

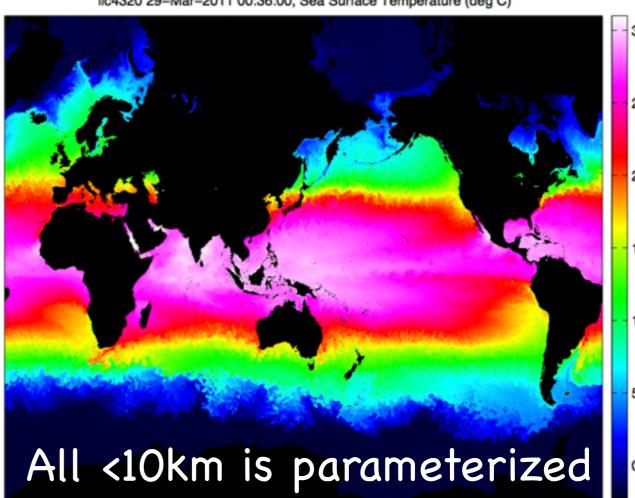
The Earth's Climate System is forced by the Sun on a global scale (20,000km pole to pole)



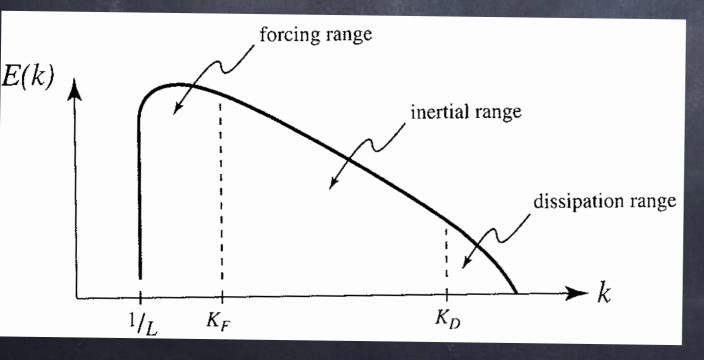
Next-gen. ocean climate models simulate globe to 10km:

Mesoscale Ocean Large Eddy Simulations (MOLES)

lic4320 29-Mar-2011 00:36:00, Sea Surface Temperature (deg C)



Turbulence cascades to scales about 10 billion times smaller



Transmitting a single variable from ocean DNS: 325,000(a year of the internet).

Key Concept for



Mesoscale Ocean Large Eddy Simulations (MOLES): Gridscale Nondimensional Parameters

Gridscale Reynolds¹:
$$Re^* = \frac{U^* \Delta x}{\nu^*}$$

Gridscale Péclet
1
:
$$Pe^* = \frac{U^* \Delta x}{\kappa^*}$$

Gridscale Rossby:

$$Ro^* = \frac{U^*}{f\Delta x}$$

Gridscale Richardson:

$$Ri^* = \frac{\Delta b^{*2} \Delta z}{\Delta U^{*2}}$$

Gridscale Burger:
$$Bu^* = \frac{N^{*2}\Delta z^2}{f^2\Delta x^2} \sim Ro^{*2}Ri^*$$

Asterisks denote *resolved* quantities, rather than true values

¹ Gridscale Reynolds and Péclet numbers MUST be O(1) for numerical stability

B. Fox-Kemper and D. Menemenlis. Can large eddy simulation techniques improve mesoscale-rich ocean models? In M. Hecht and H. Hasumi, editors, Ocean Modeling in an Eddying Regime, volume 177, pages 319-338. AGU Geophysical Monograph Series, 2008.

A Global Parameterization of Mixed Layer Eddy Flow & Scale Aware Restratification validated against simulations

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. Ocean Modelling, 39:61-78, 2011.

$$egin{aligned} \overline{\mathbf{u}'b'} &\equiv \mathbf{\Psi} imes
abla ar{b} \ \mathbf{\Psi} &= egin{bmatrix} \Delta x \ L_f \end{bmatrix} rac{C_e H^2 \mu(z)}{\sqrt{f^2 + au^{-2}}}
abla ar{b} imes \hat{\mathbf{z}} \end{aligned}$$

Compare to the original singular, unrescaled version

$$\Psi = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{\mathbf{z}}$$

New version handles the equator, and averages over many fronts

A Global Parameterization of Mixed Layer Eddy Flow & Scale Aware Restratification validated against simulations

