From Climate to Kolmogorov: upper ocean variability across scales

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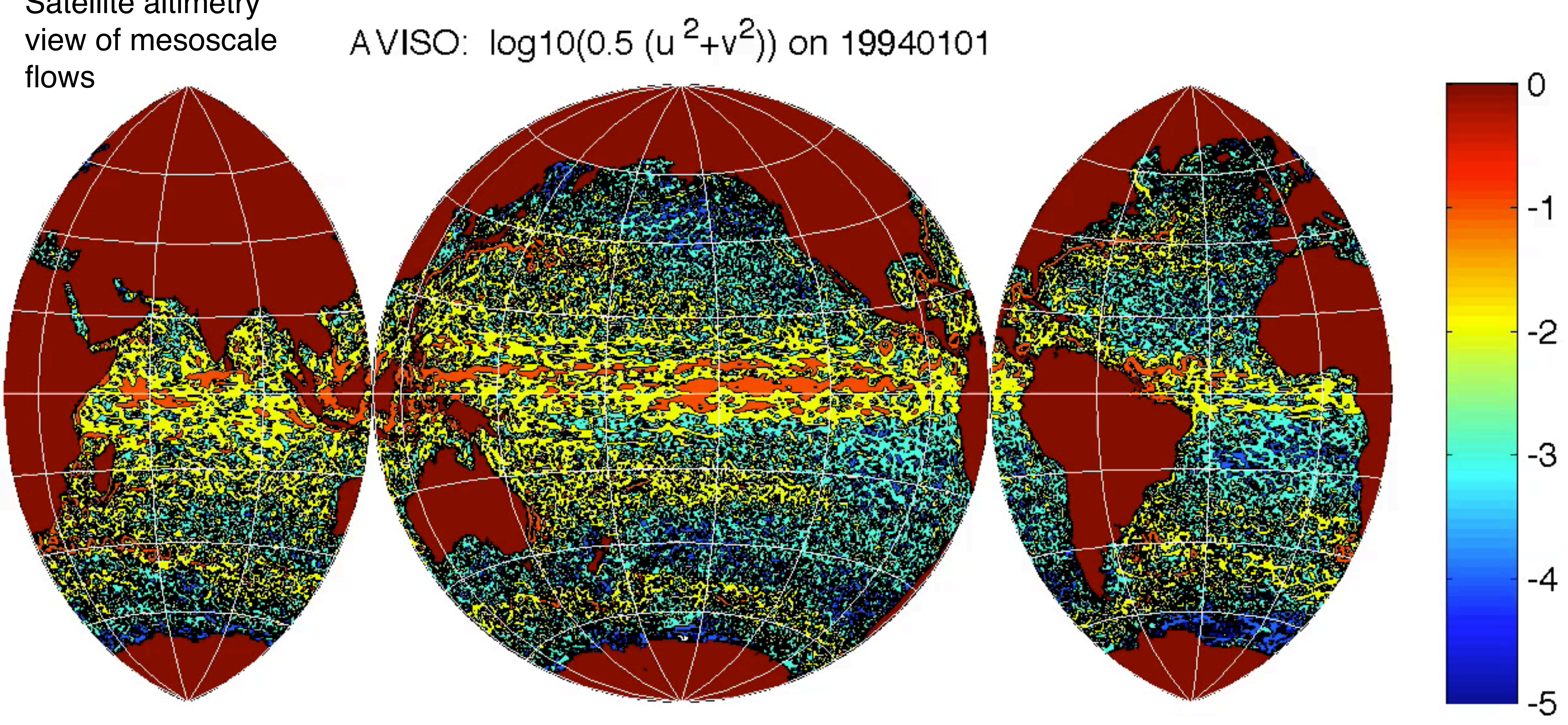
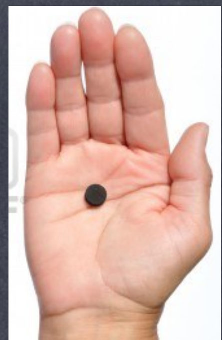
with Brodie Pearson (Brown), Frank O. Bryan (NCAR), and S. Bachman (DAMTP)

Brandeis IGERT Seminar 2/8/17, Sponsor: NSF 1350795

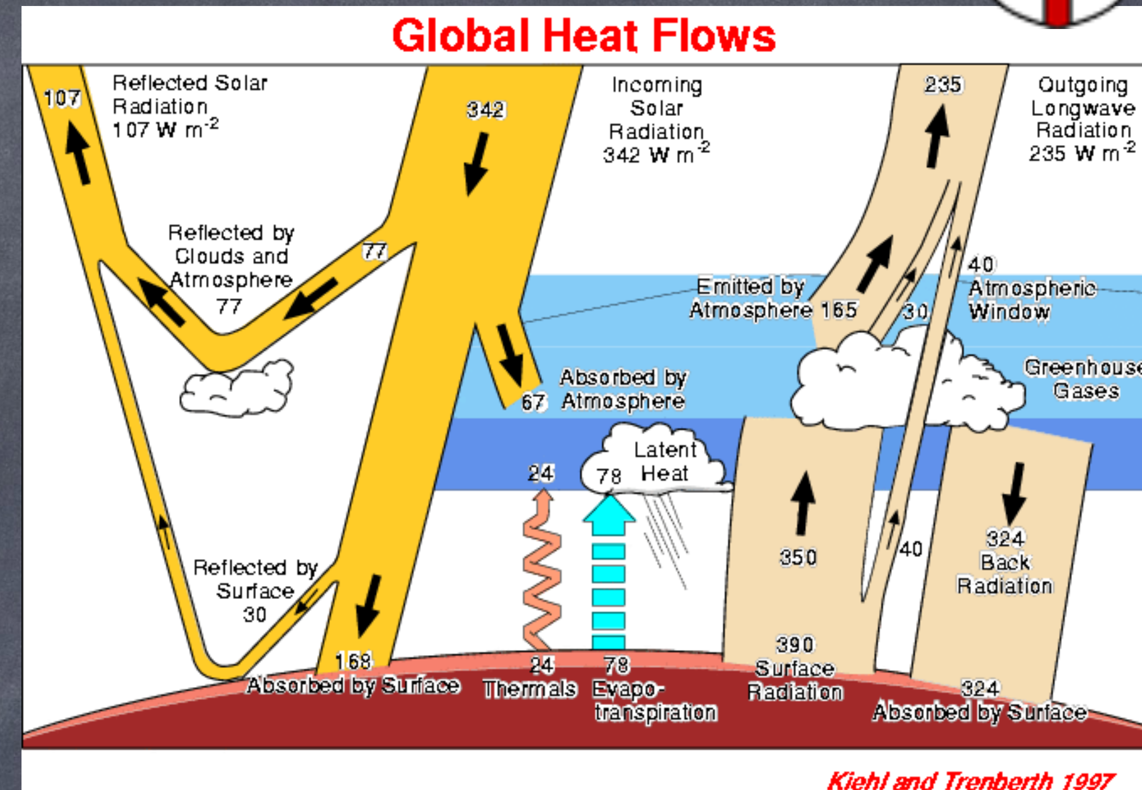
Satellite altimetry view of mesoscale flows

