

READY TO
RESOLVE:
SUBGRID

PARAMETERIZATION

FOR
TOMORROW'S
CLIMATE
MODELS

Baylor Fox-Kemper
Brown University



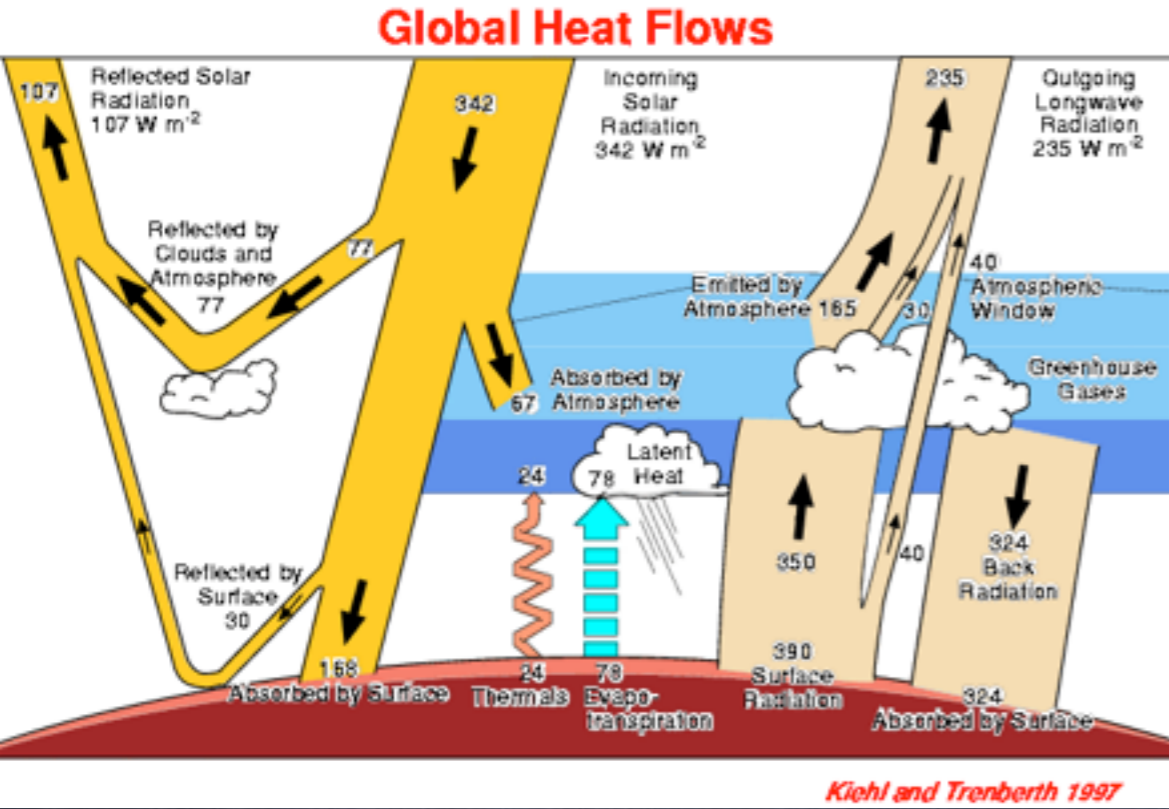
contributions from Scott Bachman (DAMTP),
Dimitris Menemenlis (JPL), MITgcm Group

AGU Ocean Sciences Meeting

Feb 25, 2014, 09:15-09:30, Session #:010

Sponsors: NSF 1350795, 0825614

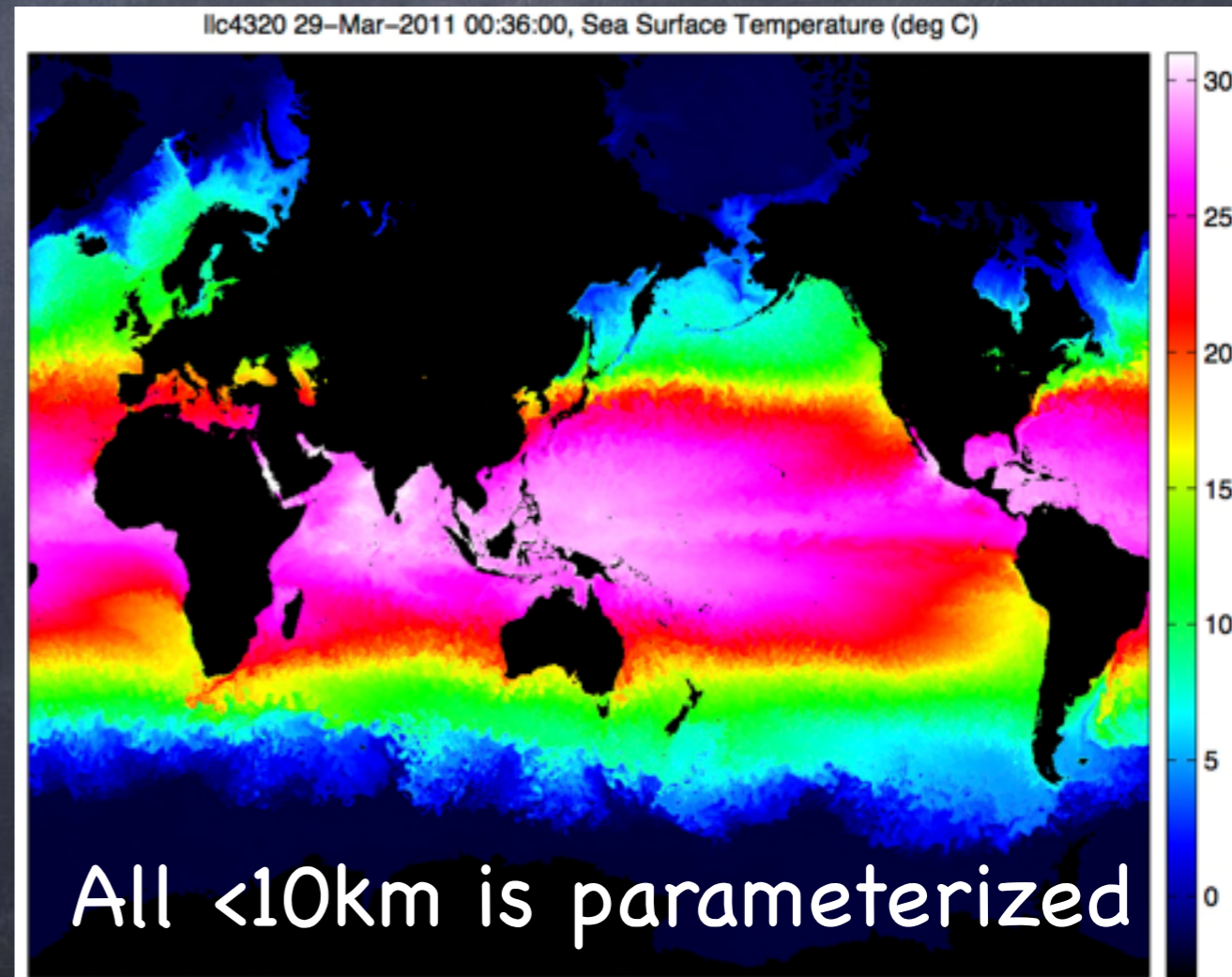
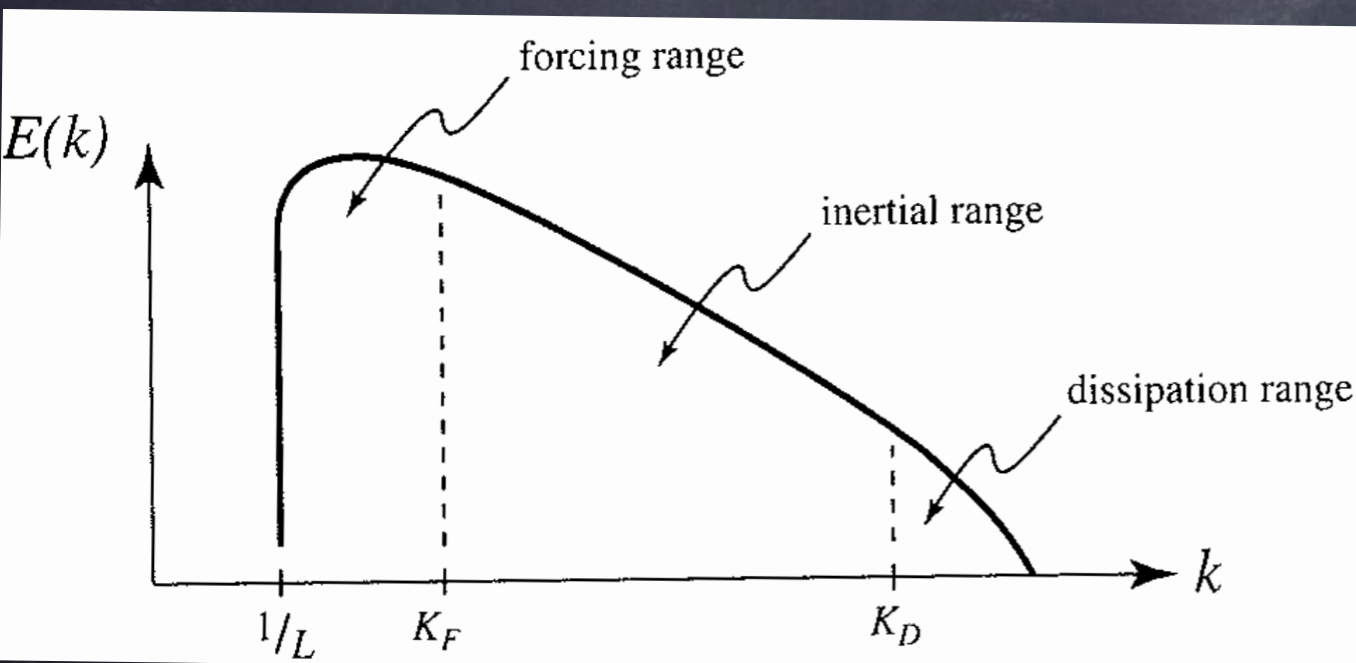
Idea: Finally, computers are fast enough that we can resolve eddies in fully coupled climate simulations, but our subgrid models are suspect or inappropriate. What to do?



