
Baylor Fox-Kemper (Brown University)
with Scott Bachman (DAMTP)
ECCO Workshop
Jan 22-24, 2014
Sponsors: NSF 1258907, 1245944, 0934737, 0825614, NASA NNX09AF38G

Scale-aware subgrid closures for models that partly resolve the mesoscale and submesoscale

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A Global Parameterization of Mixed Layer Eddy Flow & Scale Aware Restratification validated against simulations

B. Fox-Kemper, G. Danabasoglu, R. Ferrari, S. M. Griffies, R. W. Hallberg, M. M. Holland, M. E. Maltrud, S. Peacock, and B. L. Samuels. Parameterization of mixed layer eddies. III: Implementation and impact in global ocean climate simulations. Ocean Modelling, 39:61-78, 2011.

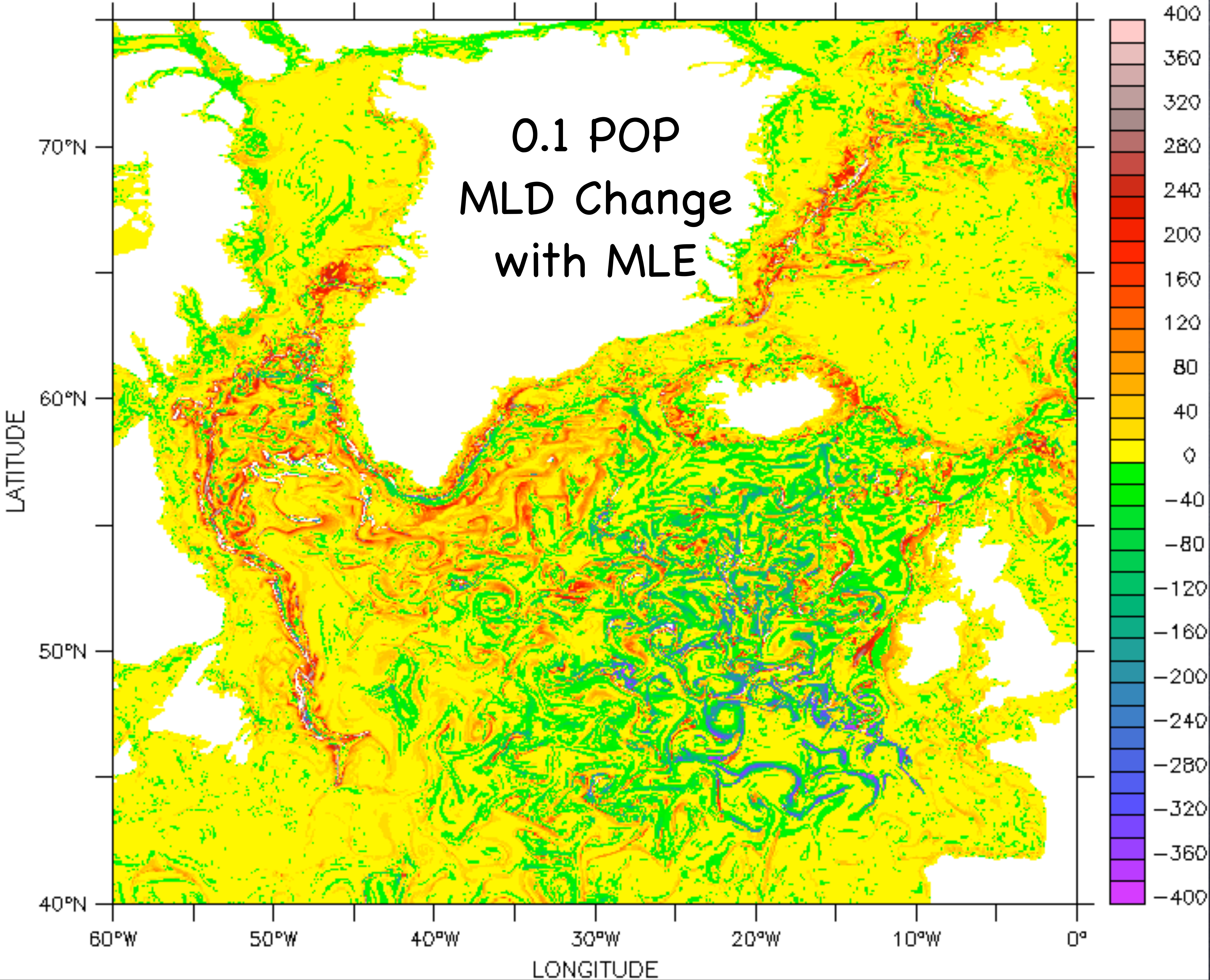
$$\overline{\mathbf{u}'b'} \equiv \mathbf{\Psi} \times \nabla \bar{b}$$

$$\mathbf{\Psi} = \left[\frac{\Delta x}{L_f}\right] \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla \bar{b} \times \hat{\mathbf{z}}$$

Compare to the original singular, unrescaled version

$$\mathbf{\Psi} = \frac{C_e H^2 \mu(z)}{|f|} \nabla \bar{b} \times \hat{\mathbf{z}}$$

New version handles the equator, and averages over many fronts

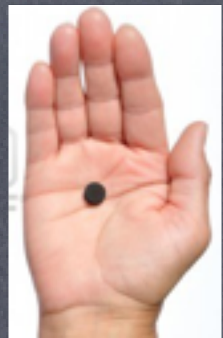


0.1 POP
MLD Change
with MLE
70°N
60°N
50°N
40°N
LATITUDE
60°W
50°W
40°W
30°W
20°W
10°W
0°
LONGITUDE
400
360
320
280
240
200
160
120
80
40
0
−40
−80
−120
−160
−200
−240
−280
−320
−360
−400

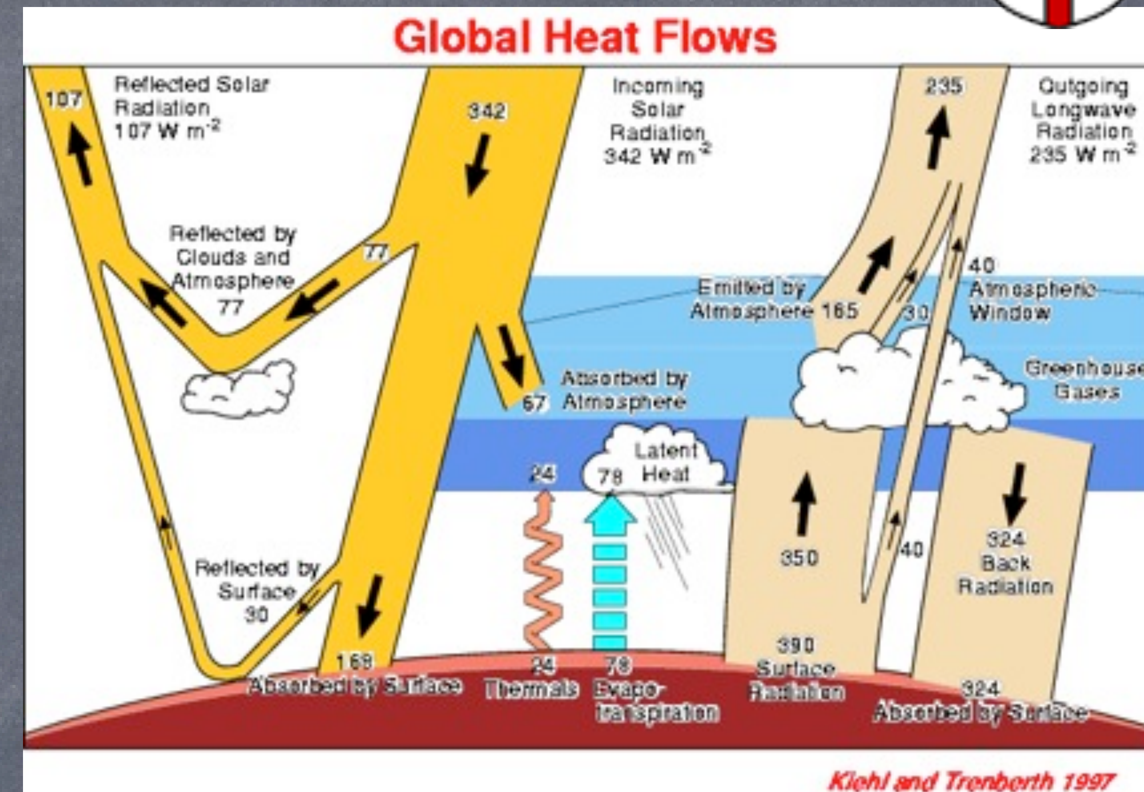
Submeso closure is already scale-aware!

- You can use the global/gcm form at any resolution.
- It turns itself off as the ML def. radius becomes resolved.

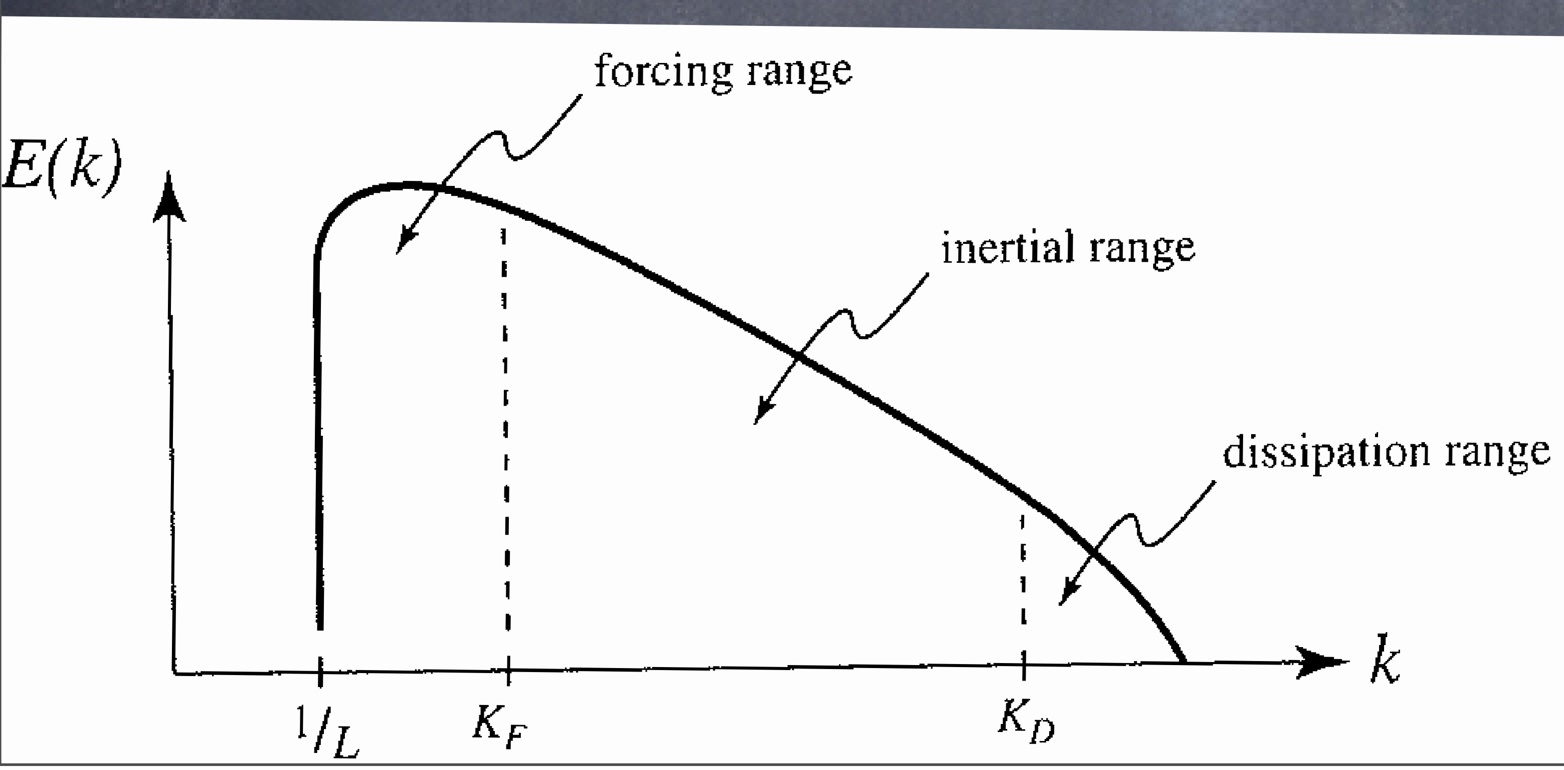
$$\boldsymbol{\Psi} = \left[\frac{\Delta x}{L_f}\right] \frac{C_e H^2 \mu(z)}{\sqrt{f^2 + \tau^{-2}}} \nabla \bar{b} \times \hat{\mathbf{z}}$$