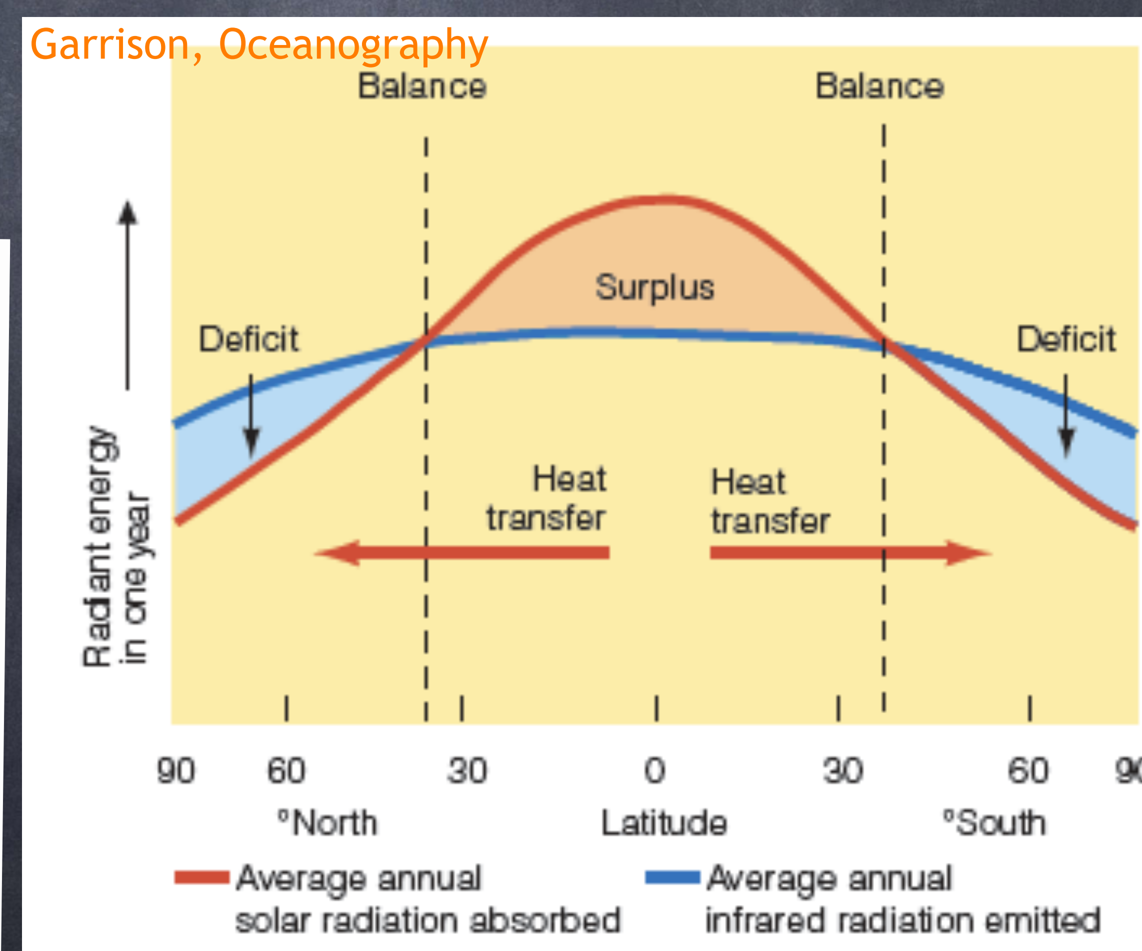


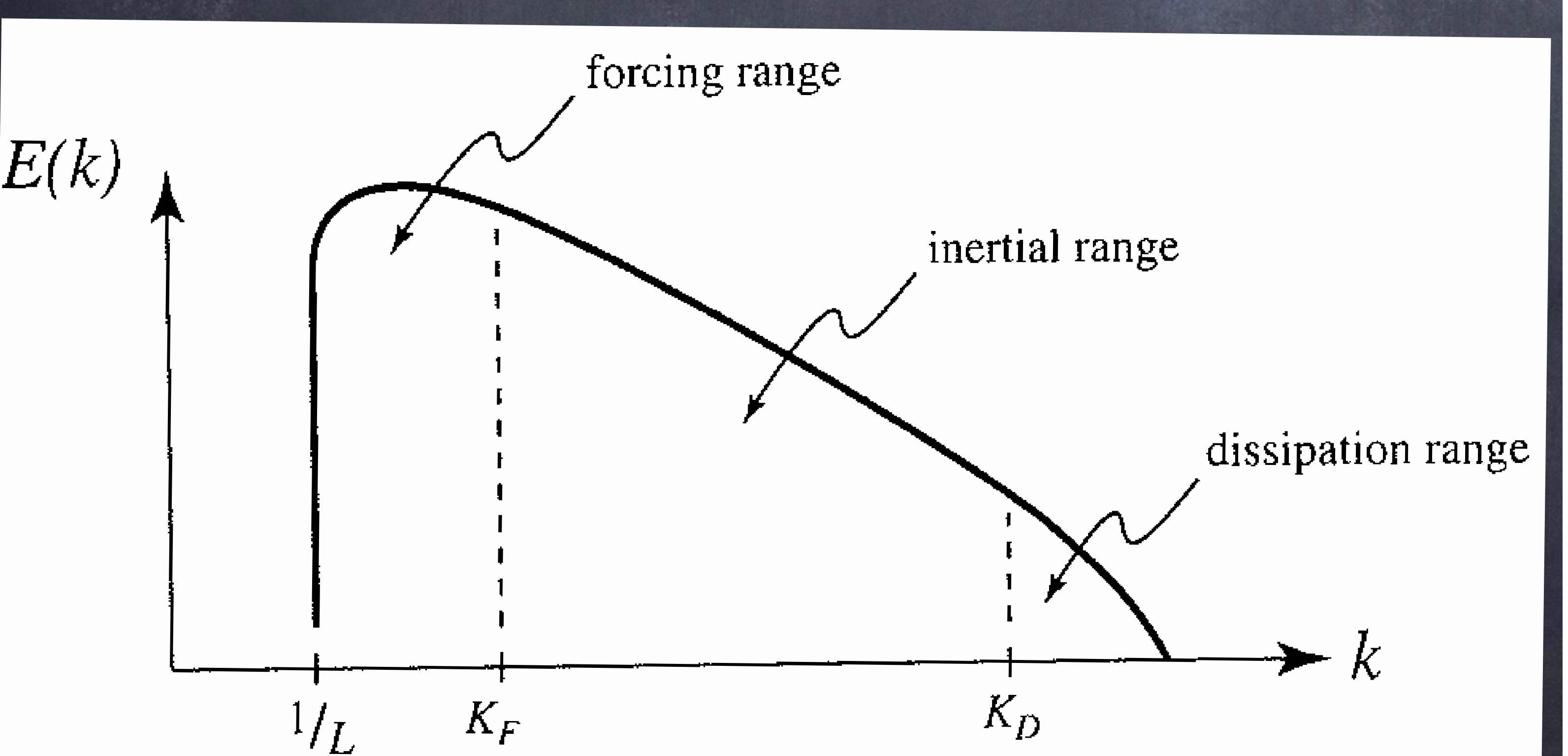


The Earth's Climate System is driven by the Sun's light (minus outgoing infrared) on a global scale

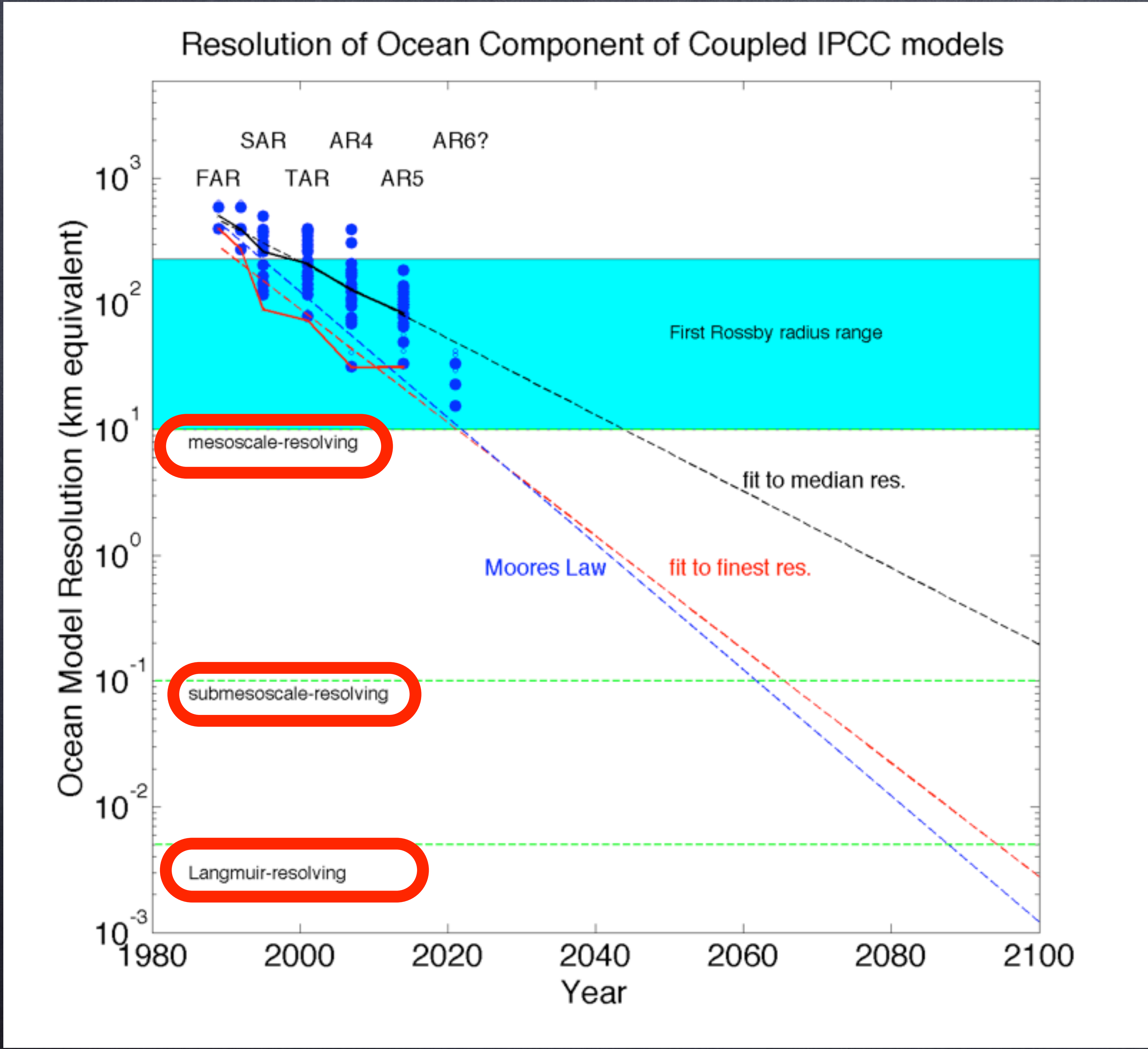
Dissipation concludes turbulence cascades to scales about a billion times smaller







Resolution will be an issue for centuries to come!



IPCC is a UN body that collates climate simulations from centers worldwide

If we can't resolve a process, we need to develop a parameterization or subgrid model of its effect

BFK, S. Bachman, B. Pearson, and S. Reckinger, 2014: Principles and advances in sub- grid modeling for eddy-rich simulations. CLIVAR Exchanges, 19(2):42–46.

Boussinesq Equations of Fluid Motion

Approx. Incompressible

$$\nabla_i u_i = 0, \quad b = \widetilde{b}(S, \Theta, P_0 - \rho_0 g z), \tag{8.2}$$

$$\frac{\partial S}{\partial t} + \nabla_i (u_i S) = \dot{S}, \quad \frac{\partial c}{\partial t} + \nabla_i (u_i c) = \dot{c}, \quad \frac{\partial}{\partial t}\Theta + \nabla_i (u_i \Theta) = \dot{\Theta}, \tag{8.3}$$

$$\frac{\partial u_j}{\partial t} + \epsilon_{jik} 2\Omega_i u_k + \nabla_j (u_j u_i) + \nabla_i p - b\delta_{zj} = \dot{u}_j$$

Coriolis

Pressure Grad. Force

buoyancy (gravity)

other

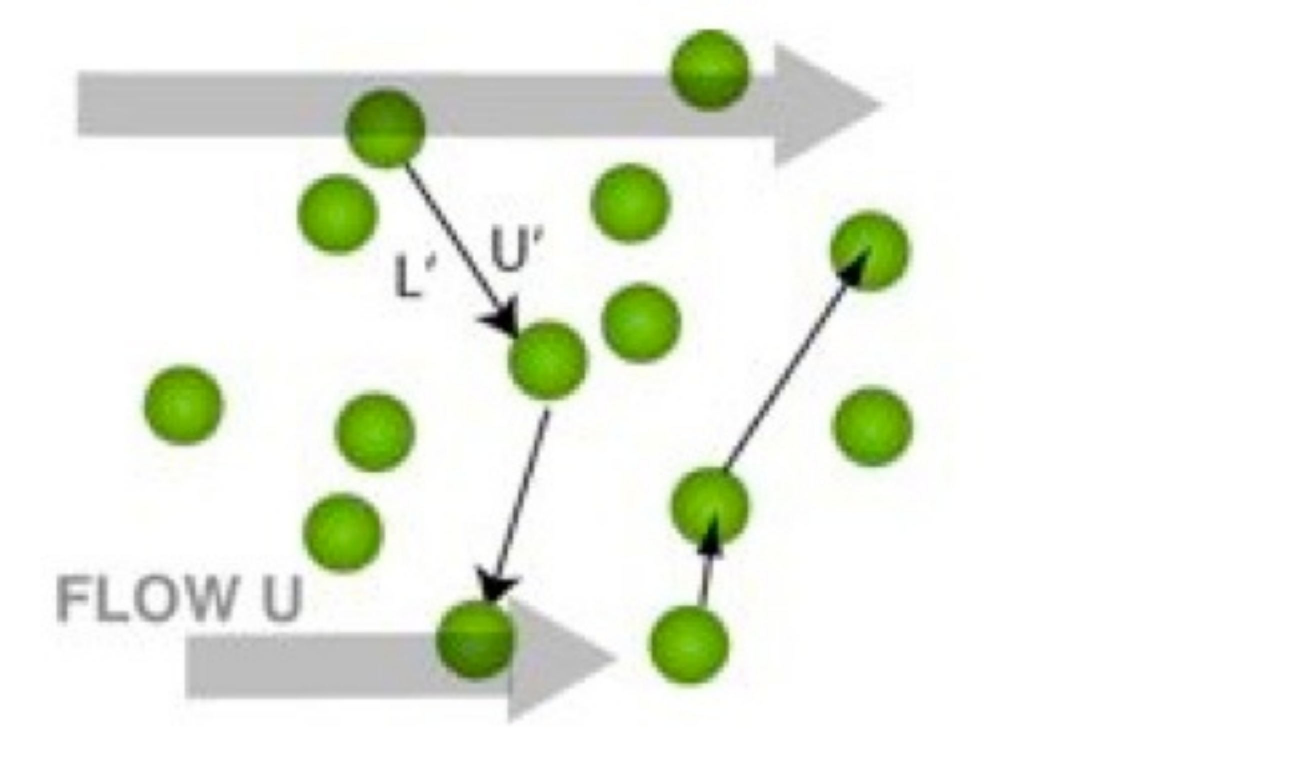
Choices are made in model representations...

- Subgrid parameterizations
  - "Do no harm" vs. "approximate unresolved scales"
- Resolution
  - "Permitting", "Resolving", Etc.
- These choices amount to establishing the "other" terms in the equations of motion relevant for large-scale motions.

E.G.: Molecular Viscosity

$$\mathbf{F} = \nabla \cdot \nu \nabla \mathbf{v}$$

Divergence Viscous Flux



$$\mathbf{F} \approx \nu \nabla^2 \mathbf{v}$$

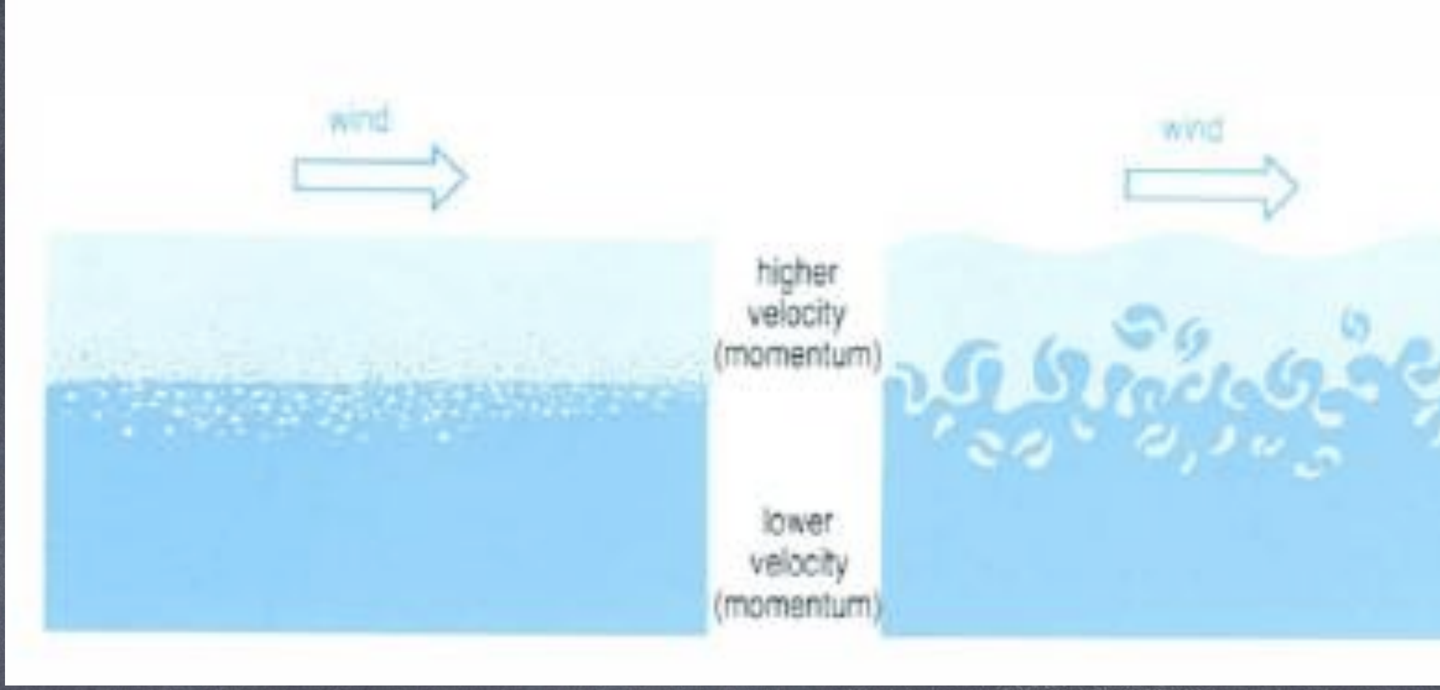
nearly constant viscosity

Laplacian is deviation from average of the neighbors

$$\mathbf{F} \propto \frac{\mathbf{v}(x+\Delta x, y) + \mathbf{v}(x-\Delta x, y) - 2\mathbf{v}(x,y)}{2\Delta x^2} + \frac{\mathbf{v}(x, y+\Delta y) + \mathbf{v}(x, y-\Delta y) - 2\mathbf{v}(x,y)}{2\Delta y^2}$$

Emery et al

Reynolds Stresses and eddy viscosity



$$\mathbf{F} = \nabla \cdot \nu \nabla \mathbf{v}$$

Divergence Viscous Flux

$$\mathbf{F_e} = -\overline{\mathbf{v}' \cdot \nabla \mathbf{v}'} = \nabla_i \cdot -\overline{\mathbf{v_i'} \mathbf{v_j'}}$$

Advection term Divergence Eddy Flux

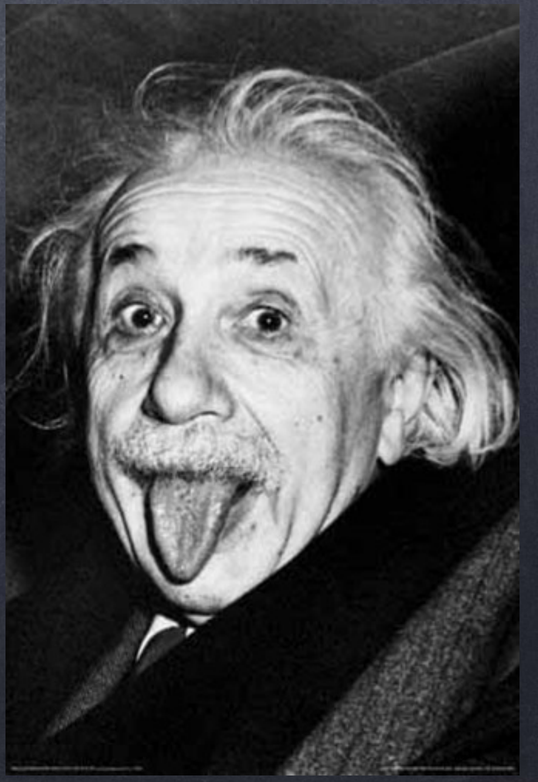
If eddies are like molecules, then

$$\mathbf{F_e} = \nabla_i \cdot -\overline{\mathbf{v_i'} \mathbf{v_j'}} \approx \nabla_i \cdot \nu_e \nabla \overline{\mathbf{v}}$$

That is, we could approximate turbulence with a bigger than molecular viscosity, the 'eddy viscosity'

Parameterizations

- Anyone who doesn't take truth seriously in small matters cannot be trusted in large ones either.
  - —A.E.



Different Uses, Different Needs

- MORANS (e.g., CESM; >50km)
- Mesoscale Ocean Reynolds-Averaged Navier-Stokes
- No small-scale instabilities resolved, all instabilities to be parameterized
- MOLES = SMORANS (e.g., grid 5-50km)
- Mesoscale Ocean Large Eddy Simulation
- Submesoscale Ocean Reynolds-Averaged Navier-Stokes
- Same Resolution, Different Parameterizations!
- SMOLES = BLORANS (e.g., grid 100m-1km)
- Submesoscale Ocean Reynolds-Averaged Navier-Stokes
- Boundary Layer Ocean Reynolds-Averaged Navier-Stokes
- BLOLES (e.g., grid 1-5m)
- Boundary Layer Ocean Large Eddy Simulation

ECCO2 Model

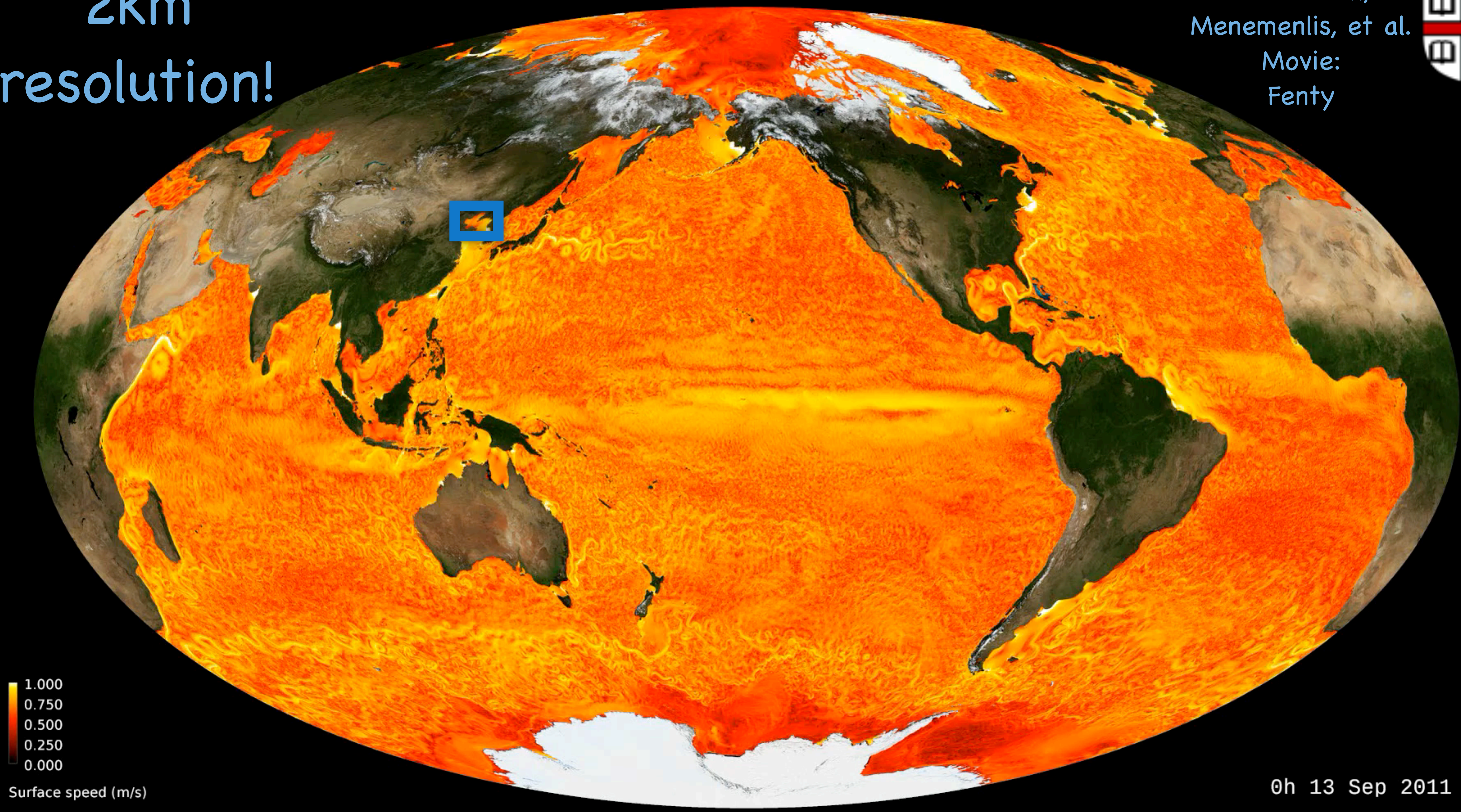
Viscosity Scheme: BFK and D. Menemenlis. Can large eddy simulation techniques improve mesoscale-rich ocean models? In M. Hecht and H. Hasumi, editors, Ocean Modeling in an Eddying Regime, volume 177, pages 319-338. AGU Geophysical Monograph Series, 2008.