The Earth's Climate System is forced by the Sun on a global scale ($24,000\text{km}$)

Next-gen. ocean climate models simulate globe to 10km : Mesoscale Ocean Large Eddy Simulations (MOLES)

Turbulence cascades to scales about 10 billion times smaller





Key Concept for

Mesoscale Ocean Large Eddy Simulations (MOLES): Gridscale Nondimensional Parameters

Gridscale Reynolds¹:

$$Re^* = \frac{U^* \Delta x}{\nu^*}$$

Gridscale Péclet¹:

$$Pe^* = \frac{U^* \Delta x}{\kappa^*}$$

Gridscale Rossby:

$$Ro^* = \frac{U^*}{f \Delta x}$$

Gridscale Richardson:

$$Ri^* = \frac{\Delta b^{*2} \Delta z}{\Delta U^{*2}}$$

Gridscale Burger:

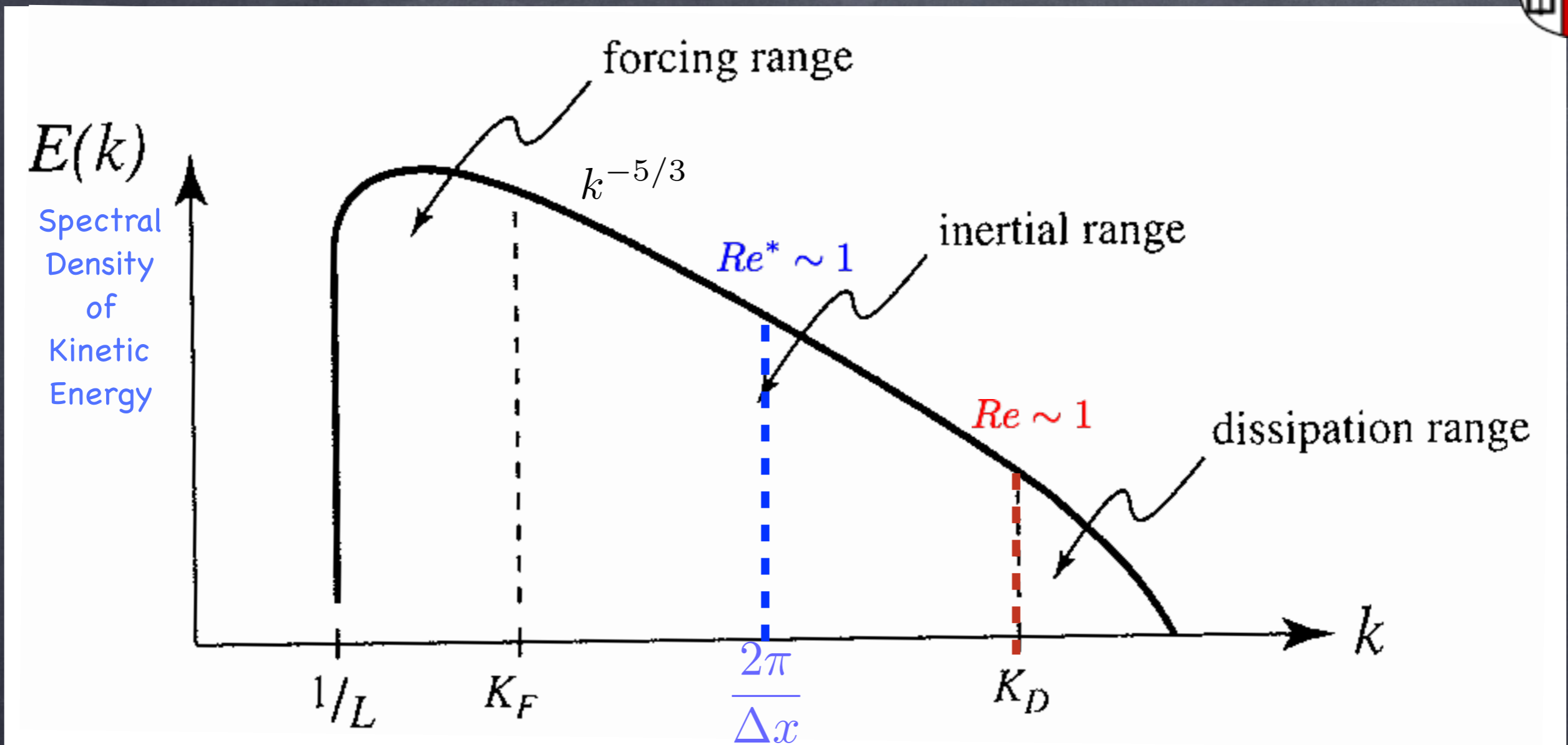
$$Bu^* = \frac{N^{*2} \Delta z^2}{f^2 \Delta x^2} \sim Ro^{*2} Ri^*$$

Asterisks denote *resolved* quantities, rather than true values

¹ Gridscale Reynolds and Péclet numbers MUST be $O(1)$ for numerical stability

3D Turbulence Cascade

Kolmogorov, 1941



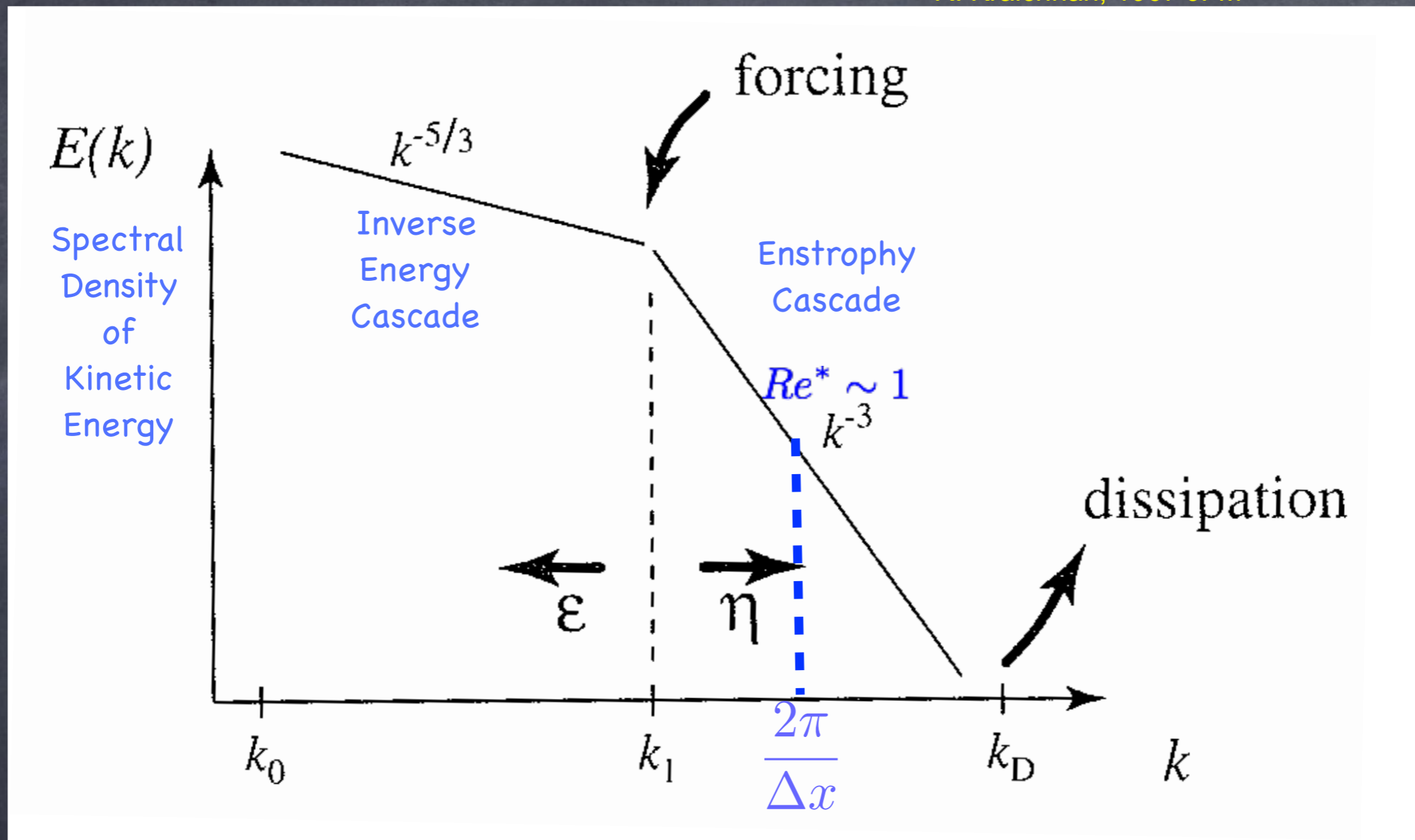
Smagorinsky (1963) Scale & Flow Aware Viscosity Scaling,
 So the Energy Cascade is Preserved, and $Re^* = \frac{U^* \Delta x}{\nu^*} = O(1)$

$$\mathbf{v}_{*h} = \left(\frac{\Upsilon_h \Delta x}{\pi} \right)^2 \sqrt{\left(\frac{\partial u_*}{\partial x} - \frac{\partial v_*}{\partial y} \right)^2 + \left(\frac{\partial u_*}{\partial y} + \frac{\partial v_*}{\partial x} \right)^2}$$

$$\Upsilon_h \approx 1$$

2D Turbulence Differs

R. Kraichnan, 1967 JFM



Leith (1996) Devises Viscosity Scaling,
 So that the Enstrophy Cascade is preserved, and $Re^* = \frac{U^* \Delta x}{\nu^*} = O(1)$

$$\nu_{2d}^* = \left(\frac{\Lambda_{2d} \Delta x}{\pi} \right)^3 |\nabla_h q_{2d}^*|$$

$$q_{2d}^* = \frac{\partial u^*}{\partial y} - \frac{\partial v^*}{\partial x}$$

F-K & Menemenlis (08): Revise Leith viscosity to quasi-2d, by damping diverging, vorticity-free, modes, too.

$$\mathbf{v}_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\underbrace{\Lambda^6 |\nabla_h q_{2d}|^2}_{\text{Leith}} + \underbrace{\Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2}_{\text{Plus}}}$$