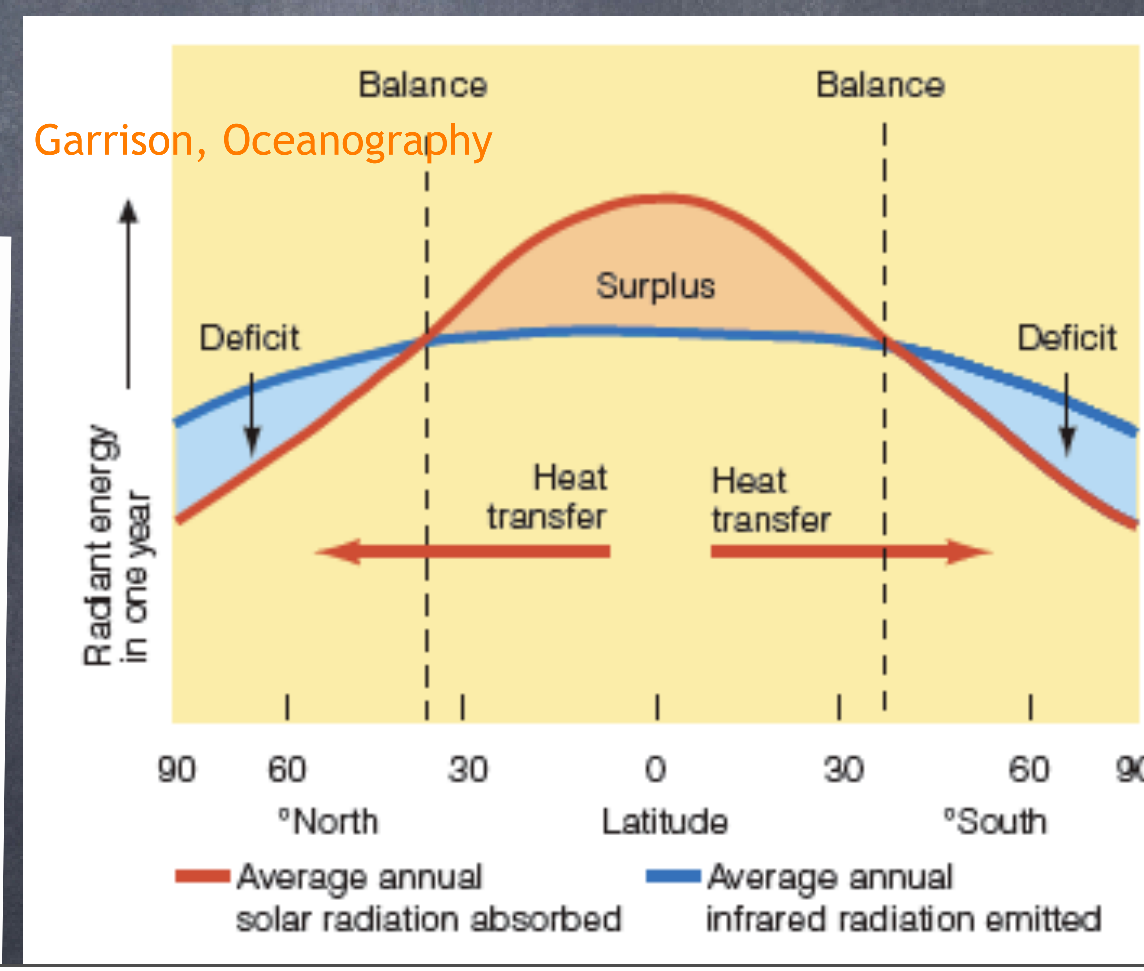


The Earth's Climate System is driven by the Sun's light (minus outgoing infrared) on a global scale

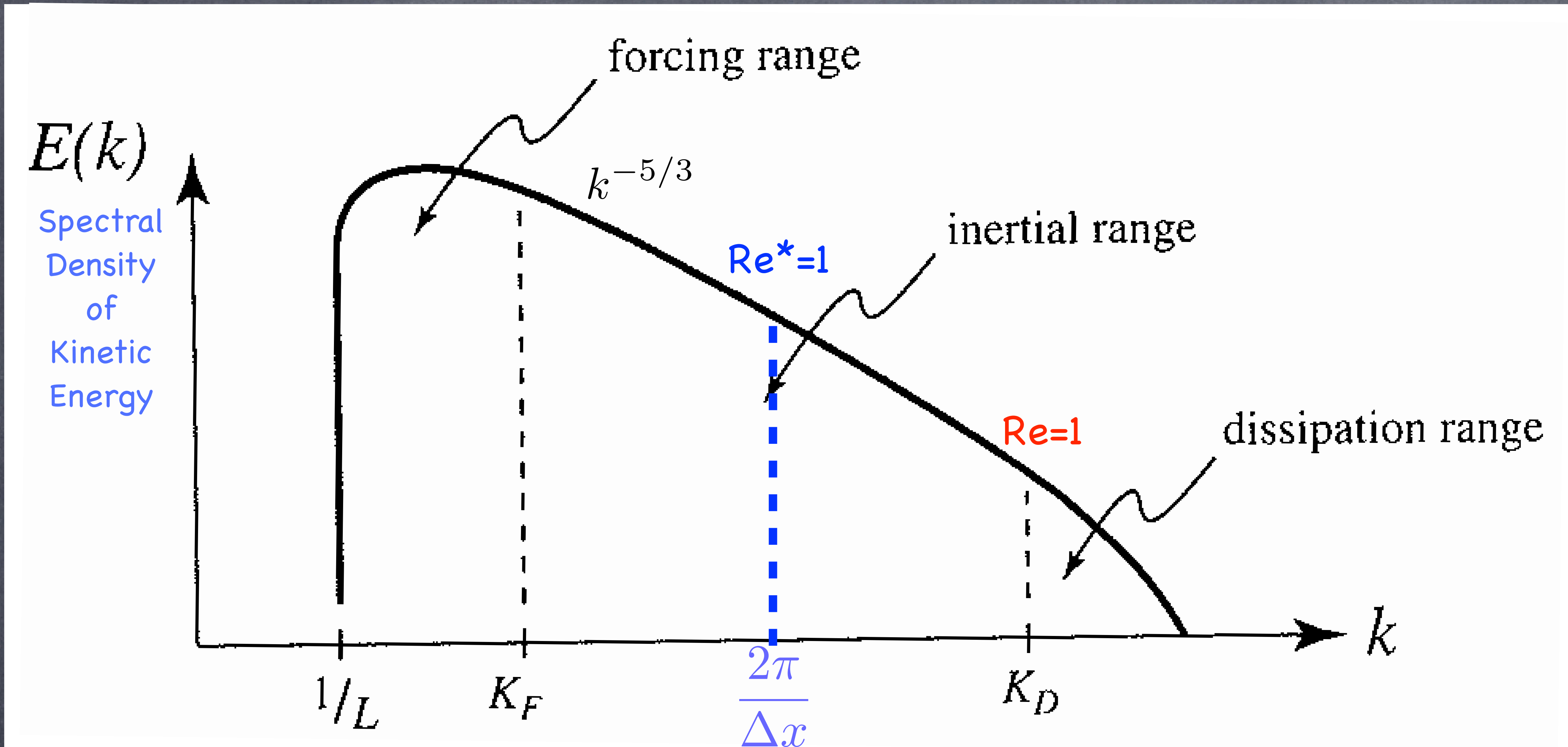


Dissipation concludes turbulence cascades to scales about a billion times smaller





3D Turbulence Cascade

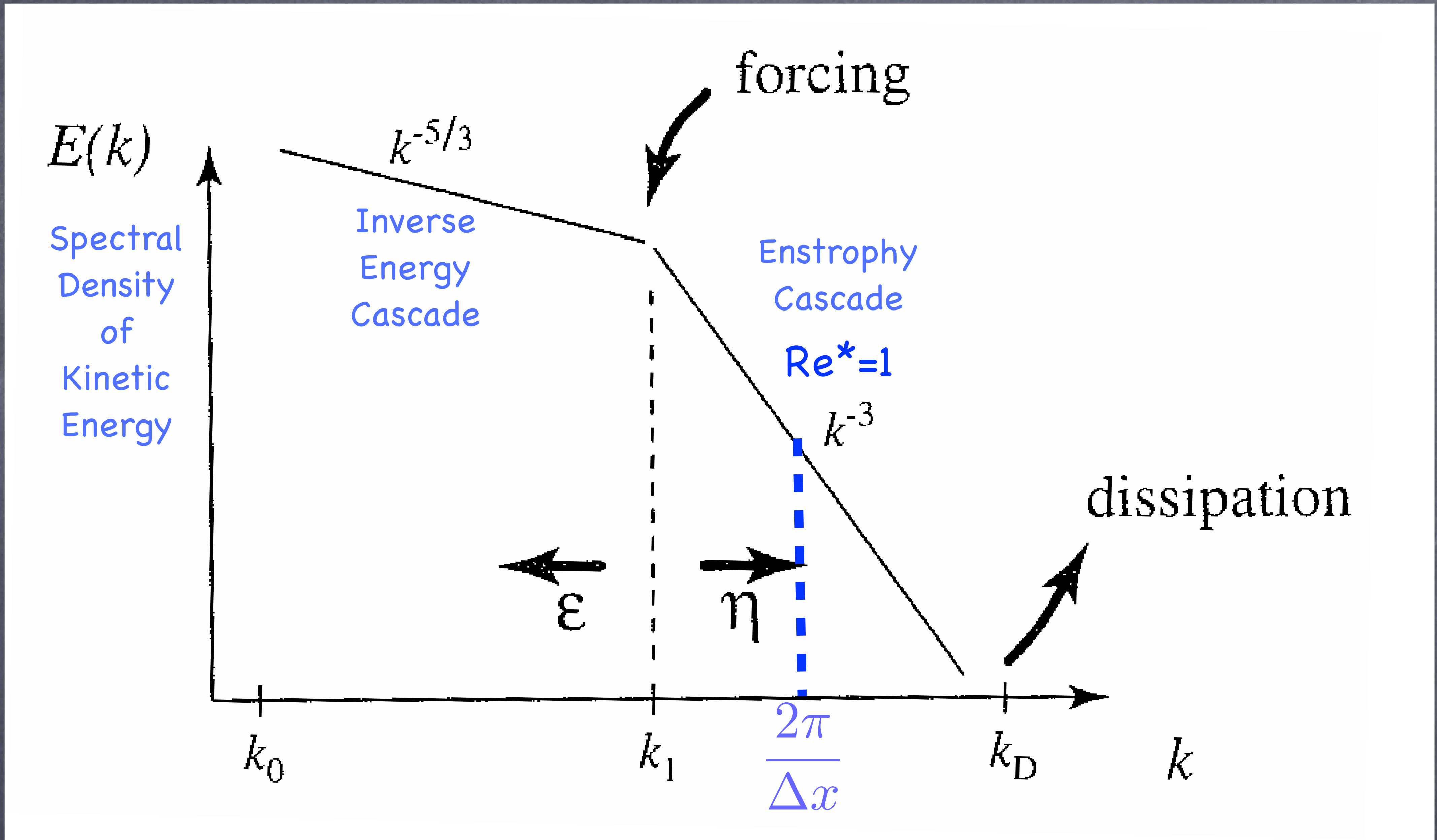


1963: Smagorinsky Scale & Flow Aware Viscosity Scaling,
So the Energy Cascade is Preserved,
but order-1 gridscale Reynolds #: $Re^* = UL/\nu_*$

$$\nu_{*h} = \left(\frac{\Upsilon_h \Delta x}{\pi}\right)^2 \sqrt{\left(\frac{\partial u_*}{\partial x} - \frac{\partial v_*}{\partial y}\right)^2 + \left(\frac{\partial u_*}{\partial y} + \frac{\partial v_*}{\partial x}\right)^2}.$$

