Thoughts on Mixed Layer Eddies

Baylor Fox-Kemper
Brown U.
Fri Feb 21, 2014

G. Boccaletti, R. Ferrari, and B. Fox-Kemper. Mixed layer instabilities and restratification. Journal of Physical Oceanography, 37(9):2228-2250, 2007.

W. A. Qazi, W. J. Emery, and B. Fox-Kemper. Computing ocean surface currents over the coastal California Current System using 30-minute lag sequential SAR images. IEEE Transactions on Geoscience and Remote Sensing, February 2013. Submitted.

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B. Fox-Kemper and R. Ferrari. Parameterization of mixed layer eddies. Part II: Prognosis and impact. Journal of Physical Oceanography, 38(6):1166-1179, 2008.

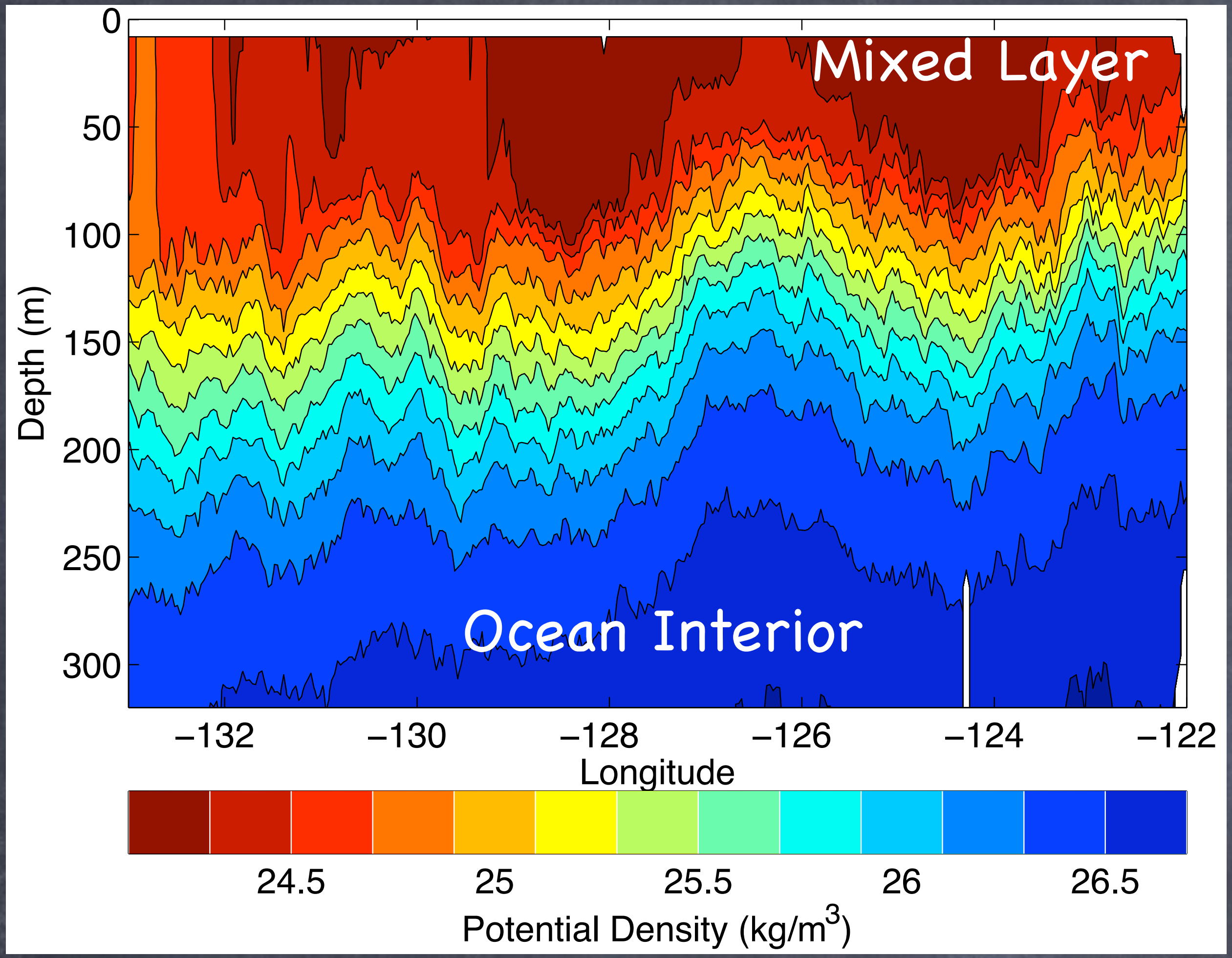
B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. Ocean Modelling, 39:61-78, 2011.

S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013.

P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, G. P. Chini. Langmuir-Submesoscale Interactions: Descriptive Analysis of Multiscale Frontal Spin-down Simulations, *JPO*, 2013. Submitted.

N. Suzuki, BFK, P. E. Hamlington, L. P. Van Roekel, S. Haney. The surface wave influence on mixed layer frontal currents and multi-scale turbulence.

Ocean Mixed Layer



Pot'l Density measured by a Seasoar along a straight section from (32.5N, 122W) to (35N, 132W) between the CA current and the subtropical gyre.

(as in Ferrari & Rudnick, 2000)

The mixed layer is not TOTALLY mixed.

Horizontal density gradients are common.

1) What does its stratification imply?

2) How does the stratification get set?

3) Why do we care?

G. Boccaletti, R. Ferrari, and B. Fox-Kemper. Mixed layer instabilities and restratification. Journal of Physical Oceanography, 37(9):2228-2250, 2007.

The Stratification Permits
Two Types of Baroclinic Instability:

Mesoscale and SubMesoscale (Boccaletti et al., 2007)

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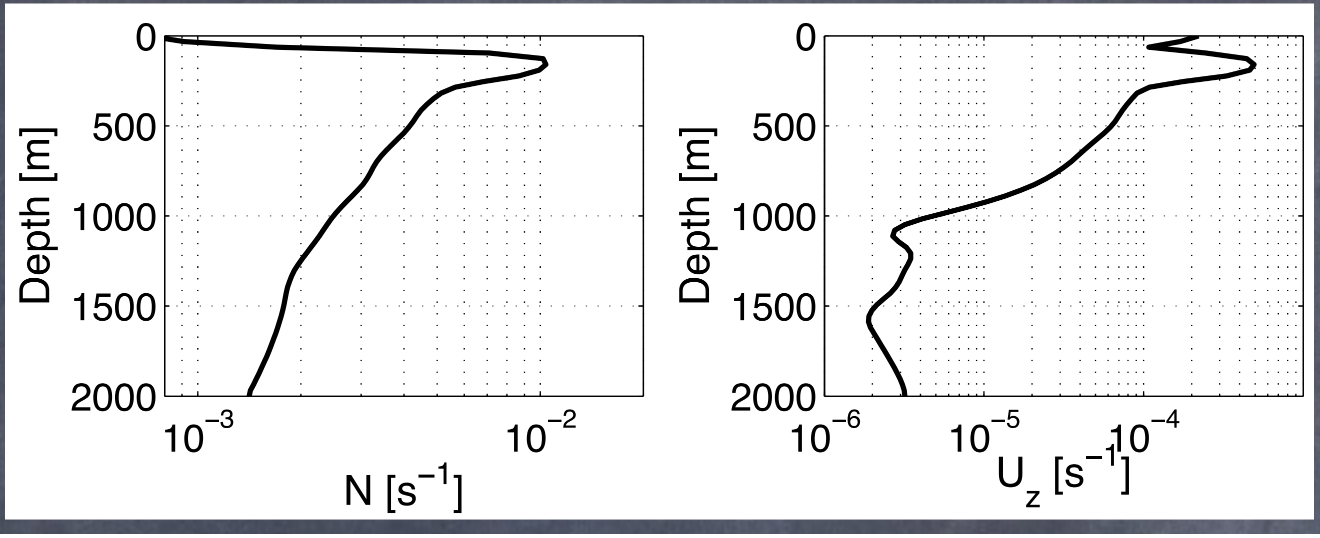
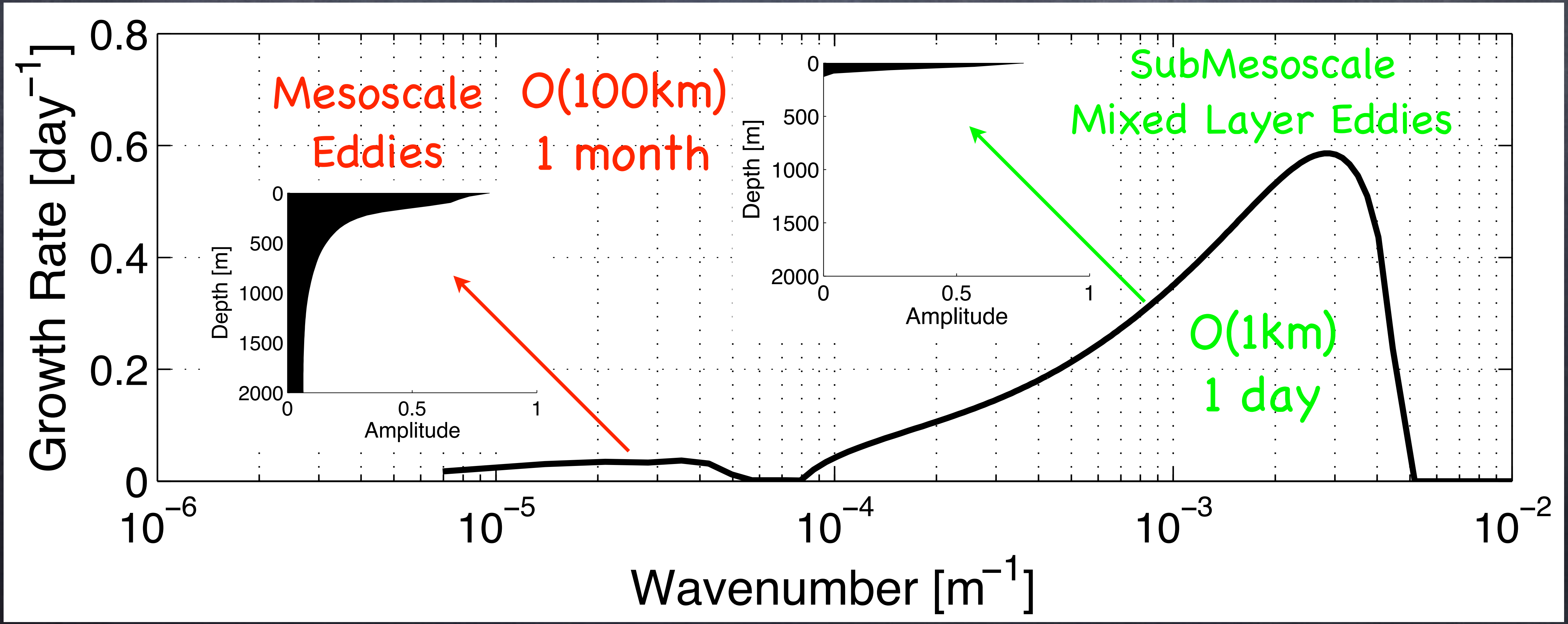




Mesoscale Eddies

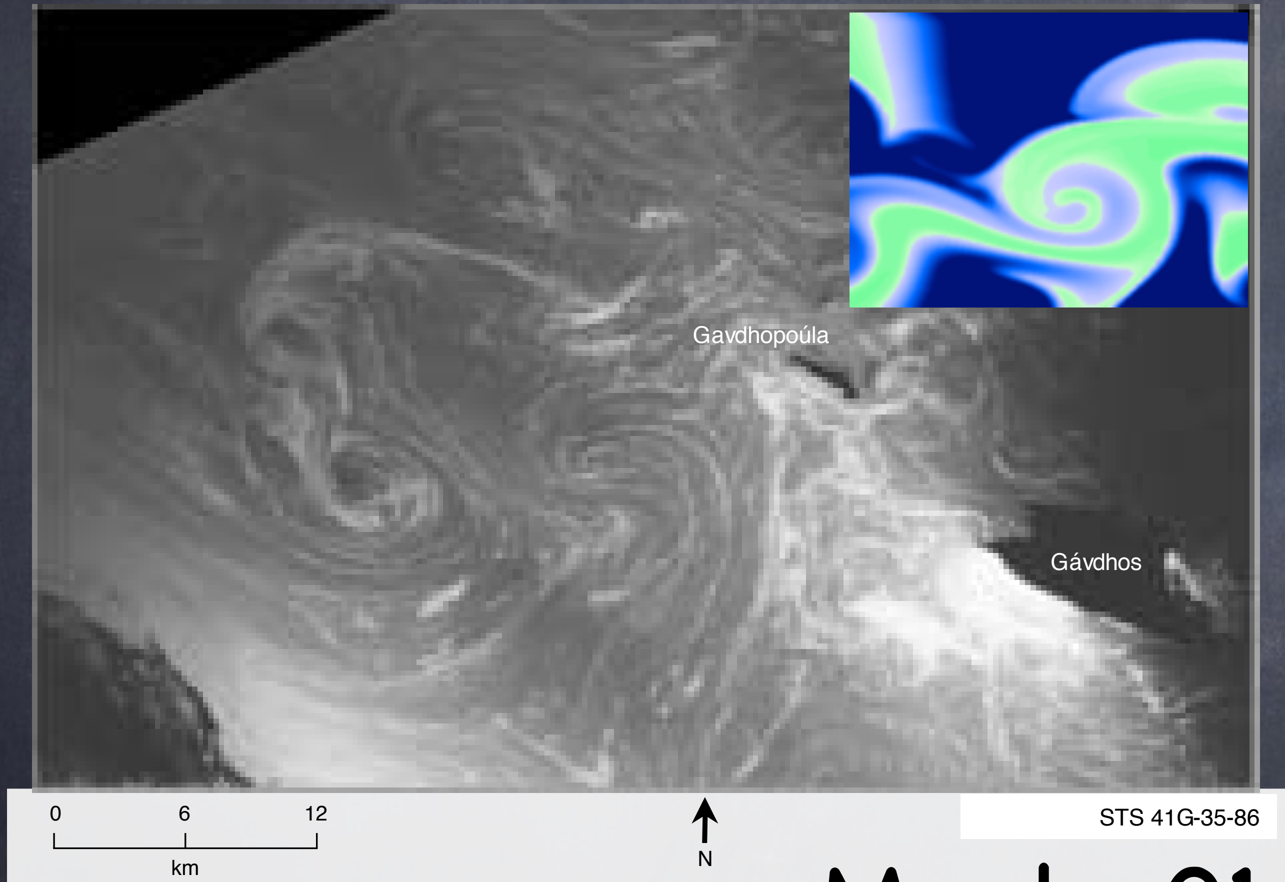
O(100km)
1 month

SubMesoscale
Mixed Layer Eddies

O(1km)
1 day

Observed:

Strongest Surface Eddies= Spirals on the Sea?



Munk, 01

Figure 1. A pair of interconnected spirals in the Mediterranean Sea south of Crete. This vortex pair has a clearly visible stagnation point between the two spirals, the cores of which are aligned with the preconditioning wind field. 7 October 1984.

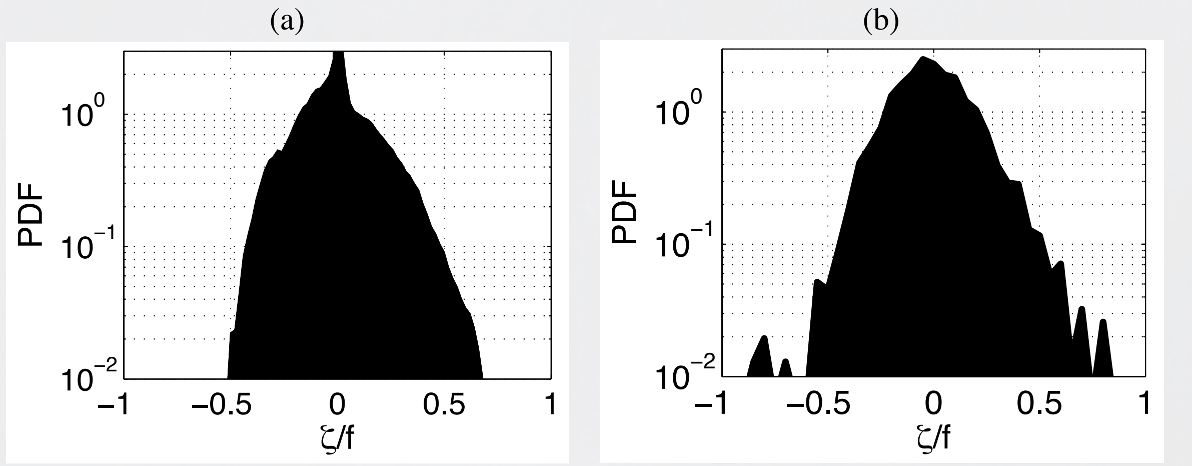


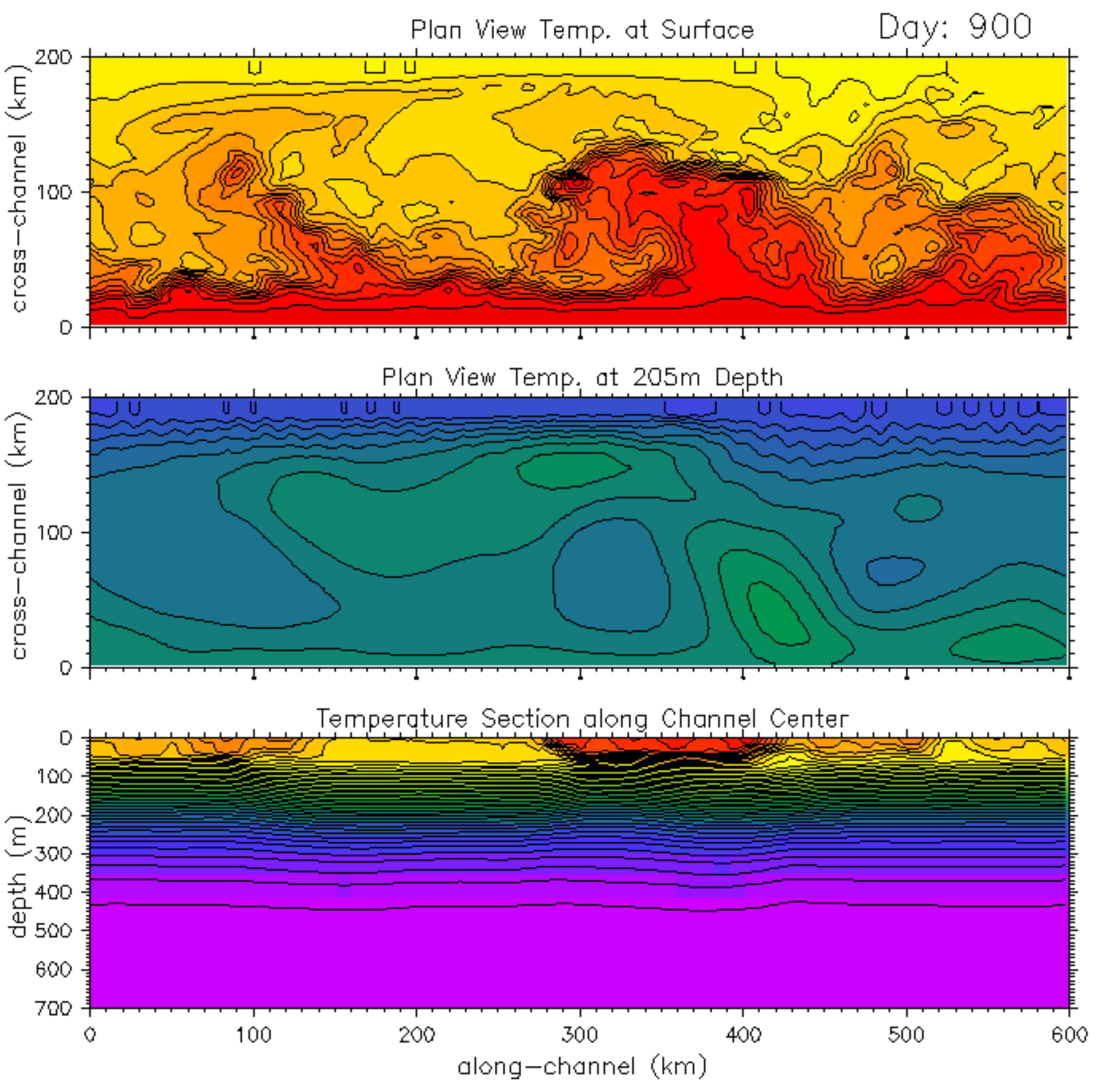
Figure 12: Probability density function of relative vorticity divided by Coriolis parameter. (a) Results from the numerical simulation of a slumping horizontal density front. ($z > 100$ only to exclude bottom Ekman layer.) The PDF is estimated using surface velocity measurements at day 25 (see also Fig. 11). A positive skewness appears as soon as the baroclinic instability enters in the nonlinear stage, and it continues to grow. Note that the peak at $\zeta/f = 0$ is due to the model's initial resting condition; that fluid has not yet been contacted by the MLI. (b) Results from ADCP measurements in the North Pacific. The PDF is calculated in bins of width 0.02.