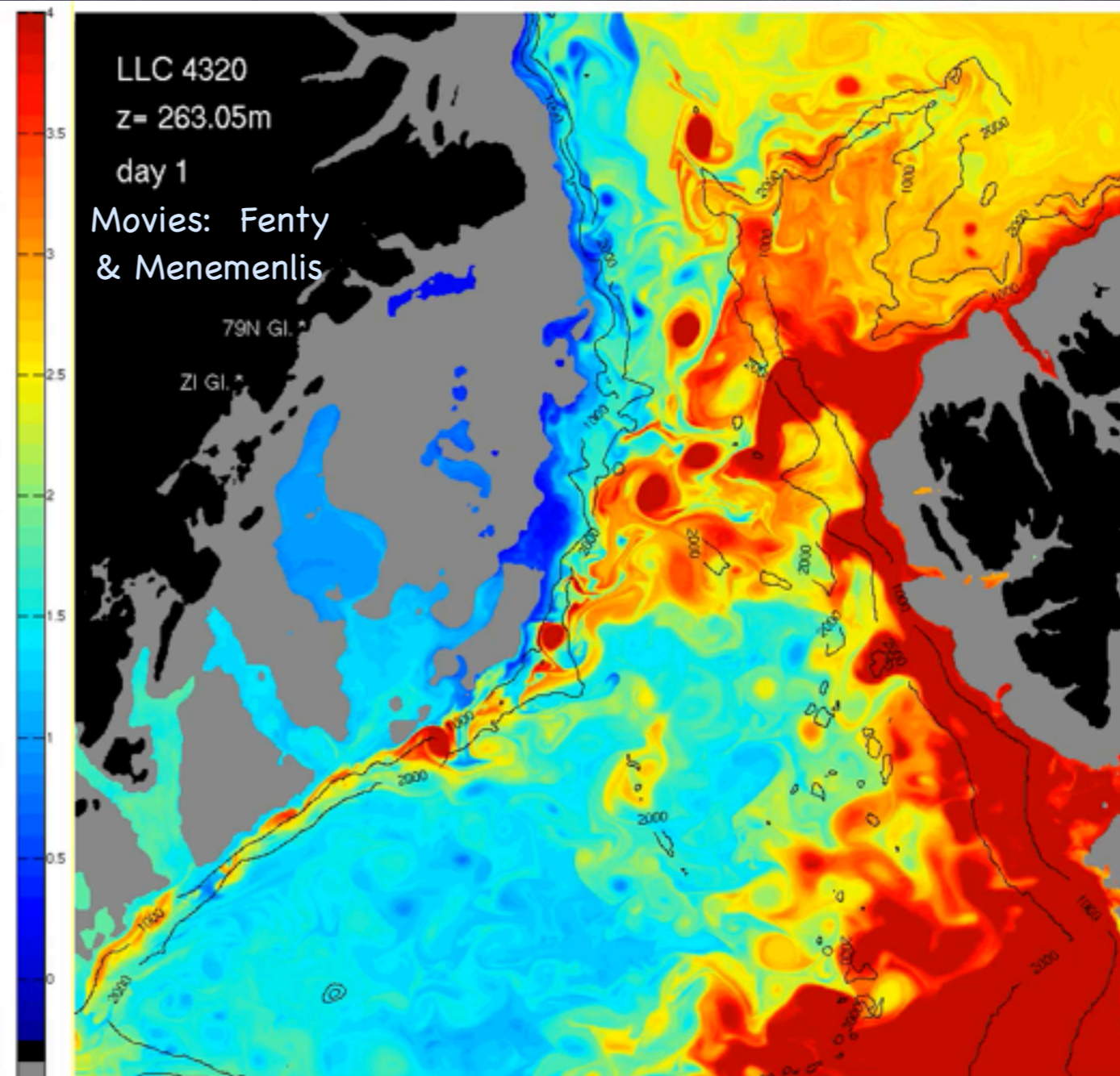
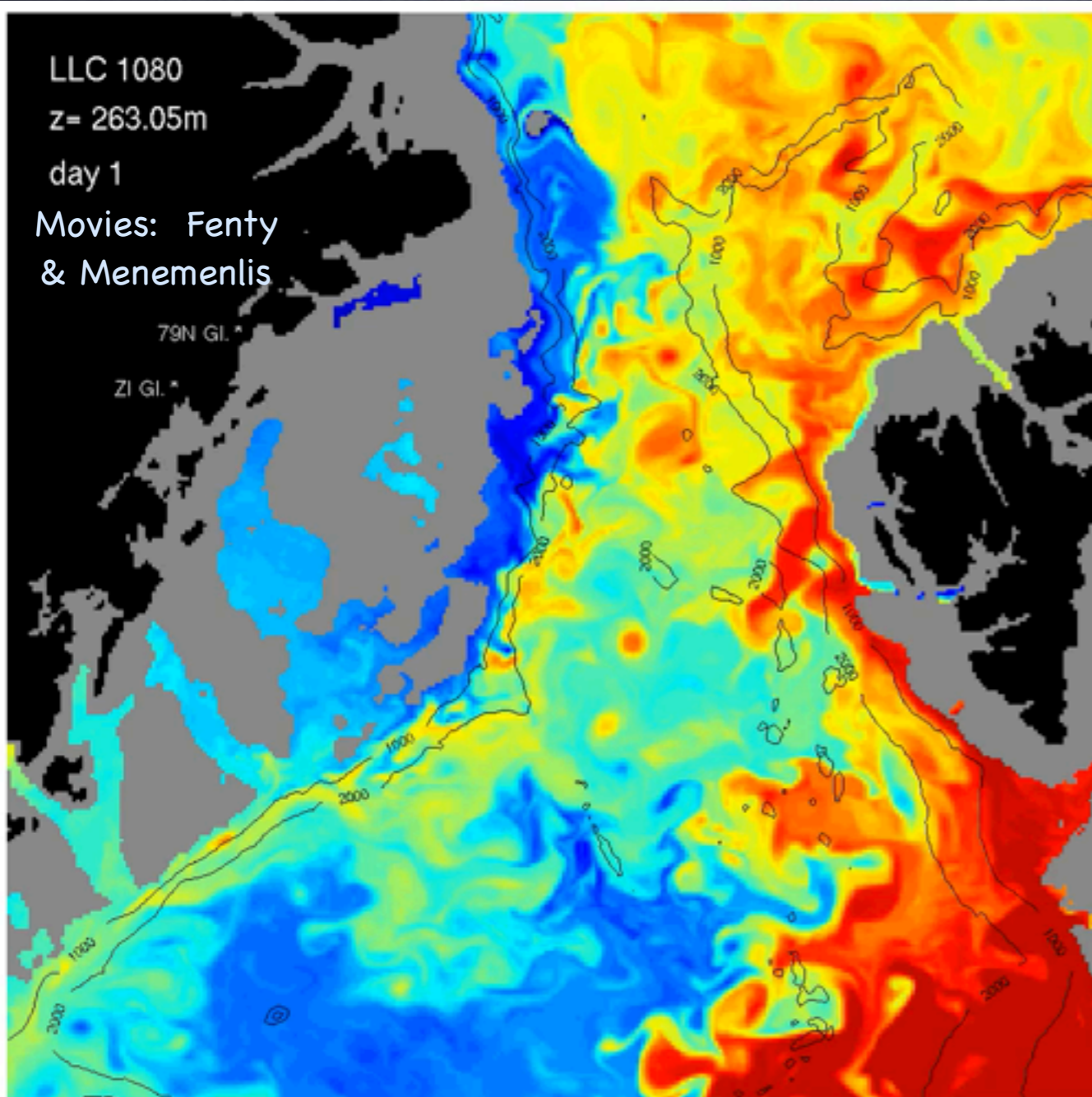


Leith-Plus Parameterization in the high-resolution ECCO runs proves stability and plug-&-play viscosity to very high resolutions without retuning:

1/12 degree

Fram Strait, Temperature at 263m

1/48 degree

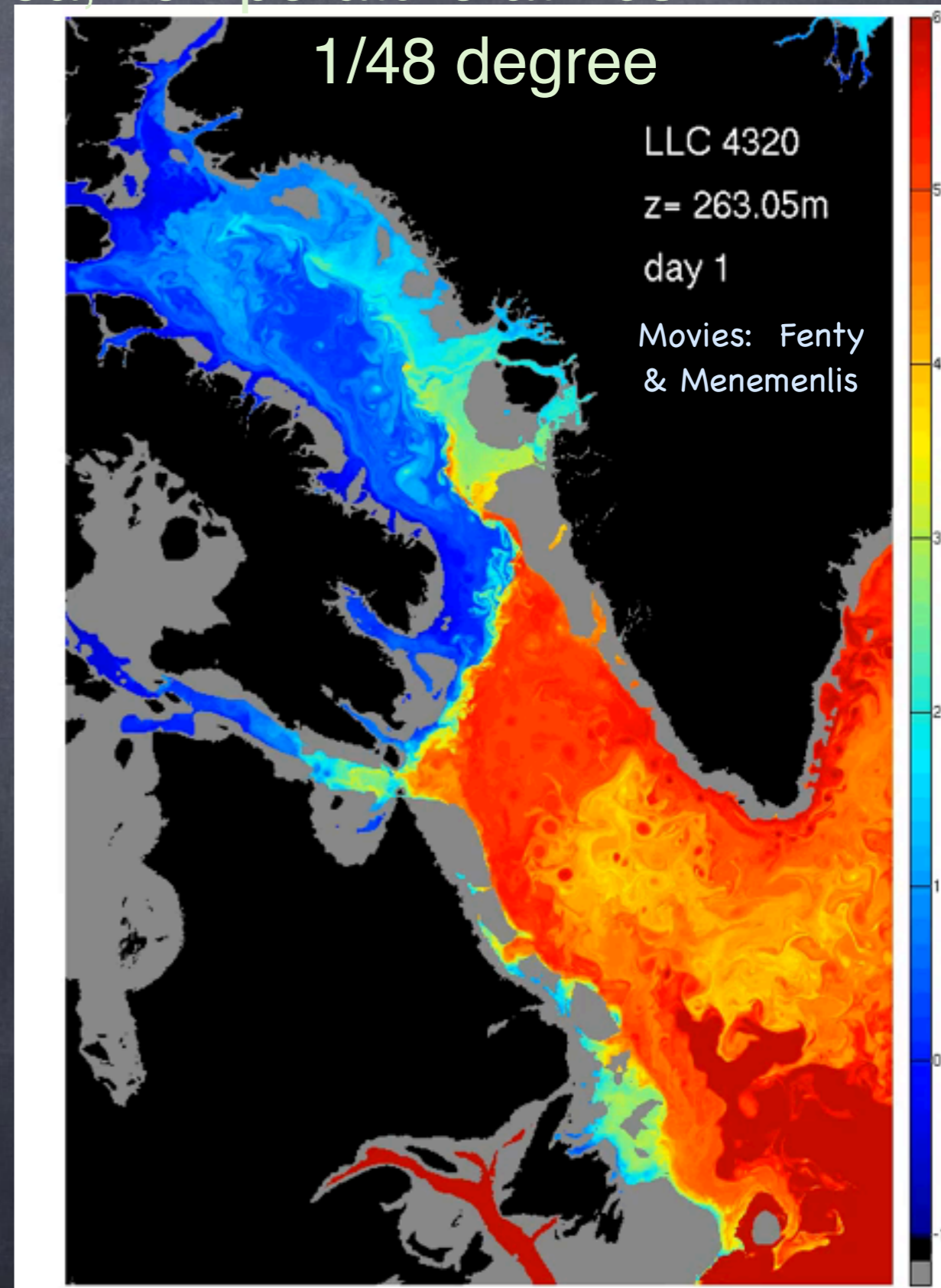
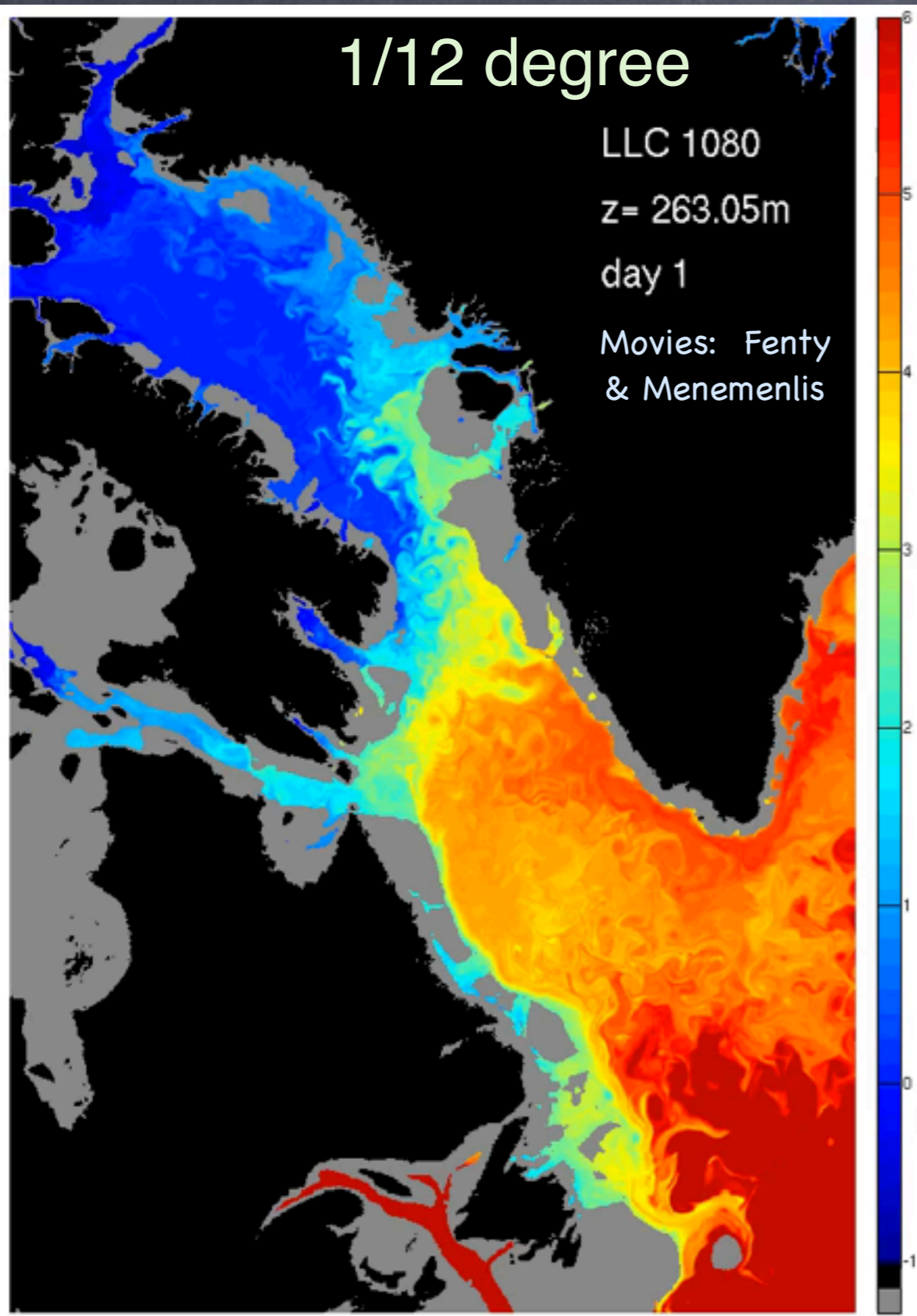


