

# Other Submesoscale Processes: Frontogenesis & Stokes Effects

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Expanding on past work with:

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Adrean Webb (TUMST), Keith Julien (CU-APPM), Greg Chini (UNH),  
Peter Sullivan (NCAR), Mark Hemer (CSIRO)

OSMOSIS Spring 2014 Workshop, Norwich, UK

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[http://hvo.wr.usgs.gov/multimedia/archive/2007/2007\\_Jan-May.html](http://hvo.wr.usgs.gov/multimedia/archive/2007/2007_Jan-May.html)

# LES of Langmuir-Submeso Interactions?

Perform large eddy simulations (LES) of Langmuir turbulence with a submesoscale temperature front using wave-averaged equations

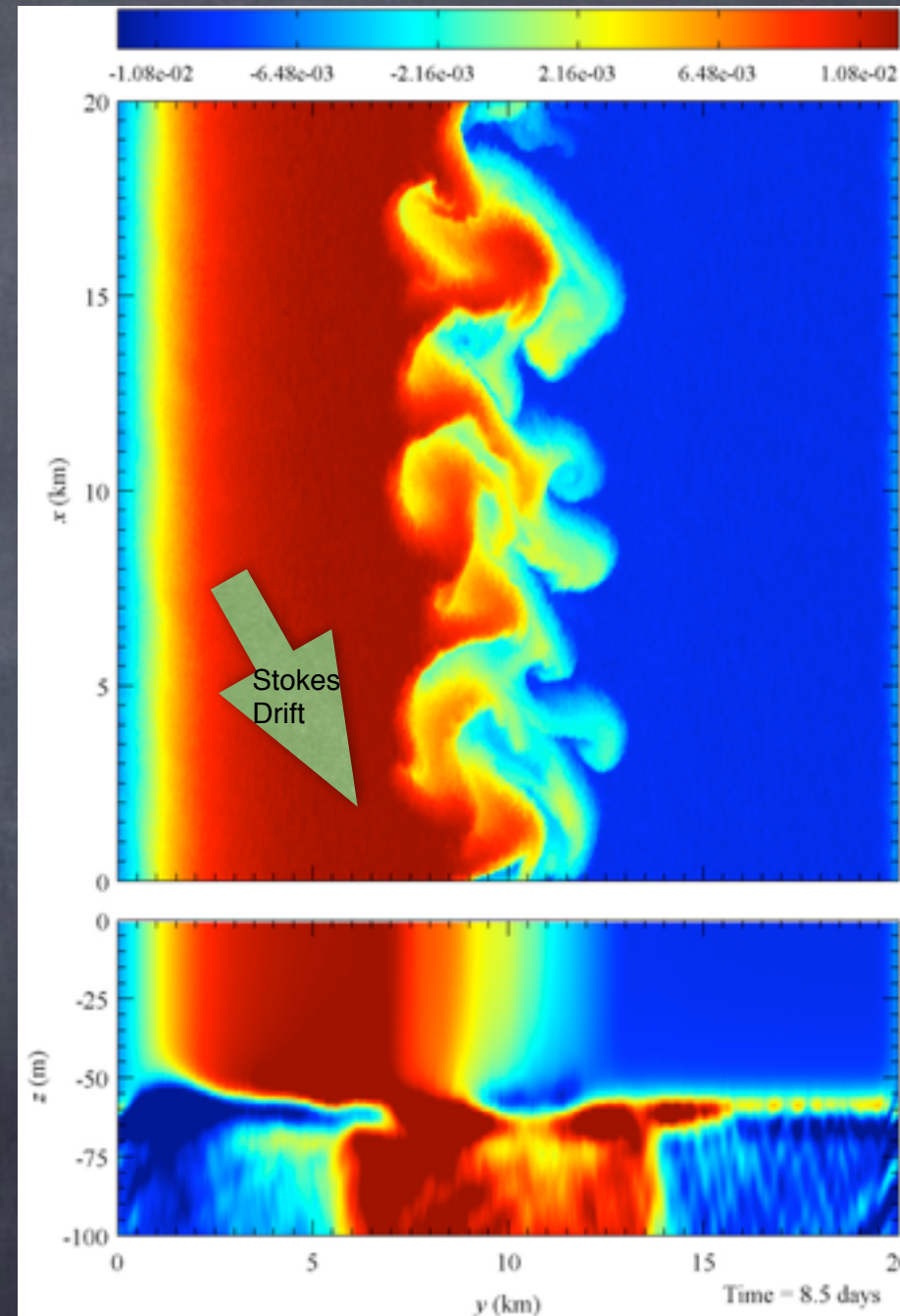
Computational parameters:

Domain size: 20km x 20km x -160m

Grid points: 4096 x 4096 x 128

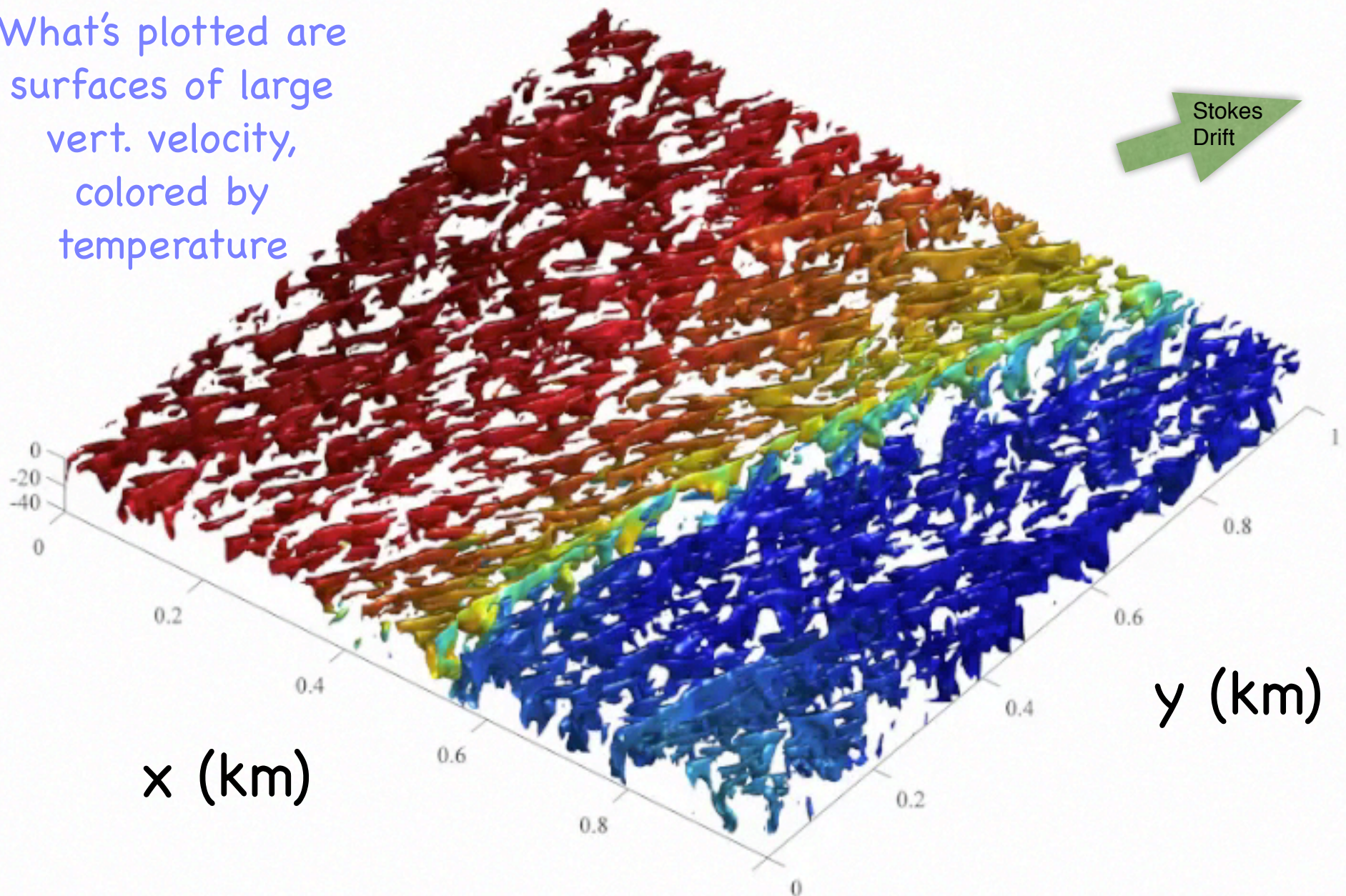
Resolution: 5m x 5m x -1.25m

1000x more gridpoints than CESM



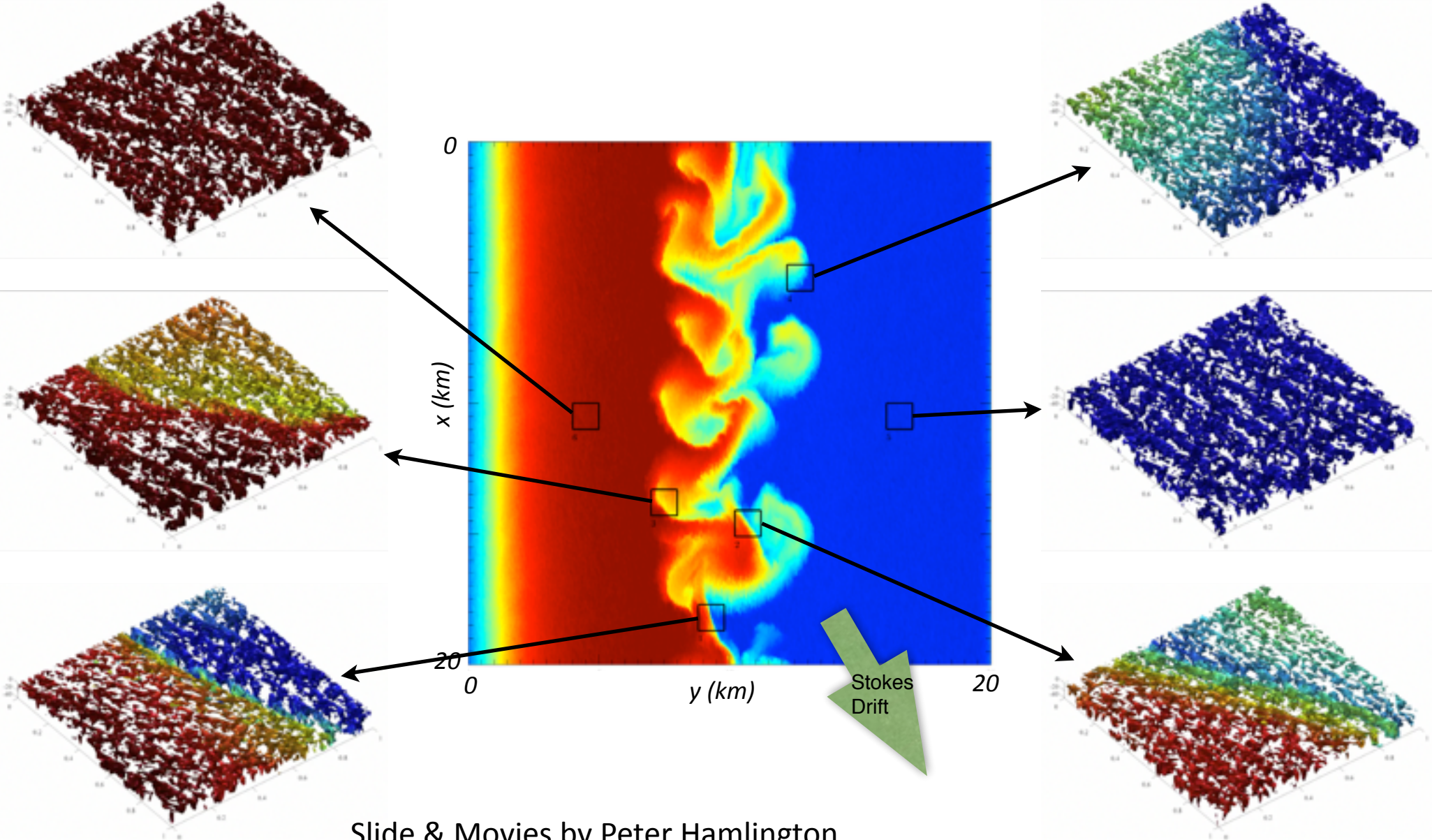
# Zoom: Submeso-Langmuir Interaction!

What's plotted are  
surfaces of large  
vert. velocity,  
colored by  
temperature



P. E. Hamlington, L. P. Van Roekel, B. Fox-Kemper, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. *Journal of Physical Oceanography*, 2014. In press.

