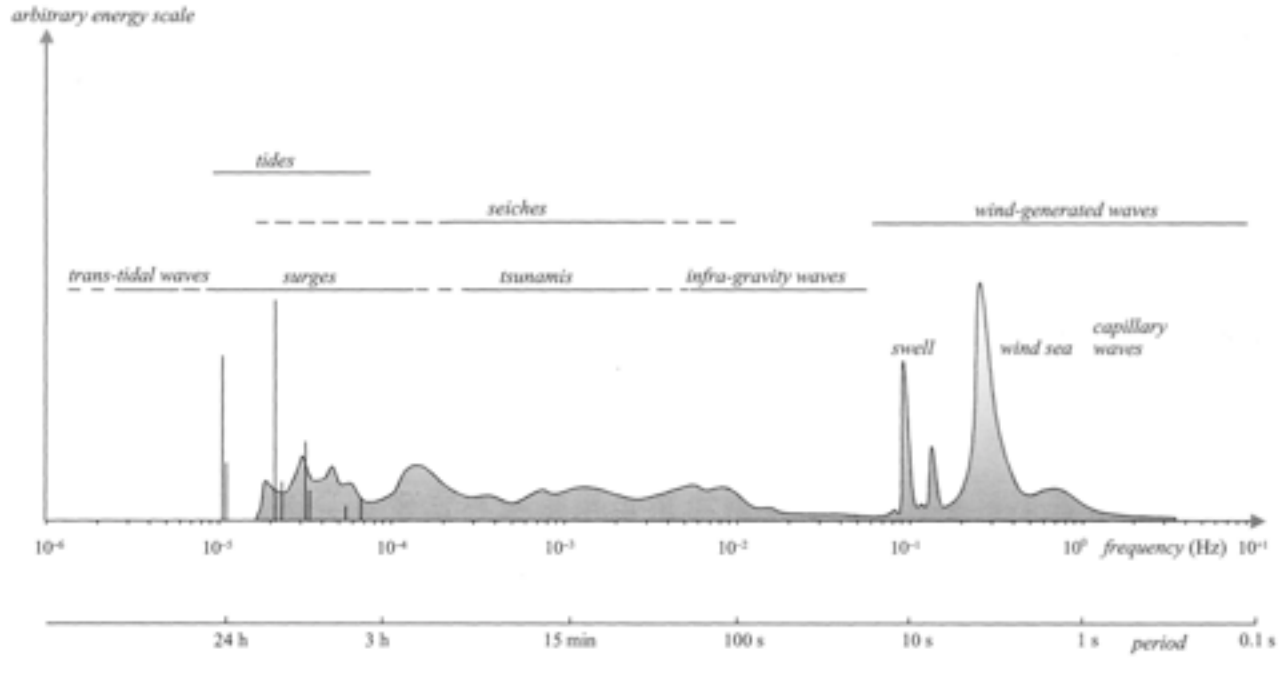


Illustration of wave spectra from different types of ocean surface waves (Holthuijsen, 2007)

The irrotational, incompressible flow obeys

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

$$u = \frac{\partial \phi}{\partial x} \qquad w = \frac{\partial \phi}{\partial z}$$

The boundary conditions are:

Solid Bottom	$w = \frac{\partial \phi}{\partial z} = 0 \quad \text{at} \quad z = -H$
Pressure Matching (dynamic)	$p = 0 \quad \text{at} \quad z = \eta$
Velocity Matching (kinematic)	$\frac{D\eta}{Dt} = w_\eta \quad \text{at} \quad z = \eta$

Surface Waves

Look for fast, small solutions of the Boussinesq Equations:

Linearized for not steep waves

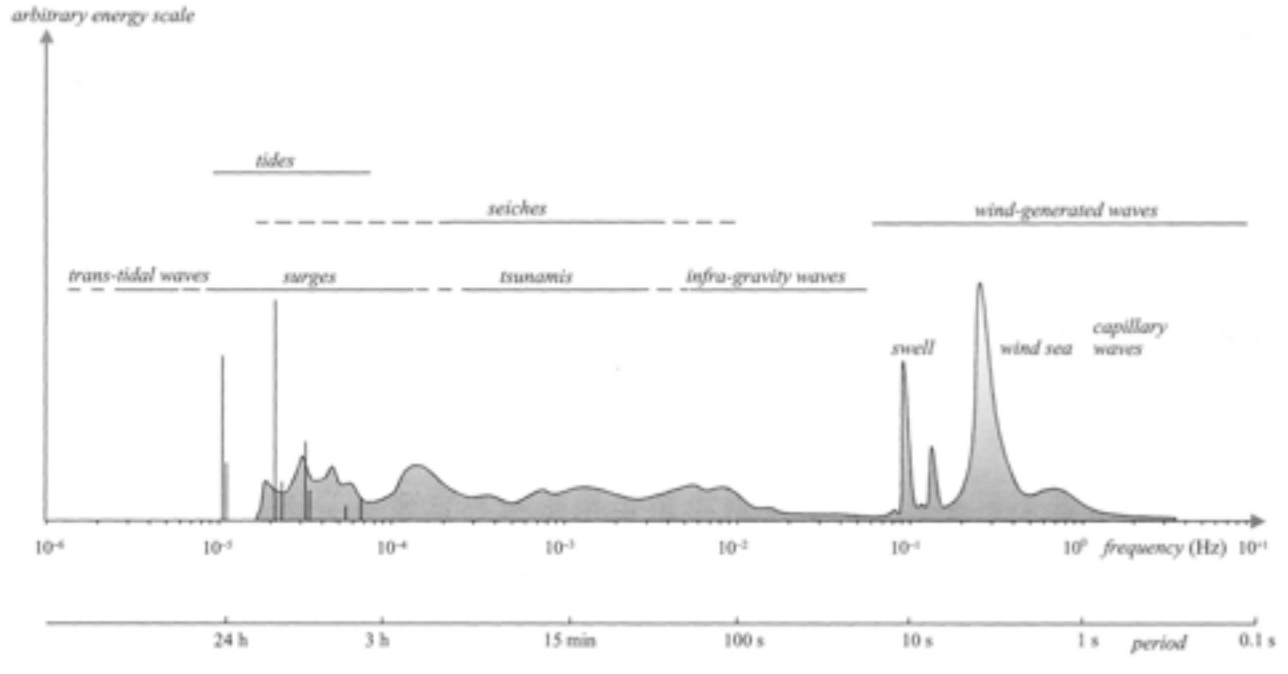


Illustration of wave spectra from different types of ocean surface waves (Holthuijsen, 2007)

The irrotational, incompressible flow obeys

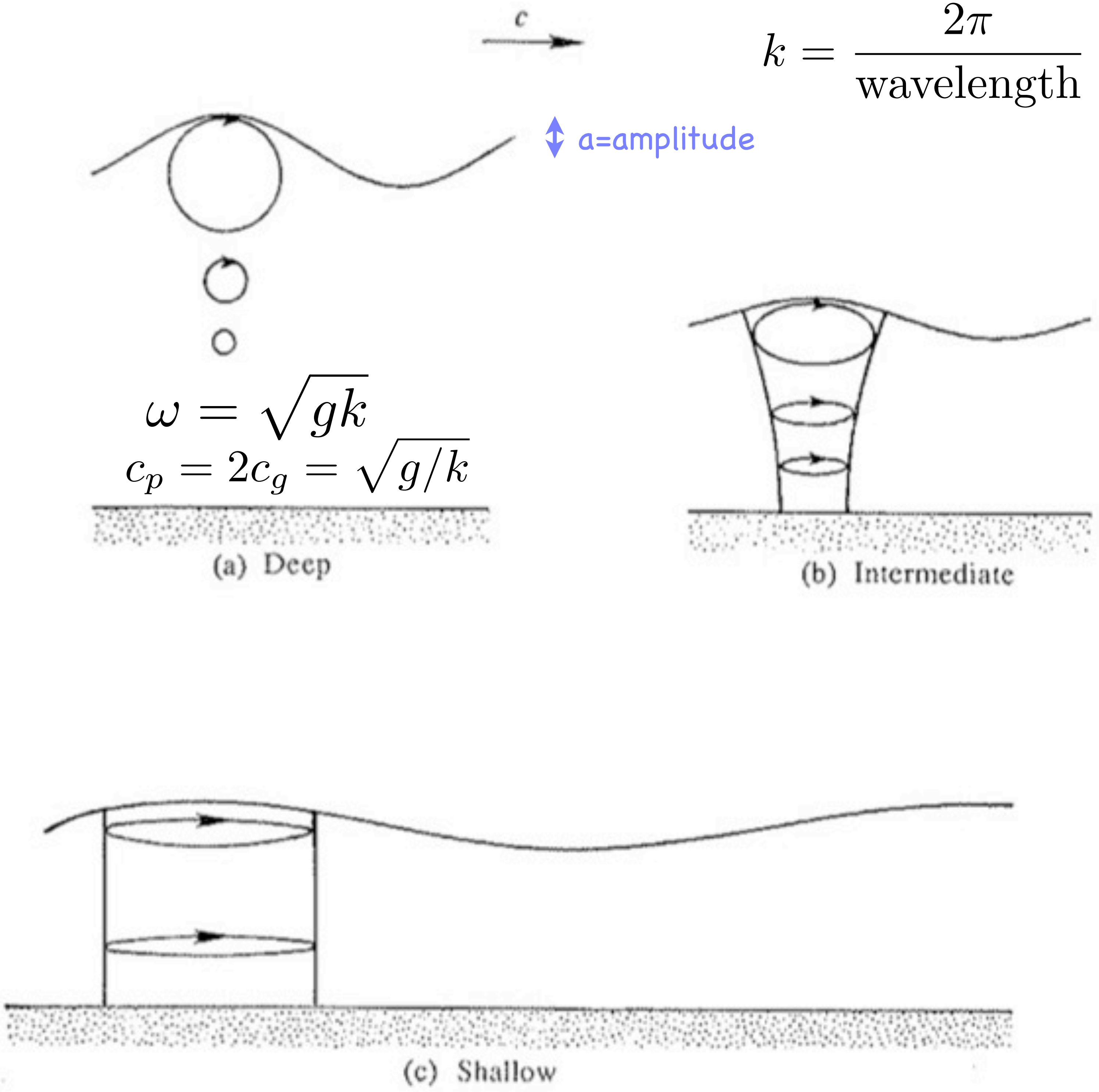
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

$$u = \frac{\partial \phi}{\partial x} \qquad w = \frac{\partial \phi}{\partial z}$$

The boundary conditions are (small steepness):

Solid Bottom	$w = \frac{\partial \phi}{\partial z} = 0$	at $z = -H$
Pressure Matching (dynamic)	$\frac{\partial \phi}{\partial t} = -g\eta$	at $z = 0$
Velocity Matching (kinematic)	$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z}$	at $z = 0$

Particle motions



$$k = \frac{2\pi}{\text{wavelength}}$$

$$\omega = \sqrt{gk}$$

$$c_p = 2c_g = \sqrt{g/k}$$

(a) Deep

(b) Intermediate

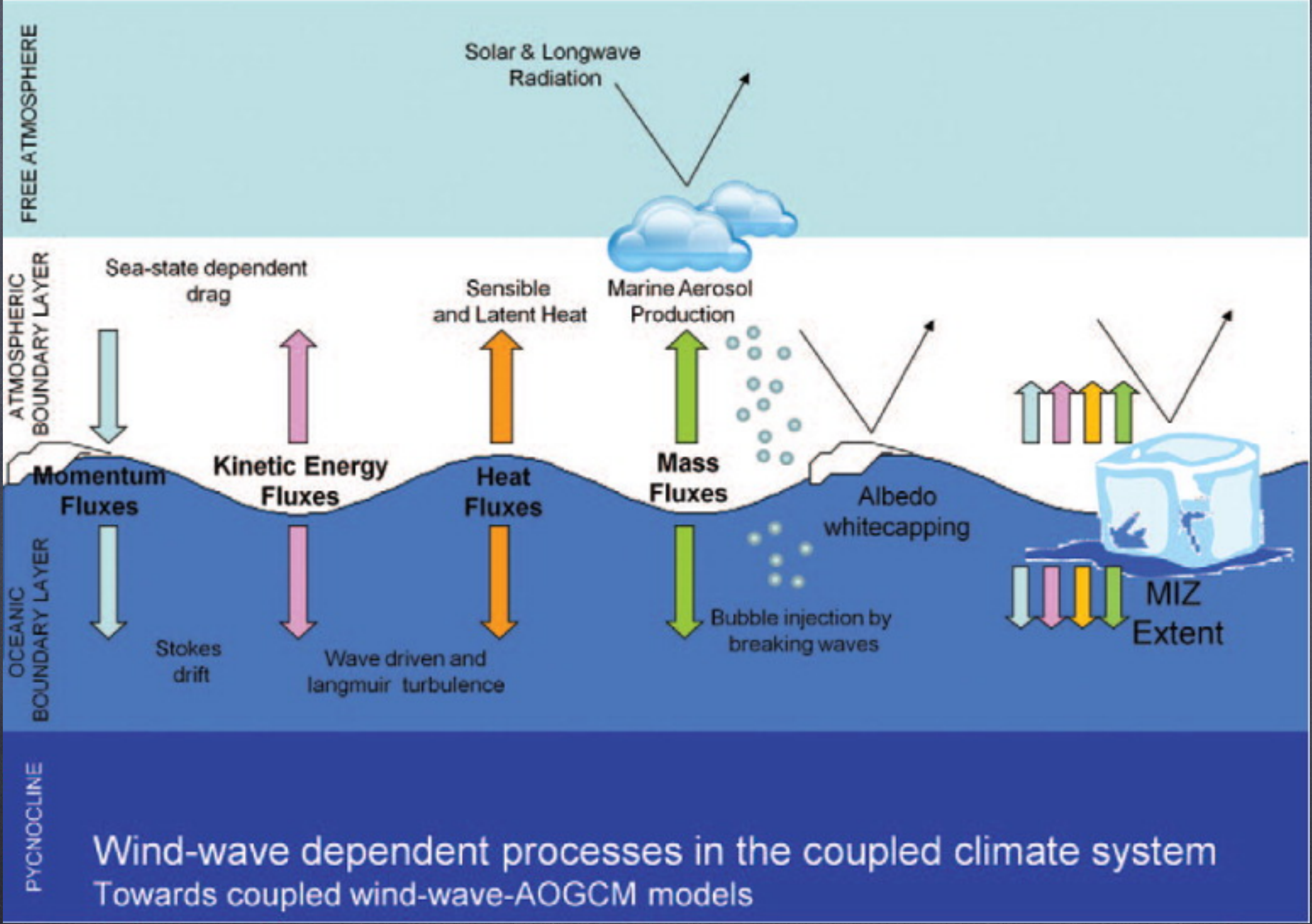
(c) Shallow

The u, v, decay exponentially toward the bottom with decay scale proportional to the wavelength.

Thus, kH is a measure of depth

ka is a measure of steepness

Deep water waves don't "feel" the bottom. Implies nonhydrostatic ($H \approx L$) & fast timescale (Ro>>1)



Wind-wave dependent processes in the coupled climate system

Towards coupled wind-wave-AOGCM models

L. Cavaleri, B. Fox-Kemper, and M. Hemer. Wind waves in the coupled climate system. Bulletin of the American Meteorological Society, 93(11):1651-1661, 2012.

The Character of the Langmuir Turbulence

image:
Thorpe, 04

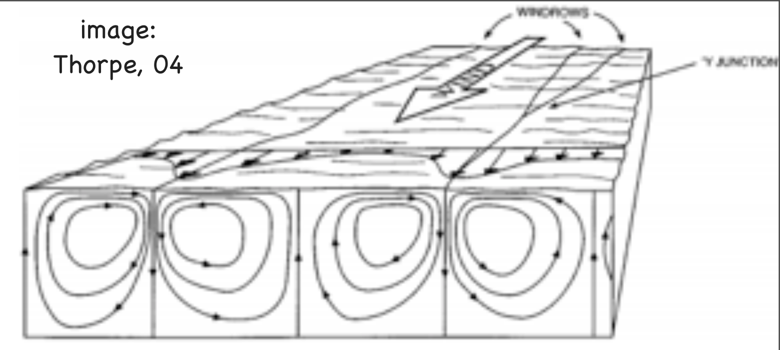


Figure 1 Sketch showing the pattern of mean flow in idealized Langmuir circulation. Th
windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). I
practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2
amalgamate and meander in space and time. Bands of bubbles or buoyant algae may for
within the downward-going (or downwelling) flow (see Figure 3).

- Near-surface
- Langmuir Cells & Langmuir Turb.
- Ro>>1
- Ri<1: Nonhydro
- 1-10m
- 10s to mins
- w, u=O(10cm/s)
- Stokes drift
- Eqtns:Craik-Leibovich
- Params: McWilliams & Sullivan, 2000, etc.

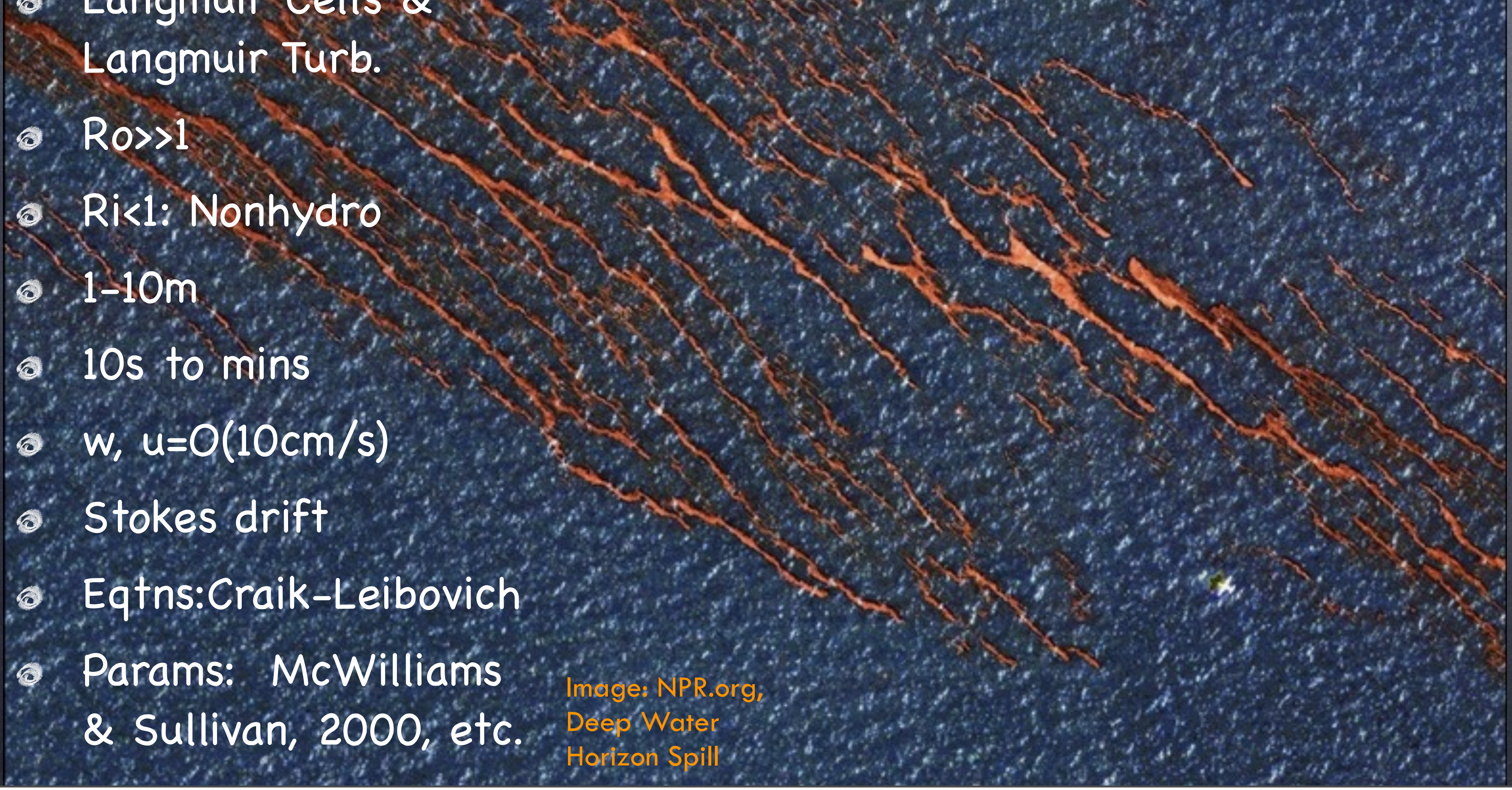
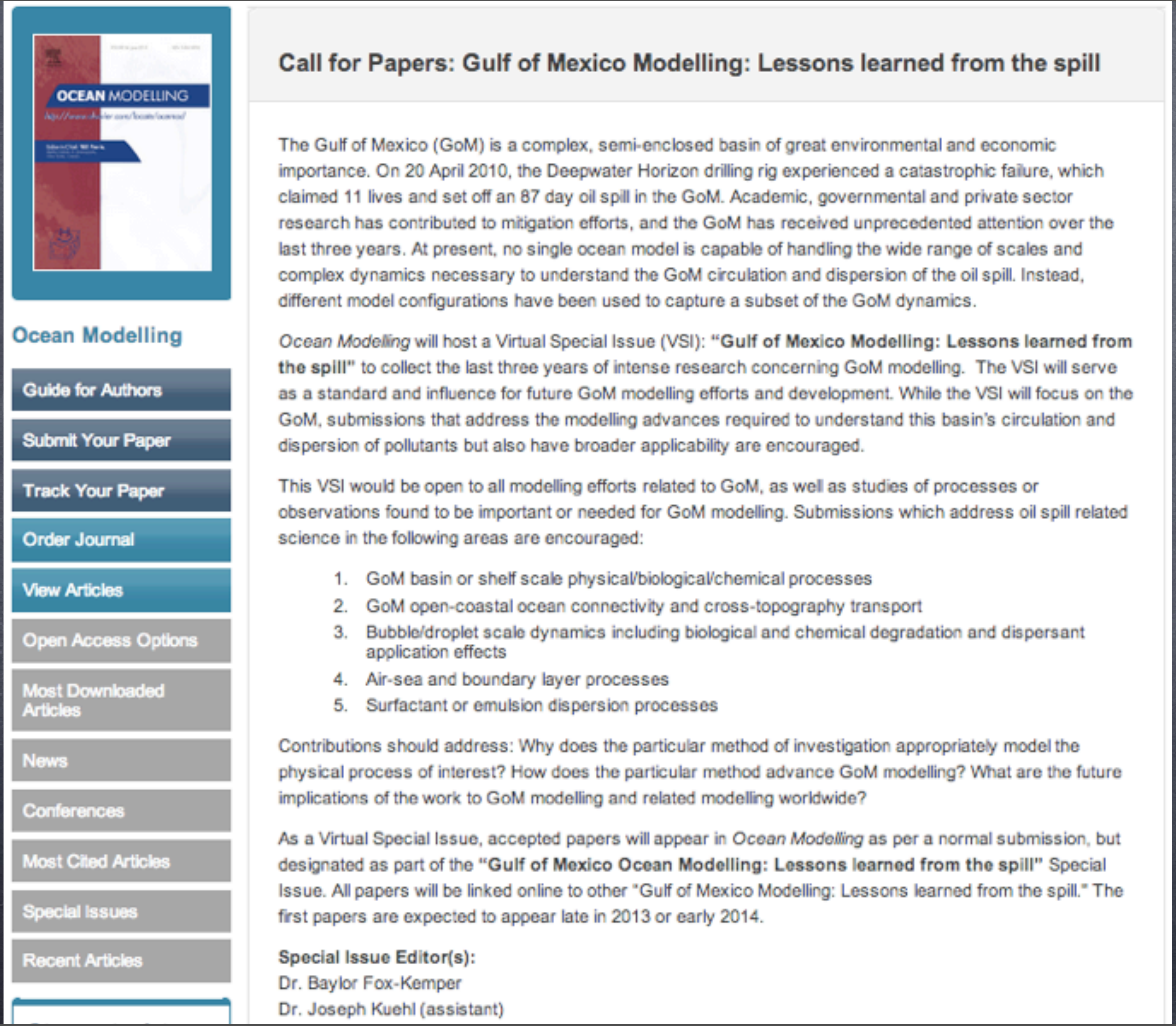


Image: NPR.org,
Deep Water
Horizon Spill



Call for Papers: Gulf of Mexico Modelling: Lessons learned from the spill

The Gulf of Mexico (GoM) is a complex, semi-enclosed basin of great environmental and economic importance. On 20 April 2010, the Deepwater Horizon drilling rig experienced a catastrophic failure, which claimed 11 lives and set off an 87 day oil spill in the GoM. Academic, governmental and private sector research has contributed to mitigation efforts, and the GoM has received unprecedented attention over the last three years. At present, no single ocean model is capable of handling the wide range of scales and complex dynamics necessary to understand the GoM circulation and dispersion of the oil spill. Instead, different model configurations have been used to capture a subset of the GoM dynamics.

Ocean Modelling will host a Virtual Special Issue (VSI): **"Gulf of Mexico Modelling: Lessons learned from the spill"** to collect the last three years of intense research concerning GoM modelling. The VSI will serve as a standard and influence for future GoM modelling efforts and development. While the VSI will focus on the GoM, submissions that address the modelling advances required to understand this basin's circulation and dispersion of pollutants but also have broader applicability are encouraged.

This VSI would be open to all modelling efforts related to GoM, as well as studies of processes or observations found to be important or needed for GoM modelling. Submissions which address oil spill related science in the following areas are encouraged:

1. GoM basin or shelf scale physical/biological/chemical processes
2. GoM open-coastal ocean connectivity and cross-topography transport
3. Bubble/droplet scale dynamics including biological and chemical degradation and dispersant application effects
4. Air-sea and boundary layer processes
5. Surfactant or emulsion dispersion processes

Contributions should address: Why does the particular method of investigation appropriately model the physical process of interest? How does the particular method advance GoM modelling? What are the future implications of the work to GoM modelling and related modelling worldwide?

As a Virtual Special Issue, accepted papers will appear in *Ocean Modelling* as per a normal submission, but designated as part of the **"Gulf of Mexico Ocean Modelling: Lessons learned from the spill"** Special Issue. All papers will be linked online to other "Gulf of Mexico Modelling: Lessons learned from the spill." The first papers are expected to appear late in 2013 or early 2014.

Special Issue Editor(s):
Dr. Baylor Fox-Kemper
Dr. Joseph Kuehl (assistant)

Craik-Leibovich Boussinesq

- Formally a multiscale asymptotic equation set:
  - 3 classes: Small, Fast; Large, Fast; Large, Slow
  - Solve first 2 types of motion in the case of limited slope (ka), irrotational --> Deep Water Waves!
  - Must also assume slowly-varying wave packets
  - Average over deep water waves in space & time,
  - Arrive at Large, Slow equation set:

$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times (\mathbf{v} + \mathbf{v}_s) = -\nabla \pi^\dagger + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$

$$\frac{\partial b}{\partial t} + (\mathbf{v} + \mathbf{v}_s) \cdot \nabla b = 0 \qquad \nabla \cdot \mathbf{v} = 0$$

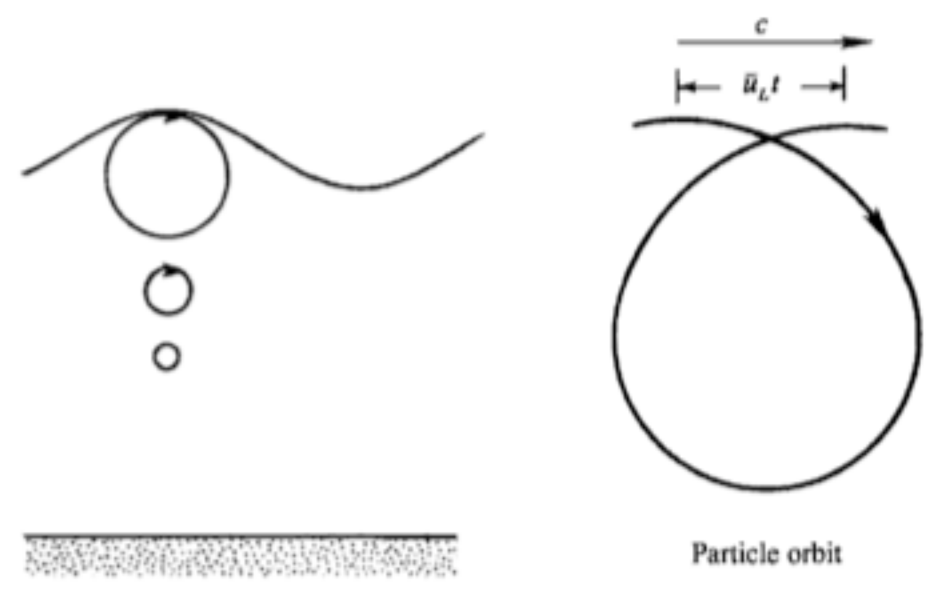
$$\mathbf{v}_s = \text{Stokes Drift}$$

Craik & Leibovich 1976; Gjaja & Holm 1996; McWilliams *et al.* 2004

What is Stokes Drift?

Take wave solns, compare the velocity of trajectories vs. Eulerian velocity, Taylor Expand, calculate:

$$
\begin{aligned}
\boldsymbol{u}^L(\boldsymbol{x}_p(t_0),t)-\boldsymbol{u}^E(\boldsymbol{x}_p(t_0),t) &\approx [\boldsymbol{x}_p(t)-\boldsymbol{x}_p(t_0)]\cdot\nabla\boldsymbol{u}^E(\boldsymbol{x}_p(t_0),t)\\
&\approx \left[\int_{t_0}^{t}\boldsymbol{u}^E(\boldsymbol{x}_p(t_0),s')ds'\right]\cdot\nabla\boldsymbol{u}^E(\boldsymbol{x}_p(t_0),t)\,.
\end{aligned}
$$



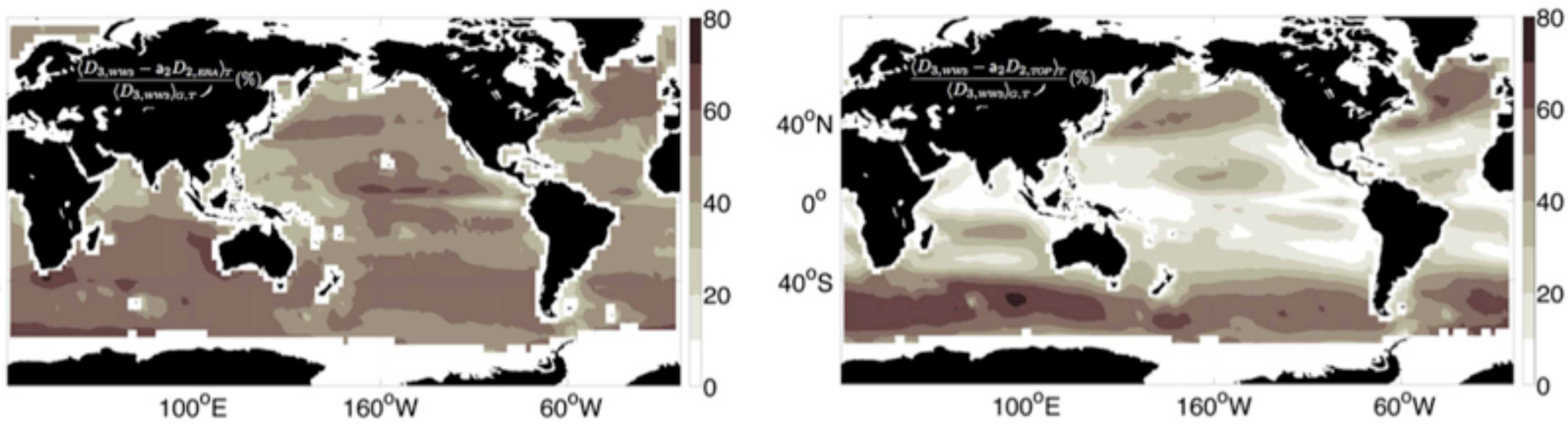
Examples:

Monochromatic: $$\boldsymbol{u}^S=\hat{\mathbf{e}}^w\frac{8\pi^3a^2f_p^3}{g}e^{\frac{8\pi^2f_p^2}{g}z}=\hat{\mathbf{e}}^w a^2\sqrt{gk^3}e^{2kz}.$$

Spectrum: $$\boldsymbol{u}^S=\frac{16\pi^3}{g}\int_0^\infty\int_{-\pi}^{\pi}(\cos\theta,\sin\theta,0)f^3S_{f\theta}(f,\theta)e^{\frac{8\pi^2f^2}{g}z}d\theta df.$$

A. Webb and B. Fox-Kemper. Wave spectral moments and Stokes drift estimation. *Ocean Modelling*, 40(3-4):273-288, 2011.

How well do we know Stokes Drift? <50% discrepancy



(e) $\langle D_{3,WW3} - a_2 D_{2,ERA}\rangle_T / \langle D_{3,WW3}\rangle_{G,T}$ (%)

(f) $\langle D_{3,WW3} - a_2 D_{2,TOP}\rangle_T / \langle D_{3,WW3}\rangle_{G,T}$ (%)

RMS error in measures of surface Stokes drift, 2 wave models (left), model vs. altimeter (right)

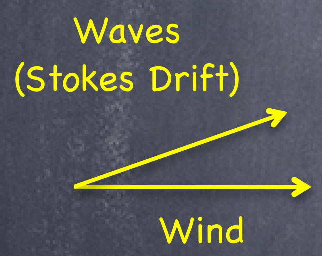
Year 2000 data & models

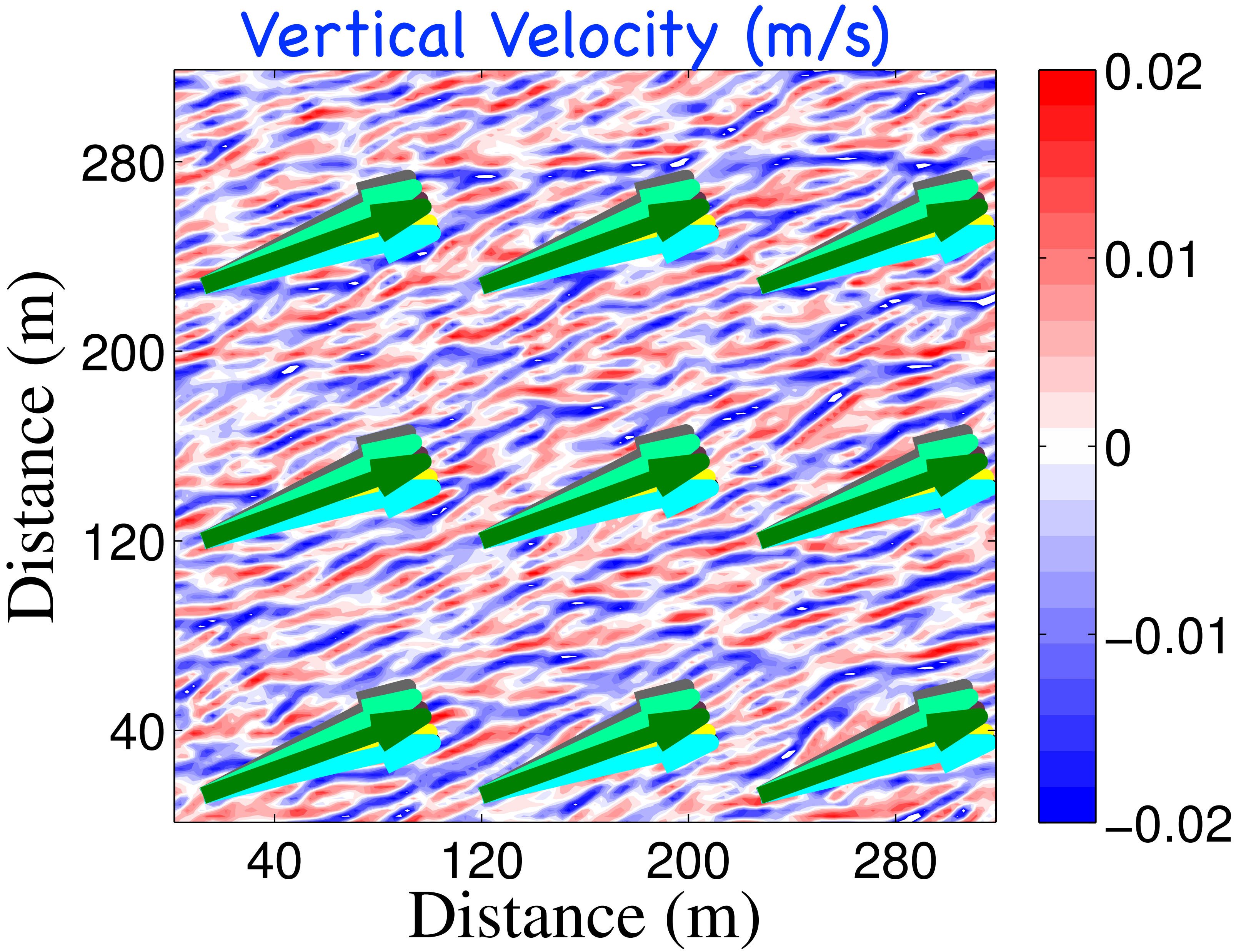
A. Webb and B. Fox-Kemper. Wave spectral moments and Stokes drift estimation. *Ocean Modelling*, 40(3-4):273-288, 2011.

Now, we've got the CLB equations & estimated global Stokes, what to do?

- 1) Stokes-driven small-scale turbulence (Large Eddy Simulations of CLB)
- 2) Laminar submesoscale flow with Stokes Coriolis & Stokes Vortex forces (Analytic Solns of CLB)
- 3) Wave-driven turbulence interacting with submesoscale flow (Multiscale LES of CLB)

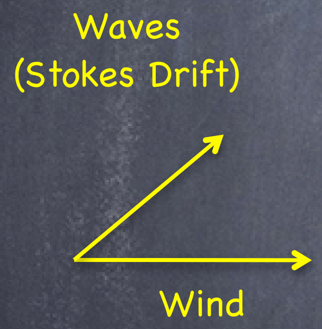
CLB as equations for Large Eddy Simulations: Tricky: Misaligned Wind & Waves





L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. Journal of Geophysical Research-Oceans, 117:C05001, 22pp, May 2012.

Tricky: Misaligned Wind & Waves



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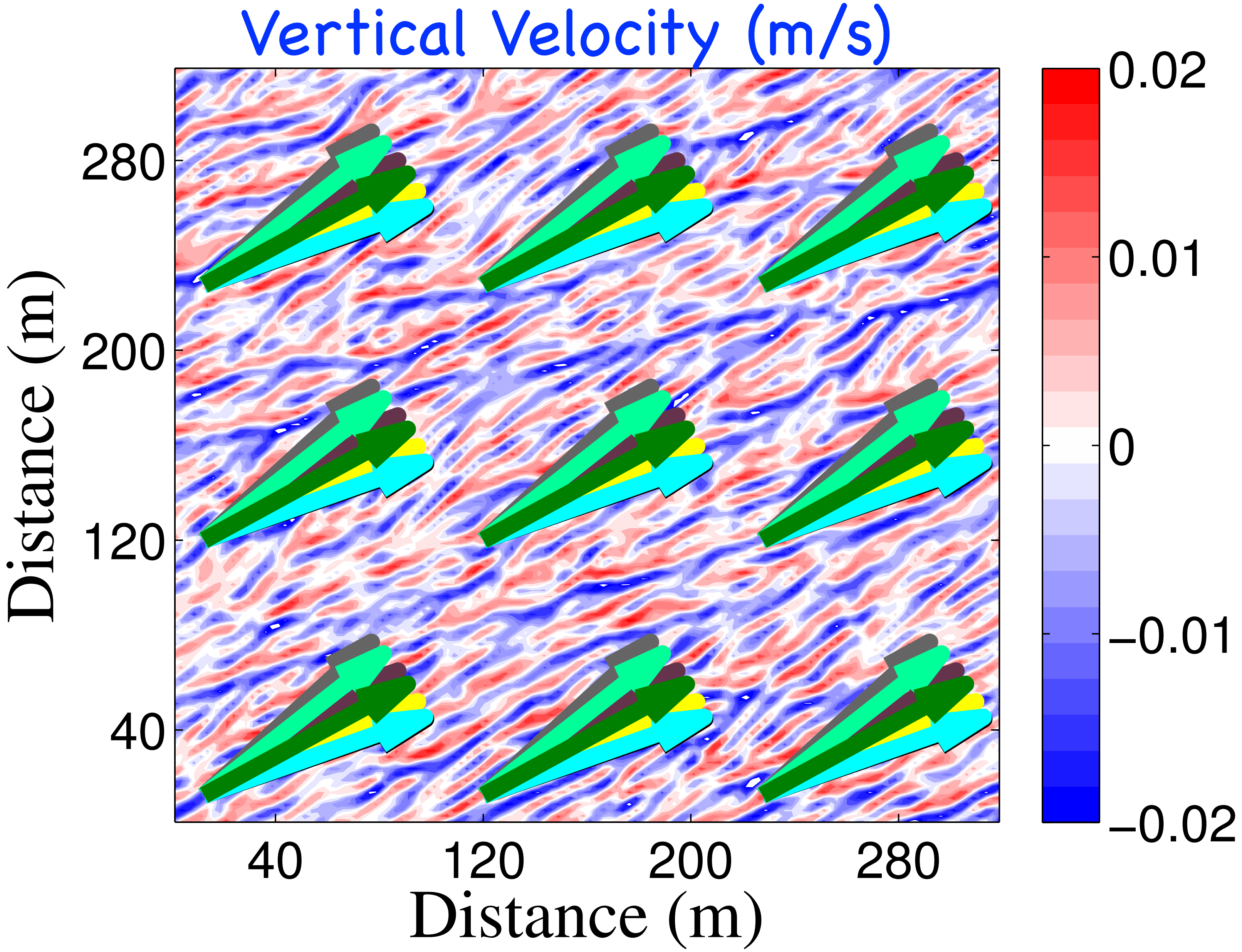

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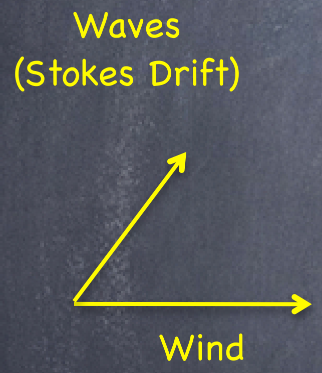

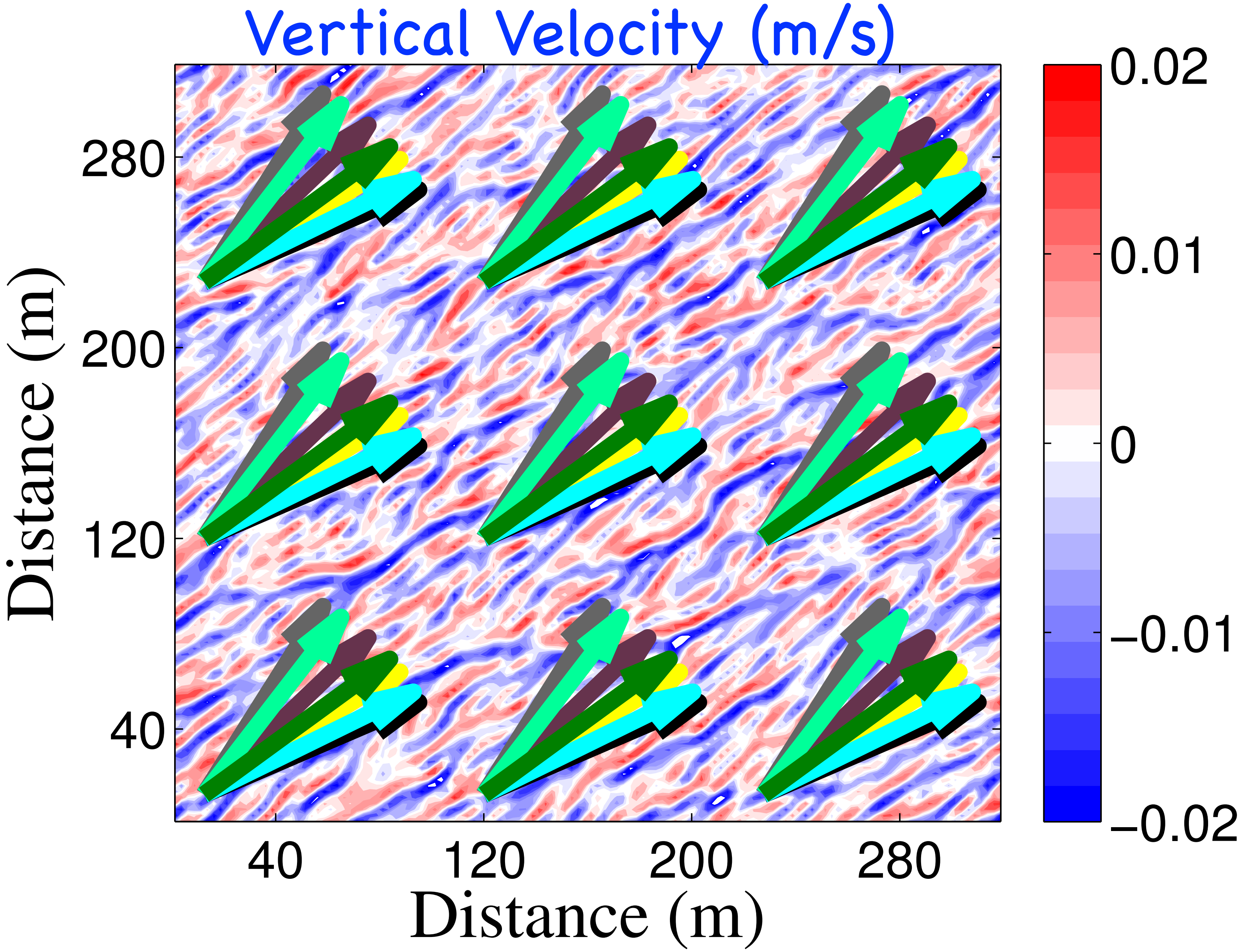

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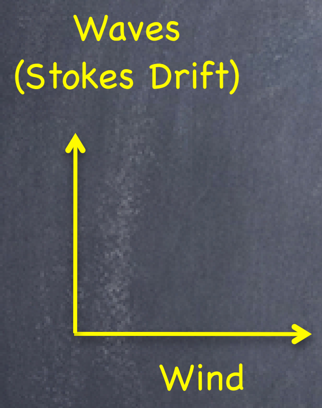

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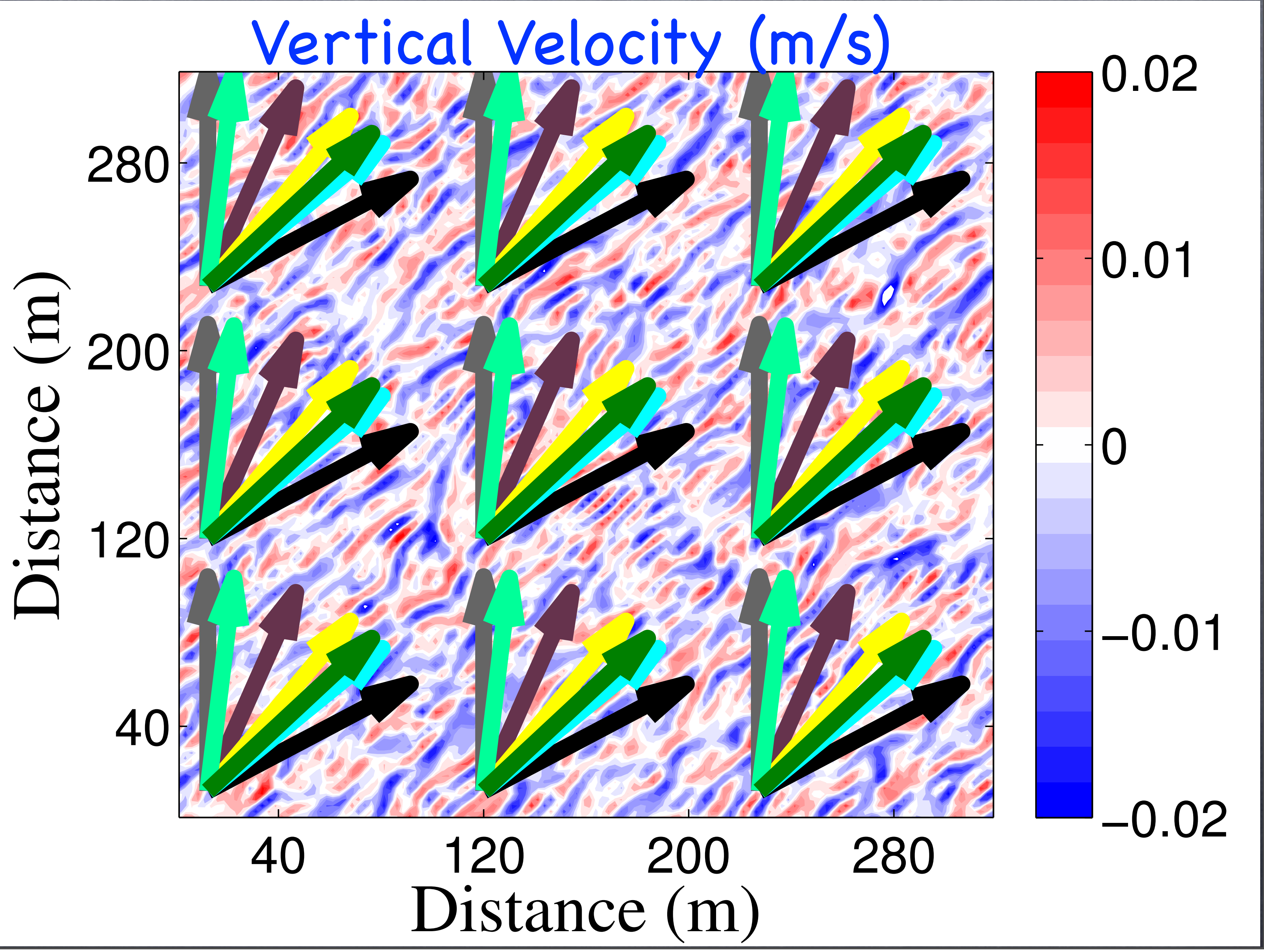


Why? Vortex Tilting Mechanism

In CLB: Tilting occurs in direction of Lagrangian shear : $\mathbf{u}_L = \mathbf{v} + \mathbf{v}_s$

$$\frac{\partial \xi}{\partial t} + \underbrace{(\mathbf{u}_L \cdot \nabla)\xi}_{\text{AD}} = \underbrace{(\omega_a \cdot \nabla)(\mathbf{u}_L \cdot \hat{\mathbf{x}}')}_{\text{TS}} + \underbrace{(\nabla b \times \hat{\mathbf{z}}) \cdot \hat{\mathbf{x}}'}_{\text{BV}} + \text{SGS},$$

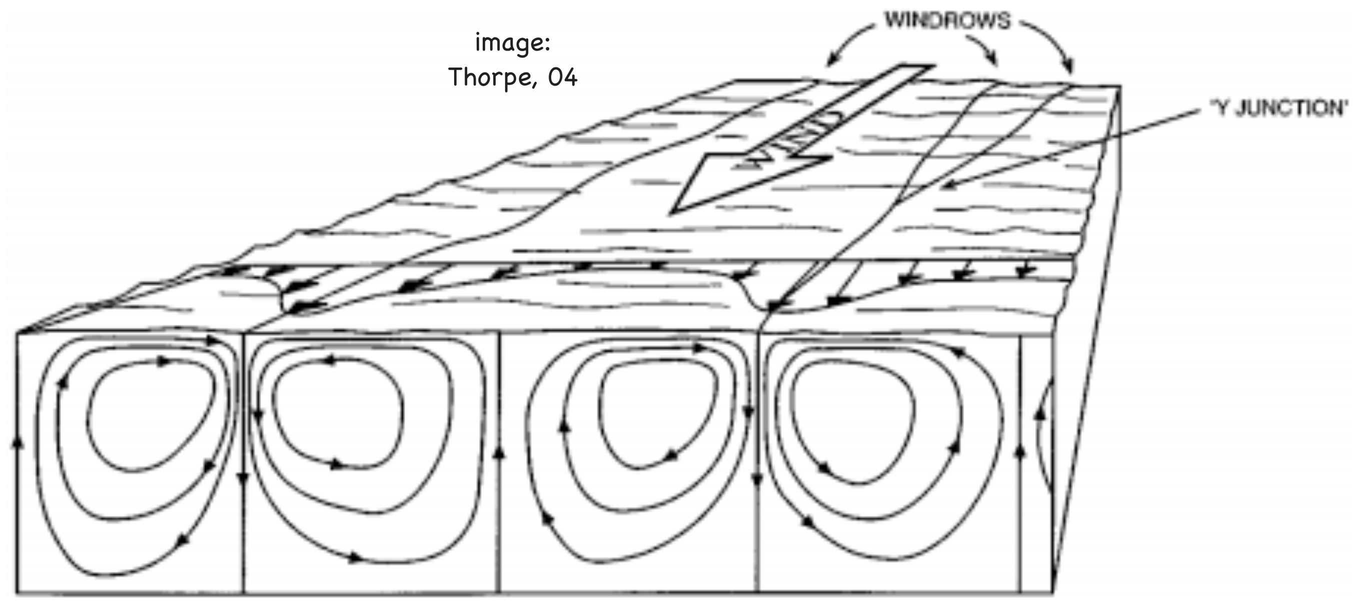


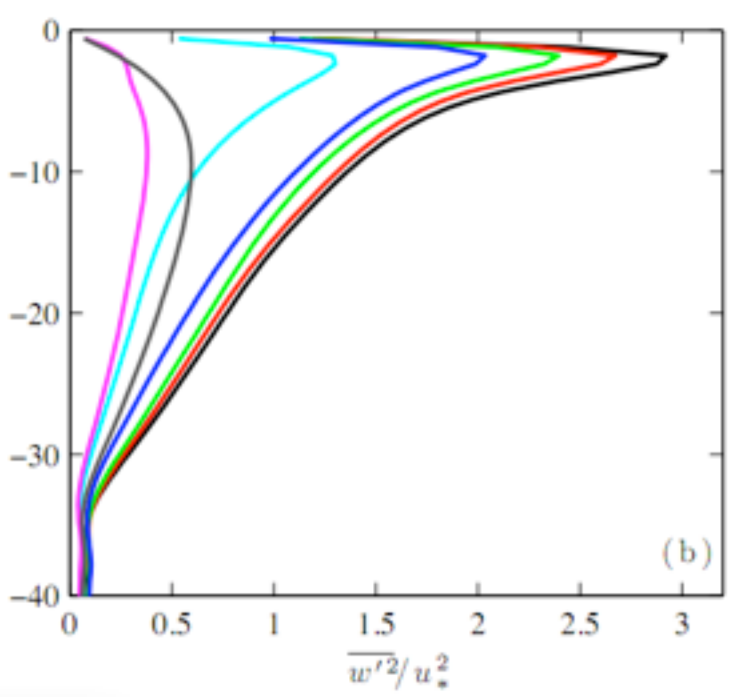
image:
Thorpe, 04

Figure 1 Sketch showing the pattern of mean flow in idealized Langmuir circulation. The windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). In practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2) amalgamate and meander in space and time. Bands of bubbles or buoyant algae may form within the downward-going (or downwelling) flow (see Figure 3).

Generalized Parameters: Predict & Project into Lagrangian Shear Direction

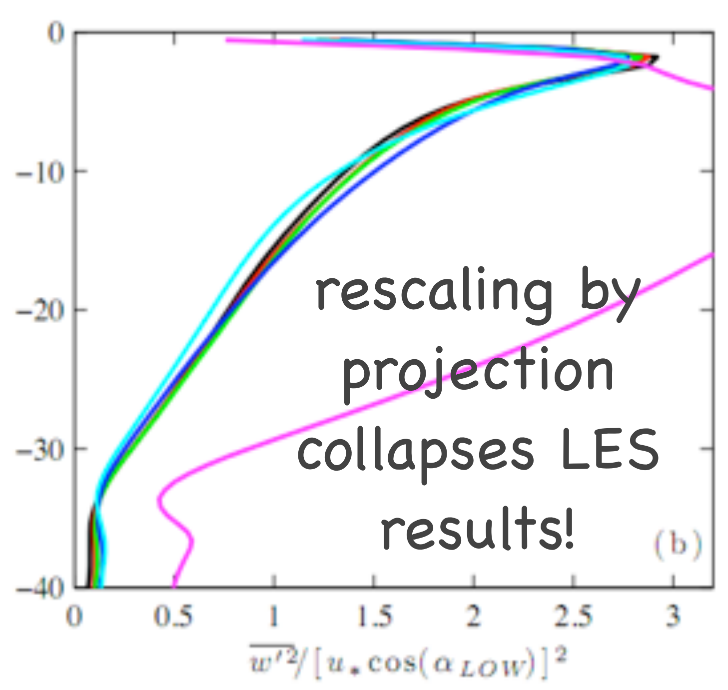
<w^2>

depth



rescaled <w^2>

depth



rescaling by projection collapses LES results!

$$\frac{\left\langle \overline{w'^2}\right\rangle_{ML}}{u_*^2} = 0.6\cos^2\left(\alpha_{LOW}\right)\left[1.0+\left(3.1La_{proj}\right)^{-2} + \left(5.4La_{proj}\right)^{-4}\right],$$

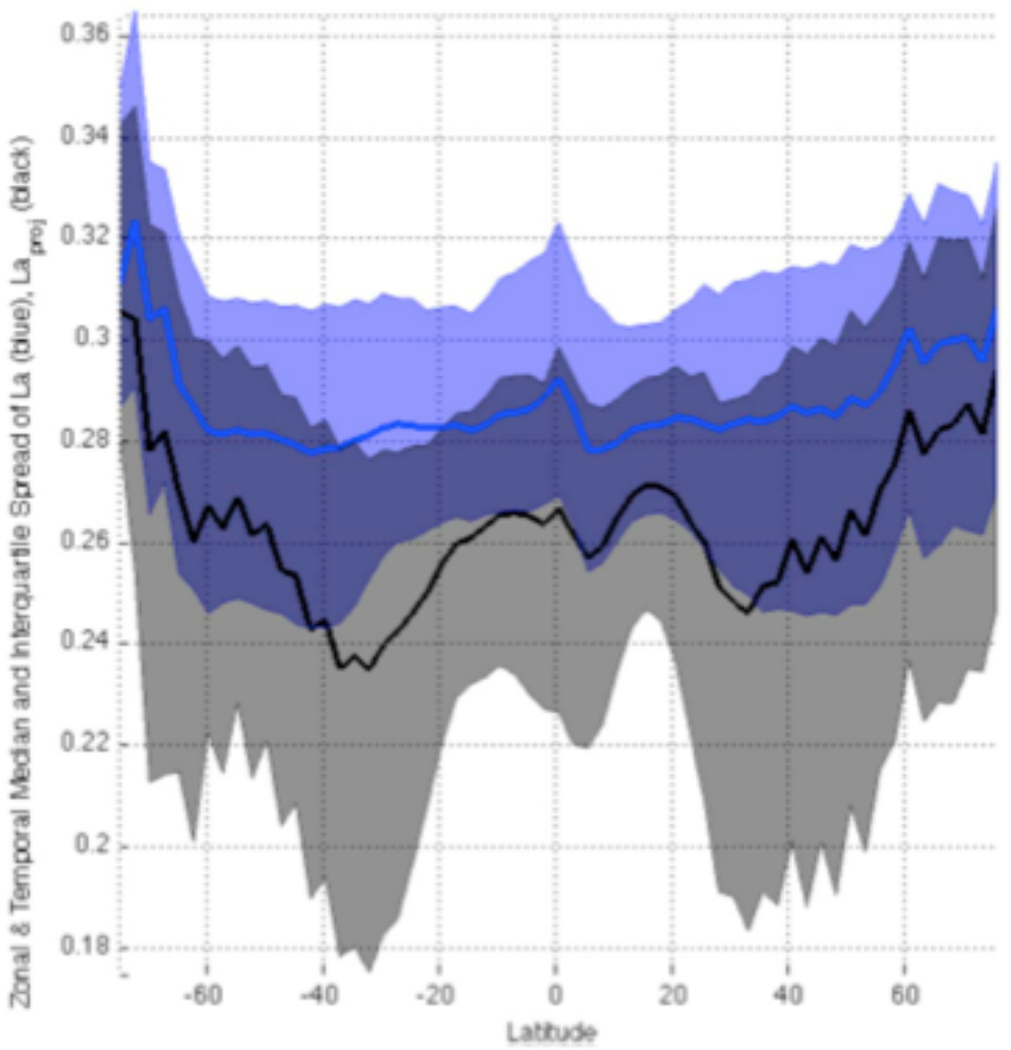
$$La_{proj}^2 = \frac{|u_*|\cos(\alpha_{LOW})}{|u_s|\cos(\theta_{ww}-\alpha_{LOW})},$$

$$\alpha_{LOW} \approx \tan^{-1}\left(\frac{\sin\left(\theta_{ww}\right)}{\frac{u_*}{u_s(0)\kappa}\ln\left(\left|\frac{H_{ML}}{z_1}\right|\right)+\cos\left(\theta_{ww}\right)}\right)$$

= parameterization for LC strength!

L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. Journal of Geophysical Research-Oceans, 117:C05001, 22pp, 2012.

Global Picture: Misalignment enhances degree to which we expect wave-driven turbulence in Boundary layer

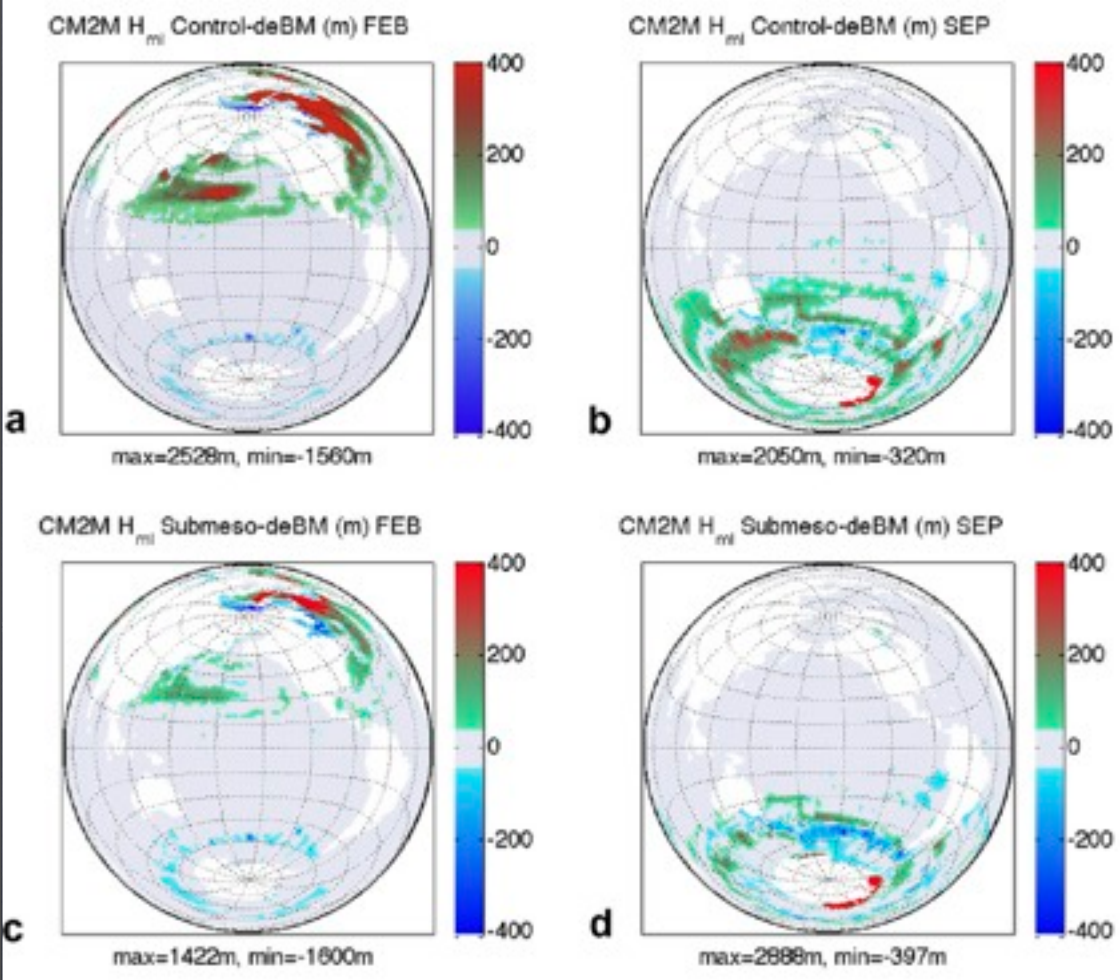


Wind-Driven

Wave-Driven

Figure 17. Temporal and zonal median and interquartile range of La_t and La_{proj} for a realistic simulation of 1994–2002 using Wave Watch III.

Recall our problem with the (submeso) Mixed Layer Eddy Restratification--Southern Ocean too shallow!



Bias
w/o
MLE

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

Sallee et al. (2013) have shown that a too shallow S. Ocean MLD is true of most* present climate models

salinity forcing or ocean physics?

*true for CMIP5 multi-model ensemble

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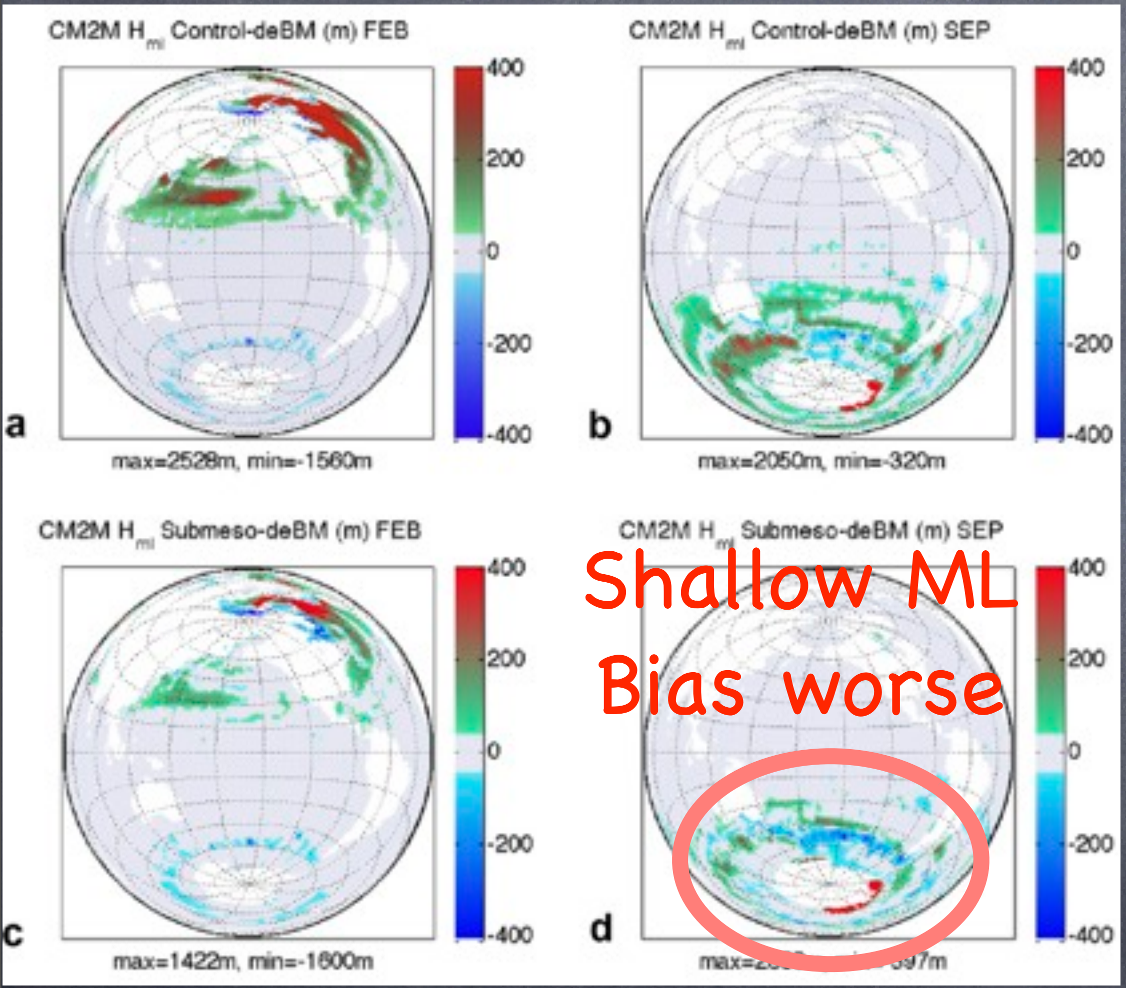

Shallow ML Bias worse

Bias w/o MLE

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

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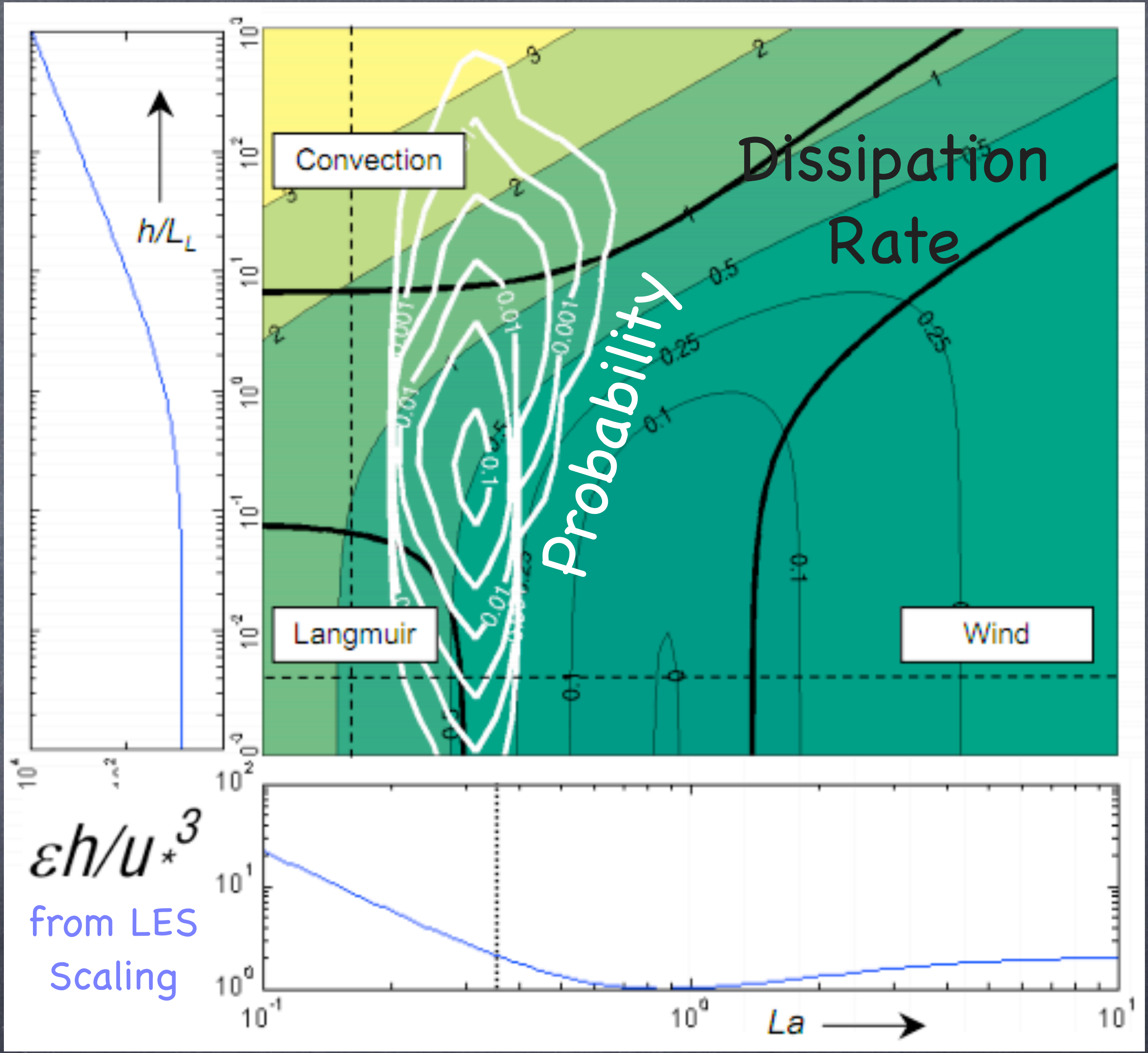
Data + LES, Southern Ocean mixing energy: Langmuir (Stokes-drift-driven) and Convective

So, waves can drive mixing via Stokes drift (combines with cooling & winds)



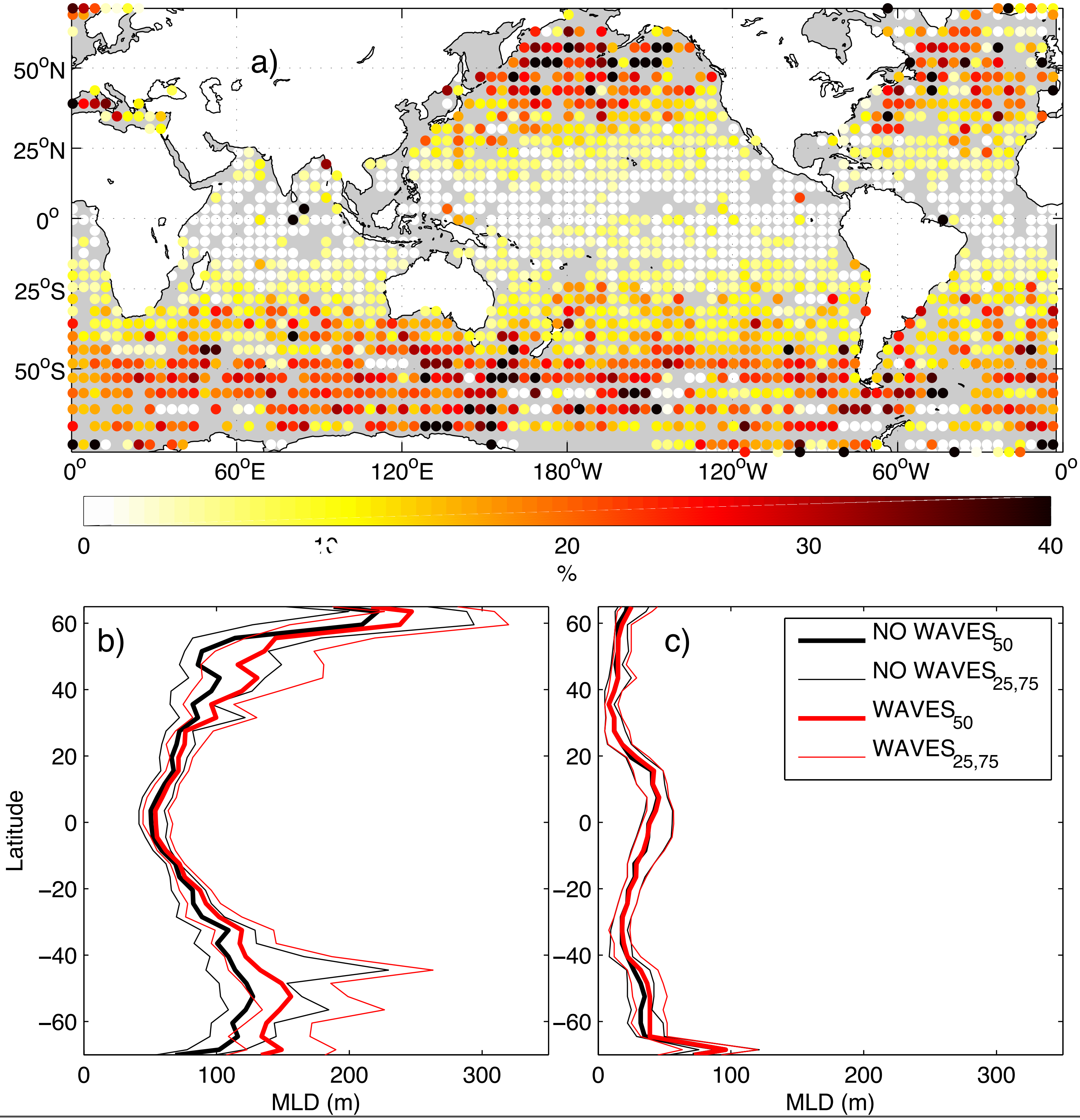
S. E. Belcher, A. A. L. M. Grant, K. E. Hanley, B. Fox-Kemper, L. Van Roekel, P. P. Sullivan, W. G. Large, A. Brown, A. Hines, D. Calvert, A. Rutgersson, H. Petterson, J. Bidlot, P. A. E. M. Janssen, and J. A. Polton. A global perspective on Langmuir turbulence in the ocean surface boundary layer. Geophysical Research Letters, 39(18):L18605, 9pp, 2012.

Including Wave-driven Mixing

(Harcourt 2013 parameterization, includes misalignment & nonequiliubrium waves)

Deepens the Mixed Layer!

M. A. Hemer, B. Fox-Kemper, & R. R. Harcourt. Quantifying the effects of wind waves the the coupled climate system, in prep. 2013.



So, Waves can Drive turbulence that affects large scale:

What about direct effects of waves on larger scales?
Stokes Coriolis & Stokes Vortex Forces on Submesoscales

Recall, Subinertial Boussinesq Equations Dominated by:

(Combined) Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b$$

So, Waves can Drive turbulence that affects large scale:

What about direct effects of waves on larger scales?
Stokes Coriolis & Stokes Vortex Forces on Submesoscales

Craik-Leibovich Boussinesq Subinertial Dominated By:

(Combined) Lagrangian Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial}{\partial z}(\mathbf{v} + \mathbf{v}_s) = \mathbf{f} \times \frac{\partial \mathbf{v}_L}{\partial z} = -\nabla b$$

Now the temperature gradients govern the Lagrangian flow, not the not the Eulerian!

J. C. McWilliams and B. Fox-Kemper. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

Craik-Leibovich Boussinesq Subinertial Dominated By:

(Combined) Lagrangian Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial \mathbf{v}_L}{\partial z} = -\nabla b$$

Now the temperature gradients govern the Lagrangian flow, not the not the Eulerian!

Buoyancy & PV also advected by Lagrangian Flow!

All GFD is for the Lagrangian Flow??

J. C. McWilliams and B. Fox-Kemper. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

Can we just forget the whole thing and interpret large scales as Lagrangian velocities?

$$[\mathbf{f} + \nabla \times \mathbf{v}] \times \frac{\partial}{\partial z}(\mathbf{v} + \mathbf{v}_s) = -\nabla b$$

Not quite, because Vortex Forces are different!

The “Ro” for waves, is big *more often* than Ro is, especially for wide, shallow currents in a mixed layer

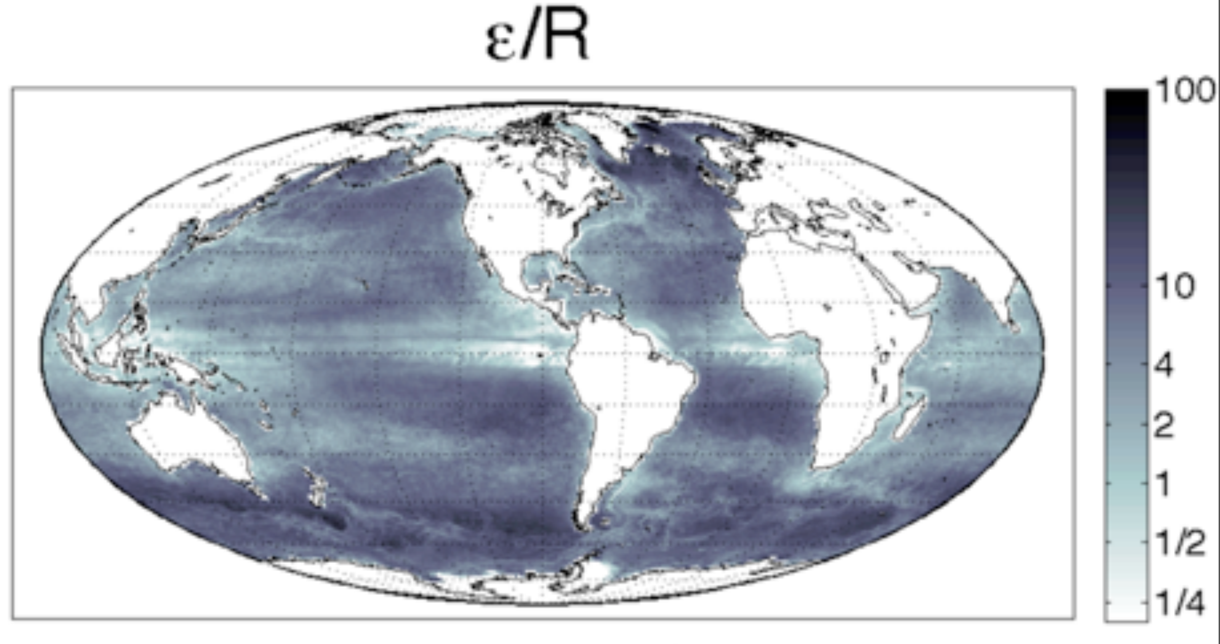
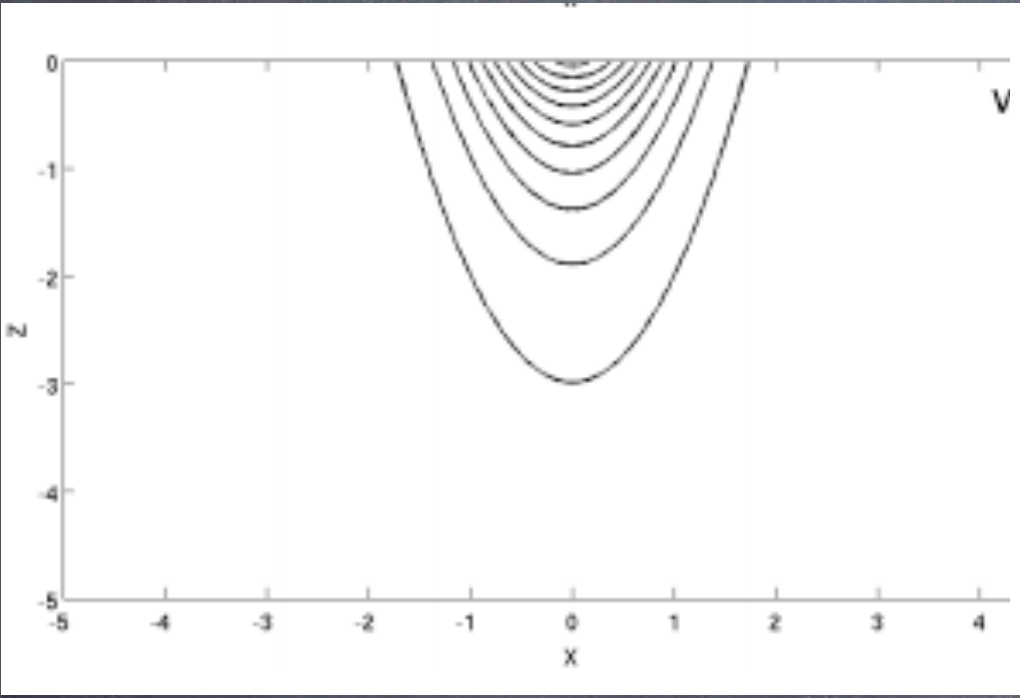
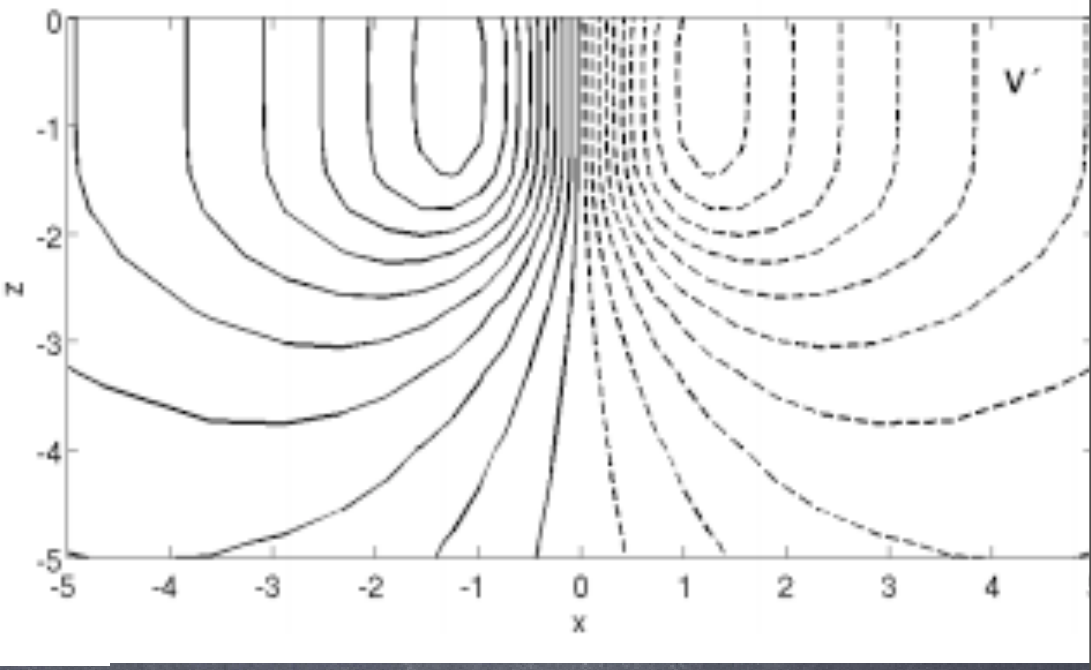


FIGURE 1. Estimated ratio $\epsilon/\mathcal{R} \approx (|\mathbf{u}_s \cdot \mathbf{u}|h)/(|\mathbf{u}|^2 h_s)$ governing the relative importance of Stokes effects versus nonlinearity. Eulerian velocity ($\mathbf{u}$) is taken as the AVISO weekly satellite geostrophic velocity or $-\mathbf{u}_s$ (for anti-Stokes flow) if $|\mathbf{u}_s| > |\mathbf{u}|$. The front/filament depth (h) is estimated as the mixed layer depth from the de Boyer Montégut *et al.* (2004) climatology. An exponential fit to the Stokes drift of the upper 9m projected onto the AVISO geostrophic velocity provides $\mathbf{u}_s \cdot \mathbf{u}$ and h_s. Stokes drift is taken from the WaveWatch-3 simulation described in Webb & Fox-Kemper (2011). $\mathbf{u}$, $\mathbf{u}_s$, and h_s are all for the year 2000, while h is from a climatology of observations over 1961-2008. The year 2000 average of $\epsilon/\mathcal{R}$ is shown.

J. C. McWilliams and B. Fox-Kemper. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

Waves (Stokes Vortex Force)
example of wave-balancing Submeso flow





Initial Submeso Front

Contours: 0.1

Perturbation on that scale due to waves

Contours: 0.014

J. C. McWilliams and B. Fox-Kemper. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

What about Langmuir-Submeso Interactions?

Movie: P. Hamlington

Perform large eddy simulations (LES) of CLB with a submesoscale temperature front--runs with and without Stokes drift

$$\frac{\partial \rho}{\partial t} + \mathbf{u}_L \cdot \nabla \rho = \mathrm{SGS}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\boldsymbol{\omega} + f\hat{\mathbf{z}}) \times \mathbf{u}_L = -\nabla \pi - \frac{g\rho\hat{\mathbf{z}}}{\rho_0} + \mathrm{SGS}$$

Computational parameters:

Domain size: 20km x 20km x -160m

Grid points: 4096 x 4096 x 128

Resolution: 5m x 5m x -1.25m

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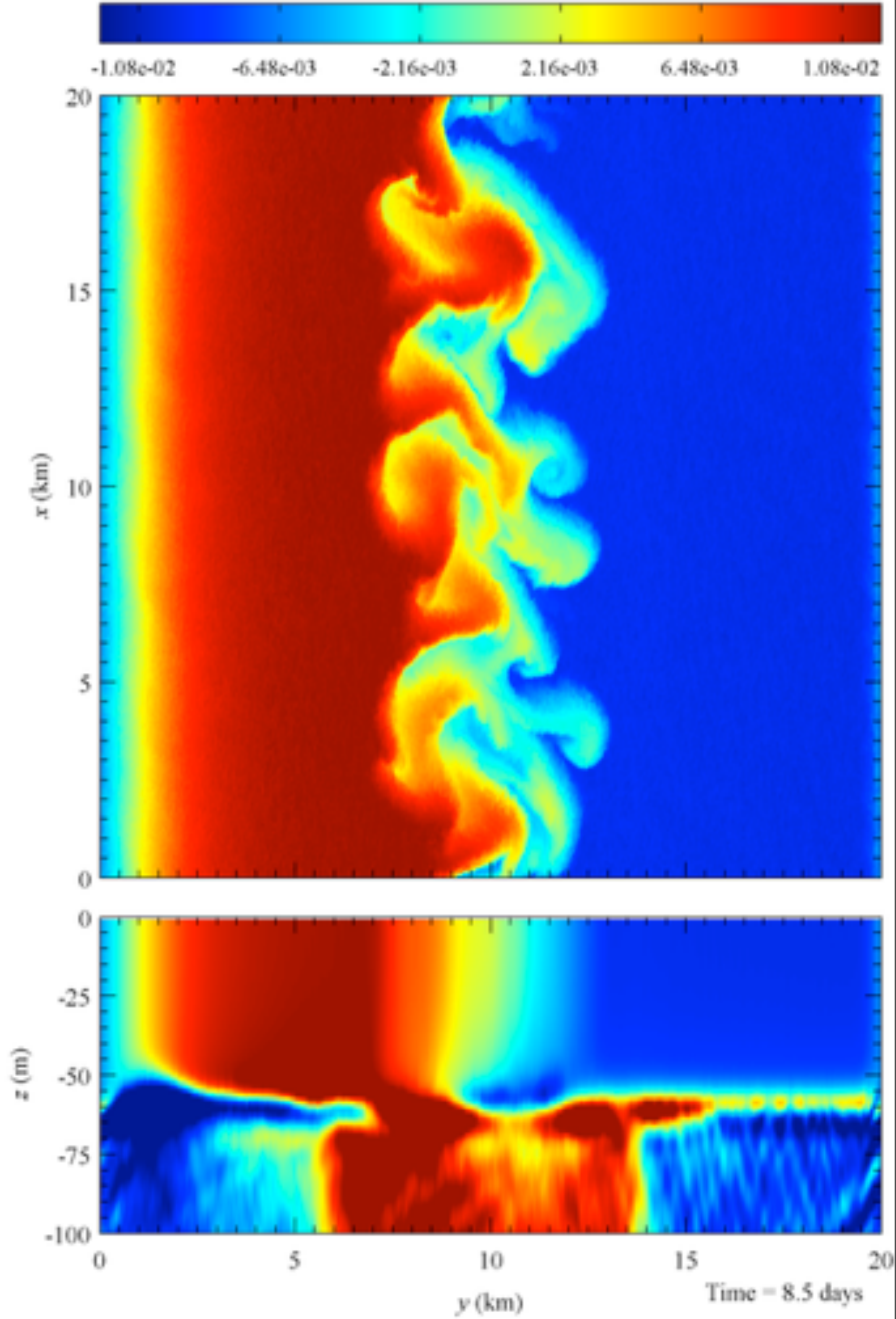
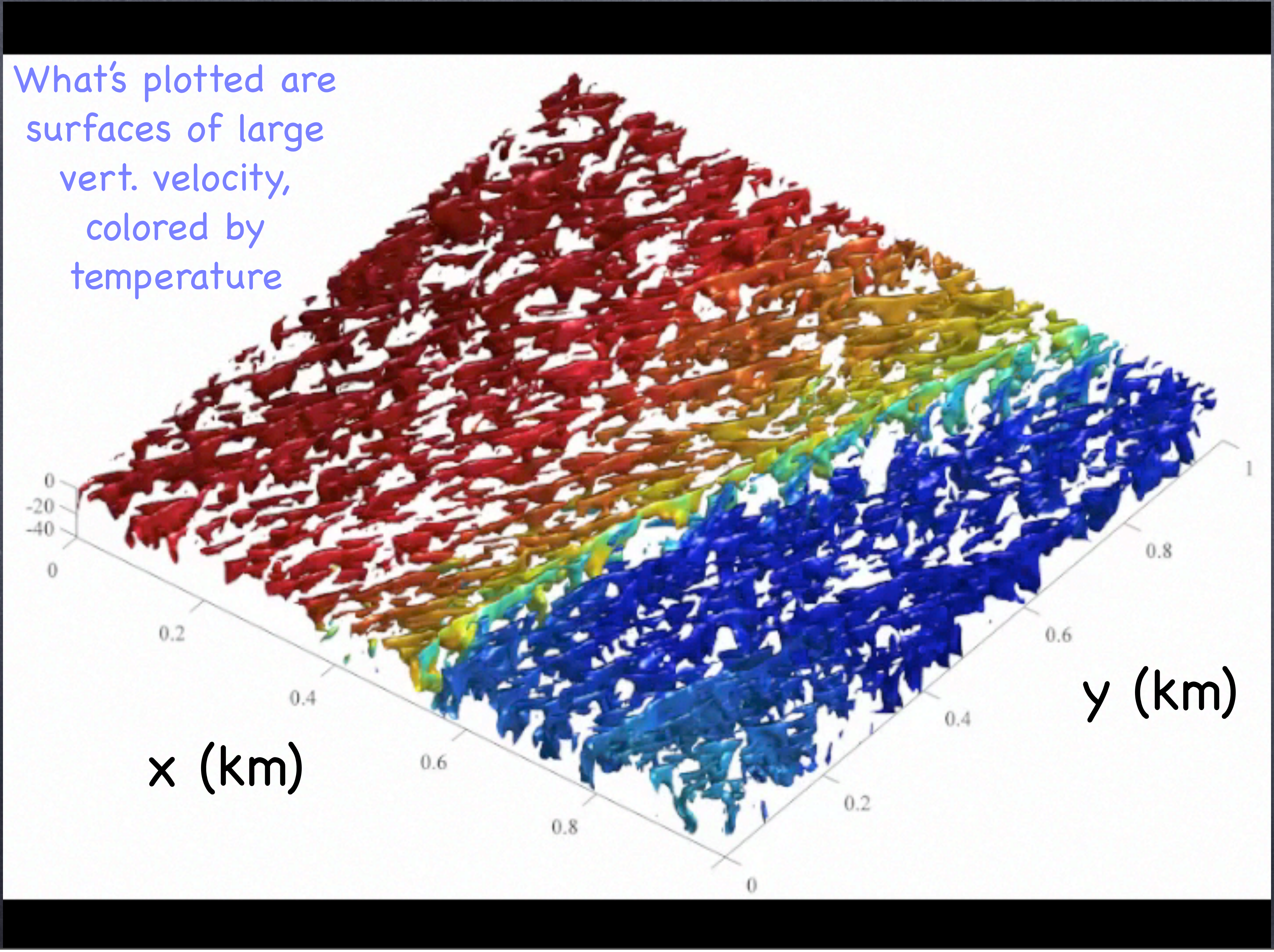


Overall results

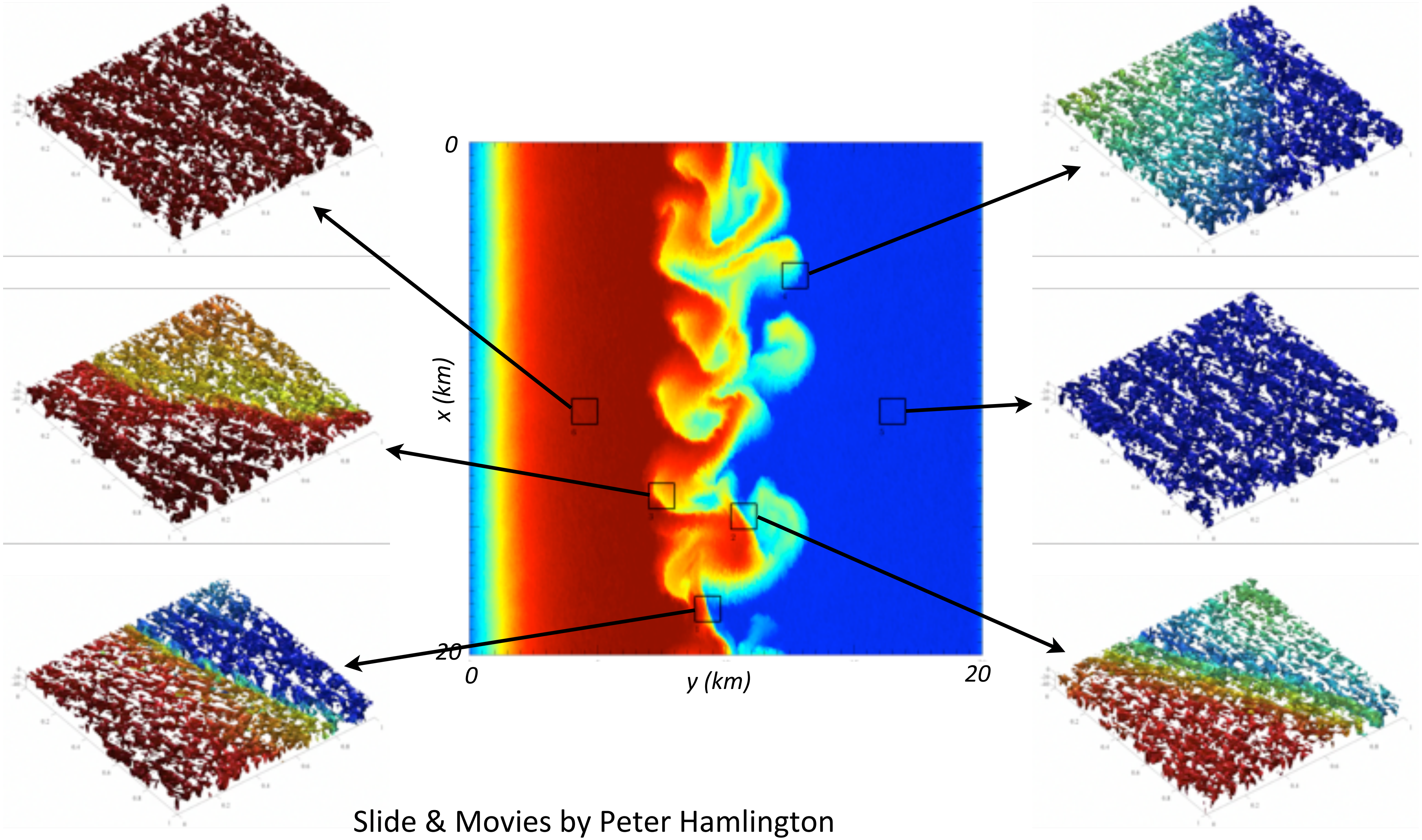
- Submesoscale flow is affected directly by Stokes Vortex Force--not in Eulerian Thermal Wind Balance!
- Strong interactions between small & large scales are rare in this configuration
- Two relatively independent turbulent spectral cascades near the surface.
- Presence of waves greatly changes small scale instability instability from symmetric instability to gravitational

P. E. Hamlington, L. P. Van Roekel, B. Fox-Kemper, K. Julien, G. P. Chini. Langmuir-Submesoscale Interactions: Descriptive Analysis of Multiscale Frontal Spin-down Simulations, *JGR-Oceans*, 2013. In prep.

Zoom: Submeso-Langmuir Interaction!



Diverse types of interaction



Slide & Movies by Peter Hamlington

P. E. Hamlington, L. P. Van Roekel, B. Fox-Kemper, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale simulations. In preparation, 2012.

Big Picture Conclusions

- Climate modeling is challenging partly due to the vast and diverse scales of fluid motions
- In the upper ocean, horizontal scales as big as basins, and as small as centimeters contribute non-negligibly
- Process models are needed to study these connections and improve subgrid models.
- Interesting are the submeso to Langmuir scales, as nonhydro. & ageostrophic effects become dominant
- The CLB are good for LES & analysis in this range, but cannot capture some effects of small, steep waves (breaking, spray, nearshore, etc.)

So, no problems? Just crunch away with CLB?

- Let’s revisit our assumptions for scale separation:
  - CLB wave equations require limited *wave steepness* and irrotational flow
  - Real wind-waves are not monochromatic, but incorporate a spectrum of waves, and...

Power Spectrum of wave height

$$\langle \eta^2 \rangle = \int_0^\infty E(k)dk = C_0 + \int_{k_h}^\infty C_1 k^{-2} dk$$

Power Spectrum of wave steepness: INFINITE!

$$\langle k^2 \eta^2 \rangle = \int_0^\infty k^2 E(k)dk = D_0 + \int_{k_h}^\infty D_1 dk$$

Steep waves break->vortex motion & small scale turbulence!

So, no problems? Just crunch away with CLB?

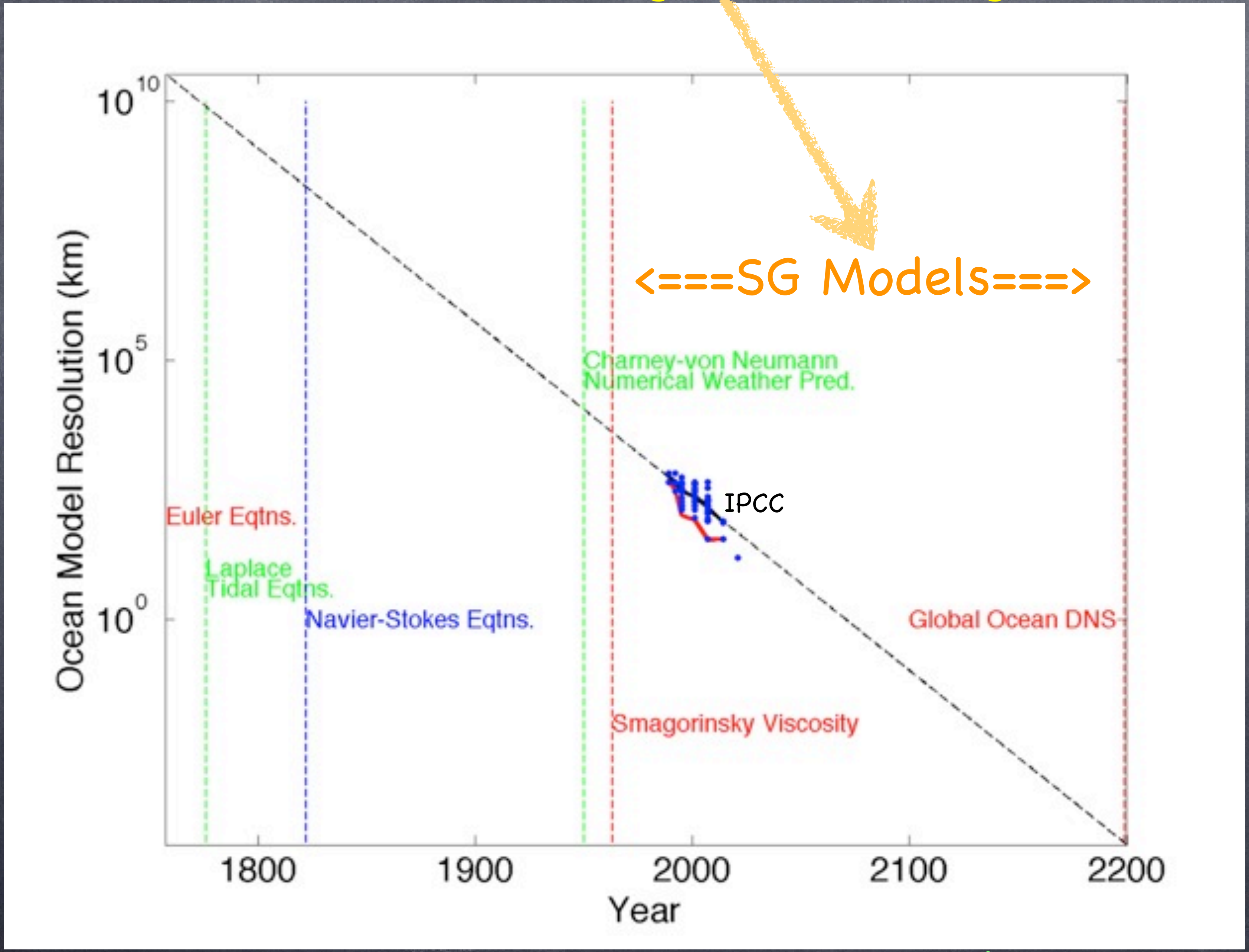
- Let's revisit our assumptions for scale separation:
  - Also, what about finite wave packets?
  - What about co-evolution of the submesoscale flow and wave packets?
  - What about steep wave effects? Breaking?
  - Are there other ways for waves to drive turbulence?

Steep waves break->vortex motion & small scale turbulence!

CLB Conclusions

- Waves are a dominant feature of the upper ocean on short timescales
- On longer timescales, rectified effects of waves--in CLB the Stokes drift--changes boundary layer and submesoscale dynamics
- Critical concept: Lagrangian shear takes over for Eulerian
- Wave effects are particularly important when waves are *not fully developed* which is most of the time for long fetch (i.e., open ocean)

Extrapolate for historical perspective: The Golden Era of Subgrid Modeling is Now!



All papers at: fox-kemper.com/research

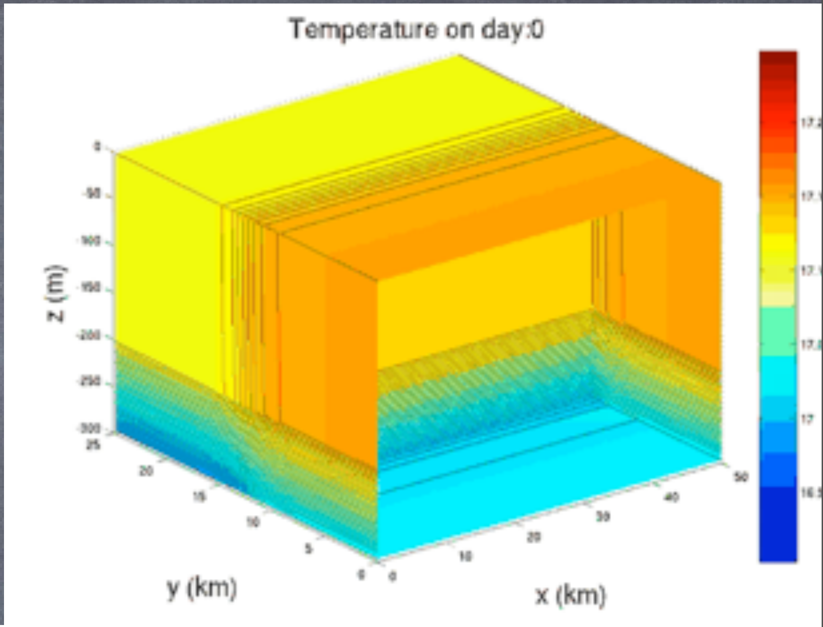
Mixed Layer Eddy Restratification

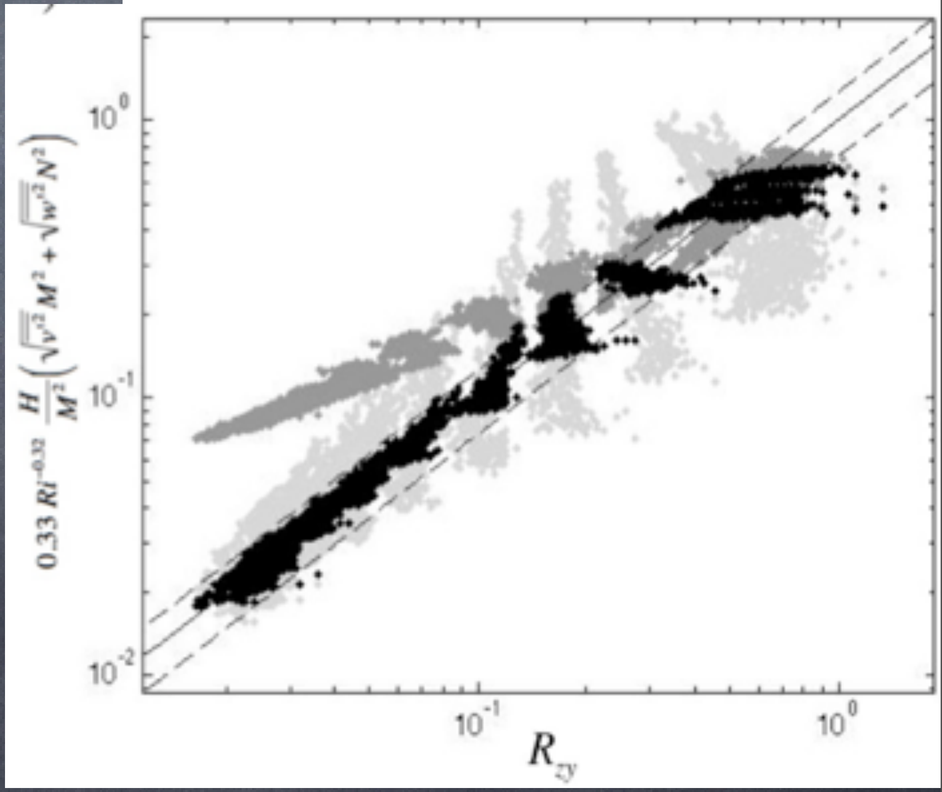
Estimating eddy buoyancy/density fluxes:

$$\overline{\mathbf{u}'b'} \equiv \mathbf{\Psi} \times \nabla\bar{b}$$

A submeso eddy-induced overturning:

$$\mathbf{\Psi} = \frac{C_e H^2 \mu(z)}{|f|}\nabla\bar{b} \times \hat{\mathbf{z}}$$





S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013

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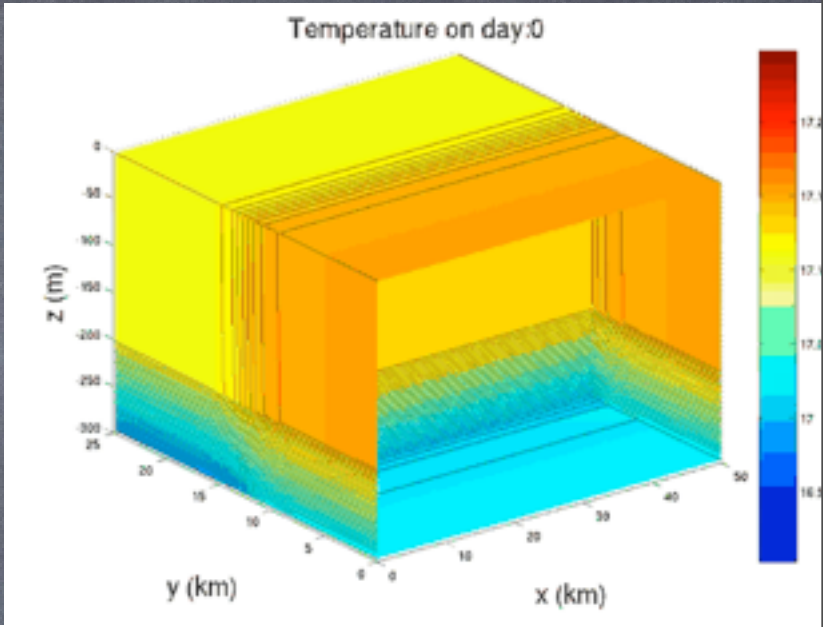
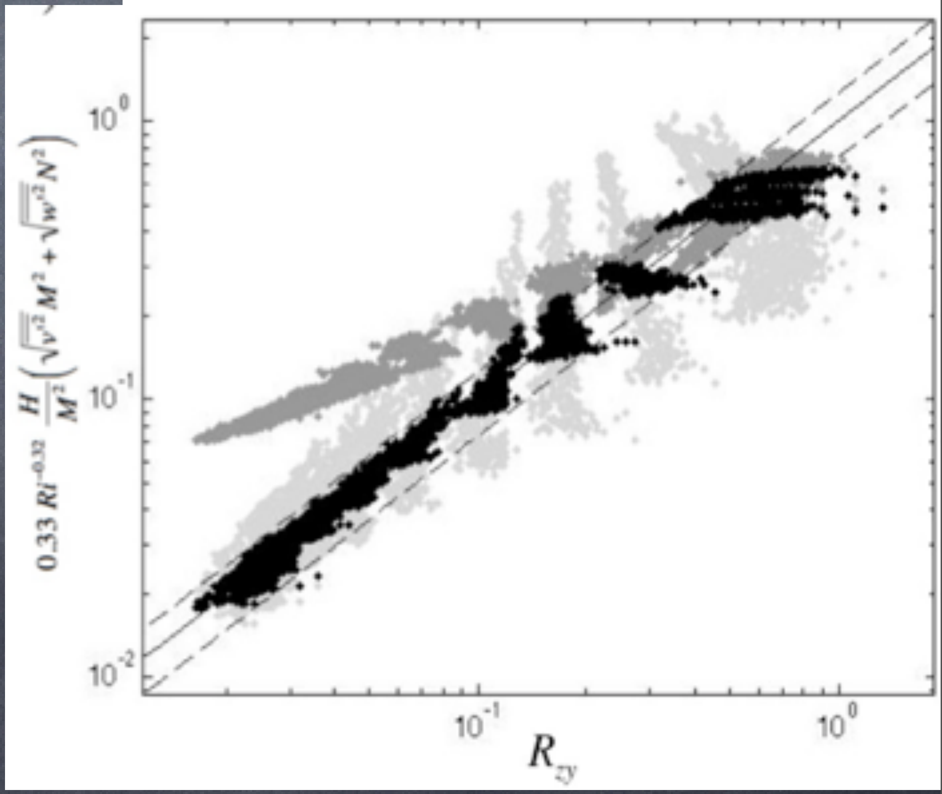
in ML only:

$$\mu(z) = 0 \text{ if } z < -H$$





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For a consistently restratifying,

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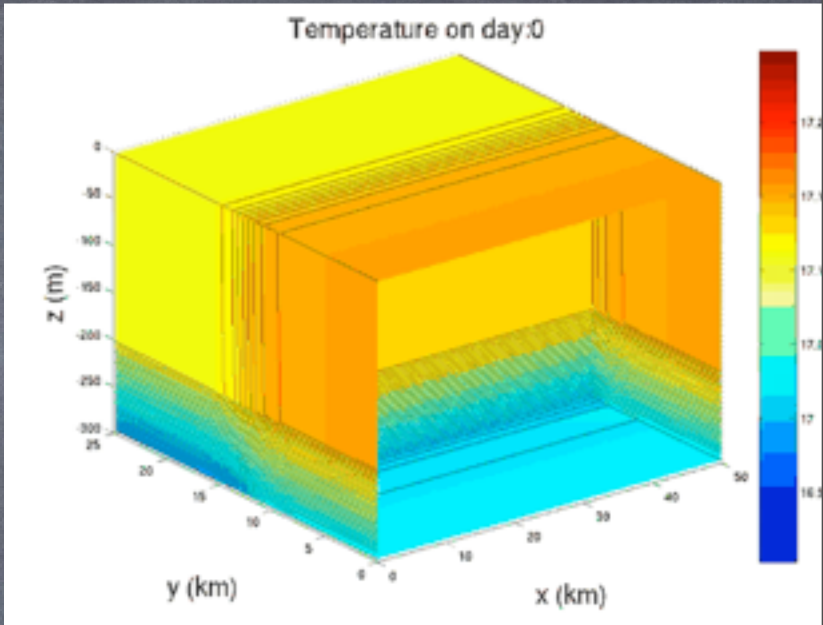

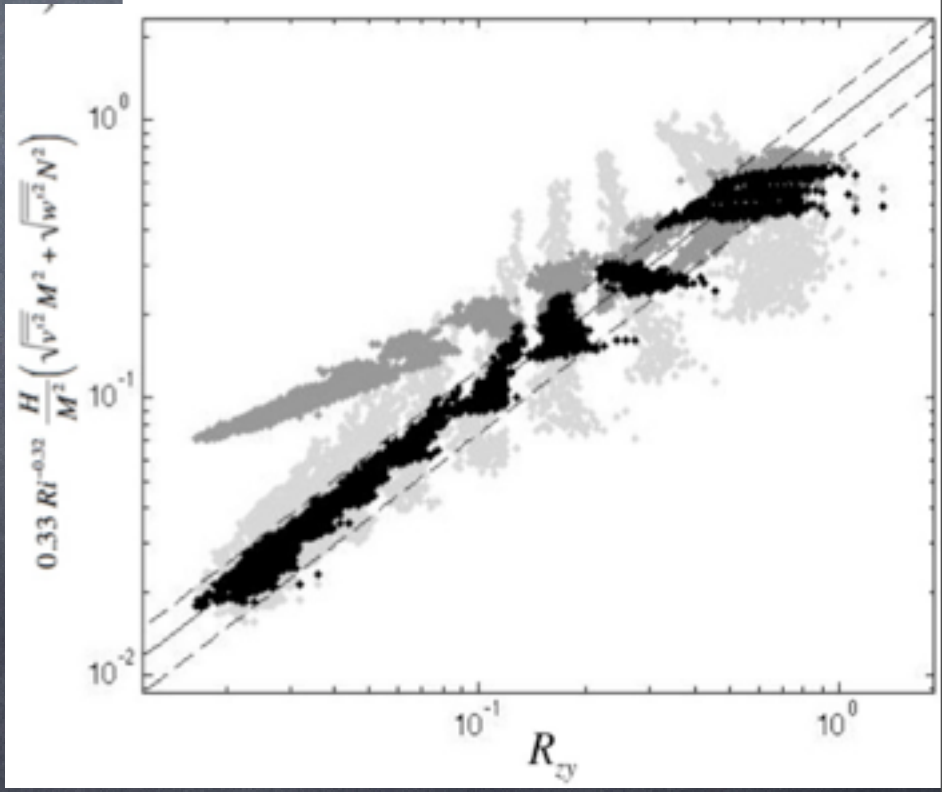

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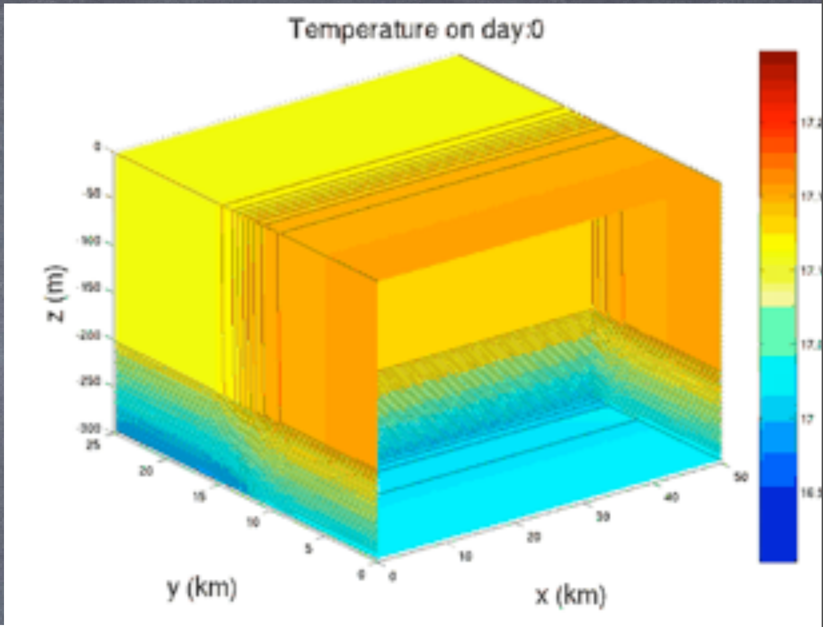
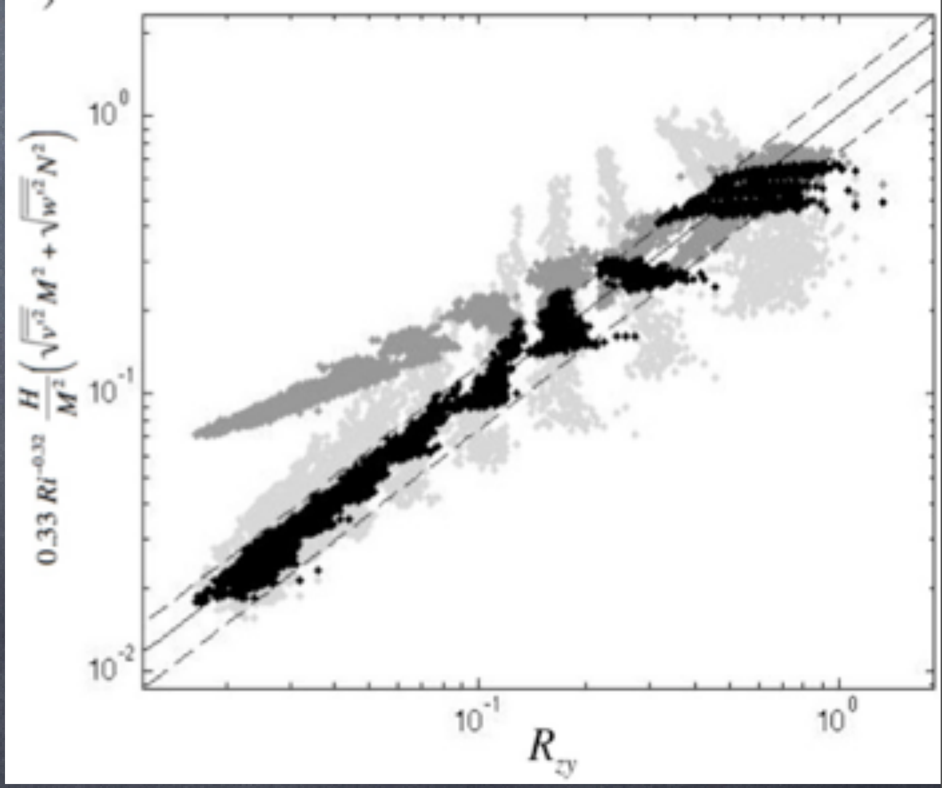
For a consistently restratifying,

$$\overline{w'b'} \propto \frac{H^2}{|f|} \left|\nabla_H \bar{b}\right|^2$$

and horizontally downgradient flux.

$$\overline{\mathbf{u}'_H b'} \propto \frac{-H^2 \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla_H \bar{b}$$





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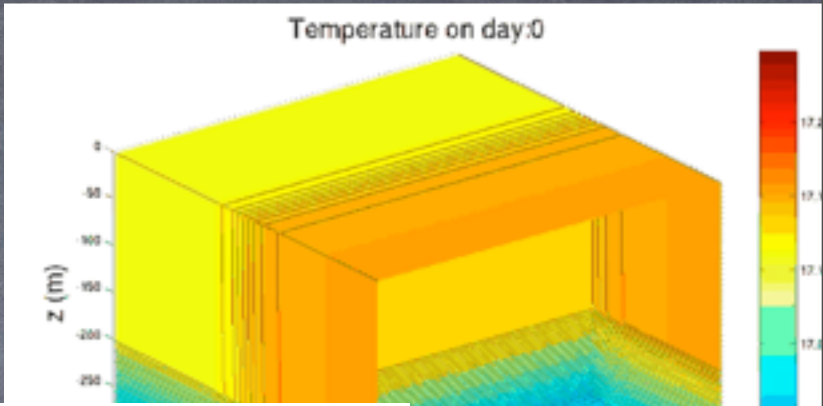
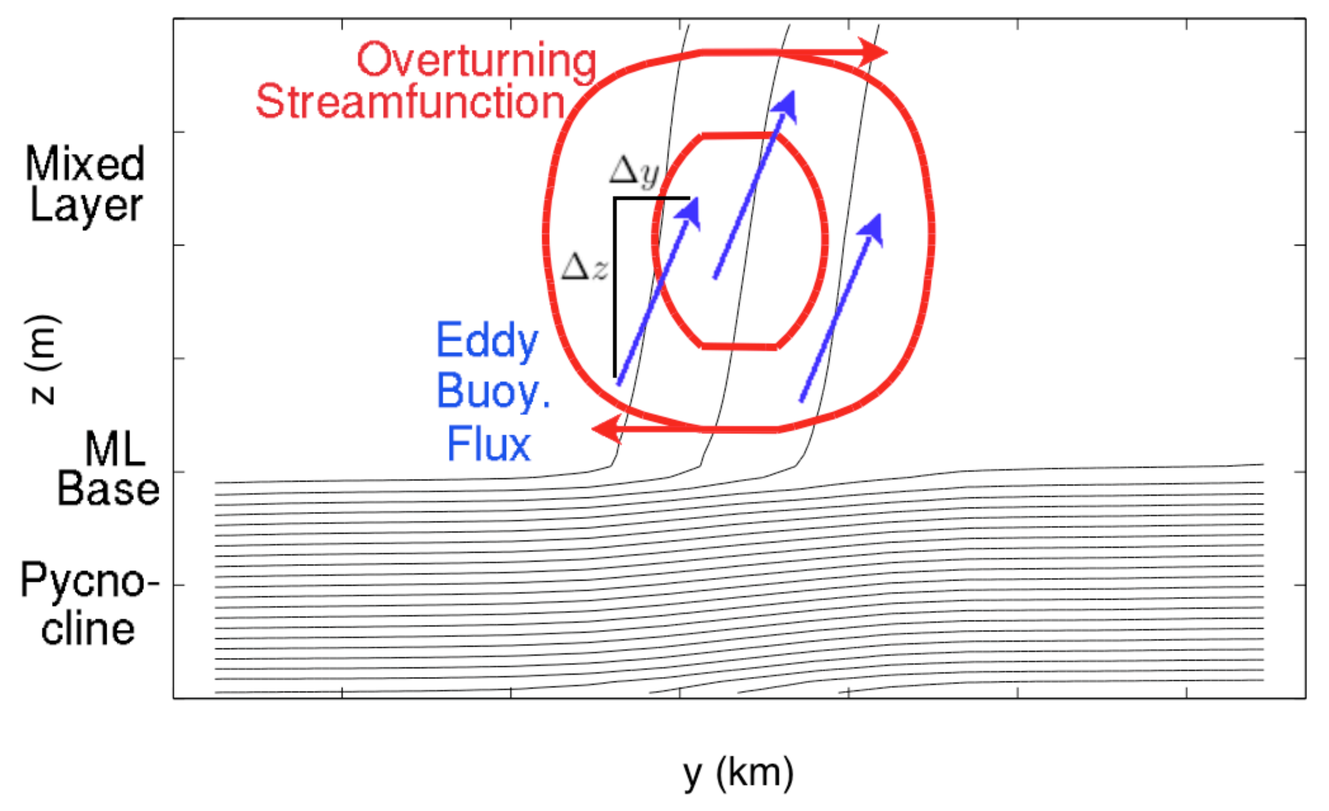
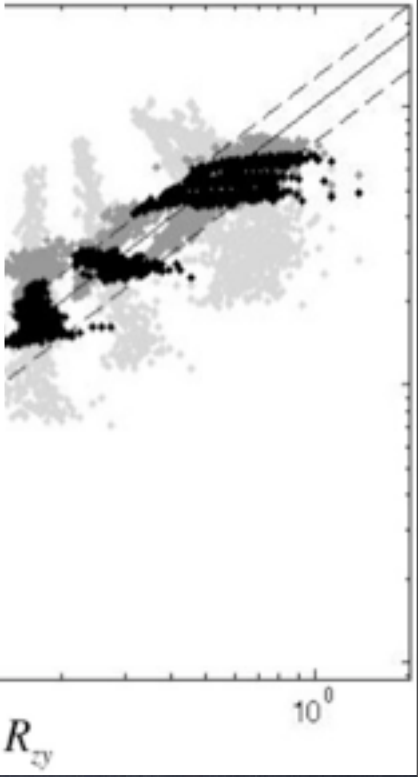


A submes

$$\mathbf{\Psi} =$$





$$\mu(z)$$

For a cor

$$\overline{w'b'}$$

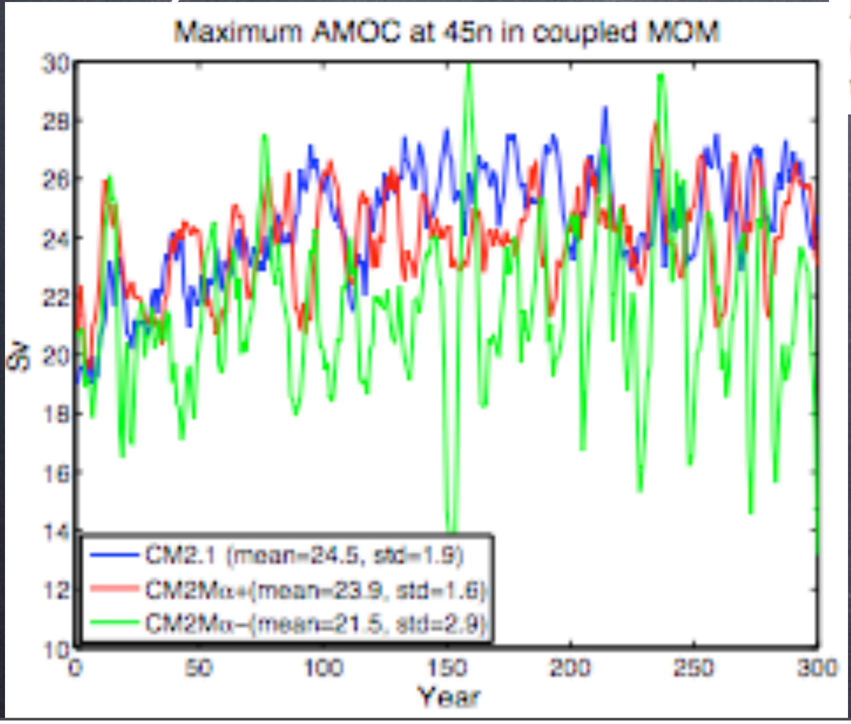
and horizontally downgradient flux.

$$\overline{\mathbf{u}'_H b'} \propto \frac{-H^2 \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla_H \bar{b}$$

S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013

Sensitivity of Climate to Submeso: AMOC & Cryosphere Impacts

May Stabilize AMOC



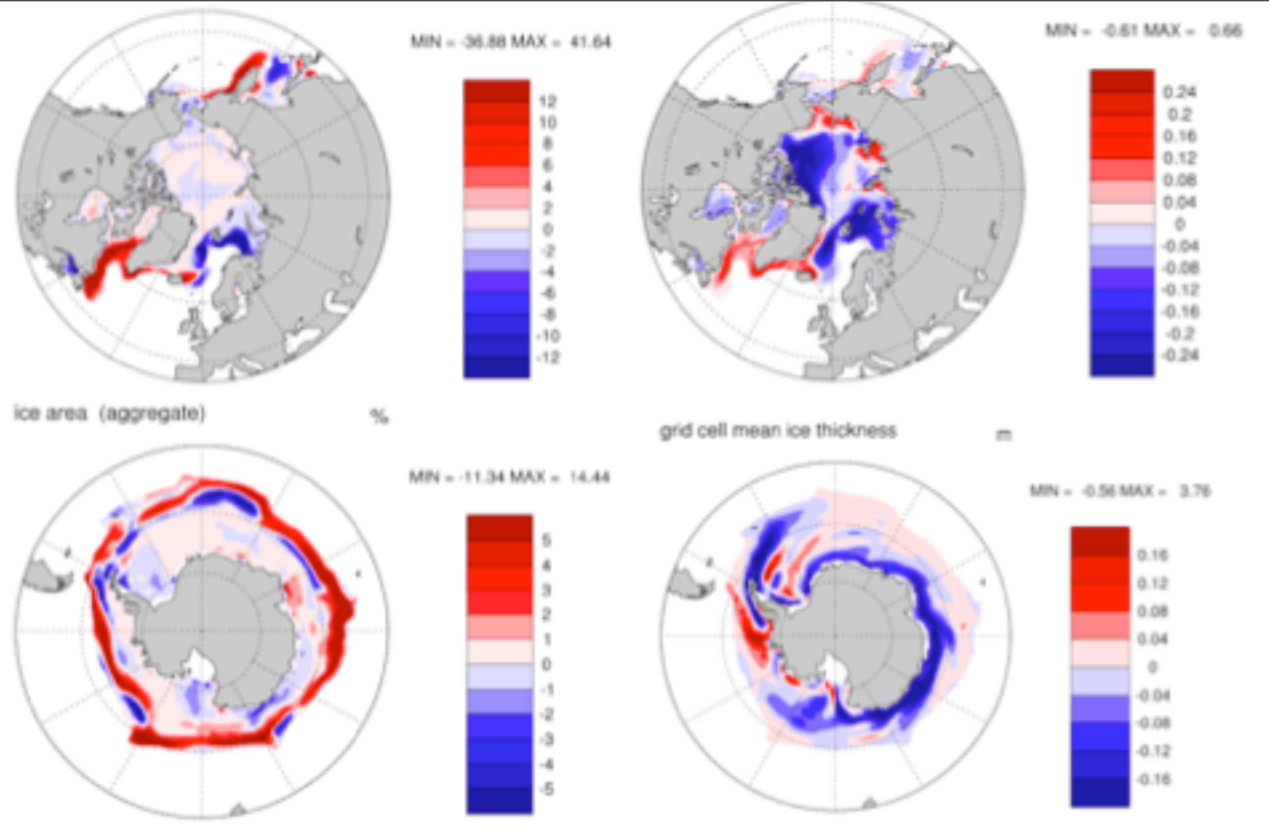


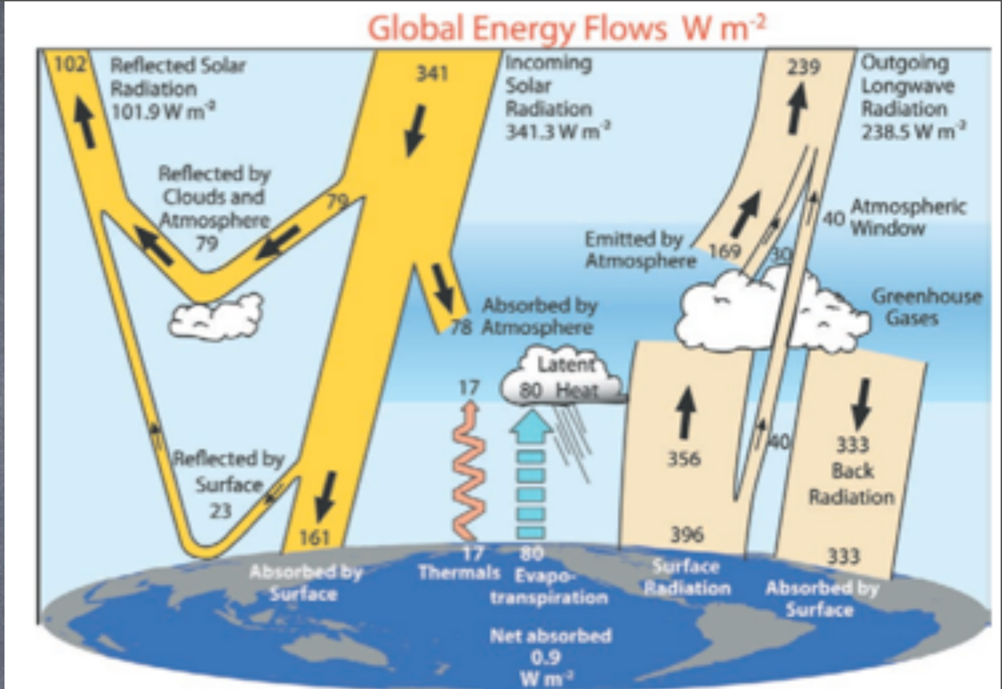
Figure 10: Wintertime sea ice sensitivity to introduction of MLE parameterization ($CCSM^+$ minus $CCSM^-$): January to March Northern Hemisphere a) ice area and b) thickness and July to September Southern Hemisphere c) ice area and d) thickness.

Affects sea ice

NO RETUNING NEEDED!!!

These are impacts:
bias change unknown

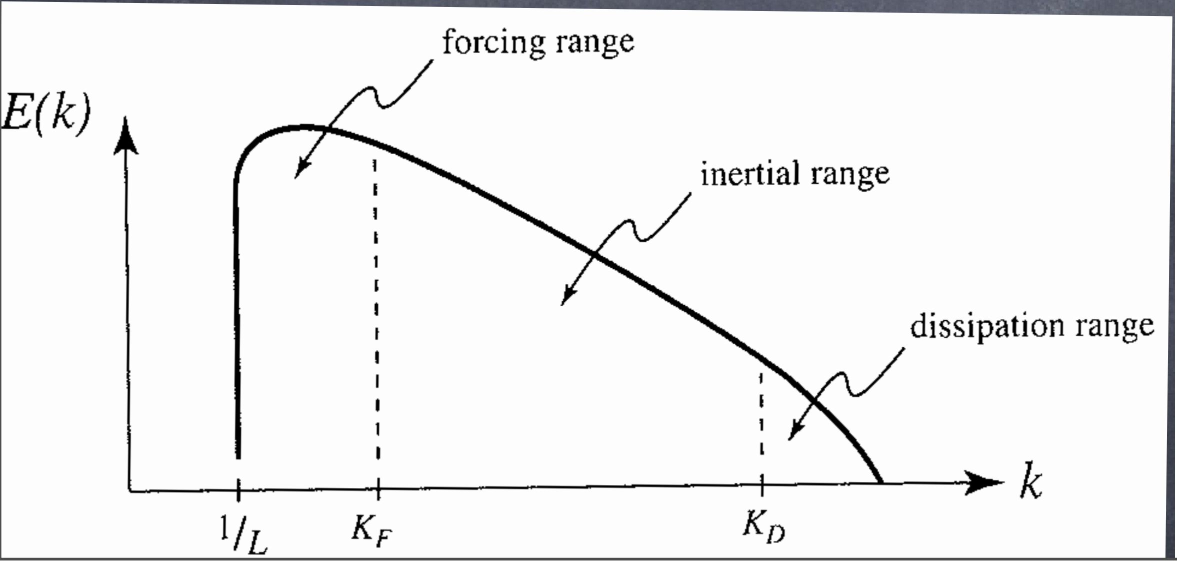
The Earth's Climate System is driven by the Sun's light (minus outgoing infrared) on a global scale

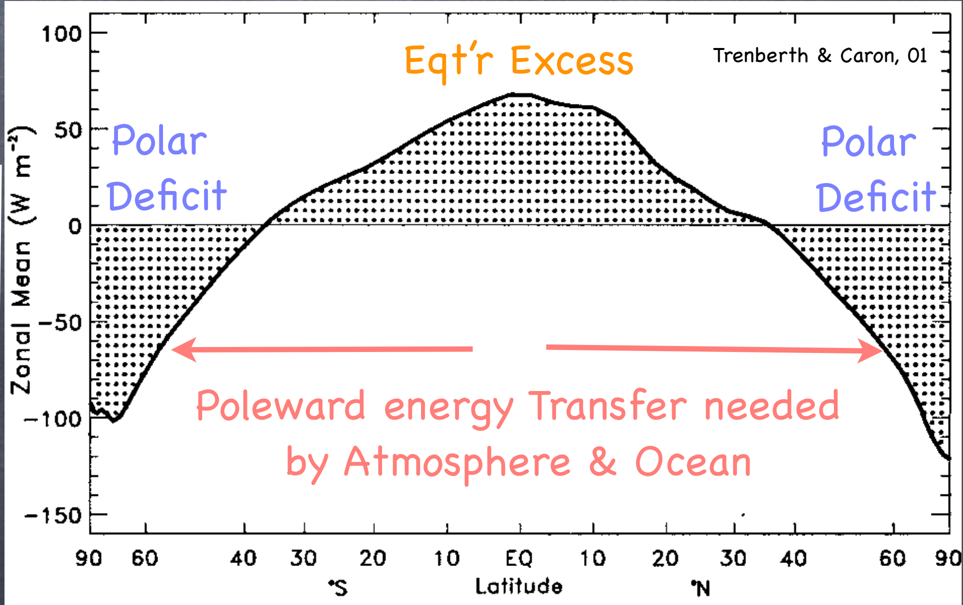


Trenberth & Fasullo, 09

Fig. 1. The global annual mean Earth's energy budget for the Mar 2000 to May 2004 period (W m⁻¹). The broad arrows indicate the schematic flow of energy in proportion to their importance.

Dissipation concludes turbulent cascades on scales about a trillion times smaller





Eqt'r Excess

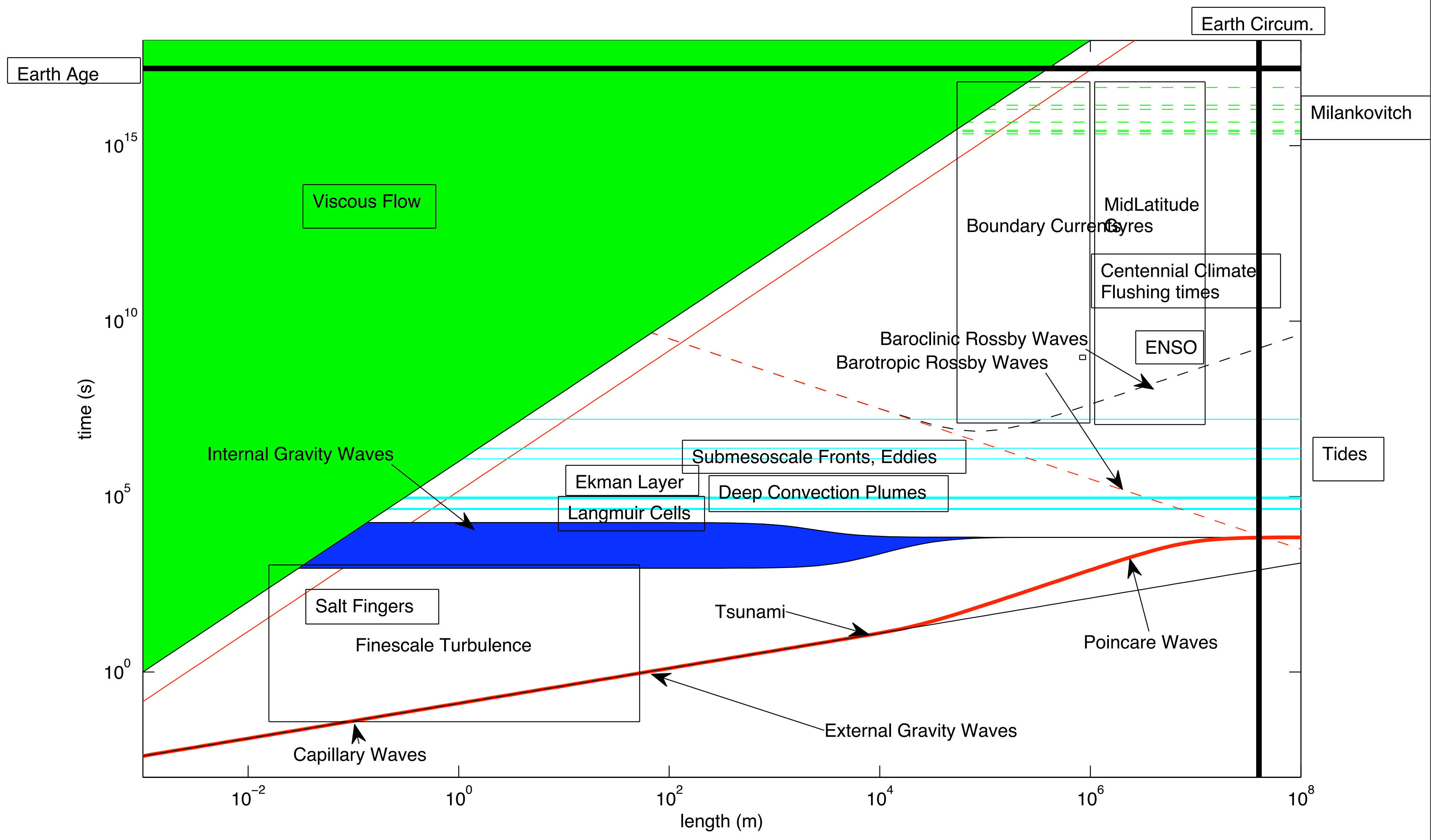
Polar Deficit

Polar Deficit

Poleward energy Transfer needed by Atmosphere & Ocean

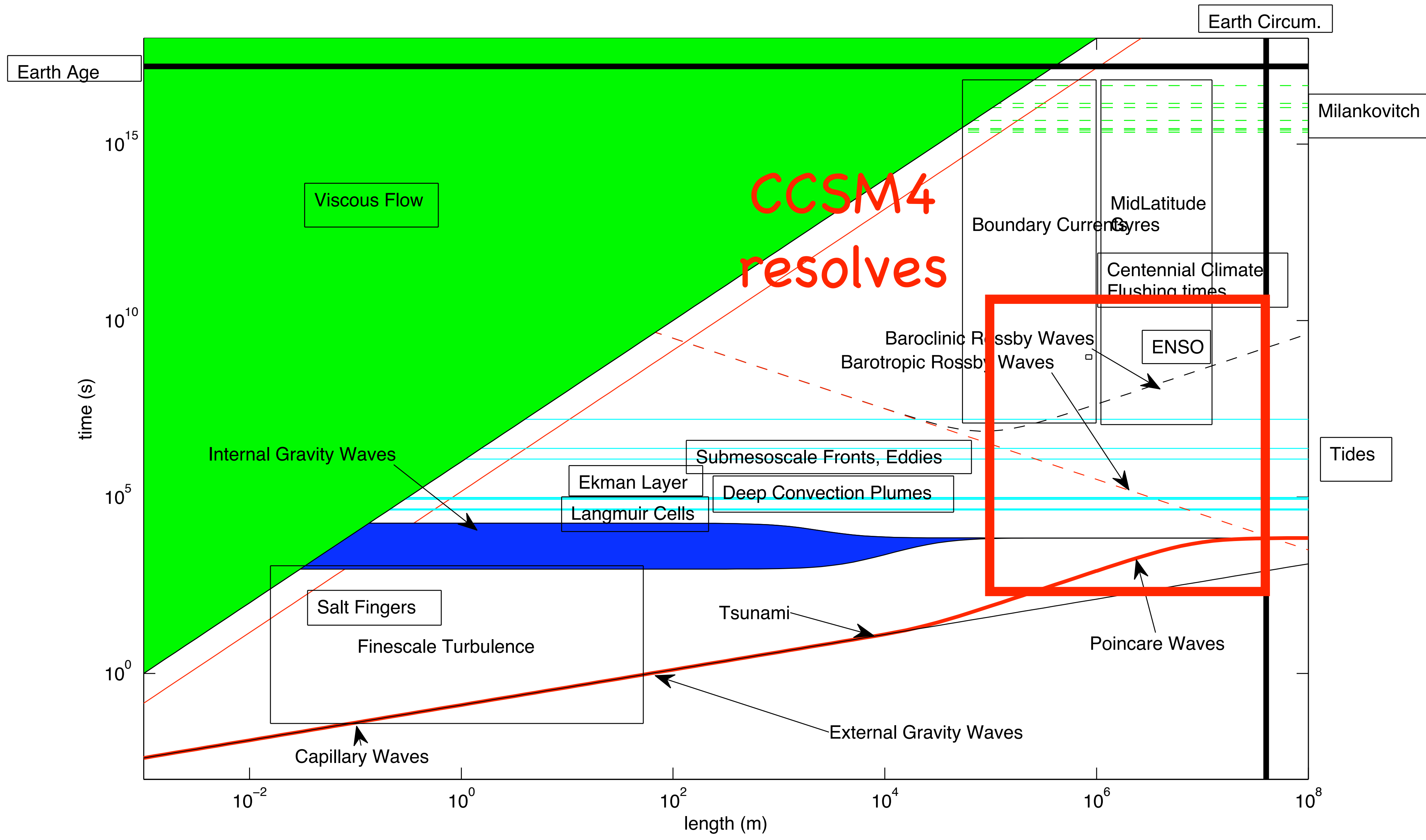
FIG. 1. TOA annualized ERBE zonal mean net radiation (W m^{-2}) for Feb 1985–Apr 1989.

The Ocean is Vast & Diverse:
Q: What processes to parameterize?
Today's A: Unresolved Upper Ocean with Air-Sea Impact



Friday, May 31, 13

The Ocean is Vast & Diverse:
Q: What processes to parameterize?
Today's A: Unresolved Upper Ocean with Air-Sea Impact



Craik-Leibovich Boussinesq

Old Boussinesq (written in vortex force form)

$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times \mathbf{v} = -\nabla \pi + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$

$$\frac{\partial b}{\partial t} + \mathbf{v} \cdot \nabla b = 0 \qquad \nabla \cdot \mathbf{v} = 0$$

Craik-Leibovich Boussinesq

$\mathbf{v}_s = \text{Stokes Drift}$

$$\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{f} + \nabla \times \mathbf{v}] \times (\mathbf{v} + \mathbf{v}_s) = -\nabla \pi^\dagger + b\mathbf{k} + \nu \nabla^2 \mathbf{v}$$

$$\frac{\partial b}{\partial t} + (\mathbf{v} + \mathbf{v}_s) \cdot \nabla b = 0$$

$$\nabla \cdot \mathbf{v} = 0$$