Leith-Plus Parameterization in the high-resolution ECCO runs proves stability and plug-&-play viscosity to very high resolutions without retuning:

$$\mathbf{v}_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\underbrace{\Lambda^6 |\nabla_h q_{2d}|^2}_{\text{Leith}} + \underbrace{\Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2}_{\text{Plus}}}$$



Lab. Sea, Temperature at 263m



2d (SWE) test of MOLES Subgrid models

Pietarila Graham &
Ringler, 2013

- Harmonic/Biharmonic/Numerical
 - Many. Often not scale- or flow-aware
 - Griffies & Hallberg, 2000, is one aware example
- Fox-Kemper & Menemenlis, 2008. ECCO2.
- Chen, Q., Gunzburger, M., Ringler, T., 2011
 - Anticipated Potential Vorticity of Sadourny
- San, Staples, Iliescu (2011, 2013)
 - Approximate Deconvolution Method
- Stochastic & Statistical Parameterizations

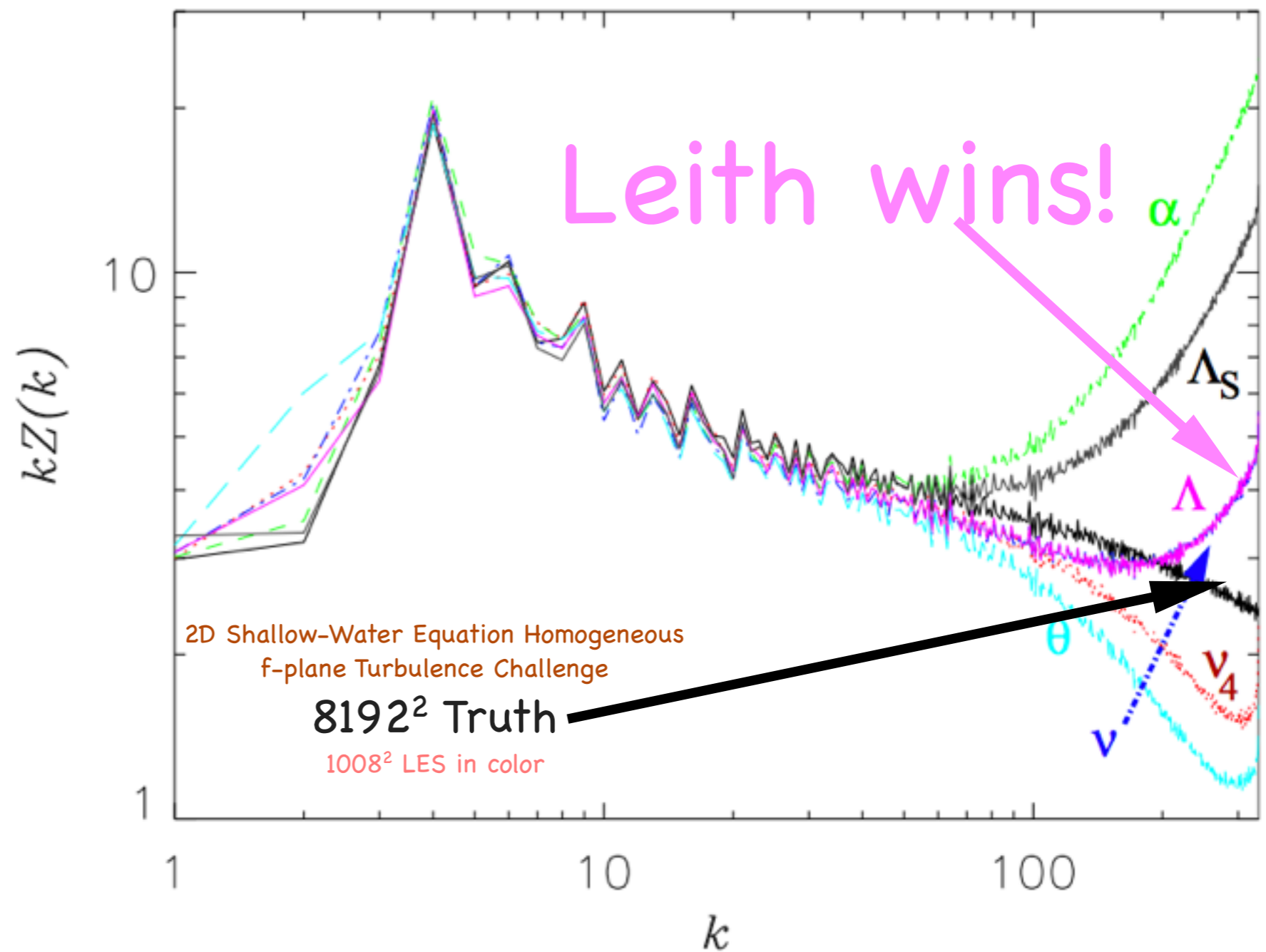


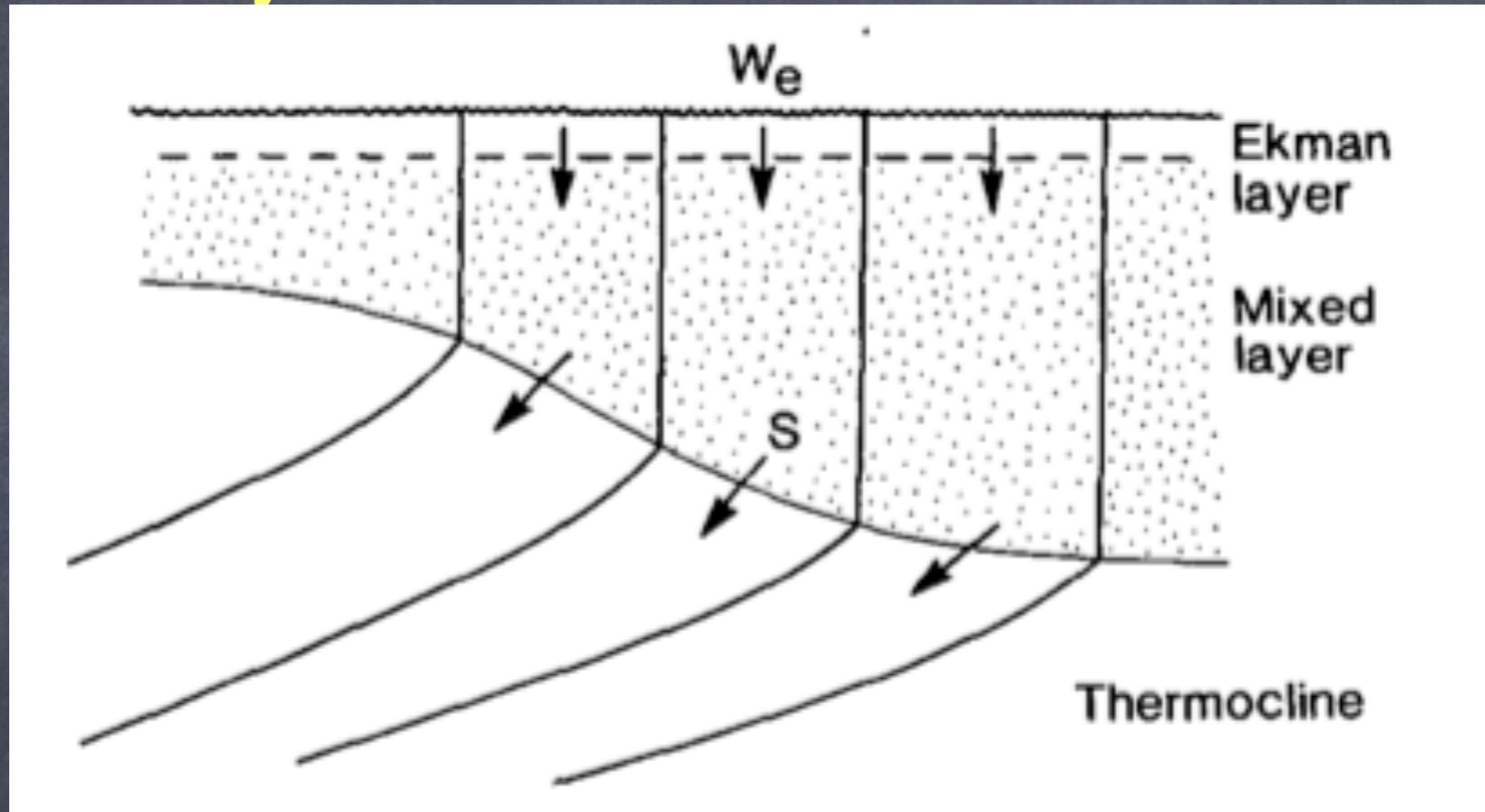
Fig. 21. Enstrophy spectra for benchmark (solid black), hyper-viscous

In the Graham & Ringler comparison,
Leith wins!

over tuned harmonic, tuned biharmonic, Smagorinsky,
LANS-alpha, & Anticipated PV



Is 2D Turbulence a good proxy for neutral flow?