2D Turbulence Differs

R. Kraichnan, 1967 JFM

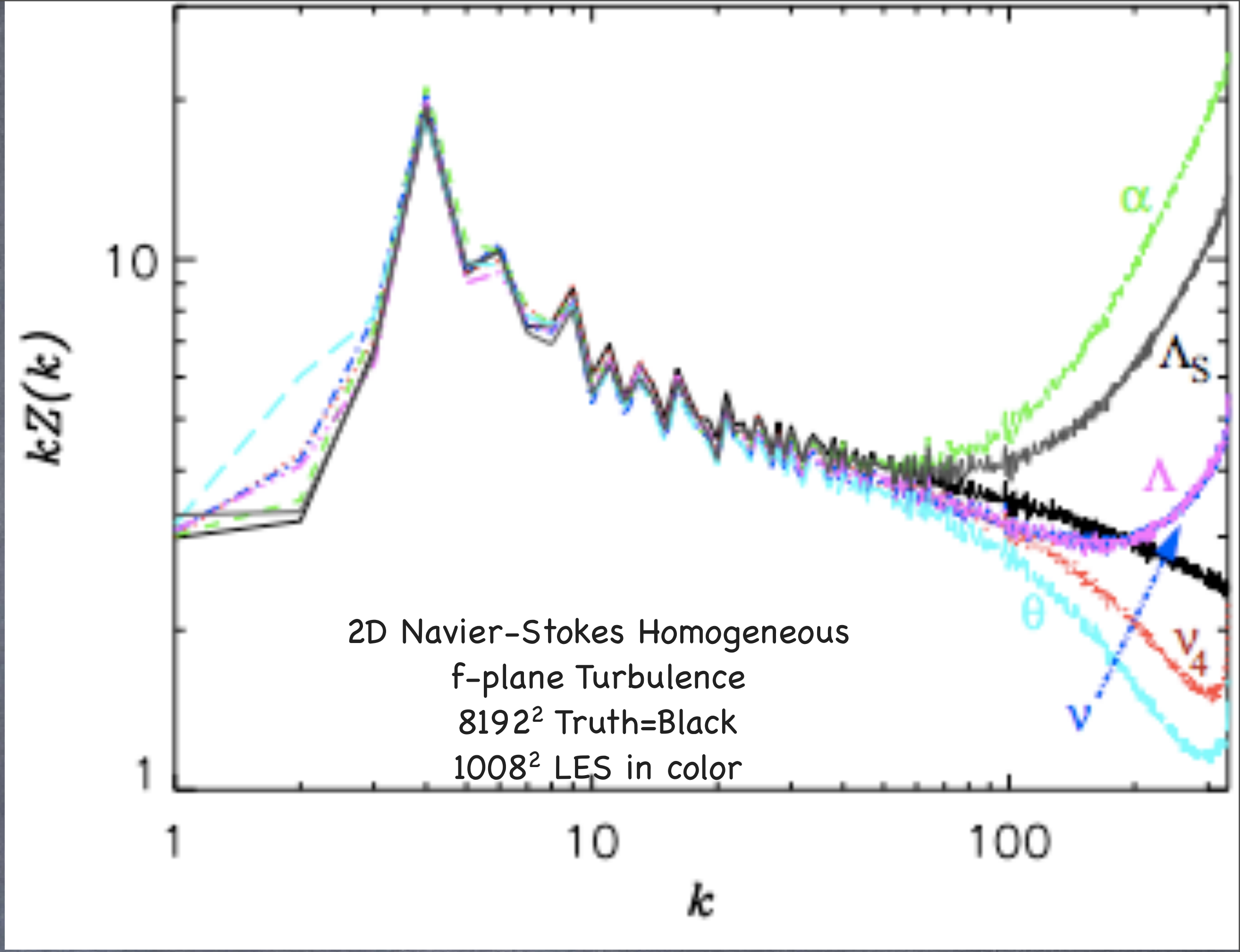


1996: Leith Devises Viscosity Scaling,
So that the Enstrophy (vorticity^2) Cascade is Preserved

$$\nu_* = \left(\frac{\Lambda\Delta x}{\pi}\right)^3 \left|\nabla_h\left(\frac{\partial u_*}{\partial y} - \frac{\partial v_*}{\partial x}\right)\right|$$

Barotropic or stacked layers

Some MOLES Truncation Methods In Use 2d (SWE) test



Graham & Ringler, 2013 Ocean Modelling

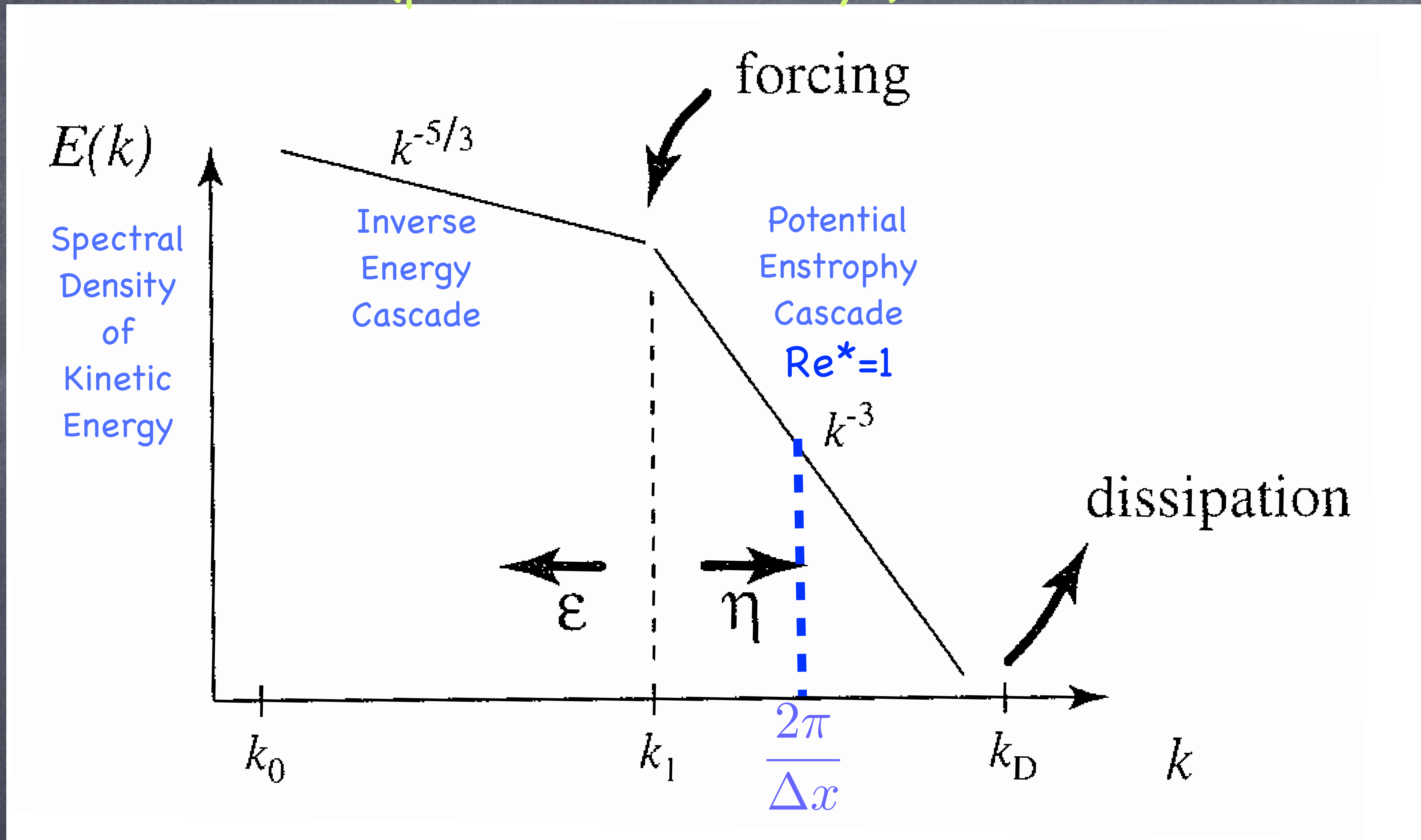
- Harmonic/Biharmonic/Numerical
  - Many. Often not scale- or flow-aware
  - Griffies & Hallberg, 2000, is one aware example
- Fox-Kemper & Menemenlis, 2008. ECCO2.
  - Leith Viscosity (2d Enstrophy Scaling)
- Chen, Q., Gunzburger, M., Ringler, T., 2011
  - Anticipated Potential Vorticity of Sadourny
- San, Staples, Iliescu (2011, 2013)
  - Approximate Deconvolution Method
- Stochastic & Statistical Parameterizations
  - Other session going on now in Y10

In this comparison, untuned Leith beats: tuned harmonic, tuned biharmonic, Smagorinsky, LANS-alpha, & Anticipated PV

See also Ramachandran et al, 2013 Ocean Modelling for SMOLES

QG Turbulence: Pot'l Enstrophy cascade (potential vorticity^2)

J. Charney, 1971 JAS



F-K & Menemenlis '08: Revise Leith Viscosity Scaling, So that diverging, vorticity-free, modes are also damped

$$\nu_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6|\nabla_h q_{2d}|^2 + \Lambda_d^6|\nabla_h(\nabla_h \cdot \mathbf{u}_*)|^2}$$

B. Fox-Kemper and D. Menemenlis. Can large eddy simulation techniques improve mesoscale-rich ocean models? In M. Hecht and H. Hasumi, editors, Ocean Modeling in an Eddying Regime, volume 177, pages 319-338. AGU Geophysical Monograph Series, 2008.