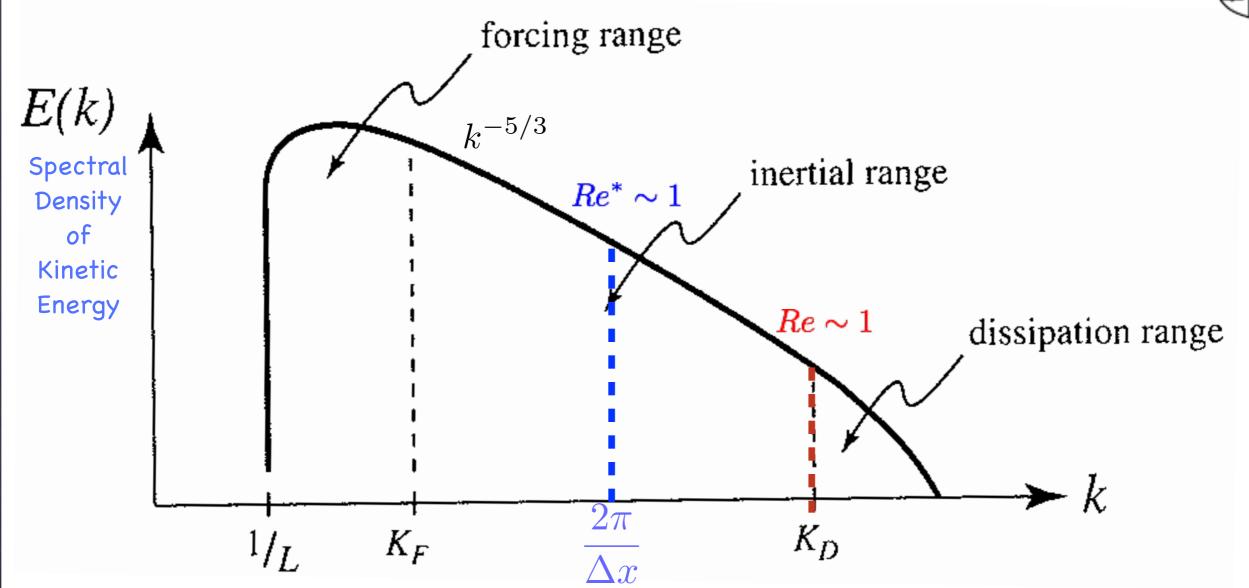
- Purely advective, gets correct rate of restratification (akin to Gent-McWilliams 90).
- In adiabatic situations, should pair nicely with an isopycnal diffusivity of equal magnitude (akin to Redi, see Bachman & F-K 13).
- In diabatic situations (it is the mixed layer), may need diabatic parameterization (Brüggemann & Eden).
- Tested in global 10km and finer regional simulations, turns itself off fairly appropriately in submeso-permitting regime

What about the mesoscale-permitting?

- What do we do when some mesoscale eddies are resolved and some are not?
- This regime is fairly compared to Large Eddy Simulations, typical of much smaller scales.
- How do we adapt the technology of LES to these scales?
- Should we just turn off GM to allow partiallyresolved eddies to get all APE (e.g., Hallberg, 2013)?

3D Turbulence Cascade

Kolmogorov, 1941



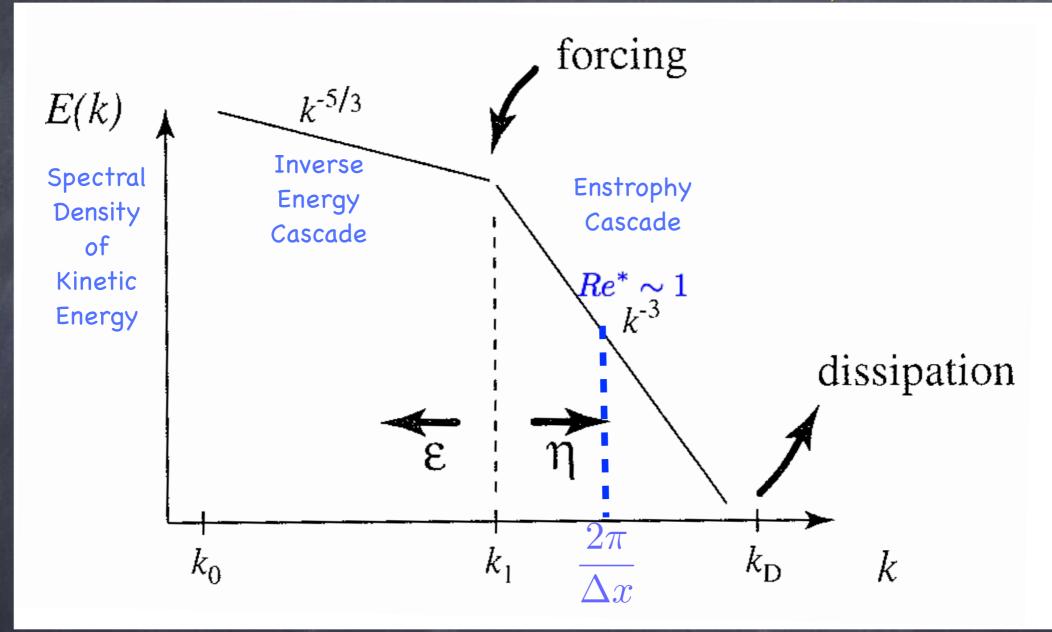
Smagorinsky (1963) Scale & Flow Aware Viscosity Scaling, So the Energy Cascade is Preserved, and $Re^* = \frac{U^* \Delta x}{\nu^*} = O(1)$

$$\mathbf{v}_{*h} = \left(\frac{\Upsilon_h \Delta x}{\pi}\right)^2 \sqrt{\left(\frac{\partial u_*}{\partial x} - \frac{\partial v_*}{\partial y}\right)^2 + \left(\frac{\partial u_*}{\partial y} + \frac{\partial v_*}{\partial x}\right)^2}.$$

 $\Upsilon_h \approx 1$

2D Turbulence Differs

R. Kraichnan, 1967 JFM



Leith (1996) Devises Viscosity Scaling, So that the Enstrophy Cascade is preserved, and $Re^* = \frac{U^*\Delta x}{\nu^*} = O(1)$

$$u_{2d}^* = \left(\frac{\Lambda_{2d}\Delta x}{\pi}\right)^3 |\nabla_h q_{2d}^*|$$

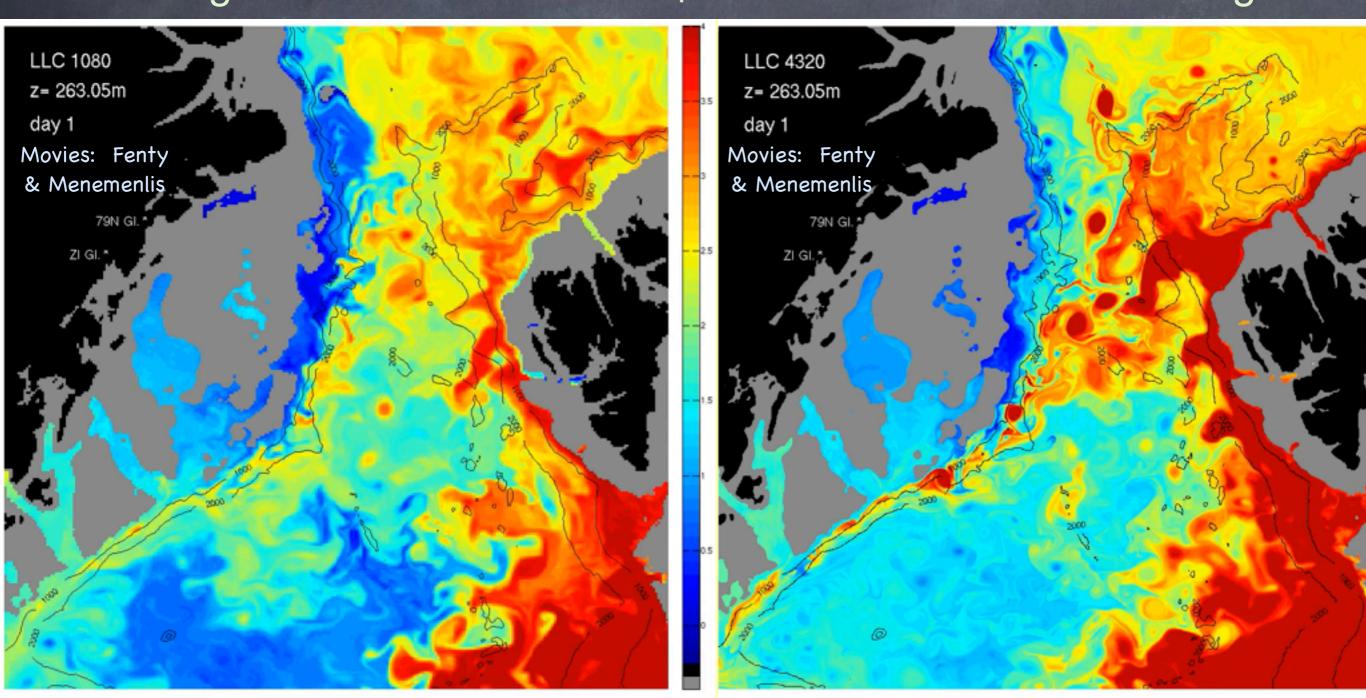
$$q_{2d}^* = \frac{\partial u^*}{\partial y} - \frac{\partial v^*}{\partial x}$$

F-K & Menemenlis (08): Revise Leith viscosity to quasi-2d, by damping diverging, vorticity-free, modes, too.

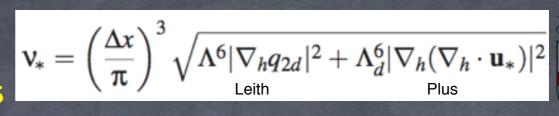
$$\mathbf{v}_* = \left(\frac{\Delta x}{\pi} \right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2}$$
 Leith Plus

Leith-Plus Parameterization in the high-resolution ECCO runs proves stability and plug-&-play viscosity to very high resolutions without retuning:

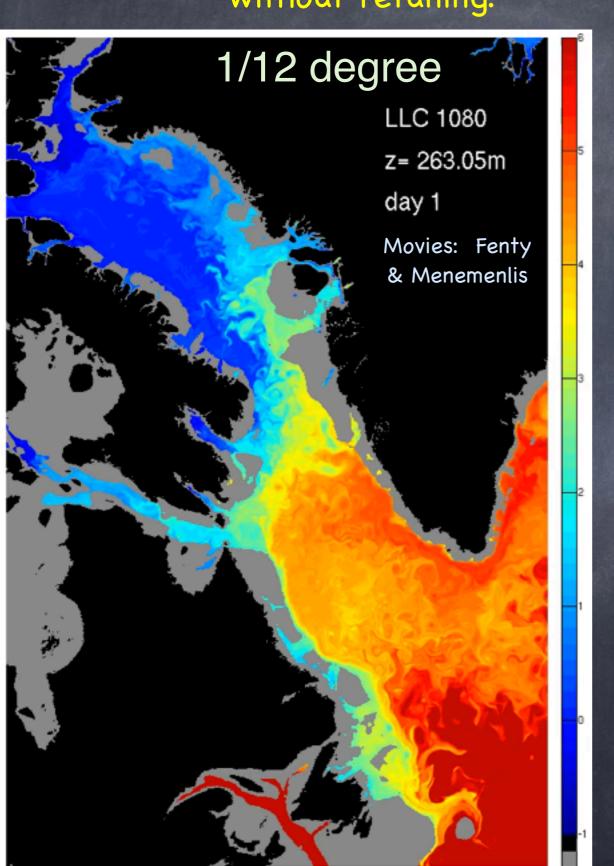
1/12 degree Fram Strait, Temperature at 263m 1/48 degree