Yes:

No:

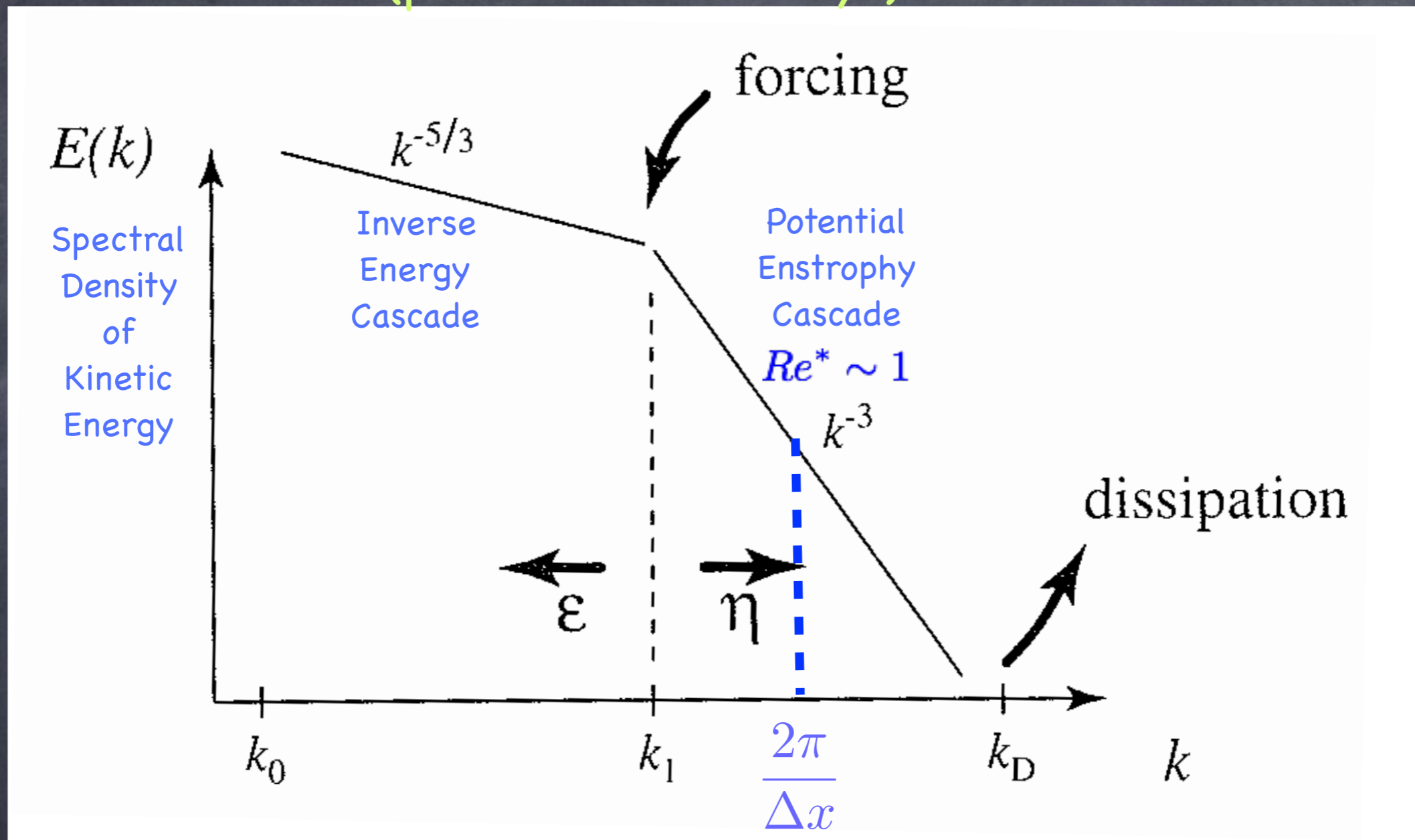
Nurser & Marshall, 1991 JPO

- For a few eddy time-scales QG & 2D AGREE (Bracco et al. '04)
- Barotropic Flow & Stratified Turbulence ($Ro \gg 1$, $Ri \gg 1$) are 2d analogs

- Bolus Fluxes-- Divergent 2d flow
- Sloped, not horiz.
- Surface Effects?

QG Turbulence: Pot'l Enstrophy cascade (potential vorticity²)

J. Charney, 1971 JAS



QG Leith:

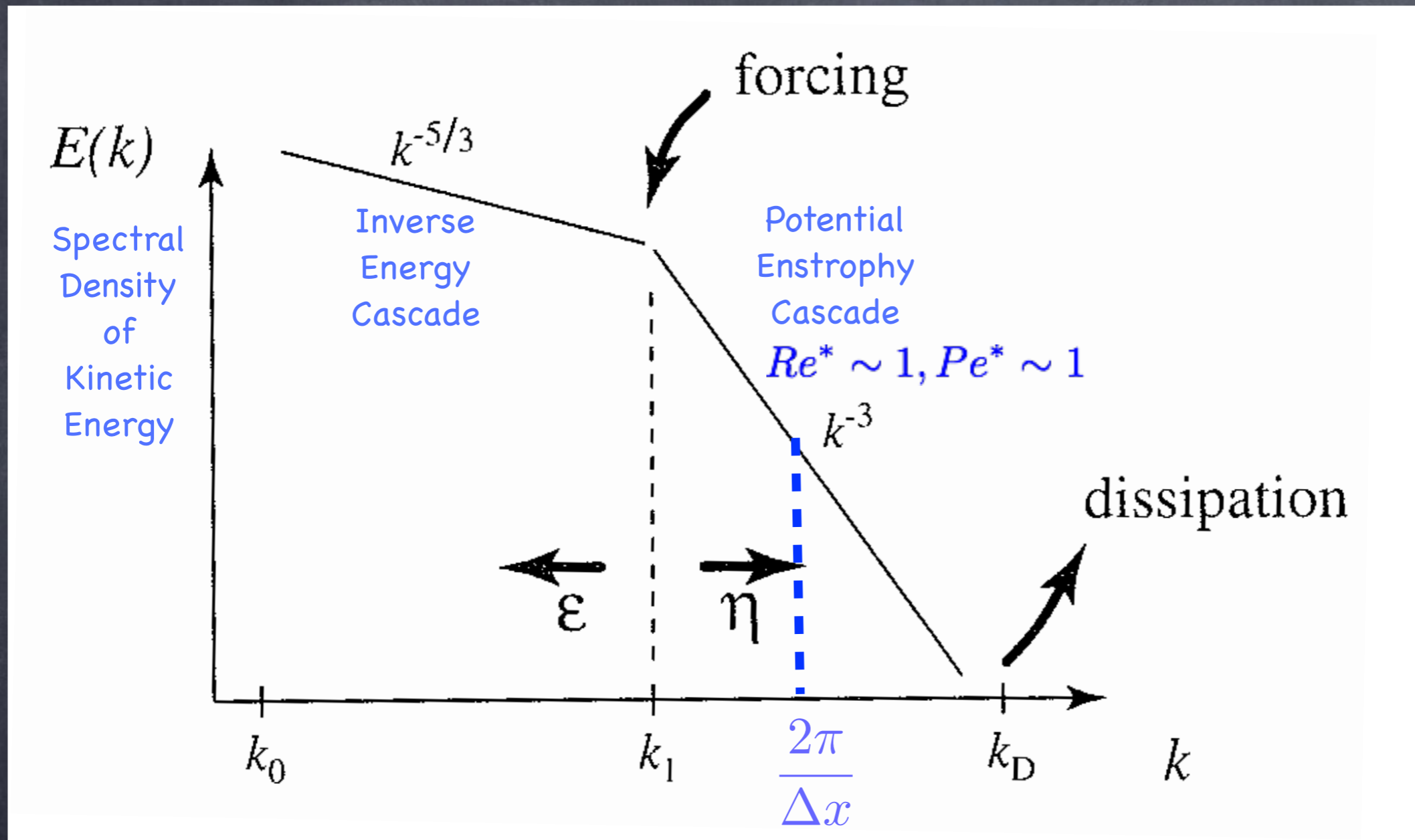
$$v_{qg} = \left(\frac{\Lambda_{qg} \Delta x}{\pi} \right)^3 |\nabla q_{qg}|$$

$$\nabla_h^2 \psi^* = q_{2d}^*$$

$$q_{qg}^* = \beta y + \nabla_h^2 \psi^* + \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^{*2}} \frac{\partial \psi^*}{\partial z} \right)$$

QG Turbulence: Pot'l Enstrophy cascade (potential vorticity²)

J. Charney, 1971 JAS



$$Re^* = \frac{U^* \Delta x}{\nu^*}$$

$$Pe^* = \frac{U^* \Delta x}{\kappa^*}$$

Consistent with QG only if scaling
applies to ALL Pot'l Enstrophy sinks—
Viscosity, Diffusivity, AND GM
Coefficient:

$$\nu_{qg} = \kappa_{Redi} = \kappa_{GM} = \left(\frac{\Lambda_{qg} \Delta x}{\pi} \right)^3 |\nabla q_{qg}|.$$



And QG pot'l enstrophy Leith is ... now working in MITgcm

- Scott Bachman (DAMTP) has implemented this QG Leith closure in the MITgcm

- Both Germano Dynamic and Fixed Coefficient

$$\Lambda_{qg} = \Lambda_{qg}(x, y, z, t) \quad \Lambda_{qg} = 1$$

$$\nu_{qg} = \left(\frac{\Lambda_{qg} \Delta x}{\pi} \right)^3 |\nabla q_{qg}| = \left(\frac{\Lambda_{qg} \Delta x}{\pi} \right)^3 \left| \nabla_h \left[\beta y + \nabla_h^2 \psi + \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2} \frac{\partial \psi}{\partial z} \right) \right] \right|.$$

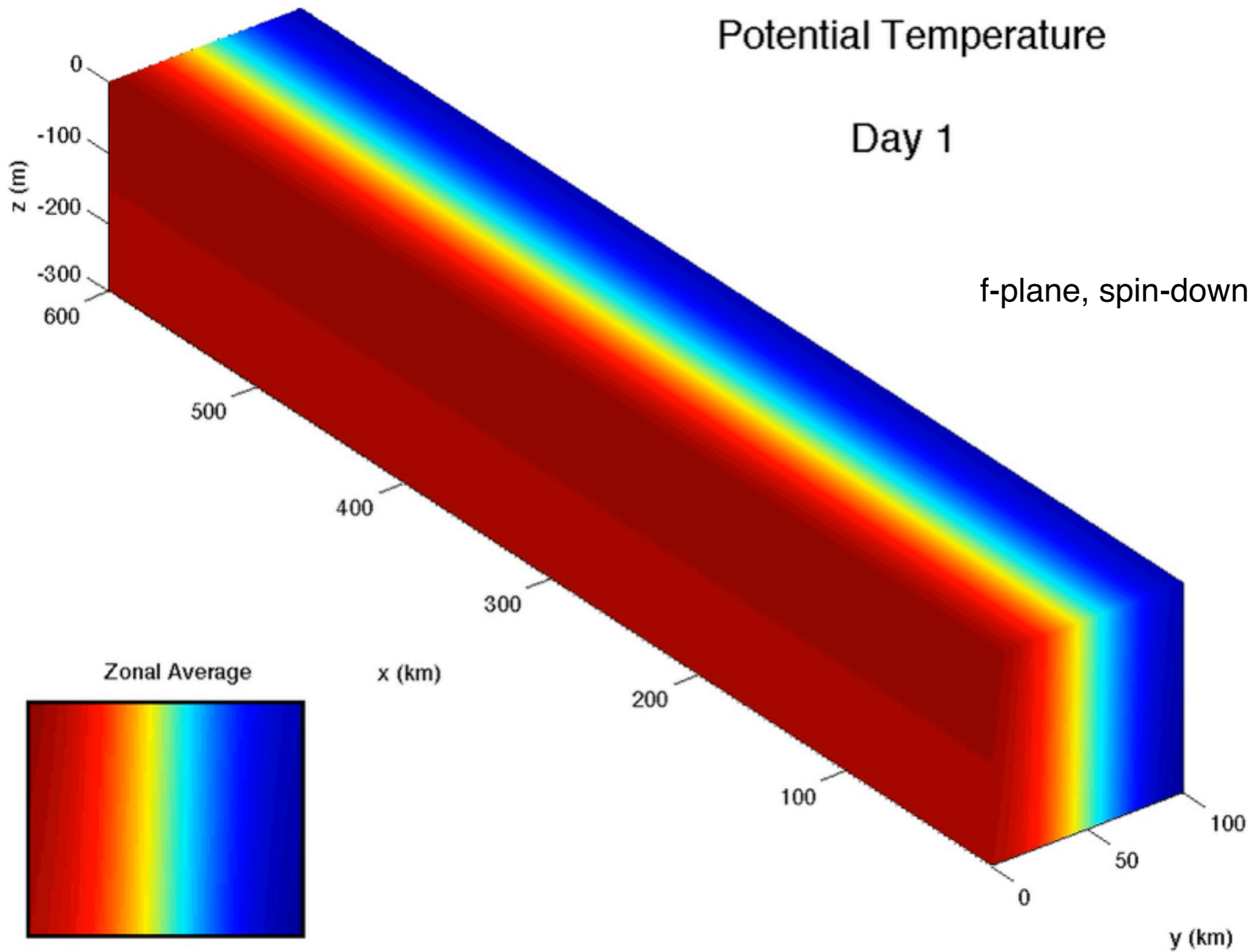
$$\nu_{qg} = \kappa_{Redi} = \kappa_{GM} = \left(\frac{\Lambda_{qg} \Delta x}{\pi} \right)^3 |\nabla q_{qg}|.$$