W. A. Qazi, W. J. Emery, and B. Fox-Kemper. Computing ocean surface currents over the coastal California Current System using 30-minute lag sequential SAR images. IEEE Transactions on Geoscience and Remote Sensing, February 2013. Submitted.

Mesoscale and SubMesoscale are Coupled Together:

ML Fronts are formed by Mesoscale Straining.

Submesoscale eddies remove PE from those fronts.



B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. Journal of Physical Oceanography, 38(6):1145-1165, 2008.

Vertical fluxes are Submesoscale and tend to restratify

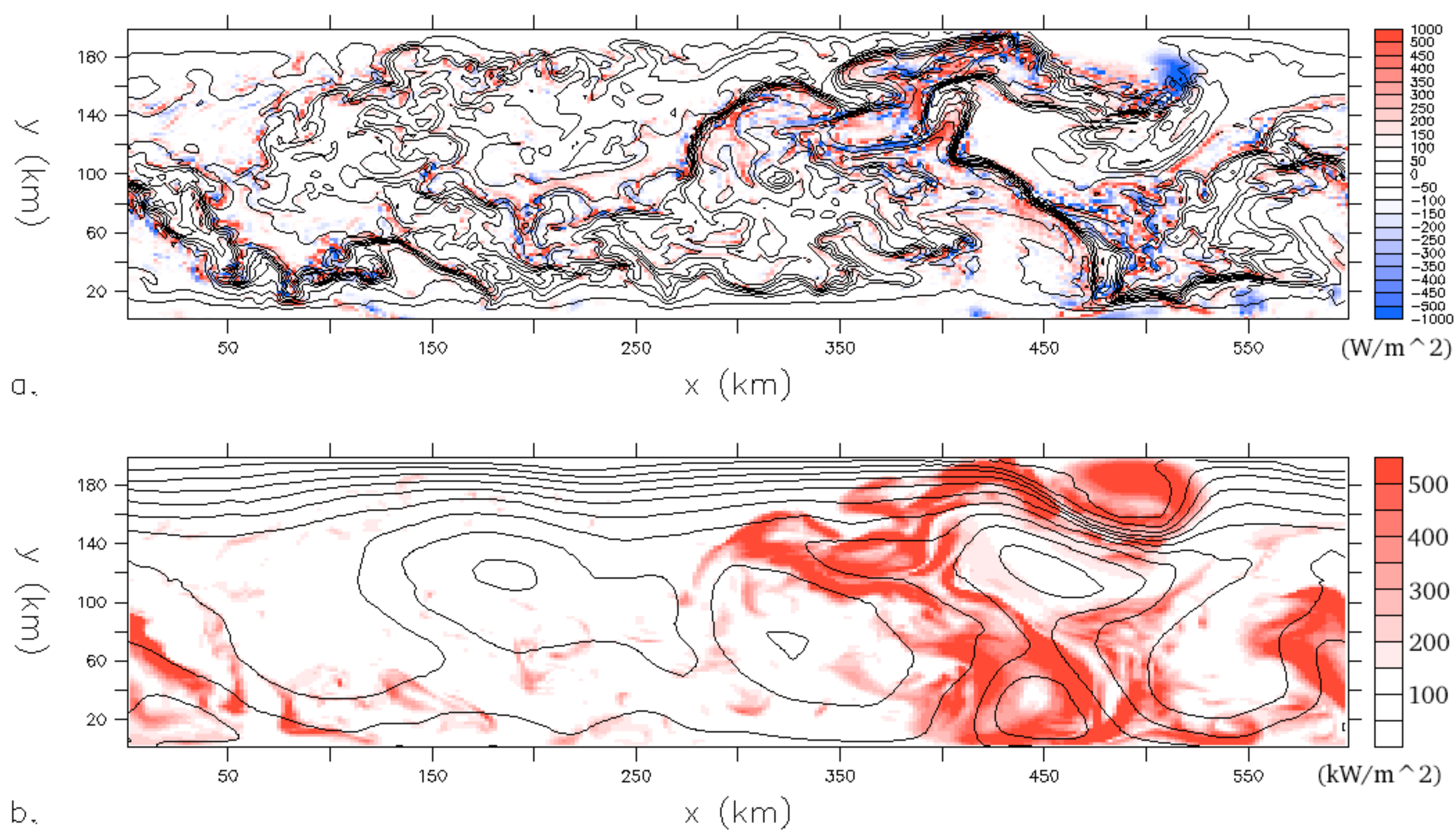


FIGURE 1: Contours of temperature at the a) surface and b) below the mixed layer base in a simulation with both mesoscale eddies and MLEs ($0.2^\circ C$ contour intervals). Shading indicates the value at the depth where $\overline{w'b'}$ (upper panel) and $|\overline{\mathbf{u}'_H b'}|$ (lower panel) take the largest magnitude.

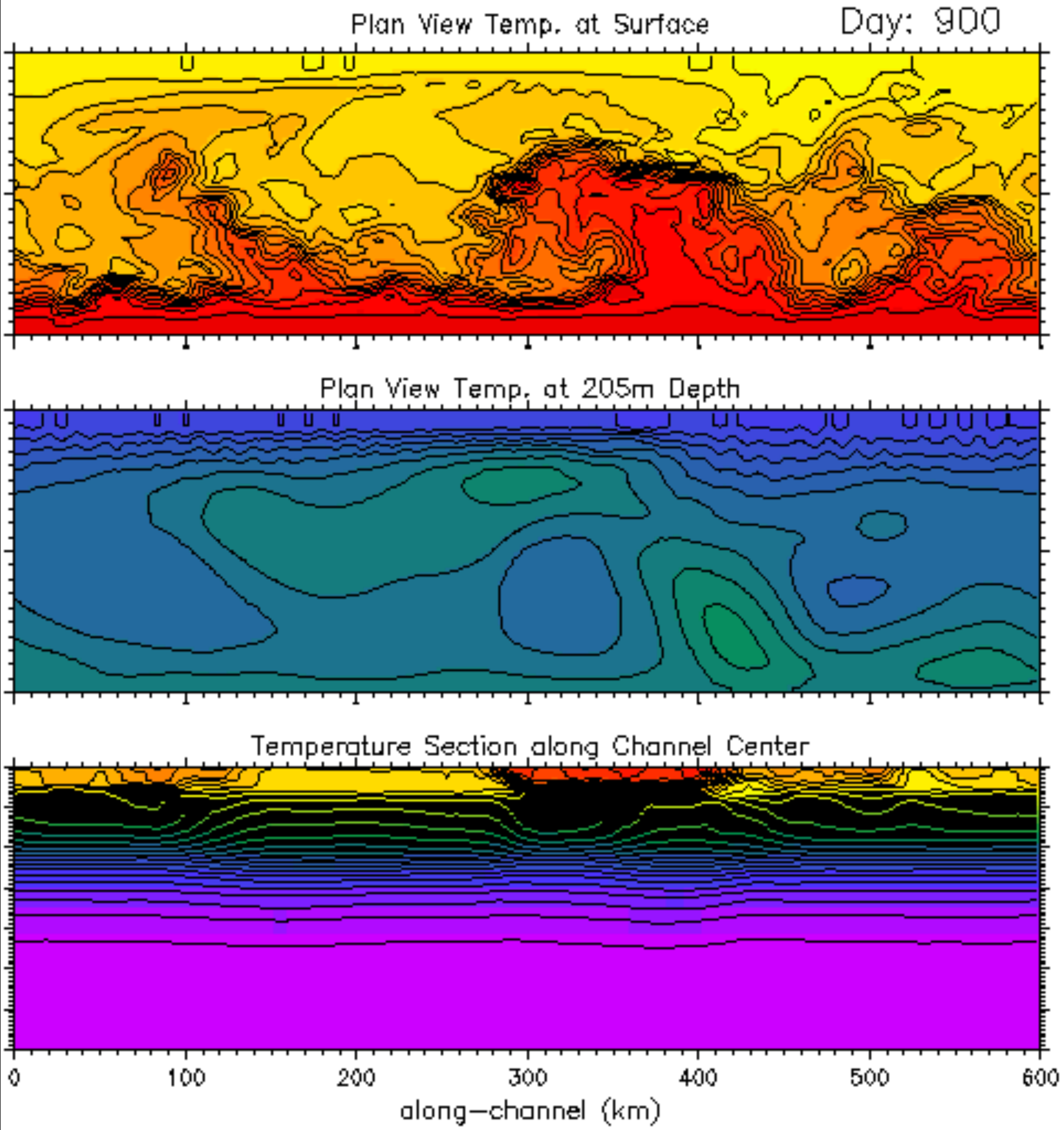
B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. Journal of Physical Oceanography, 38(6): 1145-1165, 2008.

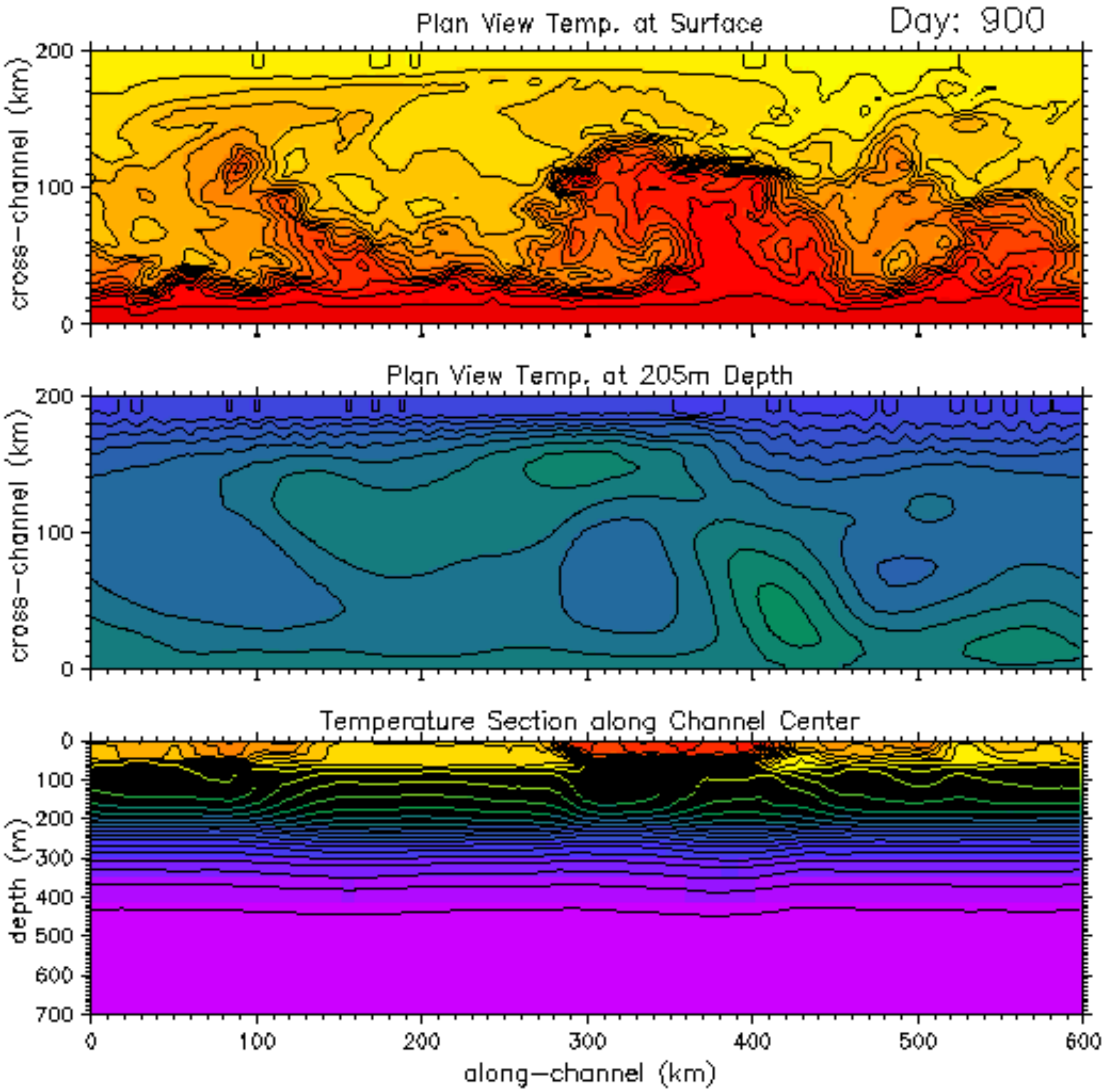
Horizontal fluxes are Mesoscale and tend to stir

B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. Journal of Physical Oceanography, 38(6):1145-1165, 2008.

Having a Mixed Layer Counts!

The vertical buoyancy flux in the ML (<w'b'>) without diurnal cycle is not less than with cycle (ML)





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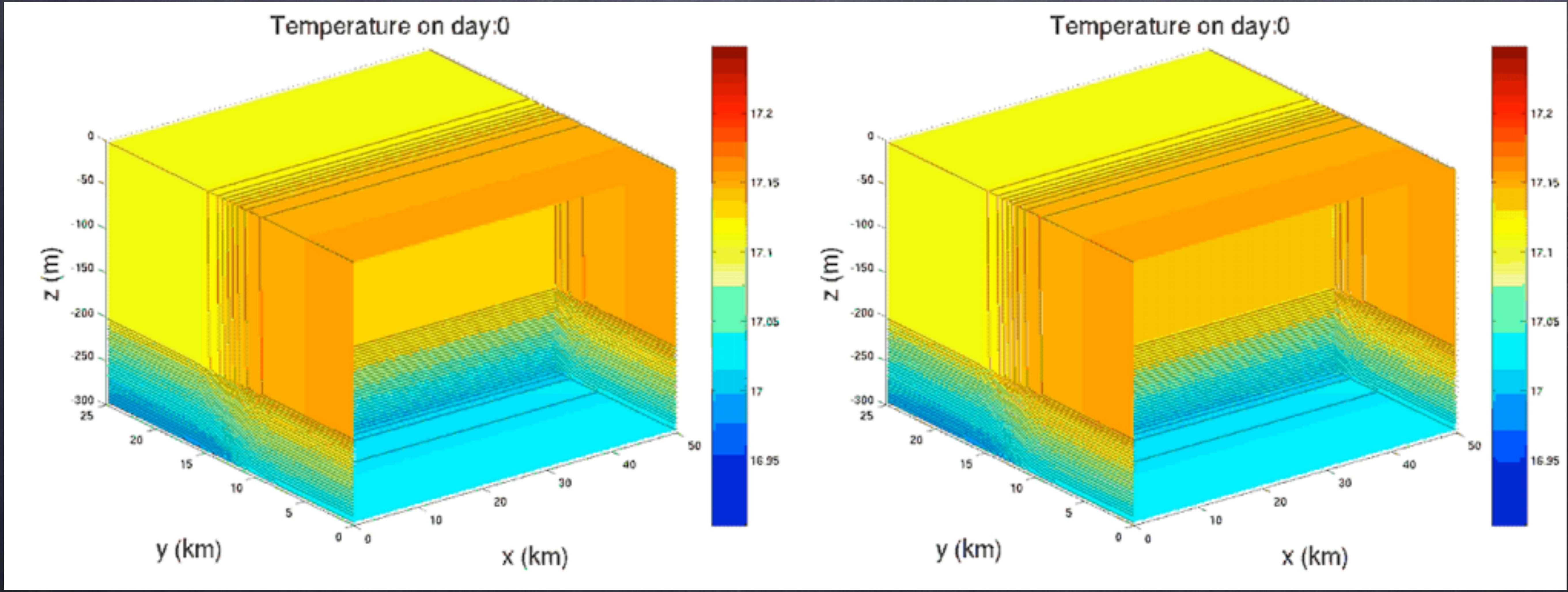
Prototype: Mixed Layer Front Adjustment