# Diverse types of interaction

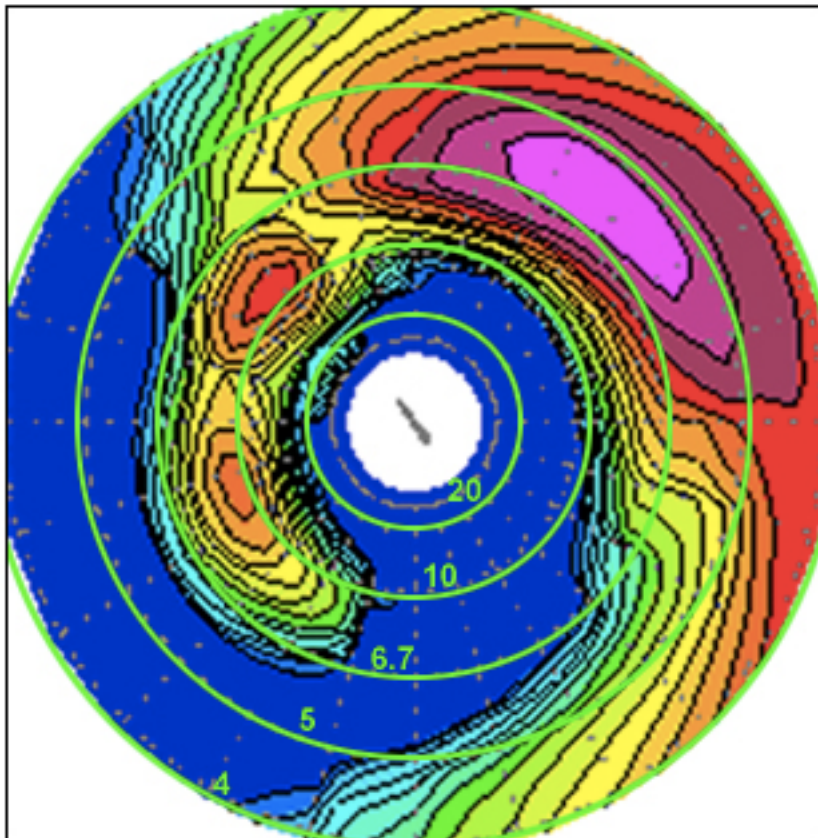


Slide & Movies by Peter Hamlington

# Surface Waves are...

fast, small, irrotational  
solutions of the  
Boussinesq Equations

NWW3 Polar Plot of Wave Energy Spectrum  
at ILM01



24 hr fcst Valid 0000 UTC 26 Apr 2002

NOAA / NWS / NCEP / MMAB

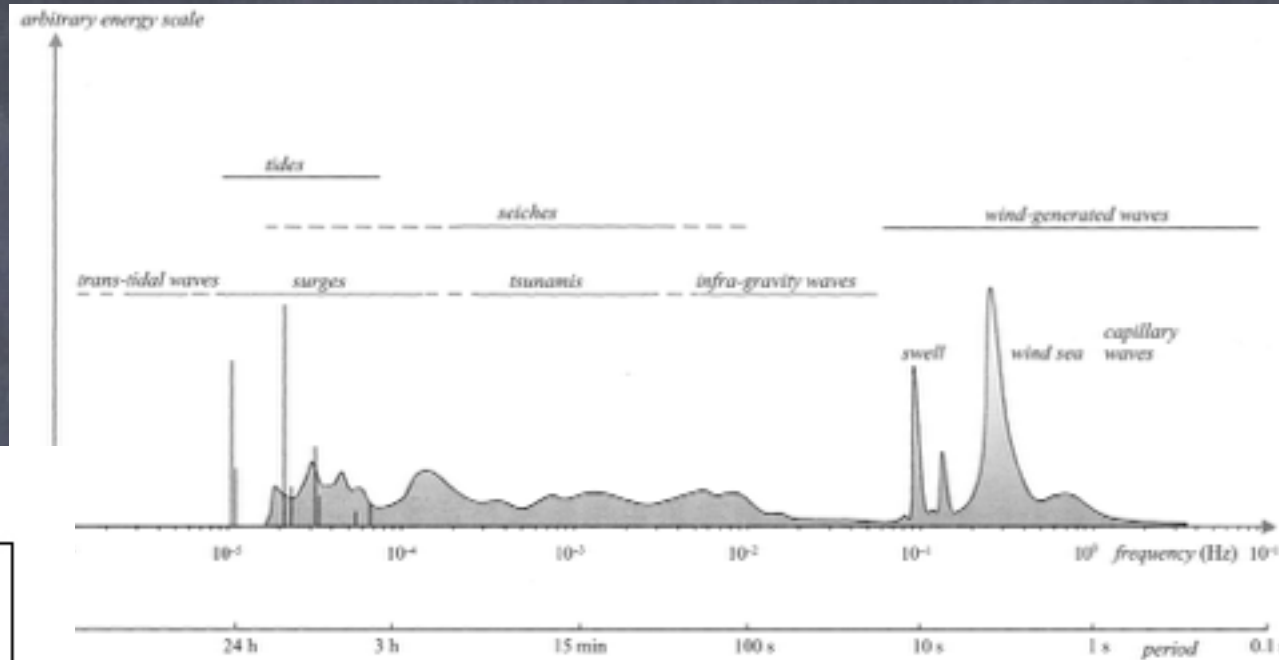


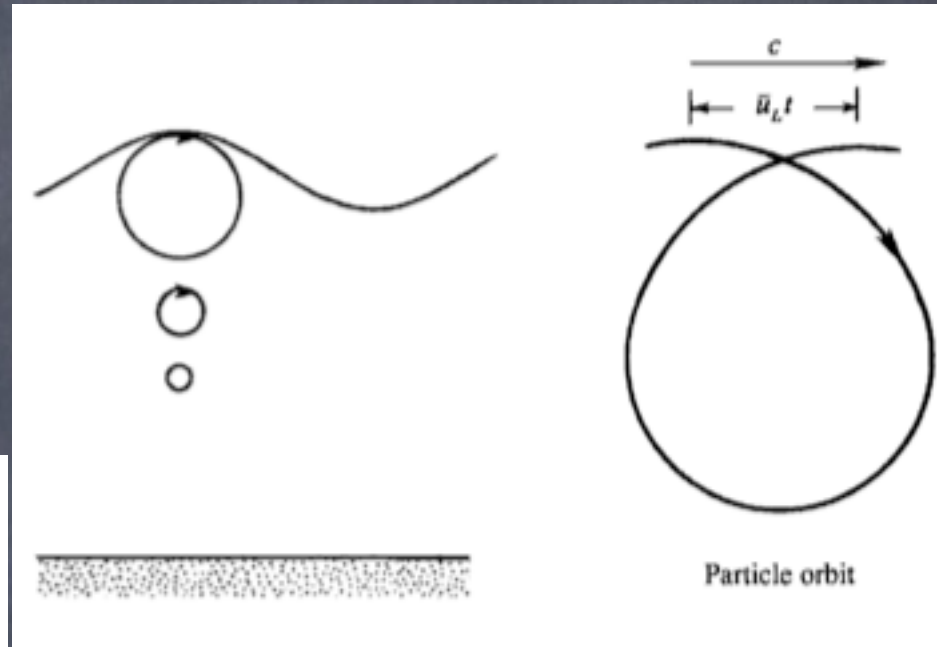
Illustration of wave spectra from different types of ocean surface waves (Holthuijsen, 2007)



# Stokes driftin' away

Take wave solns, compare the velocity of trajectories vs. Eulerian velocity, Taylor  
Expand, calculate:

$$\begin{aligned} \mathbf{u}^L(\mathbf{x}_p(t_0), t) - \mathbf{u}^E(\mathbf{x}_p(t_0), t) &\approx [\mathbf{x}_p(t) - \mathbf{x}_p(t_0)] \cdot \nabla \mathbf{u}^E(\mathbf{x}_p(t_0), t) \\ &\approx \left[ \int_{t_0}^t \mathbf{u}^E(\mathbf{x}_p(t_0), s') ds' \right] \cdot \nabla \mathbf{u}^E(\mathbf{x}_p(t_0), t). \end{aligned}$$



Monochromatic:

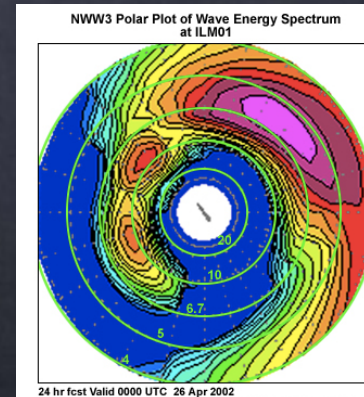
$$\mathbf{u}^S = \hat{\mathbf{e}}^w \frac{8\pi^3 a^2 f_p^3}{g} e^{\frac{8\pi^2 f_p^2}{g} z} = \hat{\mathbf{e}}^w a^2 \sqrt{gk^3} e^{2kz}.$$

Spectrum:

$$\mathbf{u}^S = \frac{16\pi^3}{g} \int_0^\infty \int_{-\pi}^\pi (\cos \theta, \sin \theta, 0) f^3 S_{f\theta}(f, \theta) e^{\frac{8\pi^2 f^2}{g} z} d\theta df.$$

Depth-Integrated:

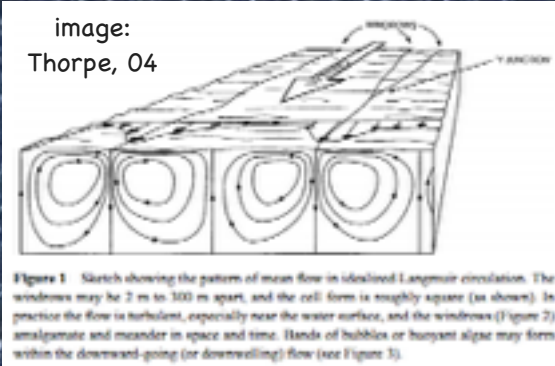
$$\mathbf{u}_*^{S-int} = \int_{-\infty}^0 \mathbf{u}_*^S(z) dz = 2\pi \int_0^\infty \mathbf{H}_*(f) f S_f(f) df$$



A. Webb and B. Fox-Kemper. Wave spectral moments and Stokes drift estimation. Ocean Modelling, 40(3-4):273-288, 2011.

A. Webb and B. Fox-Kemper. Estimating Stokes drift for random seas. In prep.

# Stokes forcing on the Langmuir Scale



- Langmuir turbulence
- $Ro \gg 1$
- $Ri < 1$ : Nonhydro
- 1-100m ( $H=L$ )
- 10s to 1hr
- Eqtns: Craik-Leibovich
- Resolved routinely in year 2170