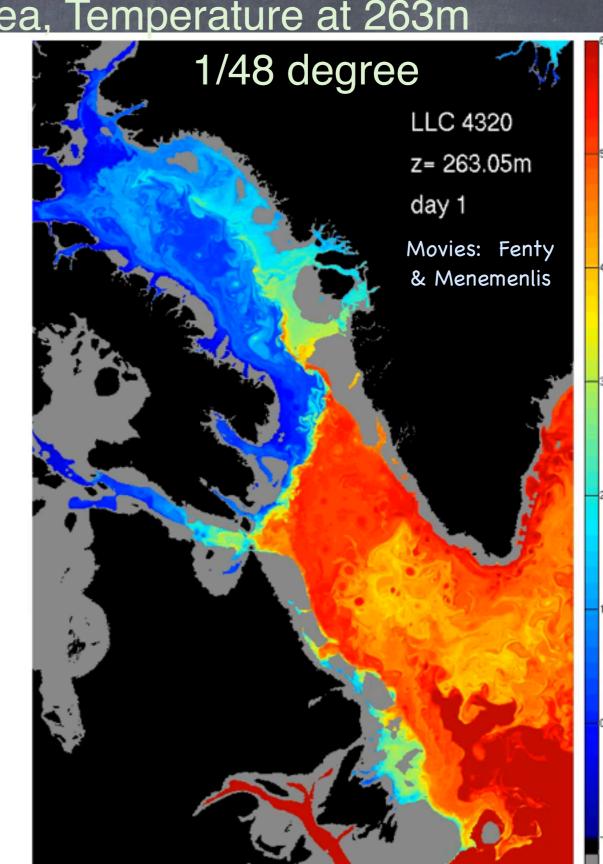


Leith-Plus Parameterization in the highresolution ECCO runs proves stability and plug-&-play viscosity to very high resolutions $v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2} \text{ and } v_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt$ without retuning:









2d (SWE) test of MOLES Subgrid models

Pietarila Graham & Ringler, 2013

- Harmonic/Biharmonic/Numerical
 - Many. Often not scale- or flowaware
 - Griffies & Hallberg, 2000, is one aware example
- Fox-Kemper & Menemenlis, 2008. ECCO2.
- Chen, Q., Gunzburger, M., Ringler, T., 2011
 - Anticipated Potential Vorticity of Sadourny
- San, Staples, Iliescu (2011, 2013)
 - Approximate Deconvolution Method
- Stochastic & Statistical Parameterizations

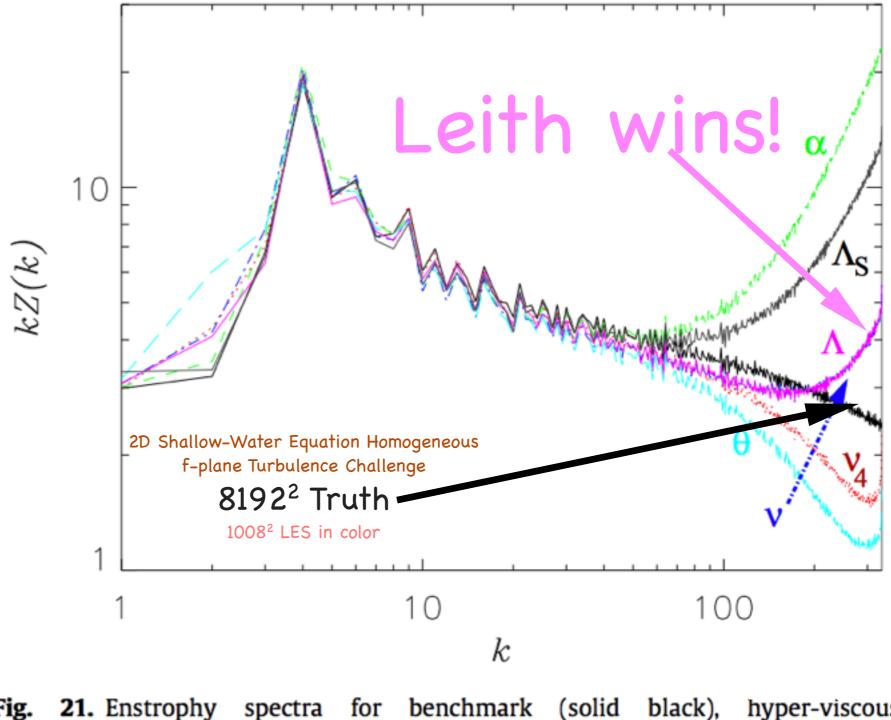


Fig. hyper-viscous

In the Graham & Ringler comparison, Leith wins!

over tuned harmonic, tuned biharmonic, Smagorinsky, LANS-alpha, & Anticipated PV

Is 2D Turbulence a good proxy for neutral flow?



We Ekman layer

Mixed layer

Thermocline

No:

Yes:

Nurser & Marshall, 1991 JPO

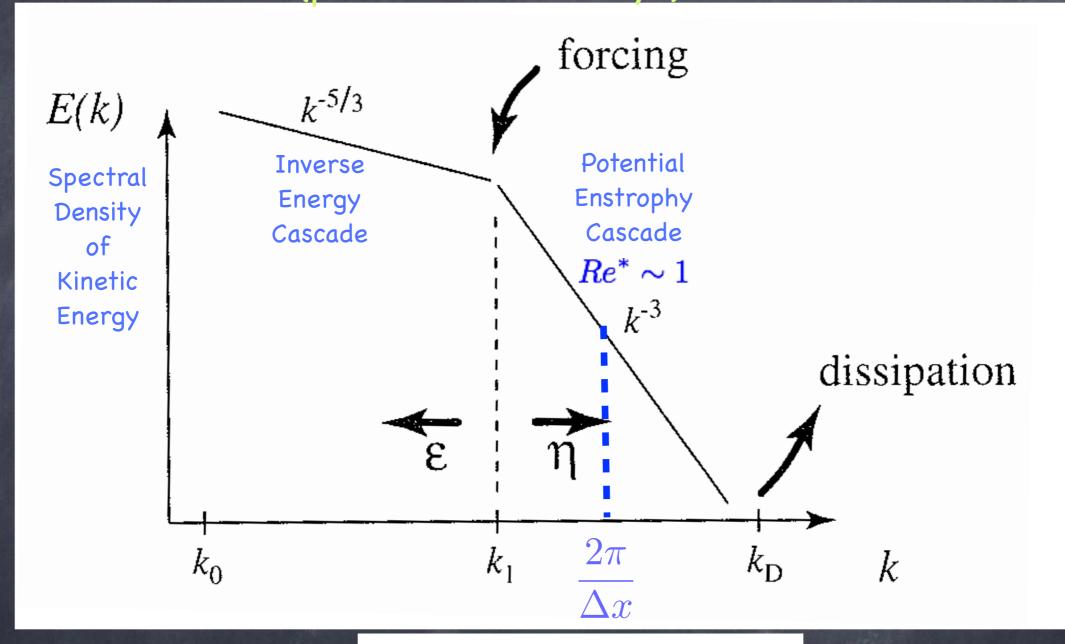
- For a few eddy time-scales
 QG & 2D AGREE (Bracco et al. '04)
- Barotropic Flow & Stratified
 Turbulence (Ro>>1, Ri>>1)
 are 2d analogs

- Bolus Fluxes--Divergent 2d flow
- Sloped, not horiz.
- Surface Effects?

QG Turbulence: Pot'l Enstrophy cascade

(potential vorticity²) J. Charney, 1971 JAS



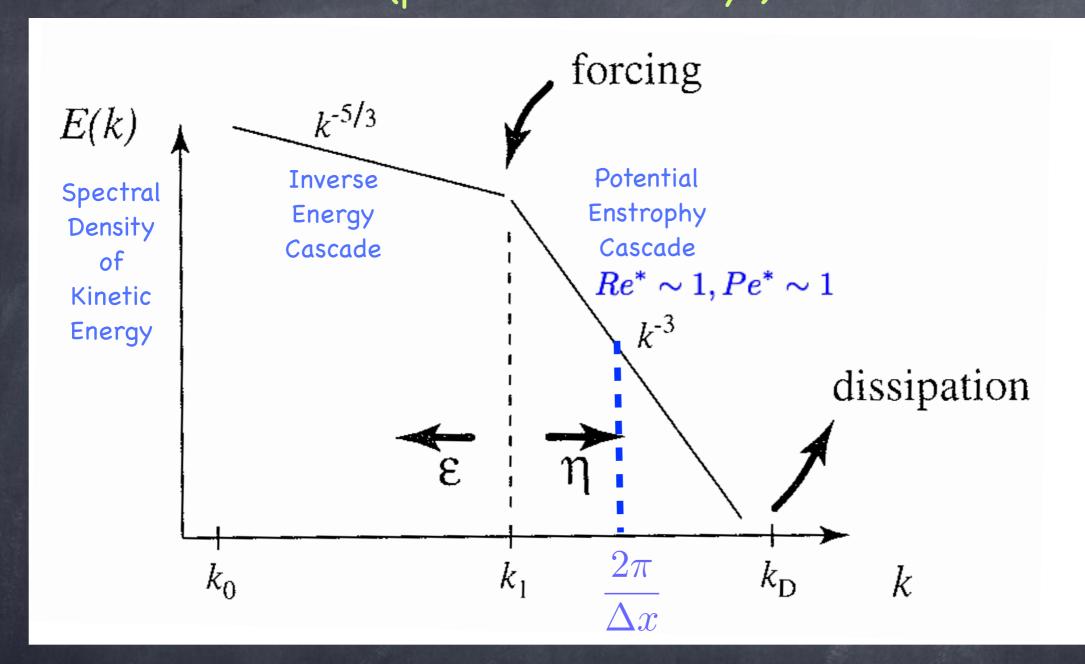


QG Leith:
$$v_{qg} = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^3 |\nabla q_{qg}|$$

$$\begin{split} \nabla_h^2 \psi^* &= q_{2d}^* \\ q_{qg}^* &= \beta y + \nabla_h^2 \psi^* + \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^{*2}} \frac{\partial \psi^*}{\partial z} \right) \end{split}$$

QG Turbulence: Pot'l Enstrophy cascade (potential vorticity²) J. Charney, 1971 JAS





$$Re^* = \frac{U^* \Delta x}{\nu^*}$$

$$Pe^* = \frac{U^* \Delta x}{\nu^*}$$

Consistent with QG only if scaling applies to ALL Pot'l Enstrophy sinks—

Viscosity, Diffusivity, AND GM Coefficient:

$$u_{qg} = \kappa_{Redi} = \kappa_{GM} = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^3 \left|\nabla q_{qg}\right|.$$

And QG pot'l enstrophy Leith is ... now working in MITgcm

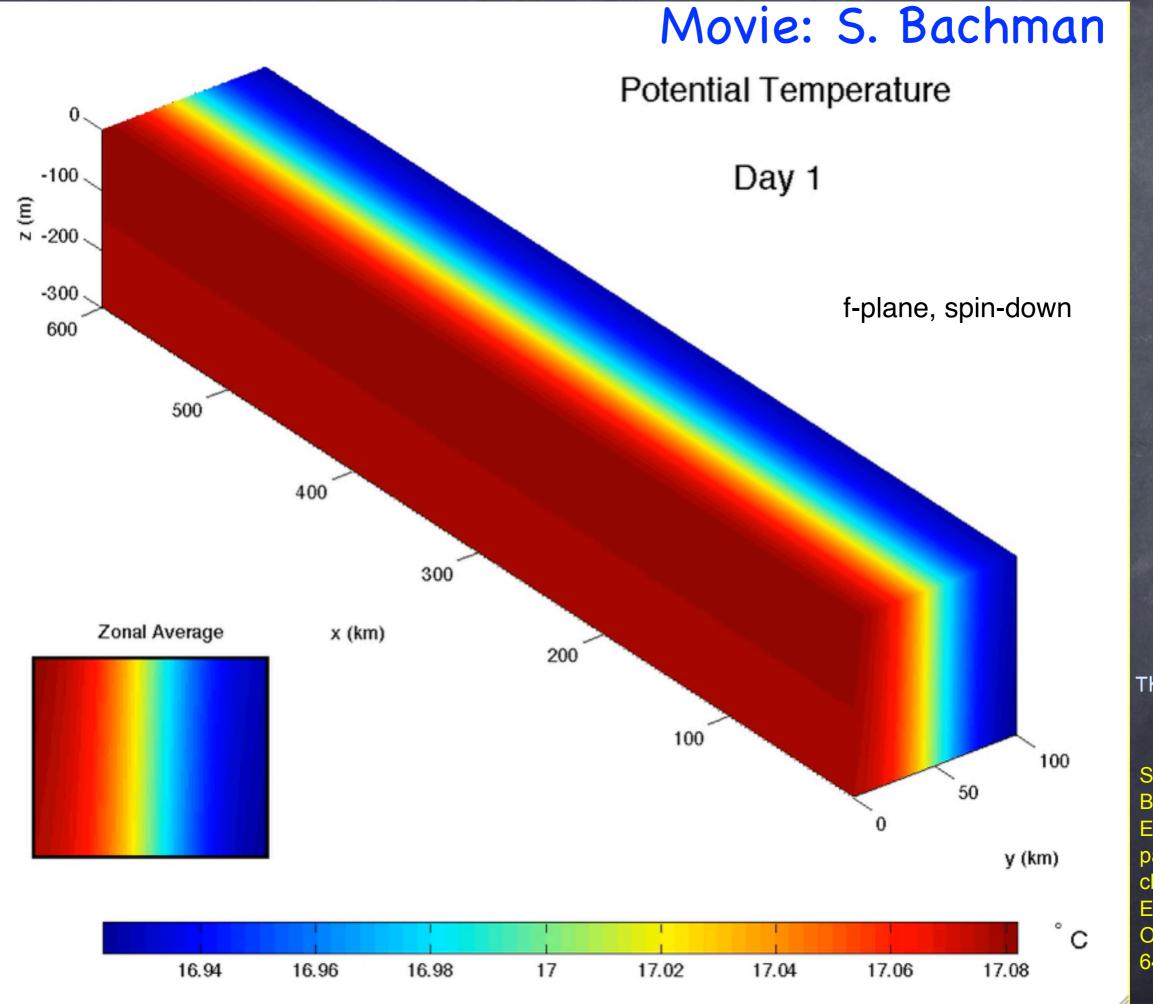


- Scott Bachman (DAMTP) has implemented this QG Leith closure in the MITgcm
 - Both Germano Dynamic and Fixed Coefficient

$$\Lambda_{qg} = \Lambda_{qg}(x, y, z, t)$$
 $\Lambda_{qg} = 1$

$$\nu_{qg} = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^{3} \left|\nabla q_{qg}\right| = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^{3} \left|\nabla_{h}\left[\beta y + \nabla_{h}^{2}\psi + \frac{\partial}{\partial z}\left(\frac{f_{0}^{2}}{N^{2}}\frac{\partial\psi}{\partial z}\right)\right]\right|.$$

$$u_{qg} = \kappa_{Redi} = \kappa_{GM} = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^3 \left|\nabla q_{qg}\right|.$$



This Slide & Movies: S. Bachman

S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013.

Does it work?

We'll test this in a channel model, using three different resolutions:

The fastest growing mode is better resolved the higher the resolution, so the spindown will be slower for the coarser runs.

But the QG dissipation / diffusivity scheme is able to compensate!

Old Method=Smagorinsky viscosity with only implicit numerical diffusivity, no GM

New Method=QG Leith

17.06 17.06 10 17.04 17.04 17.02 17.02 N 30 17 16.98 16.98 16,96 16,96 50 50 16.94 20 80 80 Frontal Spindown, no GM/Redi; time = 01.0 d Frontal Spindown with Germano; time = 01.0 d 17.06 17.06 10 17.04 17.04 17.02 17.02 N 30 16.98 16.98 16.96 16.96 10 20 30 40 10 20 30 40 Frontal Spindown with Germano; time = 01.0 d Frontal Spindown, no GM/Redi; time = 01.0 d 17.06 10 17.04 17.04 17.02 17.02 17 16.98 16.98 16.96 16.96 50 15 10 15

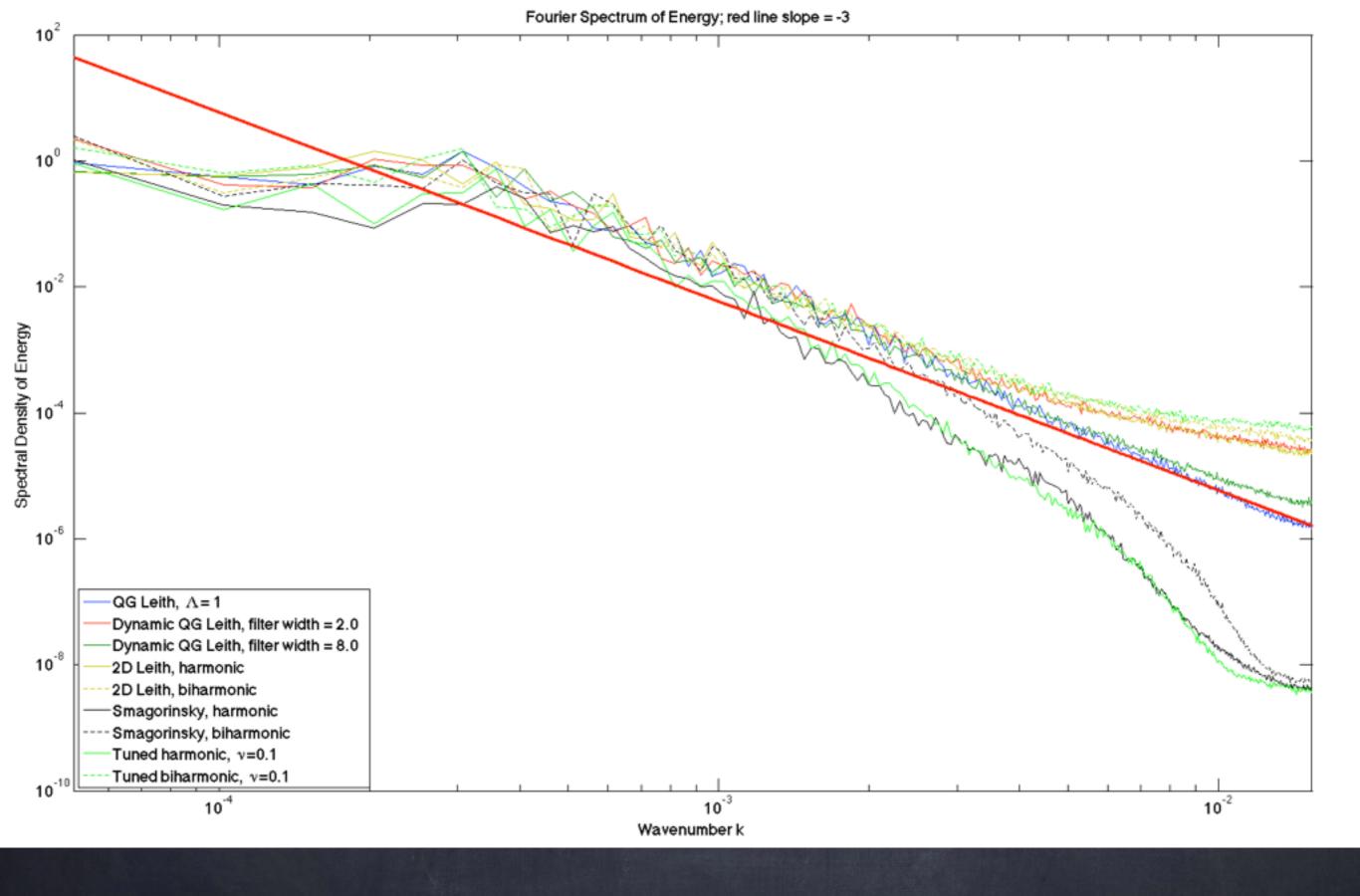
New Method

Frontal Spindown with Germano; time = 01.0 d

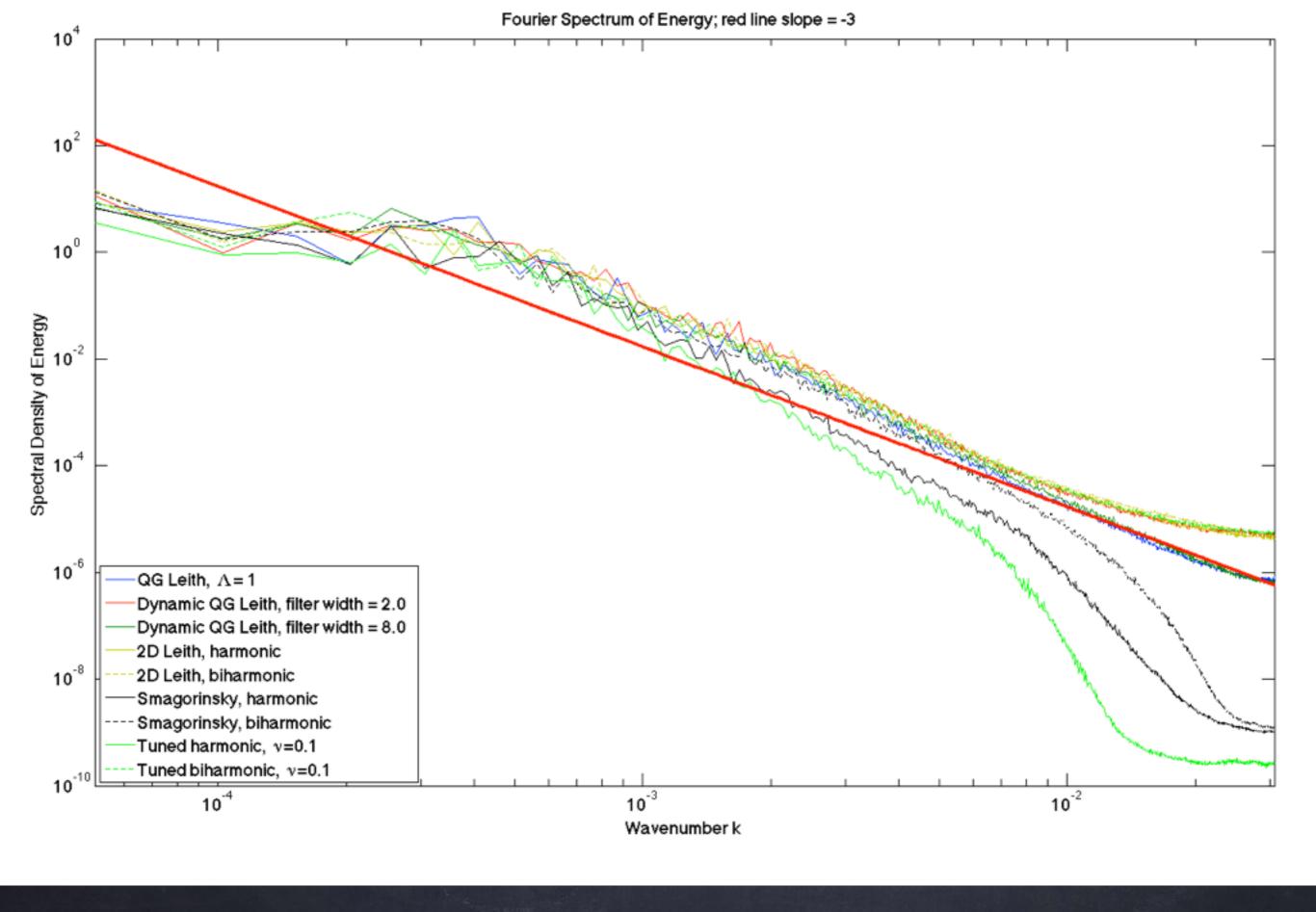
Old Method

Frontal Spindown, no GM/Redi; time = 01.0 d

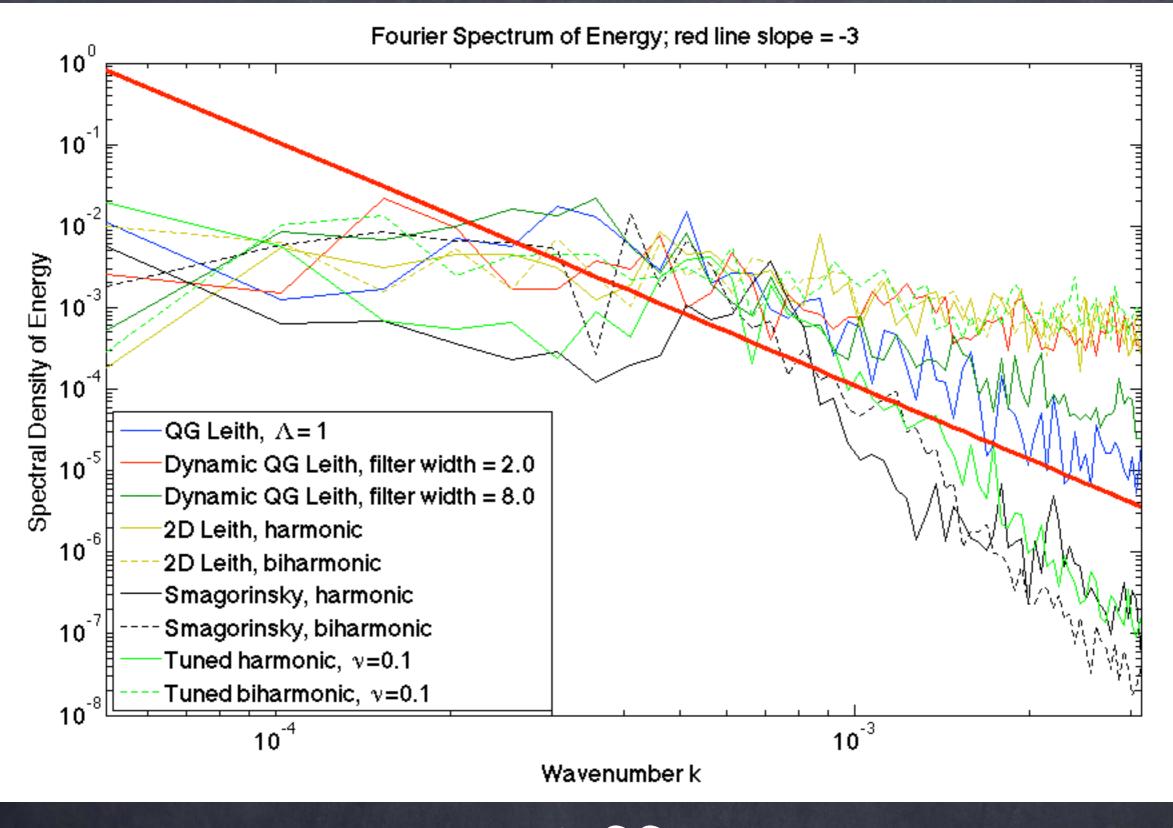
This Slide & Movies:
S. Bachman



Comparing 9 different subgrid closures with dx= 0.4 Ld



Comparing 9 different subgrid closures with dx=0.2 Ld



Comparing 9 different subgrid closures with dx=2 L_d

But...we need to be careful of when QG isn't appropriate:

Stretching term can be too large when unstratified use gridscale Burger number to determine when:

$$Bu^* = \frac{N^{*2}\Delta z^2}{f^2\Delta x^2} \sim Ro^{*2}Ri^*$$

$$\frac{\nu_{qg}^*}{\nu_{2d}^*} \approx \frac{|\nabla_h q_{qg}^*|}{|\nabla_h q_{2d}^*|} \sim 1 + Bu^* \sim 1 + Ro^{*2}Ri^*$$

Surface QG has different spectral characteristics—we have a theory, but simultaneous implementation unclear

Conclusions

Promising mesoscale method: Realistic tests next! QG Leith=viscosity, Redi diffusivity, *and* GM transfer coeff.

- - Ensures O(1) gridscale Reynolds & Péclet Nearly as suggested by Roberts & Marshall, 98, JPO
 - Revert to 2D Leith when QG is inappropriate
 - QG only if gridscale Burger near 1, gridscale Richardson>1
- Our results suggest QG Leith will deliver the proven plug&play capability of LeithPlus with improved QG-based physics-Will matter most where stretching terms or APE balance are important, e.g., WBC.
- Since it was built to keep gridscale Re and Pe O(1), we expect to have spurious diapycnal mixing small (Ilicak et al., 2012). Testing now, based on buoyancy classes and passive tracers.