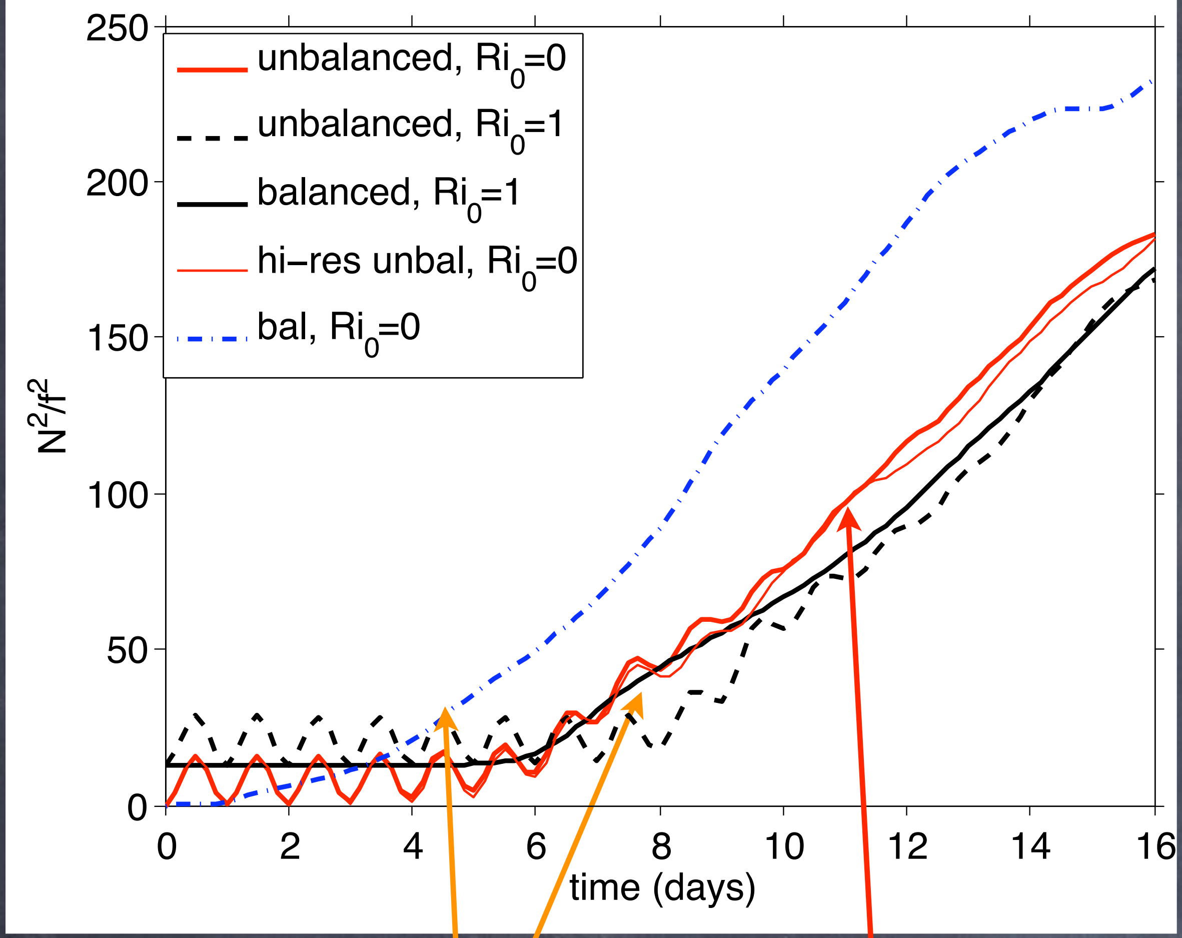


Simple Spindown

Plus, Diurnal Cycle and KPP

Note: initial geostrophic adjustment overwhelmed by eddy restratification

Parameterization of Finite Amp. Eddies: Ingredients

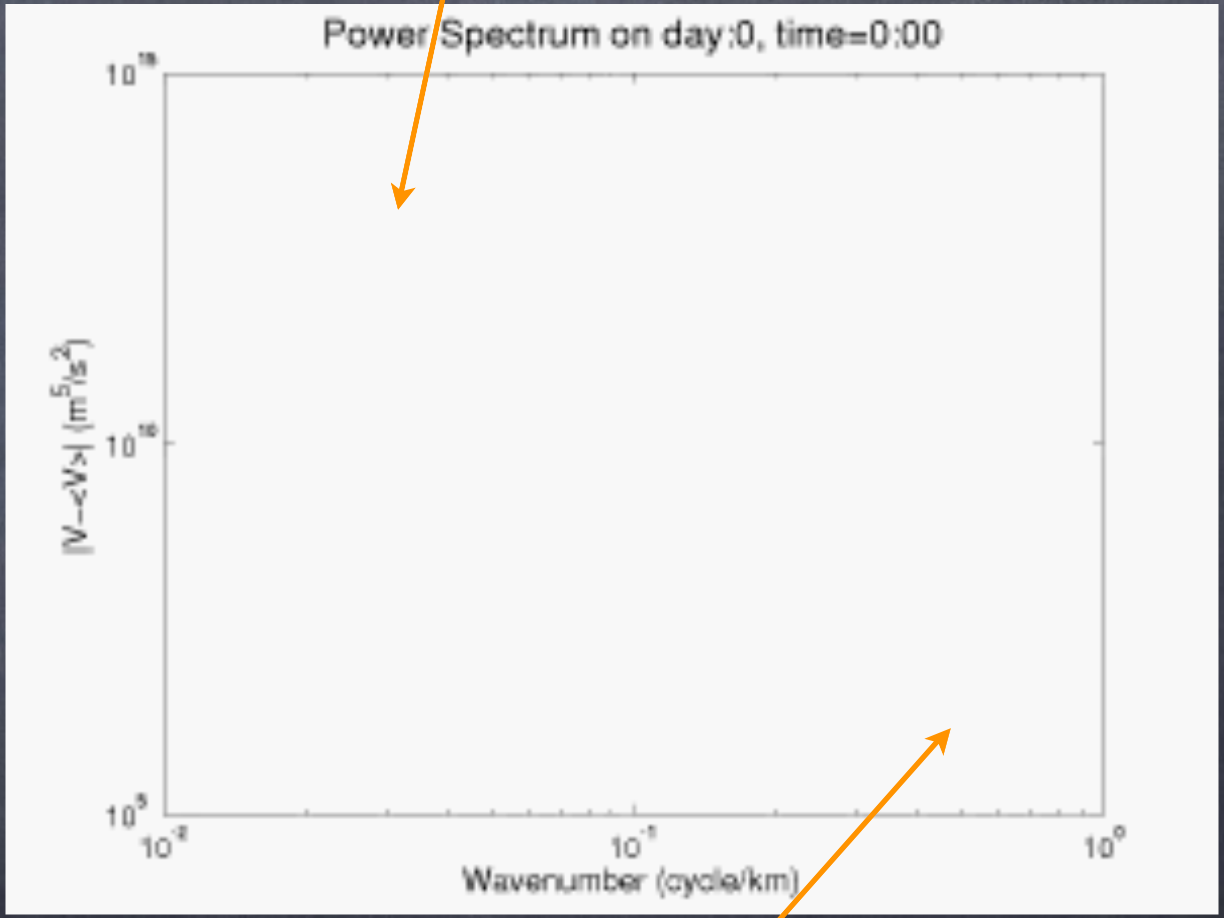


Resolution Convergence

Eddies at Finite Amplitude

Power Spectrum of KE

At Finite Amplitude Horizontal Scale Unclear



Initially, Linear Prediction of Lengthscale good

B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. Journal of Physical Oceanography, 38(6):1145-1165, 2008.

Inverse Cascade => No Results from Linear Instability Ingredients

What lengthscale dominates <w′b′>?

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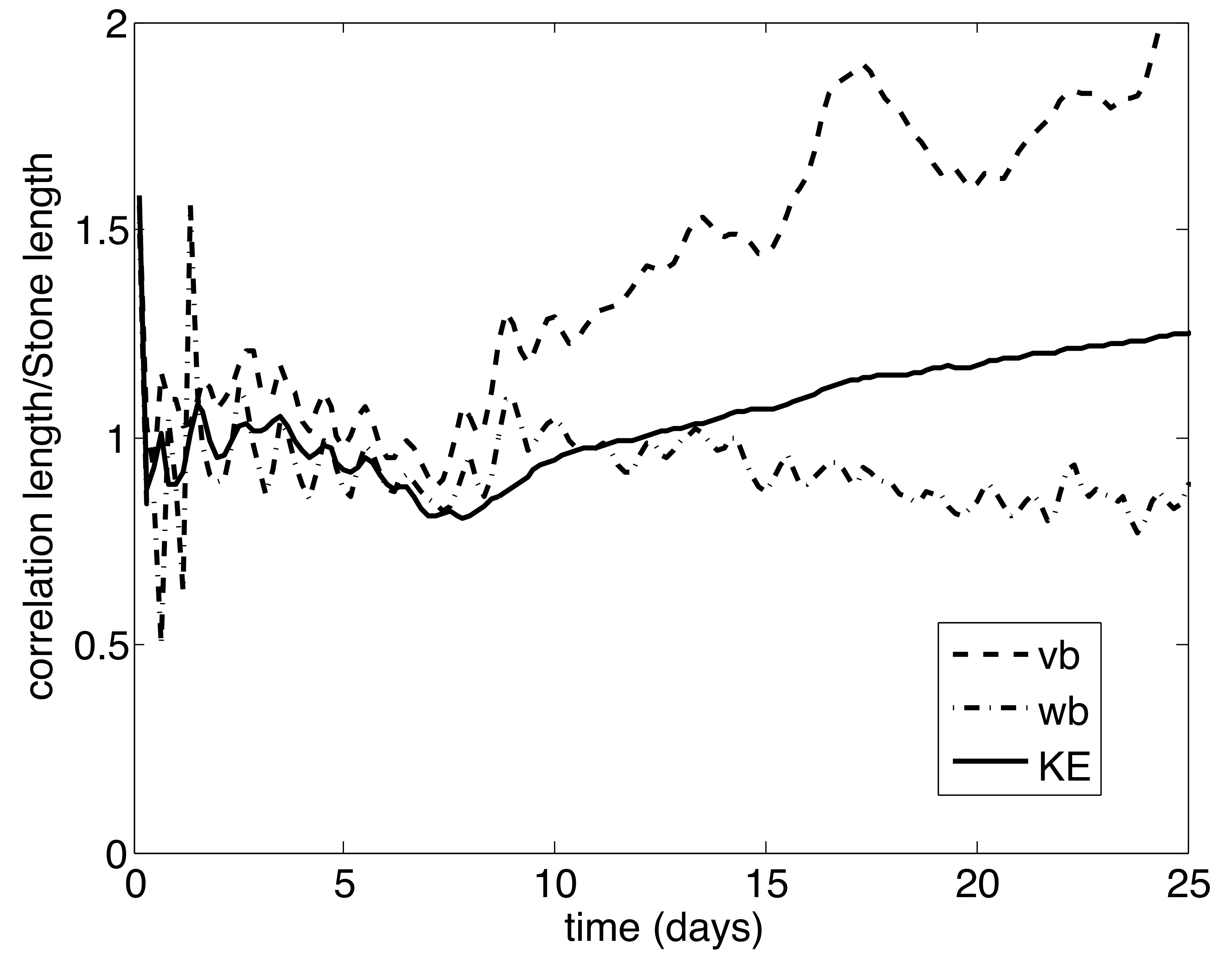
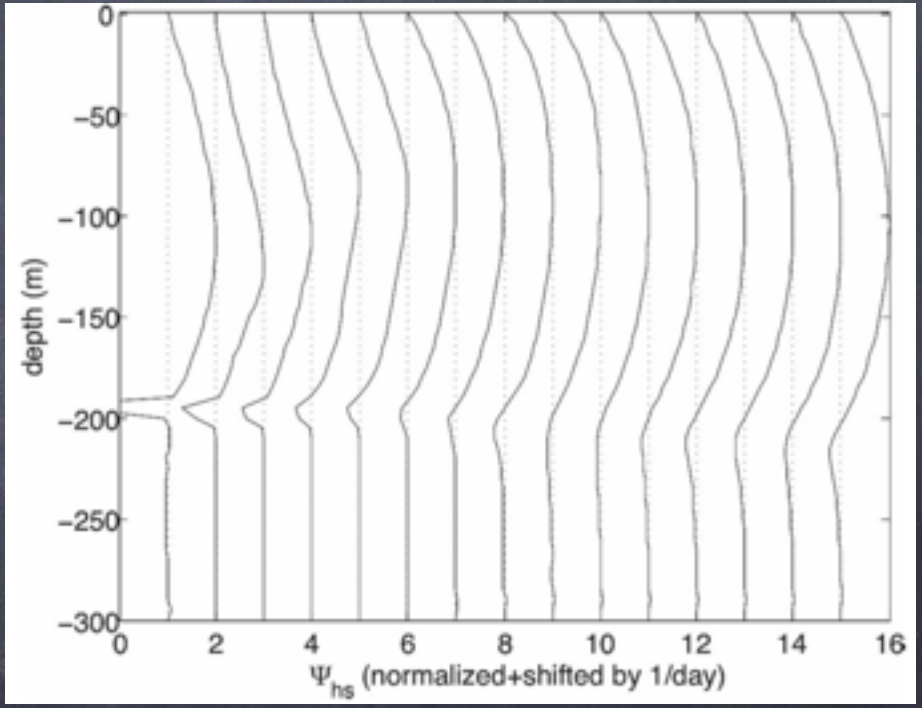


vb

KE

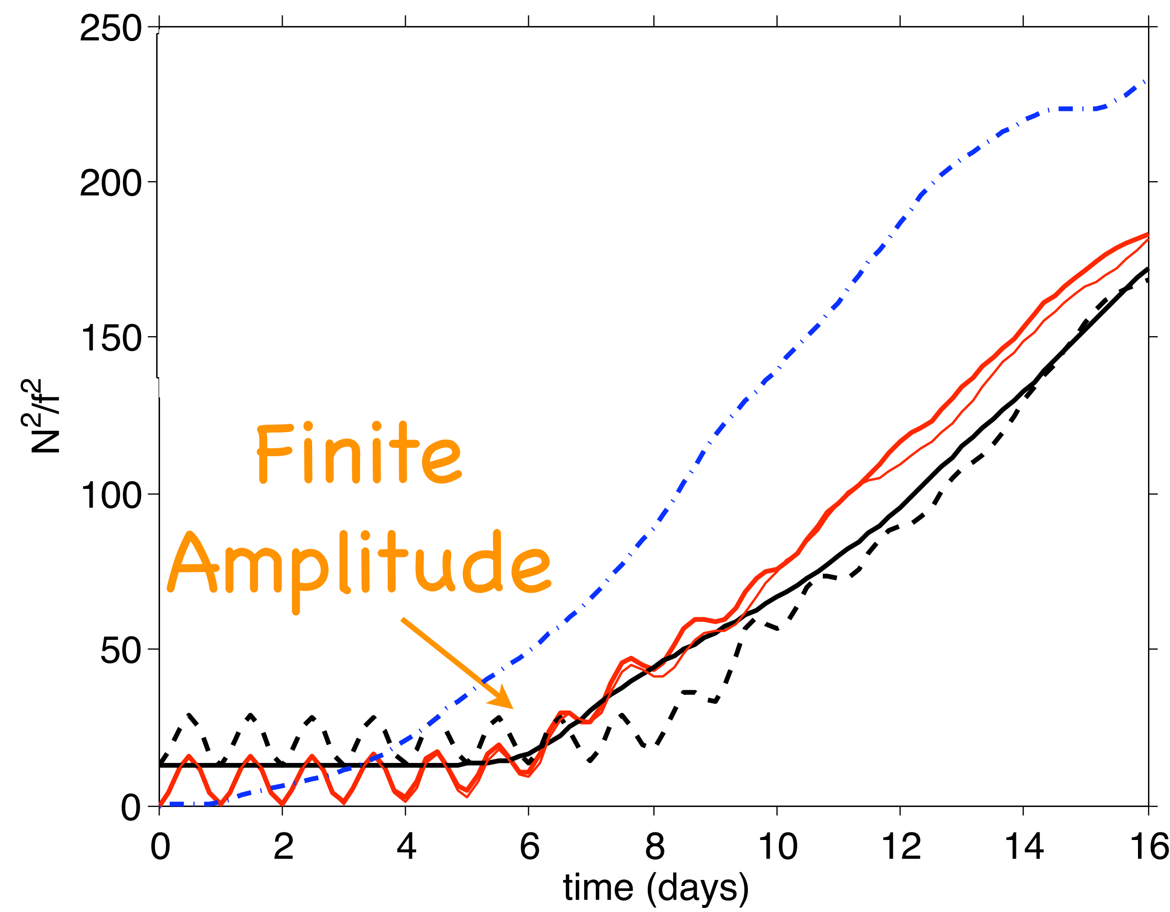
wb



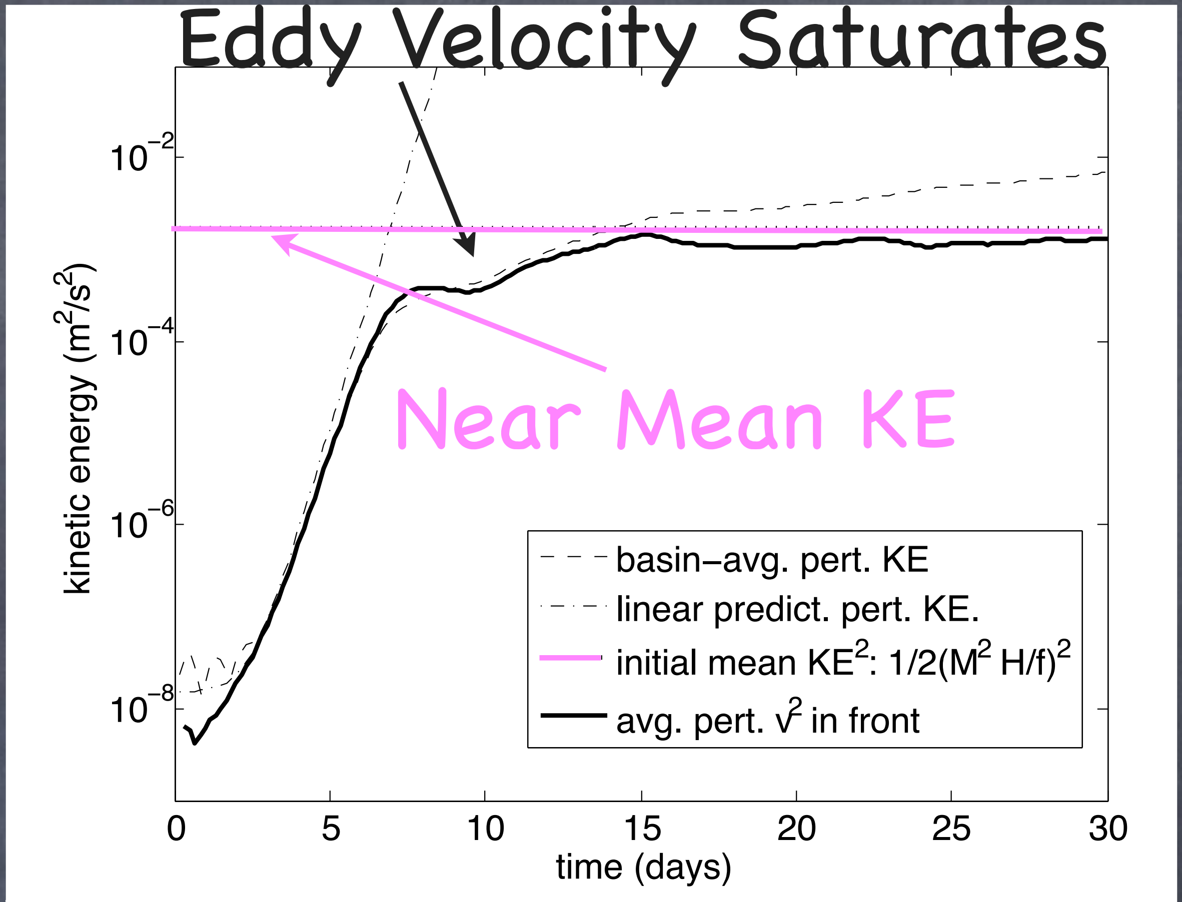
Stone fastest-mode Soln OK!

$$\mu(z) = \left[1-\left(\frac{2z}{H}+1\right)^2\right]\left[1+\frac{5}{21}\left(\frac{2z}{H}+1\right)^2\right]$$