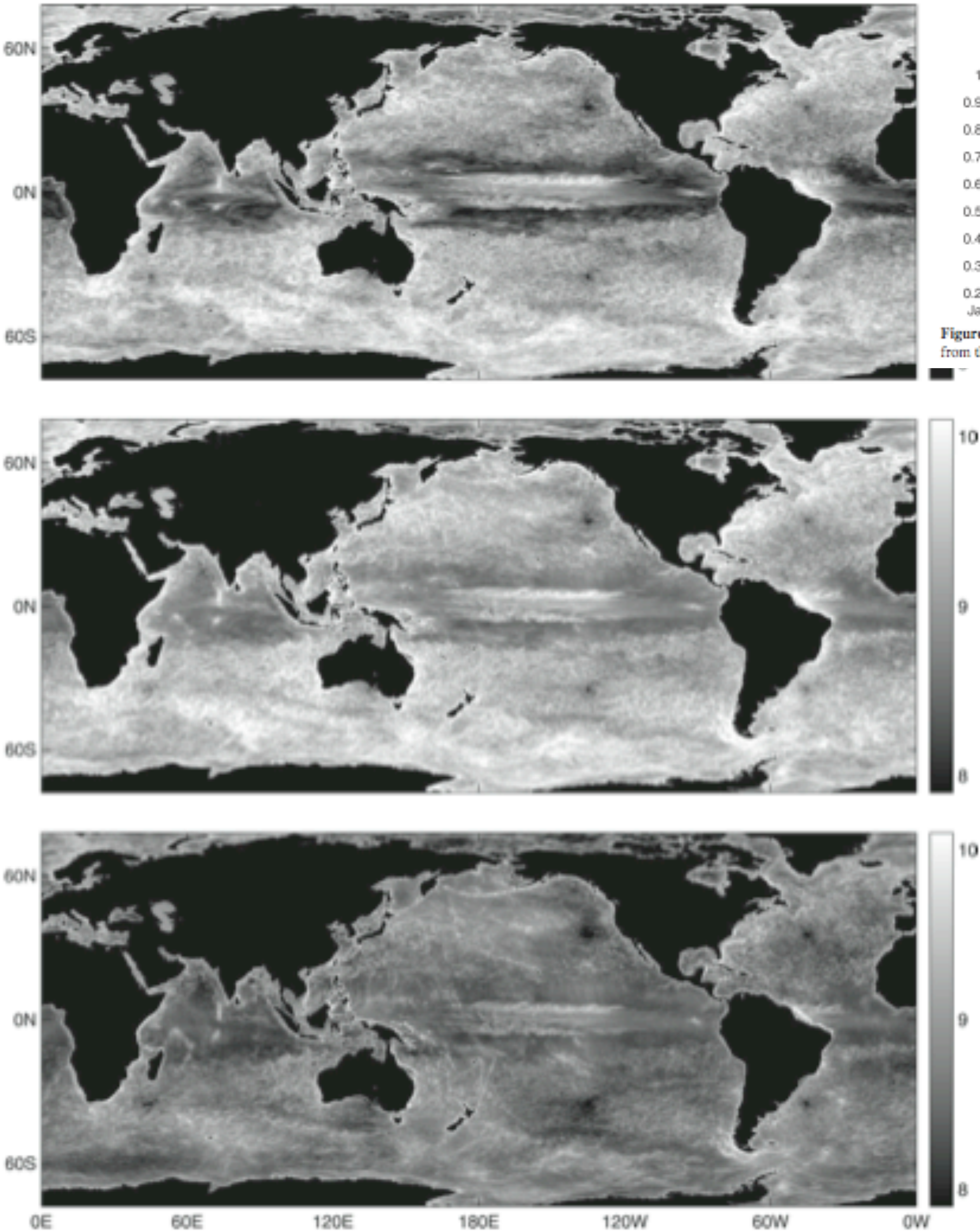


Figure 3. Monthly mean biharmonic viscosity, ν_4, in the model's surface level for December 2001. Units are $m^4\ s^{-1}$ and color scale displays $\log_{10}(\nu_4)$. Top panel is from the *LeithOnly* integration. Middle panel is from the *LeithPlus* integration. Bottom panel shows the divergent modification of the *LeithPlus* integration.

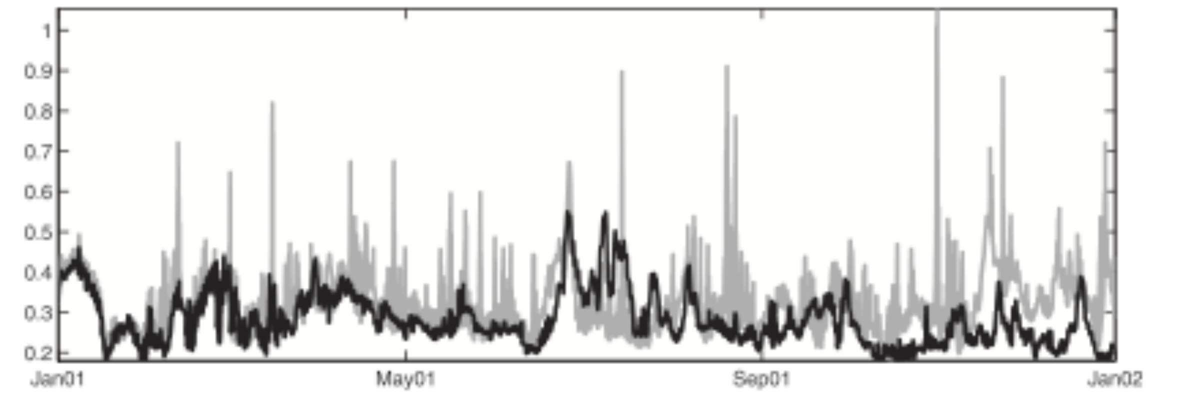
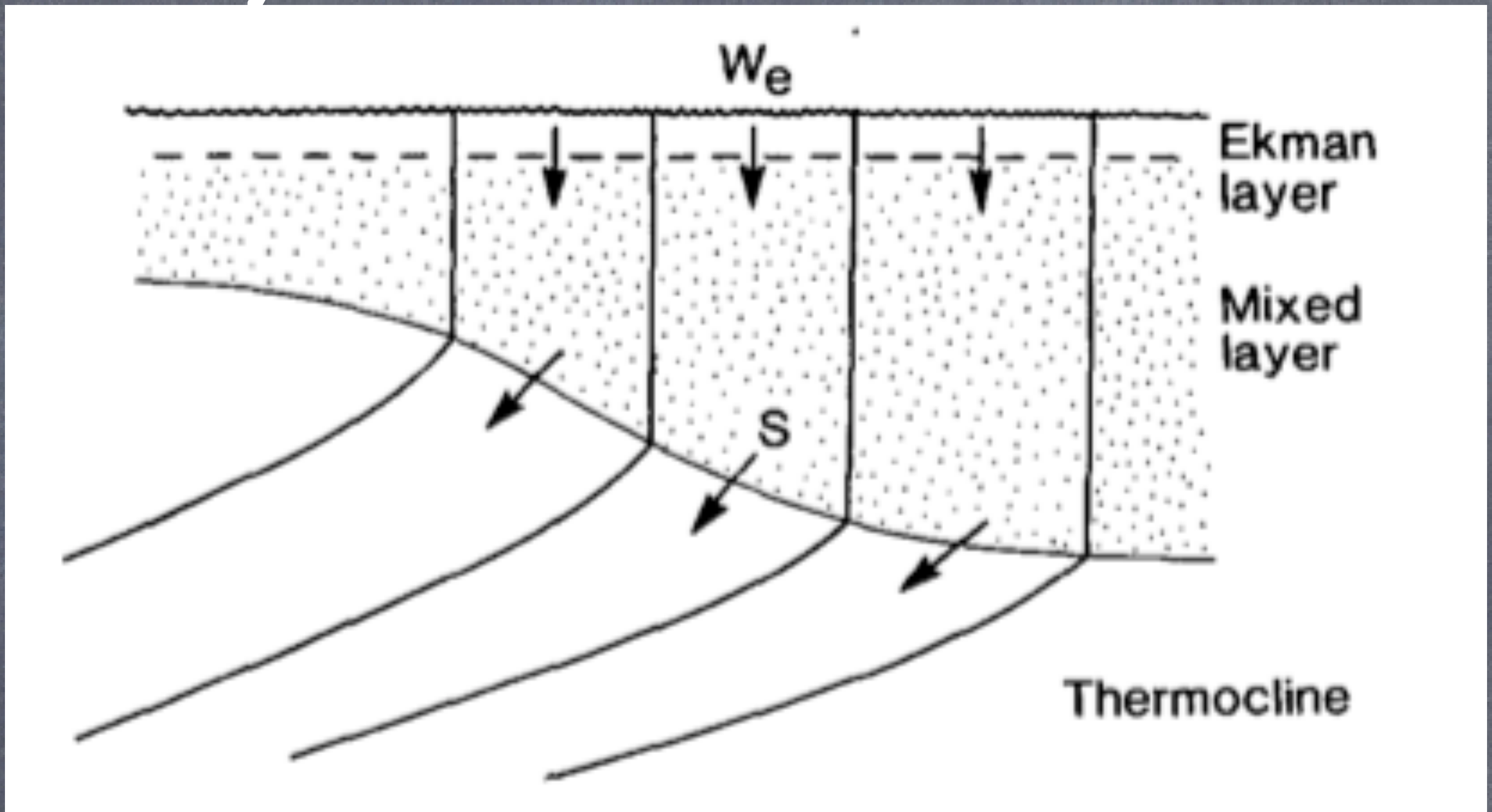


Figure 4. Maximum Courant number, $w\Delta t/\Delta z$, for vertical advection. Gray line is from the *LeithOnly* integration, and black line is from the *LeithPlus* integration.

B. Fox-Kemper and D. Menemenlis. Can large eddy simulation techniques improve mesoscale-rich ocean models? In M. Hecht and H. Hasumi, editors, Ocean Modeling in an Eddying Regime, volume 177, pages 319-338. AGU Geophysical Monograph Series, 2008.

Is 2D Turbulence a good proxy for neutral flow?



Nurser & Marshall, 1991 JPO

Yes:

- For a few eddy time-scales QG & 2D AGREE (Bracco et al. '04)
- Barotropic Flow--Obvious 2d analogue

No:

- Bolus Fluxes--Divergent 2d flow
- Sloped, not horiz.
- Surface Effects?

Friday, January 24, 14

Some Asymptotic Limits, Following McWilliams '85

- Very Small Fr, but large Ro:
  - stacked 2d layers
- Very Small H/L, small b:
  - Barotropic 2d Turbulence
- Ro, Fr, beta << 1,
- & Ro/Fr=Ld/L=1,
  - QG, SQG

The following equations result

$$Ro\left[\frac{\partial \boldsymbol{V}_h}{\partial t} + \boldsymbol{V}_H \cdot \nabla \boldsymbol{V}_H + Fr^2 \max(Ro^{-1}, 1) w \frac{\partial}{\partial z} \boldsymbol{V}_H\right] + \max(1, Ro)\nabla_H \pi + \left(1 + \frac{\beta y}{f_0}\right) \boldsymbol{k} \times \boldsymbol{V}_H = 0, \tag{2.44}$$

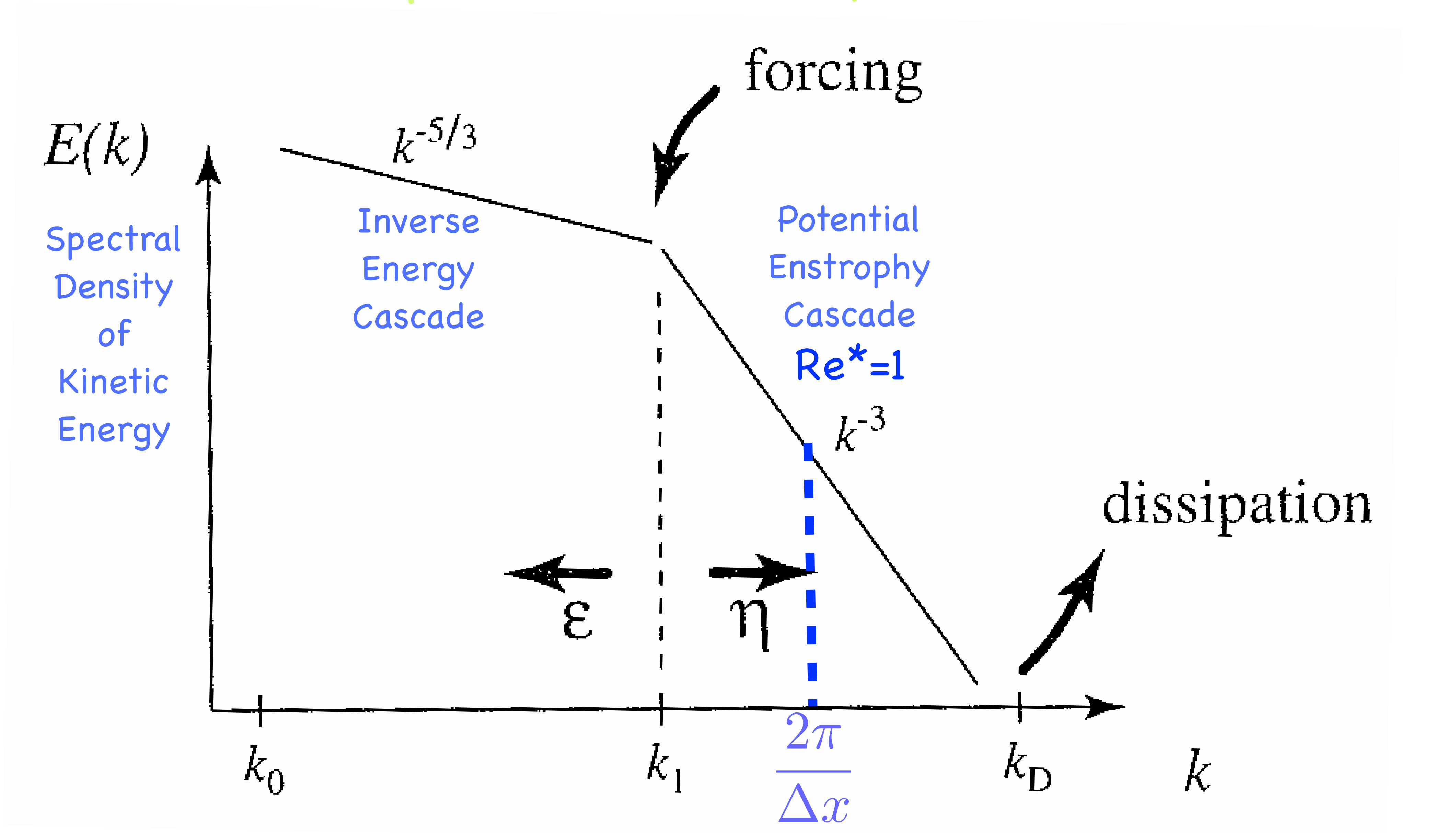
$$Fr^2 \frac{H^2}{L^2}\left[\frac{\partial w}{\partial t} + \boldsymbol{V}_H \cdot w + Fr^2 \max(Ro^{-1}, 1) w \frac{\partial}{\partial z} w\right] + \frac{\partial}{\partial z}\pi - b = 0, \tag{2.45}$$

$$\frac{\partial b}{\partial t} + \boldsymbol{V}_H \cdot \nabla b + Fr^2 \max(Ro^{-1}, 1) w \frac{\partial}{\partial z} b + w \frac{N^2}{N_0^2} = 0, \tag{2.46}$$

$$\nabla \cdot \boldsymbol{V}_H + Fr^2 \max(Ro^{-1}, 1) \frac{\partial}{\partial z} w = 0. \tag{2.47}$$

QG Turbulence: Pot'l Enstrophy cascade (potential vorticity²)

J. Charney, 1971 JAS



$$\nu_{qg} = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^3 |\nabla q_{qg}| = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^3 \left|\nabla_h \left[\beta y + \nabla_h^2 \psi + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial \psi}{\partial z}\right)\right]\right|.$$

$$\nabla_h \nabla_h^2 \psi = \nabla_h \left[\frac{\partial u_*}{\partial_y} - \frac{\partial v_*}{\partial_x}\right]$$

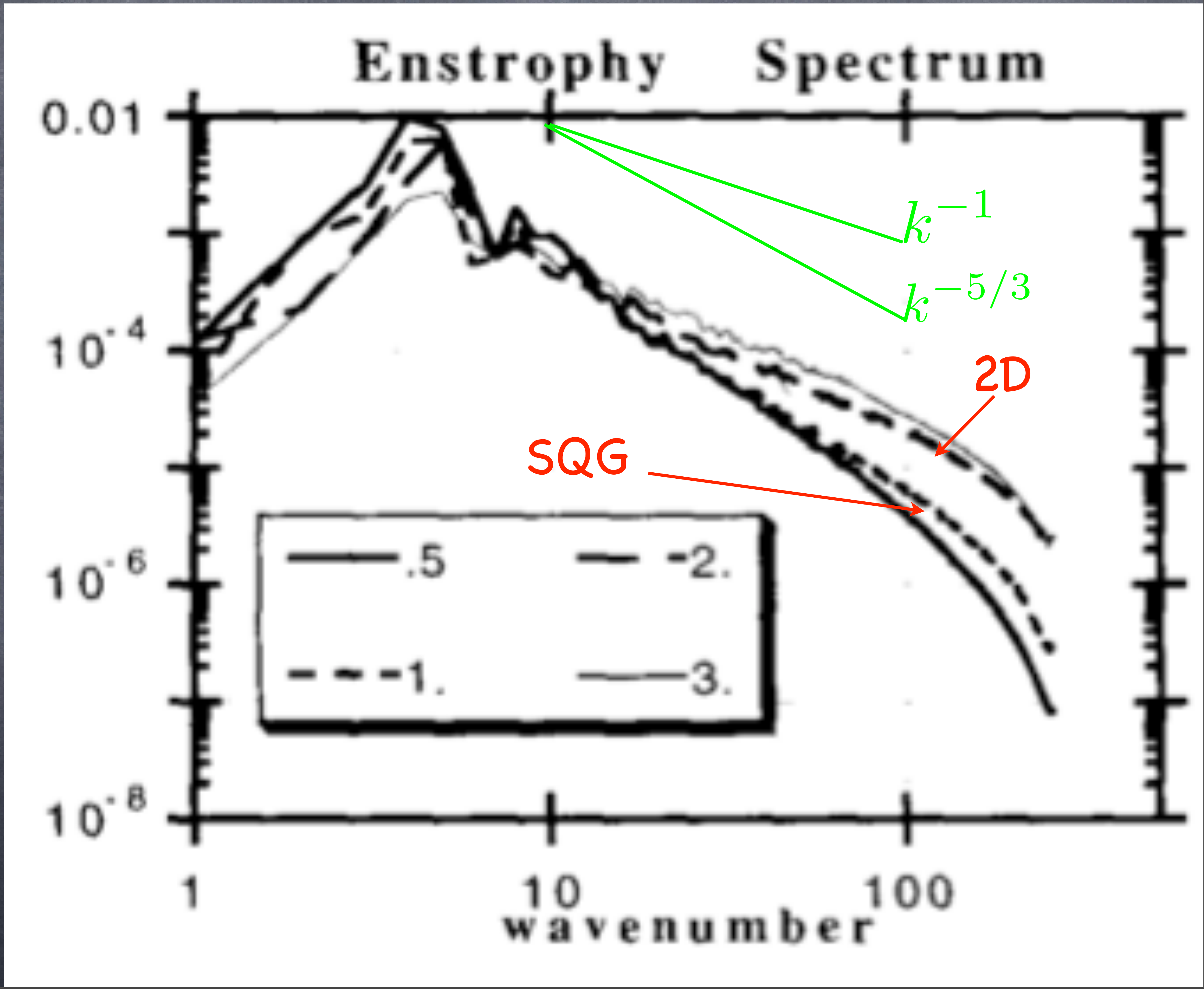
$$\nu_{qg} = \kappa_{Redi} = \kappa_{GM} = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^3 |\nabla q_{qg}|.$$

In real stratified flows, things are a bit more complex than in 2d

Even more than QG...

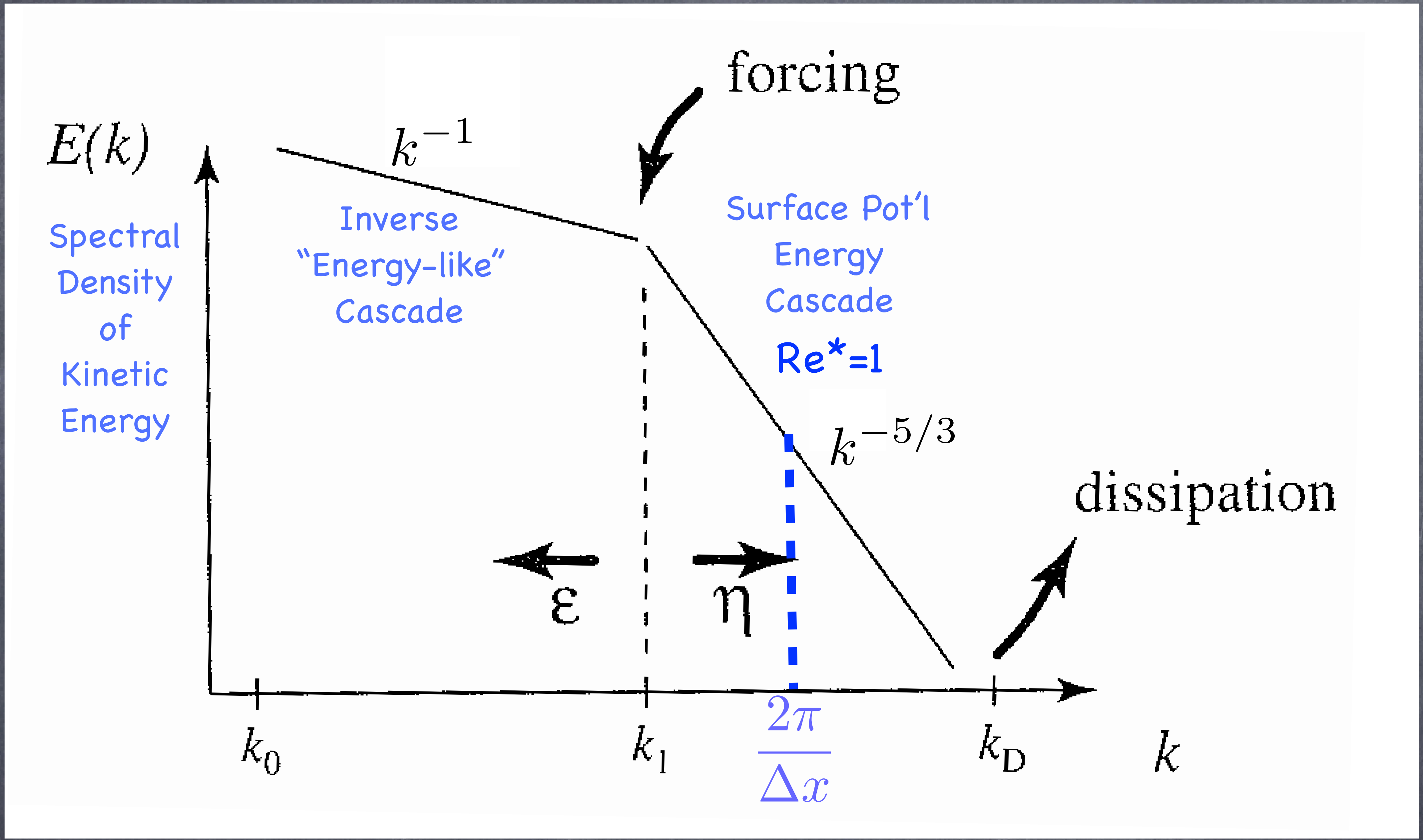
Surface Effects may dominate

Pierrehumbert, Held, Swanson, 1994 Chaos
Spectra of Local and Nonlocal Two-dimensional Turbulence



SQG Turbulence: Surface Buoyancy & Velocity cascade--scales surface horiz. diffusivity only

W. Blumen, 1978 JAS
Held et al 1995, JFM.
Smith et al. 2002, JFM



Smag-Like (Inverse):

$$\kappa_* = \left(\frac{\Upsilon \Delta x}{\pi}\right)^{4/3} \left|\frac{1}{f}\nabla_h b\right|^{2/3}$$

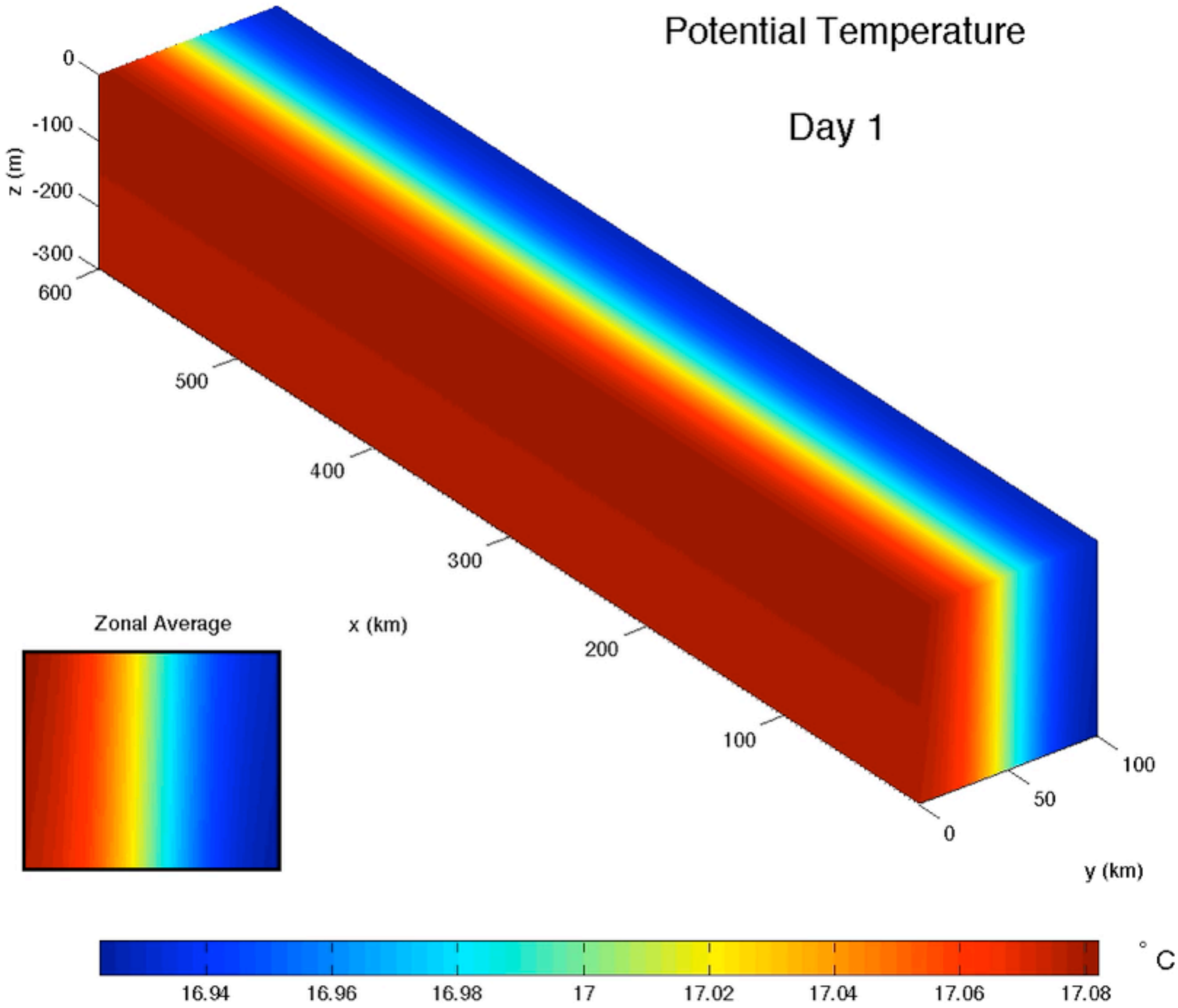
Leith-Like (Direct):

$$\kappa_* = \left(\frac{\Lambda \Delta x}{2\pi}\right)^{3/2} \left[-\frac{\partial}{\partial z}|\nabla_h \psi|^2\right]^{1/2}$$

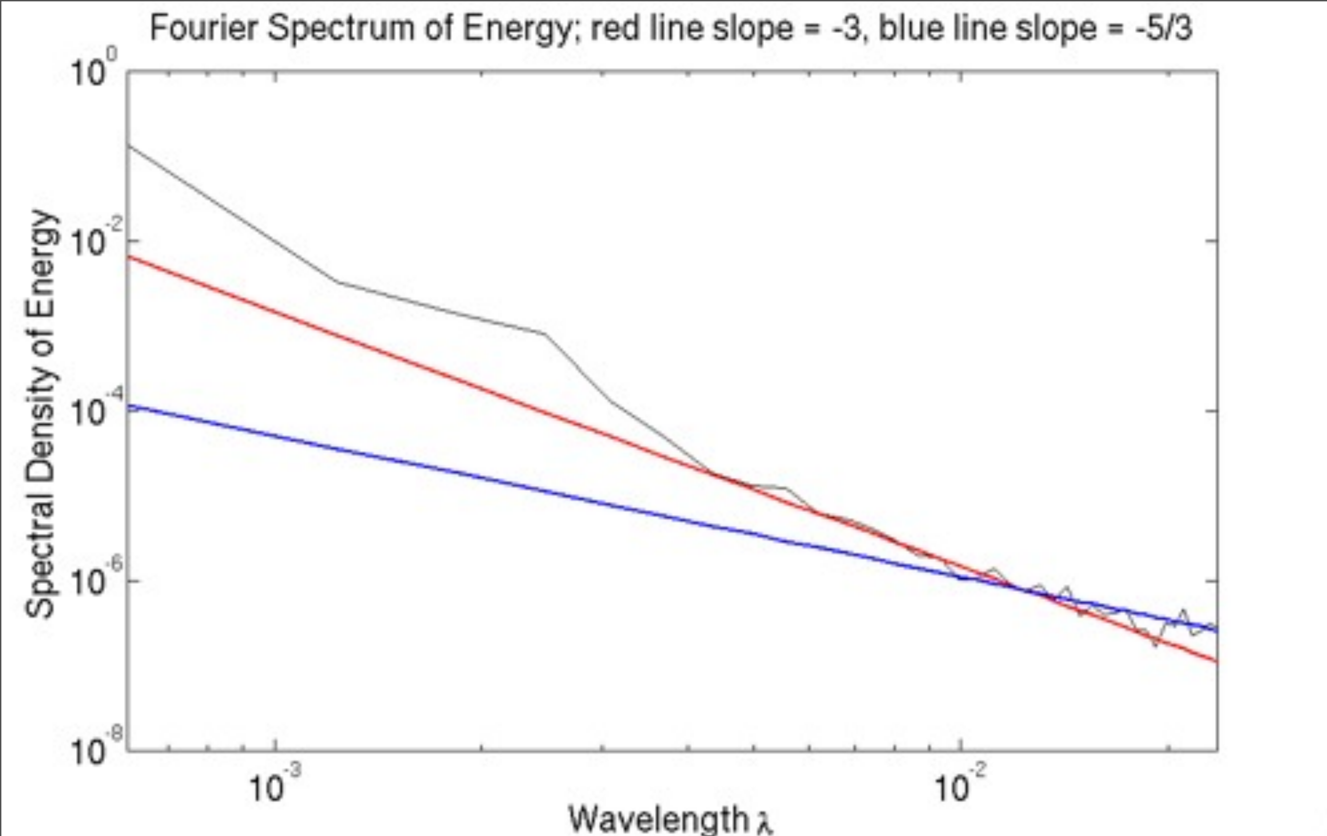
And QG pot'l enstrophy Leith is ... working in MITgcm

- Scott Bachman (DAMTP) has implemented this QG Leith closure in the MITgcm
  - Both Germano Dynamic and Fixed Coefficient
- Sets viscosity=diffusivity=GM coefficient
- Both are stable and robust, very similar (is dynamical needed?)
- Both work better than Smagorinsky, smoother spectrum to grid scale (to be shown next).
- But, we don't yet understand the spectral behavior of all test cases. 2d barotropic, QG, & SQG, equatorial are coexistent...

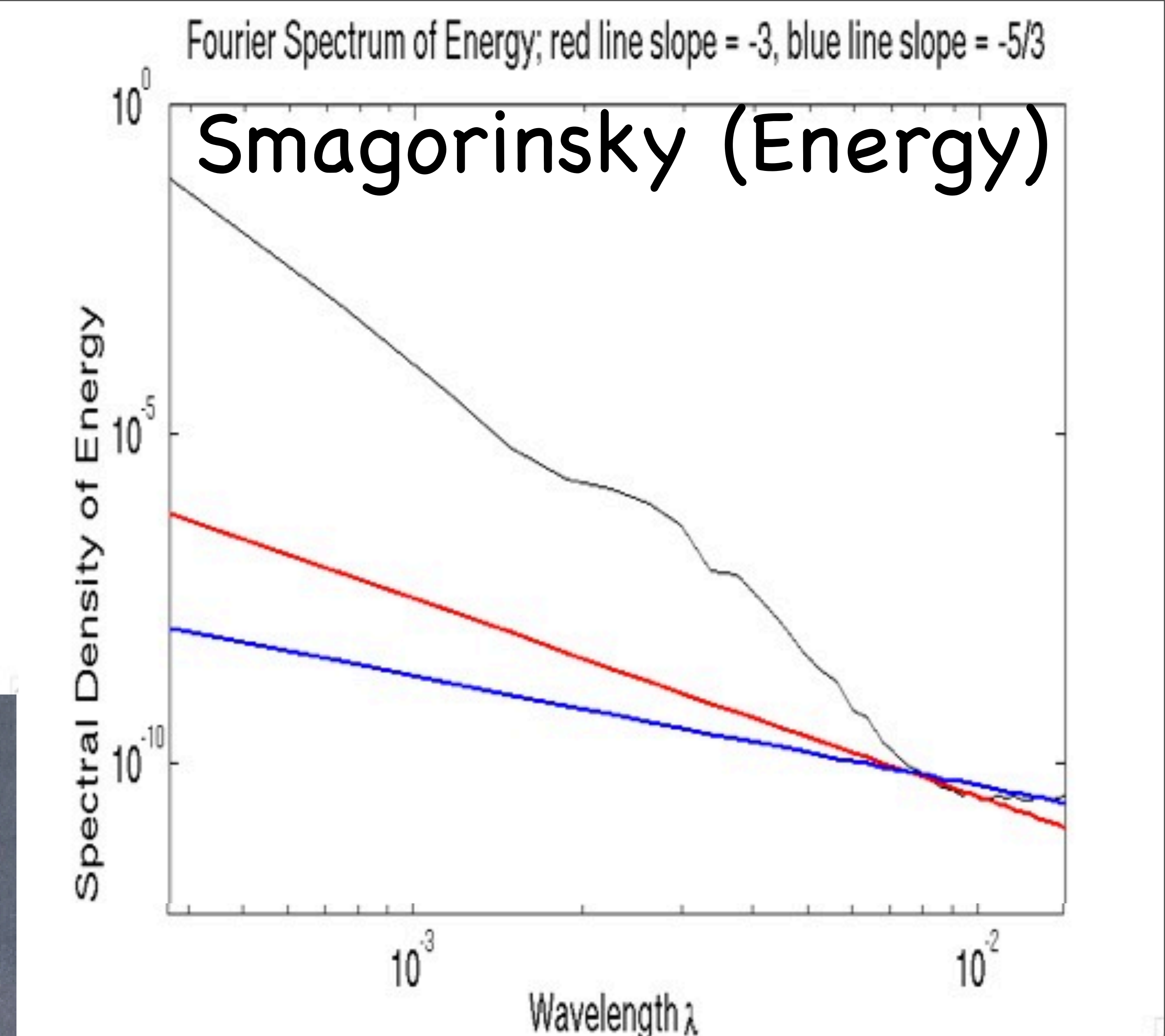
Movie: S. Bachman



S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013.



QG Leith (Pot'l Enstrophy)



Comparing the spectrum in QG Leith against another (inappropriate) LES closure, we see:

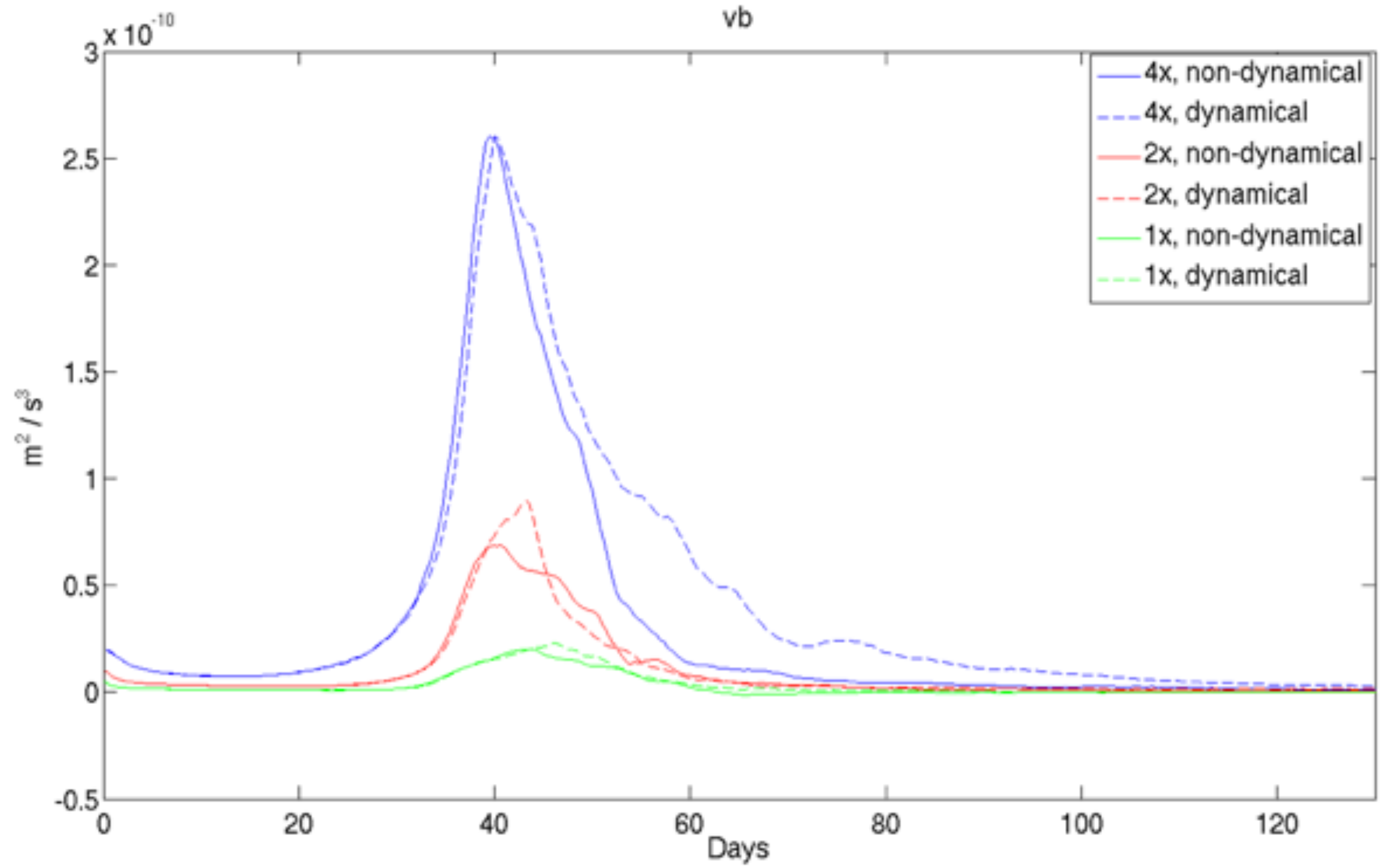
1) Better adherence to expected spectrum

2) Less "ski jump" near gridscale

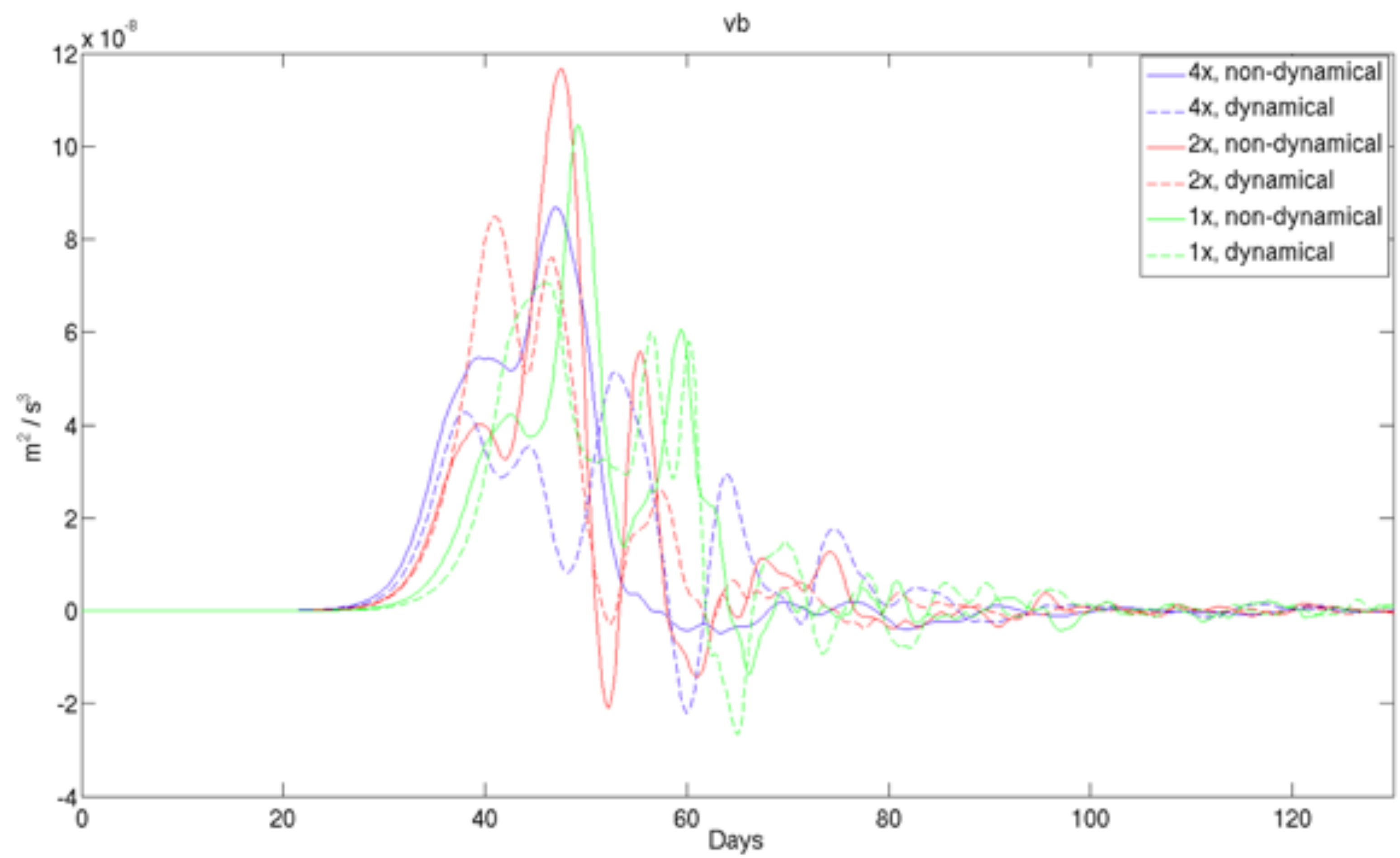
3) Effects of choice *not limited* to small scales, slope in Smag. is too steep across whole range!

Fluxes:
Horizontal Buoyancy
<vb>

Parameterized:

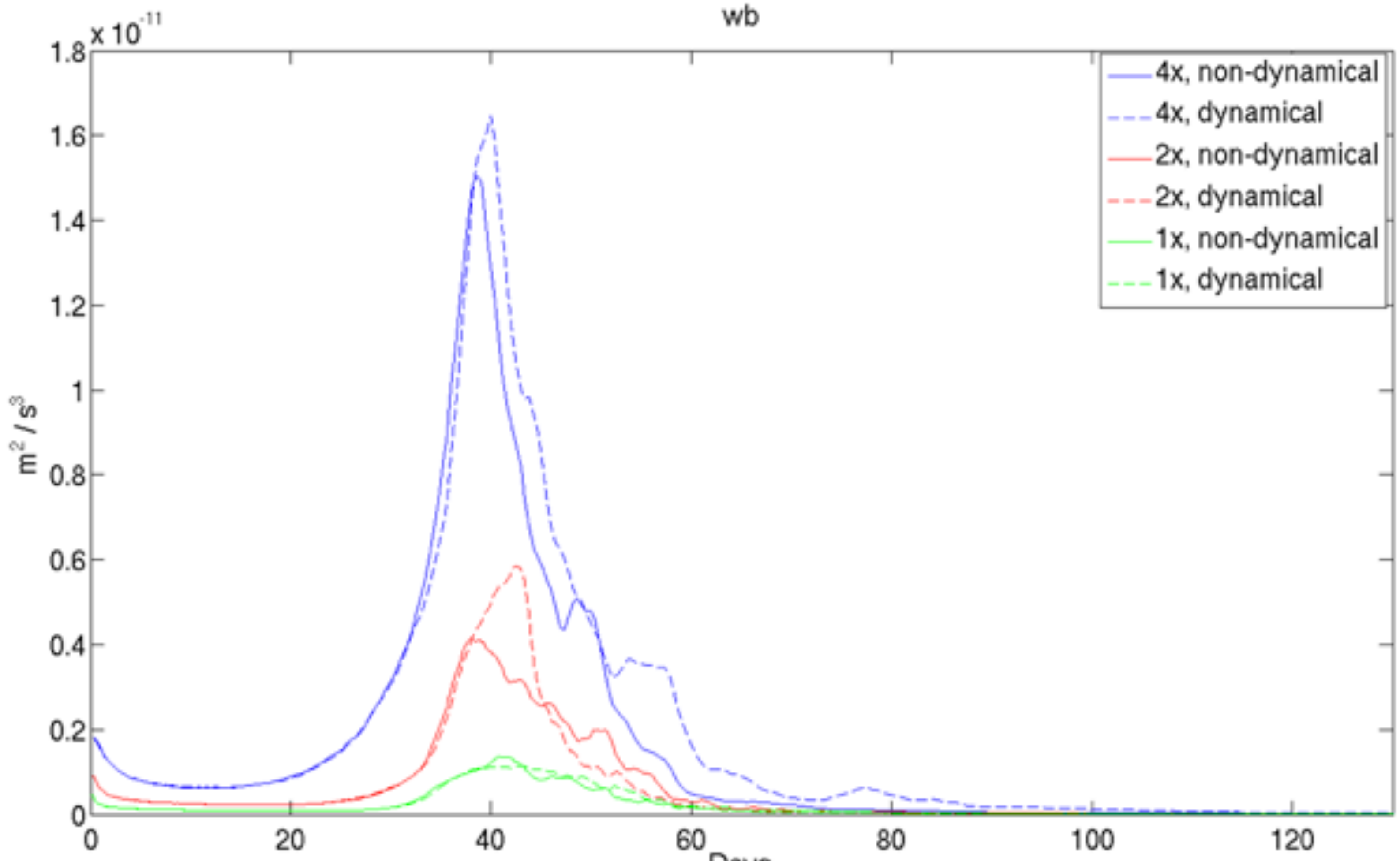


Total:

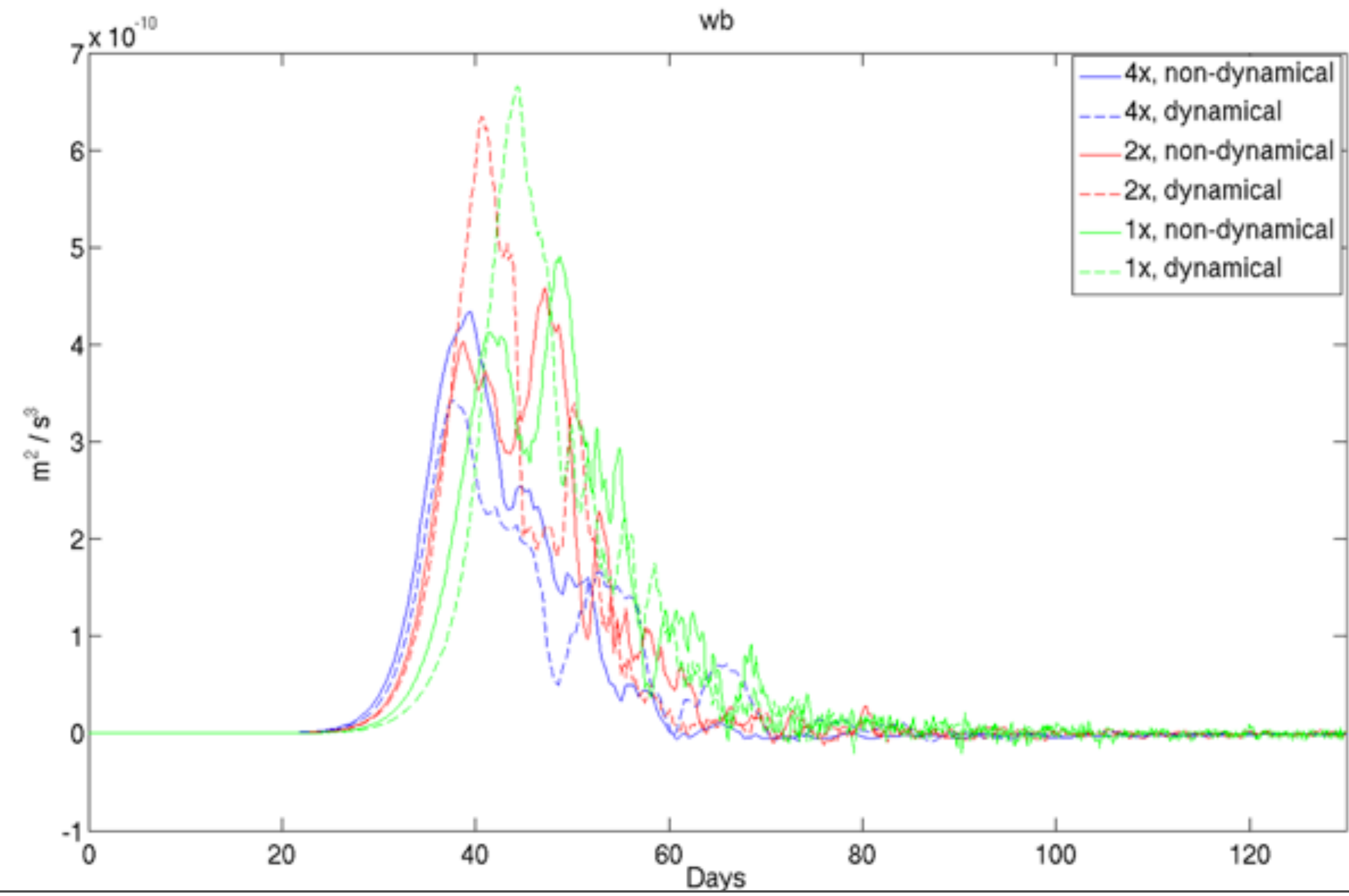


Fluxes: Vertical Buoyancy <wb>

Parameterized:

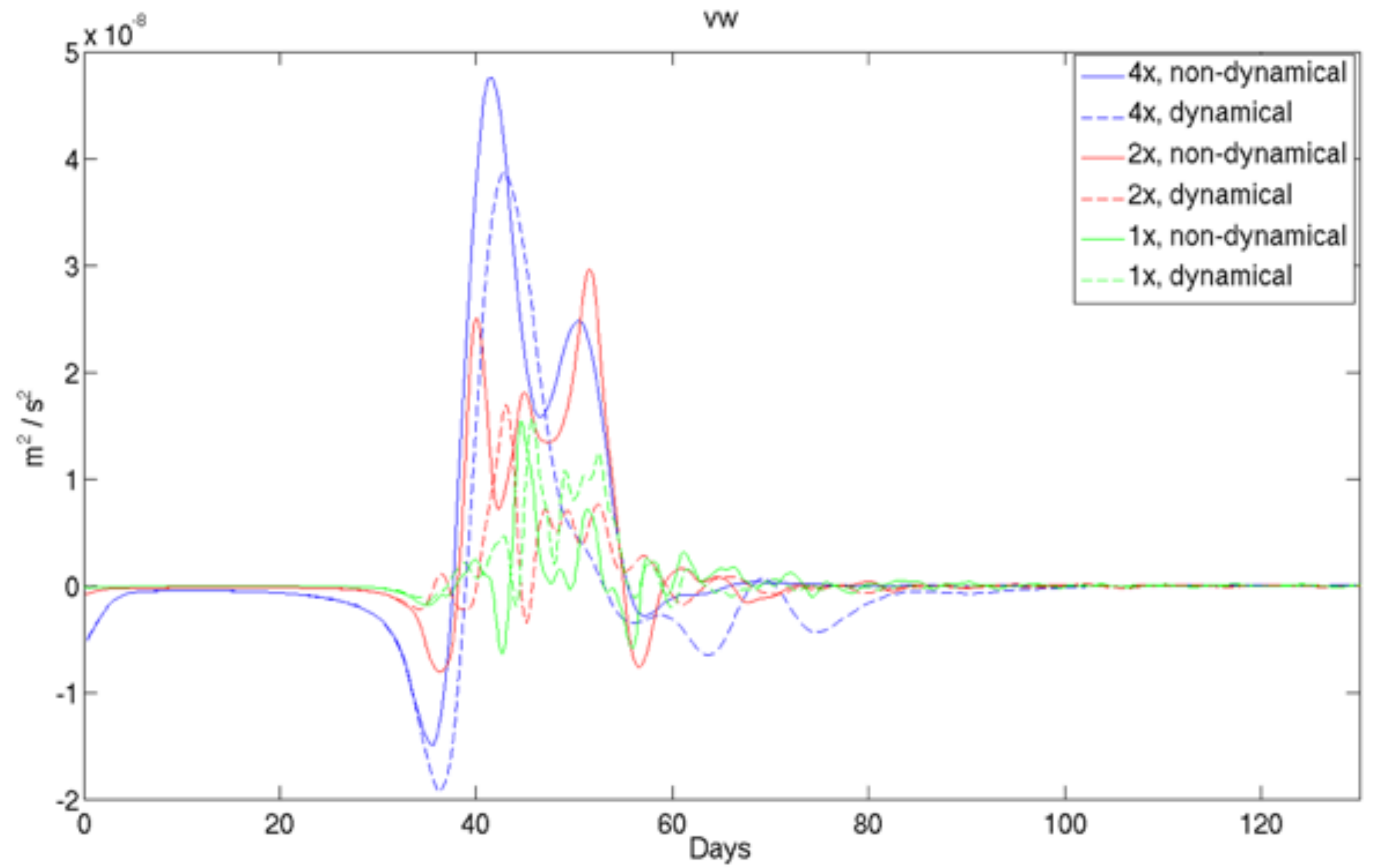


Total:

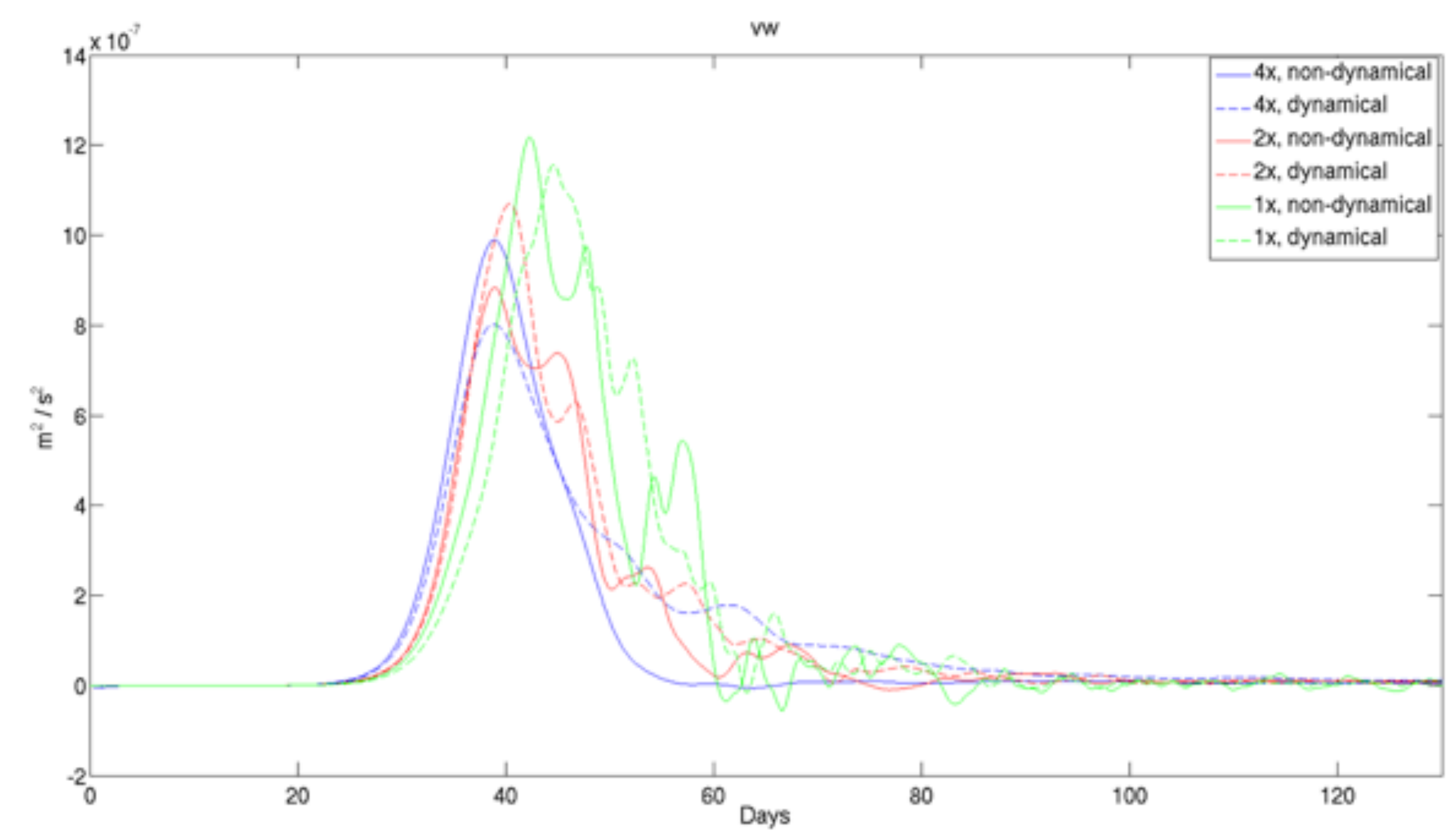


Fluxes: Momentum <vw>

Parameterized:



Total:



A Prescription for Parameterization... Needs to be checked in various regimes

- QG Leith & Potential Vorticity to generate #1 viscosity
- 2D Leith & Barotropic Vorticity to generate #2 viscosity
- SQG Leith & Surf. Buoyancy to generate #3 diffusivity
- Take max(#1, #2, #3) as viscosity, Redi diffusivity, *and* as GM transfer coeff.

  Nearly suggested by Roberts & Marshall, 98, JPO

- Note: Unlike Eddy-Free closures, e.g., Visbeck et al (97), Eddy-Rich closures take advantage of resolved eddies & instabilities, only need a boost from eddy-permitting to eddy-resolving (and for numerical stability)