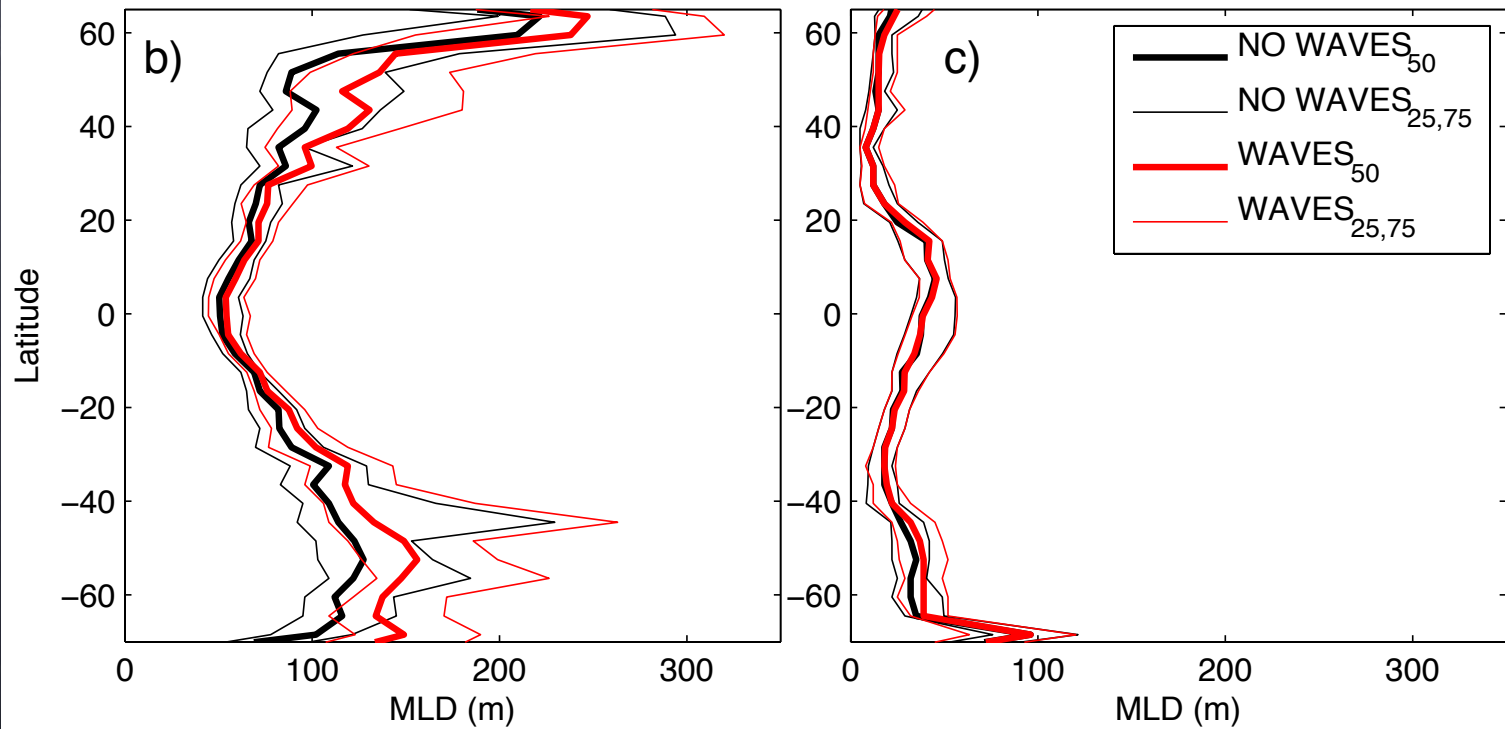
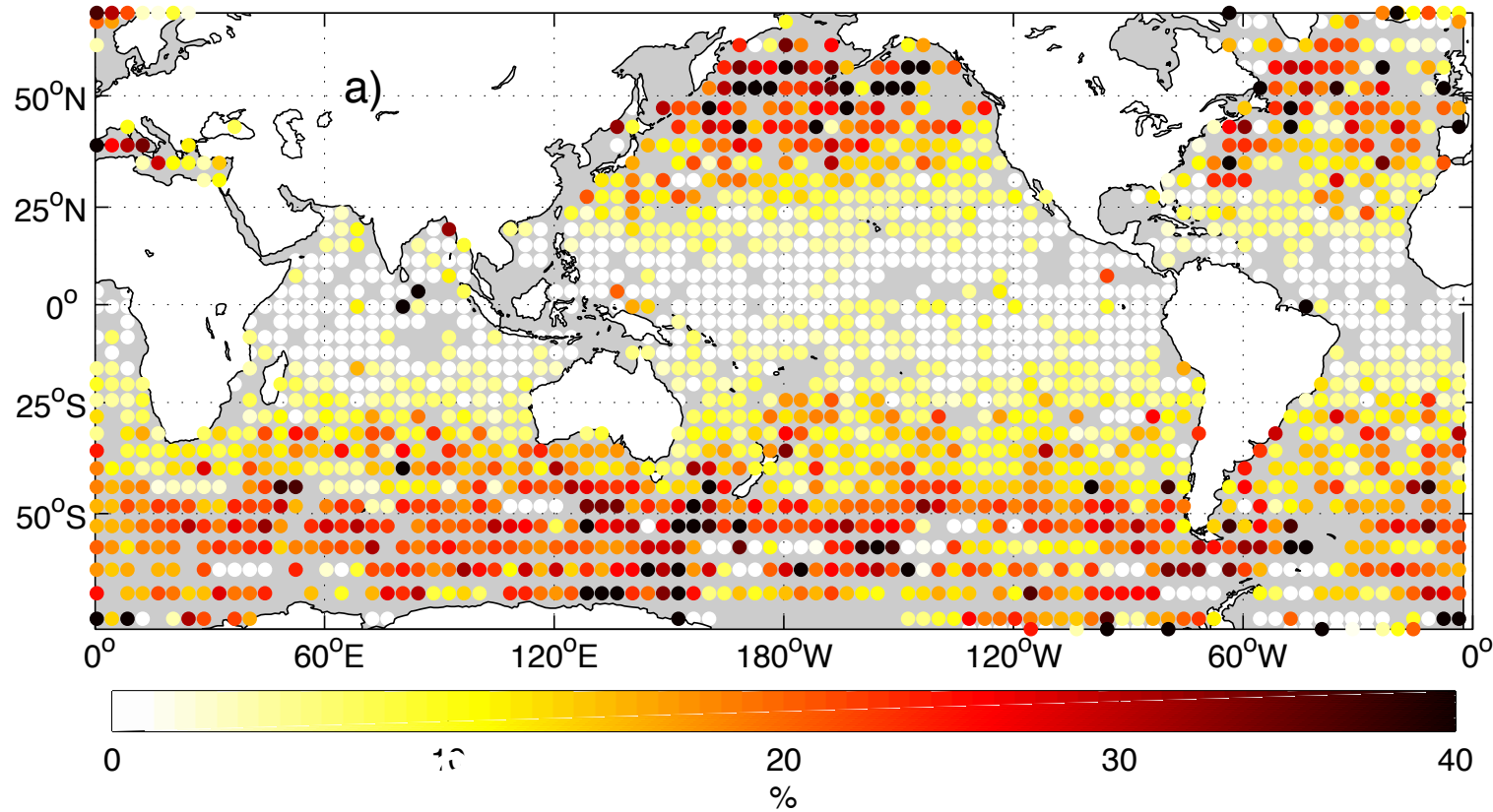
Image: NPR.org,  
Deep Water  
Horizon Spill

“Wave-forcing and hence Langmuir turbulence could be important over wide areas of the ocean and in all seasons in the Southern Ocean.”

Offline  
observation-driven  
parameterization:

Including  
Stokes-driven  
Mixing  
(Harcourt 2013)  
Deepens the  
Mixed Layer!

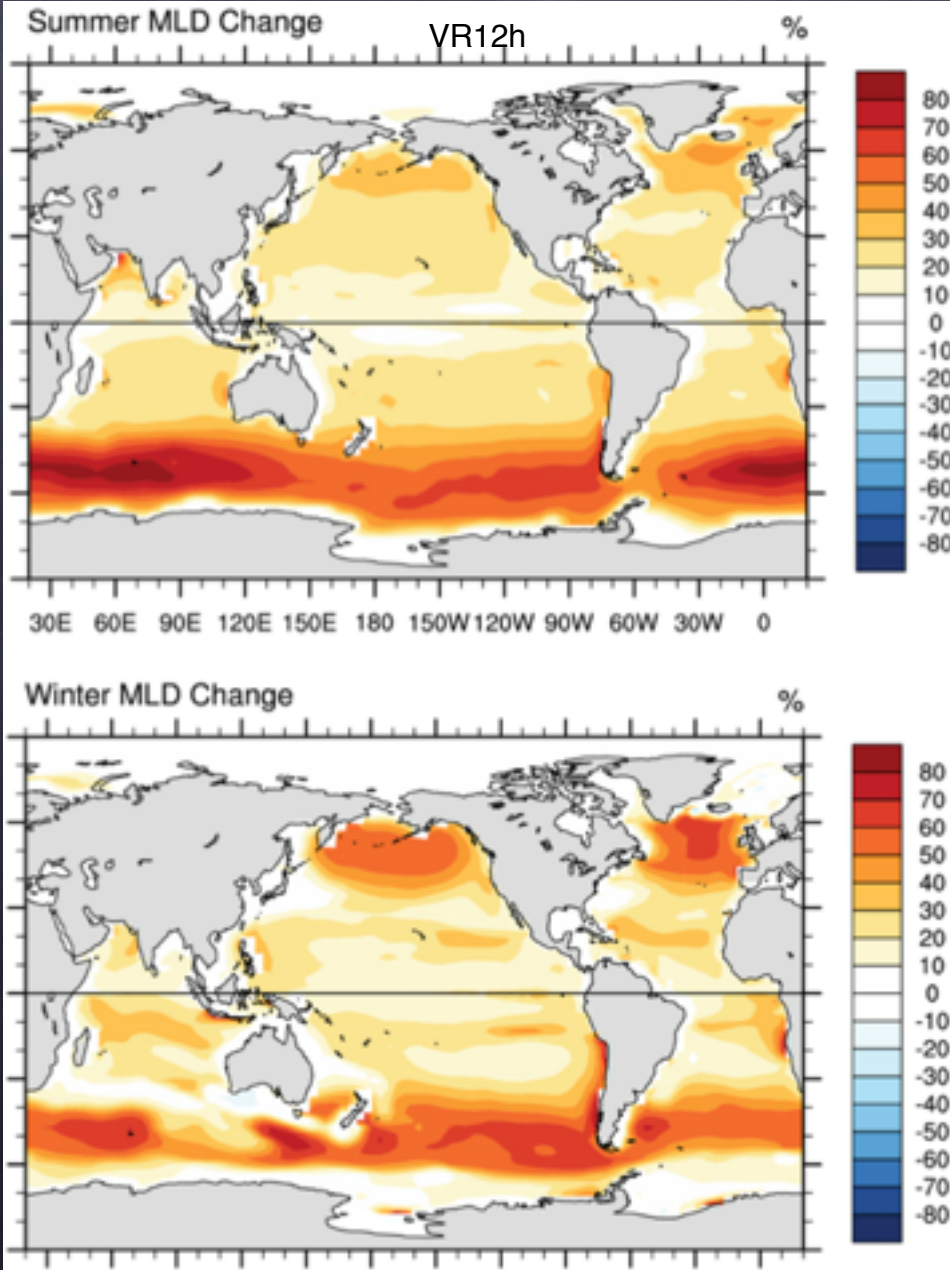
E. A. D'Asaro, J. Thomson, A. Y. Shcherbina, R. R. Harcourt, M. F. Cronin, M. A. Hemer, and B. Fox-Kemper.  
Quantifying upper ocean turbulence driven by surface waves. *Geophysical Research Letters*, 41(1): 102-107, January 2014.





# Including Stokes-driven Mixing in CESM, too!

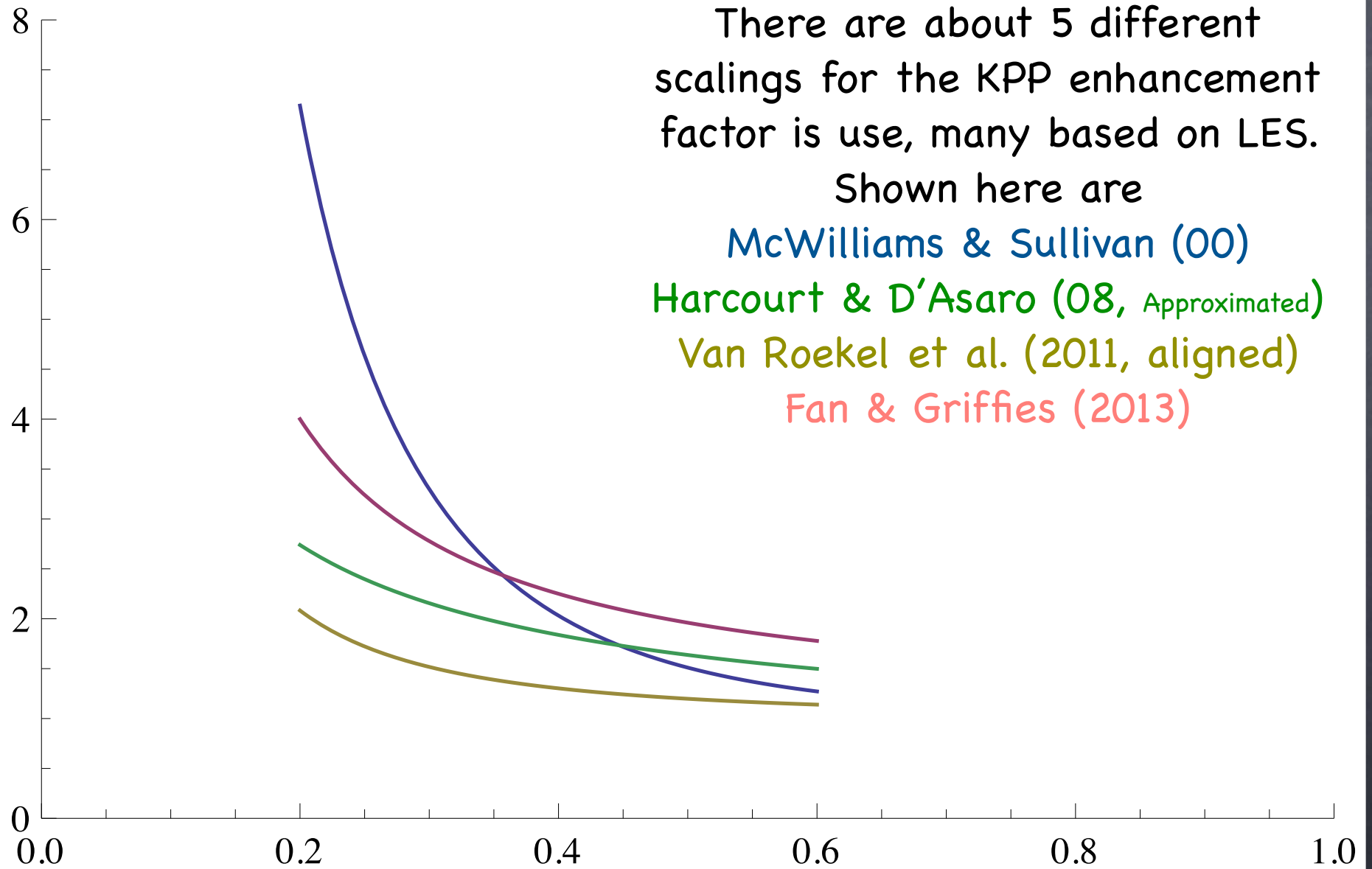
(with another param.: Harcourt & D'Asaro, 2008; Van Roekel et al, 2012)



RMSE (m)		
Summer	Global	South of 30S
CTRL	15.42	20.52
MS2K	26.92	20.15
VR12a	15.67	16.66
VR12g	15.26	16.72
VR12h	16.34	15.07
Winter	Global	South of 30S
CTRL	59.82	64.22
MS2K	135.16	181.88
VR12a	54.94	53.90
VR12g	56.63	57.57
VR12h	55.05	53.79