Parameterization of Finite Amp. Eddies: Ingredients

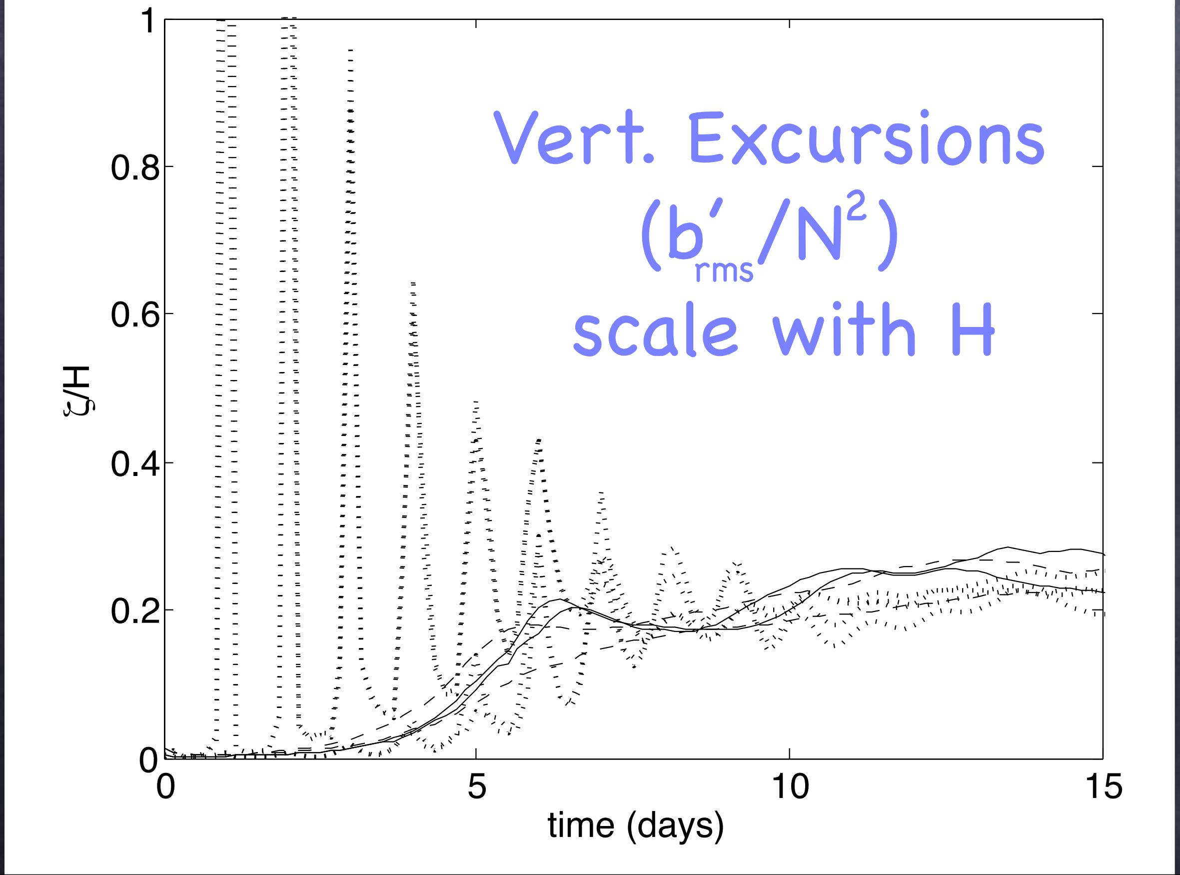


Finite Amplitude



Eddy Velocity Saturates

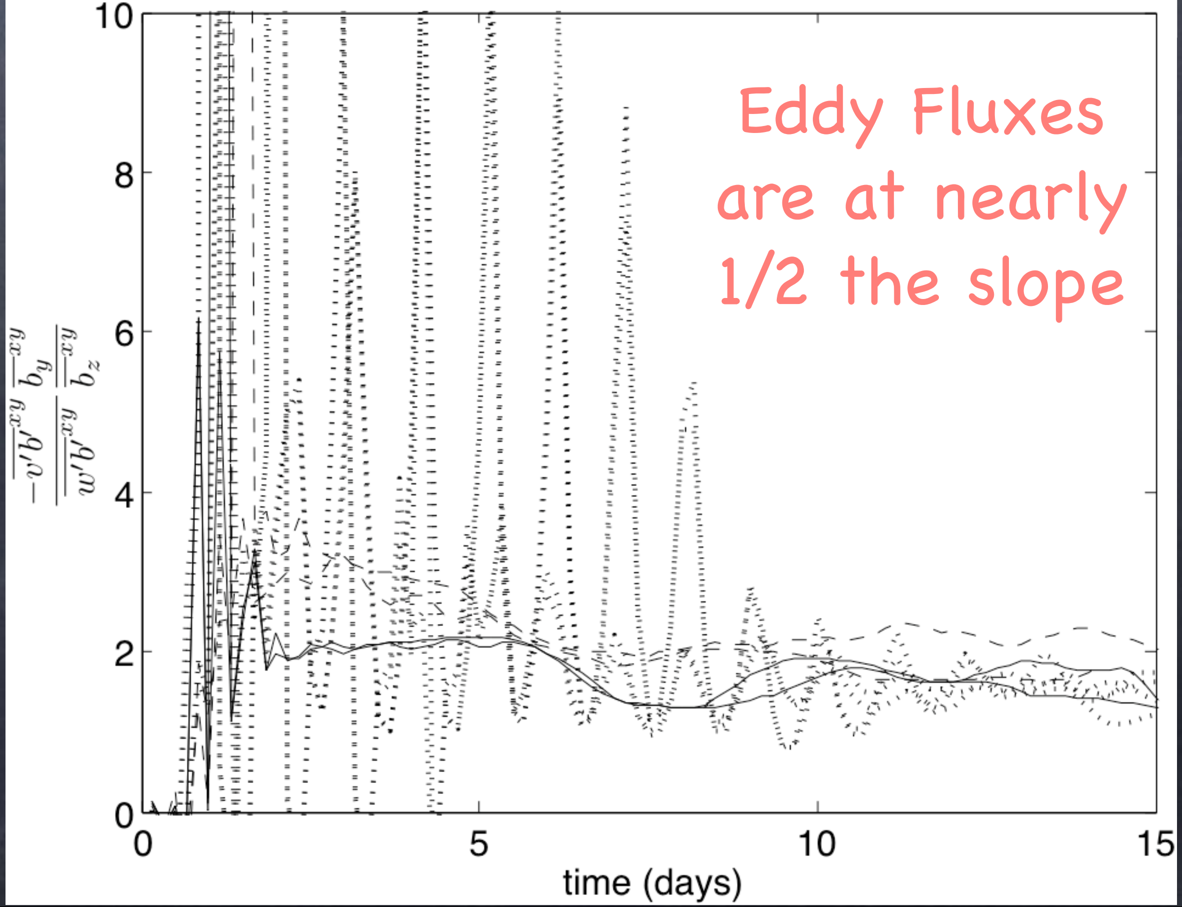
Near Mean KE



Vert. Excursions

(b'_{rms}/N^2)

scale with H



Eddy Fluxes are at nearly 1/2 the slope

B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. Journal of Physical Oceanography, 38(6):1145-1165, 2008.

Magnitude Analysis: Vert. Fluxes

Extraction of potential energy by submesoscale eddies:

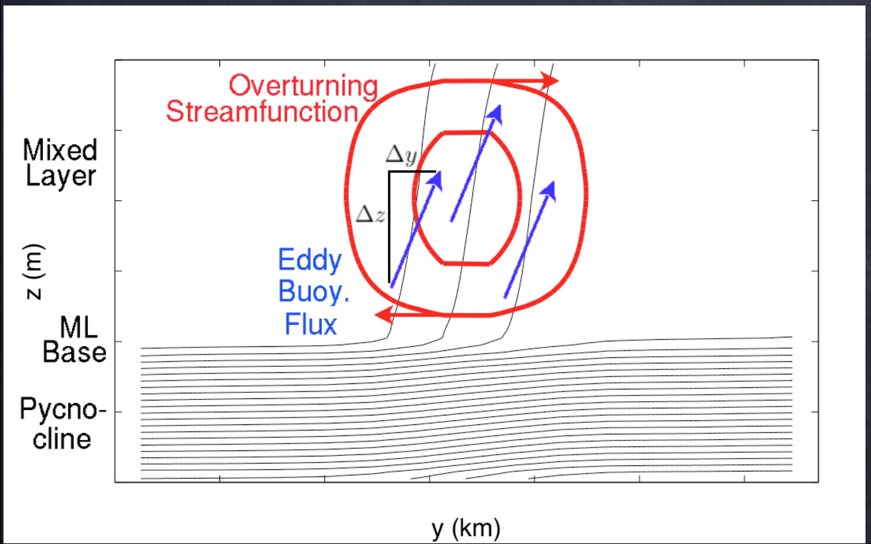
$$-\langle wb\rangle = \frac{\partial\langle PE\rangle}{\partial t} \approx \frac{\Delta PE}{\Delta t} \propto \frac{\Delta z\Delta b}{\Delta t}$$

Buoy. diff just parcel exchange of large-scale buoy.

Flux slope scales with the buoy. slope: $\frac{\Delta y}{\Delta z} \propto \frac{-\frac{\partial\bar b}{\partial z}}{\frac{\partial\bar b}{\partial y}}$

Time scale is turnover time from mean thermal wind:

Vertical scale known: $\Delta z \propto H$



$$\langle wb\rangle \propto \frac{\Delta z\left(\Delta y\frac{\partial\bar b}{\partial y} + \Delta z\frac{\partial\bar b}{\partial z}\right)}{\Delta t}$$

$$\langle wb\rangle \propto \frac{H^2}{|f|}\left[\frac{\partial\bar b}{\partial y}\right]^2$$

Fox-Kemper et al., 2007

B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. Journal of Physical Oceanography, 38(6):1145-1165, 2008.

Eddies effect a largely adiabatic transfer: thus representable by a streamfunction

$$\mathbf{\Psi} \propto \frac{H^2 \nabla \bar{b} \times \hat{\mathbf{z}}}{|f|} \longrightarrow \overline{\mathbf{u}' b'} \equiv \mathbf{\Psi} \times \nabla \bar{b}$$

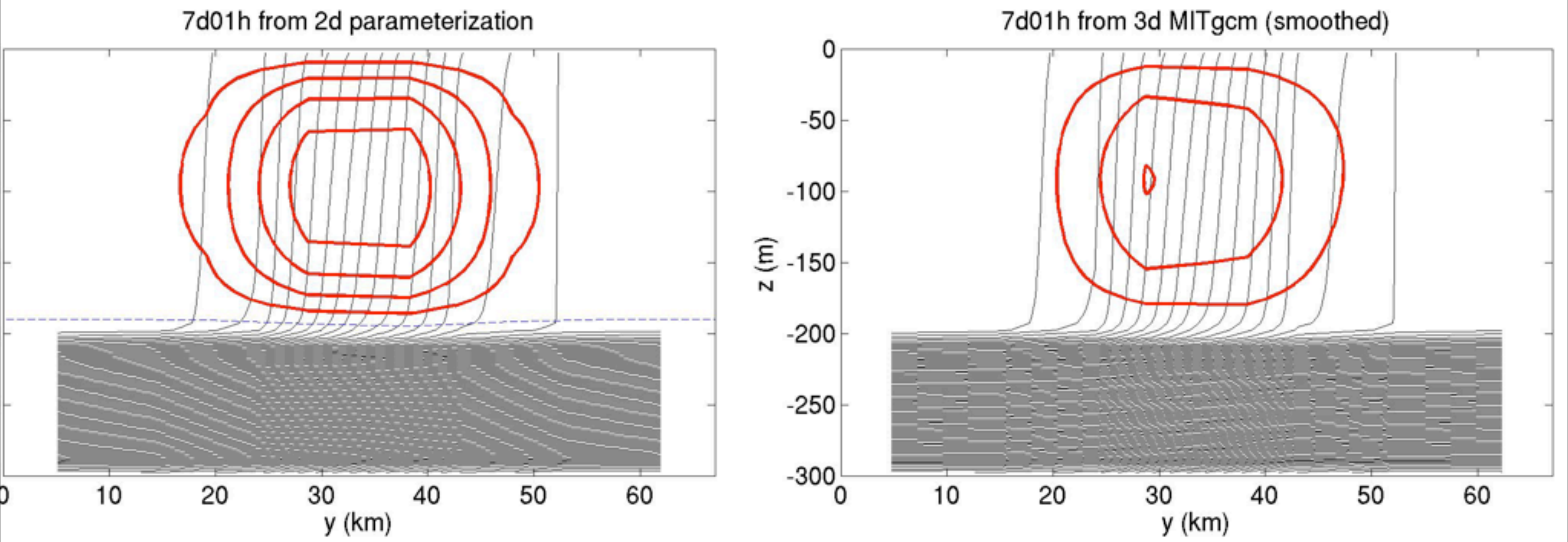
For a consistently upward,

$$\overline{w' b'} \propto \frac{H^2}{|f|} \left|\nabla_H \bar{b}\right|^2$$

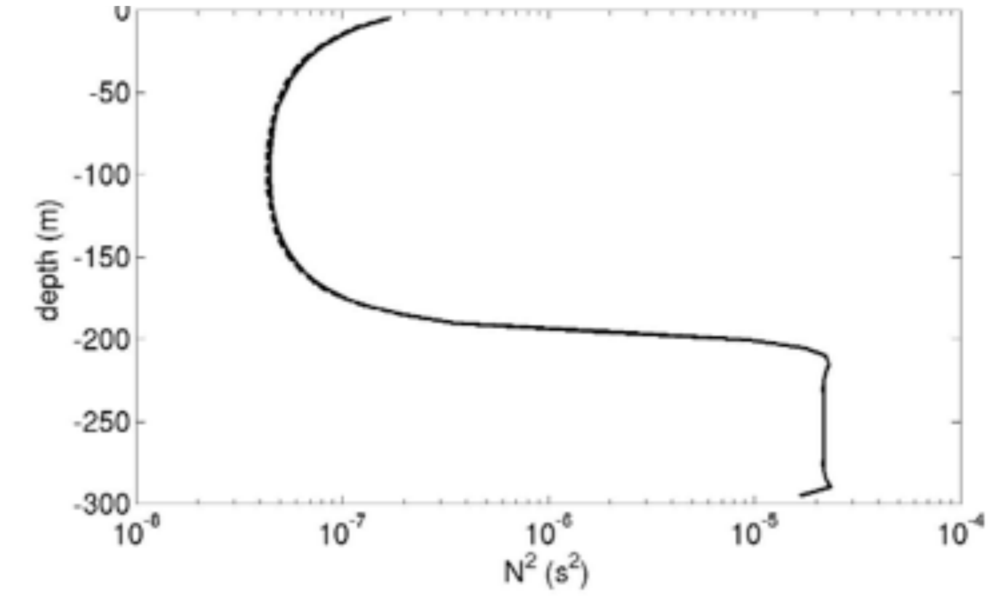
And horizontally downgradient flux.

$$\overline{\mathbf{u}'_H b'} \propto \frac{-H^2 \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla_H \bar{b}$$

What does it look like?



N^2



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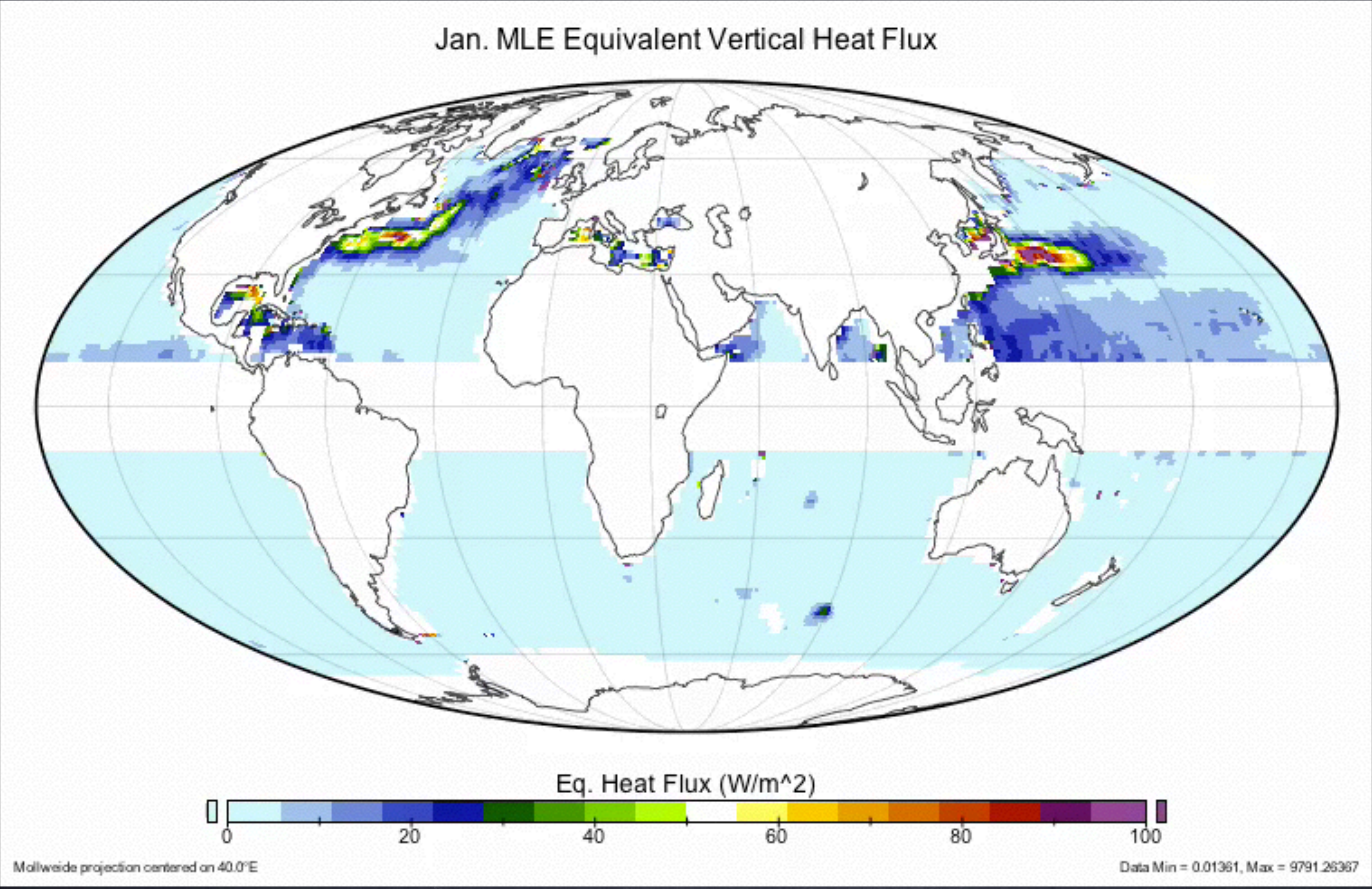

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A Global Parameterization of Mixed Layer Eddy Restratification
with scale-aware parameters
validated against simulations

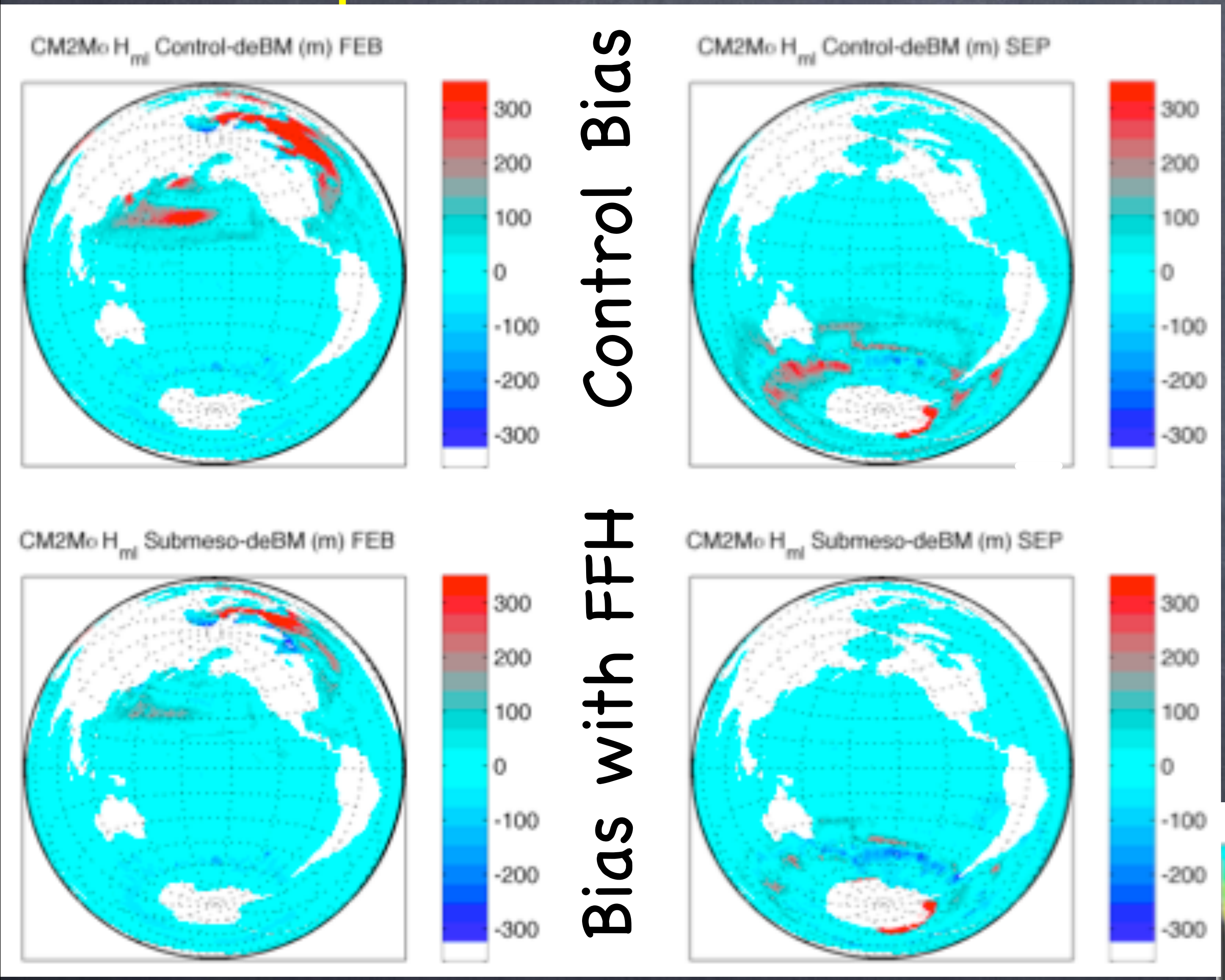
$$\boxed{\mathbf{\Psi} = \left[\frac{\Delta x}{L_f}\right] \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla \bar{b} \times \hat{\mathbf{z}}}$$

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. Ocean Modelling, 39:61-78, 2011.

$$\mu(z) = \left[1 - \left(\frac{2z}{H} + 1\right)^2\right] \left[1 + \frac{5}{21} \left(\frac{2z}{H} + 1\right)^2\right]$$

Physical Sensitivity of Ocean Climate to Submesoscale Eddy Restratification:

FFH implemented in CCSM (NCAR), CM2M & CM2G (GFDL)

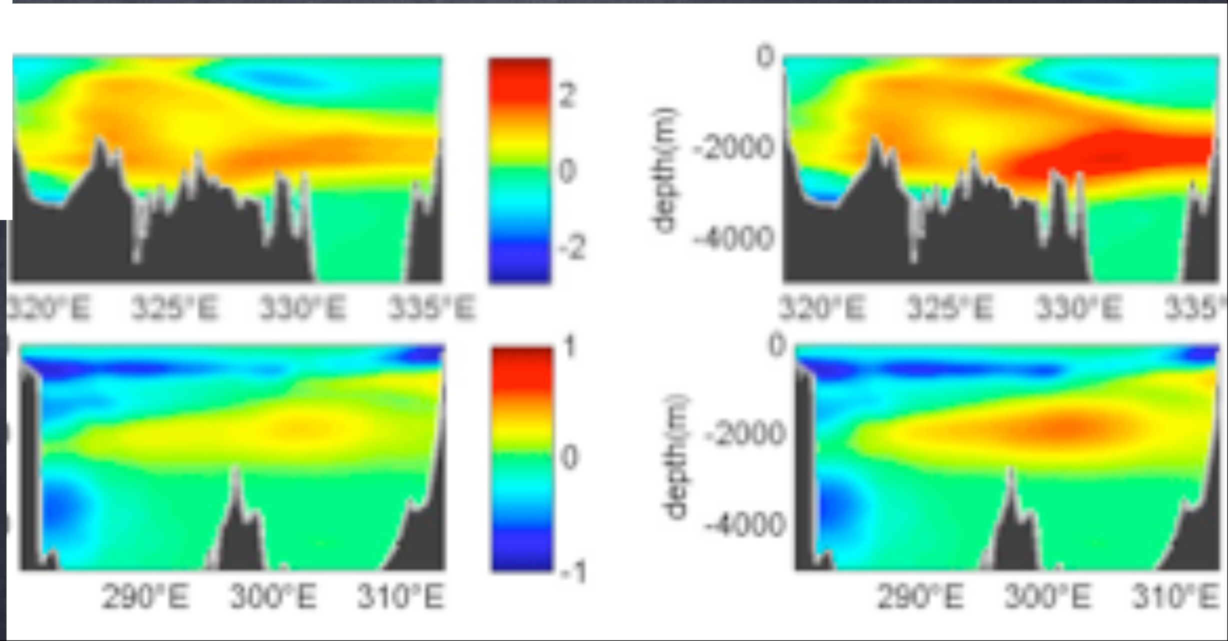


NO RETUNING NEEDED!!!

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. Ocean Modelling, 39:61-78, 2011.

Improves CFCs

Bias with FFH Control Bias

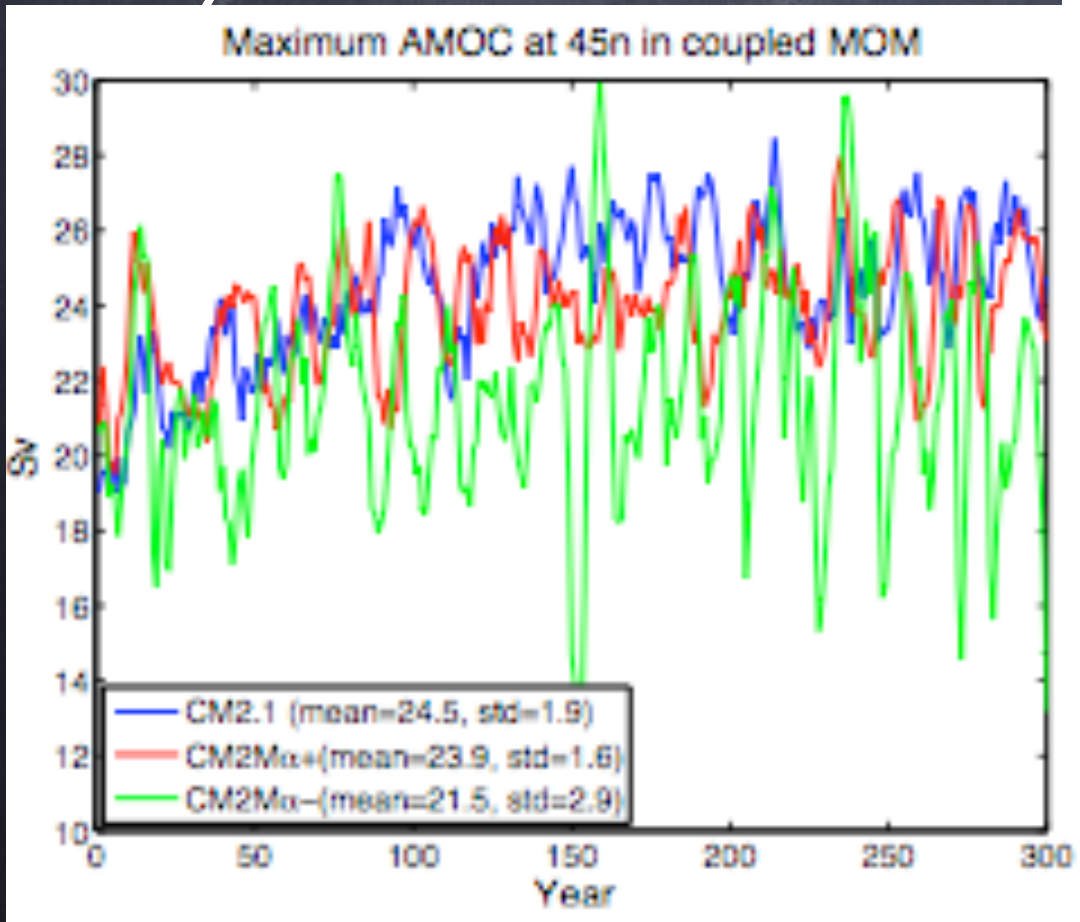


Deep ML Bias reduced

From Fox-Kemper et al., 2011

Sensitivity of Climate to Submeso: AMOC & Cryosphere Impacts

May Stabilize AMOC



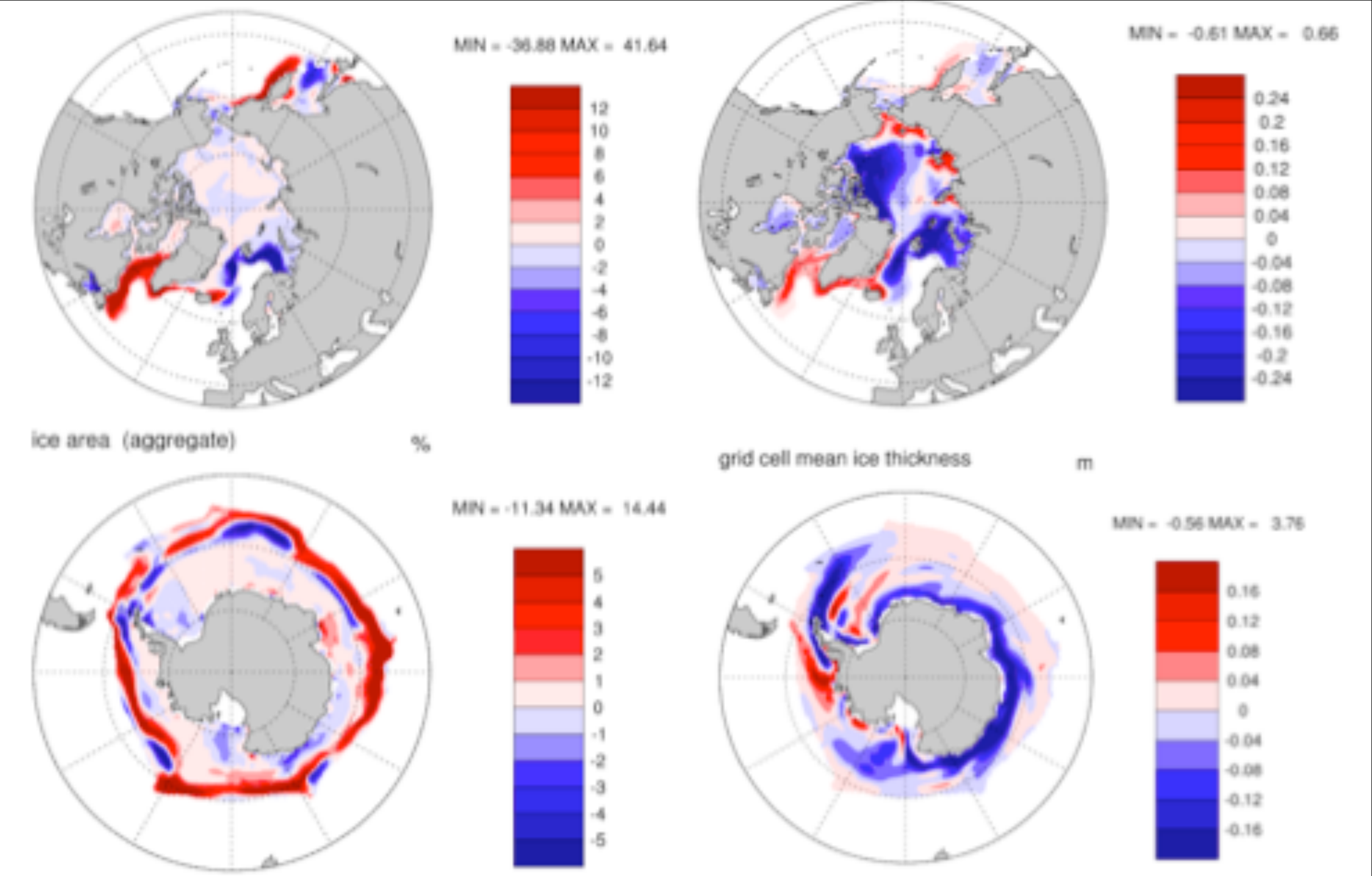


Figure 10: Wintertime sea ice sensitivity to introduction of MLE parameterization ($CCSM^+$ minus $CCSM^-$): January to March Northern Hemisphere a) ice area and b) thickness and July to September Southern Hemisphere c) ice area and d) thickness.

Affects sea ice

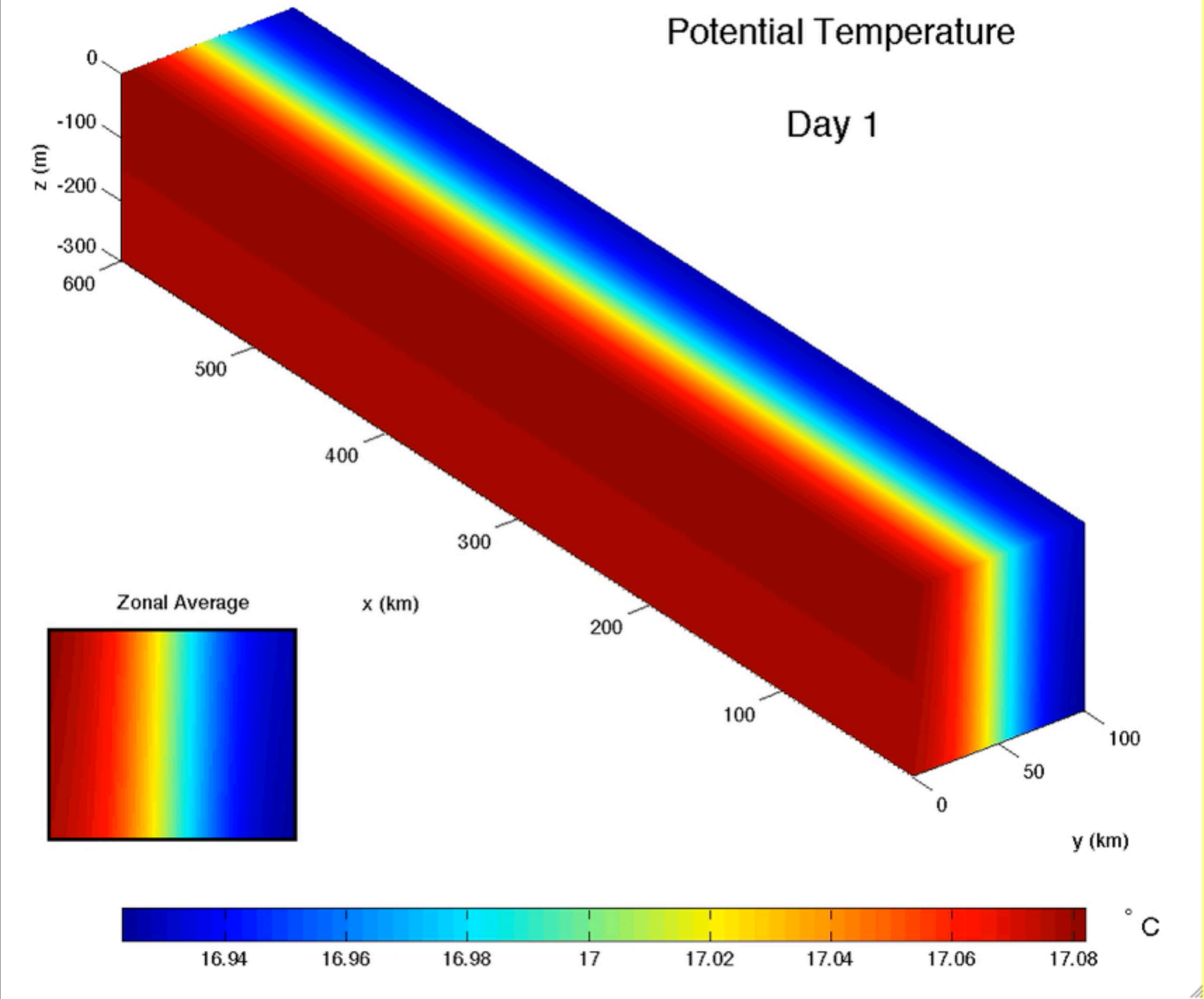
B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. Ocean Modelling, 39:61-78, 2011.

NO RETUNING NEEDED!!!

These are impacts: bias change unknown

Next few slides are
all from S. Bachman

Movie: S. Bachman



This Slide & Movies:
S. Bachman

S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013.

How do we solve for R ?

There is a fundamental issue in trying to solve for a tensor…

What happens if we have only one tracer?

$$\boxed{\overline{u'b'} = -R\nabla\overline{b}}$$

Take a zonal average, and
write the system out in full:

$$\boxed{\begin{aligned}\overline{v'b'} &= -R_{11}\overline{b}_y - R_{12}\overline{b}_z \\ \overline{w'b'} &= -R_{21}\overline{b}_y - R_{22}\overline{b}_z\end{aligned}}$$

2 Equations…

4 Unknowns!

Underdetermined! (not unique)

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How do we solve for R ?

To overcome this issue…

Use multiple tracers:

$$\overline{u_i'\tau_\pi'} = -R_{ij}\nabla_j\bar{\tau}_\pi$$

$$\overline{u_i'\tau_\pi'}(\nabla_j\bar{\tau}_\pi)^{-1} = -R_{ij}$$

> 4 Equations…

4 Unknowns!

Overdetermined!

Moore-Penrose pseudoinverse (least-squares fit)

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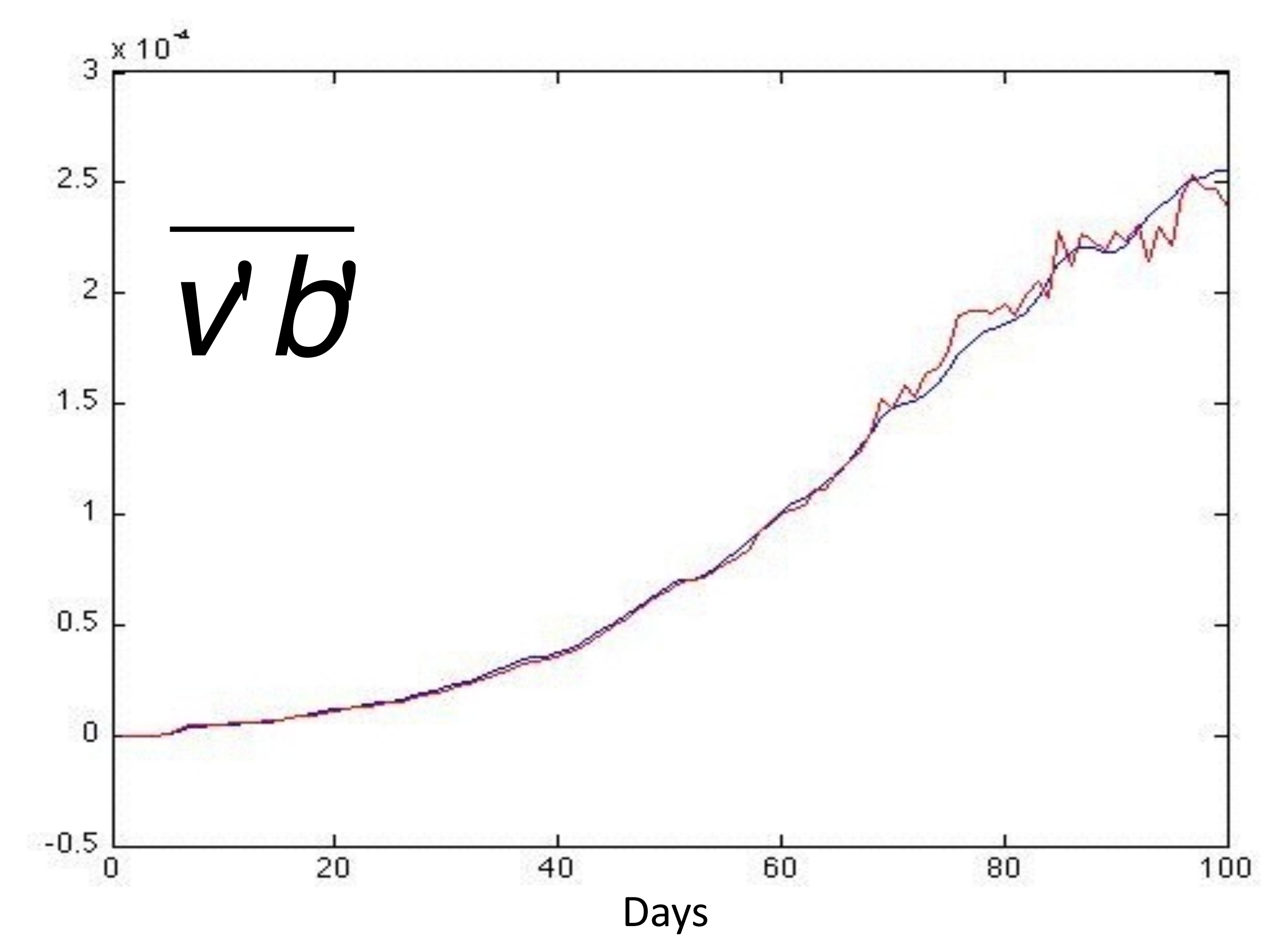
Tracer gradients less aligned = better LS fit!

Overdetermining the system is appropriate to reduce degrees of freedom in the zonal average.

The reconstruction is excellent.



🟦 Original fluxes
🟥 Reconstructed fluxes

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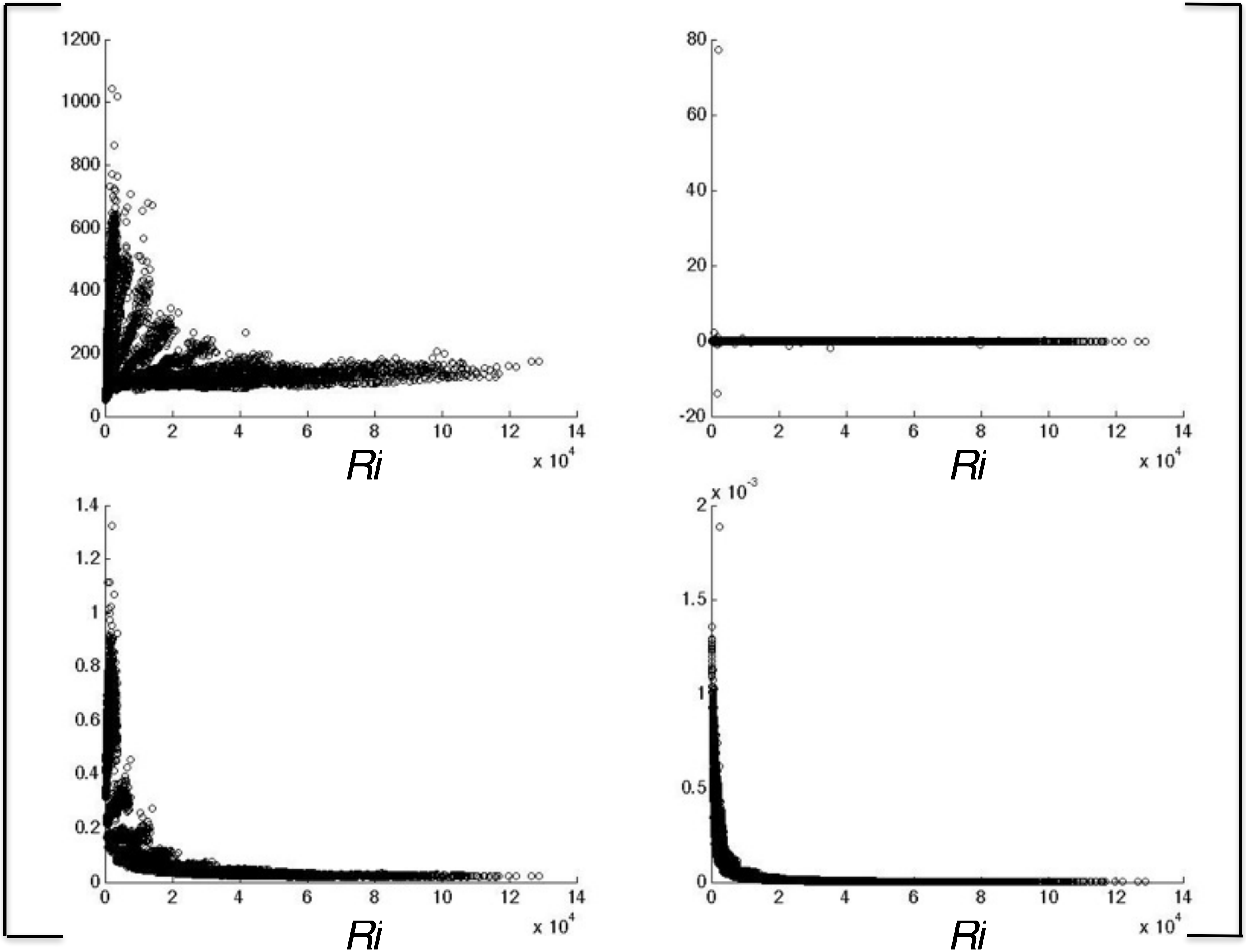
Estimates of these buoyancy fluxes have improved substantially (error is now < 10%)*… Used to be that getting error within a factor of two was the best we could do!

* - We can get it to around 1% if we use lots of tracers!

RAW OUTPUT

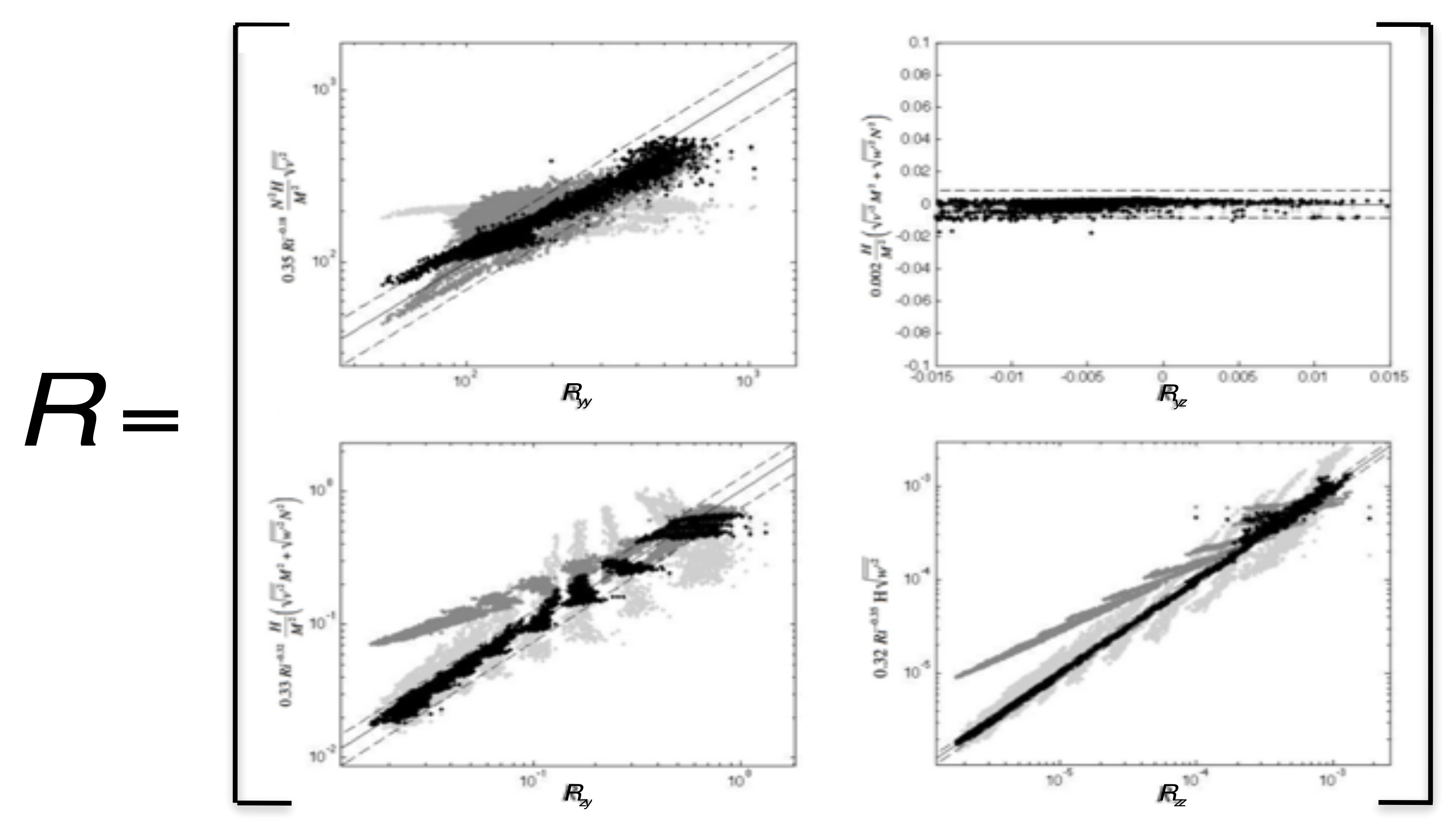
S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, April 2013.

$R=$



SCALED OUTPUT

S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, April 2013.



Black dots – Best possible fit, with velocity variances and power of *Ri*

Dark grey dots – Best possible fit with velocity variance, w/o power of *Ri*

Light grey dots – FFH08 scaling with power of *Ri*; no velocity variance

SCALED OUTPUT

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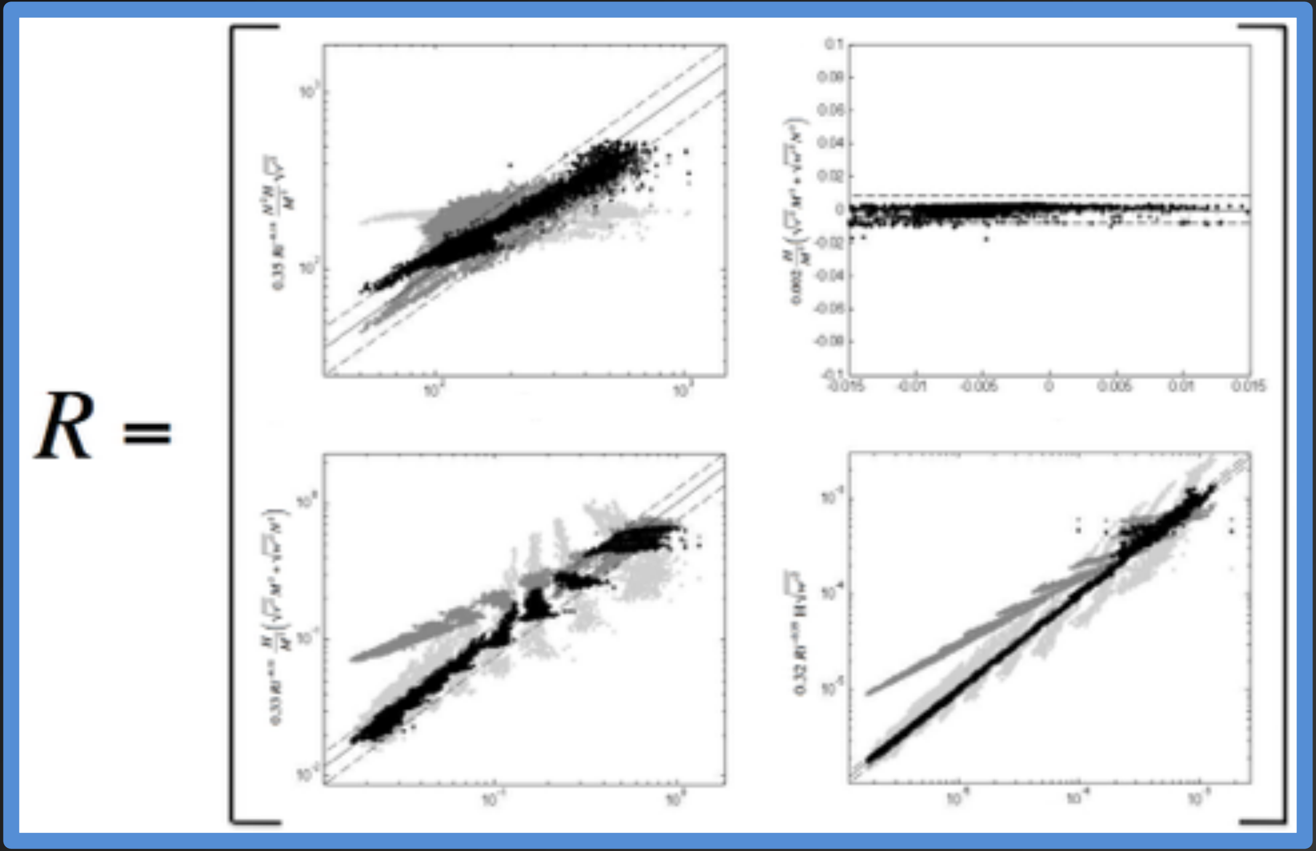
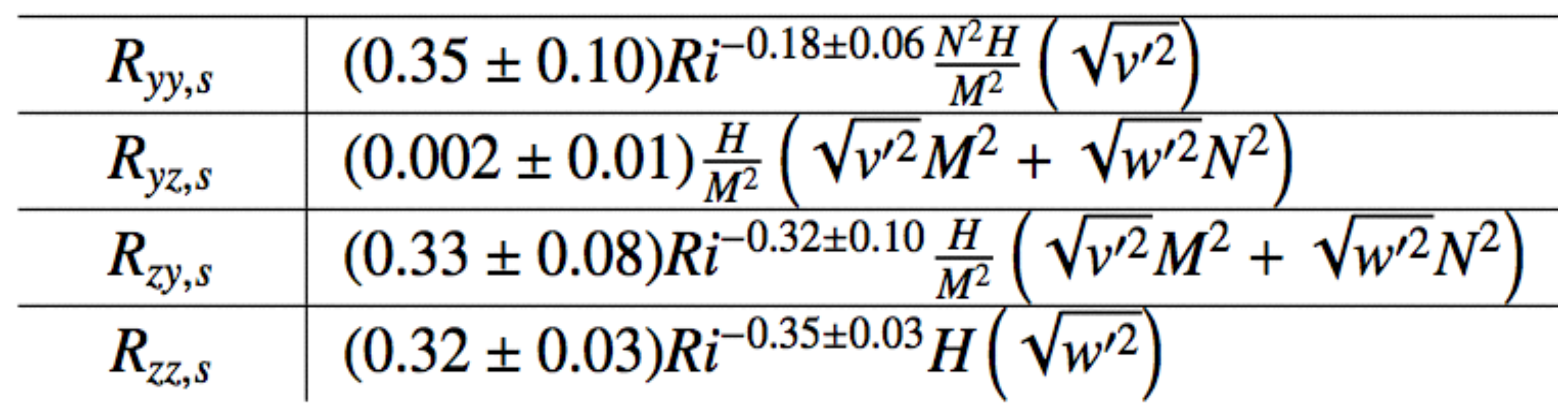
Our 69 simulations (~5000 data points) suggest scaling for R like so:





$R_{yy,s}$	$(0.35 \pm 0.10)Ri^{-0.18\pm0.06}\frac{N^2H}{M^2}\left(\sqrt{v'^2}\right)$
$R_{yz,s}$	$(0.002 \pm 0.01)\frac{H}{M^2}\left(\sqrt{v'^2}M^2 + \sqrt{w'^2}N^2\right)$
$R_{zy,s}$	$(0.33 \pm 0.08)Ri^{-0.32\pm0.10}\frac{H}{M^2}\left(\sqrt{v'^2}M^2 + \sqrt{w'^2}N^2\right)$
$R_{zz,s}$	$(0.32 \pm 0.03)Ri^{-0.35\pm0.03}H\left(\sqrt{w'^2}\right)$

So, Waves can Drive turbulence that affects large scale:

What about direct effects of waves on larger scales?

Stokes Coriolis & Stokes Vortex Forces on Submesoscales

Recall, Subinertial Boussinesq Equations Dominated by:

(Combined) Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b$$

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

So, Waves can Drive turbulence that affects large scale:

What about direct effects of waves on larger scales?
Stokes Coriolis & Stokes Vortex Forces on Submesoscales

Craik-Leibovich Boussinesq Subinertial Dominated By:

(Combined) Lagrangian Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial}{\partial z}(\mathbf{v} + \mathbf{v}_s) = \mathbf{f} \times \frac{\partial \mathbf{v}_L}{\partial z} = -\nabla b$$

Now the buoyancy gradients govern the Lagrangian flow, not the not the Eulerian!

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

Craik-Leibovich Boussinesq Subinertial Dominated By:

(Combined) Lagrangian Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial \mathbf{v}_L}{\partial z} = -\nabla b$$

Now the buoyancy gradients govern the Lagrangian flow, not the not the Eulerian!

Buoyancy & PV also advected by Lagrangian Flow!

All GFD is for the Lagrangian Flow??

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

Can we just forget the whole thing and interpret large scales as Lagrangian velocities?

$$[\mathbf{f} + \nabla \times \mathbf{v}] \times \frac{\partial}{\partial z}(\mathbf{v} + \mathbf{v}_s) = -\nabla b$$

No, because vortex force is different!

The "Rossby #" for waves, is big *more often* than Ro is

Talk to Haney for more!!!

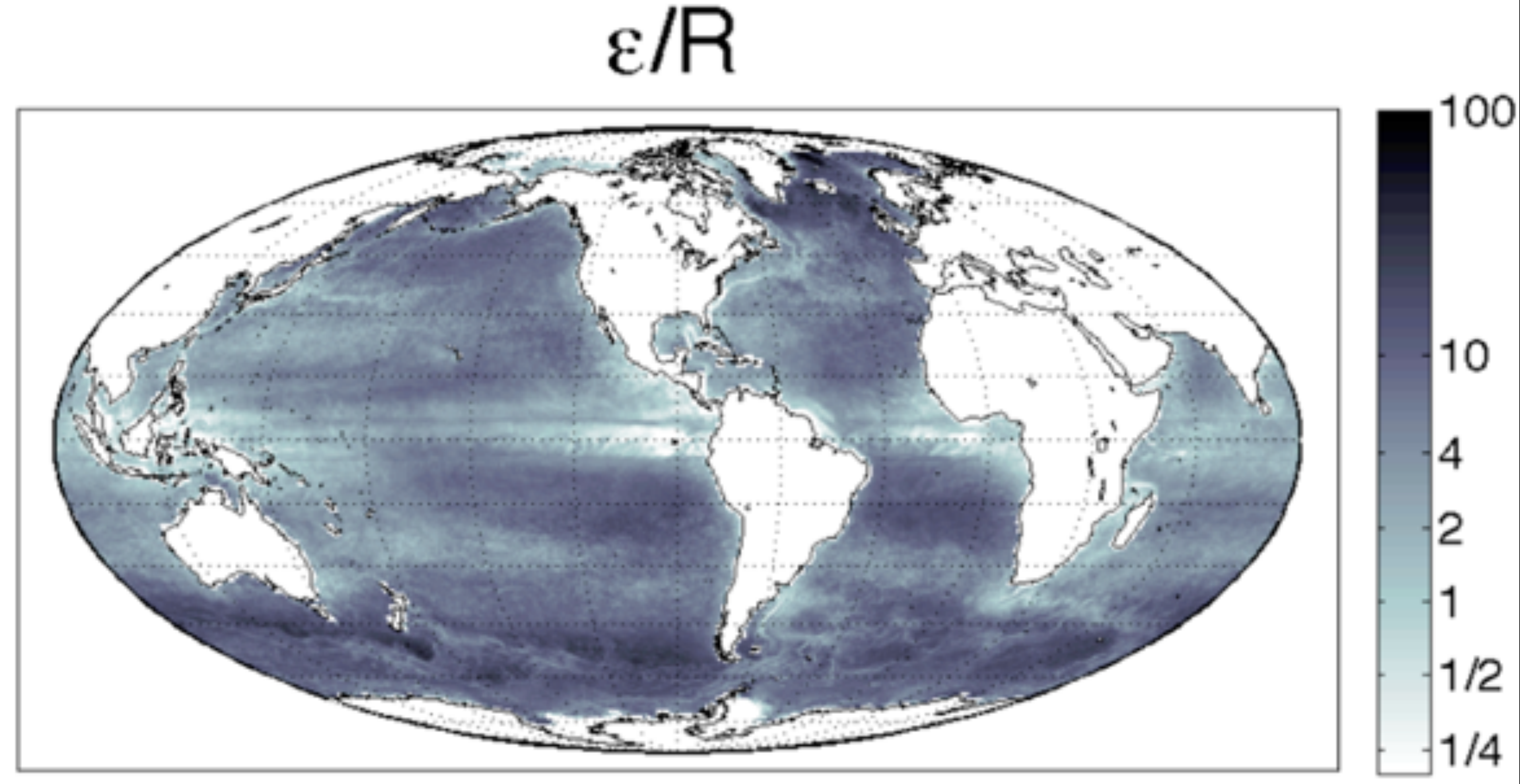


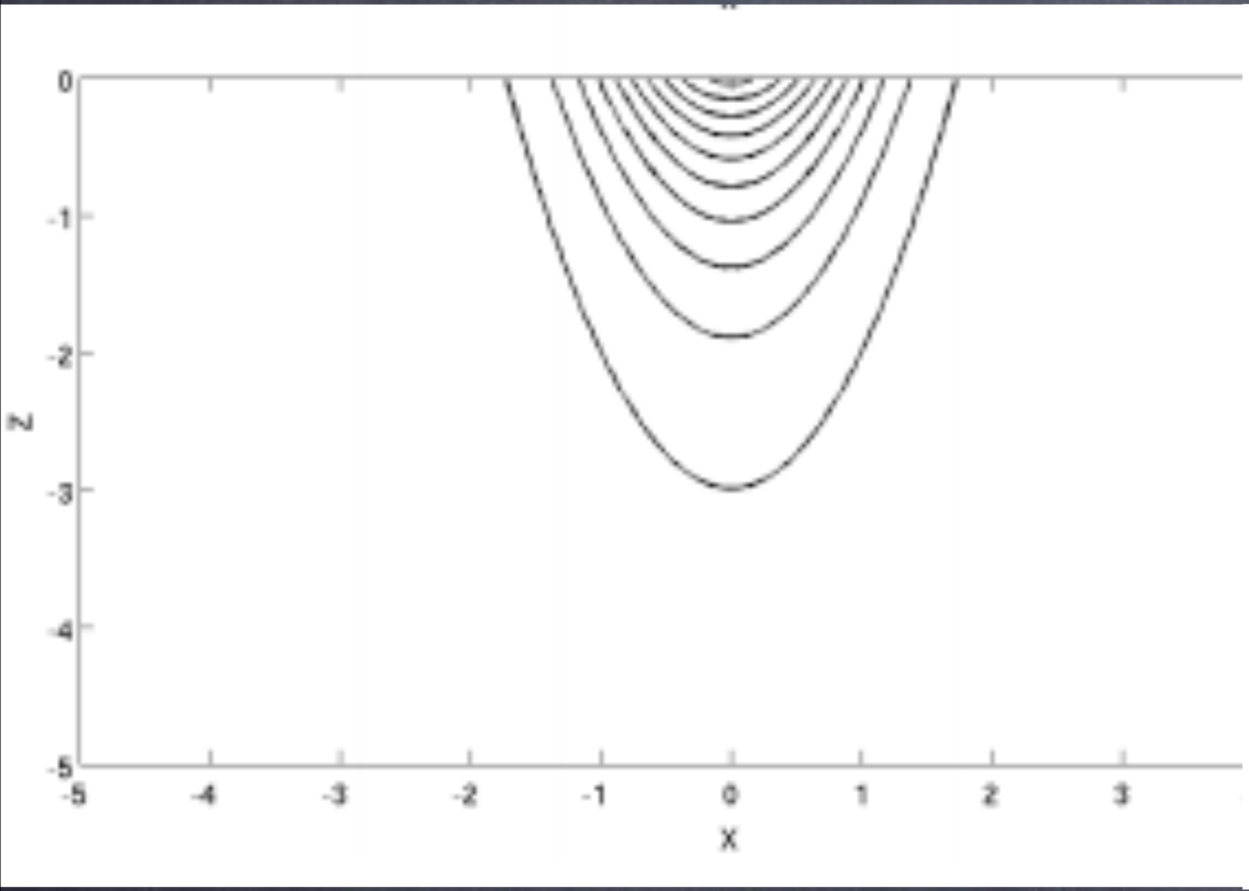
FIGURE 1. Estimated ratio $\epsilon/\mathcal{R} \approx (|\mathbf{u}_s \cdot \mathbf{u}|h)/(|\mathbf{u}|^2 h_s)$ governing the relative importance of Stokes effects versus nonlinearity. Eulerian velocity ($\mathbf{u}$) is taken as the AVISO weekly satellite geostrophic velocity or $-\mathbf{u}_s$ (for anti-Stokes flow) if $|\mathbf{u}_s| > |\mathbf{u}|$. The front/filament depth (h) is estimated as the mixed layer depth from the de Boyer Montégut *et al.* (2004) climatology. An exponential fit to the Stokes drift of the upper 9m projected onto the AVISO geostrophic velocity provides $\mathbf{u}_s \cdot \mathbf{u}$ and h_s. Stokes drift is taken from the WaveWatch-3 simulation described in Webb & Fox-Kemper (2011). $\mathbf{u}$, $\mathbf{u}_s$, and h_s are all for the year 2000, while h is from a climatology of observations over 1961-2008. The year 2000 average of $\epsilon/\mathcal{R}$ is shown.

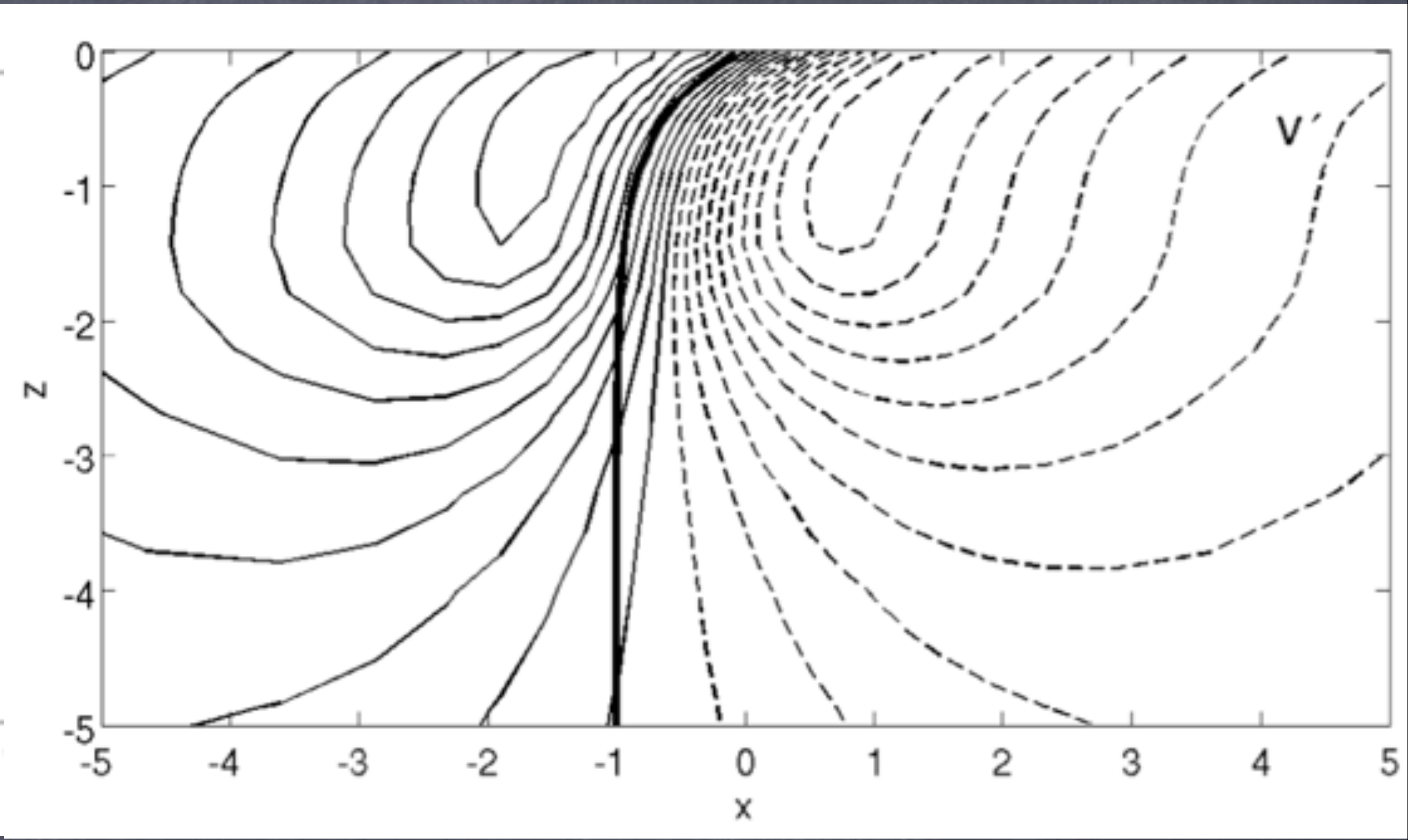
J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

Waves (Stokes Vortex Force) example of wave-balanced Submeso flow

$\epsilon = 2, \epsilon \gg \mathcal{R}$

Near the "sweet spot"





Initial Submeso Front

Contours: 0.1

Perturbation on that scale due to waves

Contours: 1.4

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 2013. Submitted.

What about Langmuir-Submeso Interactions?

Perform large eddy simulations (LES) of CLB with a submesoscale temperature front with winds--
with and without Stokes drift

$$\frac{\partial \rho}{\partial t} + \mathbf{u}_L \cdot \nabla \rho = \mathrm{SGS}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\boldsymbol{\omega} + f\hat{\mathbf{z}}) \times \mathbf{u}_L = -\nabla \pi - \frac{g\rho\hat{\mathbf{z}}}{\rho_0} + \mathrm{SGS}$$

Computational parameters:

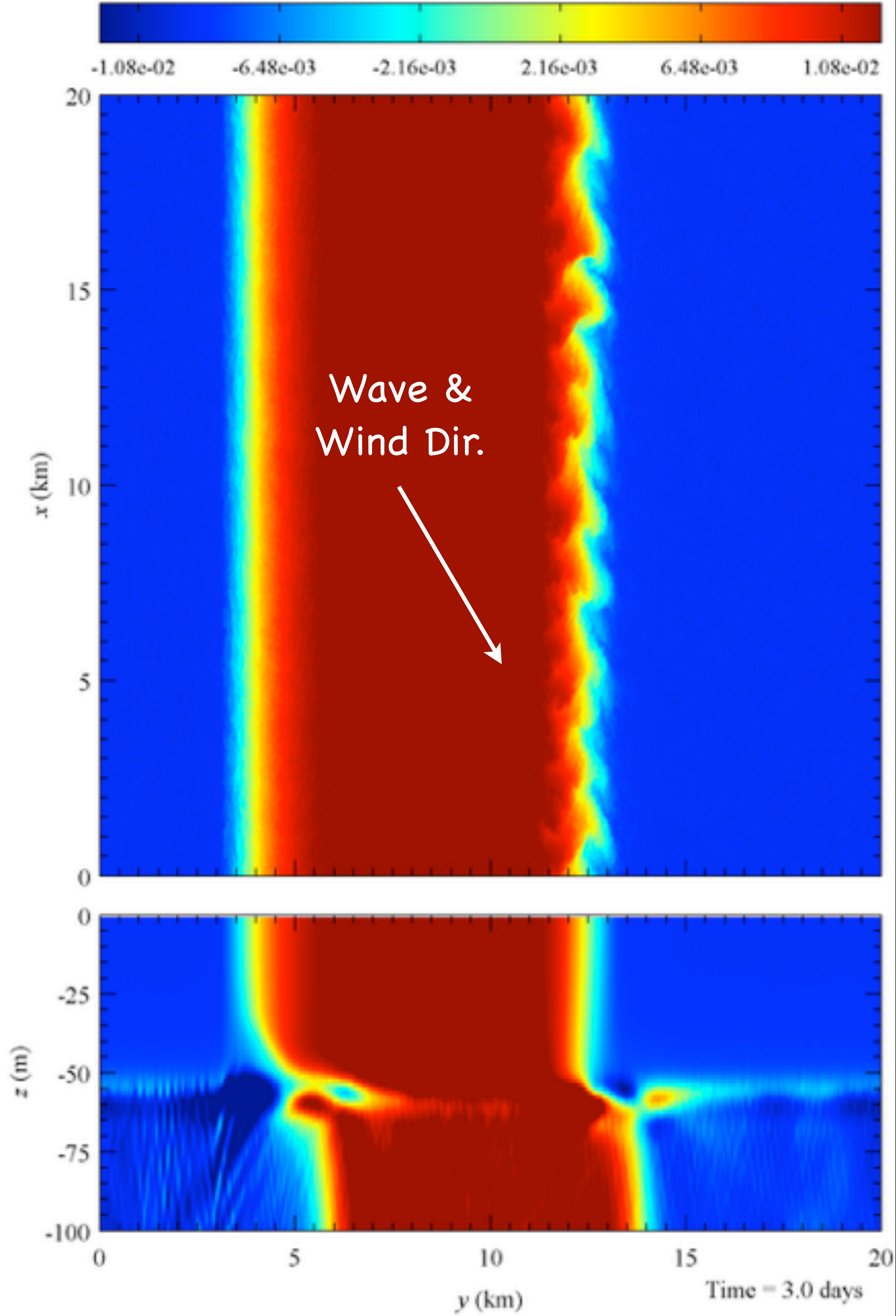
Domain size: 20km x 20km x -160m

Grid points: 4096 x 4096 x 128

Resolution: 5m x 5m x -1.25m

Movie: P. Hamlington

Talk to him for more!!

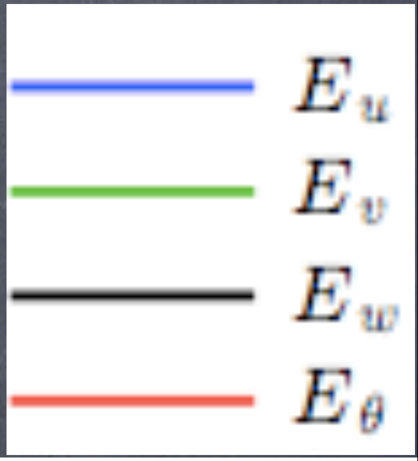


Wave & Wind Dir.

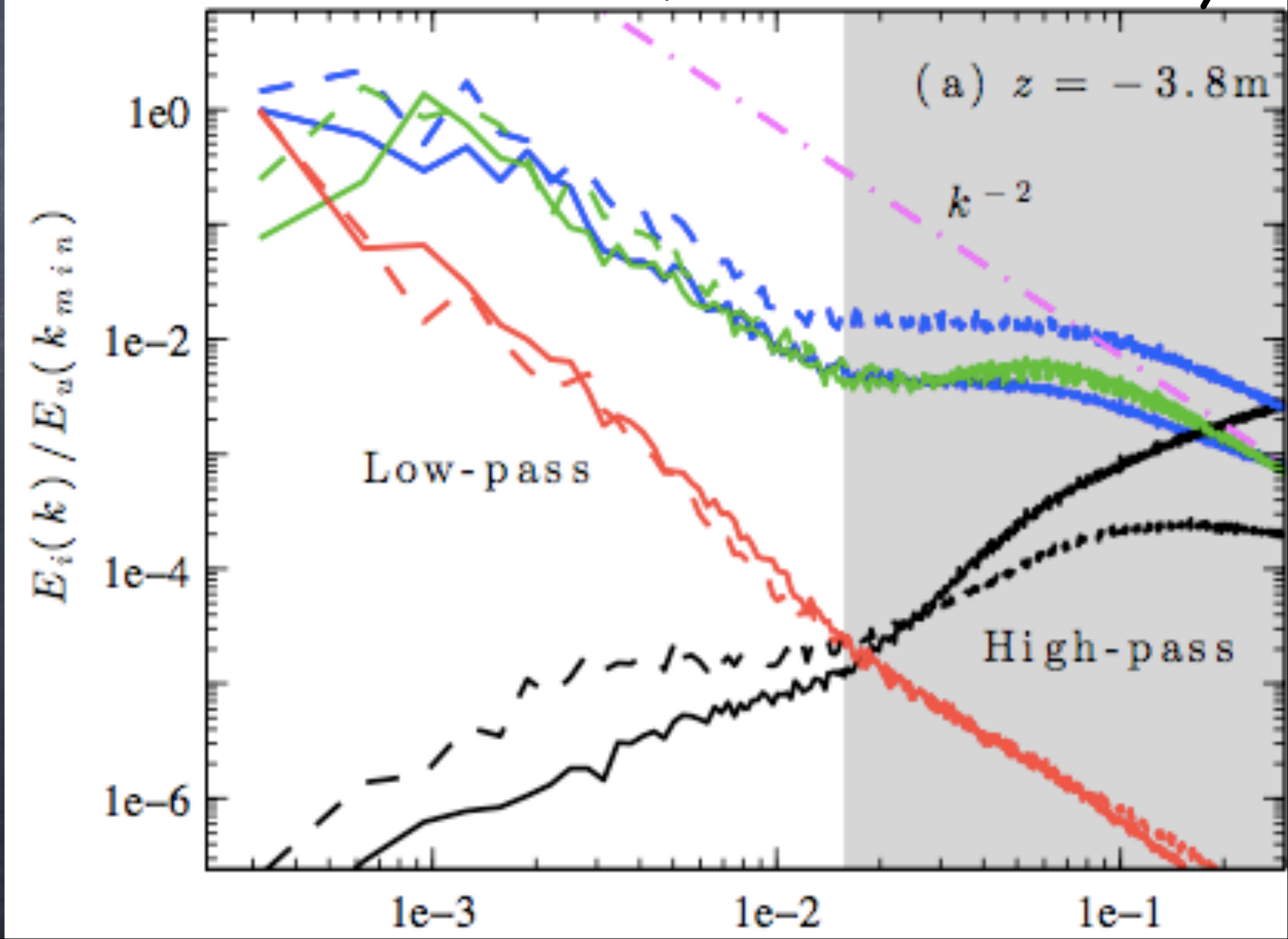
Overall results from multiscale LES

- Submesoscale flow is affected by wave-balance and enhanced <u'w'> (weaker surf. w/ Stokes)
- Strong two-way turbulent interactions are rare for this configuration
- Two turbulent cascades.
- Presence of waves greatly changes small scale from symmetric instability to gravitational

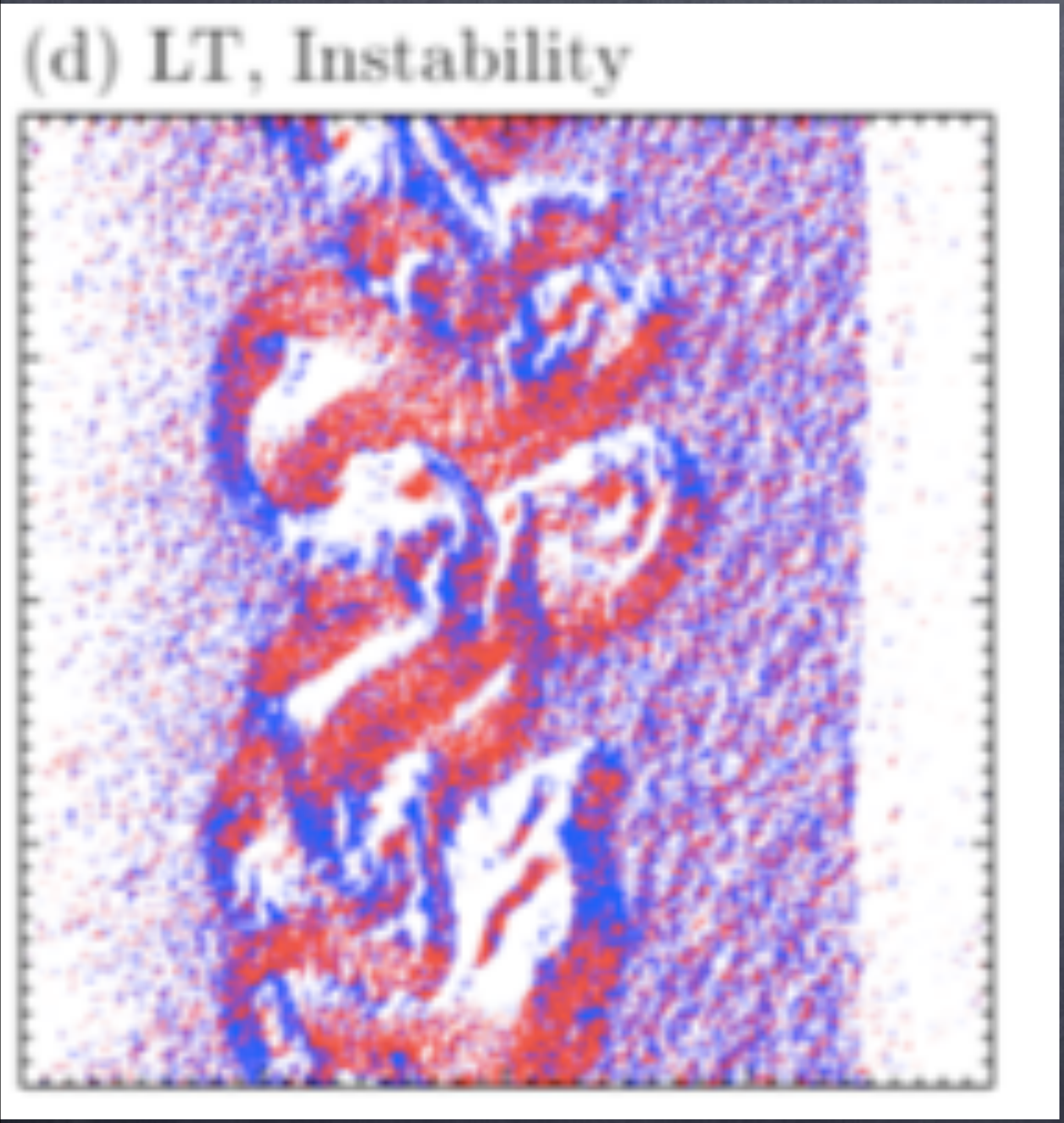
P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, G. P. Chini. Langmuir-Submesoscale Interactions: Descriptive Analysis of Multiscale Frontal Spin-down Simulations, *JPO*, 2013. Submitted.