Movie: S. Bachman

Potential Temperature

Day 1

f-plane, spin-down



This Slide & Movies:
S. Bachman

S. Bachman and
B. Fox-Kemper.
Eddy
parameterization
challenge suite. I:
Eady spindown.
Ocean Modelling,
64:12-28, 2013.

Does it work?

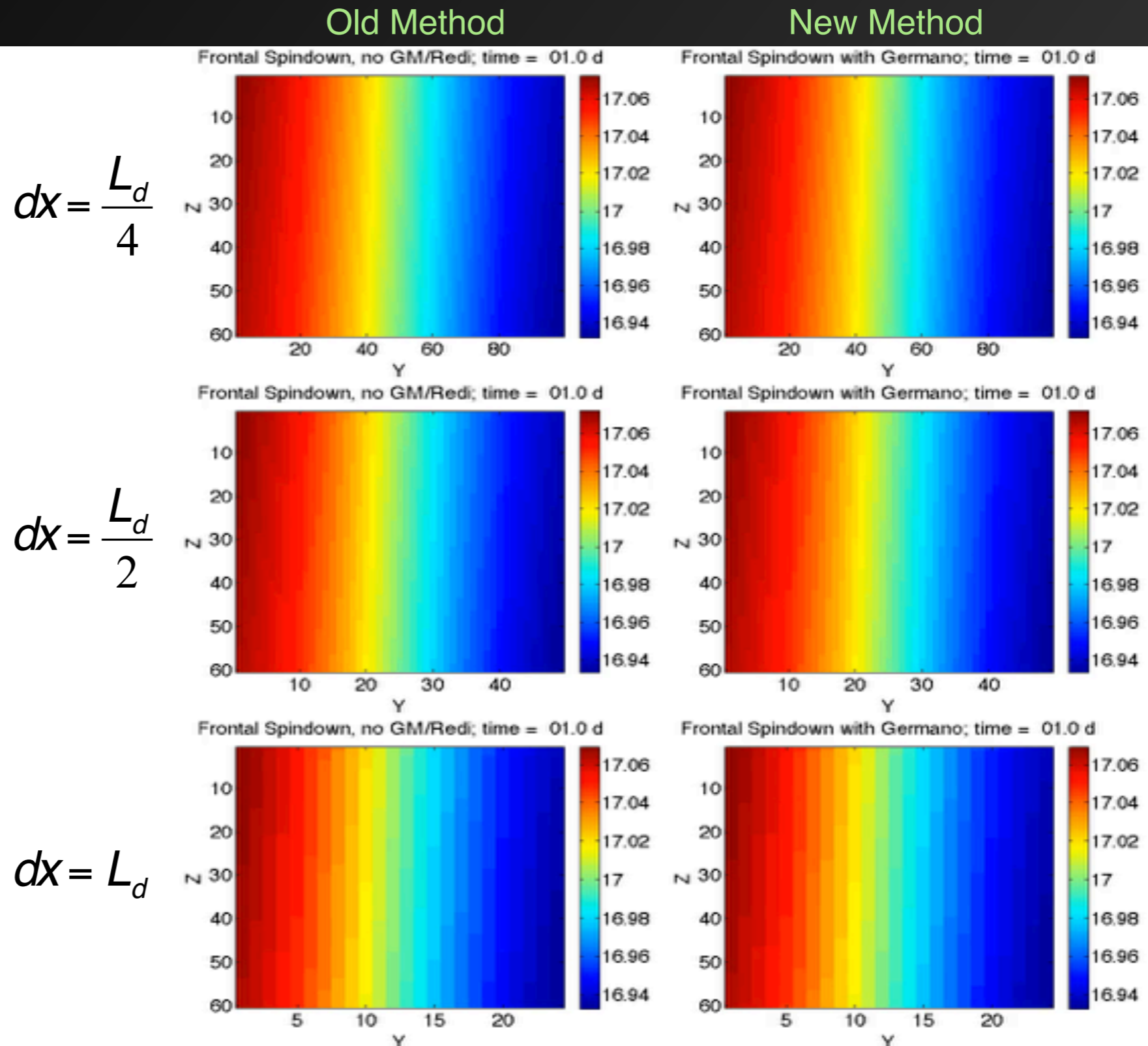
We'll test this in a channel model, using three different resolutions:

The fastest growing mode is better resolved the higher the resolution, so the spindown will be slower for the coarser runs.

But the QG dissipation / diffusivity scheme is able to compensate!

Old Method=Smagorinsky viscosity with only implicit numerical diffusivity, no GM

New Method=QG Leith



Do the spectra behave?

Old Method=Smagorinsky
viscosity with only implicit
numerical diffusivity, no GM

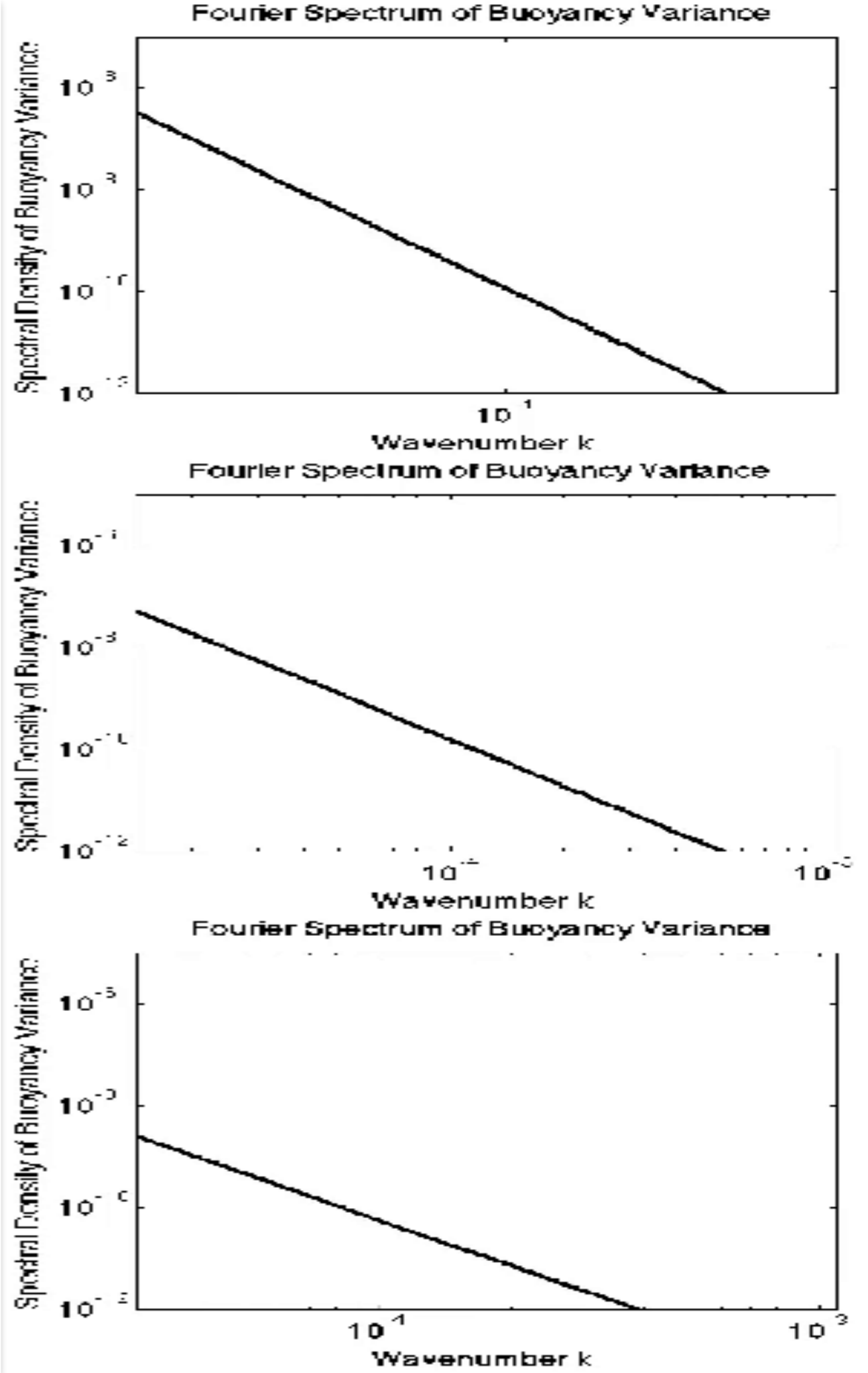
Old Method in Red

New Method=QG Leith
Blue – dynamic QG Leith

$$dx = \frac{L_d}{4}$$

$$dx = \frac{L_d}{2}$$

$$dx = L_d$$



But...we need to be careful of when QG isn't appropriate:

- Stretching term can be too large when unstratified—use gridscale Burger number to determine when:

$$Bu^* = \frac{N^{*2} \Delta z^2}{f^2 \Delta x^2} \sim Ro^{*2} Ri^*$$

$$\frac{\nu_{qg}^*}{\nu_{2d}^*} \approx \frac{|\nabla_h q_{qg}^*|}{|\nabla_h q_{2d}^*|} \sim 1 + Bu^* \sim 1 + Ro^{*2} Ri^*$$