Remains to be co-tuned with mixed layer eddy, mesoscale, and near-inertial mixing parameterizations

W enhancement factor



La

# Dimensionless Boussinesq

## Spanning Mesoscale to Stratified Turbulence

following McWilliams (85)

$$Ro [v_{i,t} + v_j v_{i,j}] + \frac{M_{Ro}}{Ri} w v_{i,z} + \boxed{\epsilon_{izj} v_j} = -M_{Ro} \pi_{,i} + \frac{Ro}{Re} v_{i,jj}$$

geostrophic

$$\frac{\alpha^2}{Ri} \left[ w_{,t} + v_j w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = \boxed{-\pi_{,z} + b} + \frac{\alpha^2}{Re Ri} w_{,jj}$$

hydrostatic

$$b_t + v_j b_{,j} + \frac{M_{Ro}}{Ro Ri} w b_z + w = 0$$

$$v_{j,j} + \frac{M_{Ro}}{Ro Ri} w_z = 0$$

Plus boundary conditions

$$Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri = \frac{N^2}{(U_{,z})^2} \quad \alpha = H/L$$

$$M_{Ro} \equiv \max(1, Ro) \quad v = \text{horiz. vel.} \quad w = \text{vert. vel.}$$

# Wave-Averaged Equations

$$\varepsilon = \frac{V^s H}{f L H_s}$$

following Lane et al. (07), McWilliams & F-K (13)  
and Suzuki & F-K (14)

(for horizontally uniform Stokes drift)

$$Ro [v_{i,t} + v_j^L v_{i,j}] + \frac{M_{Ro}}{Ri} w v_{i,z} + \epsilon_{izj} v_j^L = -M_{Ro} \pi_{,i} + \frac{Ro}{Re} v_{i,jj}$$

$$\frac{\alpha^2}{Ri} \left[ w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = -\pi_{,z} + b + \boxed{\varepsilon v_j^L v_{j,z}^s} + \frac{\alpha^2}{Re Ri} w_{,jj}$$

$$b_t + v_j^L b_{,j} + \frac{M_{Ro}}{Ro Ri} w b_z + w = 0$$

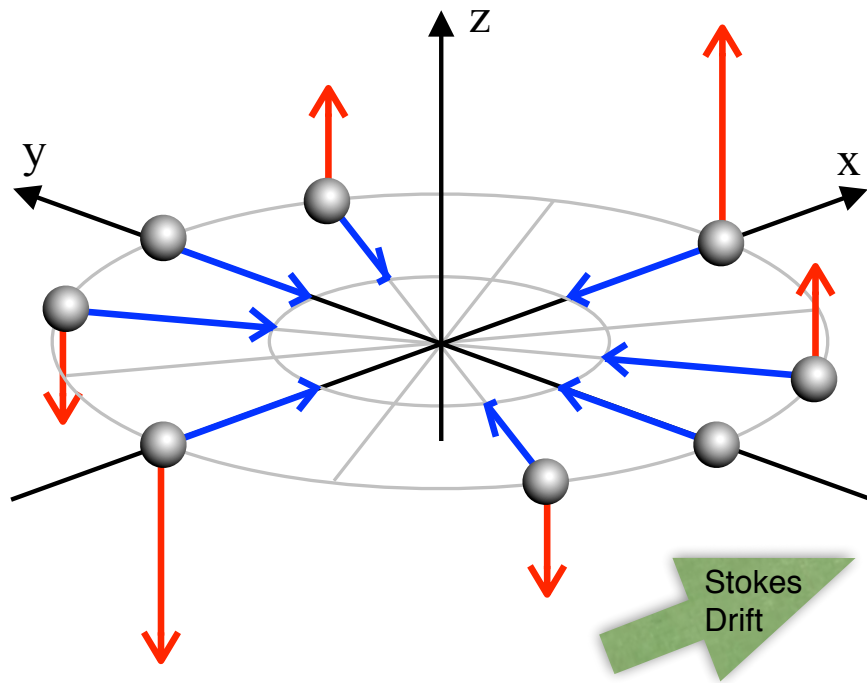
$$v_{j,j} + \frac{M_{Ro}}{Ro Ri} w_z = 0$$

Plus boundary  
conditions

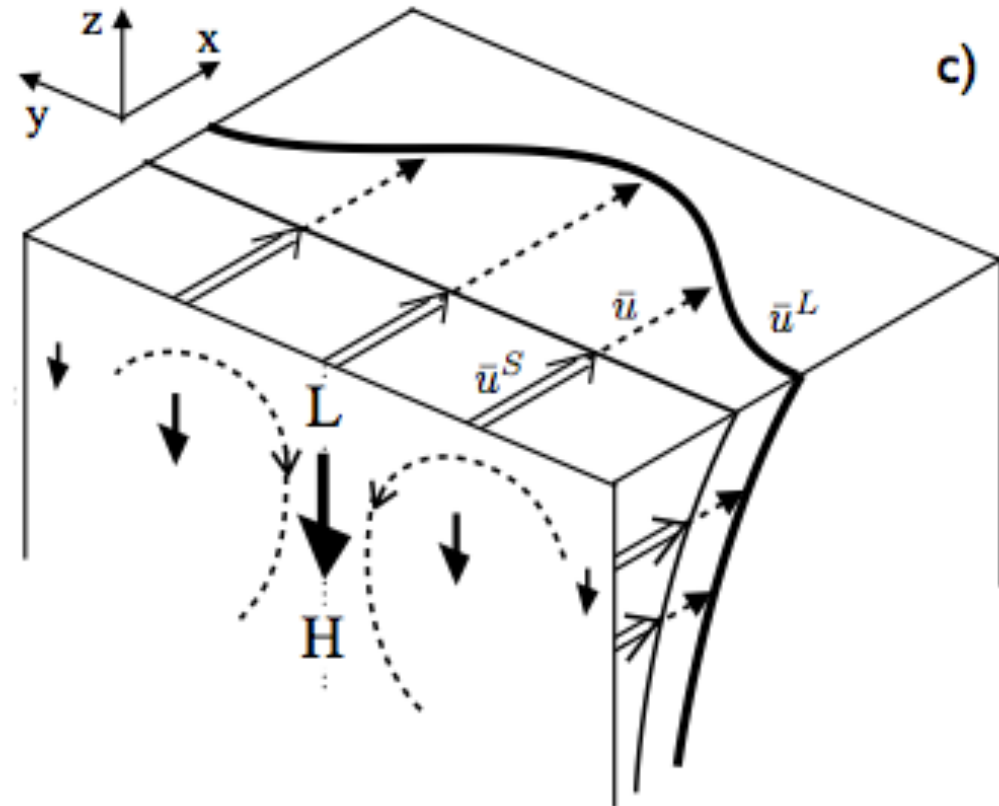
LAGRANGIAN (Eulerian+Stokes) advection & Coriolis  
Stokes shear force is NEW \*nonhydrostatic\* term in Vert. Mom.

# Stokes Shear Force

and the CL2 mechanism for Langmuir circulations  
 Flow directed along Stokes shear=downward force



← : Stokes-shear force    ● : water parcel  
← : turbulent velocity



$$\frac{\alpha^2}{Ri} \left[ w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = -\pi_{,z} + b + \epsilon v_j^L v_{j,z}^s + \frac{\alpha^2}{Re Ri} w_{,jj}$$

# Stokes forcing of the (Sub)mesoscale

← 10 km

(Capet et al., 2008)

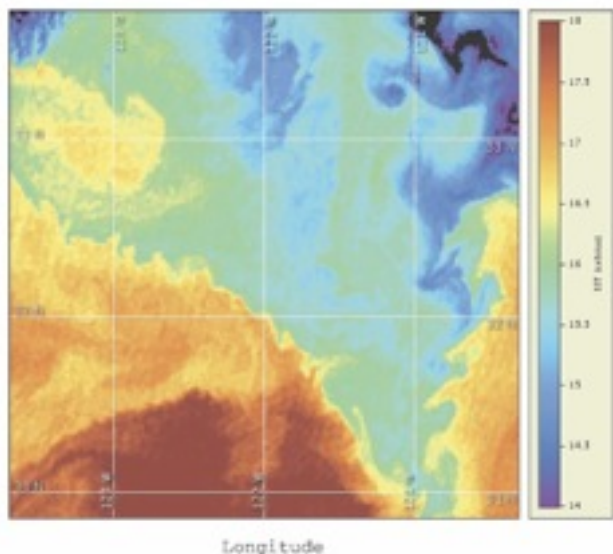
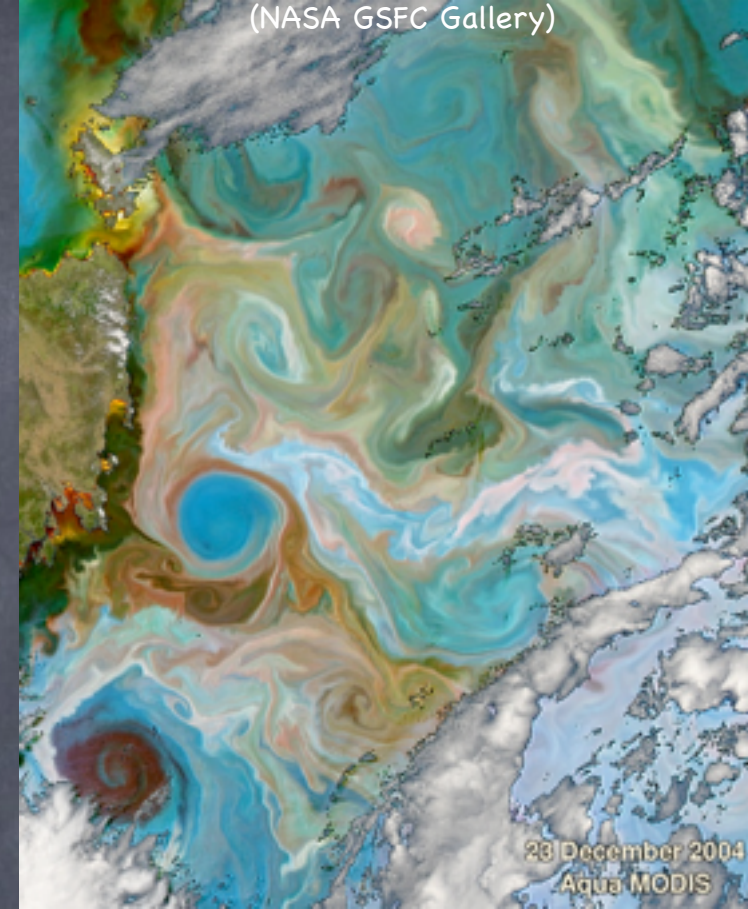
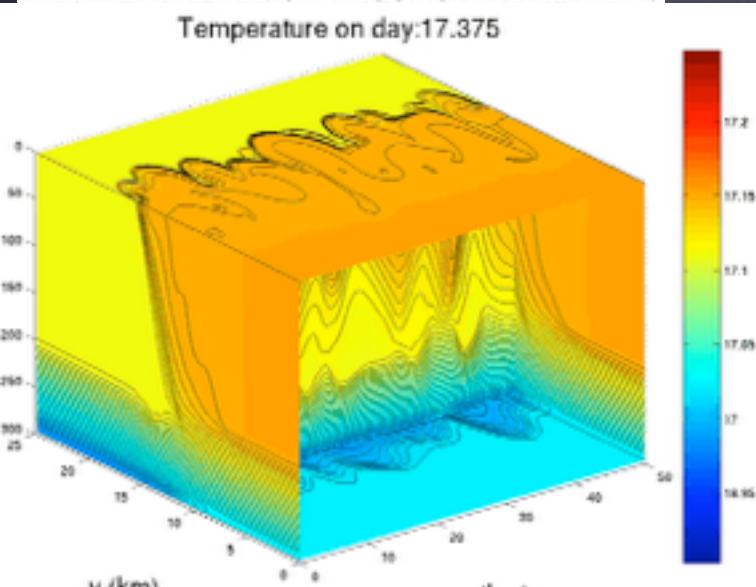


FIG. 16. Sea surface temperature measured at 1832 UTC 3 Jan 2006 off Point Conception in the California Current from CoastWatch (<http://coastwatch.pfeg.noaa.gov>). The fronts between recently

- Fronts (mesoscale & submesoscale)
- Eddies
- $Ro=O(1)$
- $Ri=O(1)$
- near-surface
- 1-10km, days
- Resolved: yr 2050 to 2100



B. Fox-Kemper, R. Ferrari, and R. W. Hallberg. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. *Journal of Physical Oceanography*, 38(6):1145-1165, 2008

S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. *Ocean Modelling*, 64:12-28, 2013

When is  $\varepsilon = \frac{V^s H}{f L H_s}$  big?

$$\varepsilon = \frac{V_s H}{f L H_s} = \frac{V_s}{\underbrace{f H_s}_{O(10-100)}} \underbrace{\frac{H}{L}}_{\text{slope}}$$

- Isopycnal slope is  $O(0.1-0.01)$  for submesoscale
- Isopycnal slope is  $O(0.0001)$  for mesoscale

Consider **perturbing** from **geostrophic, hydrostatic** soln:

$$\begin{aligned} \phi = & \phi_{00000} + \varepsilon \phi_{10000} + Ro \phi_{01000} + \frac{1}{\sqrt{Ri}} \phi_{00100} + \frac{\alpha^2}{\sqrt{Ri}} \phi_{00010} + \frac{1}{Re} \phi_{00001} \\ & + \varepsilon^2 \phi_{20000} + \varepsilon Ro \phi_{11000} + Ro^2 \phi_{02000} + \frac{Ro}{\sqrt{Ri}} \phi_{01100} + \dots \\ & + O(Ro^3) \end{aligned}$$

$$\frac{\varepsilon}{Ro} = \frac{V_s H f L}{f L H_s V} = \frac{V_s H}{V H_s}$$

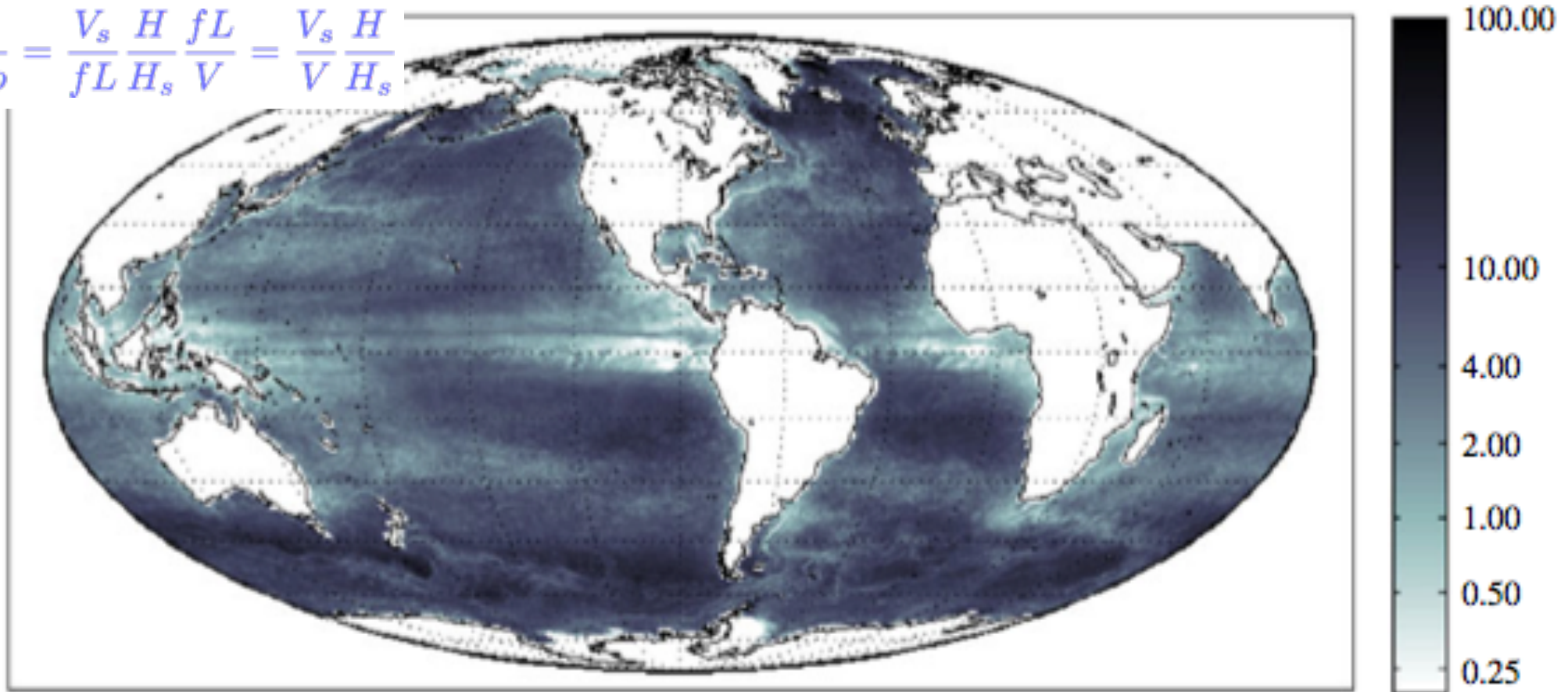
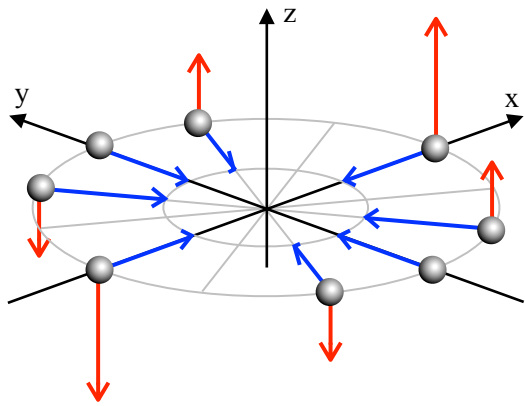


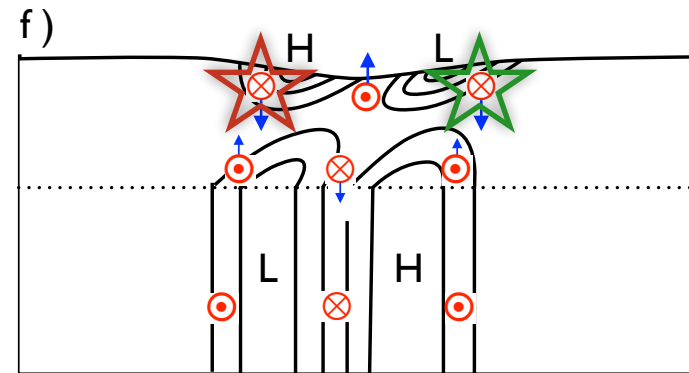
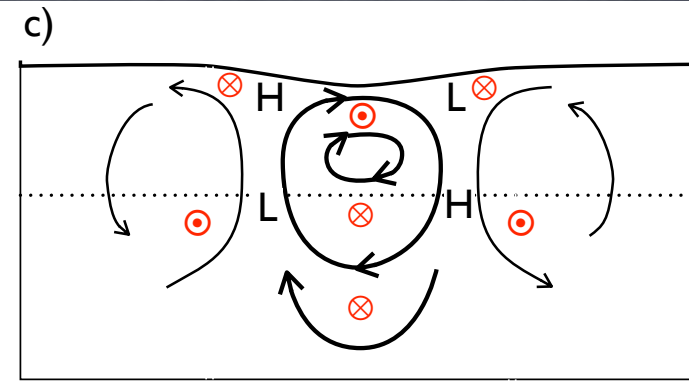
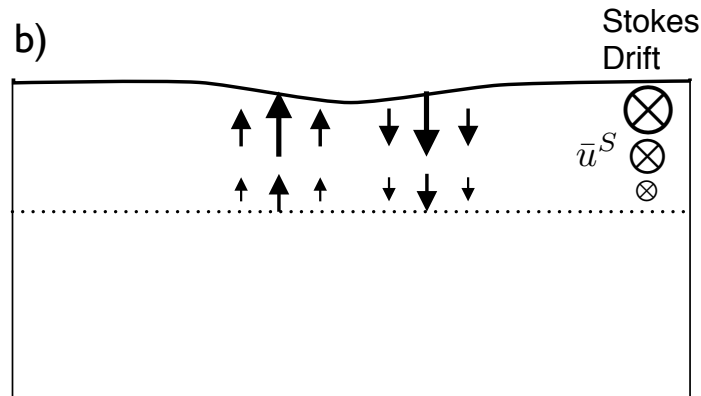
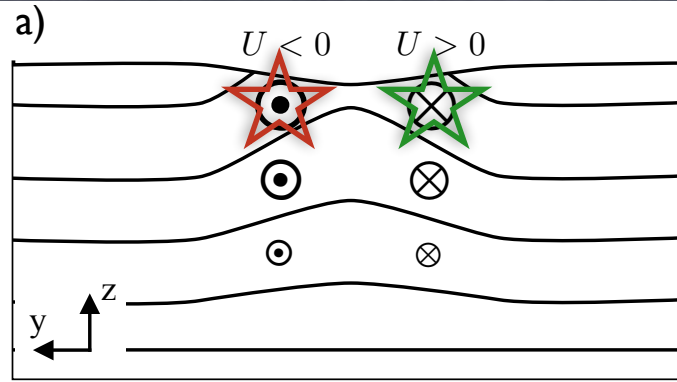
FIGURE 1. (Colour online) Estimated ratio  $\varepsilon/\mathcal{R} \approx (|\mathbf{u}_s \cdot \mathbf{u}|h) / (|\mathbf{u}|^2 h_s)$  governing the relative importance of Stokes effects versus nonlinearity. Eulerian velocity ( $\mathbf{u}$ ) is taken as the



# Stokes Shear Force on Submesoscale Cold Filament



←: Stokes-shear force    ●: water parcel  
 ←: turbulent velocity



J. C. McWilliams and B. F-K.  
 Oceanic wave-balanced surface  
 fronts and filaments. *Journal of Fluid  
 Mechanics*, 730:464-490, 2013.

N. Suzuki and B. Fox-Kemper.  
 Understanding Stokes Forces in the  
 Wave-Averaged Equations, In prep,  
 2014.

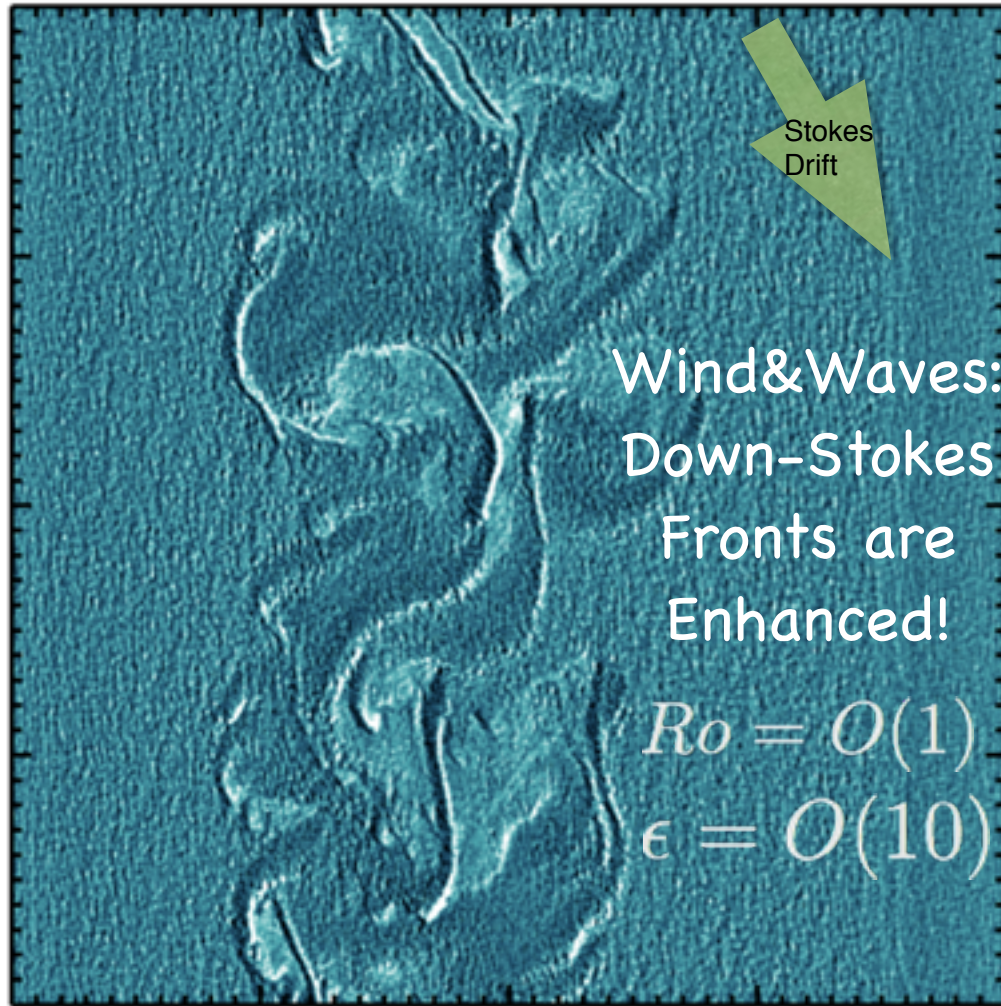
Enhances Fronts for Down-Front Stokes

Opposes Fronts for Up-Front Stokes

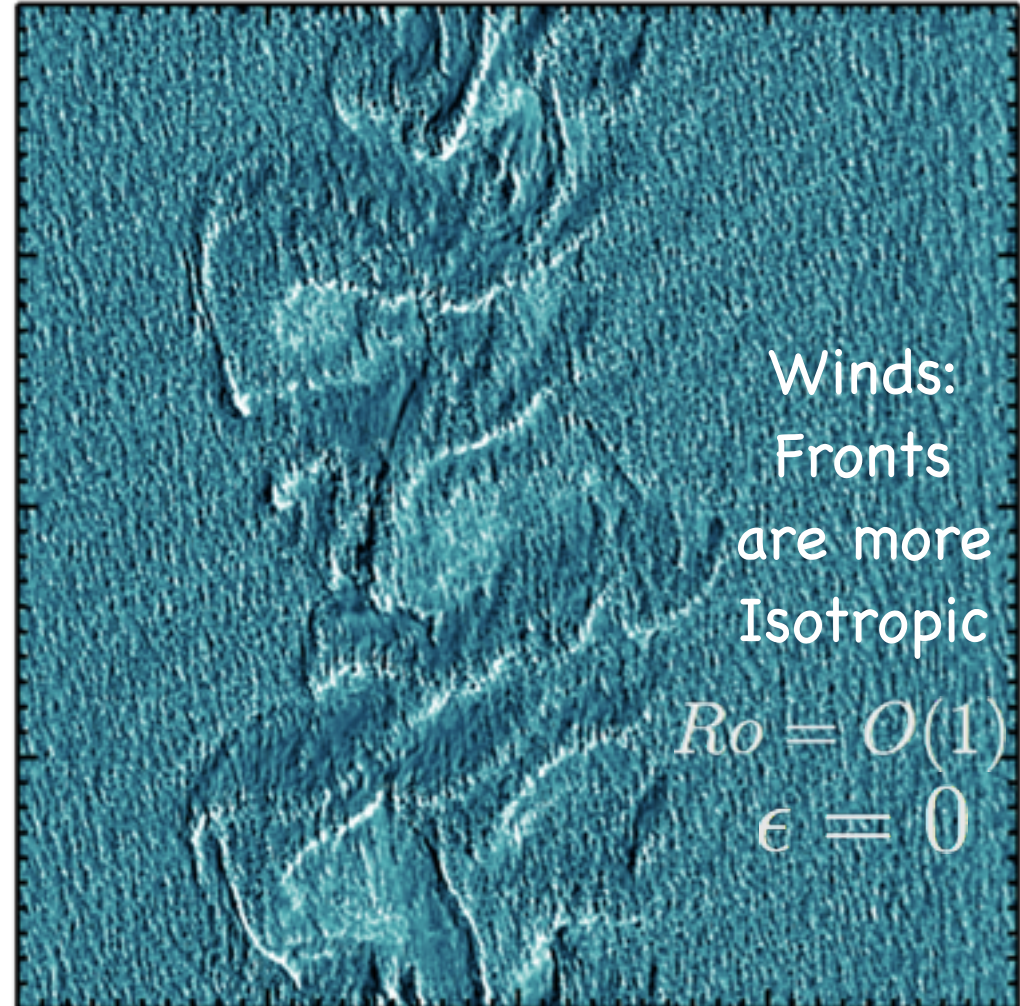
# Are Fronts and Filaments different with Stokes shear force?

$$\frac{\alpha^2}{Ri} \left[ w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = -\pi_{,z} + b + \epsilon v_j^L v_{j,z}^s + \frac{\alpha^2}{Re Ri} w_{,jj}$$

(b) LT,  $\omega_z/f$  Wind & Waves



(d) ST,  $\omega_z/f$  Wind Only



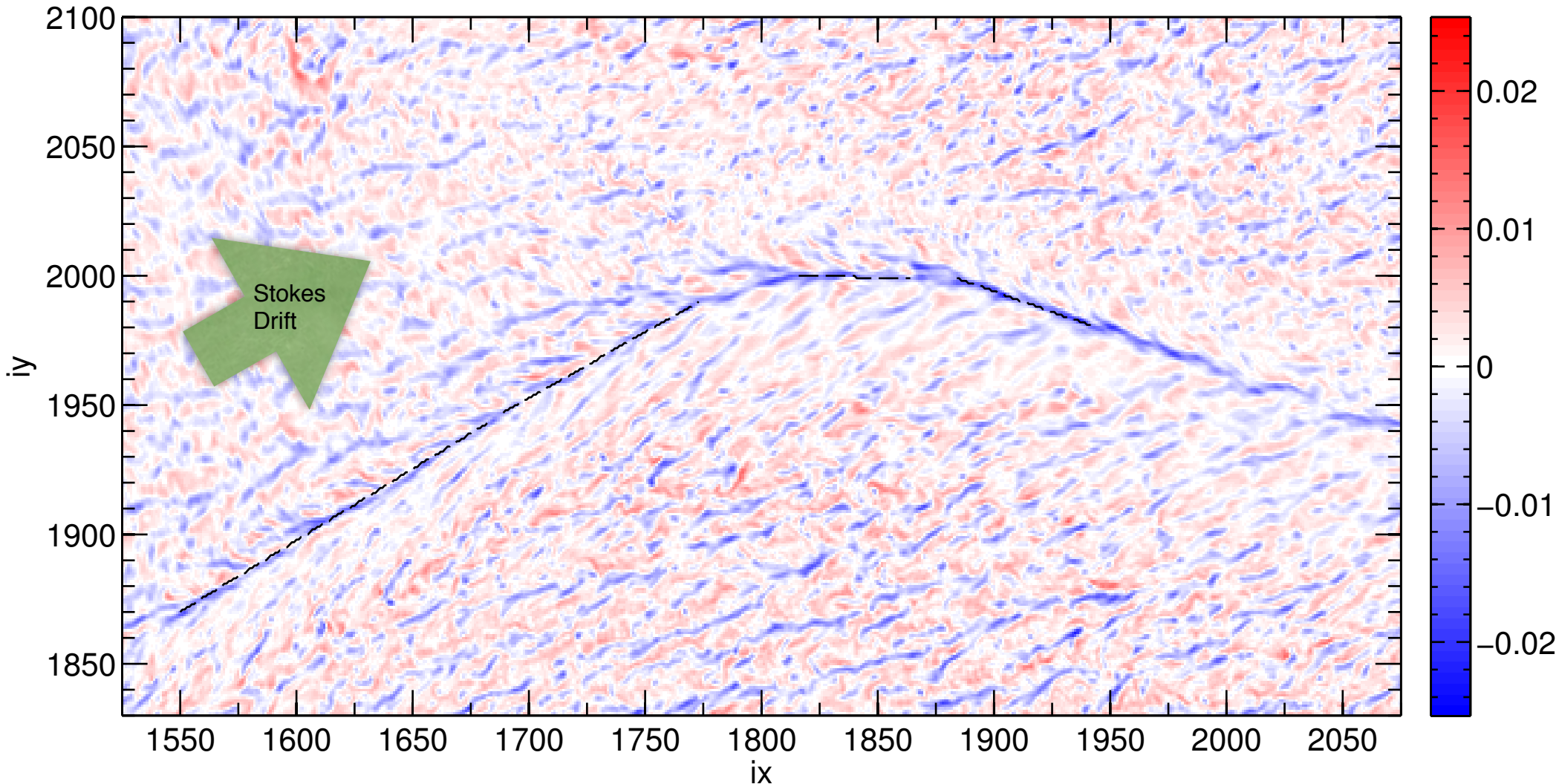
N. Suzuki and B. Fox-Kemper. Surface Wave Stokes Forces Influence Frontogenesis, JPO, in prep, 2014.

J. C. McWilliams and B. F-K. Oceanic wave-balanced surface fronts and filaments. Journal of Fluid Mechanics, 730:464-490, 2013.

P. E. Hamlington, L. P. Van Roekel, B. Fox-Kemper, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. Journal of Physical Oceanography, 2014. In press.

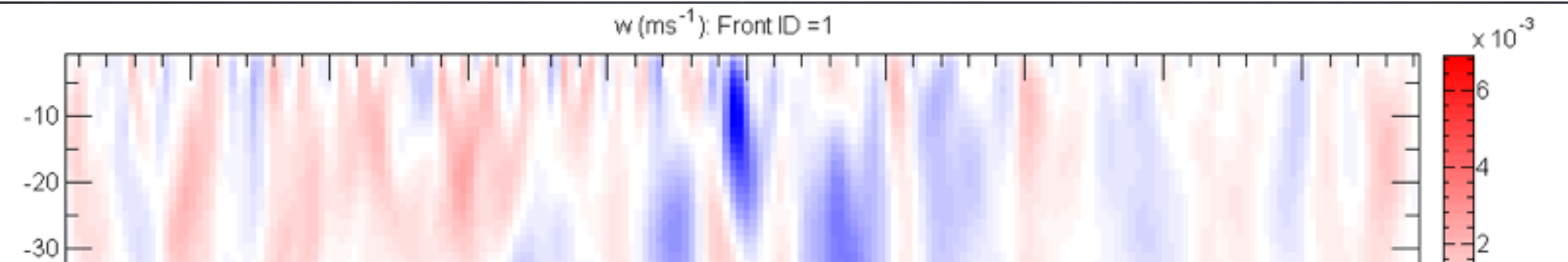
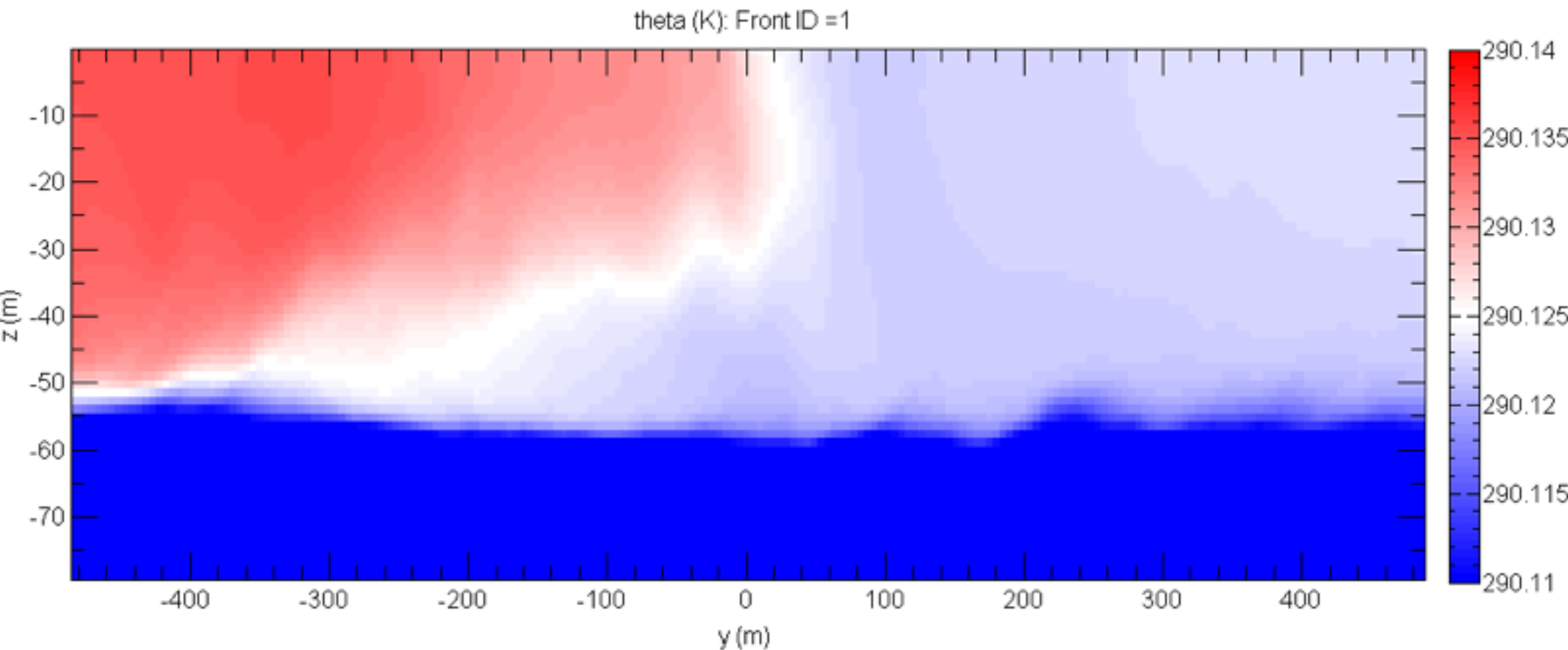
Let's examine  
a particular front with  $\varepsilon = \frac{V^s H}{f L H_s} \approx 20$

10min-ave. w ( $\text{ms}^{-1}$ ) at  $z = -12.5\text{m}$



# Along-Front and 10min Average

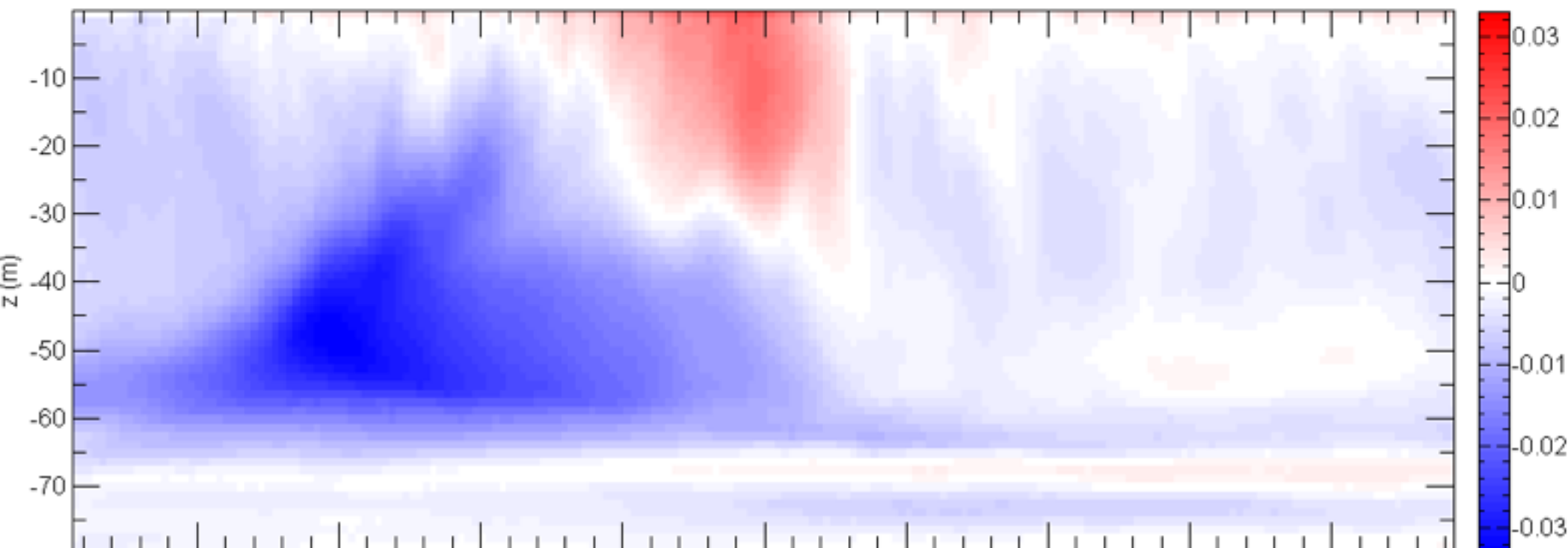
$$\varepsilon = \frac{V^s H}{f L H_s} \approx 20$$



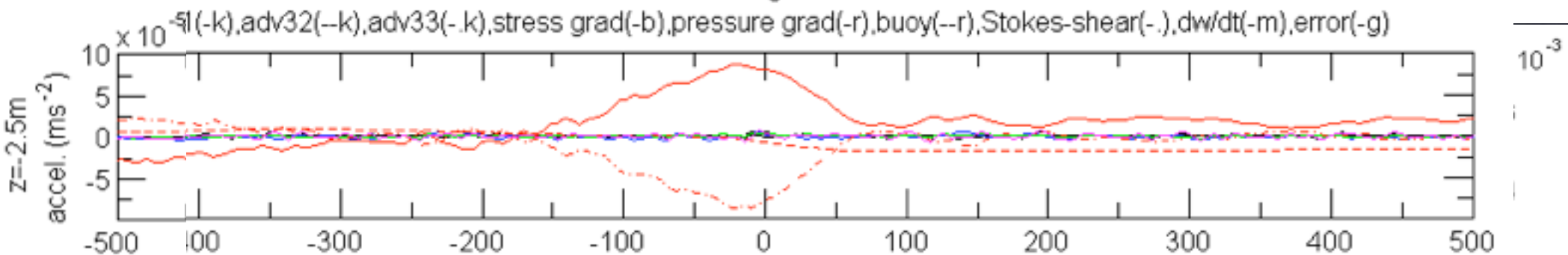
# Along-Front and 10min Average

$$\varepsilon = \frac{V^s H}{f L H_s} \approx 20$$

Eulerian u (ms<sup>-1</sup>): Front ID = 1

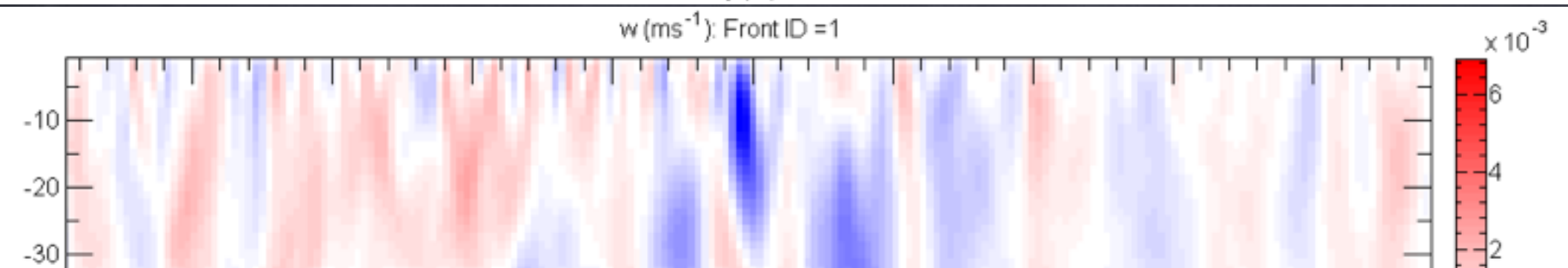
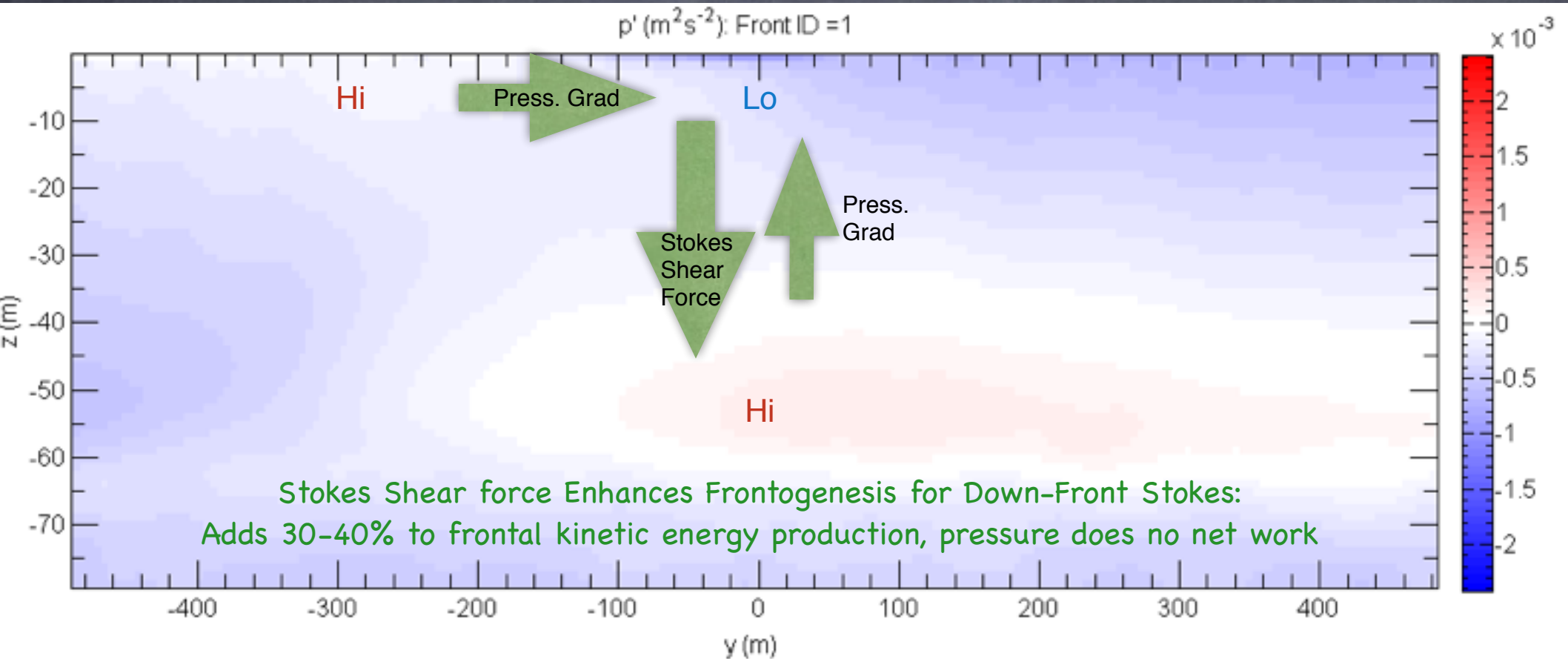


z-momentum budget: Front ID = 1

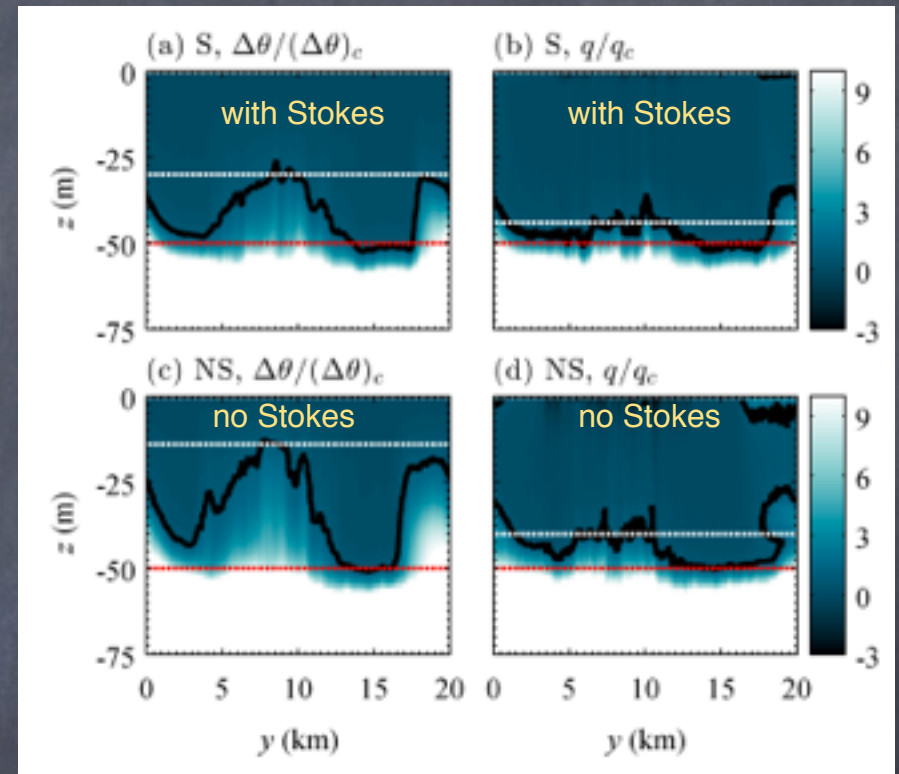
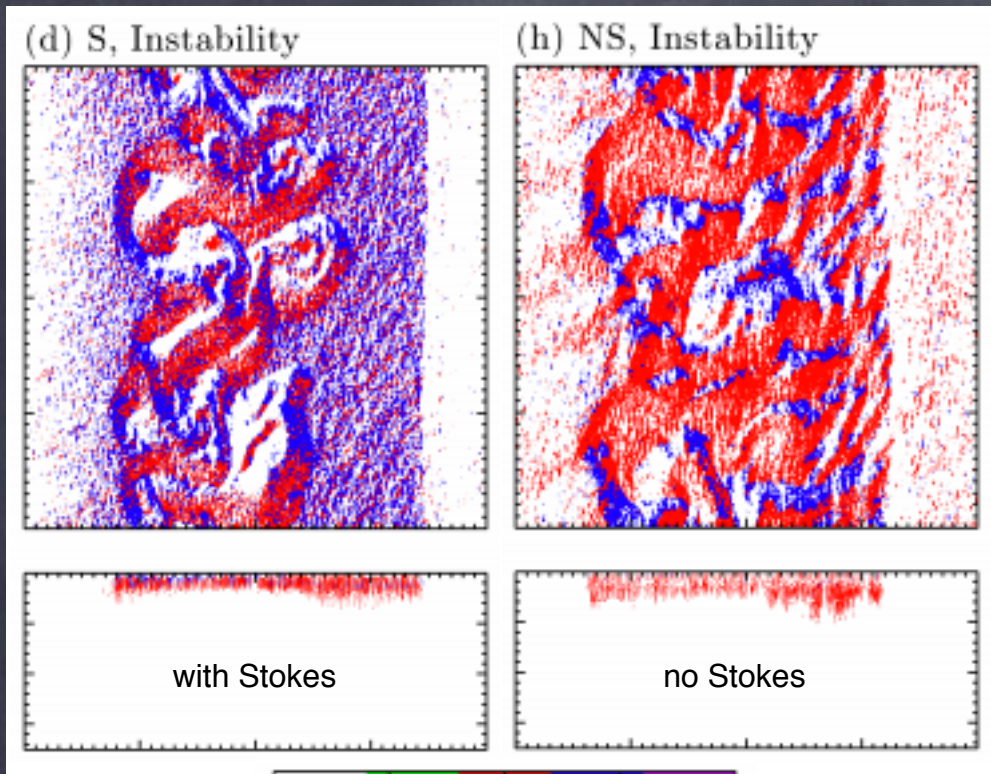


# Along-Front and 10min Average

$$\varepsilon = \frac{V^s H}{f L H_s} \approx 20$$



# Stokes also influences Submesoscale & Langmuir-scale Instabilities through Lagrangian shear (Holm '96) & Lagrangian Thermal Wind



So,  $q < 0$

$$Ri_L = \frac{N^2}{(dv^L/dz)^2} \approx \frac{N^2 f^2}{|\nabla_h b|^2}$$

$$q = (f + \nabla \times u) \cdot \nabla b \approx [f + \nabla \times (u^L - u^s)] \cdot \nabla b$$

Reinterpret Hoskins, Stone, & Charney-Stern-Pedlosky with care!

Is not the same as  $Ri < \frac{f}{\zeta}$

P. E. Hamlington, L. P. Van Roekel, B. Fox-Kemper, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. *Journal of Physical Oceanography*, 2014. In press.

S. Haney, B. Fox-Kemper, and K. Julien. Stability of the ocean mixed layer in the presence of surface gravity wave forcing. In TOS/ALSO/AGU 2014 Ocean Sciences Meeting. American Geophysical Union, 2014. Paper in prep.

# Conclusion:

KH win if

$$Ri_L < \frac{1}{4}$$

SI win if

$$\frac{1}{4} < Ri_L$$

and  
 $q < 0$

BCI win if

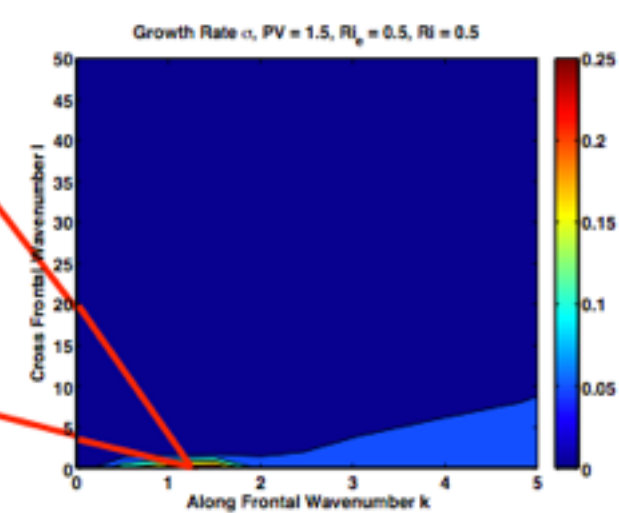
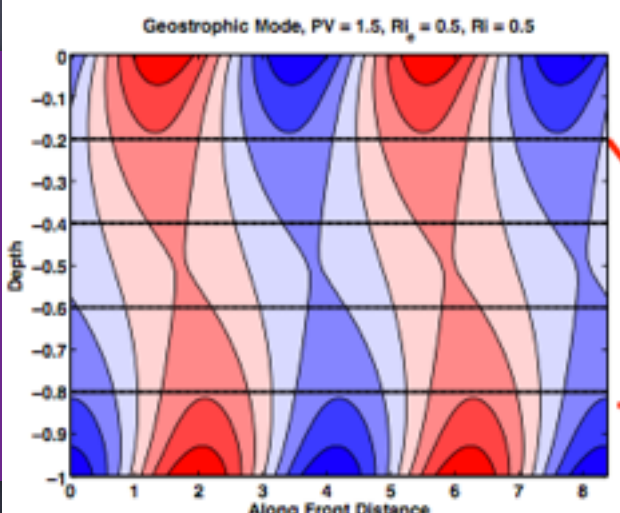
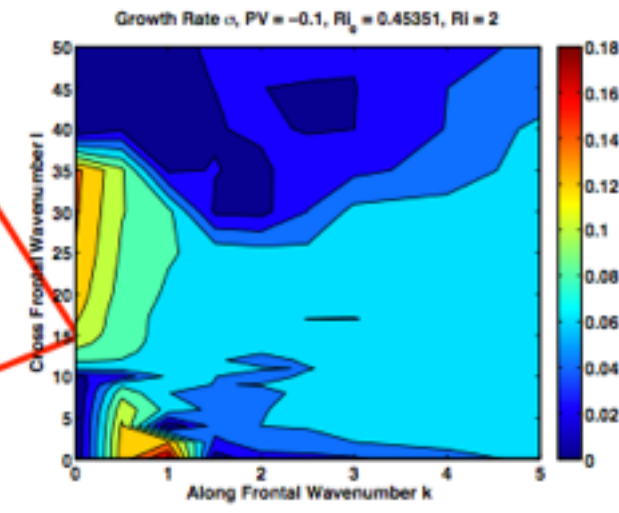
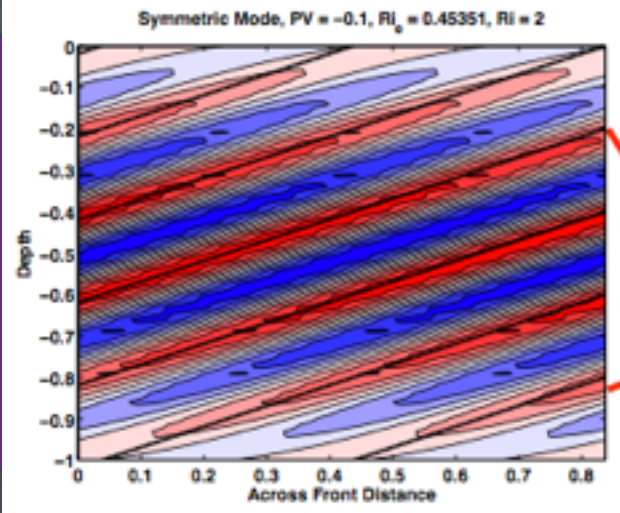
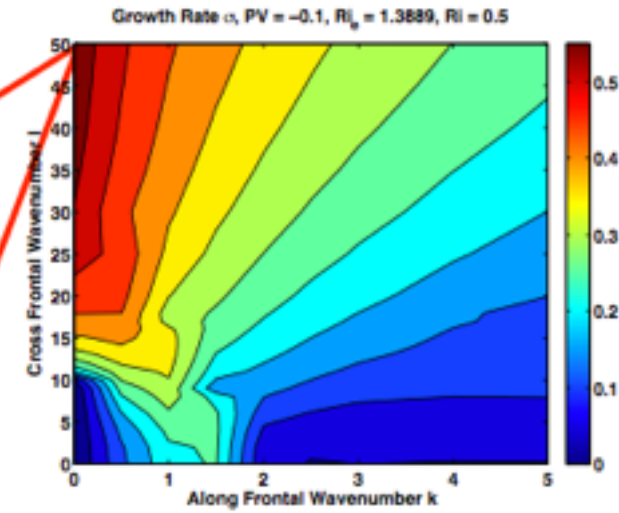
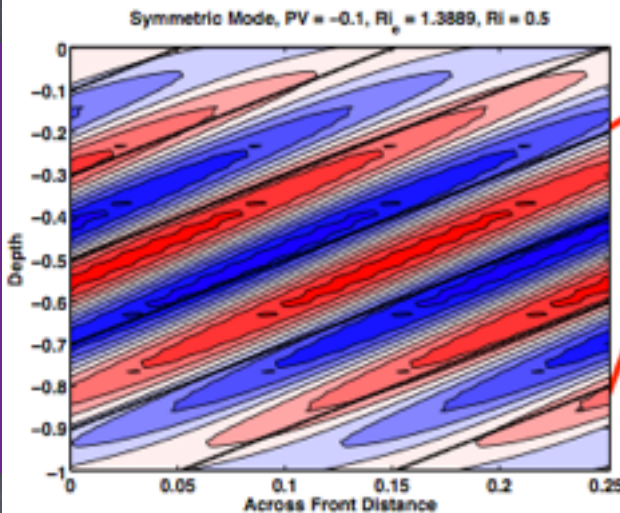
$$\frac{1}{4} < Ri_L$$

and  
 $q > 0$

$q < 0$   
 $Ri_E > 1$   
 $\frac{1}{4} < Ri_L < 1$   
**SI win**

$q < 0$   
 $Ri_E < 1$   
 $Ri_L > 1$   
**SI win**

$q > 0$   
 $Ri_E < 1$   
 $\frac{1}{4} < Ri_L < 1$   
**BCI win**



S. Haney, B. Fox-Kemper, and K. Julien.  
 Stability of the ocean mixed layer in the  
 presence of surface gravity wave forcing.  
 In TOS/ALSO/AGU 2014 Ocean  
 Sciences Meeting. American  
 Geophysical Union, 2014. Paper in prep.



# Conclusions

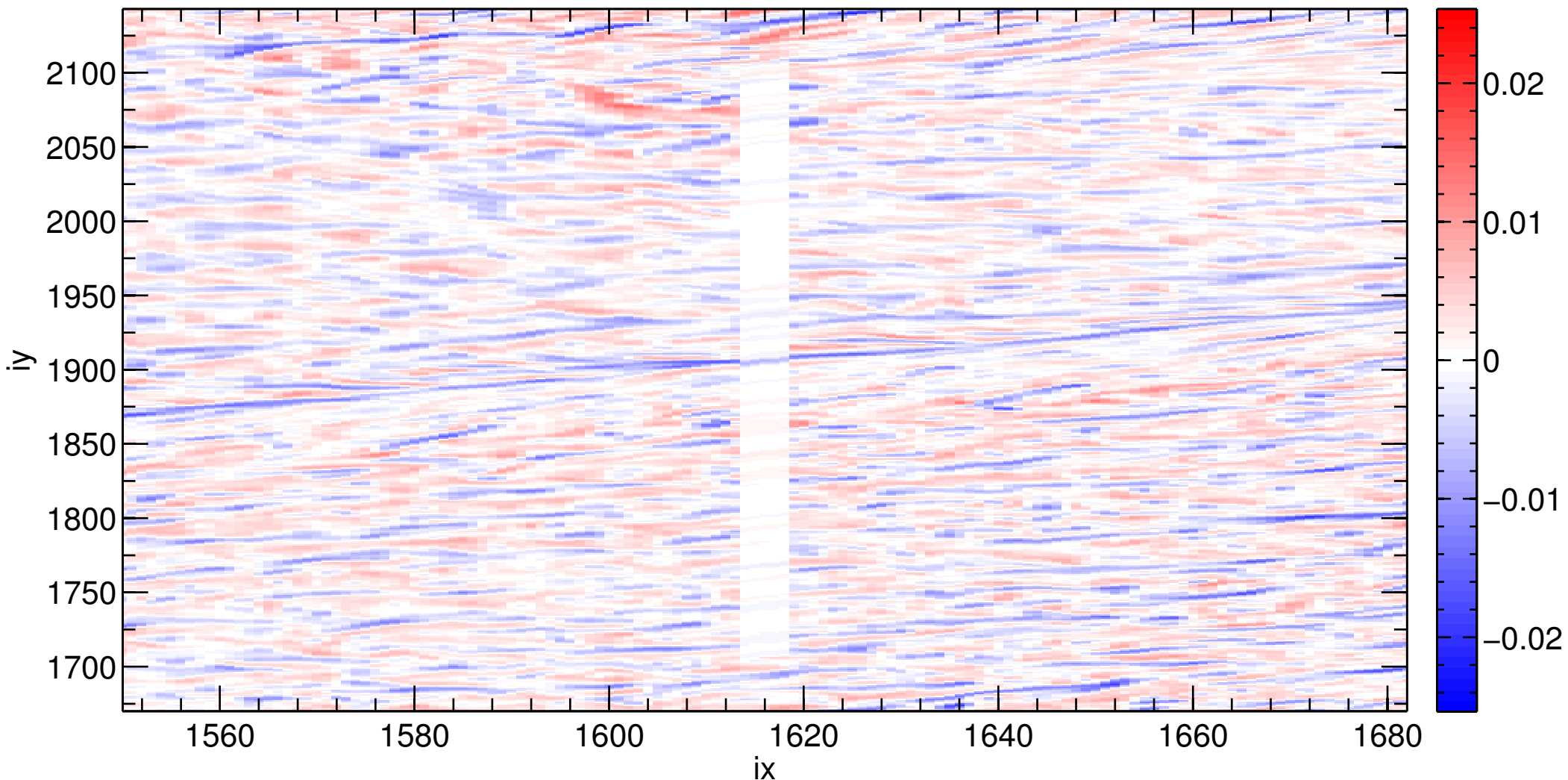
All papers at: [fox-kemper.com/pubs](http://fox-kemper.com/pubs)

- Stokes shear force affects frontogenesis. Add/subtract 30–40% of frontal KE production for downfront/upfront Stokes drift
- The controlling parameter,  $\varepsilon$ , measures nonhydrostatic frontal effects. It can dominate other nonlinear effects, such as  $O(1)$  Rossby, and is  $O(20)$  in these simulated submesoscale fronts.
- Down–Stokes fronts are sharper than those directed across or esp. against Stokes and have horizontal velocity and pressures that are not antisymmetric about the max  $w$ .
- Future/Present: Cross-frontal transport pathways, wave–mean 2–way interaction, and Stokes effects on frontal instabilities
- Overall: Stokes force can affect submesoscale dynamics as well as Langmuir turbulence.

# Along-Front and 10min Average

$$\epsilon = \frac{V^s H}{f L H_s} \approx 20$$

10min-ave w ( $\text{ms}^{-1}$ ); Front ID = 1



# Along-Front and 10min Average

$$\epsilon = \frac{V^s H}{f L H_s} \approx 20$$

10min-ave along-front velocity ( $\text{ms}^{-1}$ ); Front ID = 1