solid=waves&wind; dashed wind only

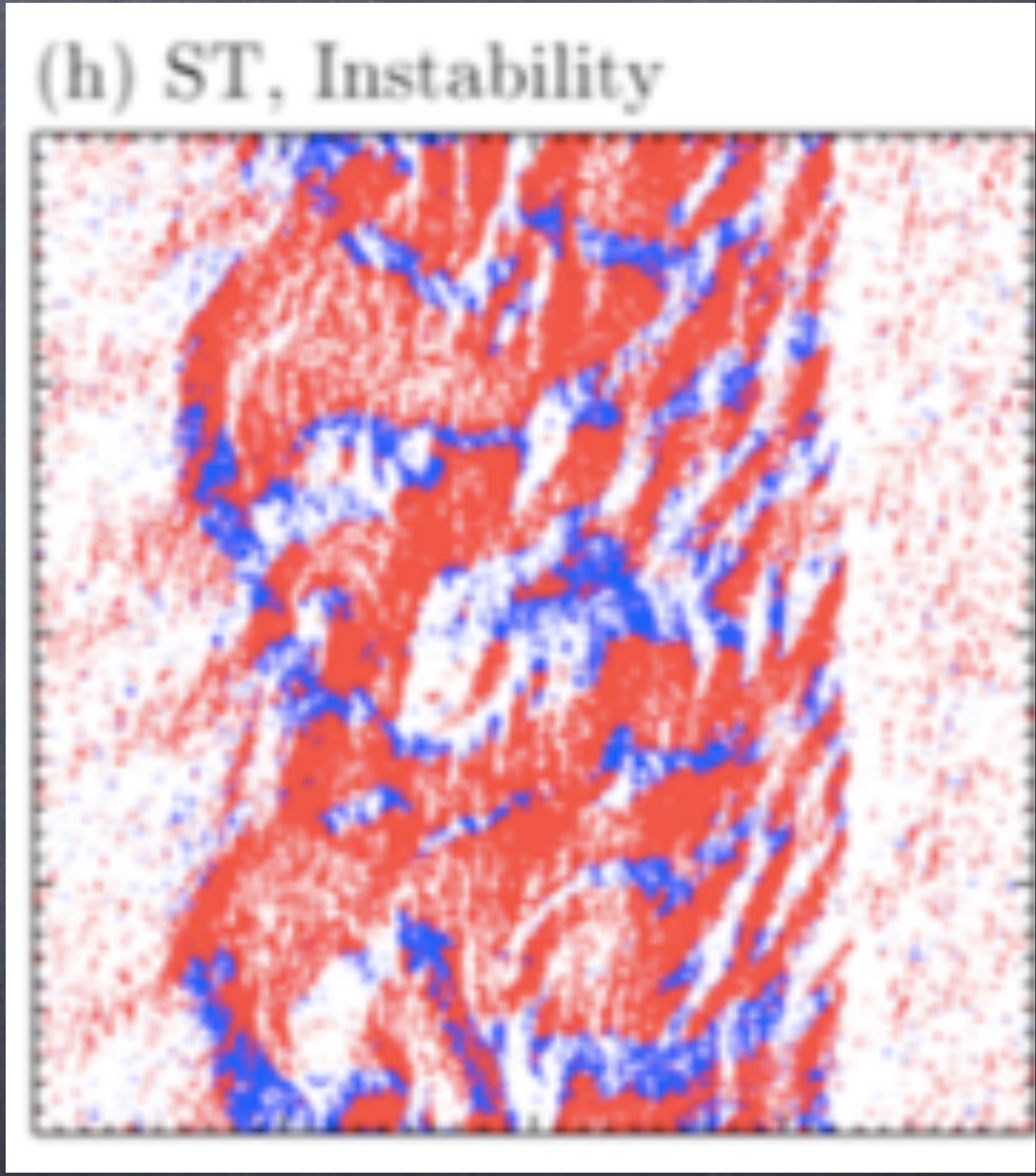


With Stokes Drift



Mostly Baroclinic &
Symmetric & Gravitational Instability

Without Stokes Drift



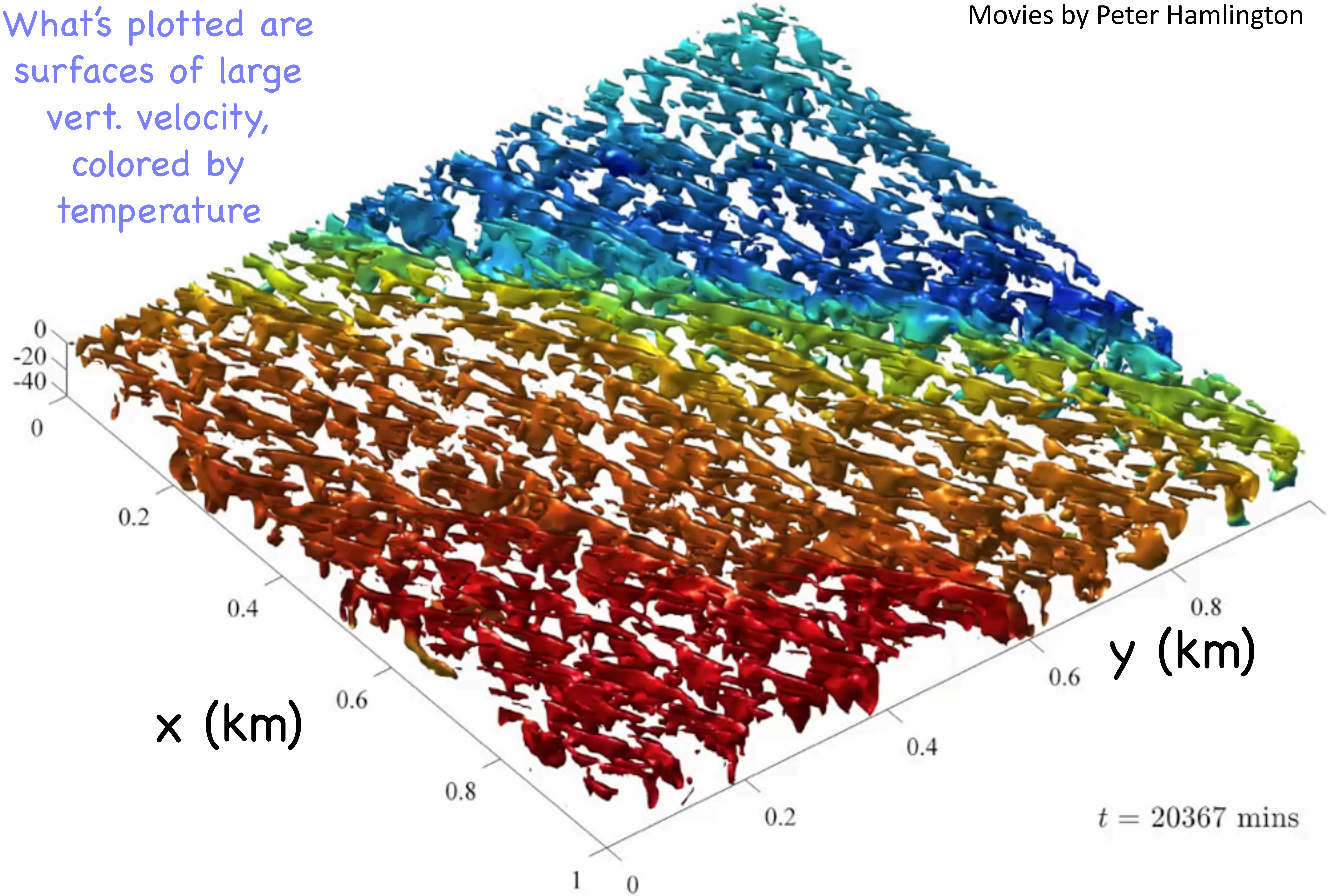
Mostly Baroclinic &
Symmetric Instability

P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, G. P. Chini. Langmuir-Submesoscale Interactions: Descriptive Analysis of Multiscale Frontal Spin-down Simulations, *JPO*, 2013. Submitted.

Zoom: Submeso-Langmuir Interaction!

What's plotted are surfaces of large vert. velocity, colored by temperature

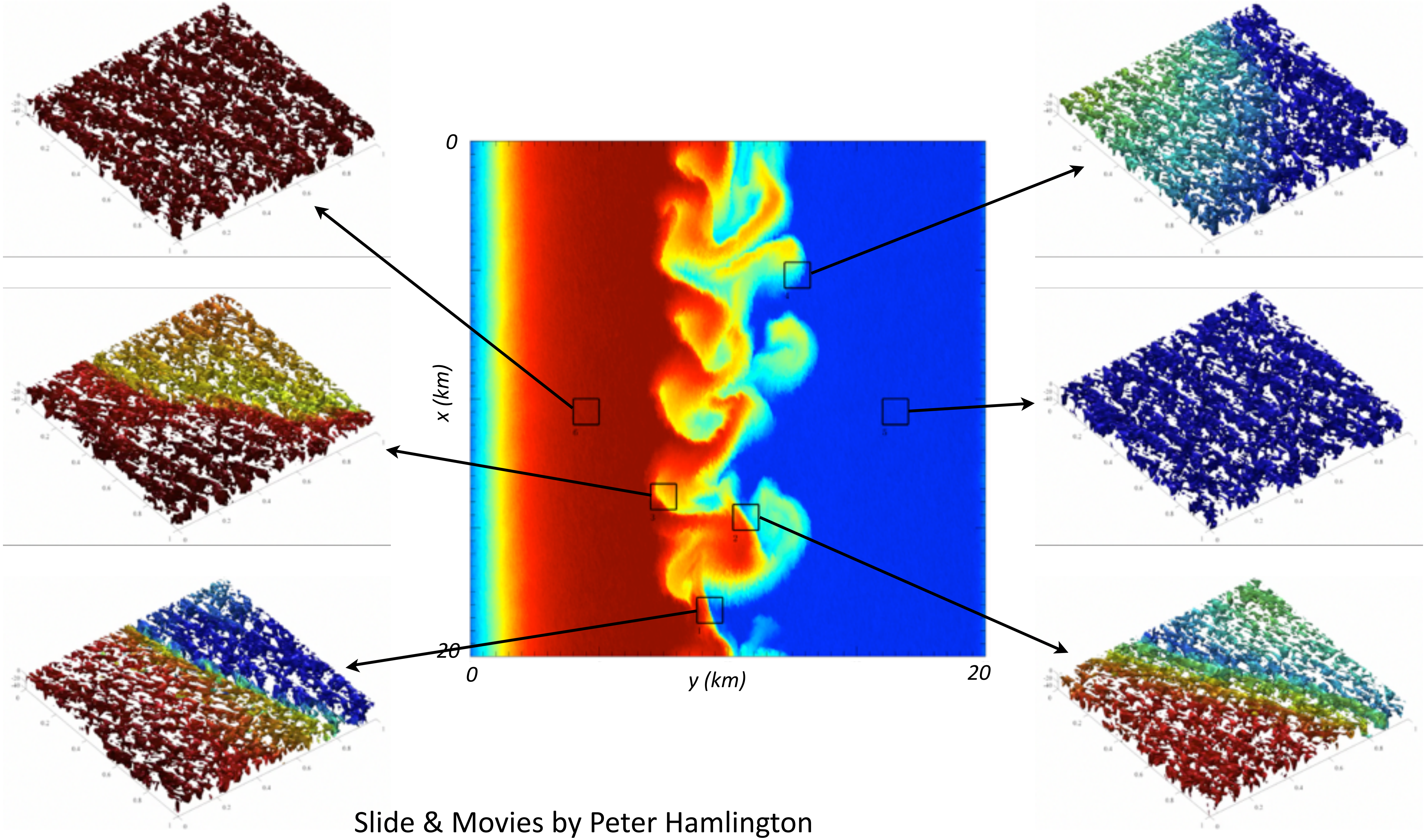
Movies by Peter Hamlington



x (km)

y (km)

Diverse types of interaction



Slide & Movies by Peter Hamlington

P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale simulations. In preparation, 2013.

The Surface Wave Influence on Mixed-Layer Frontal Currents and Multi-scale Turbulence

Nonhydrostatic, Ageostrophic, Submesoscale Frontgenesis under Wave and Wind Forcing

N. Suzuki[*1], B. Fox-Kemper[*2], P. E. Hamlington[*3], L. P. Van Roekel[*4], S. Haney[*3]

[*1] Brown University (Email: nobuhiro_suzuki@brown.edu), [*2] Brown University, [*3] University of Colorado, [*4] Northland College

A new form of the Boussinesq Craik-Leibovich eq.

$$\frac{\partial \mathbf{u}}{\partial t} + \underline{\underline{(\mathbf{u}^L\cdot\nabla)\mathbf{u}}} = -\nabla p - \mathbf{f}\times\mathbf{u}^L + b\hat{z} - \underline{\underline{u_j^L\nabla u_j^S}}$$

Lagrangian advection: responsible for the MKE-TKE conversion

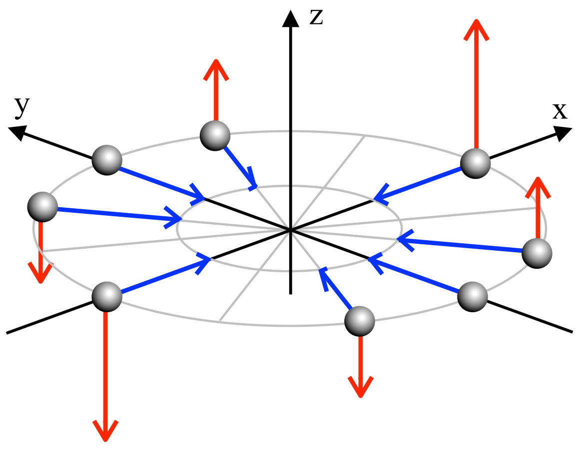
Stokes-shear force: The wave energy transfers via this term.

For horizontally uniform Stokes drift with $w^L = 0$

$$\frac{\partial u}{\partial t} + u^L\frac{\partial u}{\partial x} + v^L\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{\partial p'}{\partial x} + f v^L$$

$$\frac{\partial v}{\partial t} + u^L\frac{\partial v}{\partial x} + v^L\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p'}{\partial y} - f u^L$$

$$\frac{\partial w}{\partial t} + u^L\frac{\partial w}{\partial x} + v^L\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p'}{\partial z} + b' - u'\frac{\partial u^S}{\partial z} - v'\frac{\partial v^S}{\partial z}$$

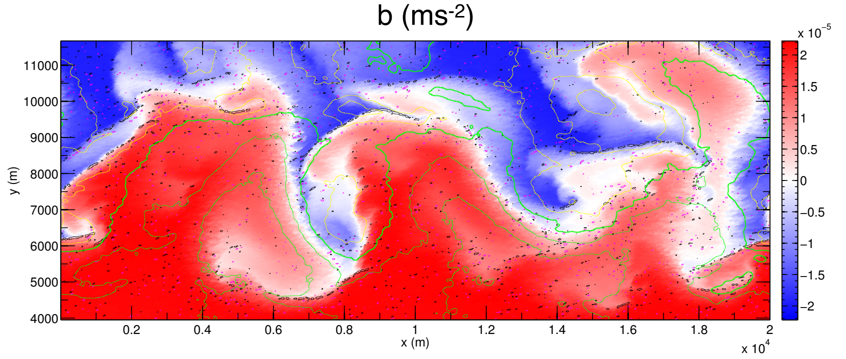


←: Stokes-shear force
●: water parcel
←: turbulent velocity

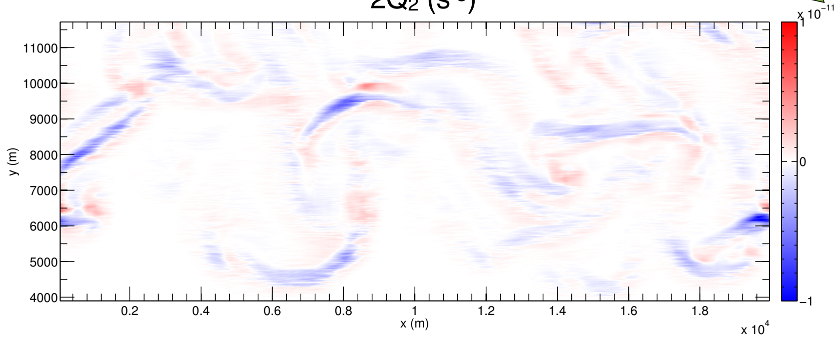
Wave-influenced Sawyer-Eliassen eq.

$$N_*^2\frac{\partial^2\psi}{\partial y^2} + F_*^2\frac{\partial^2\psi}{\partial z^2} + 2M_*^2\frac{\partial^2\psi}{\partial y\partial z} = 2Q_2 + S_{turb} + S_{SSF}$$

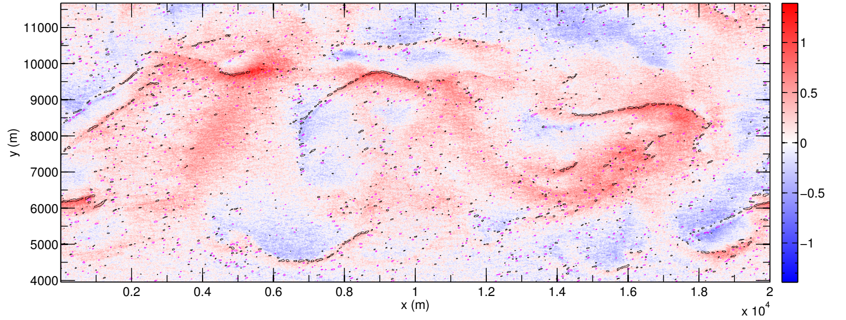
b (ms^{-2})



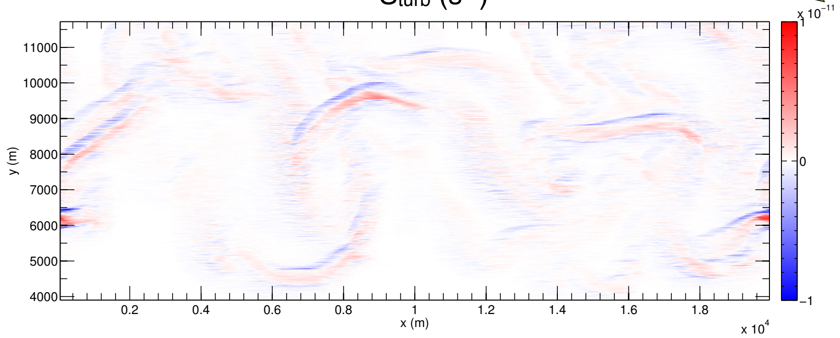
2Q$_2$ (s^{-3})



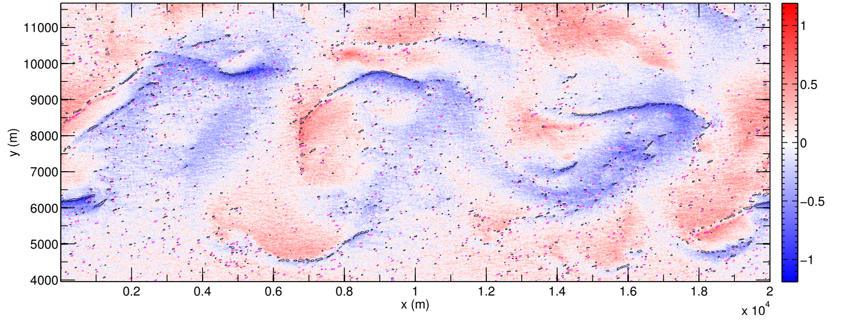
$b' - \partial_z p'$ (ms^{-2})



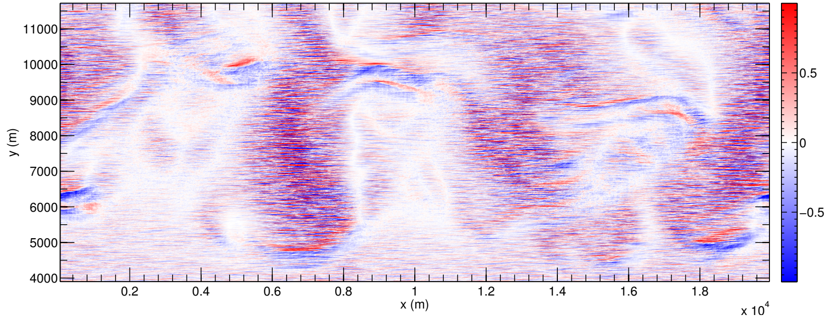
S$_{\mathrm{turb}}$ (s^{-3})



stokes-shear force (ms^{-2})



S$_{\mathrm{SSF}}$ (s^{-3})



across-front structure of b (ms^{-2}) and velocity