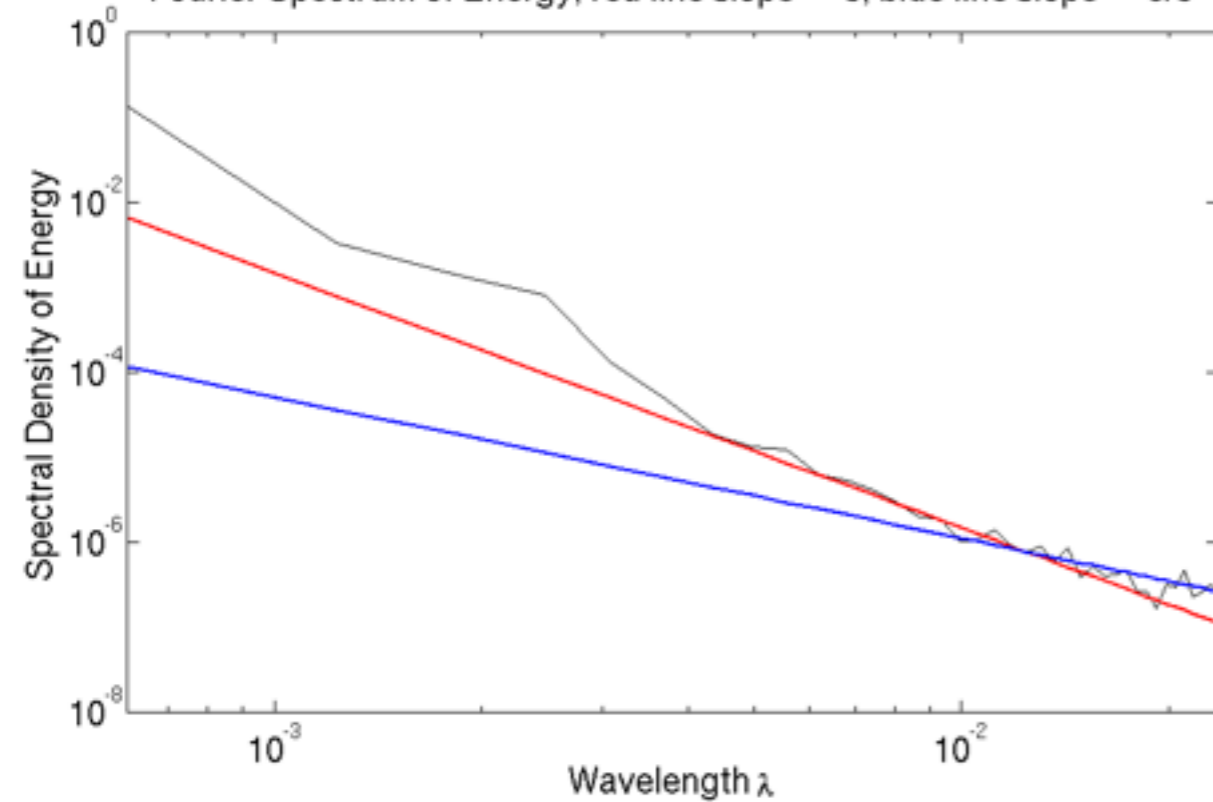
- Surface QG has different spectral characteristics—we have a theory, but simultaneous implementation unclear

Conclusions

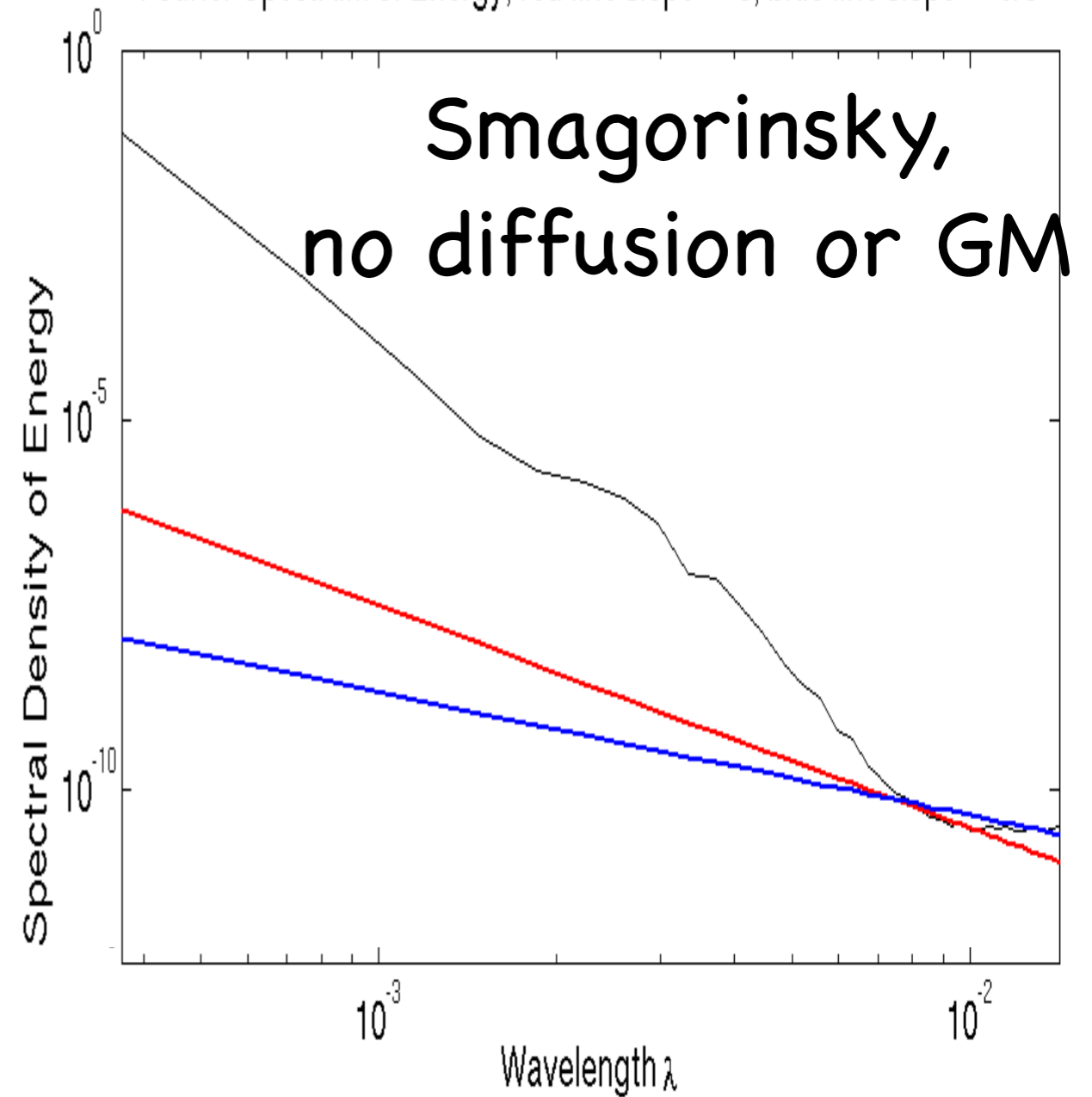
Promising method: Realistic tests next!

- QG Leith=viscosity, Redi diffusivity, *and* GM transfer coeff.
Nearly as suggested by Roberts & Marshall, 98, JPO
- Ensures $O(1)$ gridscale Reynolds & Péclet
- Revert to 2D Leith when QG is inappropriate
 - QG only if gridscale Burger near 1, gridscale Richardson >1
- Our results suggest QG Leith will deliver the proven plug&play capability of LeithPlus with improved QG-based physics—Will matter most where stretching terms or APE balance are important, e.g., WBC.

Fourier Spectrum of Energy; red line slope = -3, blue line slope = -5/3



Fourier Spectrum of Energy; red line slope = -3, blue line slope = -5/3



QG Leith (Pot'l Enstrophy)

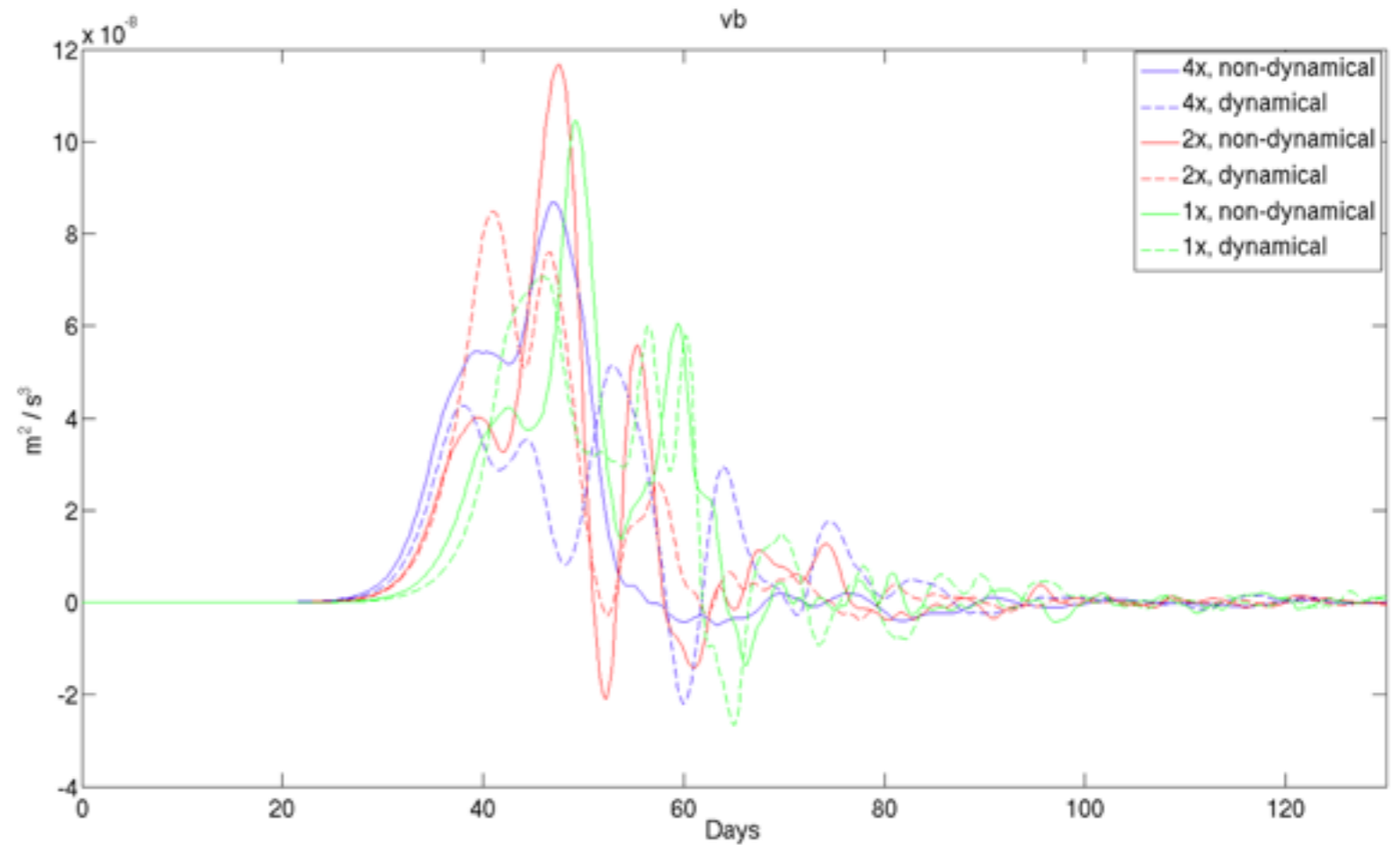
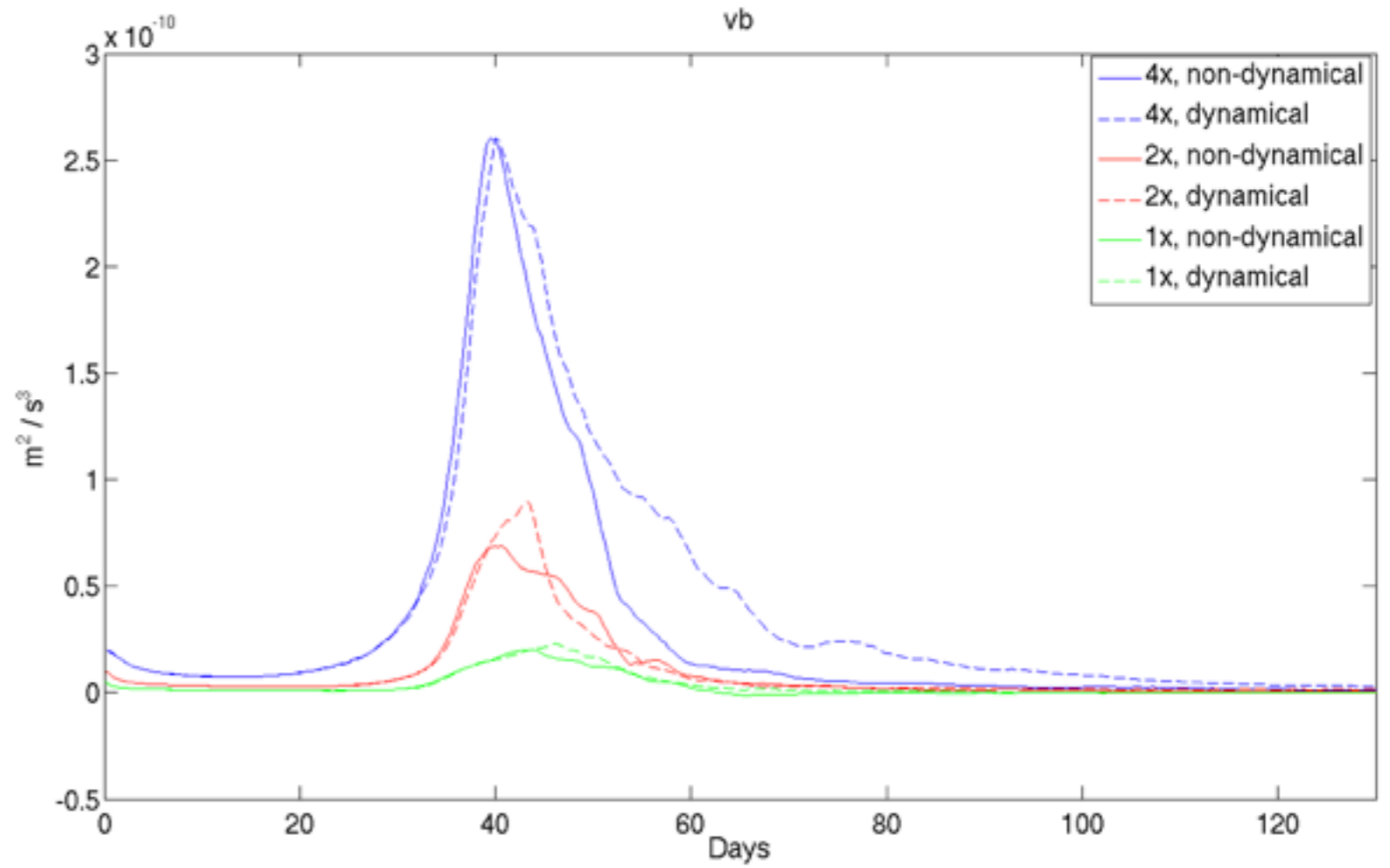
Comparing the spectrum in QG Leith against another (inappropriate) LES closure, we see:

- 1) Better adherence to expected spectrum
- 2) Less "ski jump" near gridscale
- 3) Effects of choice *not limited* to small scales, slope in Smagorinsky run is too steep across whole range!

Fluxes:
Horizontal Buoyancy
 $\langle vb \rangle$

Parameterized:

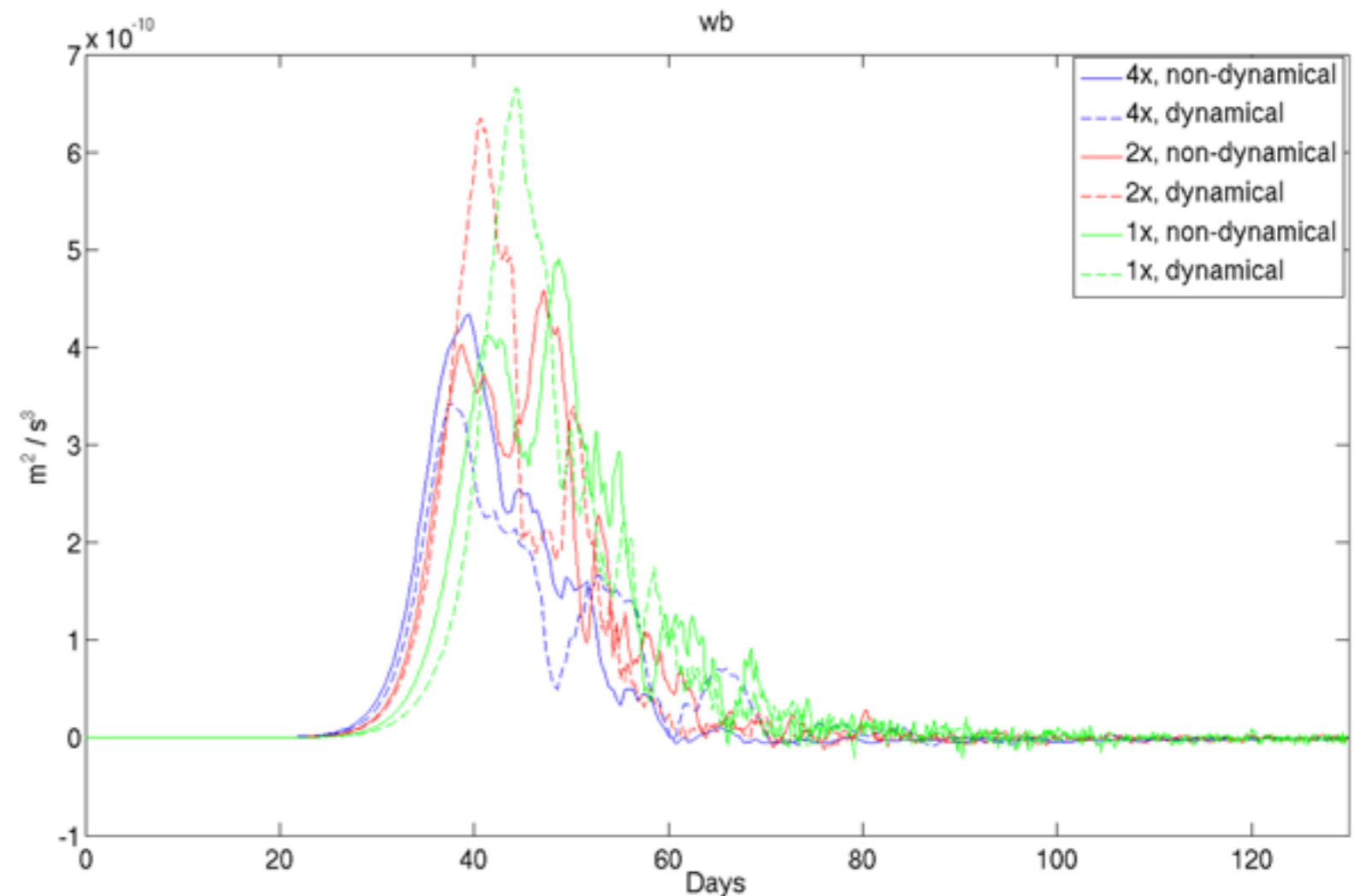
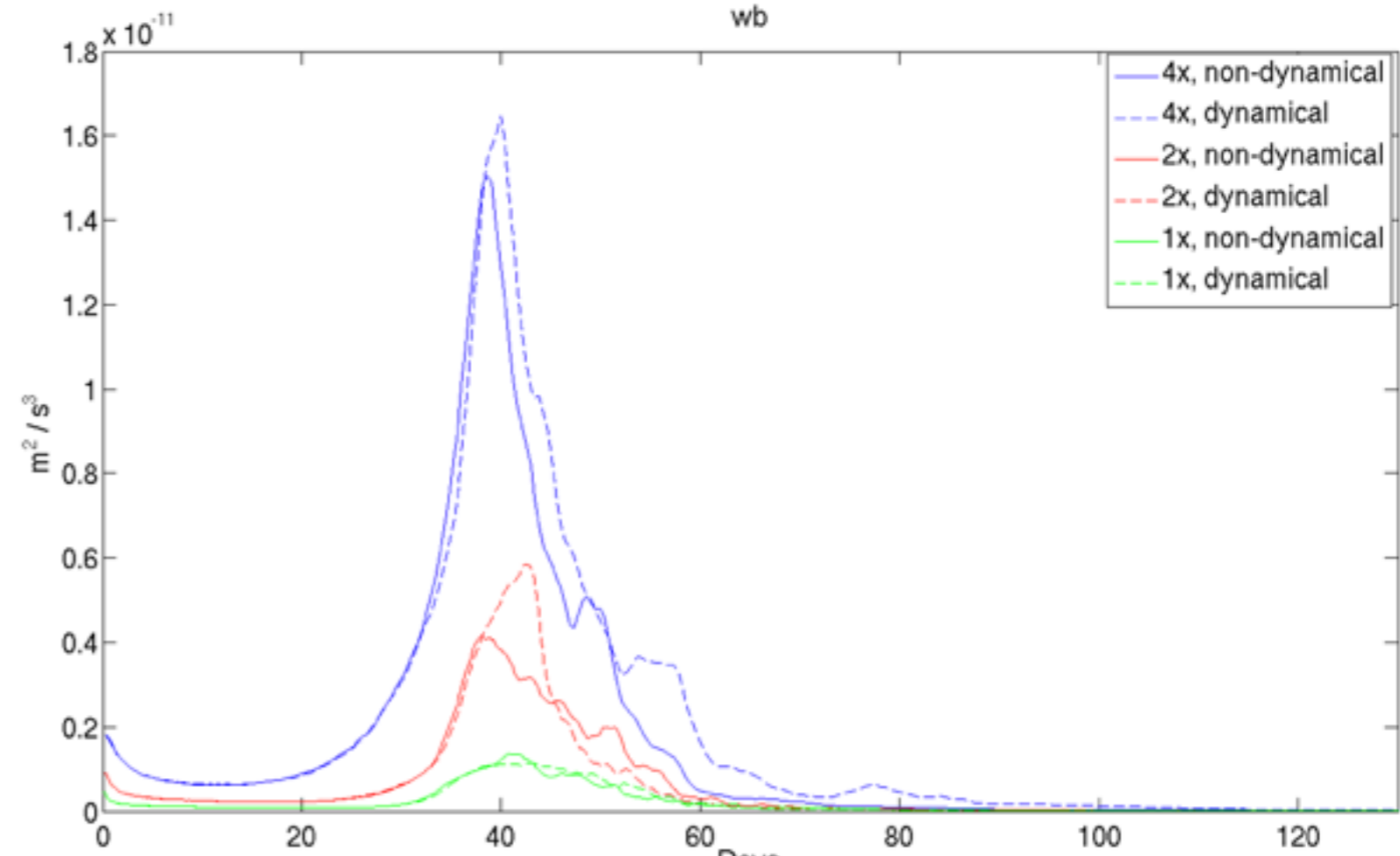
Total:



Fluxes:
Vertical Buoyancy
<wb>

Parameterized:

Total:



Fluxes:
Momentum

$\langle v w \rangle$

Parameterized:

Total:

