

# What's Waves Got to Do with It? Stokes Effects on Turbulence, Fronts, and Instabilities of the Upper Ocean

**Baylor Fox-Kemper (Brown Geo.)**

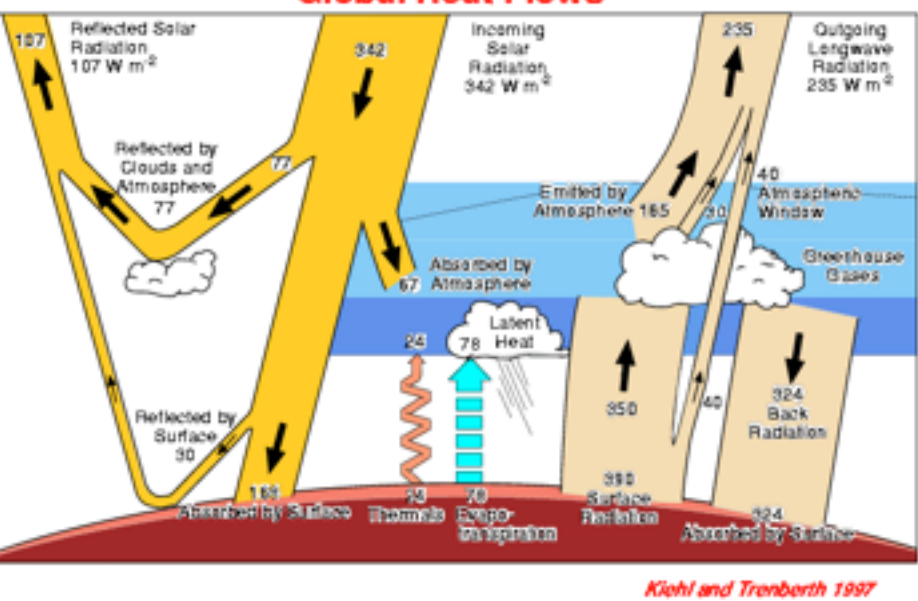
with Jim McWilliams (UCLA), Qing Li (Brown Geo), Nobu Suzuki (Brown Geo), and Sean Haney (CU-Boulder), Peter Hamlington (CU-Boulder), Luke Van Roekel (Northland College), Adrean Webb (TUMST), Keith Julien (CU-APPM), Greg Chini (UNH), E. D'Asaro & R. Harcourt (UW), Peter Sullivan (NCAR), Mark Hemer (CSIRO)

IPAM Mathematics of Turbulence, 10/27/14: 9-9:40

Sponsors: NSF 1258907, 1245944, 0934737, NASA NNX09AF38G



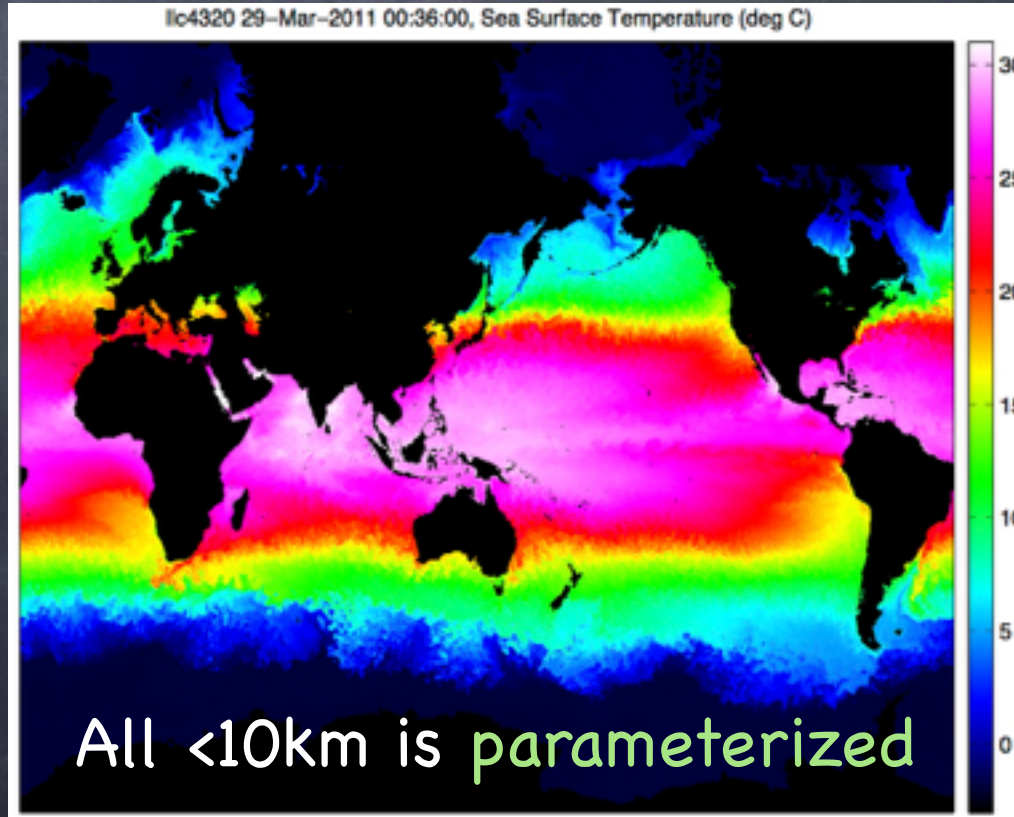
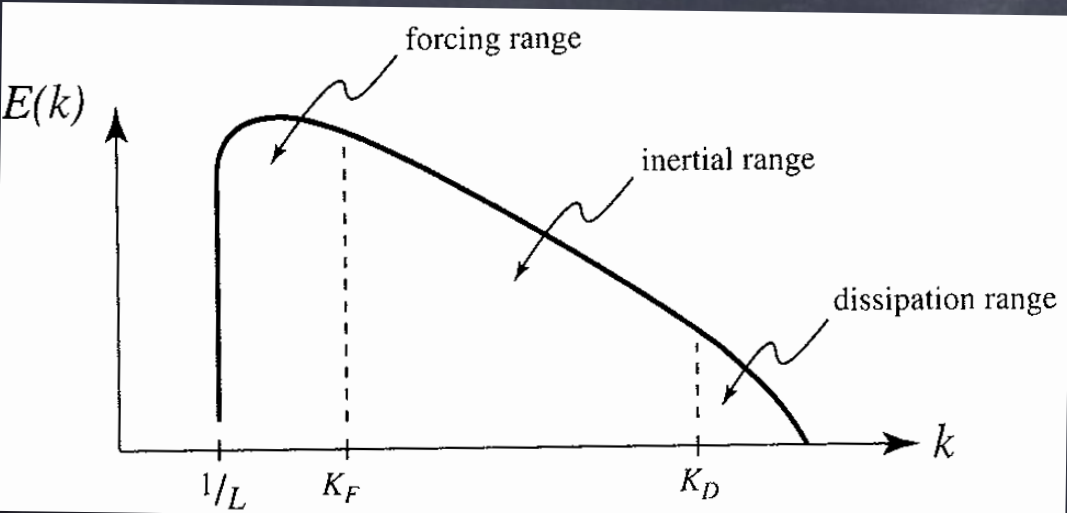
### Global Heat Flows



The Earth's Climate System is forced by the Sun on a global scale (20,000–40,000km)

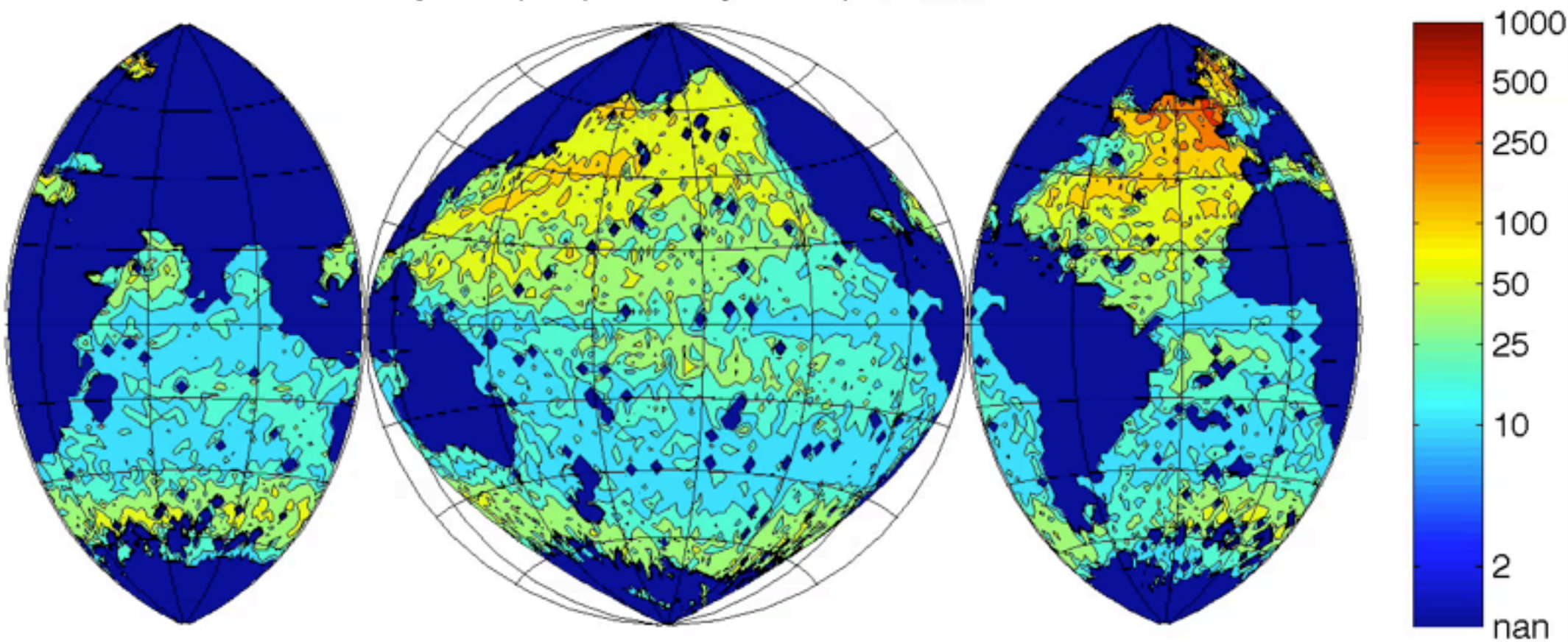
Next-gen. ocean climate models simulate globe to 10km: Mesoscale Ocean Large Eddy Simulations (MOLES)

Turbulence cascades to scales about 10 billion times smaller  $O(1mm)$



# The Ocean Mixed Layer

Mixed Layer Depth ( $\Delta$  density=0.001) in month 1



**Stommel's Demon:** ocean properties at depth set by deepest wintertime mixed layer & its properties

From Argo float data courtesy C. de Boyer-Montegut

# Dimensionless Boussinesq Eqtns.

## Spanning Global to Stratified Turbulence

following McWilliams (85)

$$Ro [v_{i,t} + v_j v_{i,j}] + \frac{M_{Ro}}{Ri} w v_{i,z} + \boxed{\epsilon_{izj} v_j} = -M_{Ro} \pi_{,i} + \frac{Ro}{Re} v_{i,jj}$$

geostrophic

$$\frac{\alpha^2}{Ri} \left[ w_{,t} + v_j w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = \boxed{-\pi_{,z} + b} + \frac{\alpha^2}{Re Ri} w_{,jj}$$

hydrostatic

$$b_t + v_j b_{,j} + \frac{M_{Ro}}{Ro Ri} w b_z + w = 0$$

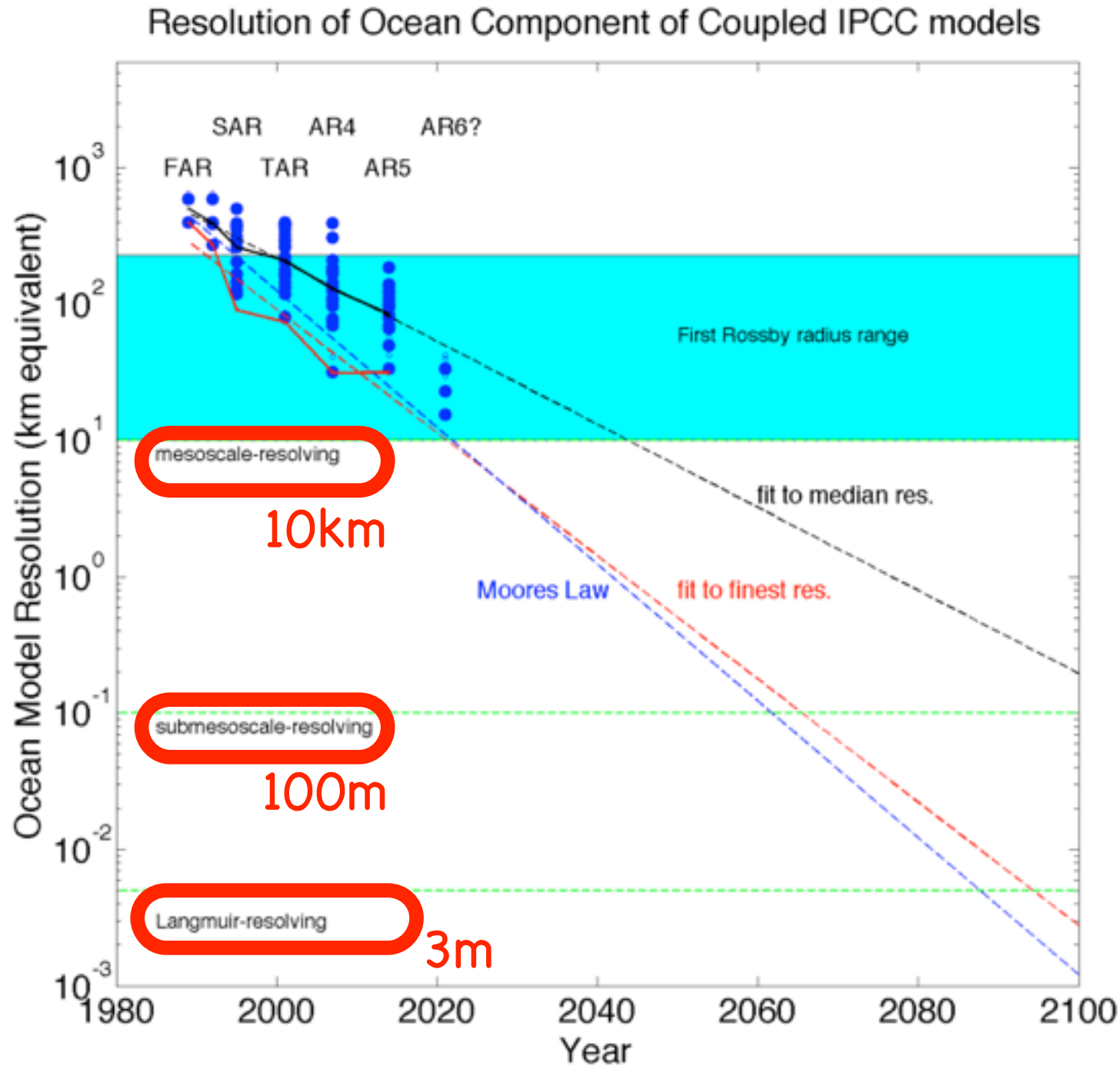
$$v_{j,j} + \frac{M_{Ro}}{Ro Ri} w_z = 0$$

Plus boundary conditions

$$Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri = \frac{N^2}{(U_{,z})^2} \quad \alpha = H/L$$

$$M_{Ro} \equiv \max(1, Ro) \quad v = \text{horiz. vel.} \quad w = \text{vert. vel.}$$

# Resolution will be an issue for centuries to come!



Here are the collection of IPCC models...

If we can't resolve a process, we need to develop a parameterization or subgrid model of its effect

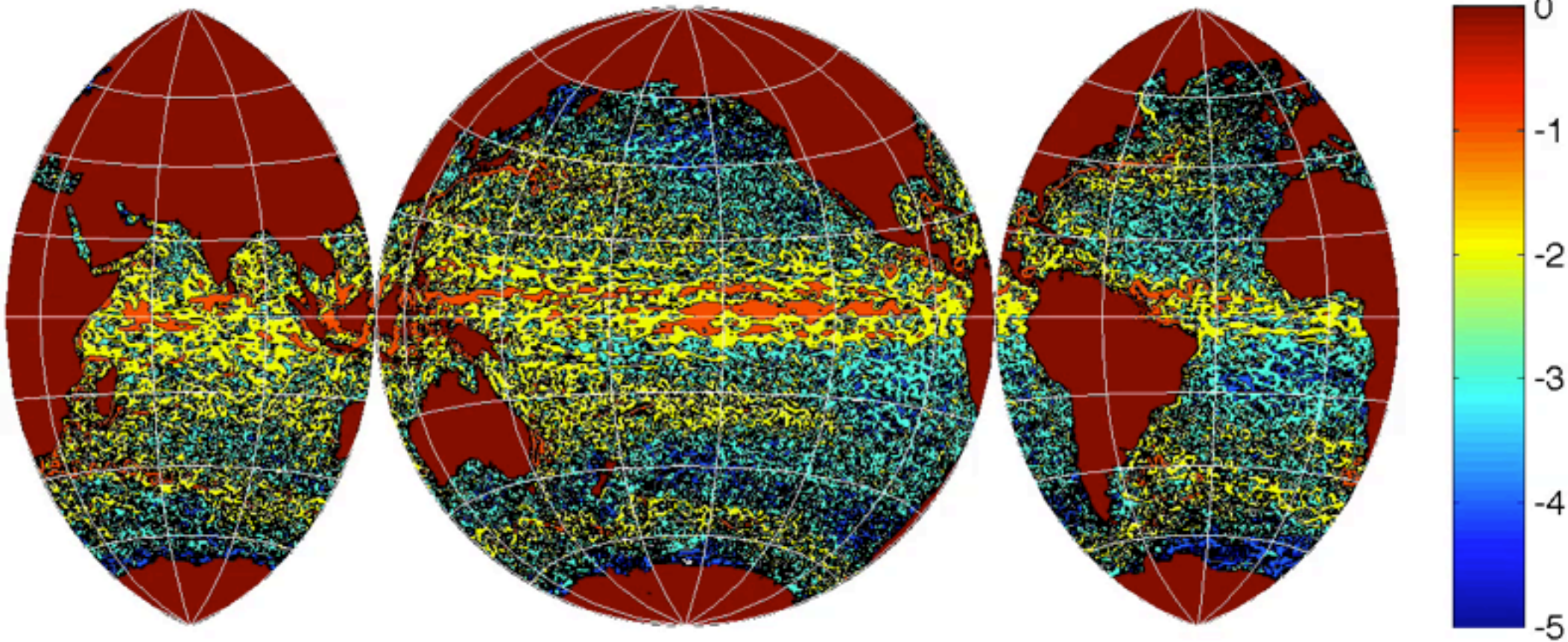
# The Character of the

← 100  
km

(NASA GSFC Gallery)



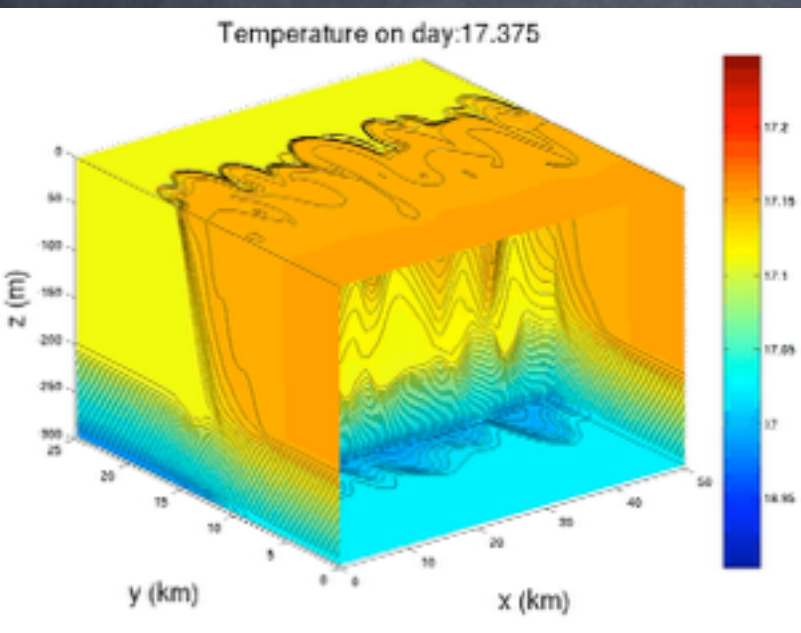
AVISO:  $\log_{10}(0.5(u^2+v^2))$  on 19940101



Eddy processes mainly **baroclinic & barotropic instability**. Parameterized (e.g., Gent-McWilliams), will be routinely resolved in climate models in 2040

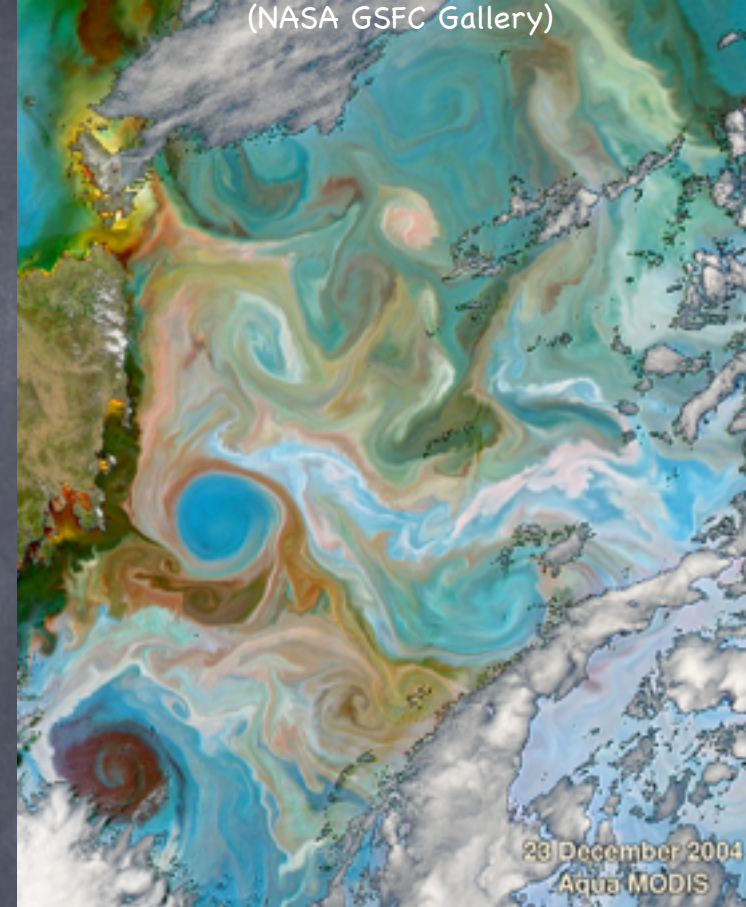
# The Character of the Submesoscale

←  
10  
km



- Fronts
- Eddies
- $Ro=O(1)$
- $Ri=O(1)$
- near-surface ( $H=100m$ )
- 1-10km, days

Routinely resolved in 2100



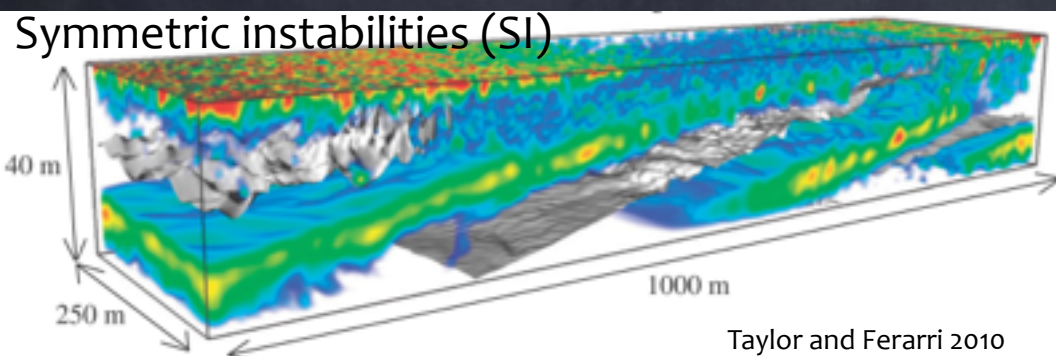
BFK, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. *Journal of Physical Oceanography*, 38(6):1145-1165, 2008

BFK, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

S. Bachman and BFK. Eddy parameterization challenge suite. I: Eady spindown. *Ocean Modelling*, 64:12-28, 2013

Instability processes often  
baroclinic instability  
symmetric instability

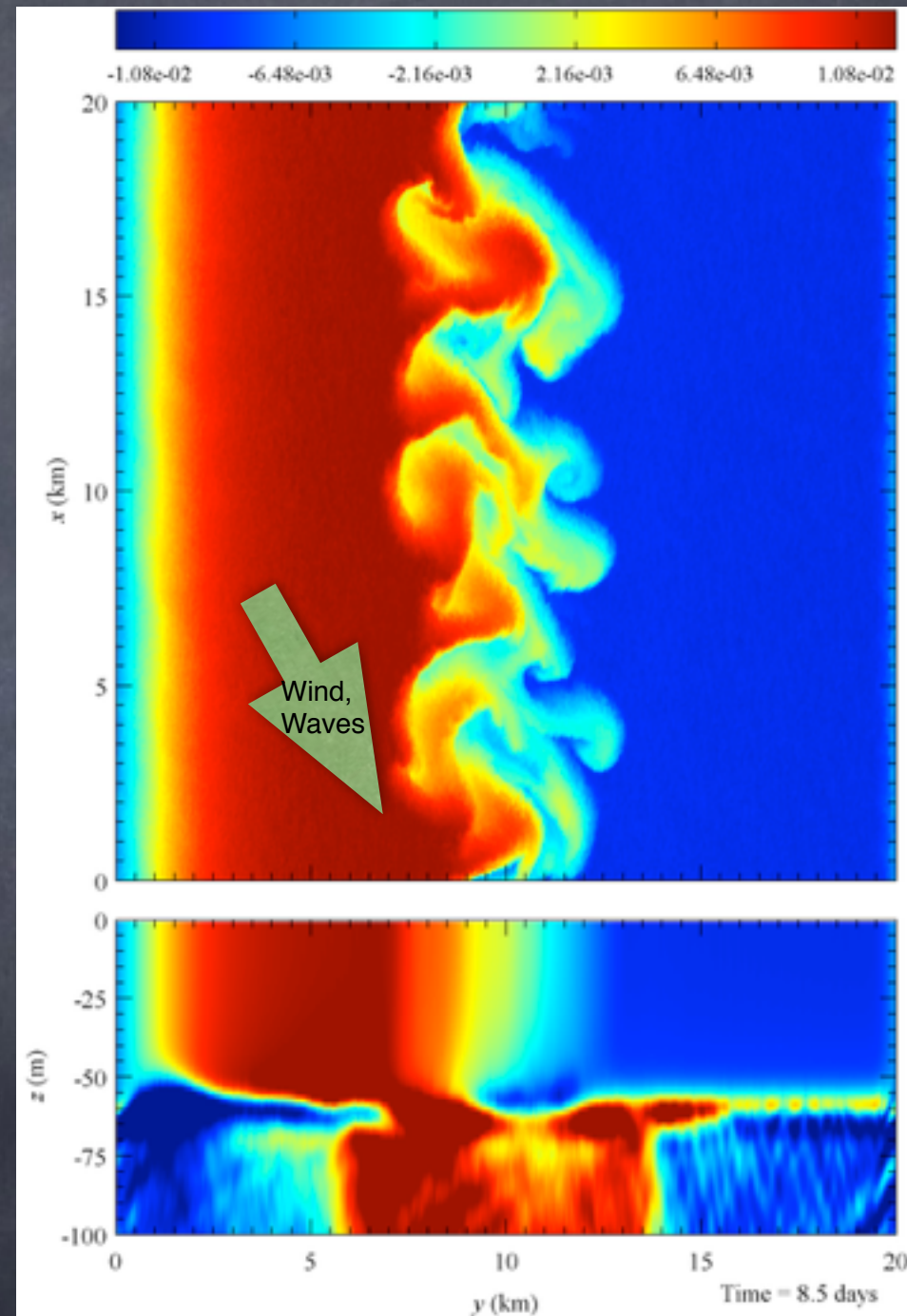
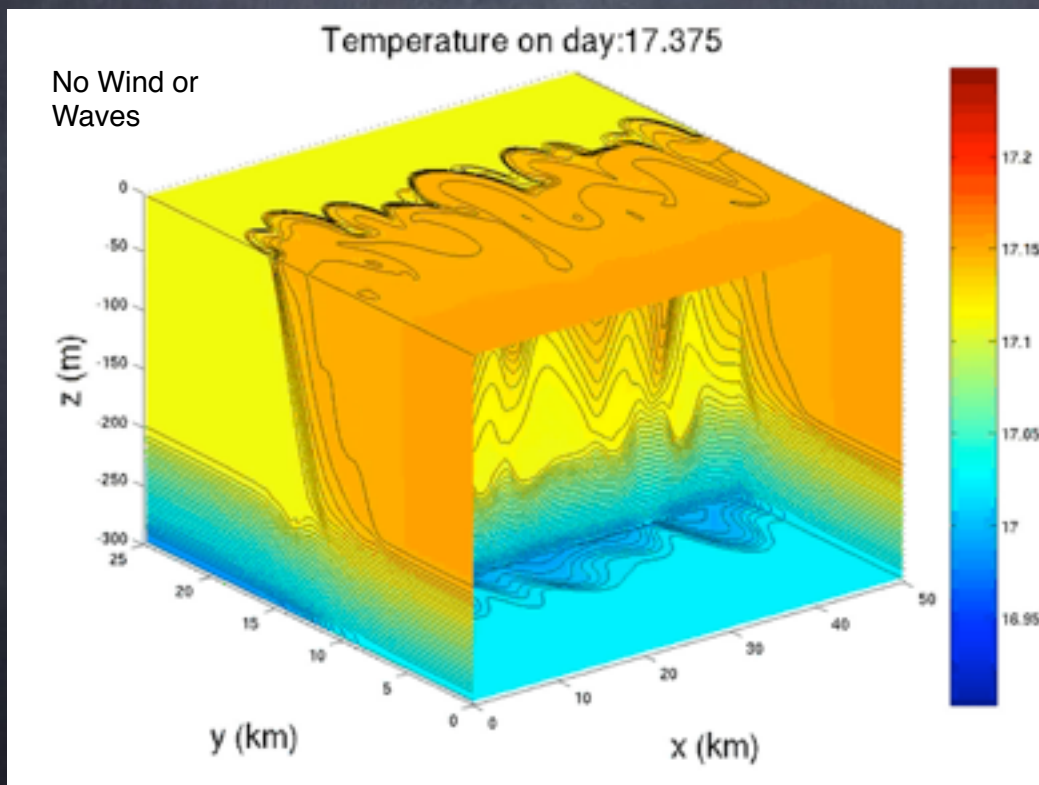
Symmetric instabilities (SI)



# Submesoscale?

Submesoscale (1–10km)  
fronts & the eddies that form  
on them help restratify the  
boundary layer

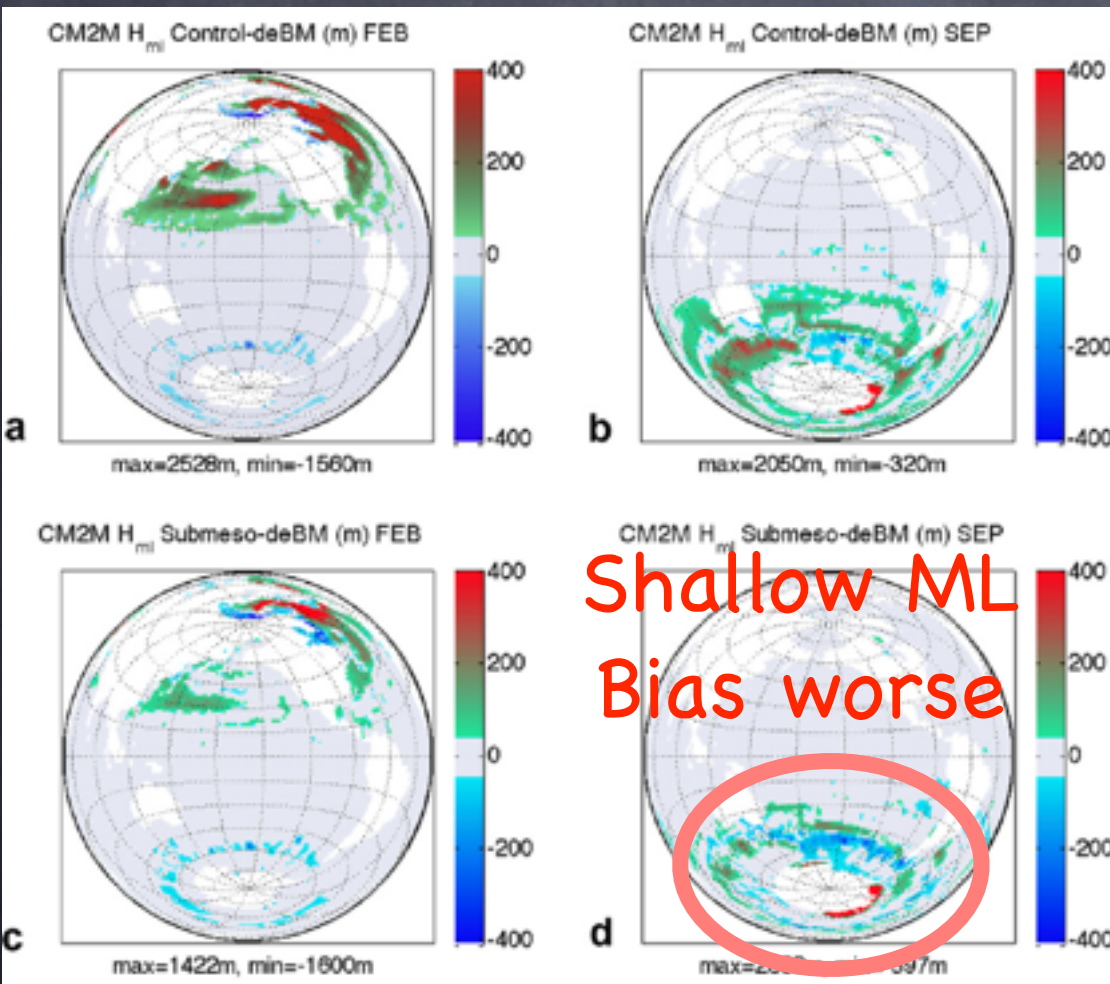
Mixing balances restratification



Movie: P. Hamlington



# A problem with Mixed Layer Eddy Restratification? Southern Ocean already too shallow!



Bias  
w/o  
MLE

Sallee et al. (2013)  
have shown that a  
too shallow S. Ocean  
MLD is true of most\*  
climate models  
even without MLE  
parameterization

BFK, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. *Ocean Modelling*, 39:61-78, 2011.

MLE not only to blame, so  
is something else missing?

A hint:



# The Character of the Langmuir Scale

image:  
Thorpe, 04

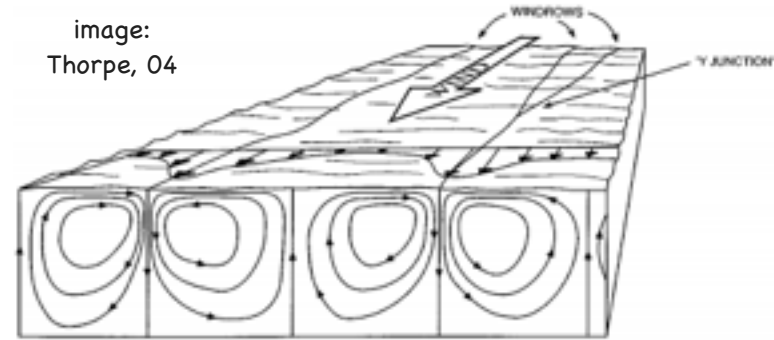


Figure 1 Sketch showing the pattern of mean flow in idealized Langmuir circulation. The windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). In practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2) amalgamate and meander in space and time. Bands of bubbles or buoyant algae may form within the downward-going (or downwelling) flow (see Figure 3).

- Near-surface

- Langmuir Cells & Langmuir Turb.

- $Ro \gg 1$

- $Ri < 1$ : Nonhydro

- 1-100m ( $H=L$ )

- 10s to 1hr

- $w, u = O(10\text{cm/s})$

- Stokes drift

- Eqtns: Wave-Avg, Craik-Leibovich

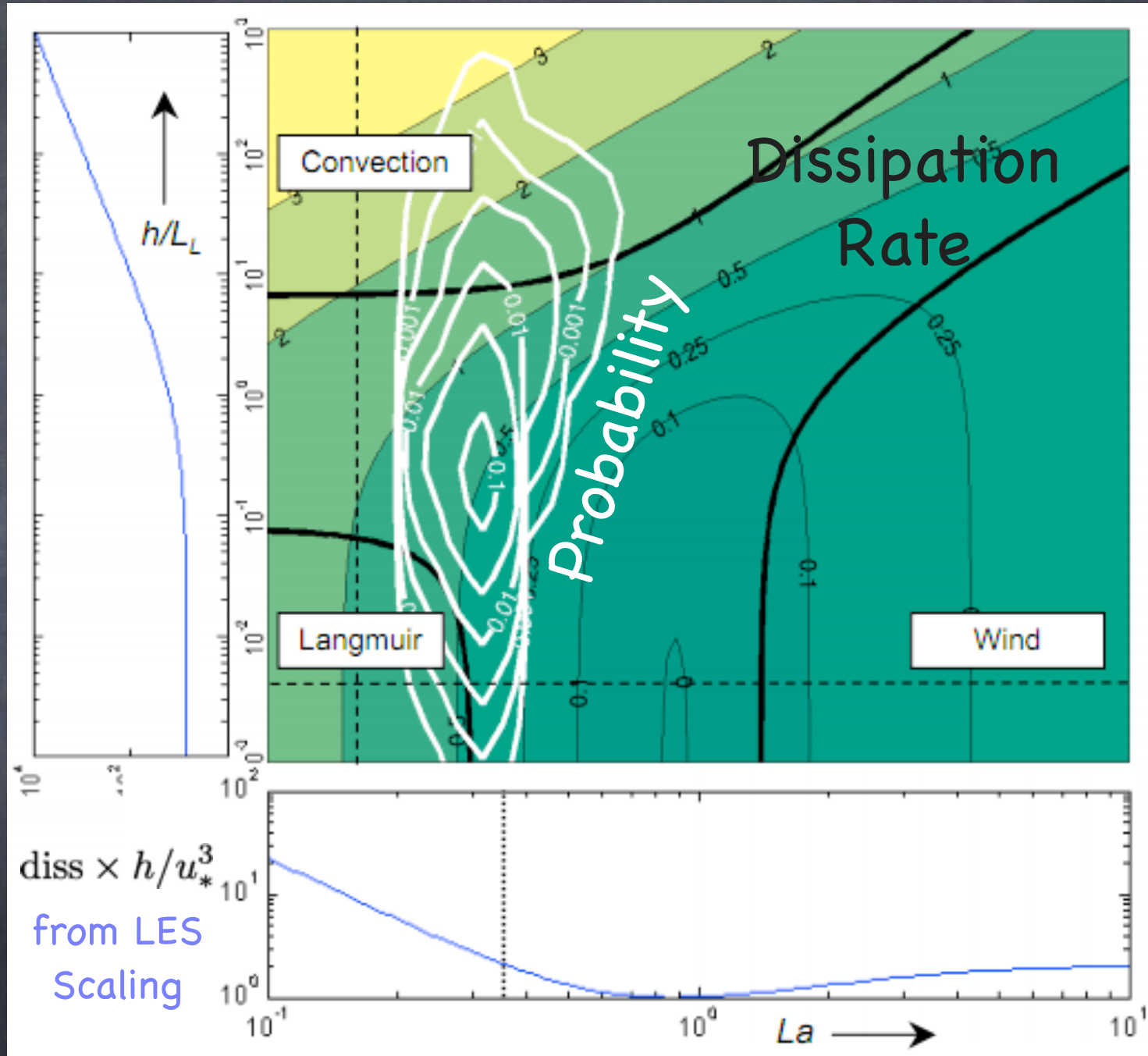
- Params: McWilliams & Sullivan, 2000, Van Roekel et al. 2011

- Resolved routinely in 2170

Image: NPR.org,  
Deep Water  
Horizon Spill

Data + Large Eddy Simulation scaling,  
Southern Ocean mixing energy:

One way to estimate  
So, waves can drive mixing via Stokes drift (combines with cooling & winds)

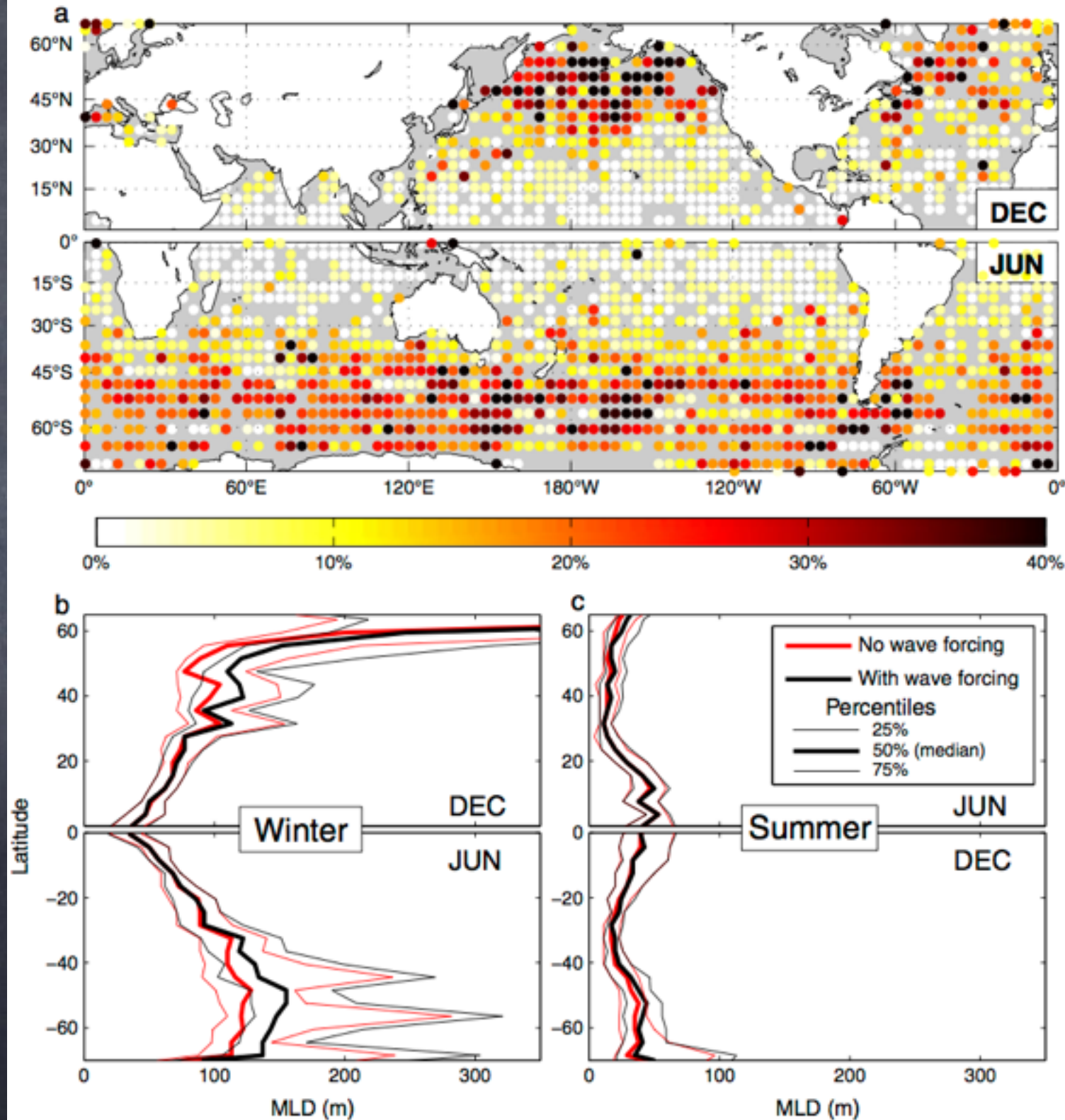


S. E. Belcher, A. A. L. M. Grant, K. E. Hanley, B. Fox-Kemper, L. Van Roekel, P. P. Sullivan, W. G. Large, A. Brown, A. Hines, D. Calvert, A. Rutgersson, H. Petterson, J. Bidlot, P. A. E. M. Janssen, and J. A. Polton. A global perspective on Langmuir turbulence in the ocean surface boundary layer. *Geophysical Research Letters*, 39(18):L18605, 9pp, 2012.

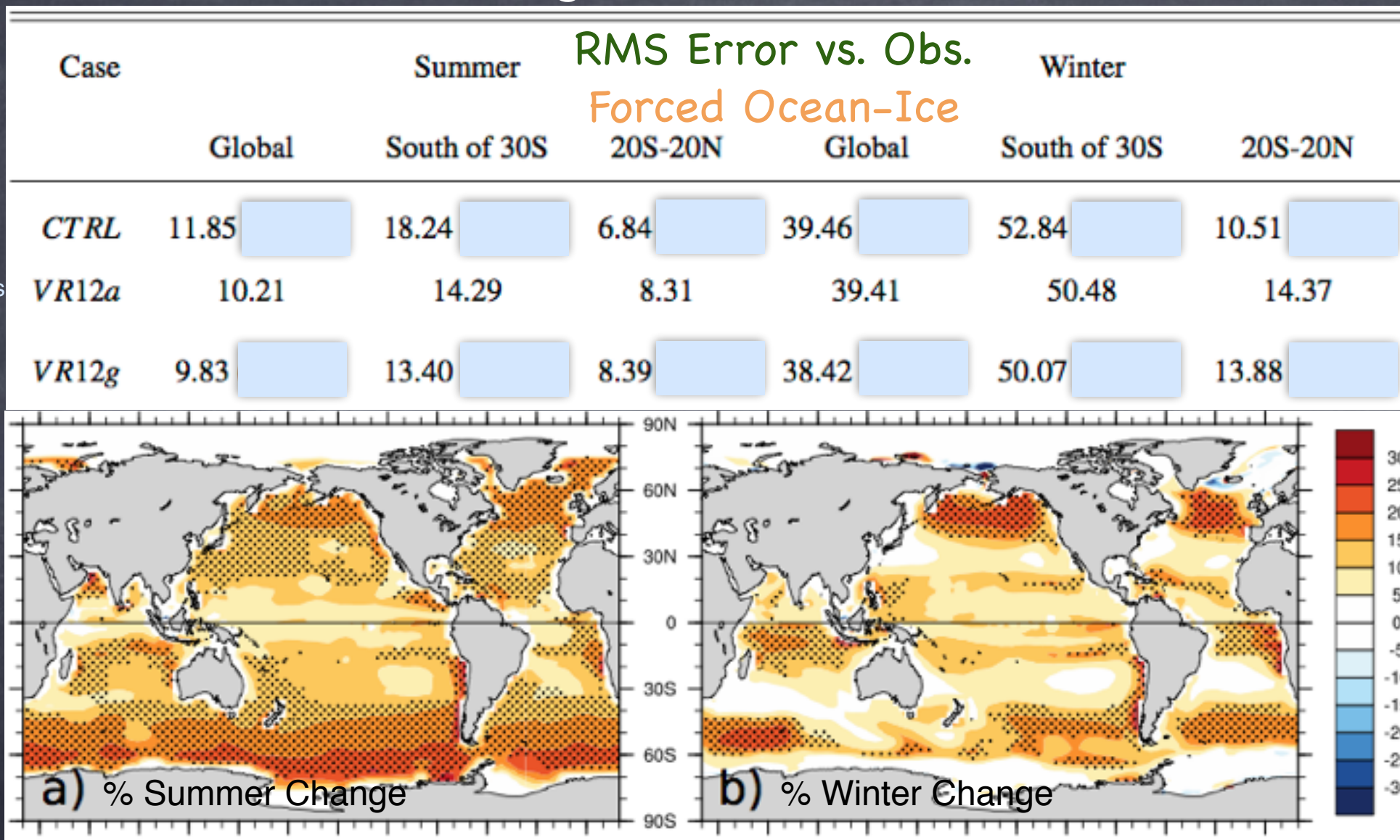
Offline  
obs-driven  
parameterization:

Including  
Stokes-driven  
Mixing  
(Harcourt 2013)  
Deepens the  
Mixed Layer!

E. A. D'Asaro, J. Thomson, A. Y. Shcherbina, R. R. Harcourt, M. F. Cronin, M. A. Hemer, and BFK. Quantifying upper ocean turbulence driven by surface waves. *Geophysical Research Letters*, 41(1): 102-107, January 2014.



# Wave-Driven Mixing in CESM Climate Model



Basic Waves

Depth & misaligned

L. P. Van Roekel, BFK, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

Q. Li, BFK, T. Arbetter, A. Webb, 2014. Assessing the Influence of Surface Wind Waves to the Global Climate by Incorporating WAVEWATCH III in CESM, 2014 AGU Ocean Sciences Meeting Poster, related paper in prep.

# Enough with Climate Models: Let's work on the dynamics!

- Including submesoscale restratification in climate models improves the boundary layer.
- Including wave-driven (Langmuir) mixing in climate models improves the boundary layer.
- But, fundamental questions remain:
  - What if these are combined? What dynamics? What interactions?

# LES of Langmuir-

## Submeso Interactions?

Perform large eddy simulations (LES)  
of Langmuir turbulence with a  
submesoscale temperature front

Use NCAR LES model to solve Wave-  
Averaged Eqtns.  
(McWilliams et al, 1997)

Computational parameters:

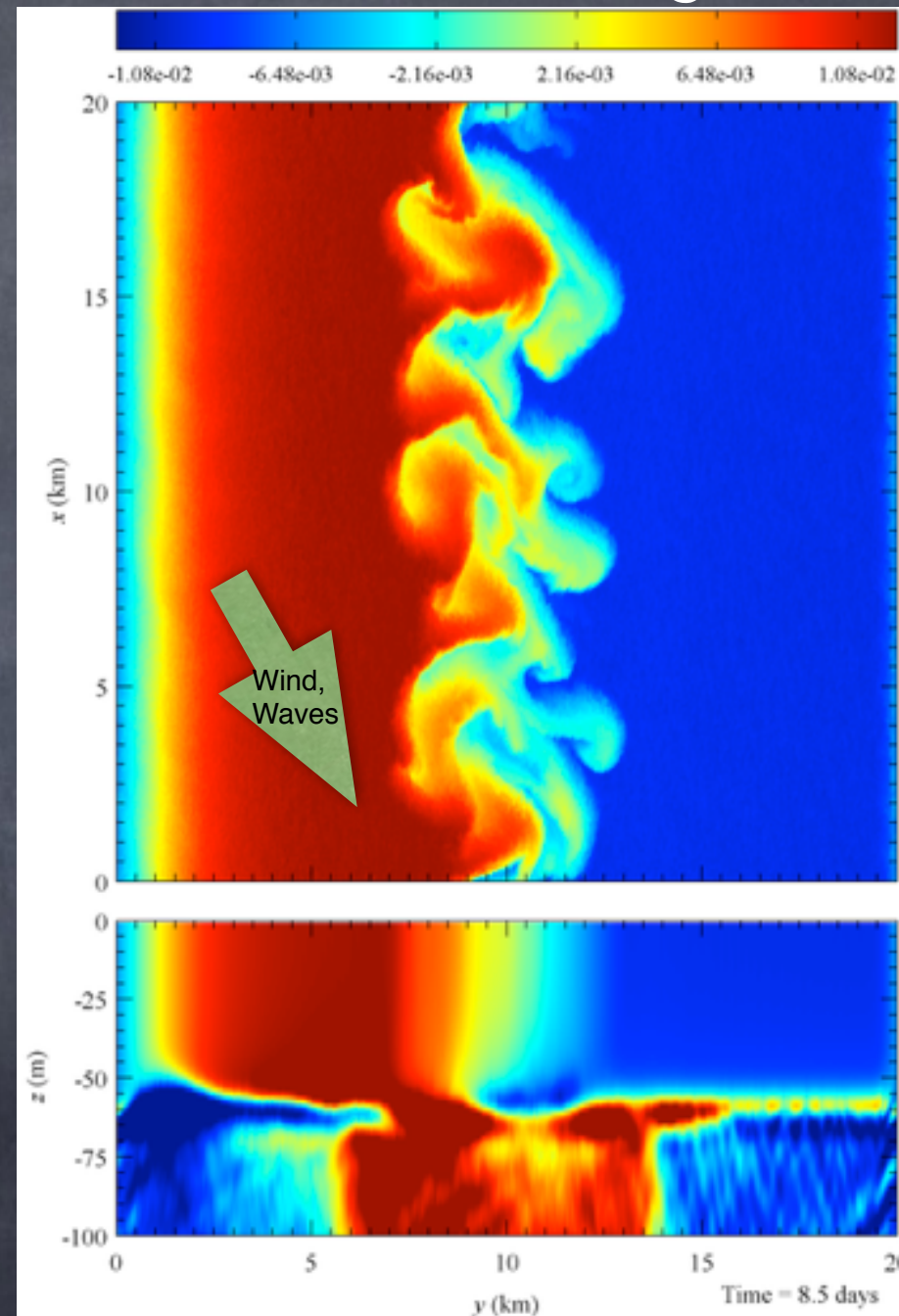
Domain size: 20km x 20km x -160m

Grid points: 4096 x 4096 x 128

Resolution: 5m x 5m x -1.25m

1000x more gridpoints than CESM

Movie: P. Hamlington

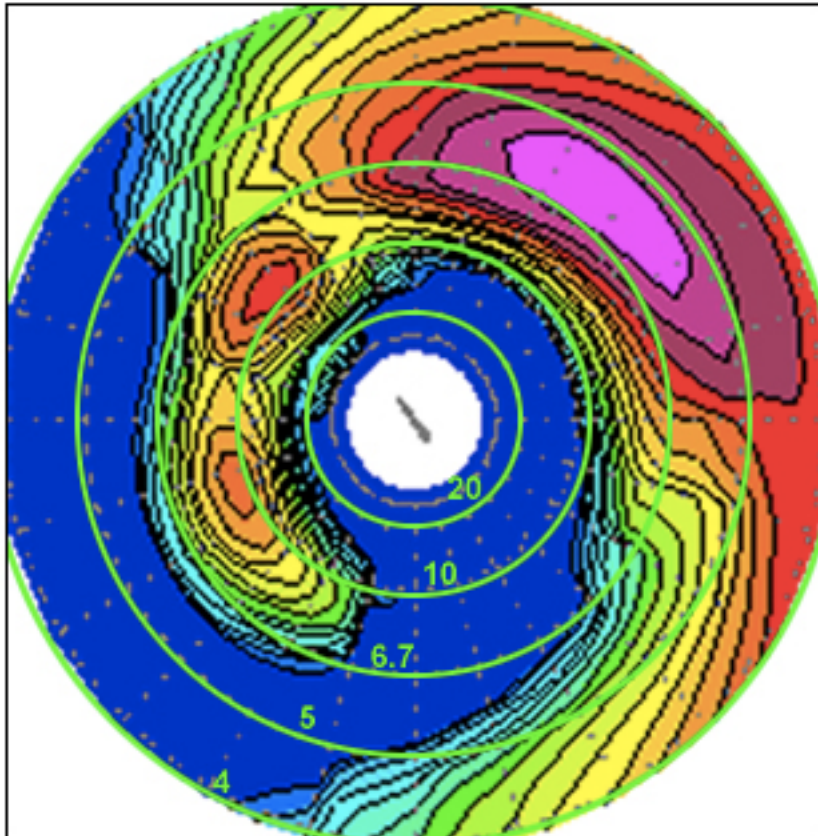




# Surface Waves are...

fast, small, irrotational  
solutions of the  
Boussinesq Equations

NWW3 Polar Plot of Wave Energy Spectrum  
at ILM01



24 hr fcst Valid 0000 UTC 26 Apr 2002

NOAA / NWS / NCEP / MMAB

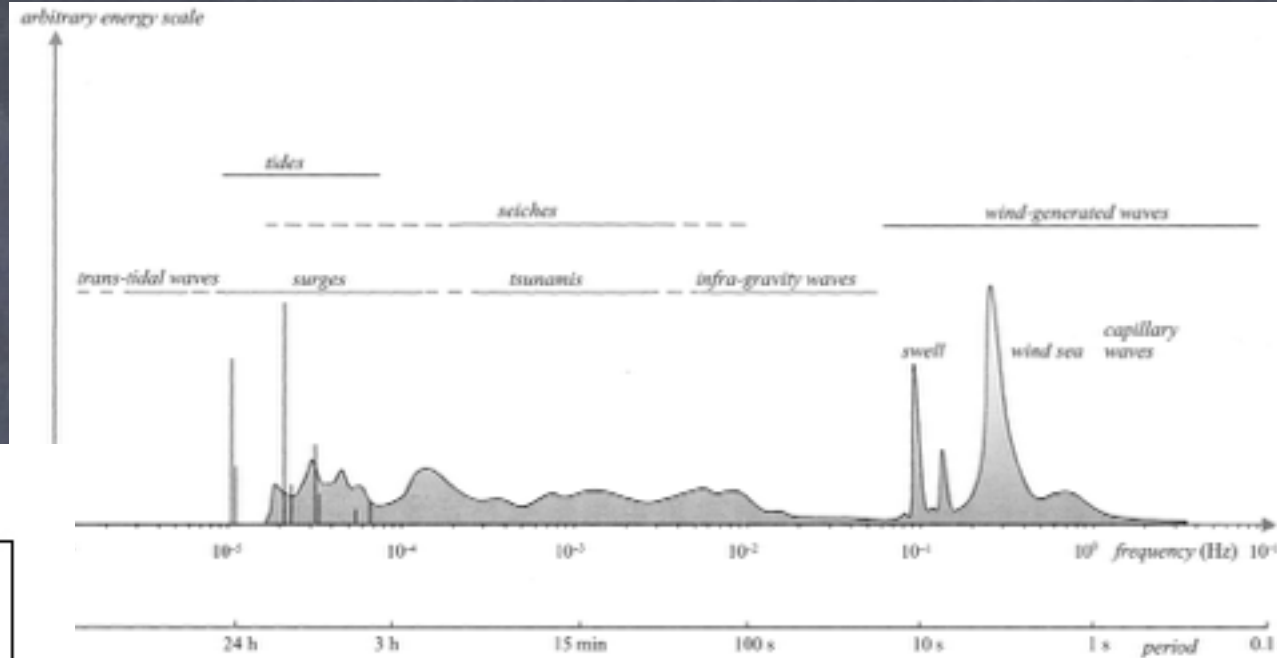


Illustration of wave spectra from different types of ocean surface waves (Holthuijsen, 2007)



# Wave-Averaged Eqtns:

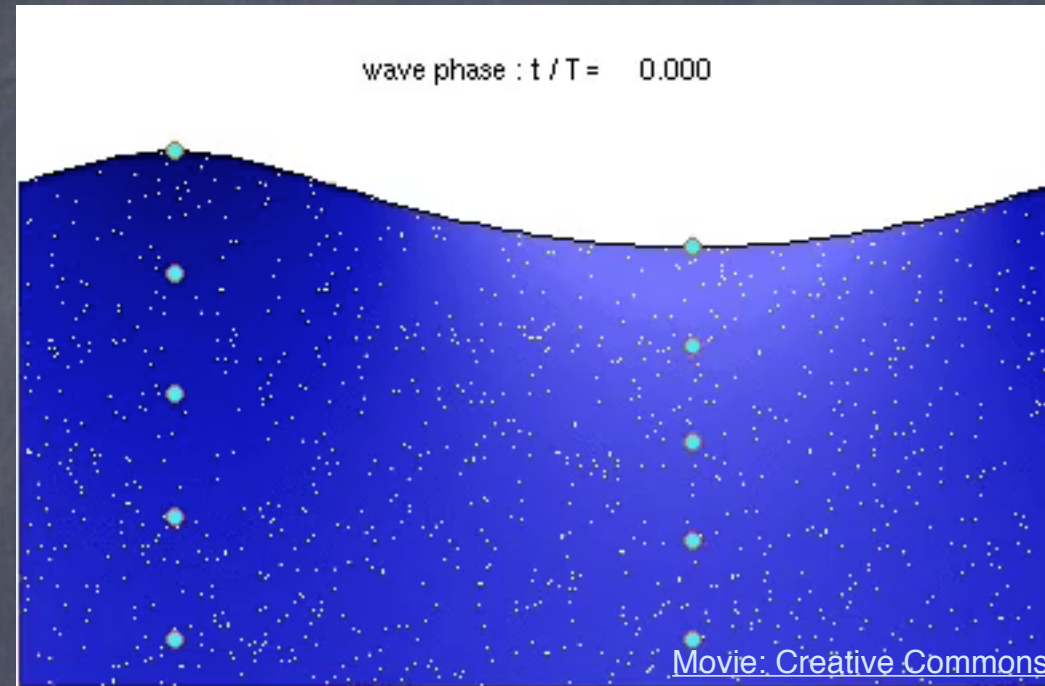
## Stokes Drift Affects Slower Phenomena

- Formally a multiscale asymptotic equation set:
  - 3 classes: Small, Fast; Large, Fast; Large, Slow
  - Solve first 2 types of motion in the case of limited slope ( $ka$ ), irrotational  $\rightarrow$  Deep Water Waves!
  - Average over deep water waves in space & time,
  - Arrive at Large, Slow equation set.

All Wave-Mean coupling terms  
involve the Stokes Drift

# Waves Provide Stokes Drift

Take wave solns, compare the velocity of trajectories vs. Eulerian velocity, leading difference=Stokes:

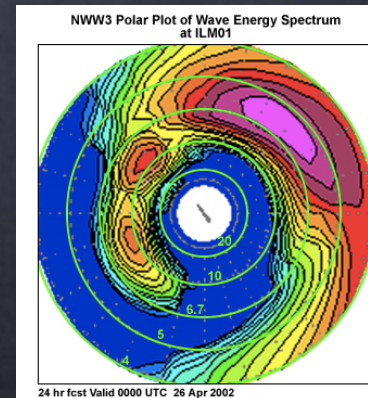


Monochromatic:

$$\mathbf{u}^S = \hat{\mathbf{e}}^w \frac{8\pi^3 a^2 f_p^3}{g} e^{\frac{8\pi^2 f_p^2}{g} p_z}$$

Wave Spectrum:

$$\mathbf{u}^S = \frac{16\pi^3}{g} \int_0^\infty \int_{-\pi}^\pi (\cos \theta, \sin \theta, 0) f^3 \mathcal{S}_{f\theta}(f, \theta) e^{\frac{8\pi^2 f^2}{g} z} d\theta df.$$



A. Webb and BFK. Wave spectral moments and Stokes drift estimation. *Ocean Modelling*, 40(3-4):273-288, 2011.

A. Webb and BFK. Estimating Stokes drift for directional random seas. *Ocean Modelling*, June 2014. Submitted.

# Wave-Averaged Equations

$$\varepsilon = \frac{V^s H}{f L H_s}$$

following Lane et al. (07), McWilliams & F-K (13)  
and Suzuki & F-K (14)

(for horizontally uniform Stokes drift)

$$Ro [v_{i,t} + v_j^L v_{i,j}] + \frac{M_{Ro}}{Ri} w v_{i,z} + \epsilon_{izj} v_j^L = -M_{Ro} \pi_{,i} + \frac{Ro}{Re} v_{i,jj}$$

$$\frac{\alpha^2}{Ri} \left[ w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = -\pi_{,z} + b - \varepsilon v_j^L v_{j,z}^s + \frac{\alpha^2}{Re Ri} w_{,jj}$$

$$b_t + v_j^L b_{,j} + \frac{M_{Ro}}{Ro Ri} w b_z + w = 0$$

$$v_{j,j} + \frac{M_{Ro}}{Ro Ri} w_z = 0$$

Plus boundary  
conditions

LAGRANGIAN (Eulerian+Stokes) advection & Coriolis

So, Waves can Drive turbulence that affect larger scales indirectly:

Stokes effects of waves on larger scales?

$$\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b$$

Becomes Lagrangian Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial}{\partial z} (\mathbf{v} + \mathbf{v}_s) = \mathbf{f} \times \frac{\partial \mathbf{v}_L}{\partial z} = -\nabla b$$

Now the temperature gradients govern the Lagrangian shear, not the Eulerian!

Lagrangian=Eulerian+Stokes

Plus, it is the Lagrangian  
Flow that transports  
tracers

(salinity, temperature,  
density, etc.)

# Analytic Stability Criteria: Geostrophic Modes

\* Charney, Stern, and Pedlosky showed, that geostrophic instability exists only if one of the following is true:

1.  $Q_y$  changes sign in the interior of the domain.
2.  $Q_y$  is the opposite sign to  $U_z^L$  at the surface.
3.  $Q_y$  is the same sign to  $U_z^L$  at the bottom.
4.  $U_z^L$  has the same sign at the surface and bottom.

Where  $Q$  is the quasi-geostrophic potential vorticity:

$$\bar{Q} = \nabla_{H^2}^2 \bar{\psi} + \beta Y + \partial_z \left( \frac{f_0^2}{N^2} \bar{\psi}_z^L \right)$$

Charney, Stern, & Pedlosky gets a tweak →  $U$  is Lagrangian sometimes!

# Wave-Averaged Equations

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following Lane et al. (07), McWilliams & F-K (13)  
and Suzuki & F-K (14)

(for horizontally uniform Stokes drift)

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$$\frac{\alpha^2}{Ri} \left[ w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = -\pi_{,z} + b - \boxed{\varepsilon v_j^L v_{j,z}^s} + \frac{\alpha^2}{Re Ri} w_{,jj}$$

$$b_t + v_j^L b_{,j} + \frac{M_{Ro}}{Ro Ri} w b_z + w = 0$$

$$v_{j,j} + \frac{M_{Ro}}{Ro Ri} w_z = 0$$

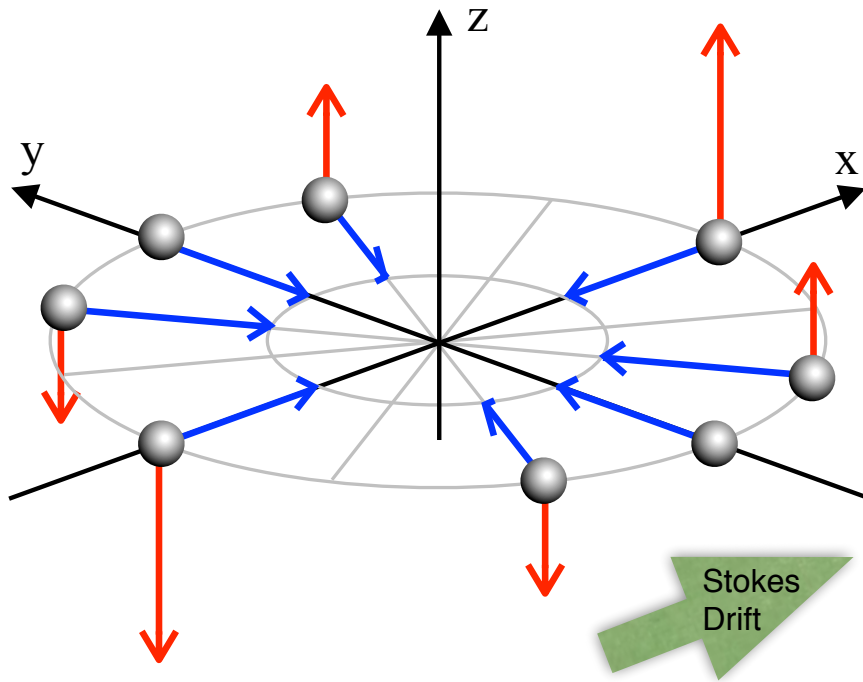
Plus boundary  
conditions

Stokes shear force is NEW \*nonhydrostatic\* term in Vert. Mom.

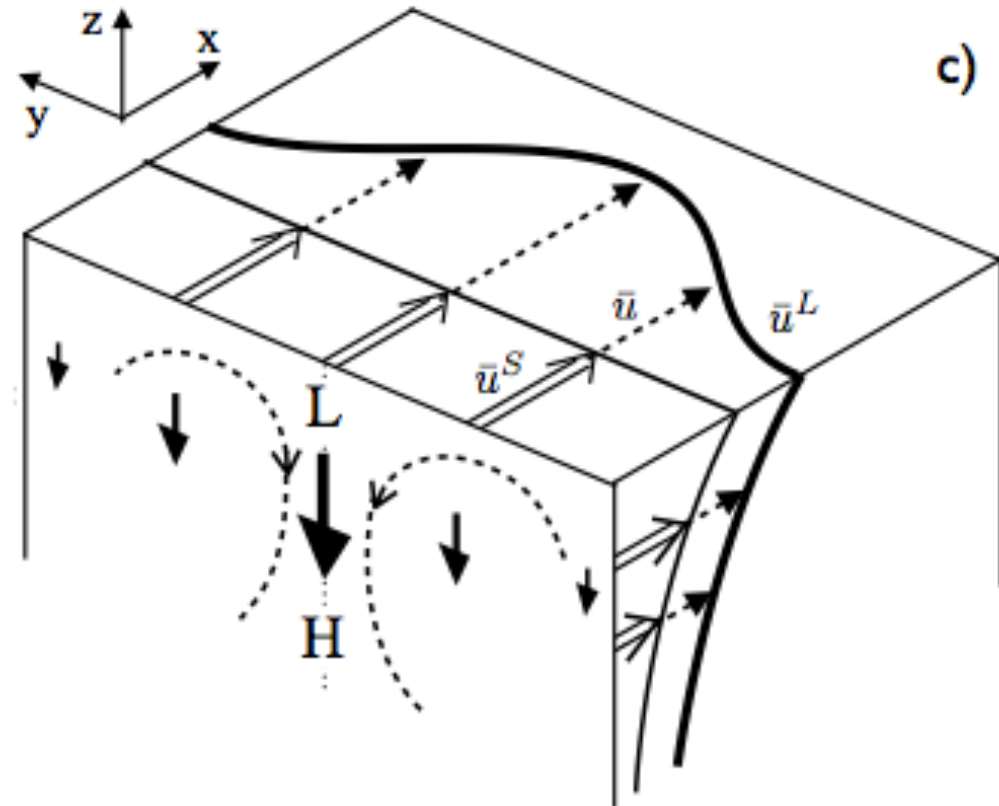


# Stokes Shear Force:

Craik-Leibovich mechanism for Langmuir circulations  
 Flow directed along Stokes shear=downward force



← : Stokes-shear force    ● : water parcel  
← : turbulent velocity



$$\frac{\alpha^2}{Ri} \left[ w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = -\pi_{,z} + b - \epsilon v_j^L v_{j,z}^s + \frac{\alpha^2}{Re Ri} w_{,jj}$$

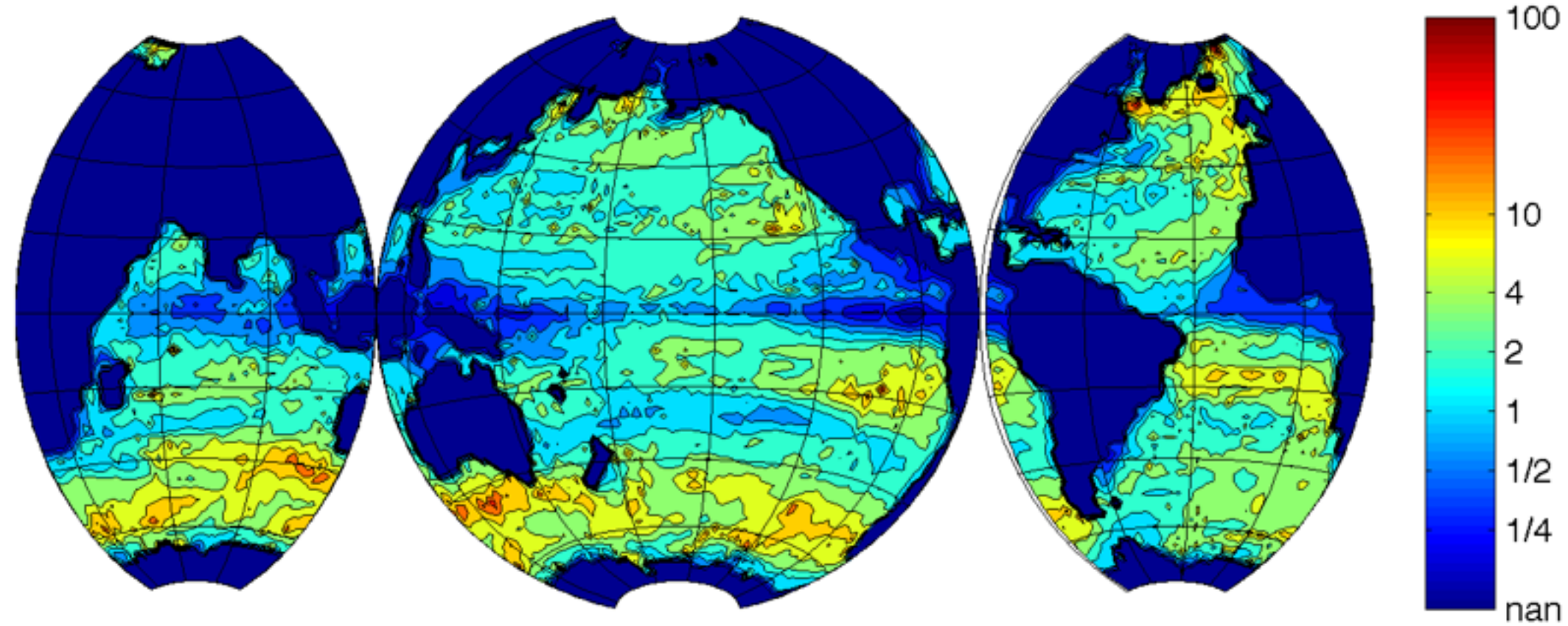
When is  $\varepsilon = \frac{V^s H}{f L H_s}$  big?

$$\varepsilon = \frac{V_s}{f L} \frac{H}{H_s} = \frac{V_s}{\underbrace{f H_s}_{O(10-100)}} \underbrace{\frac{H}{L}}_{\text{slope}}$$

- Isopycnal slope ( $H/L$ ) is  $O(0.1-0.01)$  for submesoscale
- Isopycnal slope ( $H/L$ ) is  $O(10^{-4})$  for mesoscale

# Potential Stokes effect at the (sub)mesoscale!!

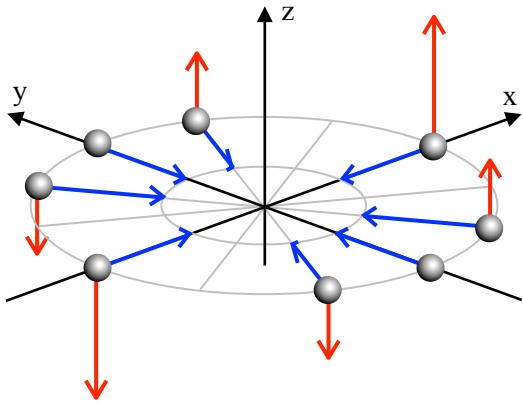
$\varepsilon/Ro$



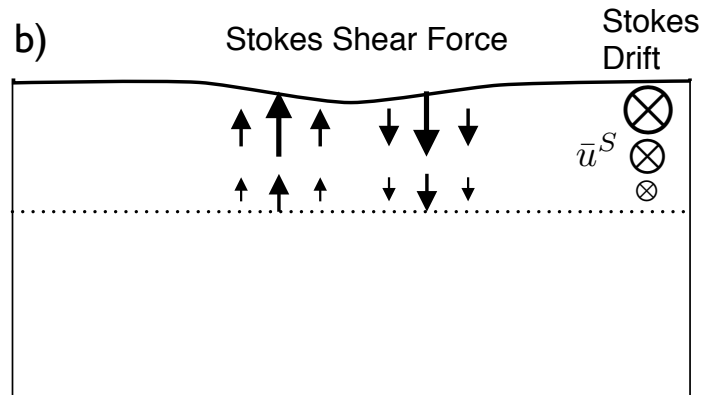
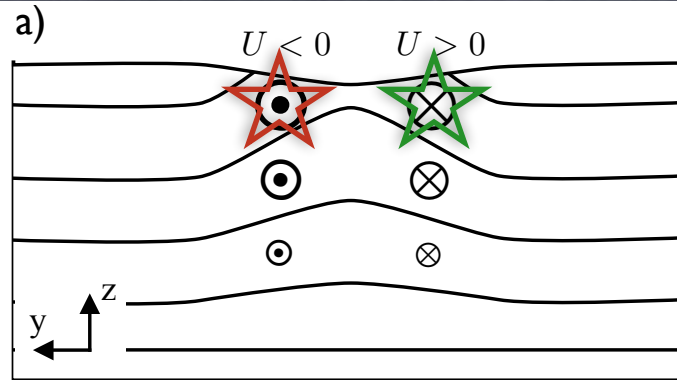
$$\frac{\varepsilon}{Ro} = \frac{V_s H f L}{f L H_s V} = \frac{V_s H}{V H_s}$$

$$\varepsilon = \frac{V^s H}{f L H_s} \quad Ro = \frac{U}{f L}$$

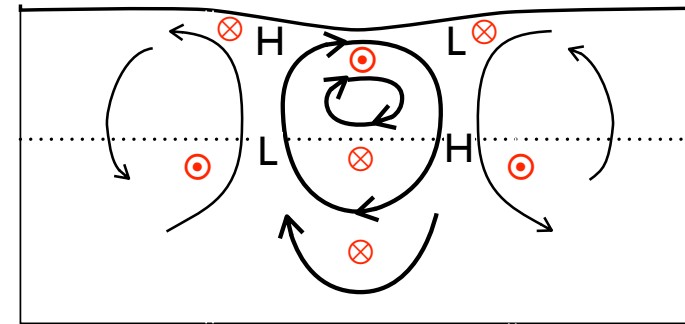
# Stokes Shear Force on Submesoscale Cold Filament



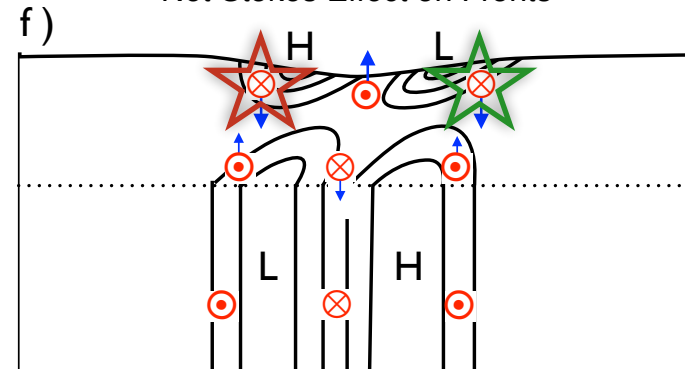
←: Stokes-shear force    ●: water parcel  
←: turbulent velocity



c) Stokes Effect on Secondary Circulation



Net Stokes Effect on Fronts



J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 730:464-490, 2013.

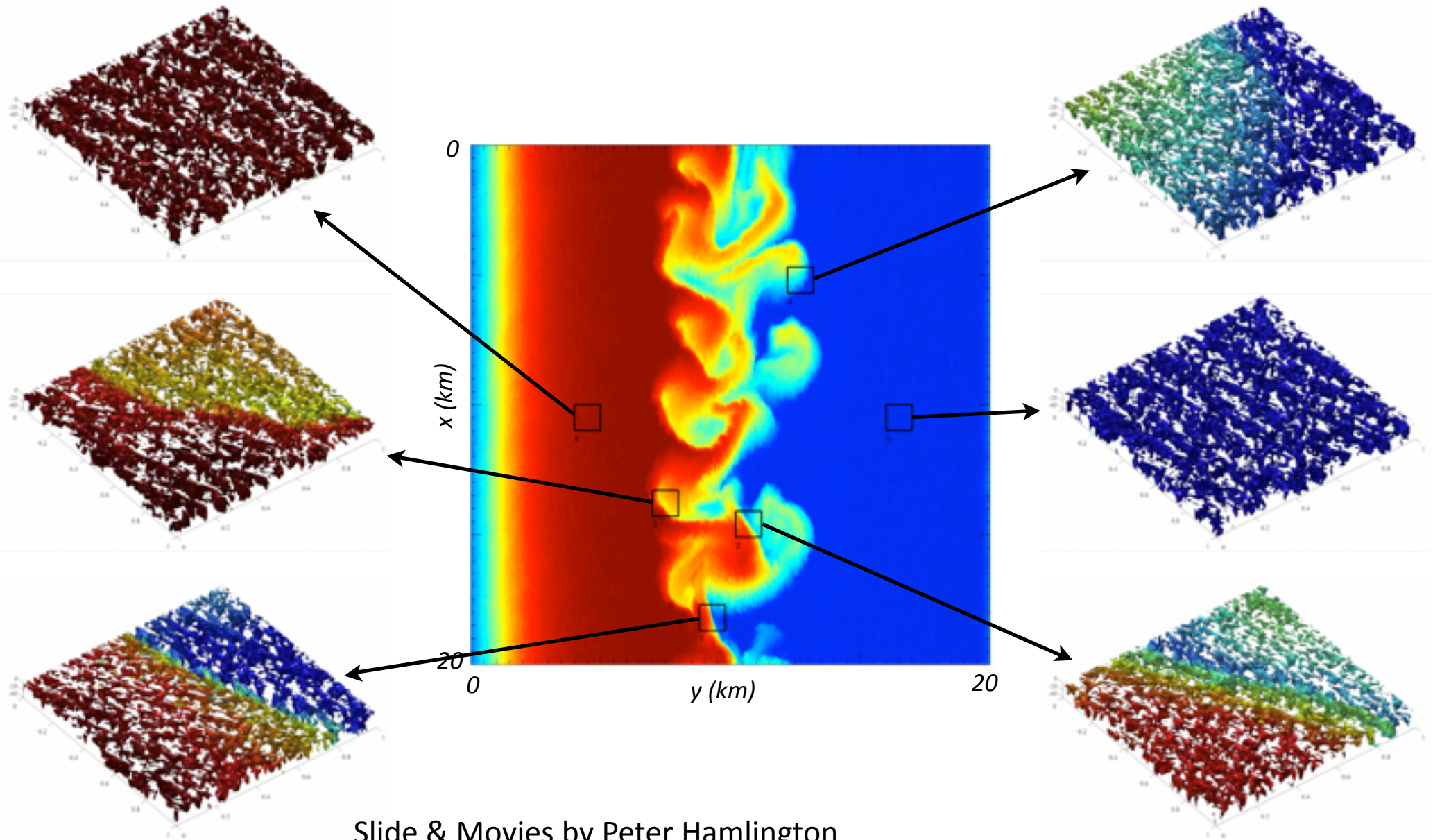
N. Suzuki and BFK. Understanding Stokes Forces in the Wave-Averaged Equations, In prep, 2014.

Enhances Fronts for Down-Front Stokes  
 Opposes Fronts for Up-Front Stokes

$$\frac{\alpha^2}{Ri} \left[ w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = -\pi_{,z} + b - \epsilon v_j^L v_{j,z}^s + \frac{\alpha^2}{Re Ri} w_{,jj}$$

Waves Give 30-40% of Power Produced at Front

# Diverse types of interaction in multiscale simulation.

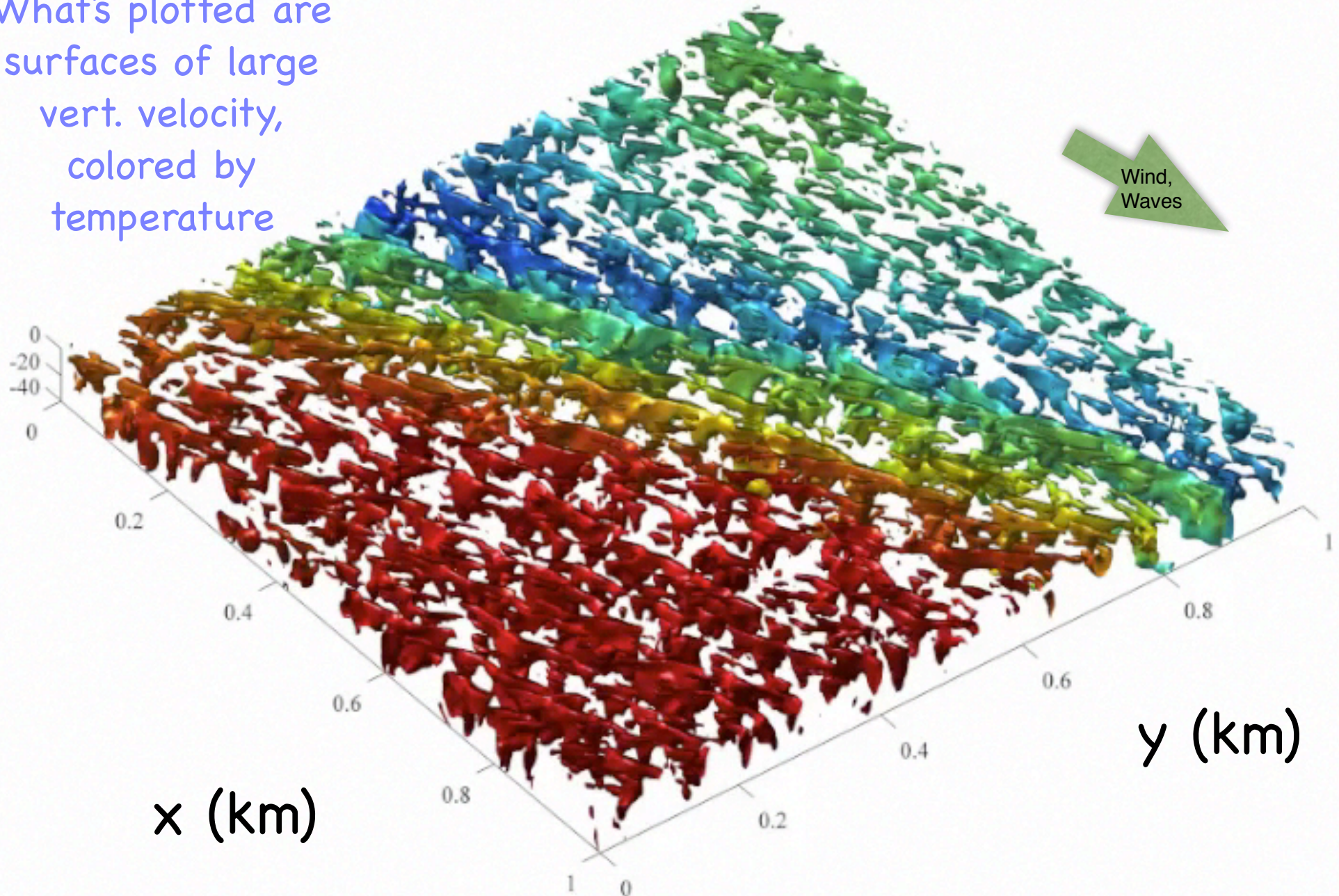


Slide & Movies by Peter Hamlington

P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. *Journal of Physical Oceanography*, 44(9):2249-2272, September 2014.

# Zoom: Submeso-Langmuir Interaction!

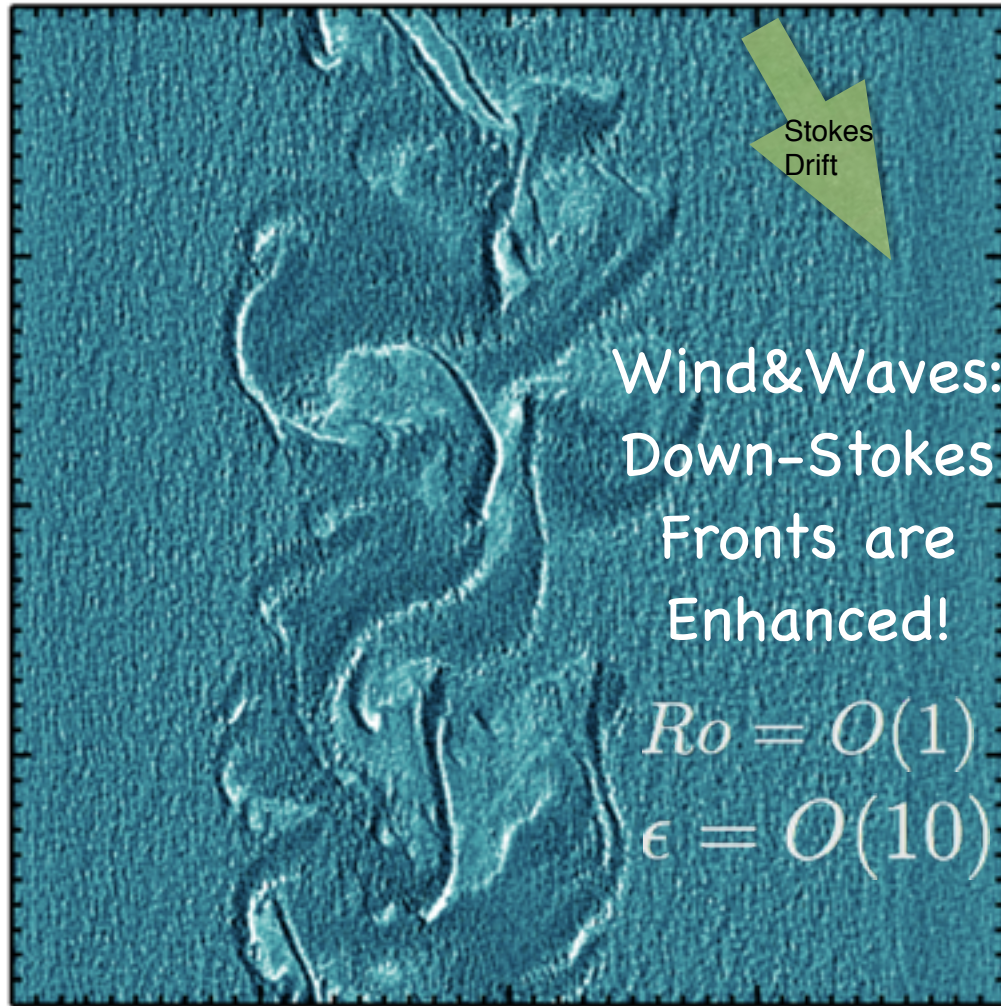
What's plotted are  
surfaces of large  
vert. velocity,  
colored by  
temperature



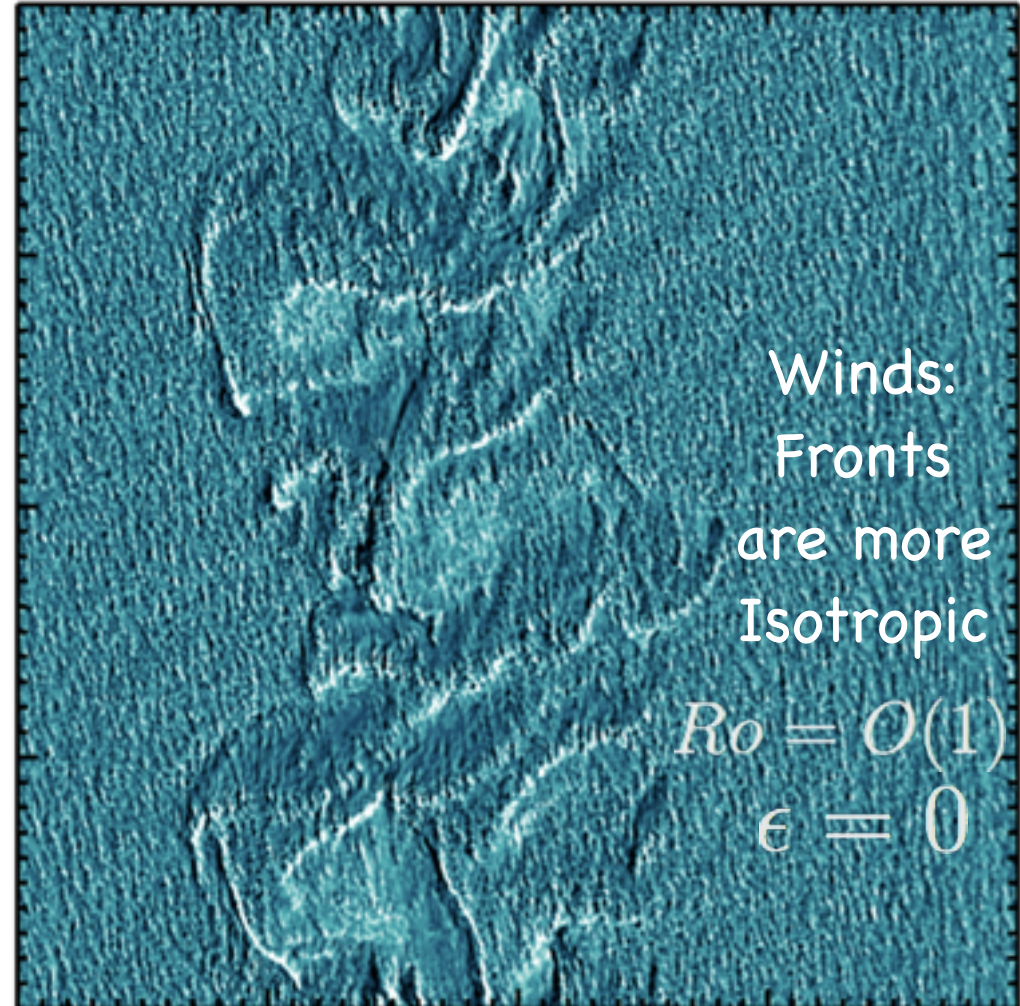
# Are Fronts and Filaments different with Stokes shear force?

$$\frac{\alpha^2}{Ri} \left[ w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = -\pi_{,z} + b - \epsilon v_j^L v_{j,z}^s + \frac{\alpha^2}{Re Ri} w_{,jj}$$

(b) LT,  $\omega_z/f$  Wind & Waves



(d) ST,  $\omega_z/f$  Wind Only



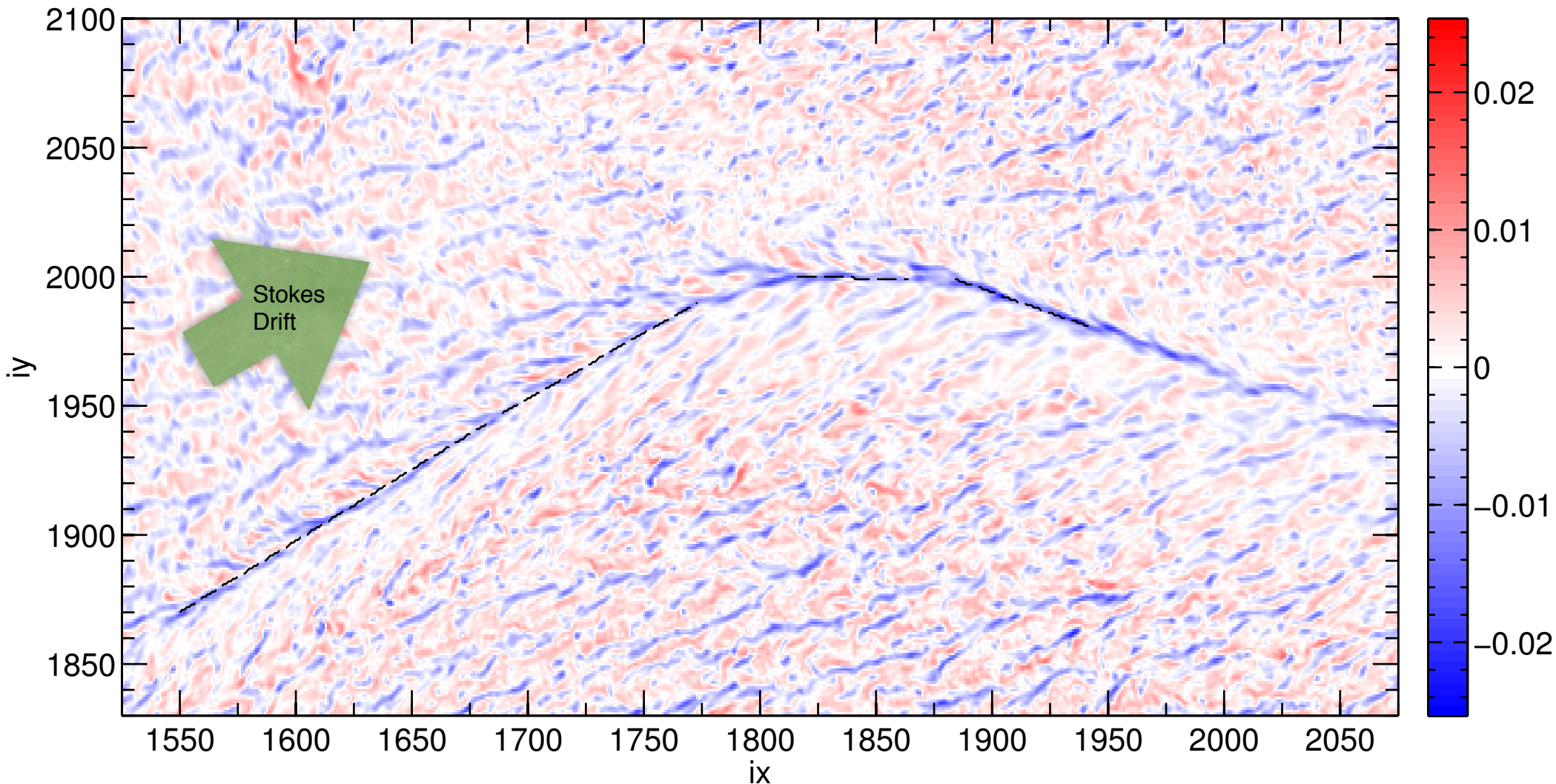
N. Suzuki and BFK. Surface Wave Stokes Forces Influence Frontogenesis, JPO, in prep, 2014.

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. Journal of Fluid Mechanics, 730:464-490, 2013.

P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. Journal of Physical Oceanography, 2014. In press.

Let's examine  
a particular front with  $\varepsilon = \frac{V^s H}{f L H_s} \approx 20$   
(Nobu Suzuki)

10min-ave. w ( $\text{ms}^{-1}$ ) at  $z = -12.5\text{m}$

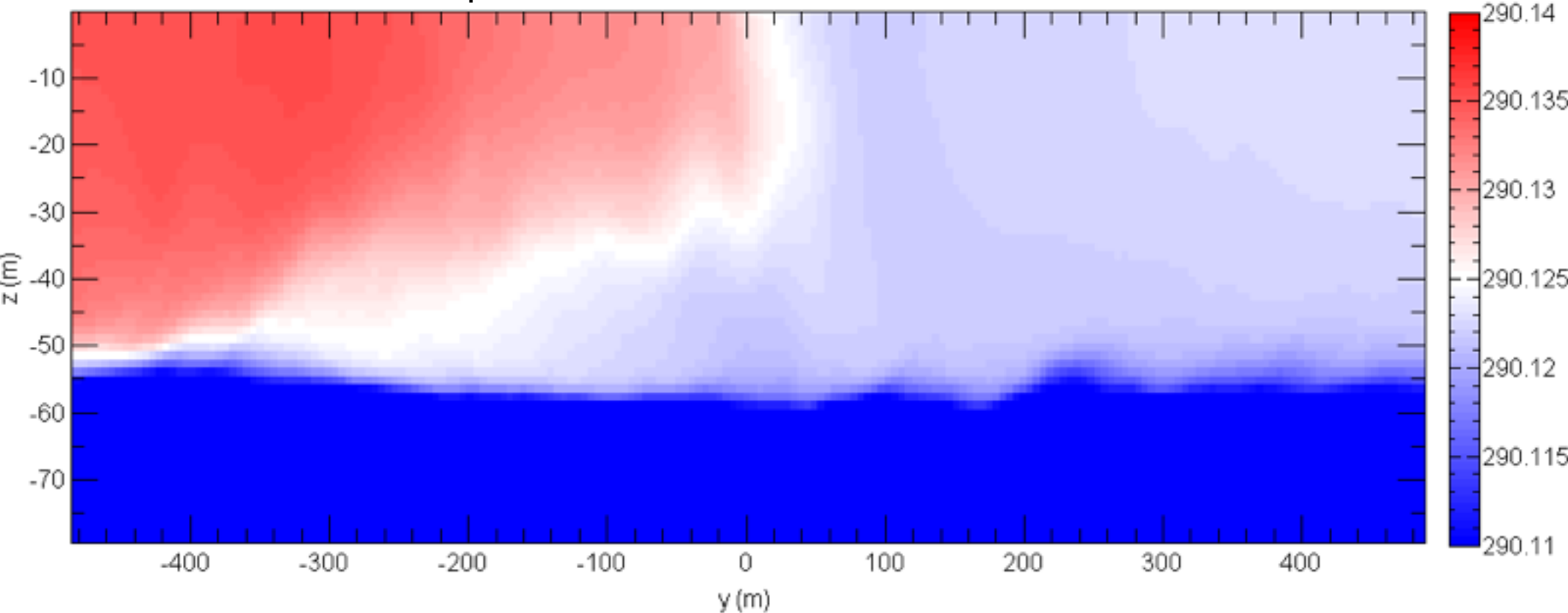




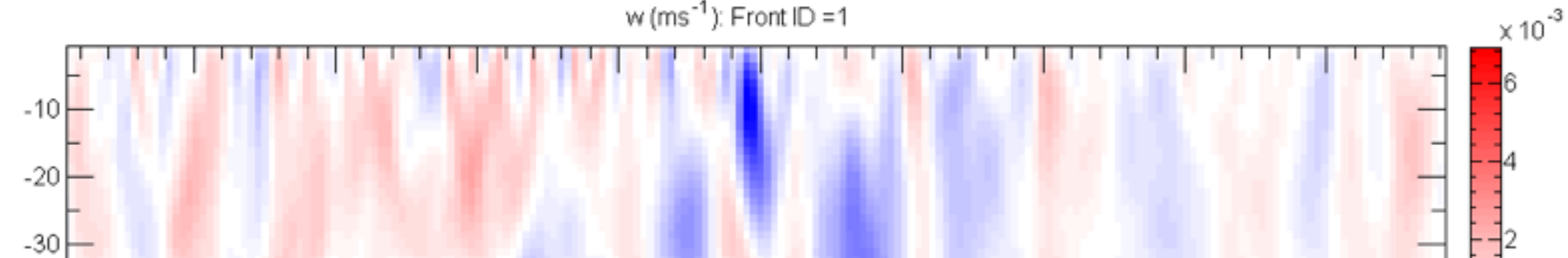
# Along-Front and 10min Average

$$\varepsilon = \frac{V^s H}{f L H_s} \approx 20$$

Temperature: theta (K): Front ID =1



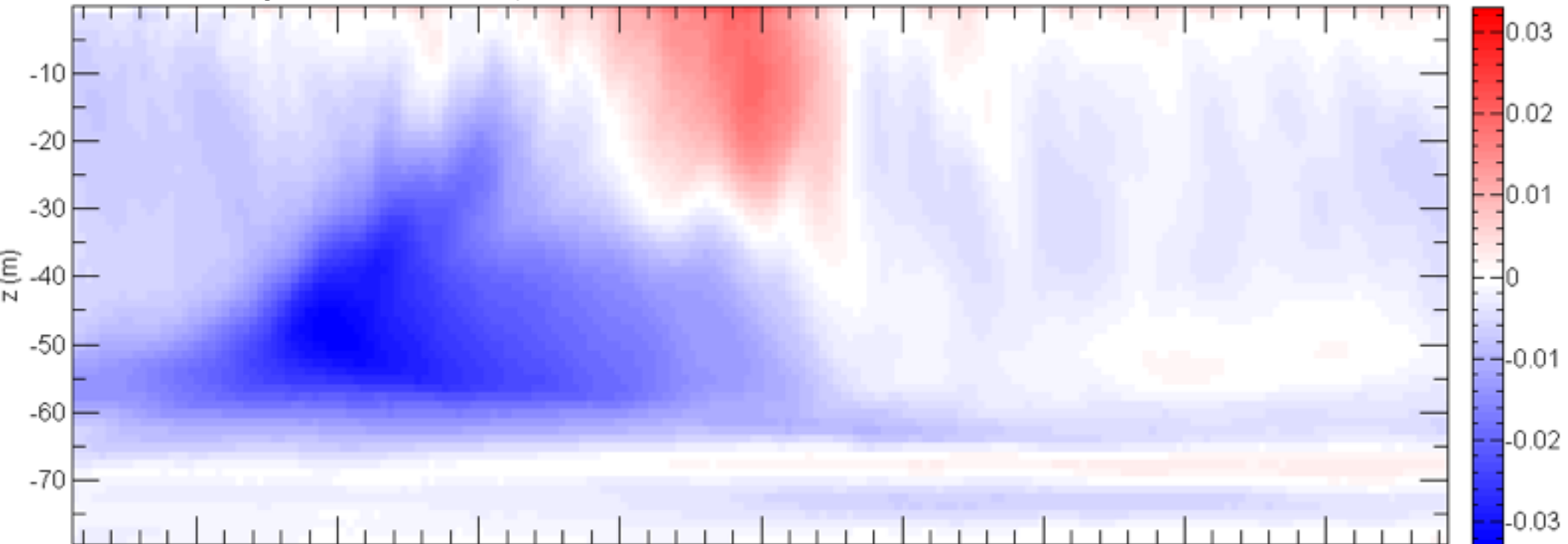
w (ms<sup>-1</sup>): Front ID =1



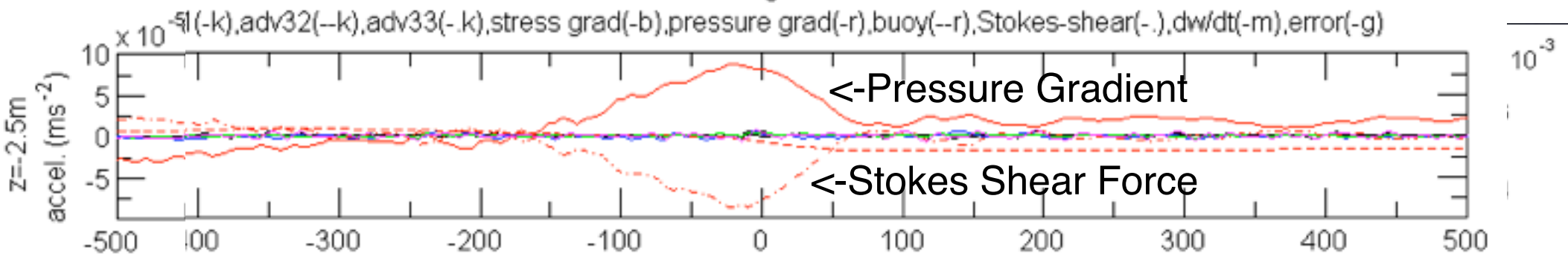
# Along-Front and 10min Average

$$\varepsilon = \frac{V^s H}{f L H_s} \approx 20$$

Along-Front Velocity: Eulerian u (ms<sup>-1</sup>): Front ID =1

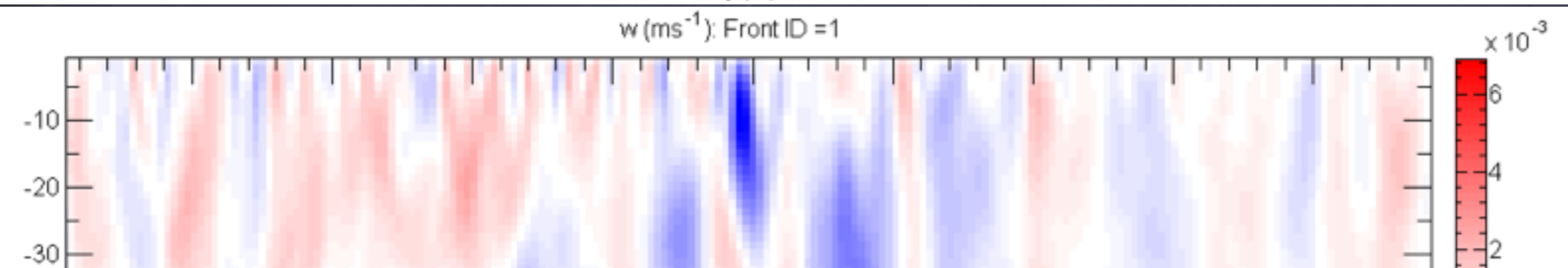
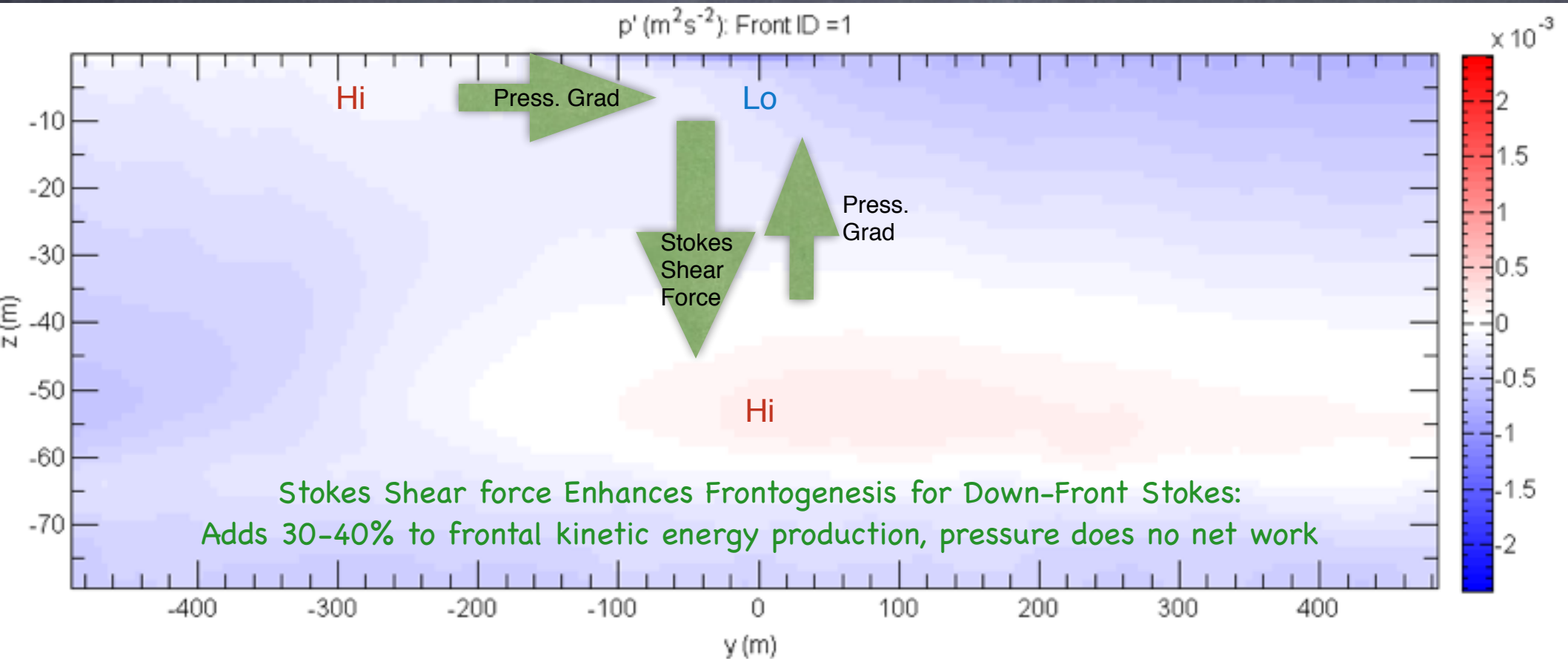


z-momentum budget: Front ID =1



# Along-Front and 10min Average

$$\varepsilon = \frac{V^s H}{f L H_s} \approx 20$$



# Are Instabilities different with Stokes shear force?

top=Stokes  
bot=no Stokes

P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. *Journal of Physical Oceanography*, 44(9): 2249-2272, September 2014.

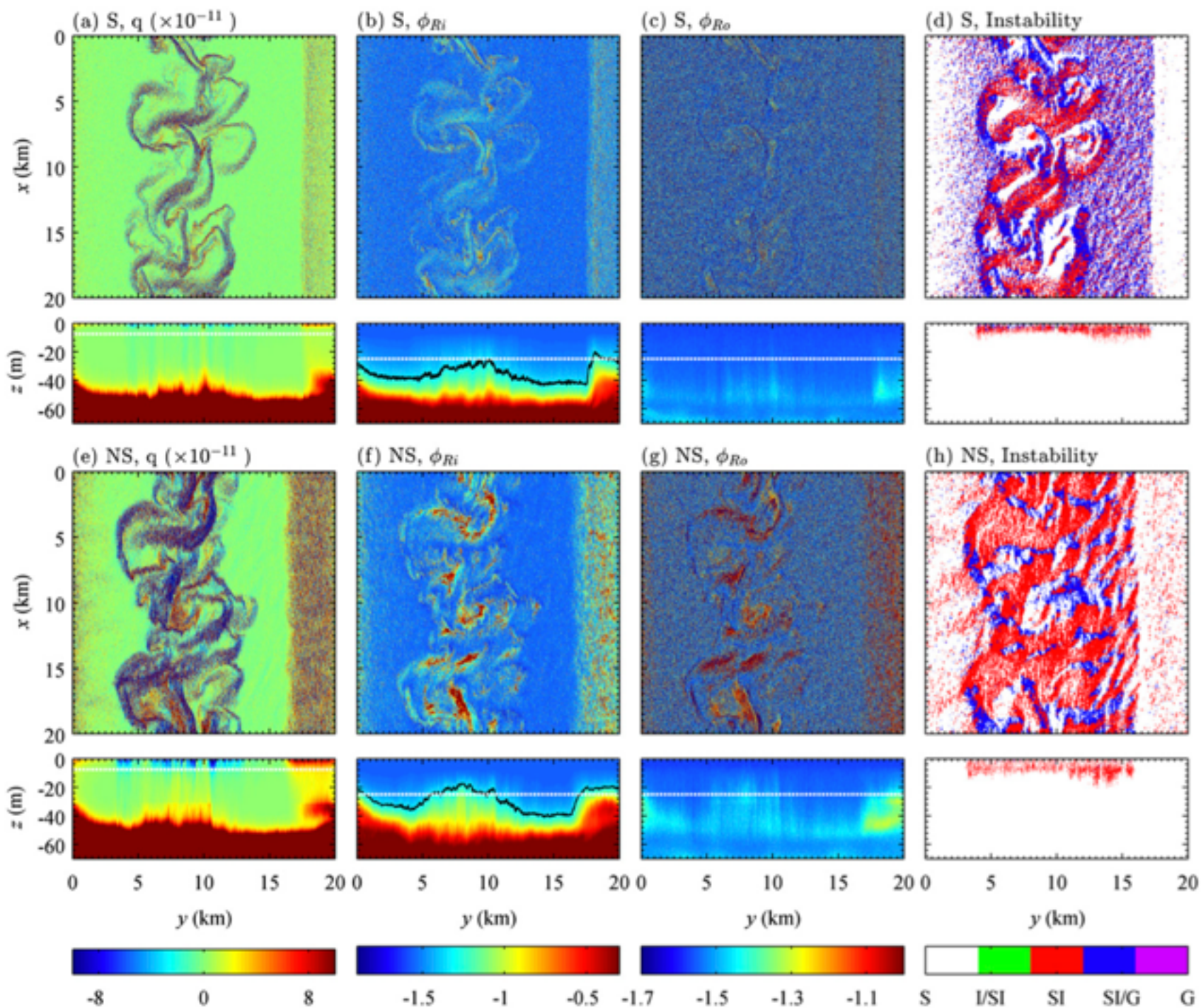
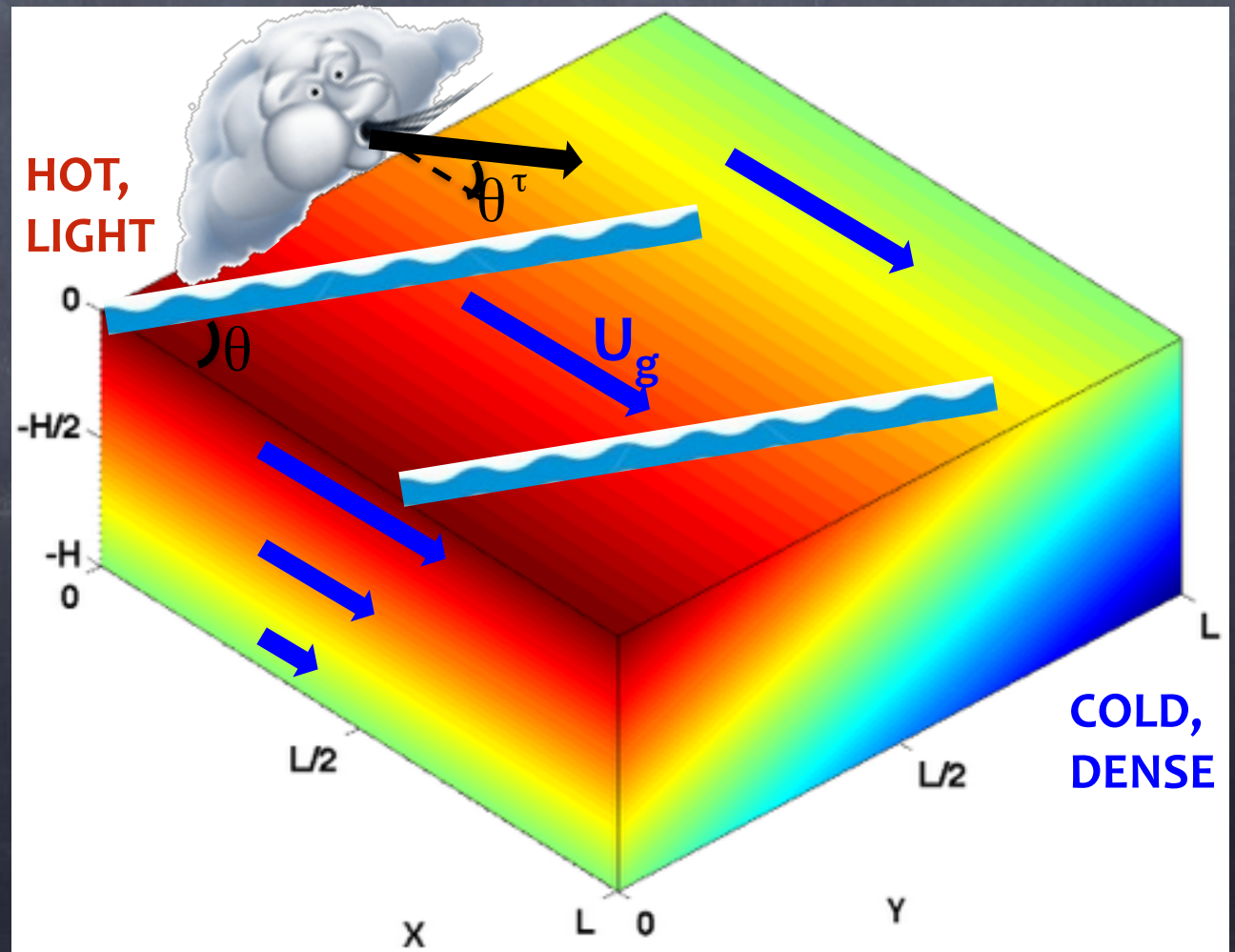


FIG. 12. (a),(e) Potential vorticity  $q$ , (b),(f) modified Richardson number  $\phi_{Ri}$ , (c),(g) modified Rossby number  $\phi_{Ro}$ , and (d),(h) instability maps in  $x$ - $y$  planes (top panels) and as a function of  $y$  and  $z$  using  $x$  averages (bottom panels) for the (a)–(d) Stokes and (e)–(h) no-Stokes simulations. Instability maps in (d) and (h) are calculated using the criteria in Table 2, and on the color axis “S” corresponds to stable regions, “I” denotes inertial instabilities, “SI” denotes symmetric instability, and “G” denotes gravitational instability.

# Stokes effects on Ocean Instabilities (Sean Haney's PhD)

- Which dynamical mixing and restratifying mechanisms, are important and under what combination of winds, waves, and fronts?
- How do the winds and waves stabilize or destabilize the typical the front?
- How does the front stabilize or destabilize the windy/wavy layer?



# Analytic Stability Criteria: Geostrophic Modes

- \* Charney, Stern, and Pedlosky showed, that geostrophic instability exists only if one of the following is true:
  1.  $Q_y$  changes sign in the interior of the domain.
  2.  $Q_y$  is the opposite sign to  $U_z^L$  at the surface.
  3.  $Q_y$  is the same sign to  $U_z^L$  at the bottom.
  4.  $U_z^L$  has the same sign at the surface and bottom.

Where  $Q$  is the quasi-geostrophic potential vorticity:

$$\bar{Q} = \nabla_{HH}^2 \bar{\psi} + \beta Y + \partial_z \left( \frac{f_0^2}{N^2} \bar{\psi}_z^L \right)$$

# Analytic Stability Criteria: Geostrophic Modes

\* Charney, Stern, and Pedlosky showed, that geostrophic instability exists only if one of the following is true:

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3.  $Q_y$  is the same sign to  $U_z^L$  at the bottom.
4.  $U_z^L$  has the same sign at the surface and bottom.

Where  $Q$  is the quasi-geostrophic potential vorticity:

$$\bar{Q} = \nabla_{\eta}^2 \bar{\psi} + \beta Y + \partial_z \left( \frac{f_0^2}{N^2} \bar{\psi}_z^L \right)$$

Charney, Stern, & Pedlosky gets a tweak →  $U$  is Lagrangian sometimes!

# Analytic Stability Criteria: Symmetric Modes

- \* Hoskins (1974) showed that symmetric instability exists only if the Ertel potential vorticity (PV) is negative.

$$PV = (\nabla \times \bar{\mathbf{U}} + f\hat{\mathbf{k}}) \cdot \nabla \bar{B} < 0 \Rightarrow SI$$

- \* Proven for constant shear Stokes drift profiles as well.
- \* The Stokes drift modifies the PV by changing the Eulerian flow that balances the pressure gradient:

Lagrangian  
Thermal Wind:

$$\bar{\mathbf{U}}_z = -\frac{\nabla_H \bar{B}}{f} - \mathbf{U}_z^S$$

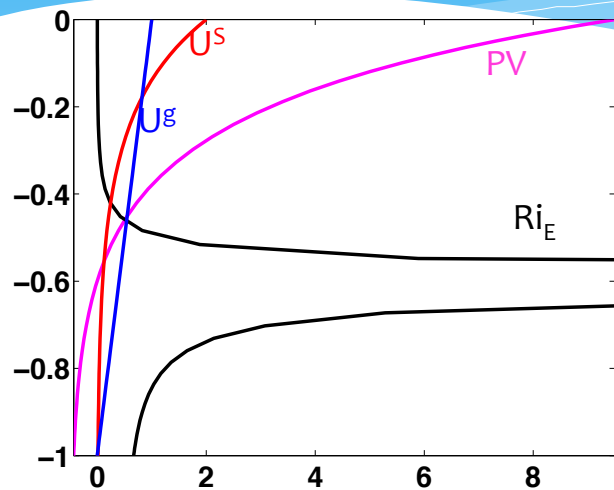
- \* The Stokes drift does not contribute directly to Ertel PV
- \* Be careful! Can't use tracer diagnosis to find Eulerian shear!



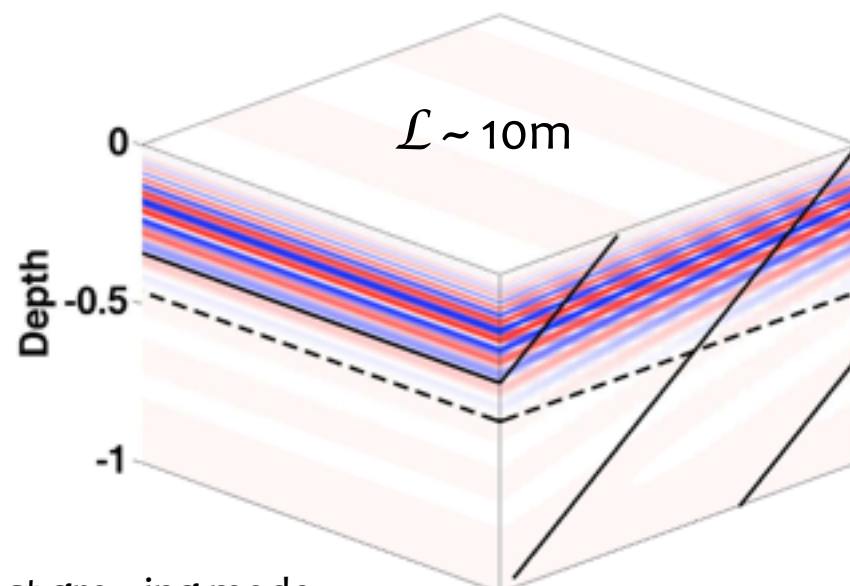
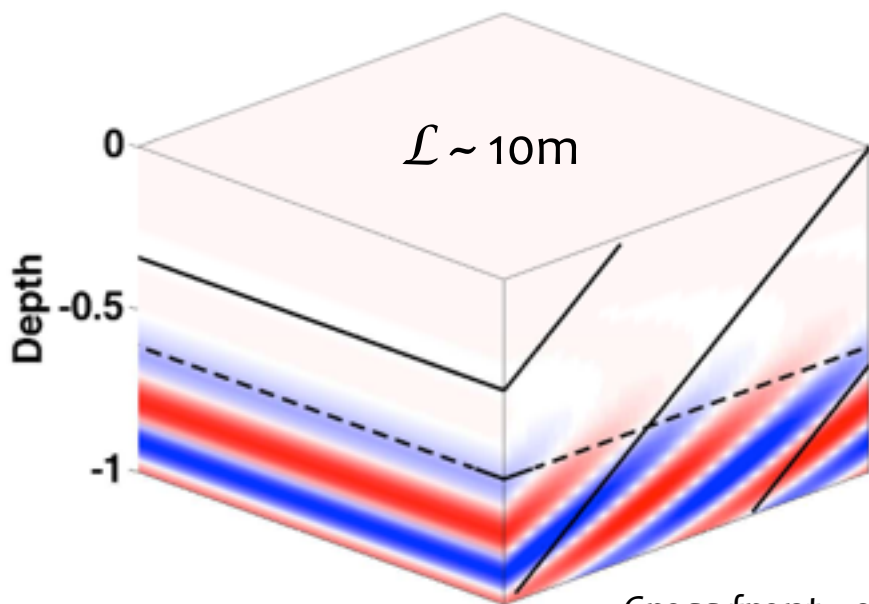
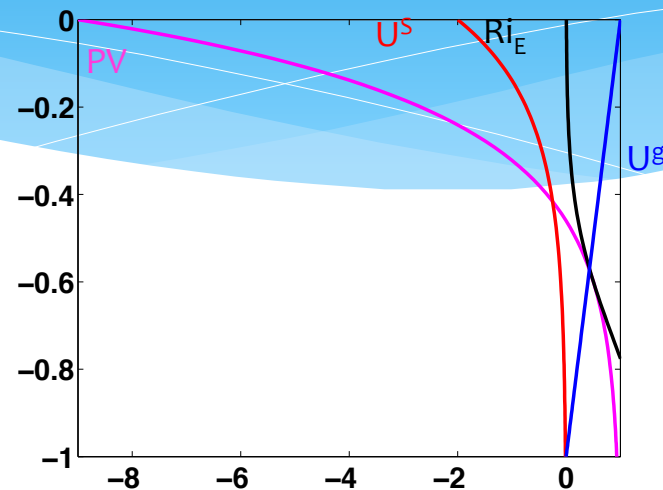
~~$Ri < 1 \Rightarrow SI$~~

$PV < 0 \Rightarrow SI$  ✓

$Ri = 0.5$



$Ri = 2$



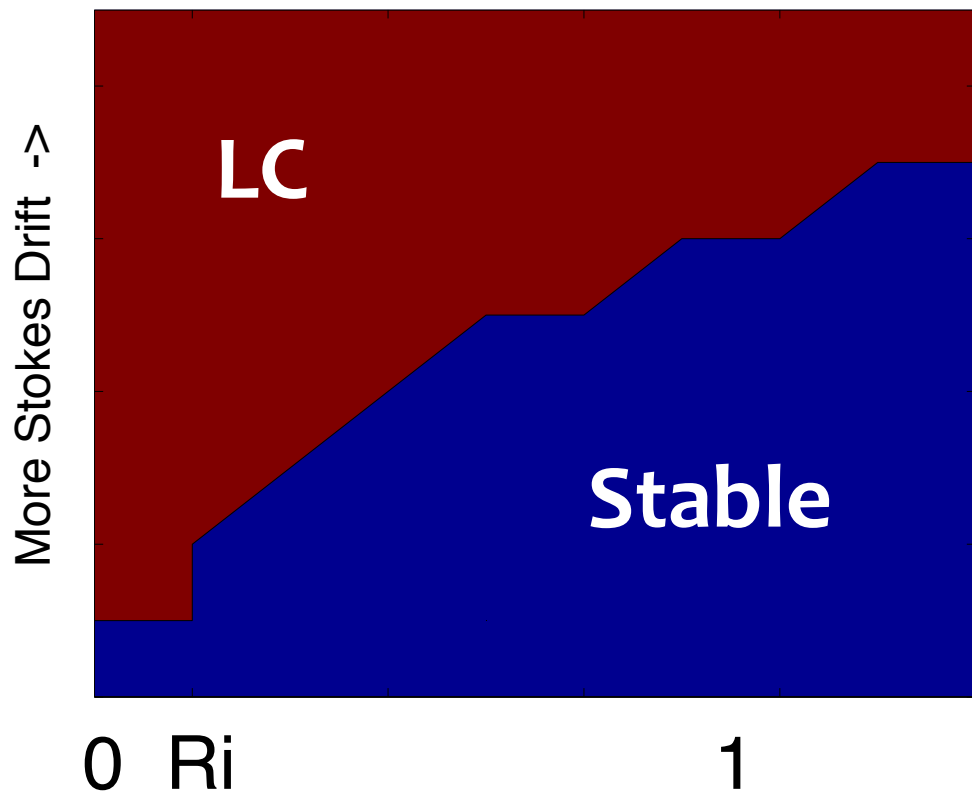
— Isopycnals

-- PV=0

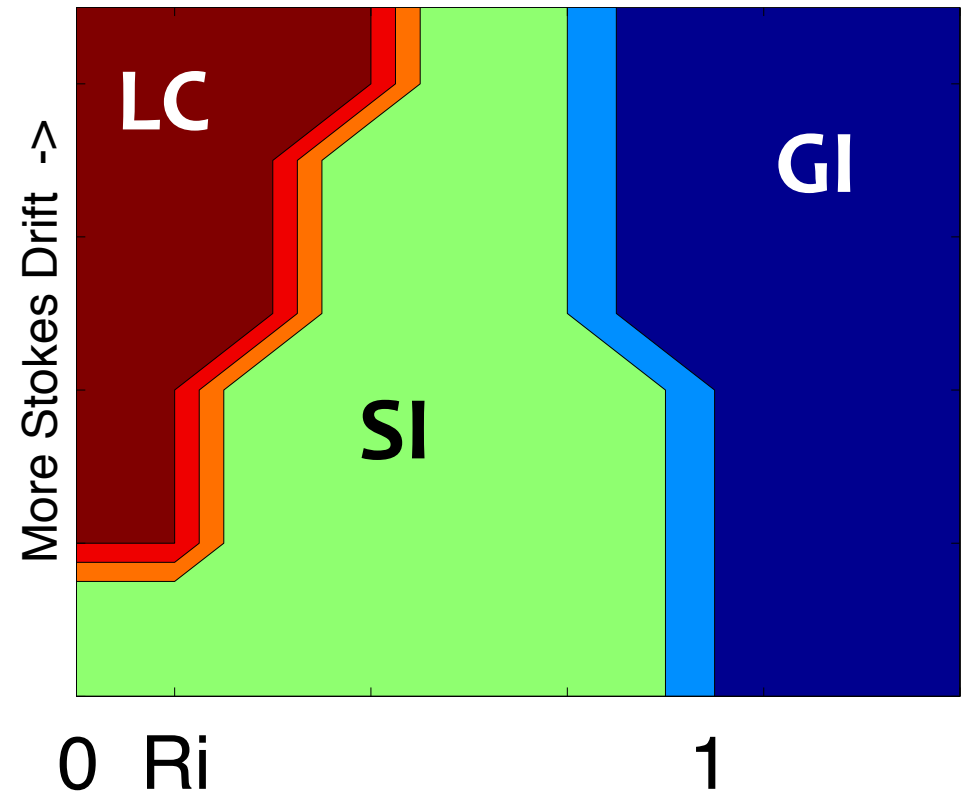
Cross front velocity for the fastest growing mode

# Linear Instability Regimes for Fastest Growing Mode

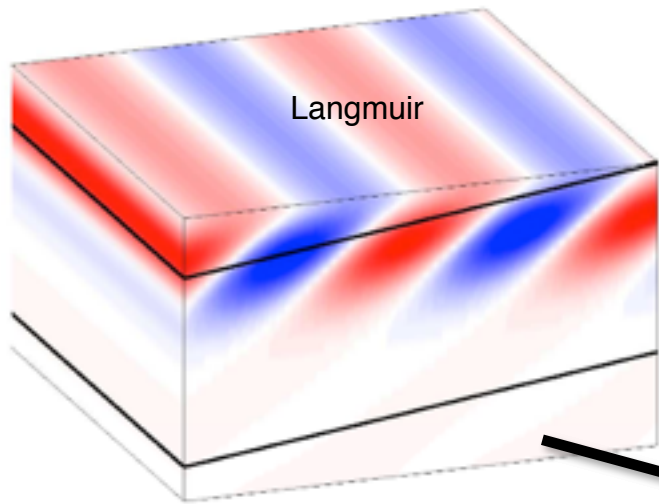
Weak Front



Strong Front



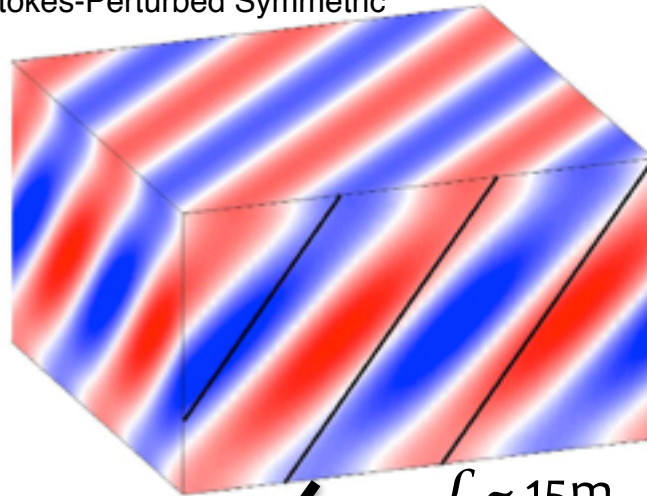
Along Front Velocity



$\mathcal{L} \sim 1\text{m}$

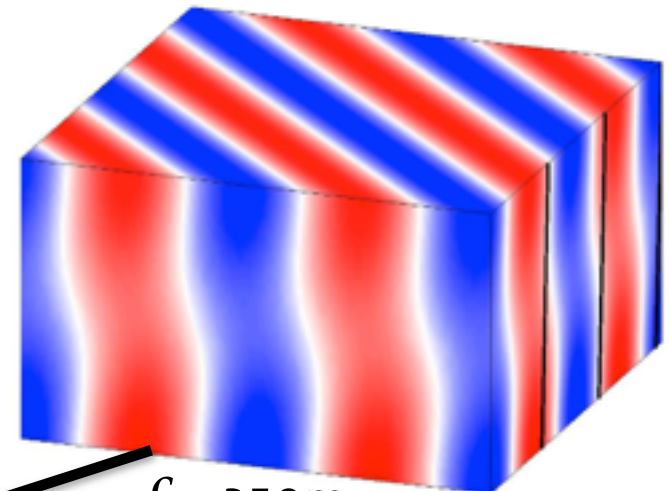
Cross Front Velocity

Stokes-Perturbed Symmetric



$\mathcal{L} \sim 15\text{m}$

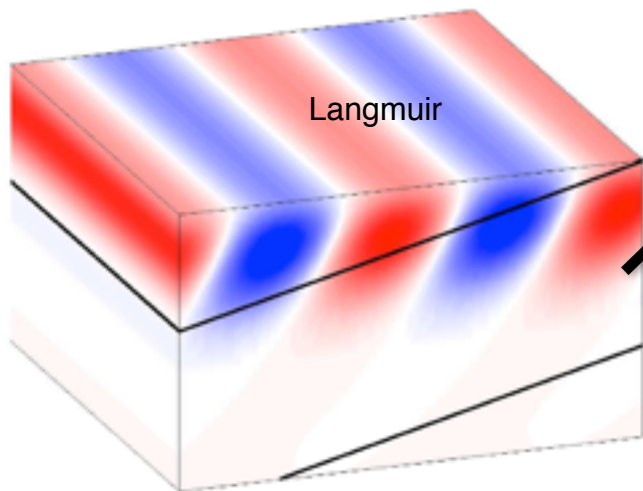
Cross Front Velocity



$\mathcal{L} \sim 250\text{m}$

Minimally  
Stokes-Perturbed  
Geostrophic

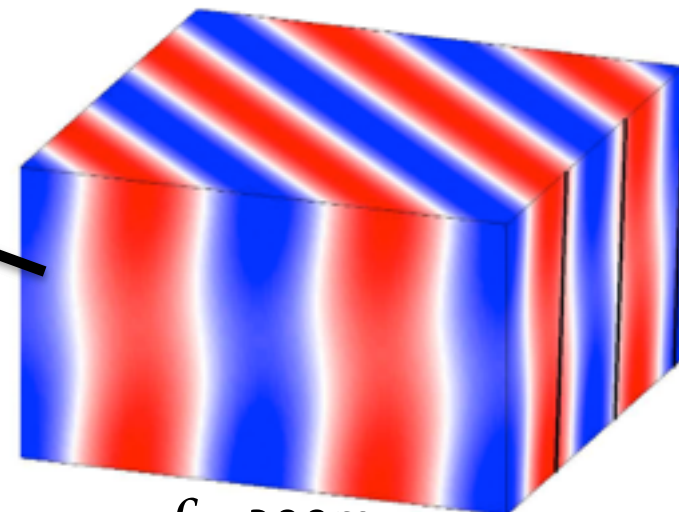
Along Front Velocity



$\mathcal{L} \sim 1\text{m}$



Cross Front Velocity

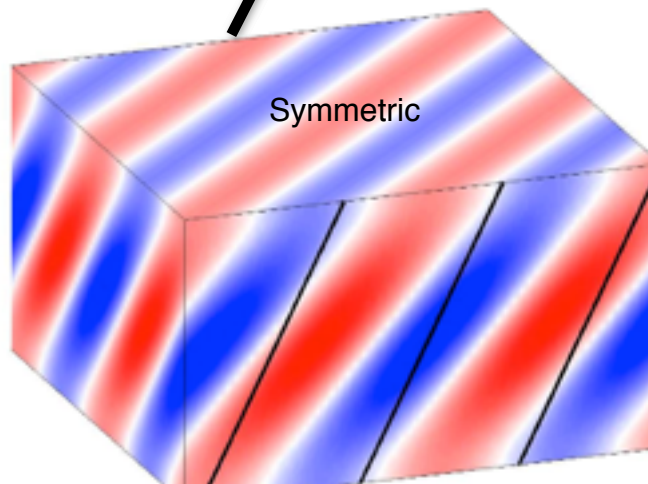


$\mathcal{L} \sim 200\text{m}$

Geostrophic

Cross Front Velocity

$\mathcal{L} \sim 50\text{m}$



# What's What?

- We have linear & nonlinear sims. with Geostrophic, Langmuir, Kelvin-Helmholtz, and Symmetric Instabilities
- How do we distinguish them?
  - Energy Source (e.g., PE→baroclinic, Stokes shear→LC)
  - By Scale & Orientation (vs. Stokes, shear, etc.)
  - By Dependence on Parameters of Growth & Scale

$$PV < 0 \Rightarrow SI$$



$U_g$

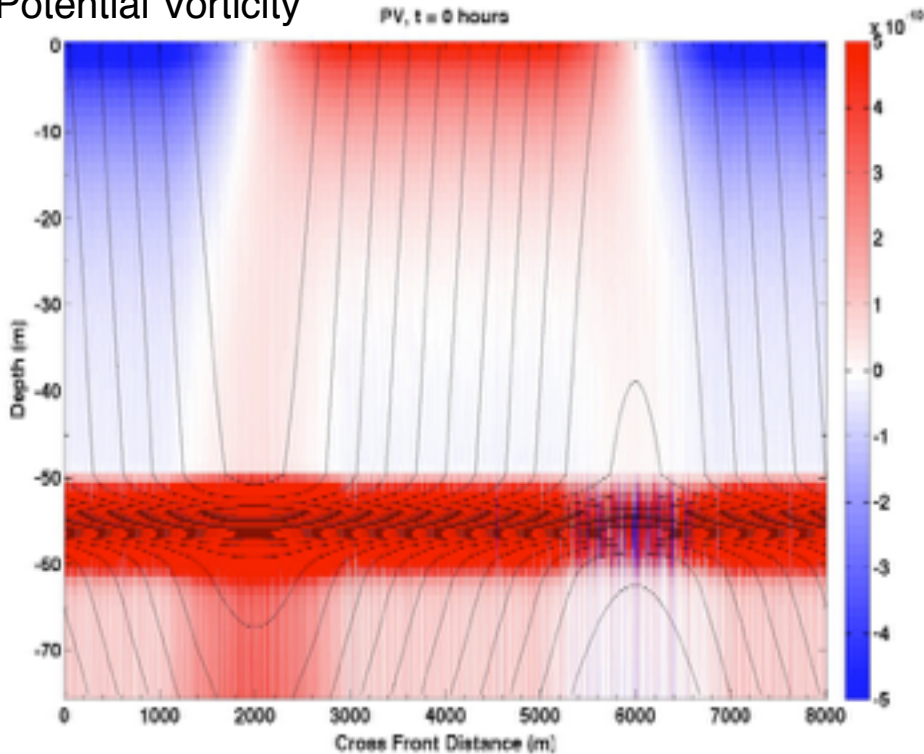


$U_s$



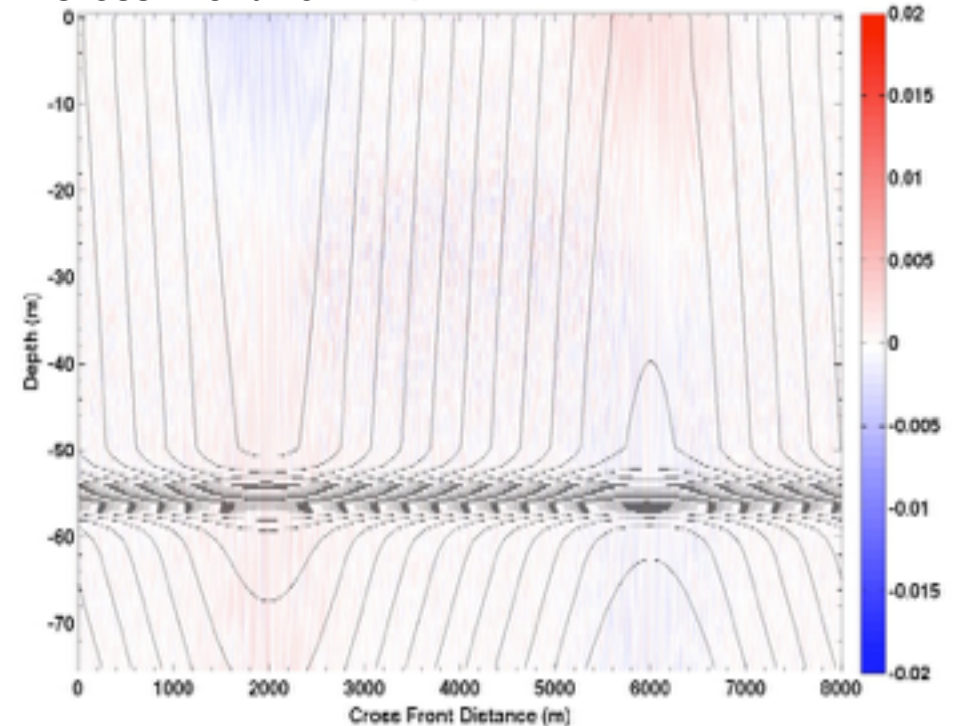
Potential Vorticity

PV, t = 0 hours



Cross-Front Vel.

V, t = 1 hours



- A “no [wind] stress” Ekman layer develops due to the the surface geostrophic stress.
- SI develop only in regions of negative PV, and are stronger for more negative PV.
- SI restore the PV to zero by exchanging negative PV for positive PV in the pycnocline.

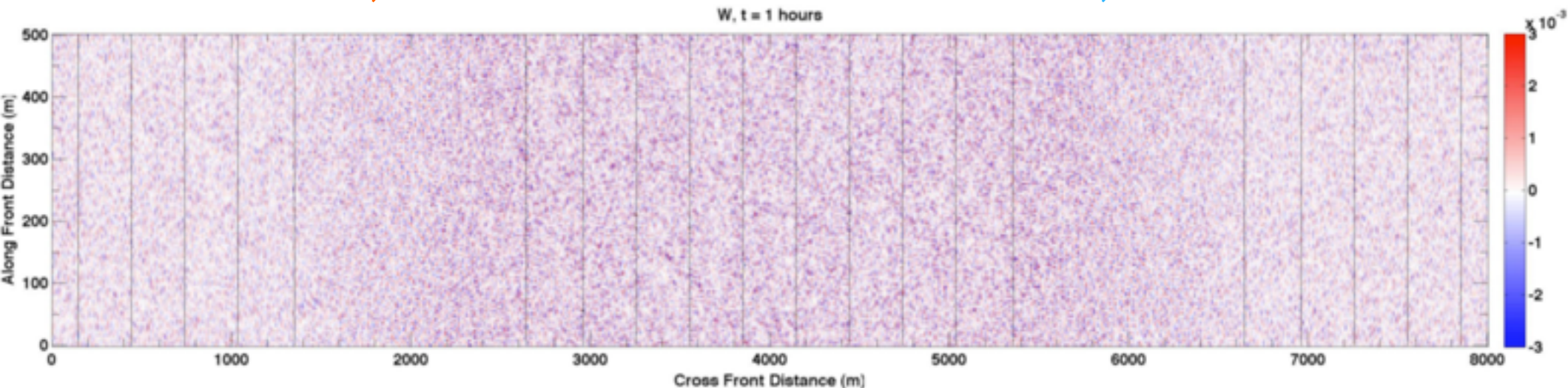
Simulation with no wind, but waves & fronts. Domain too small for mixed layer eddies.  
 It is SI, not LC that restratify. SI are strongly affected by Stokes drift.  
 Compare to Taylor & Ferrari (2010) & Li, Chini, Flierl (2012).

# Near-Surface: No SI, Fronts Slow LC & Make KH rolls

Horizontal slice of vert. vel. at ~ 5m deep.

**HOT, LIGHT**

**COLD, DENSE**

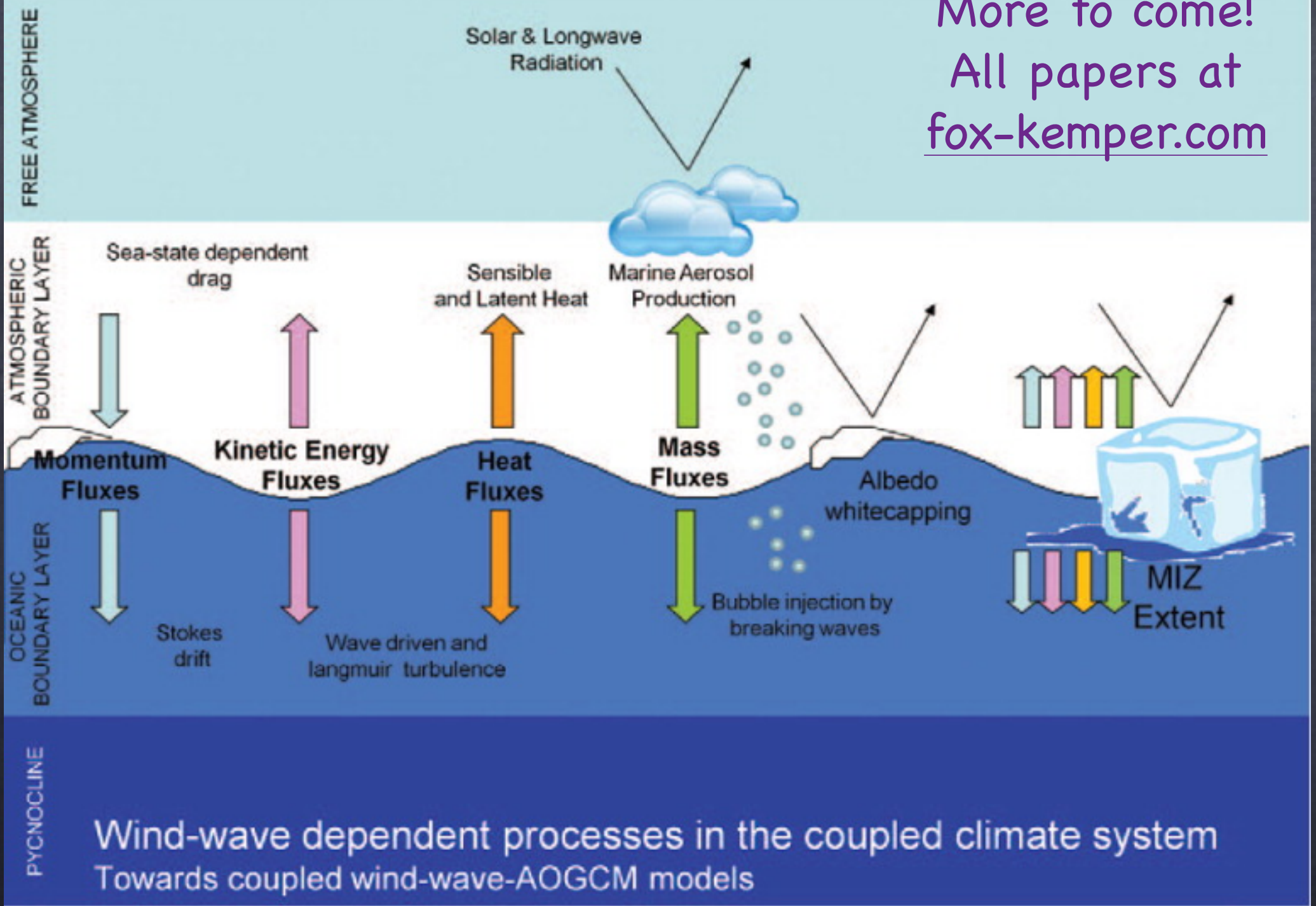


- LC develop in regions without horizontal stratification, align with Lagrangian shear direction
- Unstable stratification in central front yields convective KH rolls, perp to shear.

# Conclusions

- Climate modeling is challenging partly due to the vast and diverse scales of fluid motions
- In the upper ocean, horizontal scales as big as basins, and as small as meters contribute non-negligibly to the air-sea exchange and climate
- Interesting transition occurs on the Submeso to Langmuir scale boundary, as nonhydro. & ageostrophic effects begin to dominate
- The effects of the Stokes forces on boundary layer and submesoscale dynamics are under-appreciated.
- All papers at: [fox-kemper.com/pubs](http://fox-kemper.com/pubs)

More to come!  
All papers at  
[fox-kemper.com](http://fox-kemper.com)



Wind-wave dependent processes in the coupled climate system  
Towards coupled wind-wave-AOGCM models

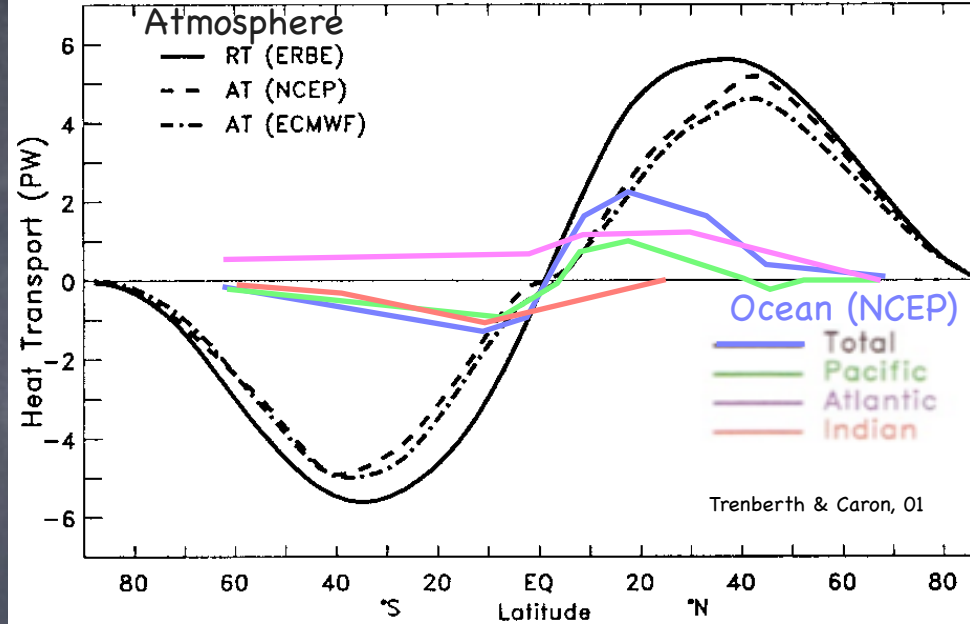


# Air-Sea Flux Errors vs. Data

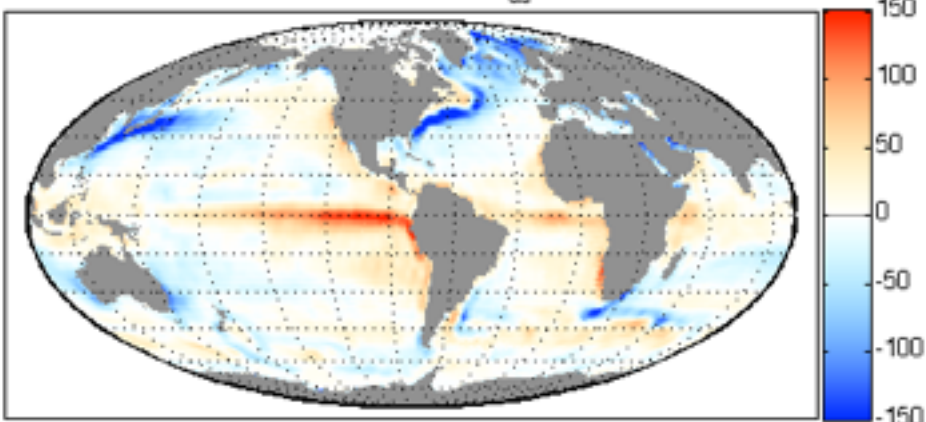
Heat capacity & mode of transport is different in A vs. O

>90% of GW is oceanic, 10m O=whole A

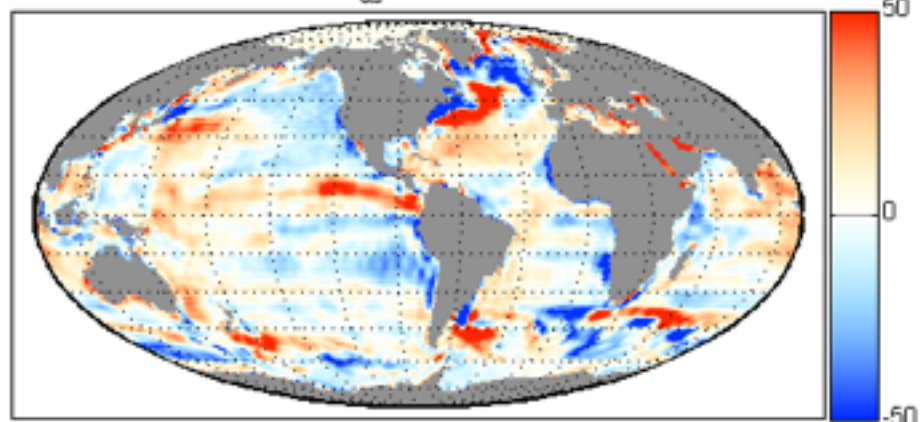
S. C. Bates, BFK, S. R. Jayne, W. G. Large, S. Stevenson, and S. G. Yeager.  
 Mean biases, variability, and trends in air-sea fluxes and SST in the  
 CCSM4. *Journal of Climate*, 25(22):7781-7801, 2012.



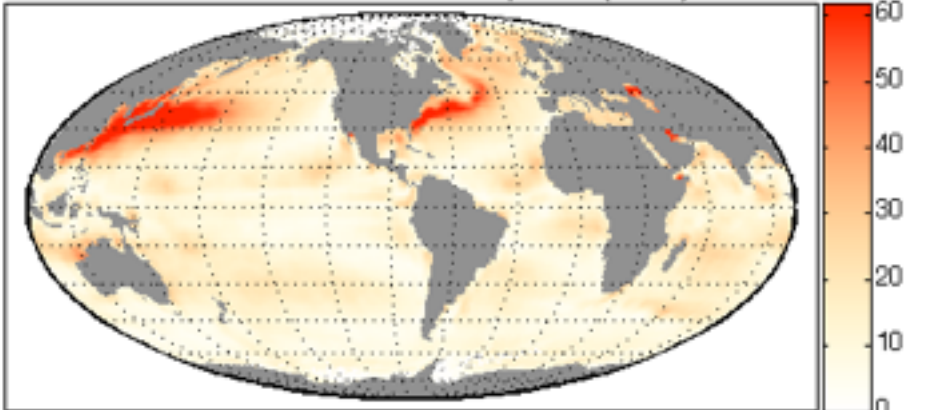
Mean of 1986-2005 CORE  $Q_{as}$  ( $W/m^2$ )



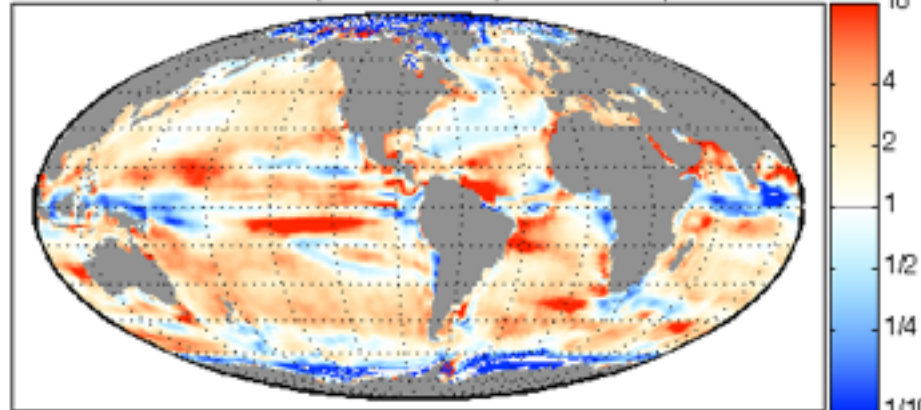
1986-2005 CCSM4-CORE  $Q_{as}$  bias, mean:1.5, rms:23 ( $W/m^2$ )



St. Dev. of CORE annual evaporation ( $W/m^2$ )



Variance ratio (CCSM4/CORE) of annual evaporation



Mean

Annual  
9-15mo

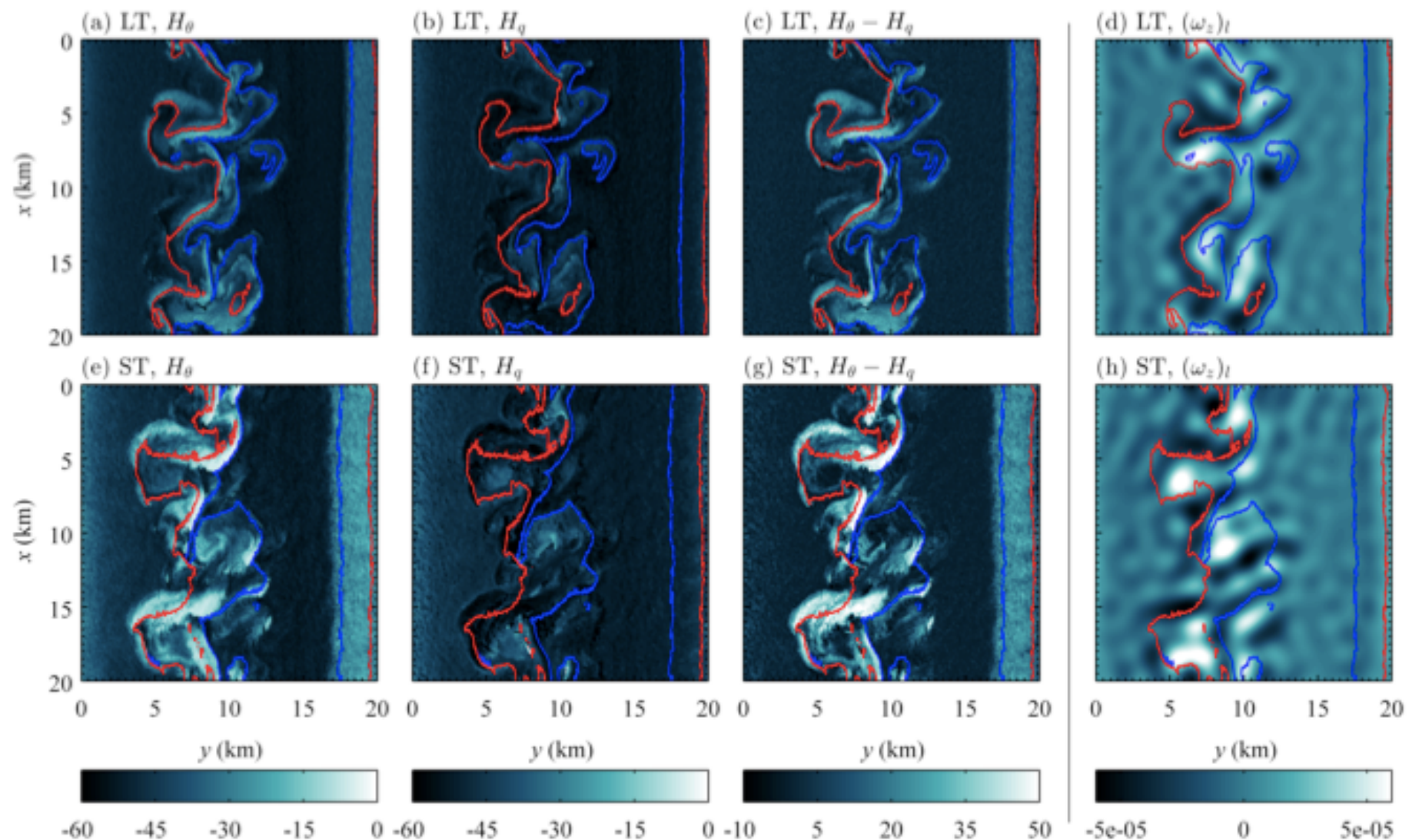


FIG. 13. Fields of the mixed layer depth (in m) based on temperature, denoted  $H_\theta$ , (a,e) and on potential vorticity, denoted  $H_q$ , (b,f) for the LT (a,b) and ST (e,f) cases. The difference  $H_\theta - H_q$  is shown in (c,g) and low-pass (submesoscale) vertical vorticity fields are shown in (d,h), where the filter cutoff for the vorticity fields is at 2km. Contour lines correspond to temperature contours taken from Figure 2.

# Mixed Layer Eddy Res

Estimating eddy buoyancy/c

$$\overline{\mathbf{u}'b'} \equiv \Psi \times \nabla \bar{b}$$

A submeso eddy-induced

$$\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times$$

in ML only:

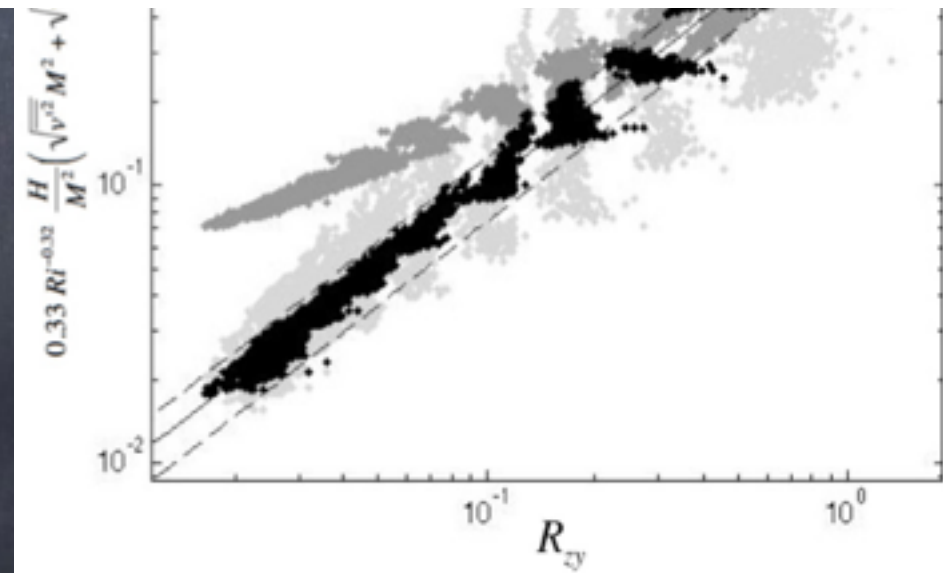
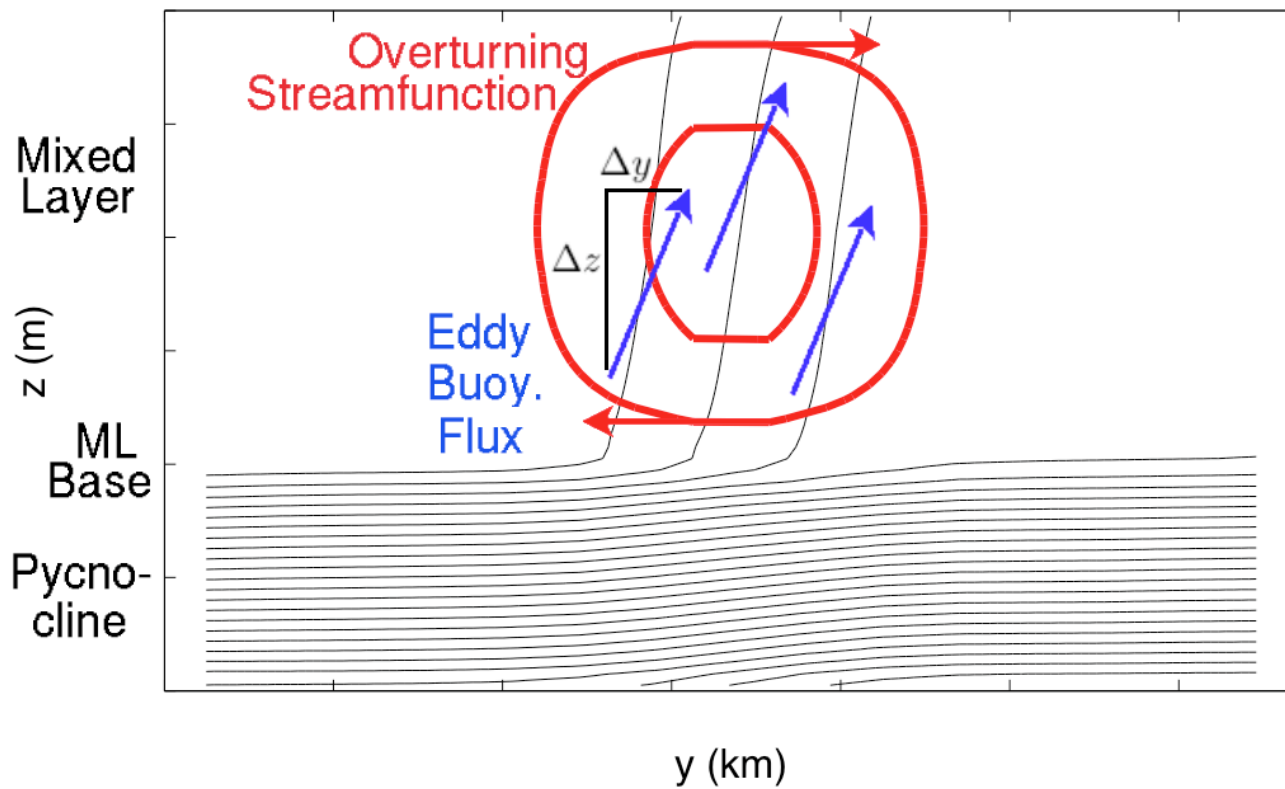
$$\mu(z) = 0 \text{ if } z < -H$$

For a consistently restratifying,

$$\overline{w'b'} \propto \frac{H^2}{|f|} |\nabla_H \bar{b}|^2$$

and horizontally downgradient flux.

$$\overline{\mathbf{u}'_H b'} \propto \frac{-H^2 \frac{\partial \bar{b}}{\partial z}}{|f|} \nabla_H \bar{b}$$



S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013

# Sensitivity of Climate to Submeso: AMOC & Cryosphere Impacts

May Stabilize AMOC

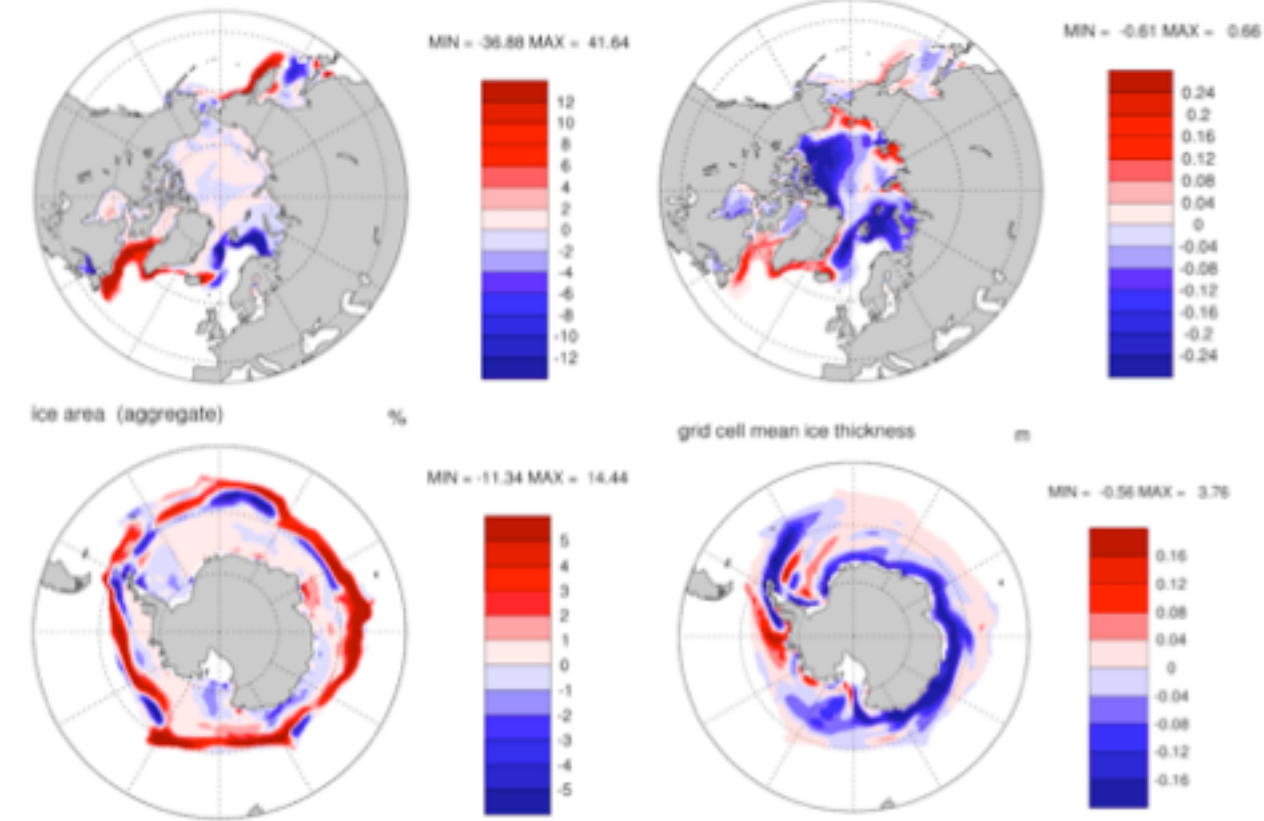
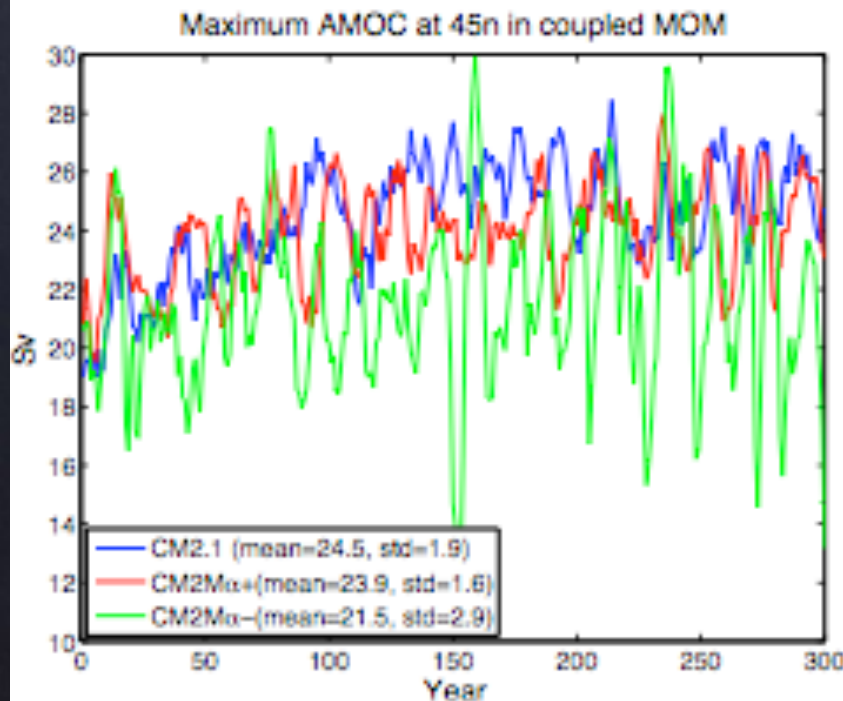


Figure 10: Wintertime sea ice sensitivity to introduction of MLE parameterization (CCSM<sup>+</sup> minus CCSM<sup>-</sup>): January to March Northern Hemisphere a) ice area and b) thickness and July to September Southern Hemisphere c) ice area and d) thickness.

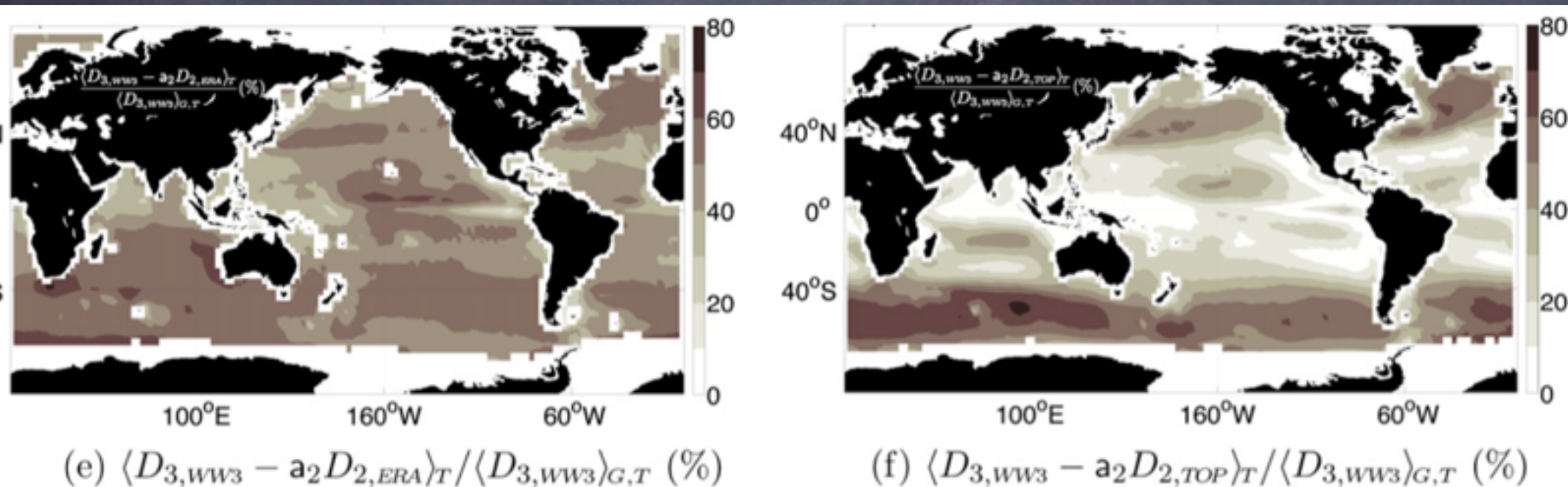


Affects sea ice

NO RETUNING  
NEEDED!!!

These are impacts:  
bias change unknown

# How well do we know Stokes Drift? <50% discrepancy



RMS error in measures of surface Stokes drift,  
2 wave models (left), model vs. altimeter (right)

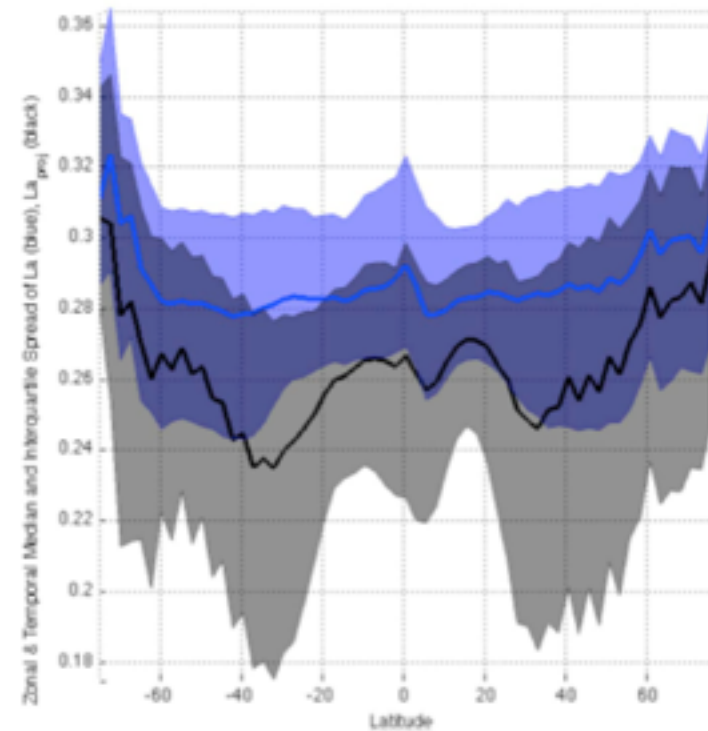
Year 2000 data & models

# Why? Vortex Tilting Mechanism

In CLB: Tilting occurs in direction of  $\mathbf{u}_L = \mathbf{v} + \mathbf{v}_s$

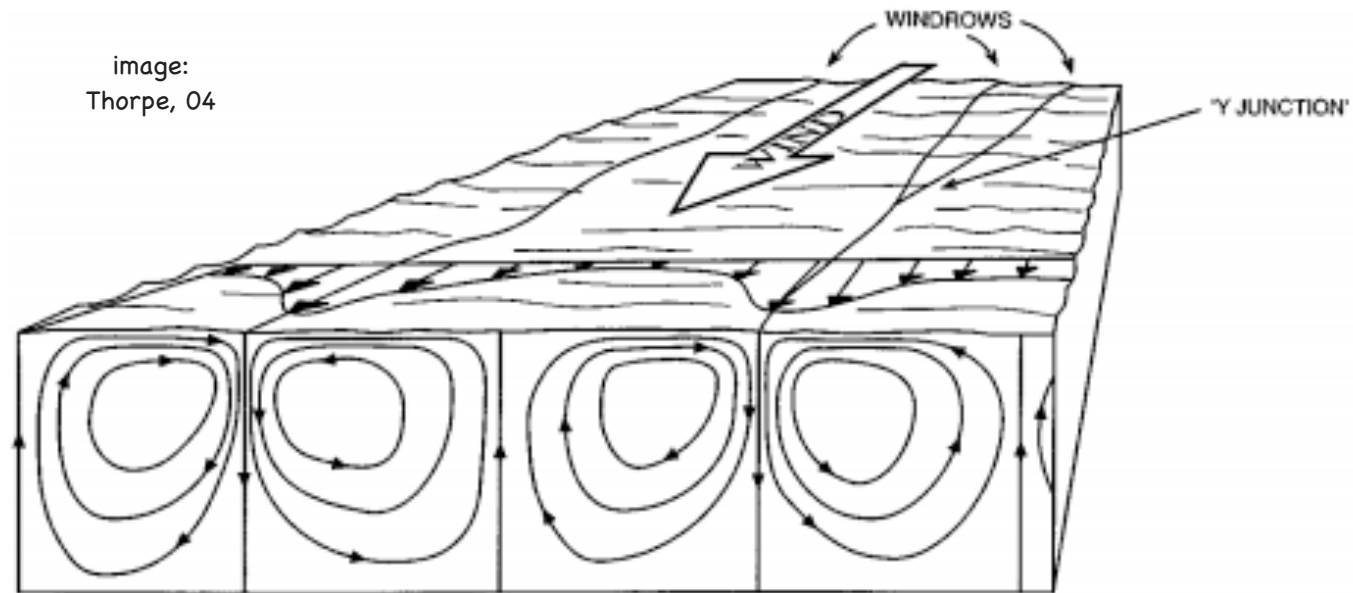
Misalignment enhances degree of wave-driven LT

$$\frac{\partial \xi}{\partial t} + \underbrace{(\mathbf{u}_L \cdot \nabla)}_{AD} \xi = \underbrace{(\boldsymbol{\omega}_a \cdot \nabla)}_{TS} (\mathbf{u}_L \cdot \hat{\mathbf{x}}') + \underbrace{(\nabla b \times \hat{\mathbf{z}})}_{BV} \cdot \hat{\mathbf{x}}' + \text{SGS},$$



**Figure 17.** Temporal and zonal median and interquartile range of  $La_t$  and  $La_{proj}$  for a realistic simulation of 1994–2002 using Wave Watch III.

image:  
Thorpe, 04



**Figure 1** Sketch showing the pattern of mean flow in idealized Langmuir circulation. The windrows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). In practice the flow is turbulent, especially near the water surface, and the windrows (Figure 2) amalgamate and meander in space and time. Bands of bubbles or buoyant algae may form within the downward-going (or downwelling) flow (see Figure 3).

# So, no problems?

## Just crunch away with CLB?

- Let's revisit our assumptions for scale separation:
  - CLB wave equations require limited \*wave steepness\* and irrotational flow
  - Real wind-waves are not monochromatic, but incorporate a spectrum of waves, and...



Power Spectrum  
of wave height

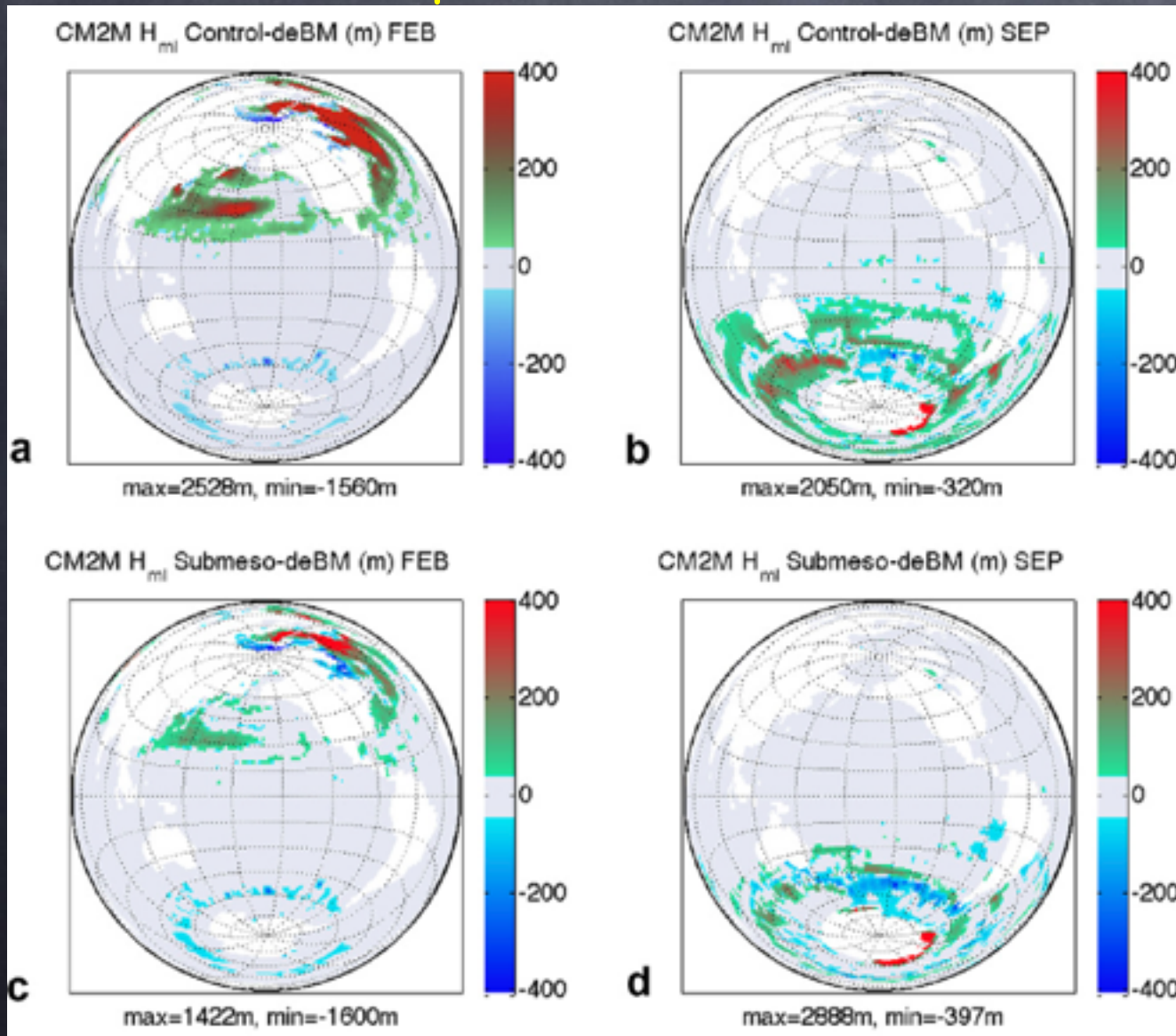
$$\langle \eta^2 \rangle = \int_0^{\infty} E(k) dk = C_0 + \int_{k_h}^{\infty} C_1 k^{-2} dk$$

Power Spectrum  
of wave  
steepness:  
INFINITE!

$$\langle k^2 \eta^2 \rangle = \int_0^{\infty} k^2 E(k) dk = D_0 + \int_{k_h}^{\infty} D_1 dk$$

Steep waves break  $\rightarrow$  vortex motion & small scale turbulence!

# Physical Sensitivity of Ocean Climate to Submesoscale Mixed Layer Eddy Restratification: MLE implemented in NCAR, GFDL, Hadley, NEMO,...



Bias  
w/o  
MLE

Deep ML  
Bias reduced

With MLE  
Parameterization



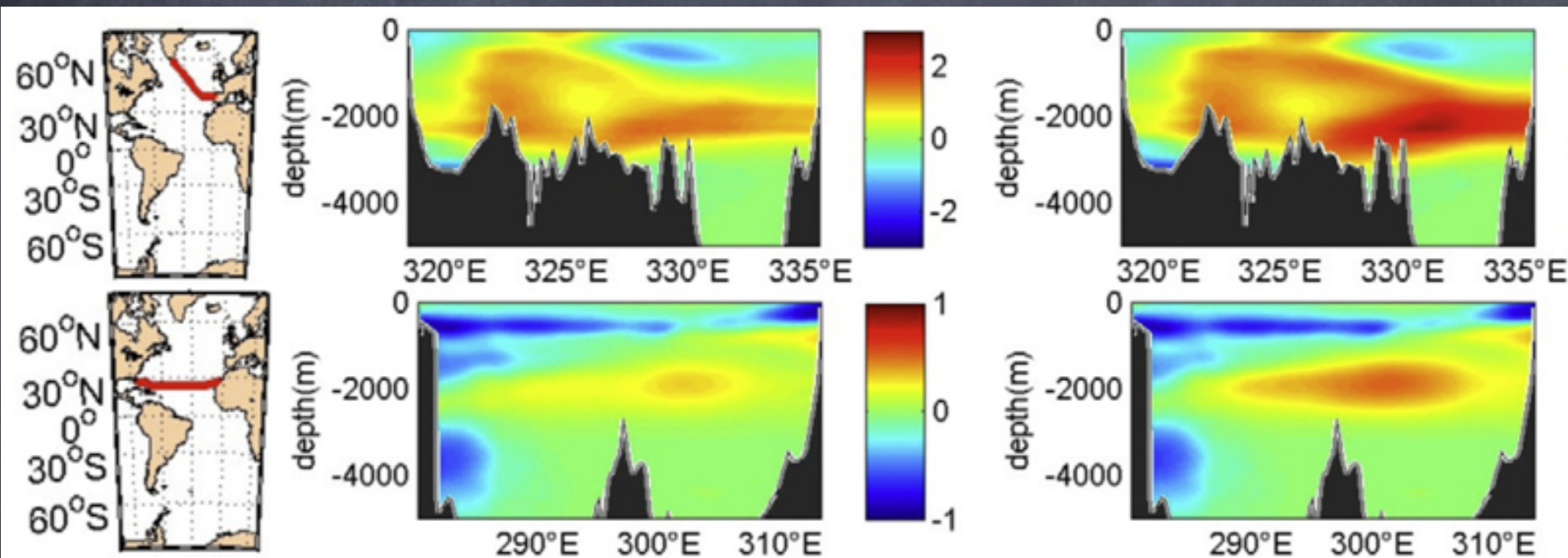
# Physical Sensitivity of Ocean Climate to Submesoscale Mixed Layer Eddy Restratification:

MLE implemented in NCAR, GFDL, Hadley, NEMO,...

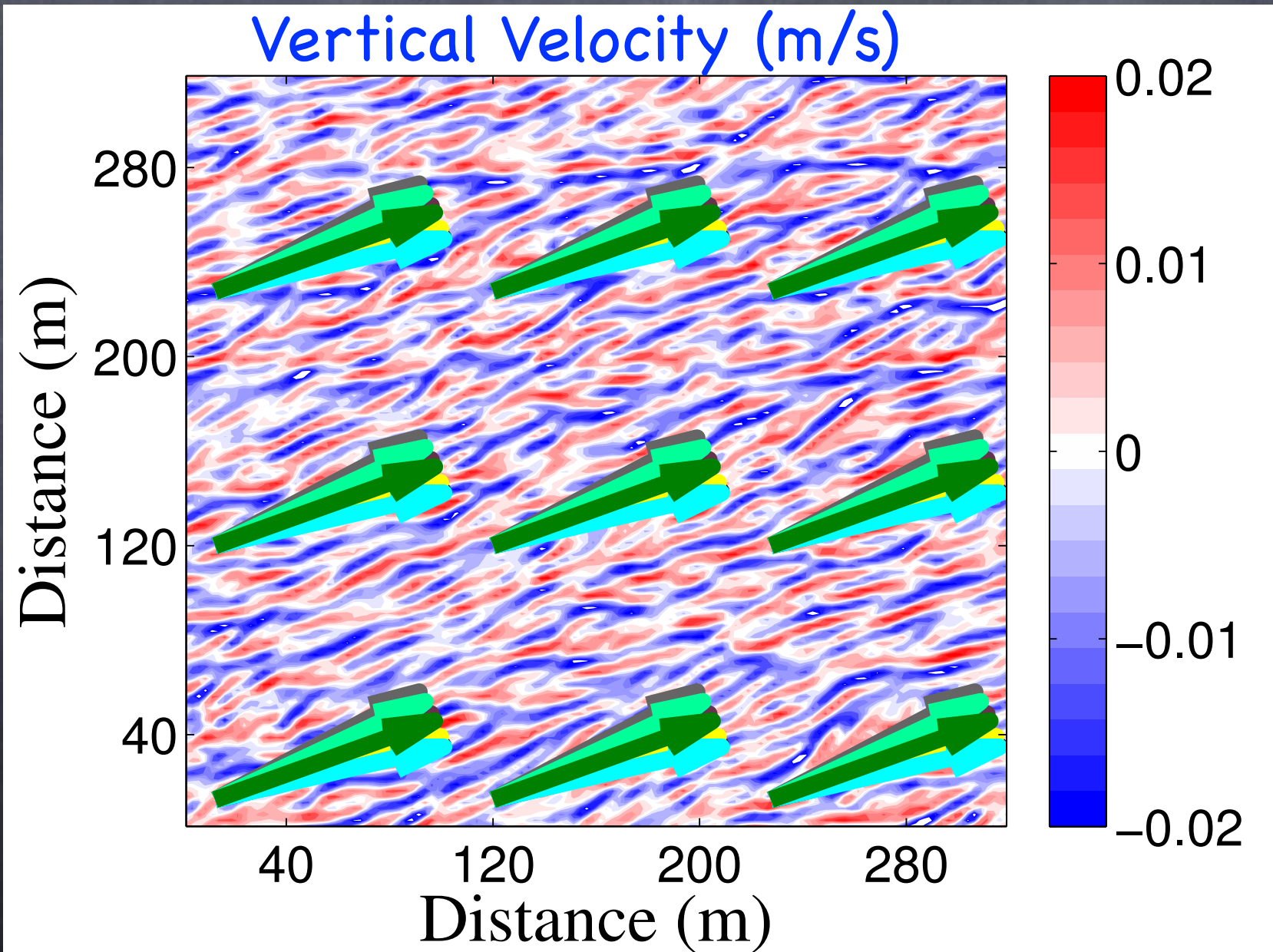
## Improves CFC uptake (water masses)

With MLE  
Parameterization

Bias w/o MLE

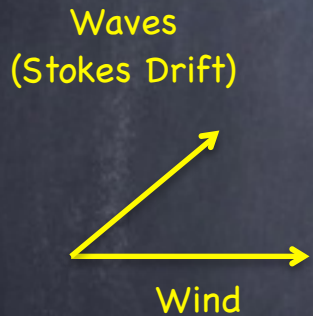
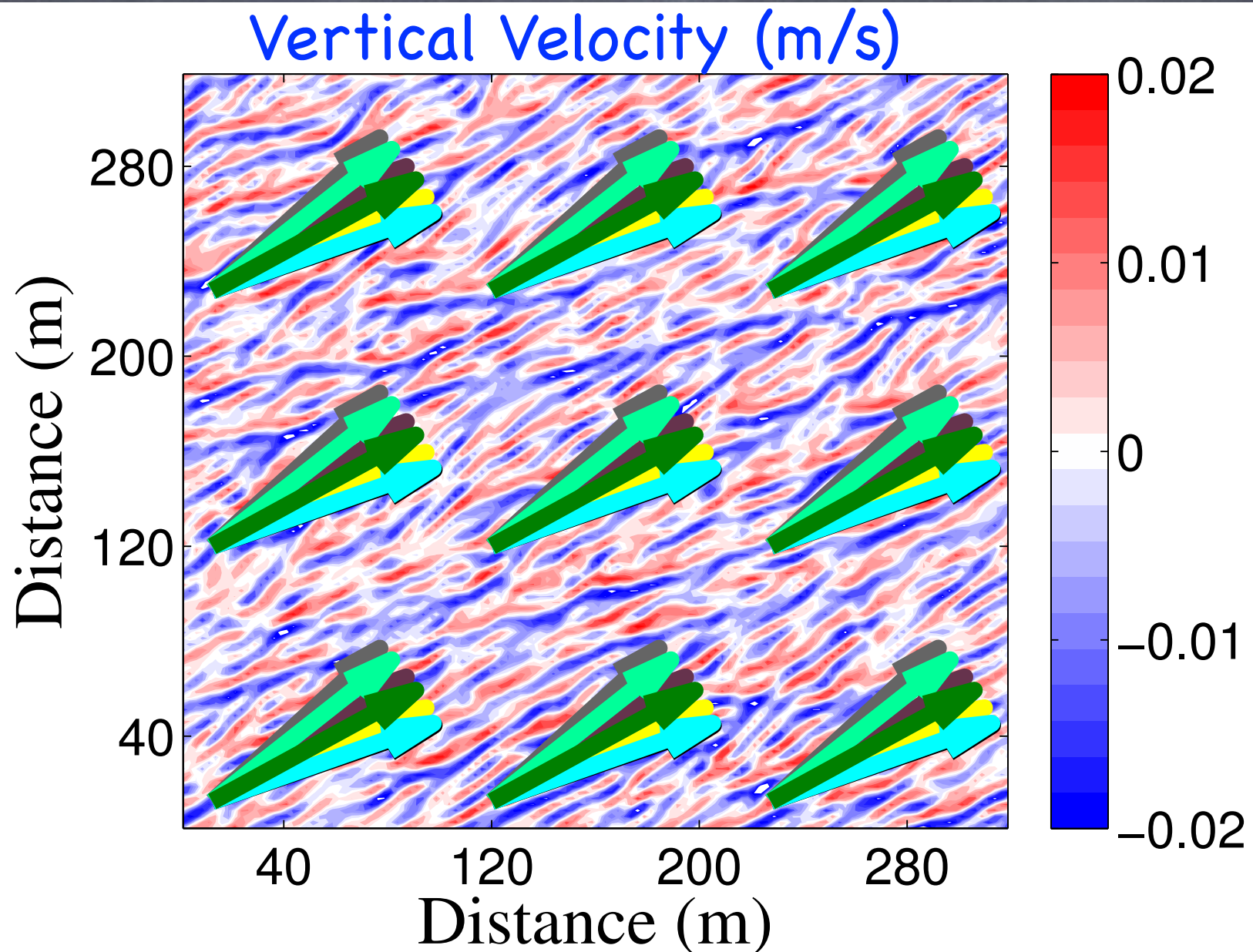


# CLB as equations for Large Eddy Simulations: Tricky: Misaligned Wind & Waves



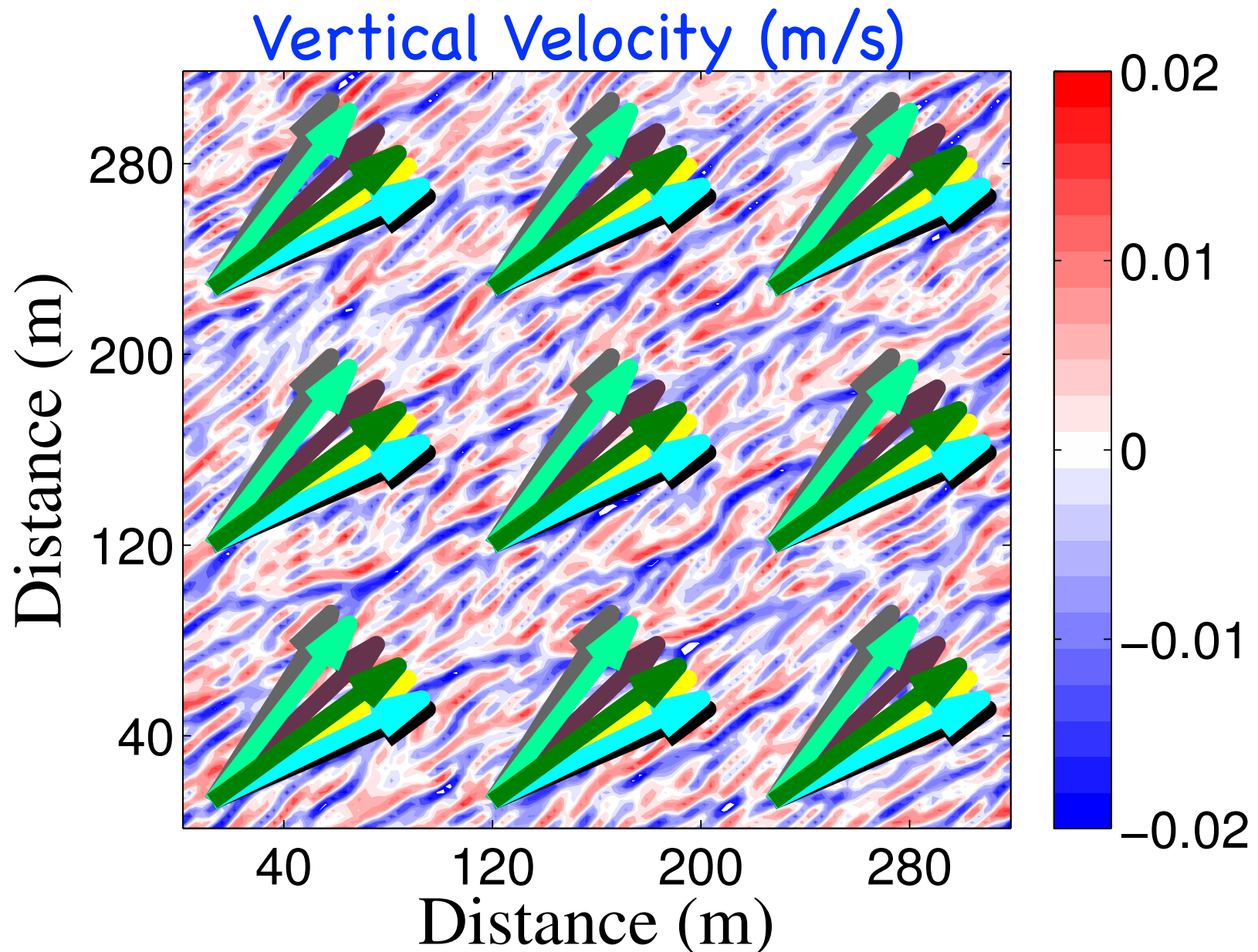
L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney.  
The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

# Tricky: Misaligned Wind & Waves



L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney.  
The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

# Tricky: Misaligned Wind & Waves

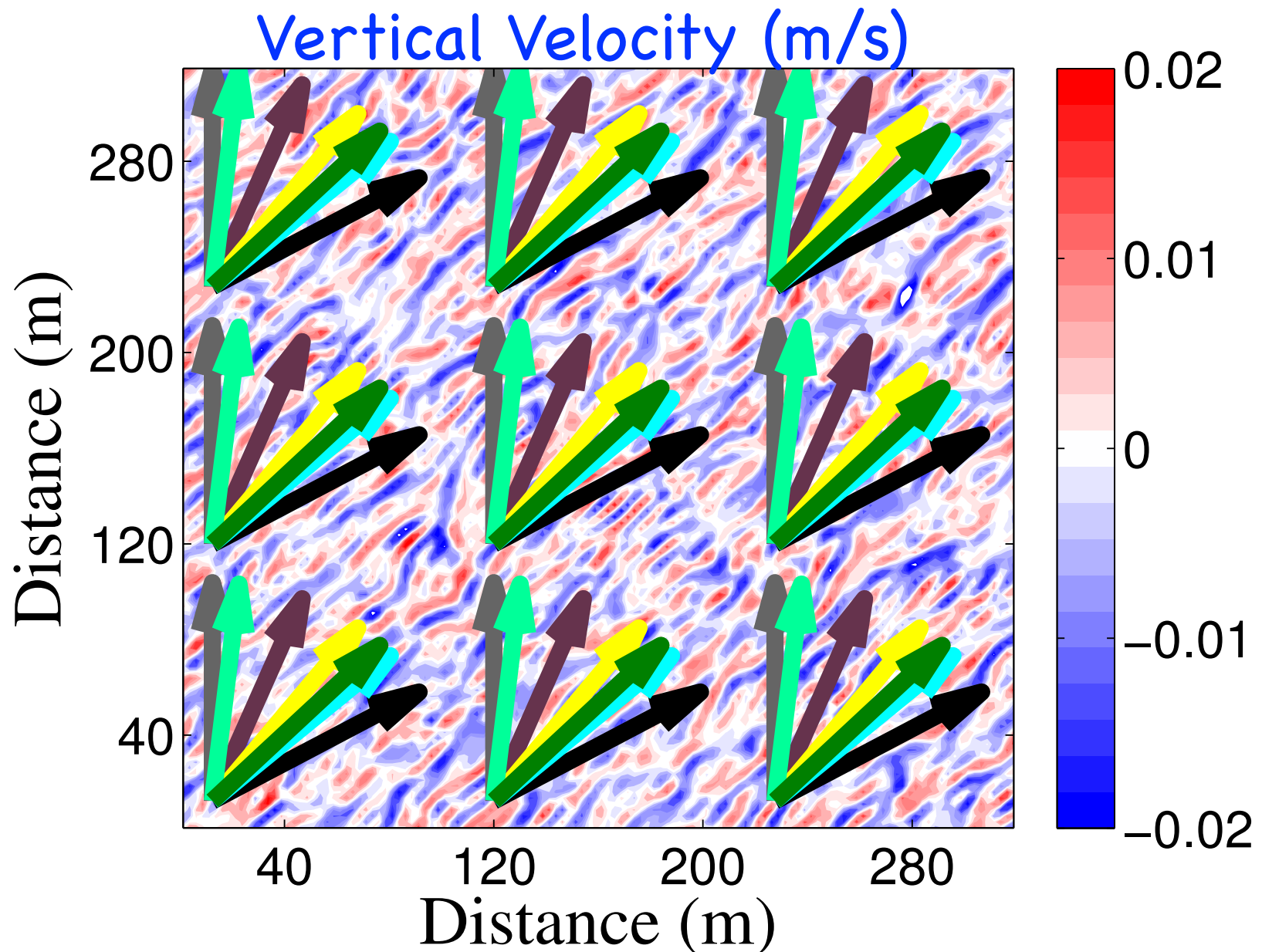


Waves  
(Stokes Drift)



L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney.  
The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

# Tricky: Misaligned Wind & Waves



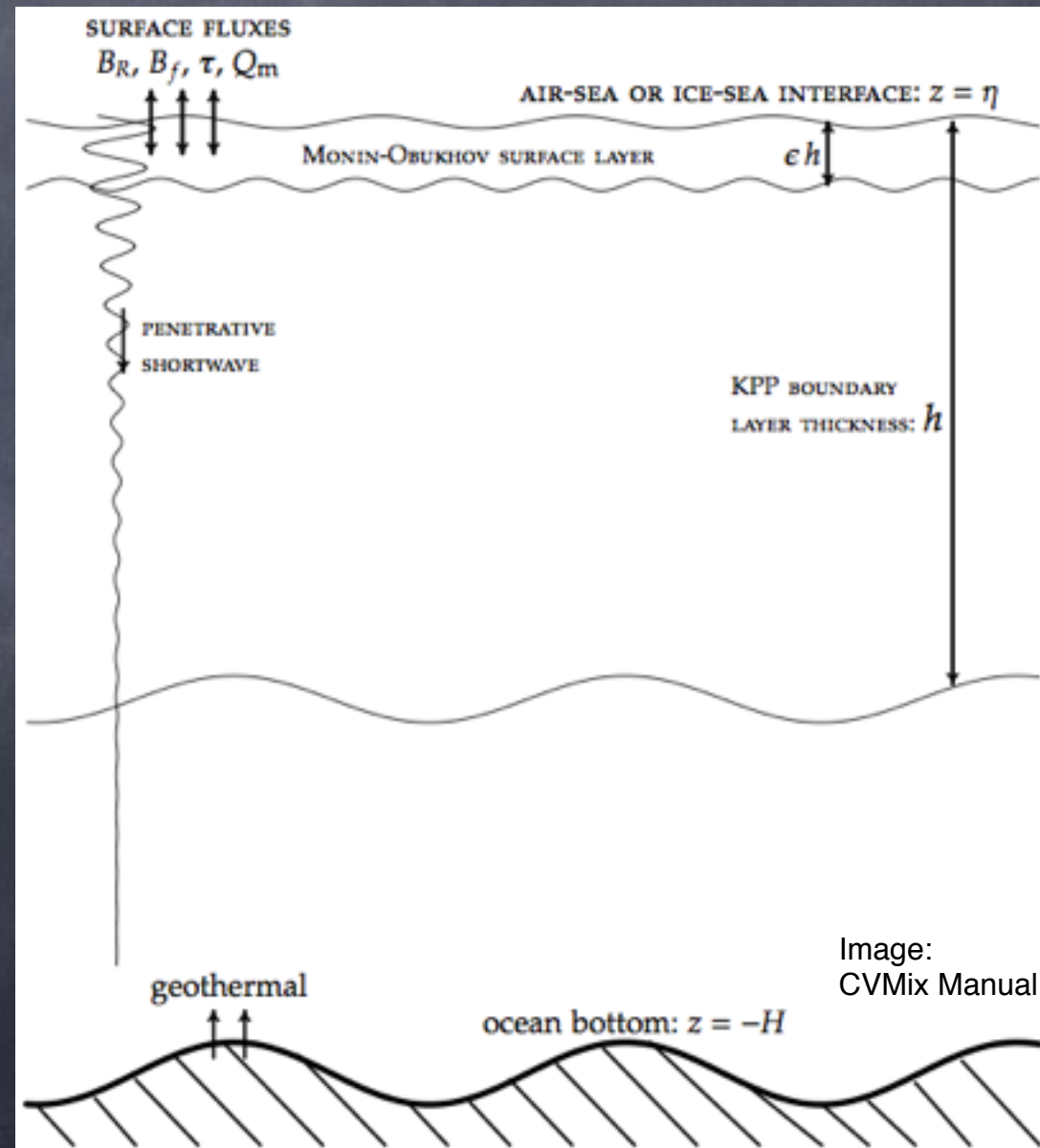
Waves  
(Stokes Drift)



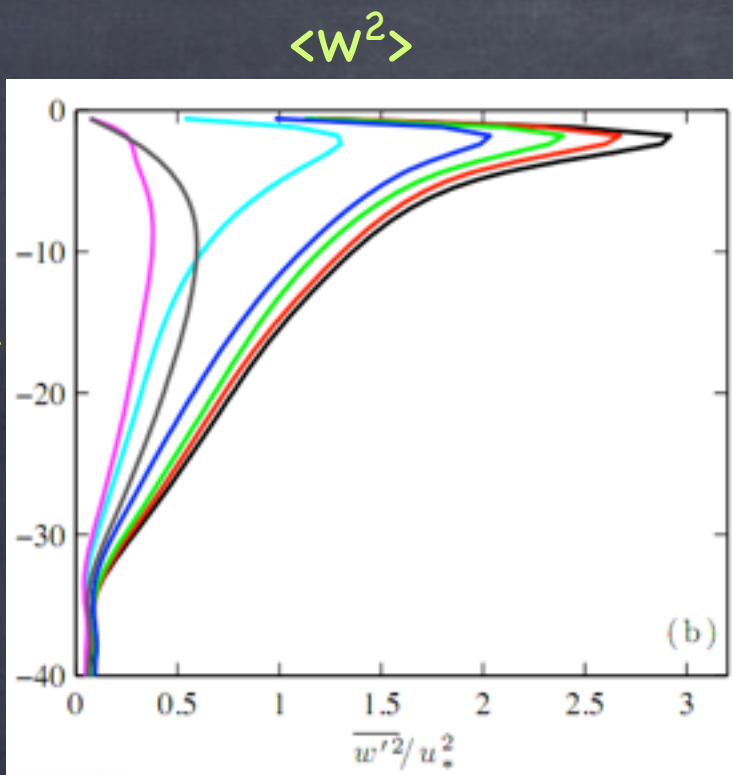
L. P. Van Roekel, B. Fox-Kemper, P. P. Sullivan, P. E. Hamlington, and S. R. Haney.  
The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

# What's in a boundary mixing parameterization?

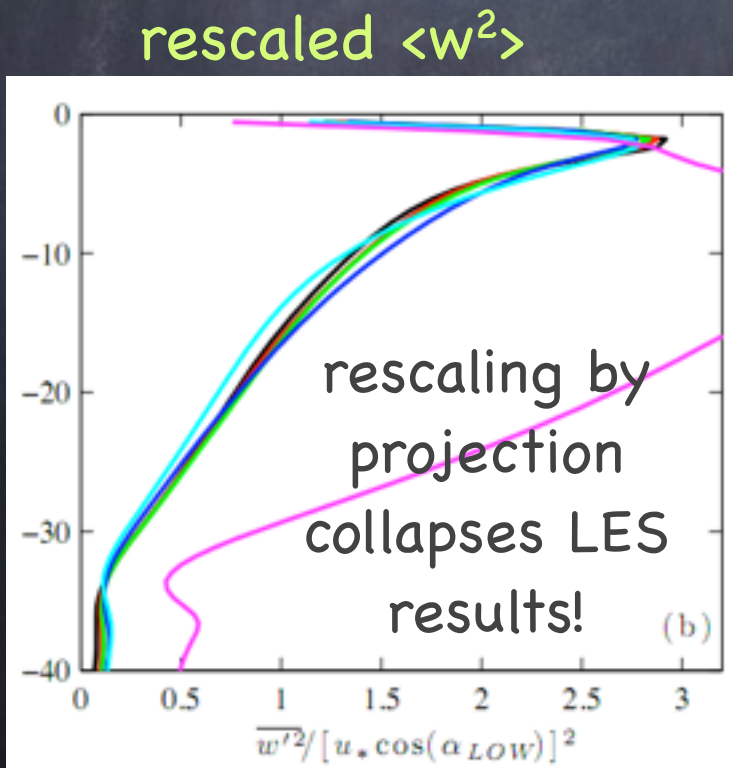
- Wind Driven vertical mixing, key:  $\kappa u^*$
- Convectively Driven vertical mixing, key:  $w_* = (-B_f h)^{1/3}$
- Boundary layer thickness, e.g.:  $Ri < 0.3$
- Non-local fluxes, etc.
- Love
- Usually not waves



depth



depth



Generalized Turbulent Langmuir No.,  
Projection of  $u^*$ ,  $u_s$  into Langmuir Direction

$$\frac{\langle \overline{w'^2} \rangle_{ML}}{u_*^2} = 0.6 \cos^2(\alpha_{LOW}) [1.0 + (3.1 La_{proj})^{-2} + (5.4 La_{proj})^{-4}],$$

$$La_{proj}^2 = \frac{|u_*| \cos(\alpha_{LOW})}{|u_s| \cos(\theta_{ww} - \alpha_{LOW})},$$

$$\alpha_{LOW} \approx \tan^{-1} \left( \frac{\sin(\theta_{ww})}{\frac{u_*}{u_s(0)\kappa} \ln \left( \left| \frac{H_{ML}}{z_1} \right| \right) + \cos(\theta_{ww})} \right)$$

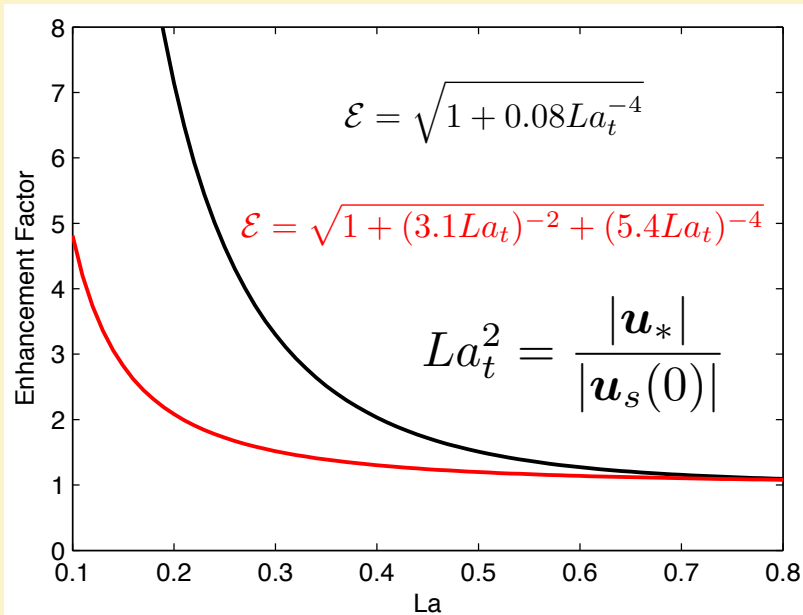
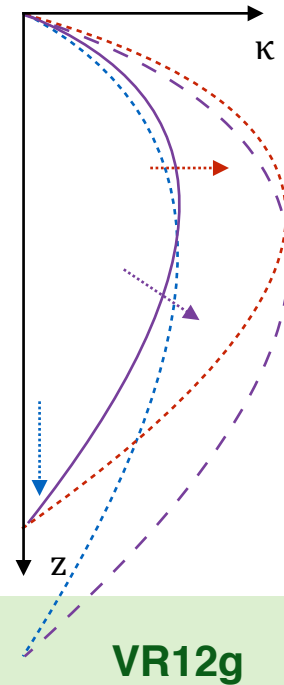
A scaling for LC  
strength & direction!

L. P. Van Roekel, BFK, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, 2012.

# Langmuir Mixing in KPP

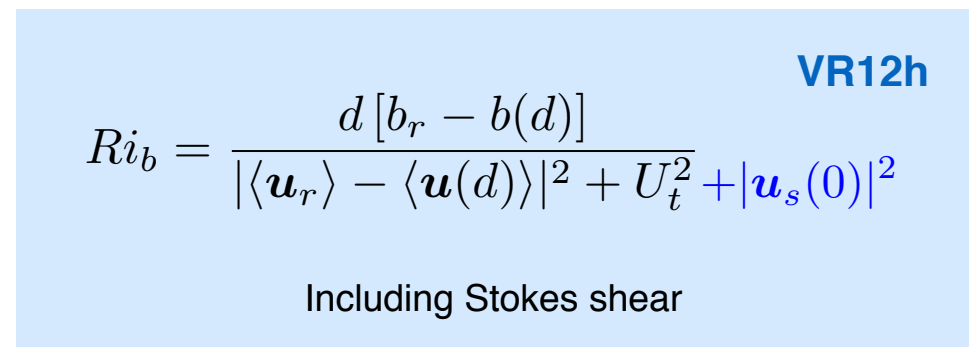
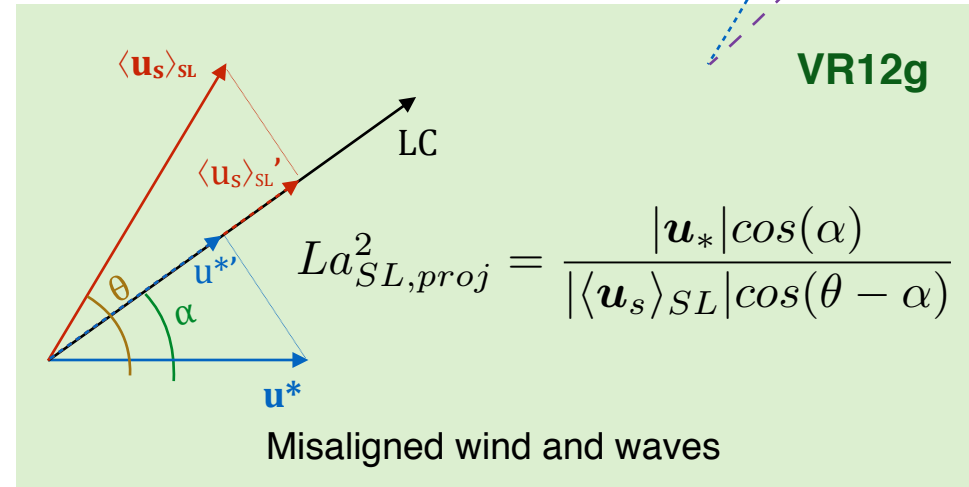
Q. Li, BFK, T. Arbetter, A. Webb, 2014. Assessing the Influence of Surface Wind Waves to the Global Climate by Incorporating WAVEWATCH III in CESM, 2014 AGU Ocean Sciences Meeting Poster, related paper in prep.

- WaveWatch-III (Stokes drift) <-> POP2 (U, T, H<sub>BL</sub>)
- CORE2 interannual forcing (Large and Yeager, 2009)
- 4 IAF cycles; average over last 50 years for climatology



**MS2K**  
**VR12a**

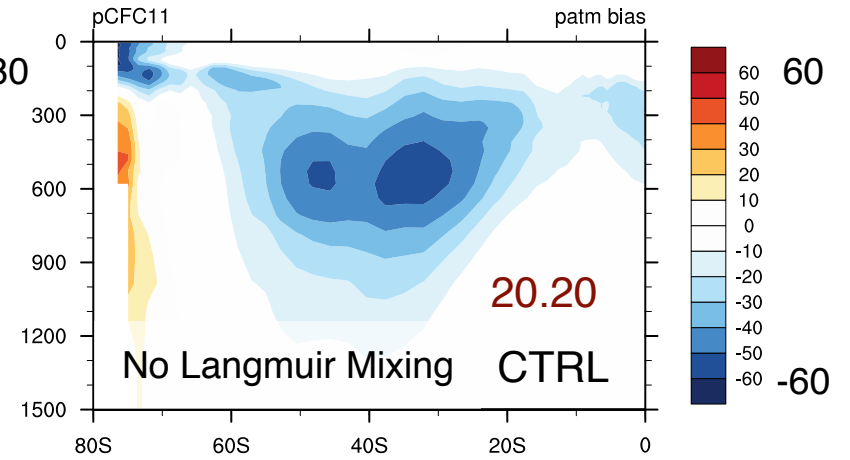
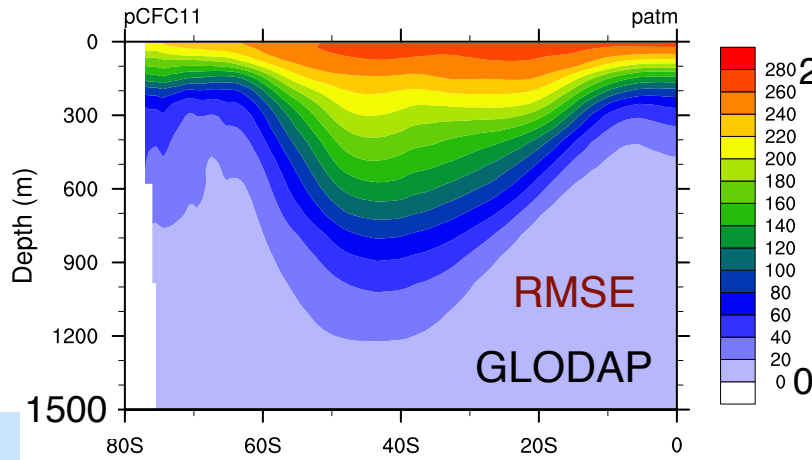
Enhancement factor to vertical velocity scale W  
Aligned wind and waves



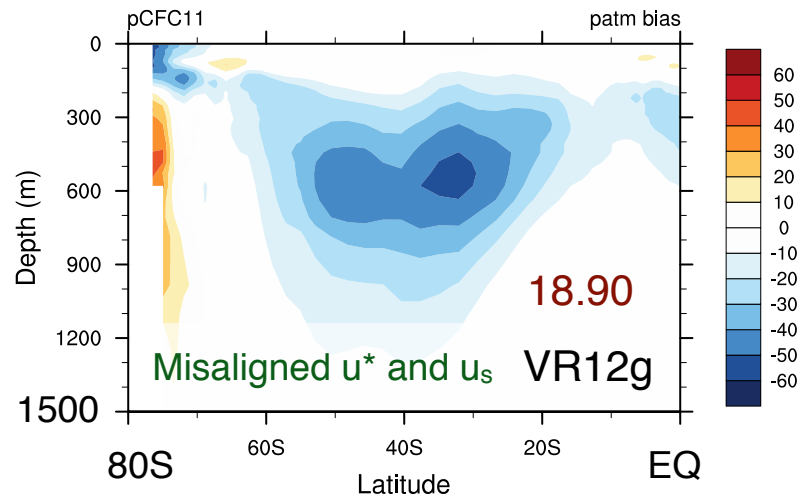


# pCFC11 Bias

Southern Hemisphere



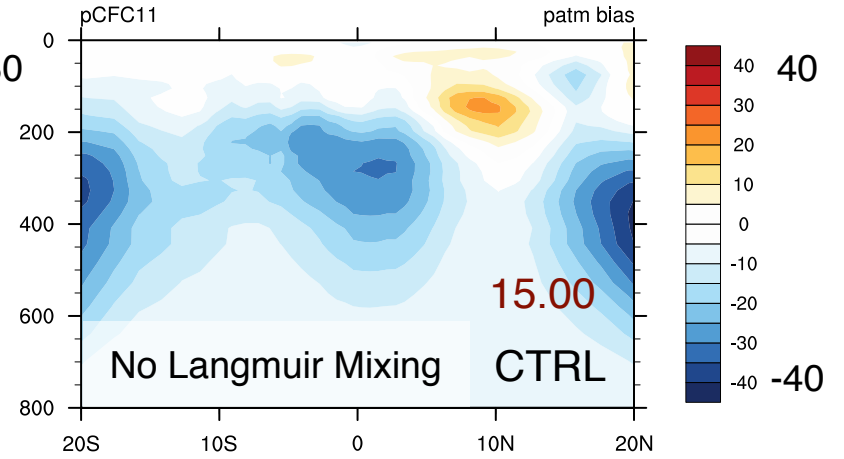
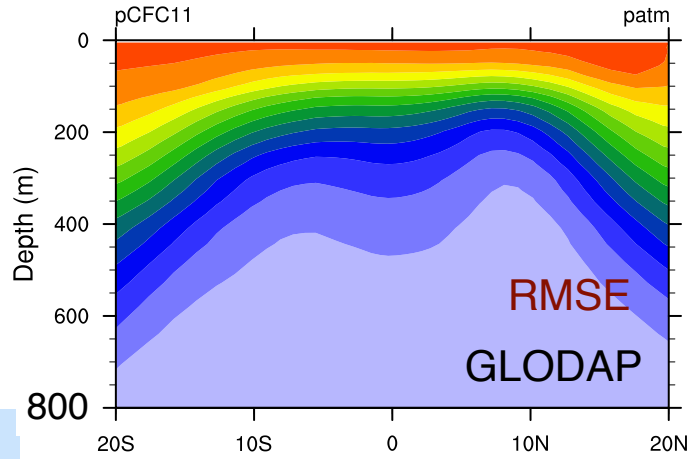
RMSEs are reduced by 6%



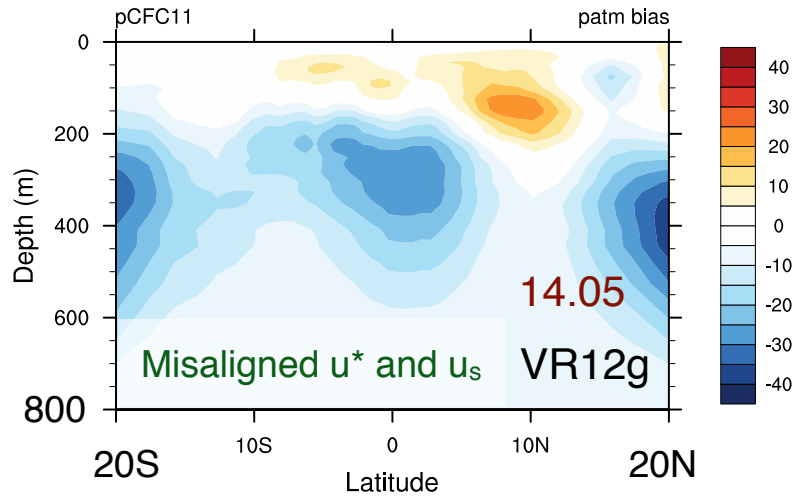
Versus observations from GLODAP: Key et al. 2004

# pCFC11 Bias

Equatorial  
Region



RMSEs are  
reduced by 6%



Versus observations from  
GLODAP: Key et al. 2004

# The Steady Background State

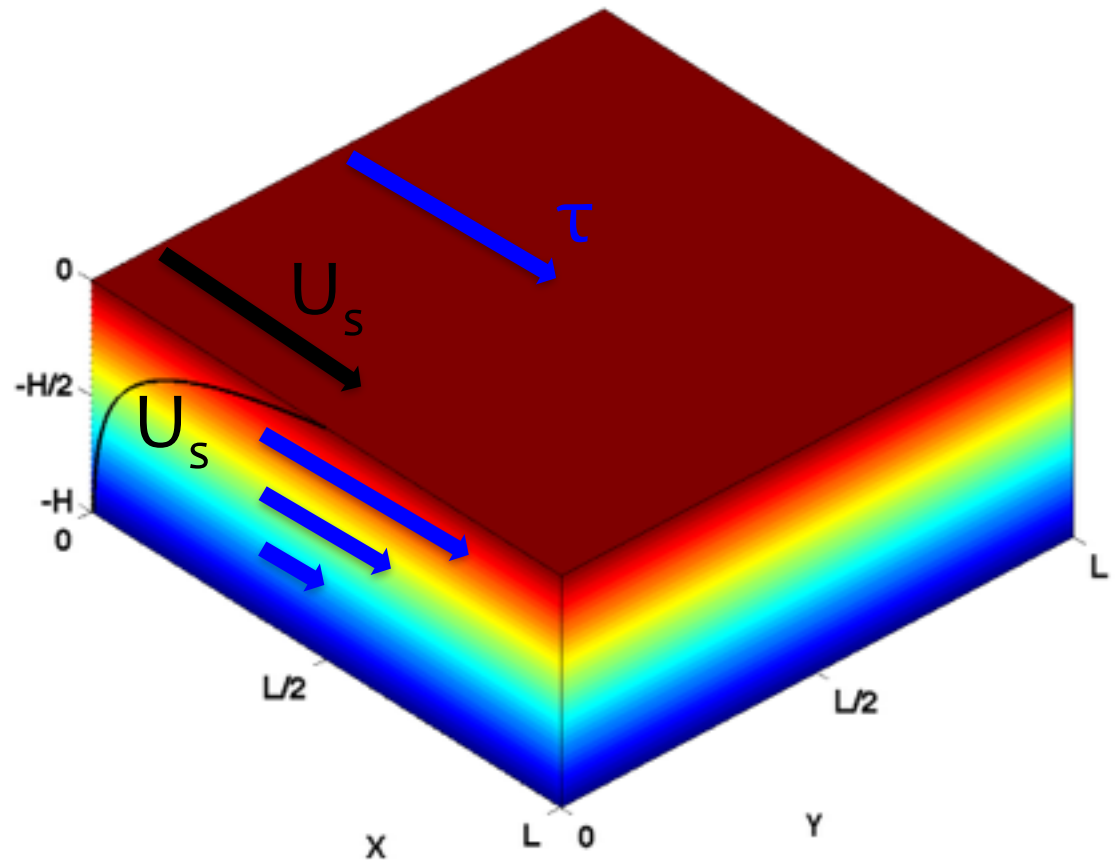
$Ro \gg 1$ ,  $Ek > 0$ ,  $\gamma = 0$   
Weak Viscid No front  
Coriolis

## Background Flow

$$\bar{U} = z \quad \bar{W} = 0$$

$$\bar{P}_z = \bar{B} \quad \left. \vphantom{\bar{P}_z} \right\} \text{Hydrostatic}$$

$$\bar{W} = 0$$



Reproduces “Classic” LC regime:  
Leibovich and Paolucci, 1980

# The Steady Background State

$Ro \ll 1$ ,  $Ek = 0$ ,  $\gamma = 1$   
 Strong Coriolis      Inviscid      Strong front

## Background Flow

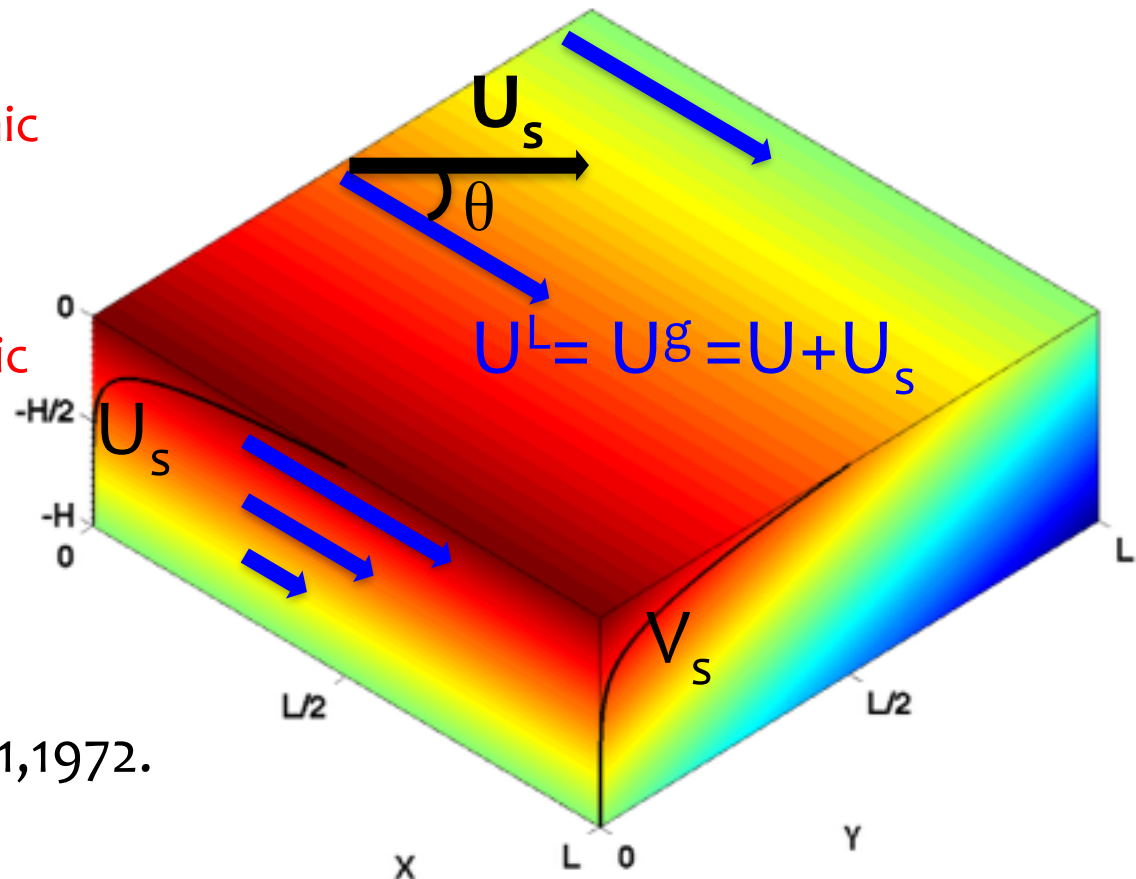
$$f(\hat{\mathbf{k}} \times \bar{\mathbf{U}}^L) = -\frac{\nabla \bar{P}}{\rho_0} \quad \left. \vphantom{\frac{\nabla \bar{P}}{\rho_0}} \right\} \text{Geostrophic}$$

$$\bar{P}_z = \bar{B} \quad \left. \vphantom{\bar{B}} \right\} \text{Hydrostatic}$$

$$f\bar{\mathbf{U}}_z^L = -\bar{B}_y \quad \left. \vphantom{\bar{B}_y} \right\} \text{Thermal Wind}$$

$$\bar{W} = 0$$

$\mathbf{U}^S \rightarrow 0 \Rightarrow$  Stone, 1966, 1970, 1971, 1972.



# The Steady Background State

$$Ro \ll 1, Ek > 0, \gamma \sim O(1)$$

Strong  
Coriolis

Viscid

Strong  
front

Background Flow

$$f(\hat{\mathbf{k}} \times \bar{\mathbf{U}}^L) + \frac{\nabla \bar{P}}{\rho_0} = \nu \bar{\mathbf{U}}_{zz}$$

Ekman-Stokes-Front Layer

$$\bar{P}_z = \bar{B} \quad \left. \vphantom{\bar{P}_z} \right\} \text{Hydrostatic}$$

$$\bar{W} = 0$$

$$\nabla \bar{B} \rightarrow 0 \Rightarrow \text{Gnanadesikan and Weller, 1995}$$