18km resolution

LLC4320 Model

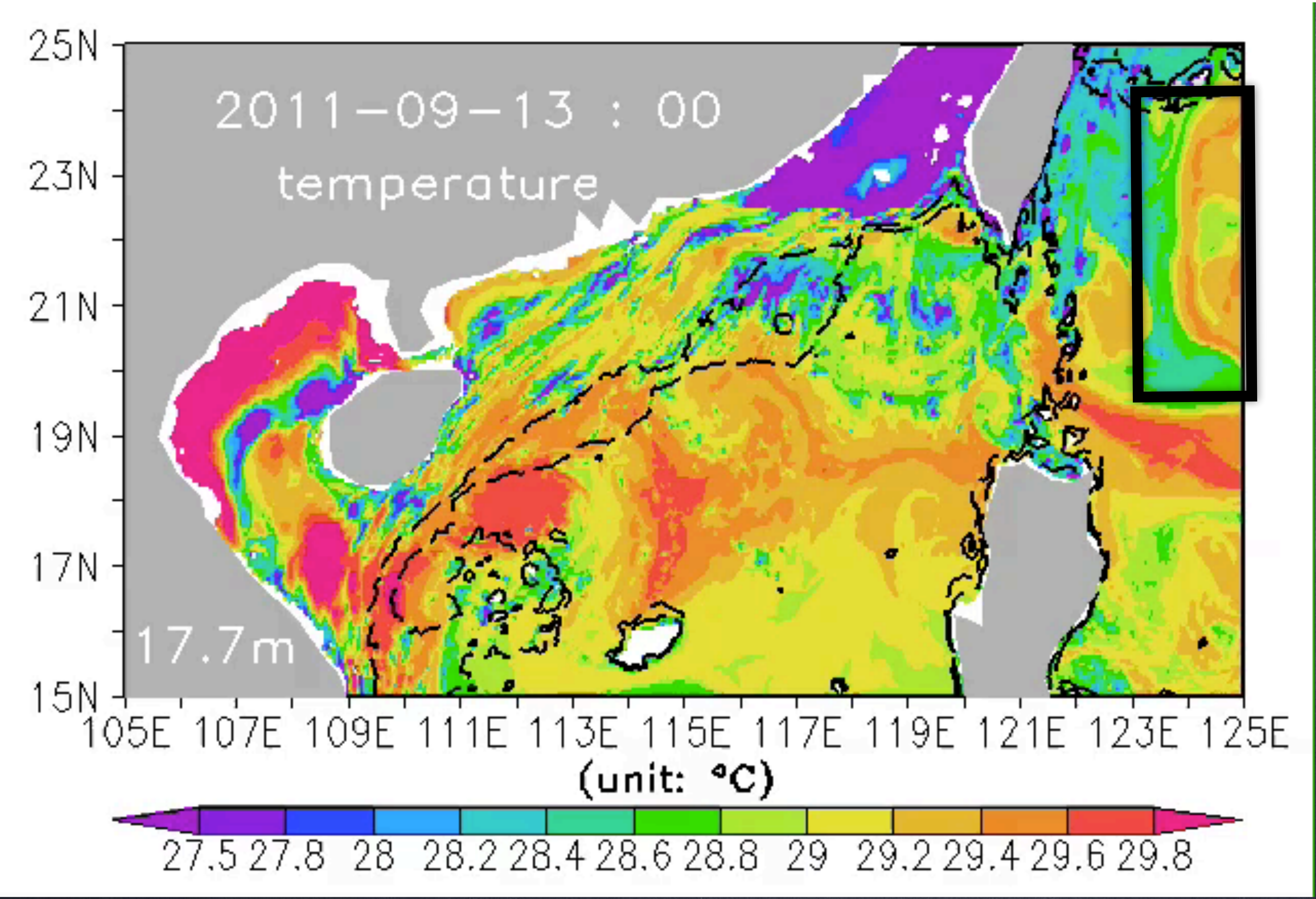
2km resolution!

Credit: Hill, Menemenlis, et al.
Movie: Fenty



B. Fox-Kemper, S. Bachman, B. Pearson, and S. Reckinger. Principles and advances in subgrid modeling for eddy-rich simulations. CLIVAR Exchanges, 19(2):42-46, July 2014.

Estimating the Circulation & Climate of the Ocean — LLC4320 Model



Movie:
Z. Jing

Brown Visitor from S. China Sea Institute of Ocean.

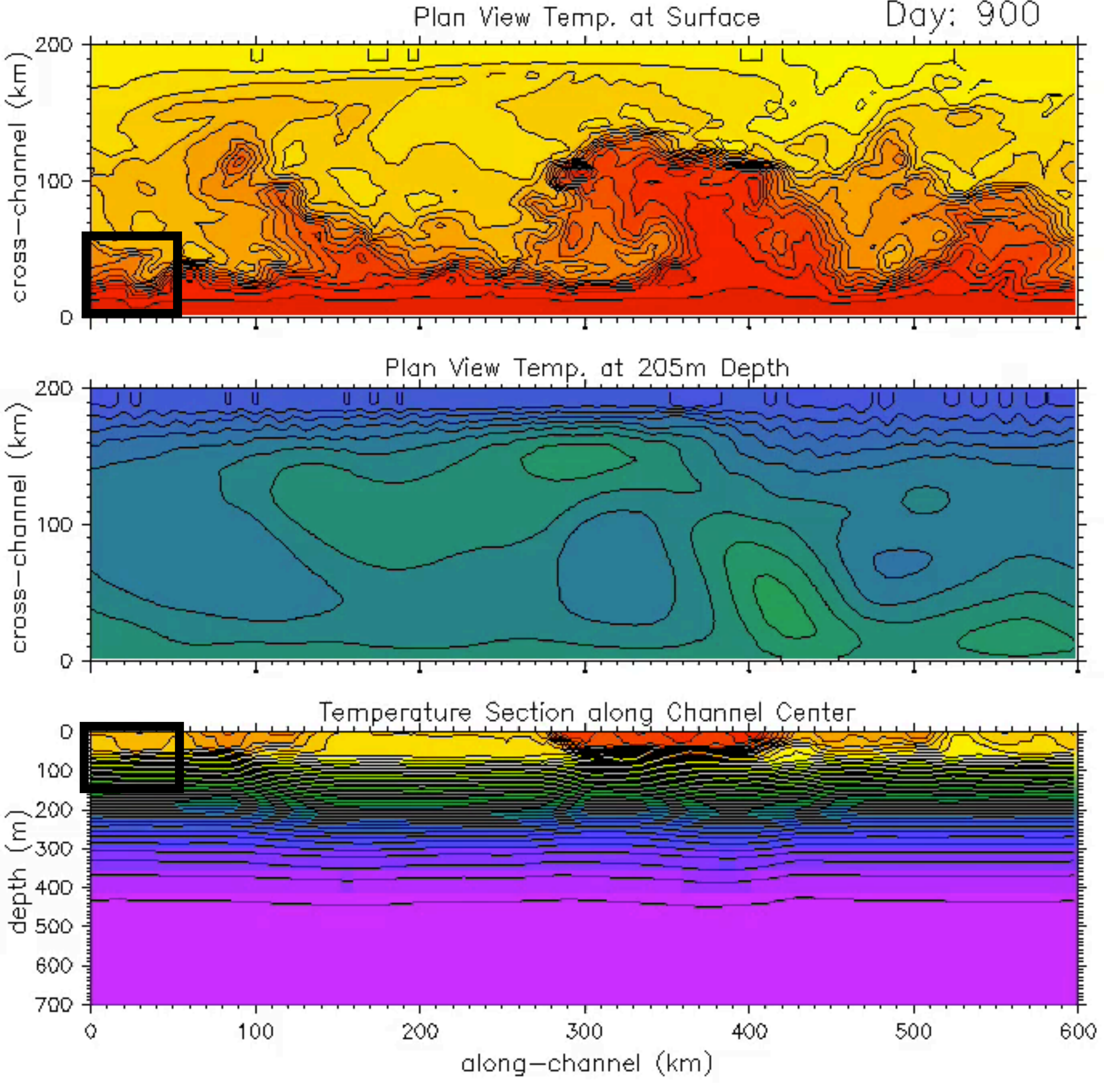
Local Analysis: Z. Jing, Y. Qi, BFK, Y. Du, and S. Lian. Seasonal thermal fronts and their associations with monsoon forcing on the continental shelf of northern South China Sea: Satellite measurements and three repeated field surveys in winter, spring and summer. Journal of Geophysical Research-Oceans, August 2015. In press.

200km x 600km x 700m domain

1000 Day Simulation

If we lose the globe, much higher resolution!

G. Boccaletti, R. Ferrari, and BFK. Mixed layer instabilities and restratification. Journal of Physical Oceanography, 37(9):2228-2250, 2007.



20km x 20km x 150m domain

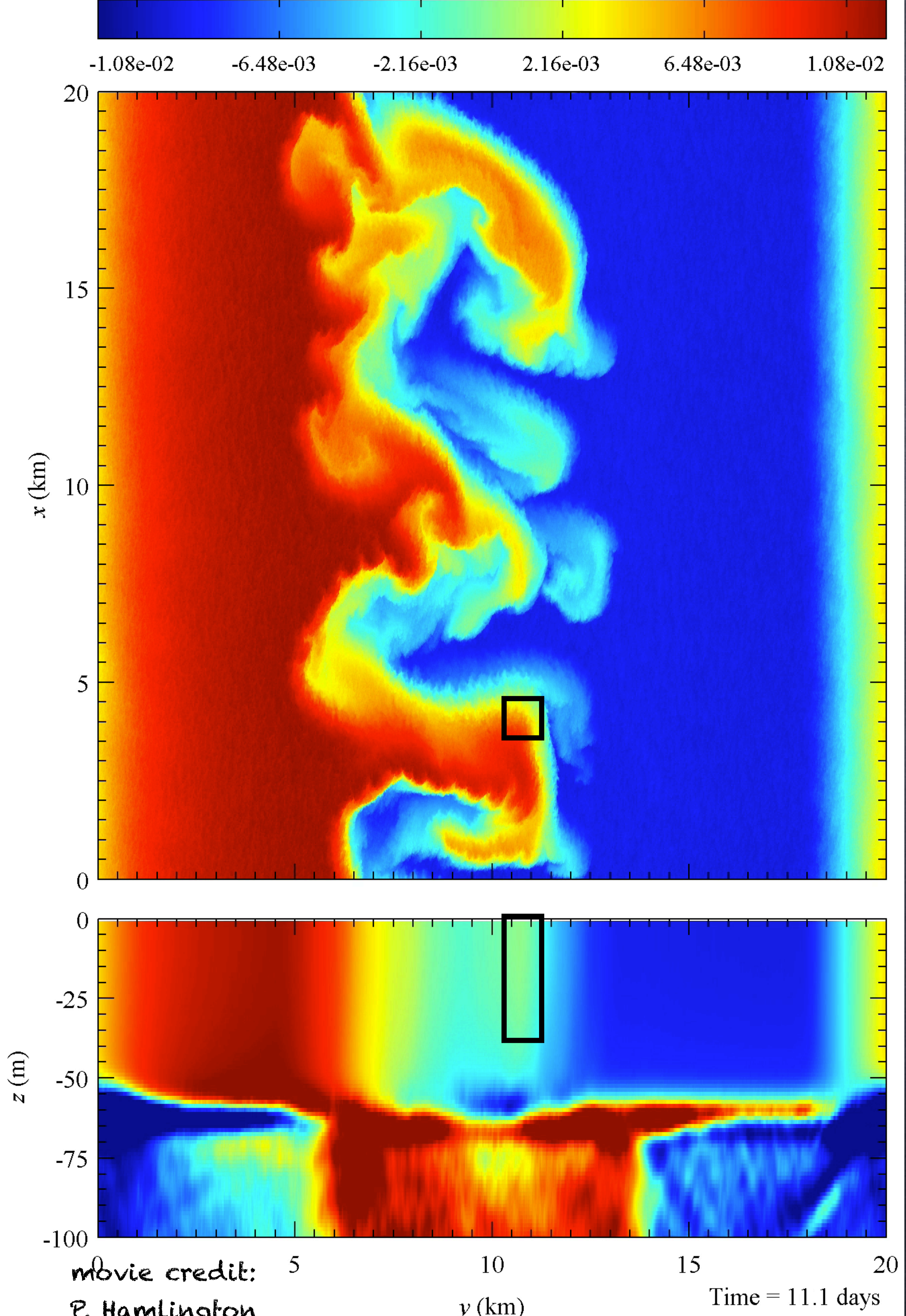
10 Day Simulation

4m x 4m x 1m Resolution


CU, now CU


CU, now LANL

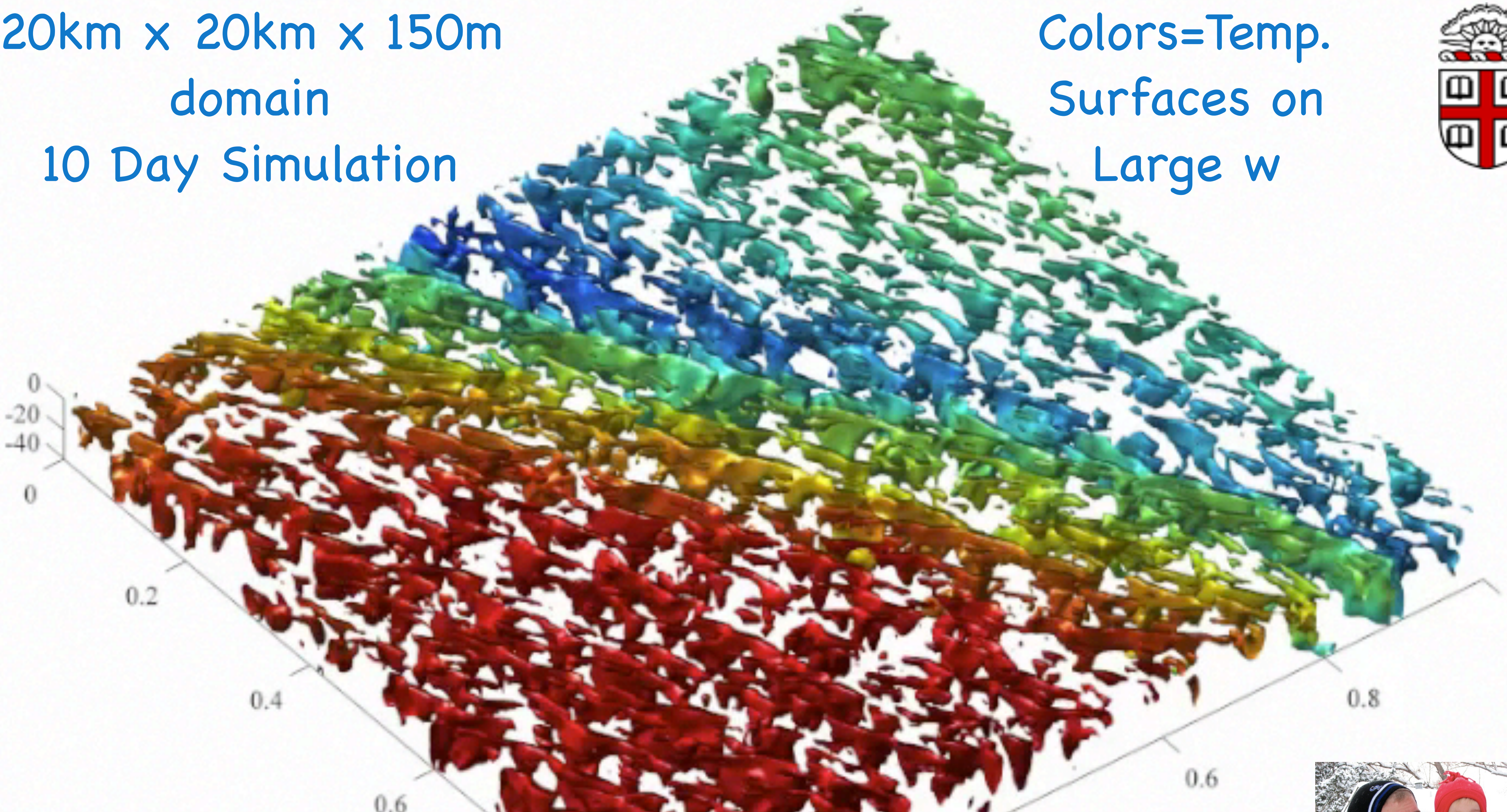
P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. Journal of Physical Oceanography, 44(9):2249-2272, September 2014.



movie credit: P. Hamlington

20km x 20km x 150m
domain
10 Day Simulation

Colors=Temp.
Surfaces on
Large w



1km x 1km x 40m
sub-domain
about 1 day shown


Brown


CU, now CU


CU, now LANL

movie credit:
P. Hamlington

N. Suzuki, BFK, P. E. Hamlington, L. P. Van Roekel. Surface waves affect frontogenesis. JGR-Oceans, 2016, submitted.

Key Concept for

Mesoscale Ocean Large Eddy Simulations (MOLES):

Gridscale* Dimensionless Parameters

Gridscale Reynolds[1]:

$$Re^* = \frac{U^* \Delta x}{\nu^*}$$

Gridscale Péclet[1]:

$$Pe^* = \frac{U^* \Delta x}{\kappa^*}$$

Gridscale Rossby:

$$Ro^* = \frac{U^*}{f \Delta x}$$

Gridscale Richardson:

$$Ri^* = \frac{\Delta b^* \Delta z}{\Delta {U^*}^2}$$

Gridscale Burger:

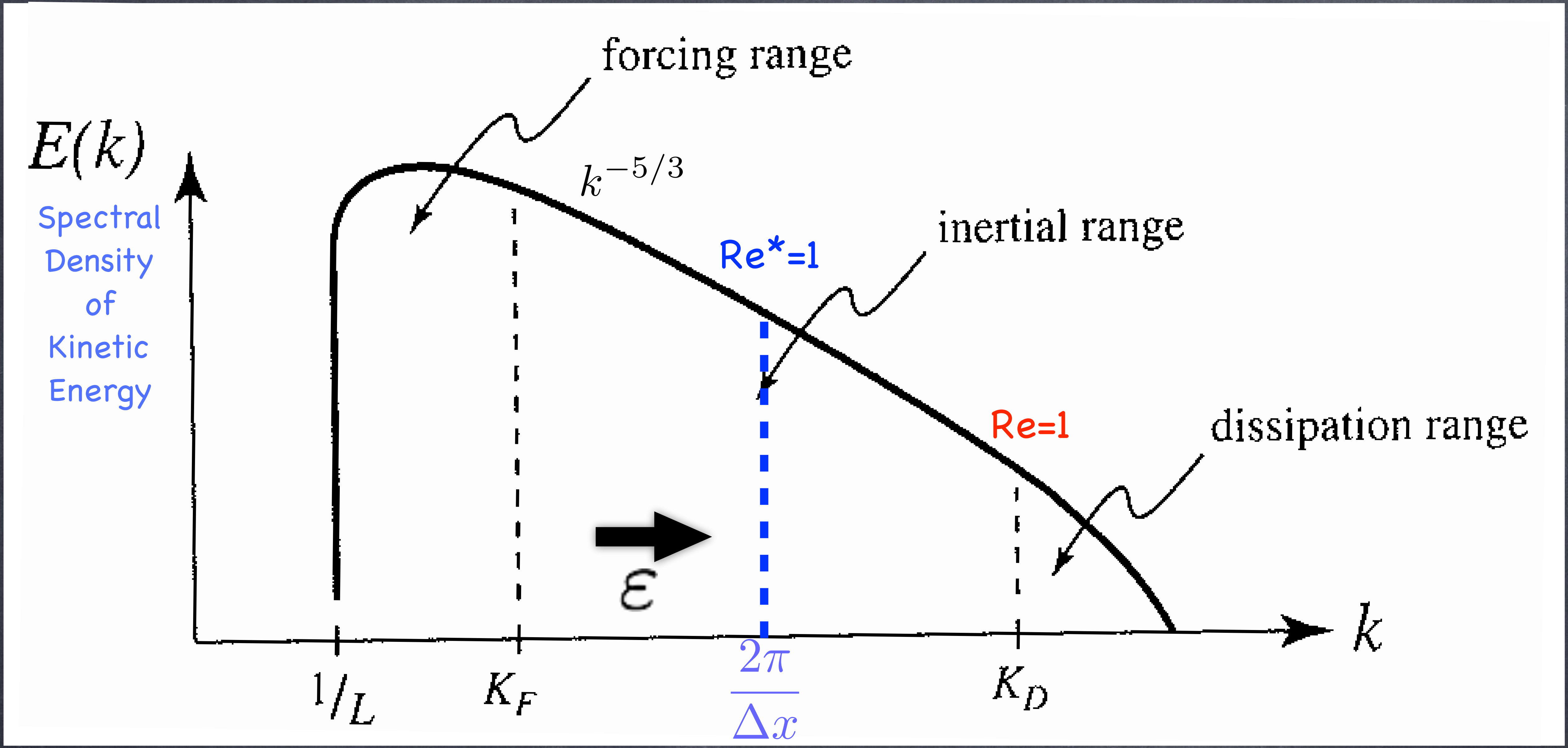
$$Bu^* = \frac{{N^*}^2 \Delta z^2}{f^2 \Delta x^2} = \frac{L_d^2}{\Delta x^2} \sim {Ro^*}^2 Ri^*$$

Asterisks denote *resolved* quantities, rather than true values

[1] Gridscale Reynolds and Péclet numbers MUST be O(1) for numerical stability

B. Fox-Kemper and D. Menemenlis. Can large eddy simulation techniques improve mesoscale-rich ocean models? In M. Hecht and H. Hasumi, editors, Ocean Modeling in an Eddying Regime, volume 177, pages 319-338. AGU Geophysical Monograph Series, 2008.

3D Turbulence Cascade



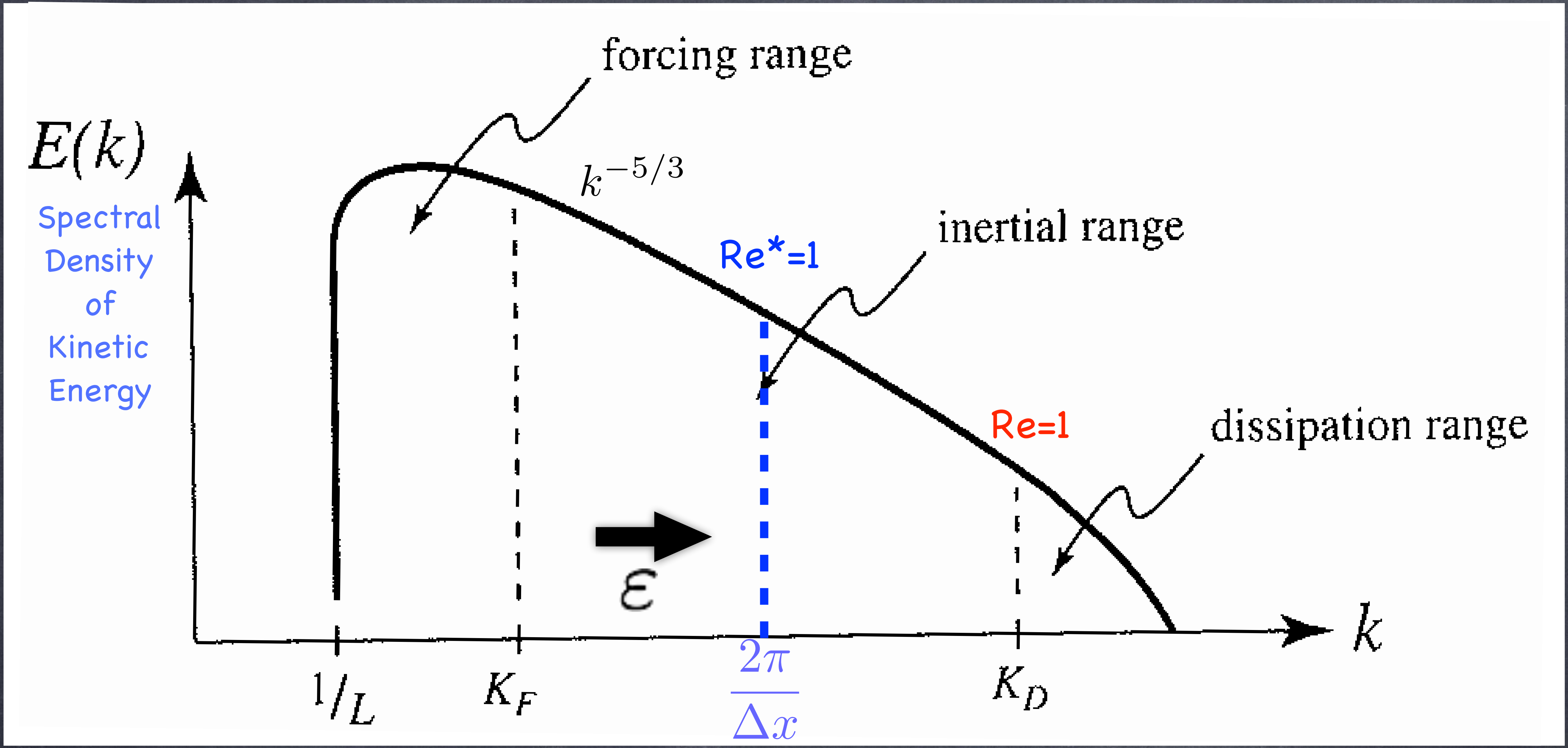
1963: Smagorinsky Scale & Flow Aware Viscosity Scaling,

So the Energy Cascade is Preserved,

but order-1 gridscale Reynolds #: $Re^* = UL/\nu_*$

$$\nu_{*h} = \left(\frac{\Upsilon_h \Delta x}{\pi}\right)^2 \sqrt{\left(\frac{\partial u_*}{\partial x} - \frac{\partial v_*}{\partial y}\right)^2 + \left(\frac{\partial u_*}{\partial y} + \frac{\partial v_*}{\partial x}\right)^2}$$

3D Turbulence Cascade



$$\dot{u}_j = -\nabla_i(\overline{u'_i u'_j}) \approx \nabla_i \nu_* \nabla_i \overline{u}_{*j}$$

$$\nu_{*h} = \left(\frac{\Upsilon_h \Delta x}{\pi}\right)^2 \sqrt{\left(\frac{\partial u_*}{\partial x} - \frac{\partial v_*}{\partial y}\right)^2 + \left(\frac{\partial u_*}{\partial y} + \frac{\partial v_*}{\partial x}\right)^2}$$

Careful to preserve symmetries!

$$\dot{u}_j = -\nabla_i(\overline{u'_i u'_j}) \approx \nabla_i \nu_* \nabla_i \overline{u}_{*j}$$

Divergence of a symmetric tensor: Req'd for conservation of angular momentum.

?

$$\text{symmetrize? : } \frac{1}{2}\left(\nabla_i \overline{u}_{*j} + \nabla_j \overline{u}_{*i}\right)$$

Mathematicians don't like this...

ICERM

The Institute for Computational and Experimental Research in Mathematics (ICERM)

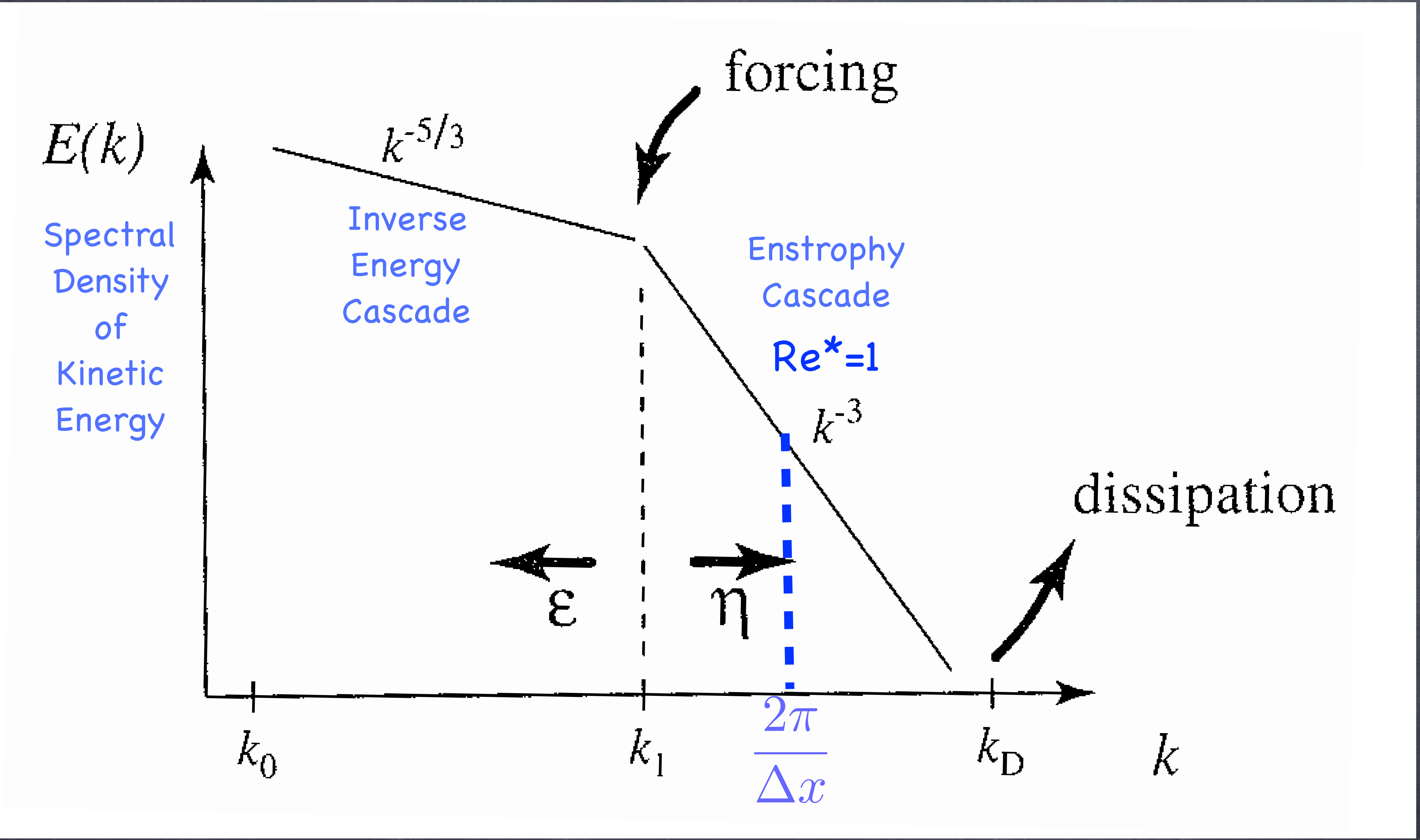
Program at ICERM, now, elaborate search for weak solutions of equations, e.g., Boussinesq equations with molecular viscosity. Analytic finesse req'd to handle nonlinear terms.

LES introduces *new nonlinearities* through the flow-aware parameterizations, which may require new analytic approaches.

$$\dot{u}_j = -\nabla_i(\overline{u'_i u'_j}) \approx \nabla_i \nu_* \nabla_i \overline{u}_{*j}$$

2D Turbulence Differs

R. Kraichnan, 1967 JFM

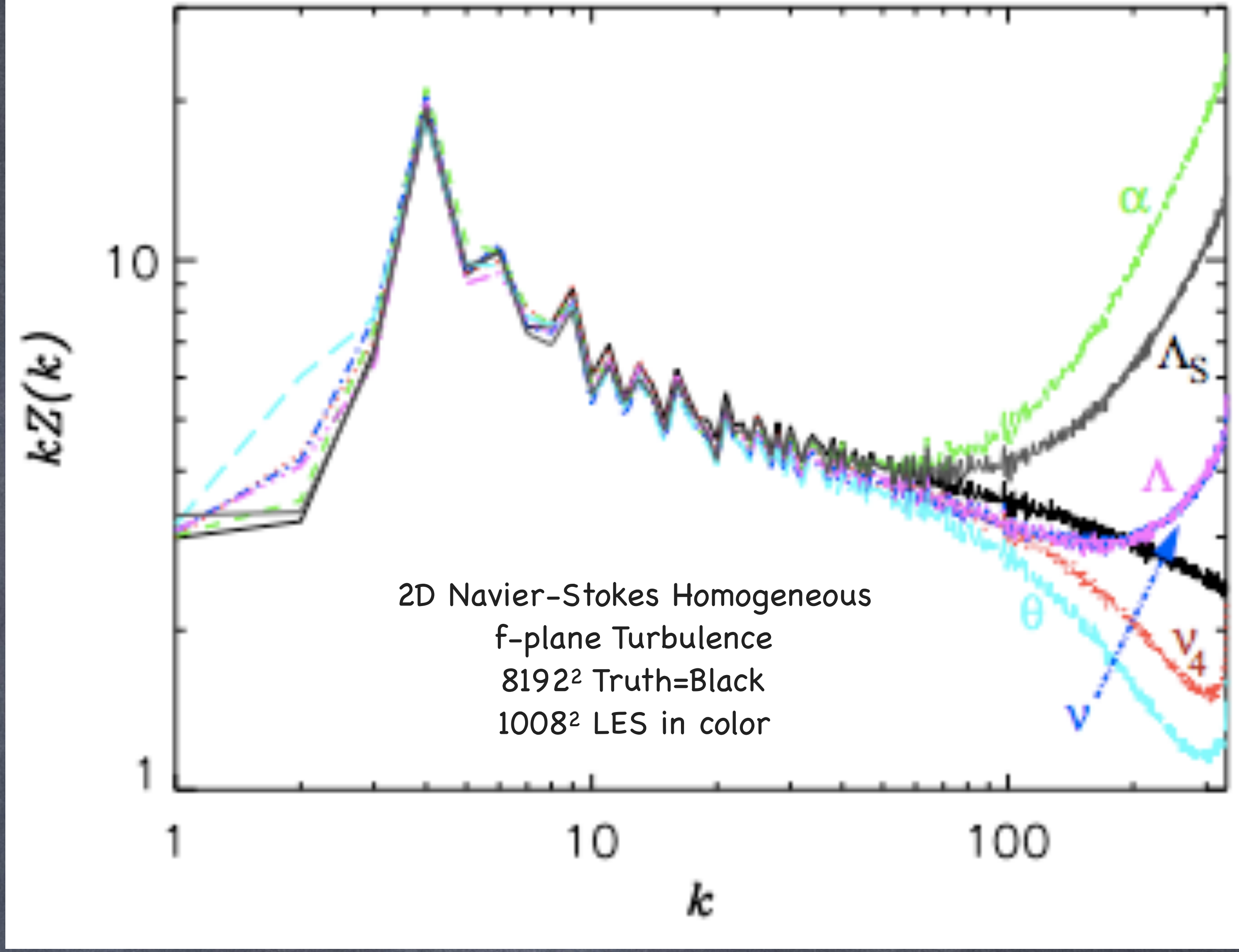


1996: Leith Devises Viscosity Scaling,
So that the Enstrophy (vorticity²) Cascade is Preserved

$$\nu_* = \left(\frac{\Lambda\Delta x}{\pi}\right)^3 \left|\nabla_h\left(\frac{\partial u_*}{\partial y} - \frac{\partial v_*}{\partial x}\right)\right|$$

Barotropic or stacked layers

Some MOLES Truncation Methods In Use 2d (shallow water) test



Graham & Ringler, 2013 Ocean Modelling

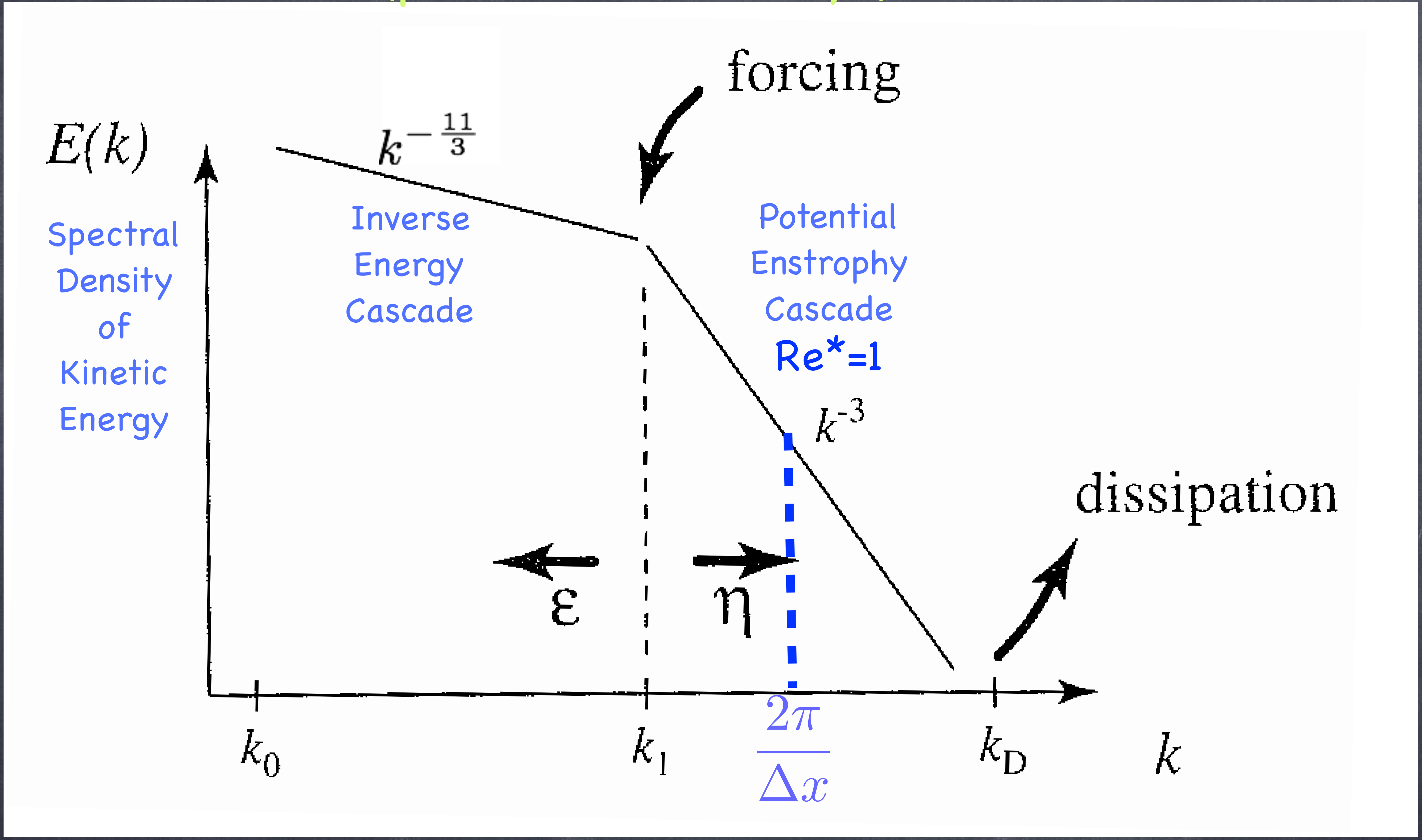
- Harmonic/Biharmonic/Numerical
  - Many. Often not scale- or flow-aware
  - Griffies & Hallberg, 2000, is one aware example
- Fox-Kemper & Menemenlis, 2008. ECCO2.
  - Leith Viscosity (2d Enstrophy Scaling)
- Chen, Q., Gunzburger, M., Ringler, T., 2011
  - Anticipated Potential Vorticity of Sadourny
- San, Staples, Iliescu (2011, 2013)
  - Approximate Deconvolution Method
- Stochastic & Statistical Parameterizations
  - Other session going on now in Y10

In this comparison, untuned Leith beats: tuned harmonic, tuned biharmonic, Smagorinsky, LANS-alpha, & Anticipated PV

See also Ramachandran et al, 2013 Ocean Modelling for SMOLES

QG Turbulence: Pot'l Enstrophy cascade (potential vorticity²)

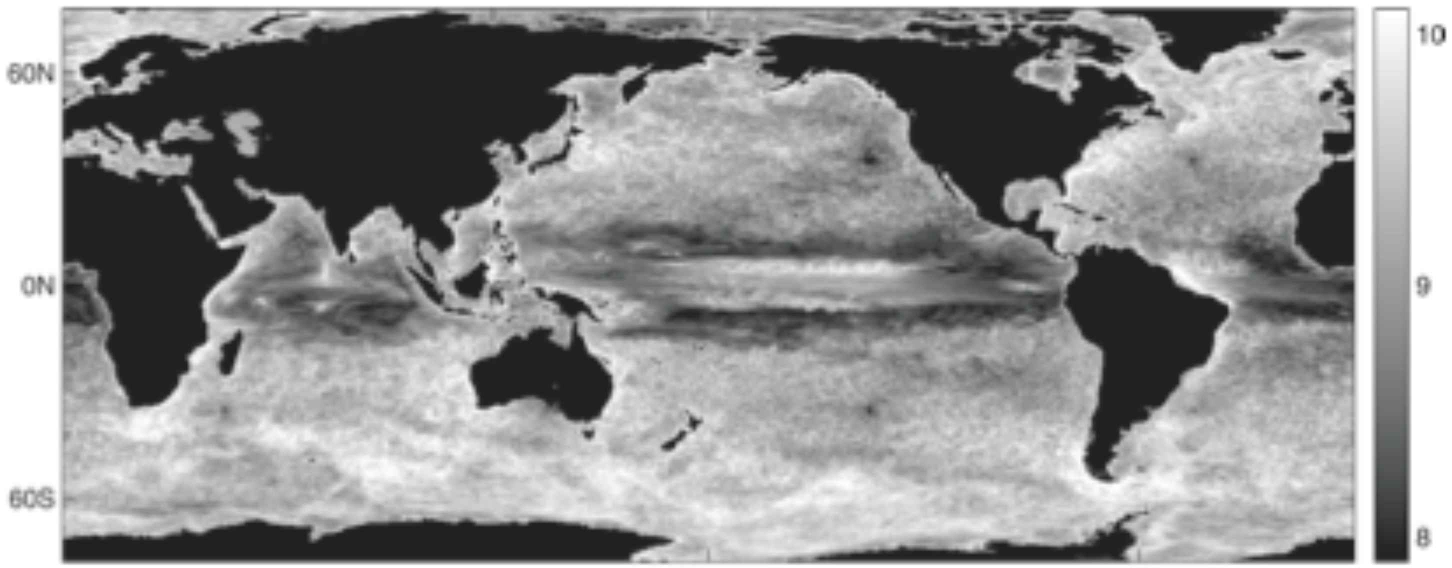
J. Charney, 1971 JAS



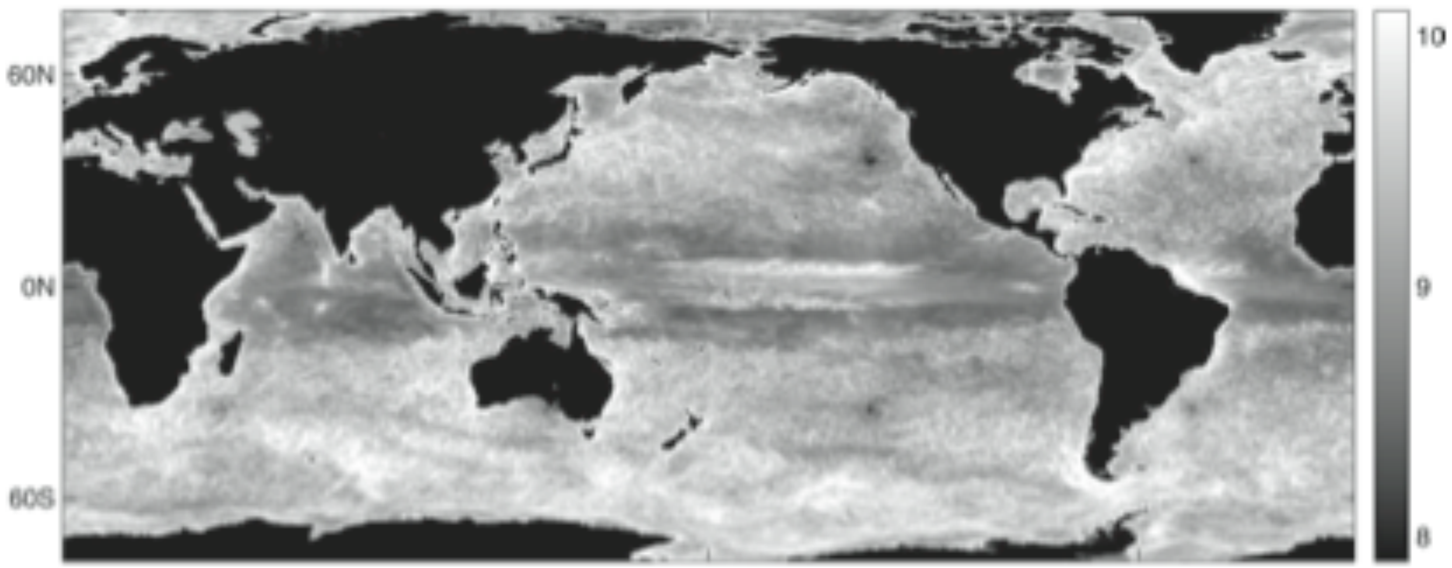
BFK & Menemenlis '08: Revise Leith Viscosity Scaling, So that diverging, vorticity-free, modes are also damped

$$\nu_* = \left(\frac{\Delta x}{\pi}\right)^3 \sqrt{\Lambda^6 |\nabla_h q_{2d}|^2 + \Lambda_d^6 |\nabla_h (\nabla_h \cdot \mathbf{u}_*)|^2}$$

BFK and D. Menemenlis. Can large eddy simulation techniques improve mesoscale-rich ocean models? In M. Hecht and H. Hasumi, editors, Ocean Modeling in an Eddying Regime, volume 177, pages 319-338. AGU Geophysical Monograph Series, 2008.



viscosity from Leith '96



viscosity from BFK & Menemenlis '08

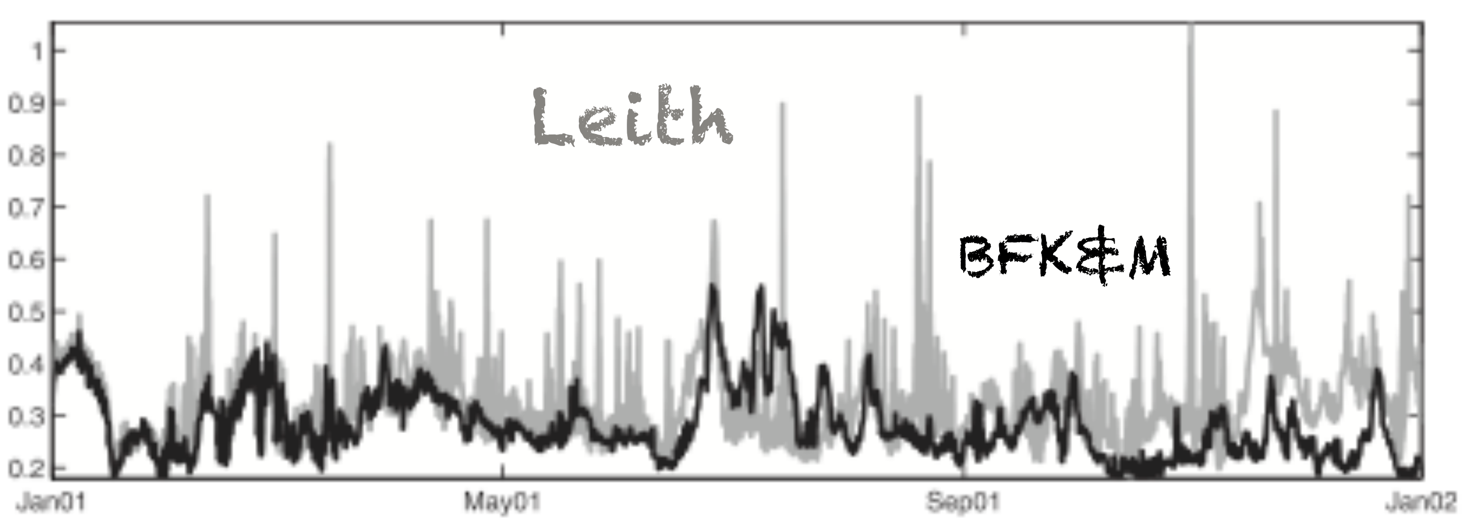


Figure 4. Maximum Courant number, $w\Delta t/\Delta z$, for vertical advection. Gray line is from the *LeithOnly* integration, and black line is from the *LeithPlus* integration.

CFL condition on vert. velocity

BFK and D. Menemenlis. Can large eddy simulation techniques improve mesoscale-rich ocean models? In M. Hecht and H. Hasumi, editors, Ocean Modeling in an Eddying Regime, volume 177, pages 319-338. AGU Geophysical Monograph Series, 2008.

ECCO2 Model

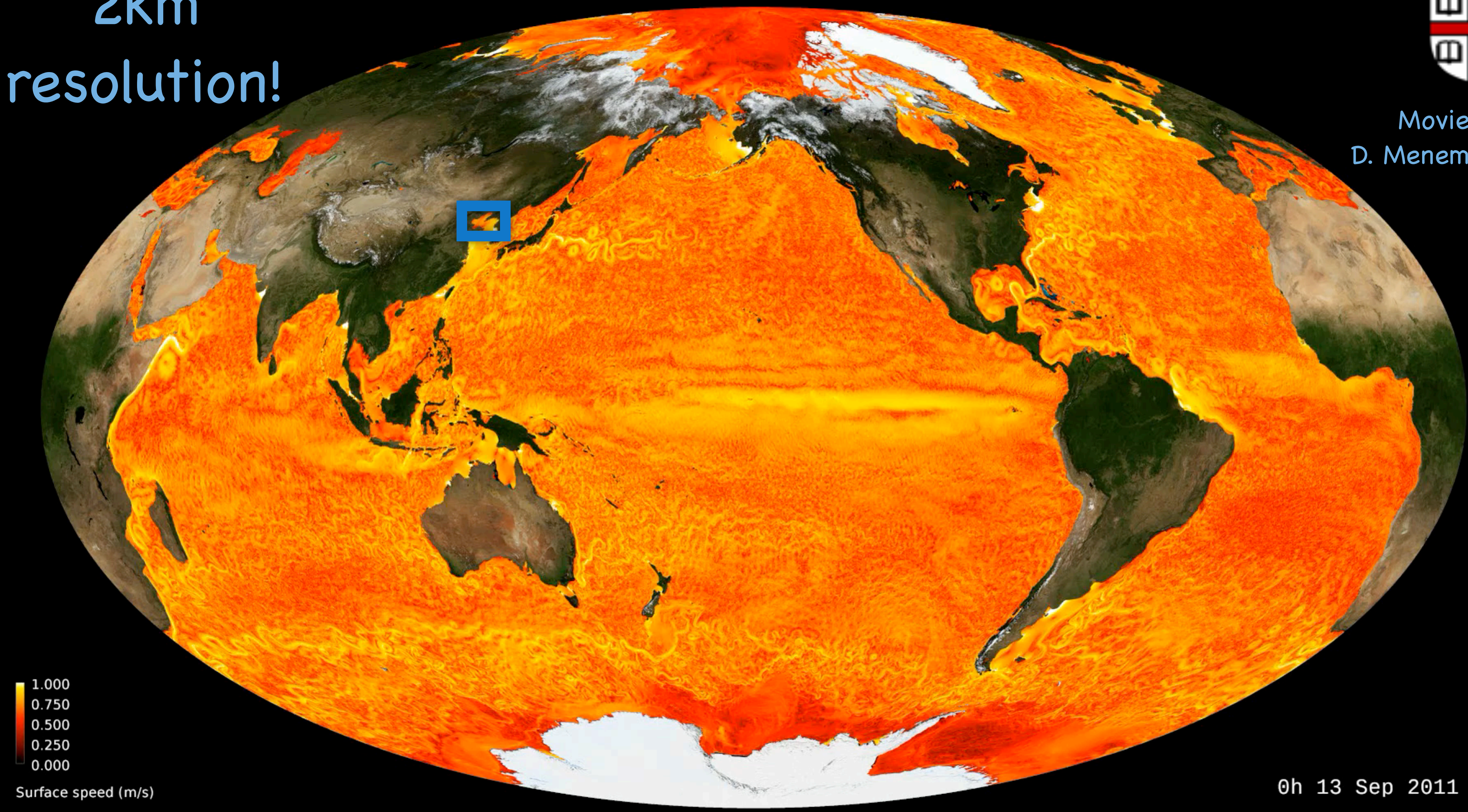
Viscosity Scheme: BFK and D. Menemenlis. Can large eddy simulation techniques improve mesoscale-rich ocean models? In M. Hecht and H. Hasumi, editors, Ocean Modeling in an Eddying Regime, volume 177, pages 319-338. AGU Geophysical Monograph Series, 2008.

18km resolution

LLC4320 Model

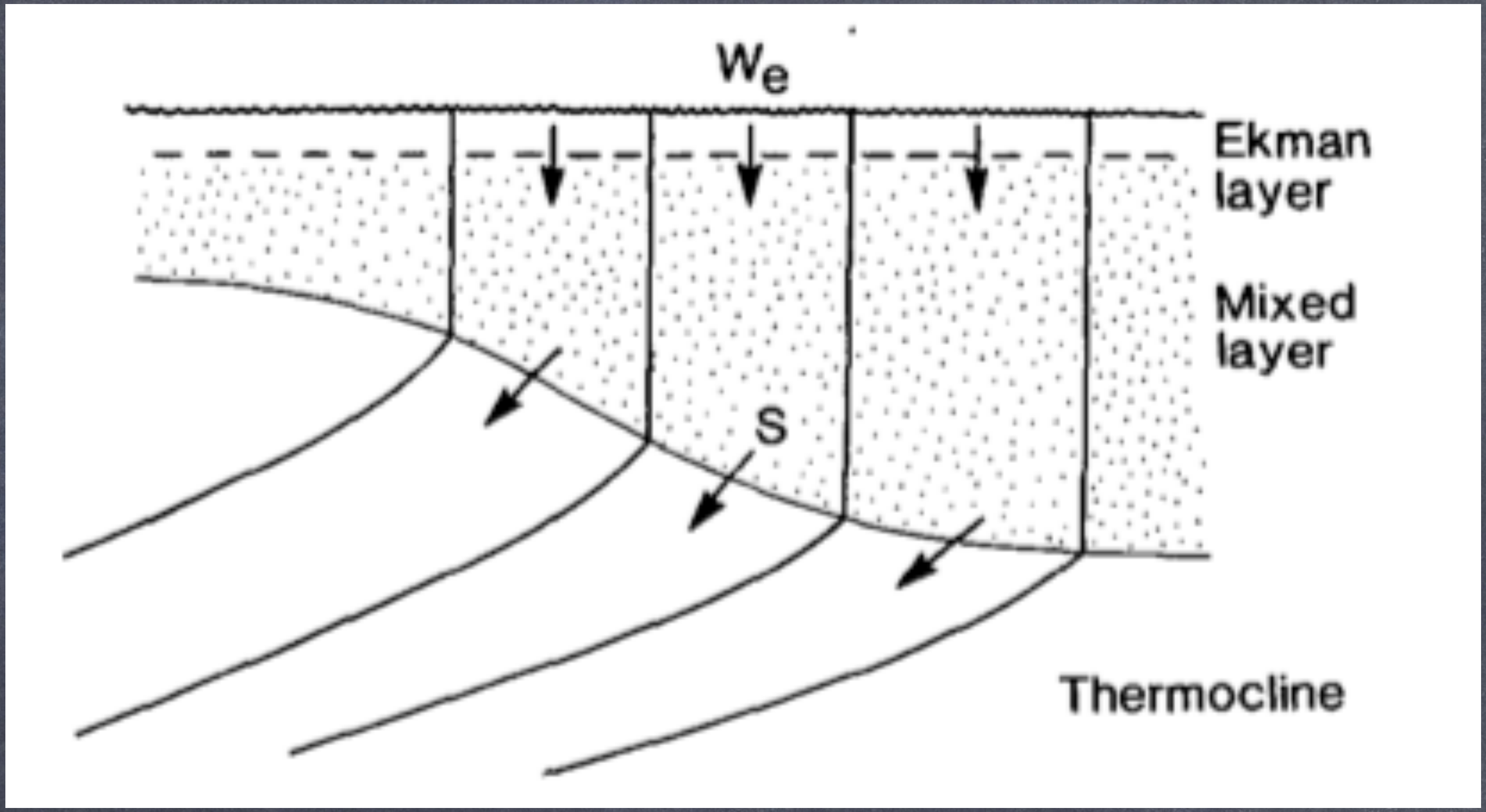
2km resolution!

Movie: D. Menemenlis



B. Fox-Kemper, S. Bachman, B. Pearson, and S. Reckinger. Principles and advances in subgrid modeling for eddy-rich simulations. CLIVAR Exchanges, 19(2):42-46, July 2014.

Is 2D Turbulence a good proxy for stratified flow?



Nurser & Marshall, 1991 JPO

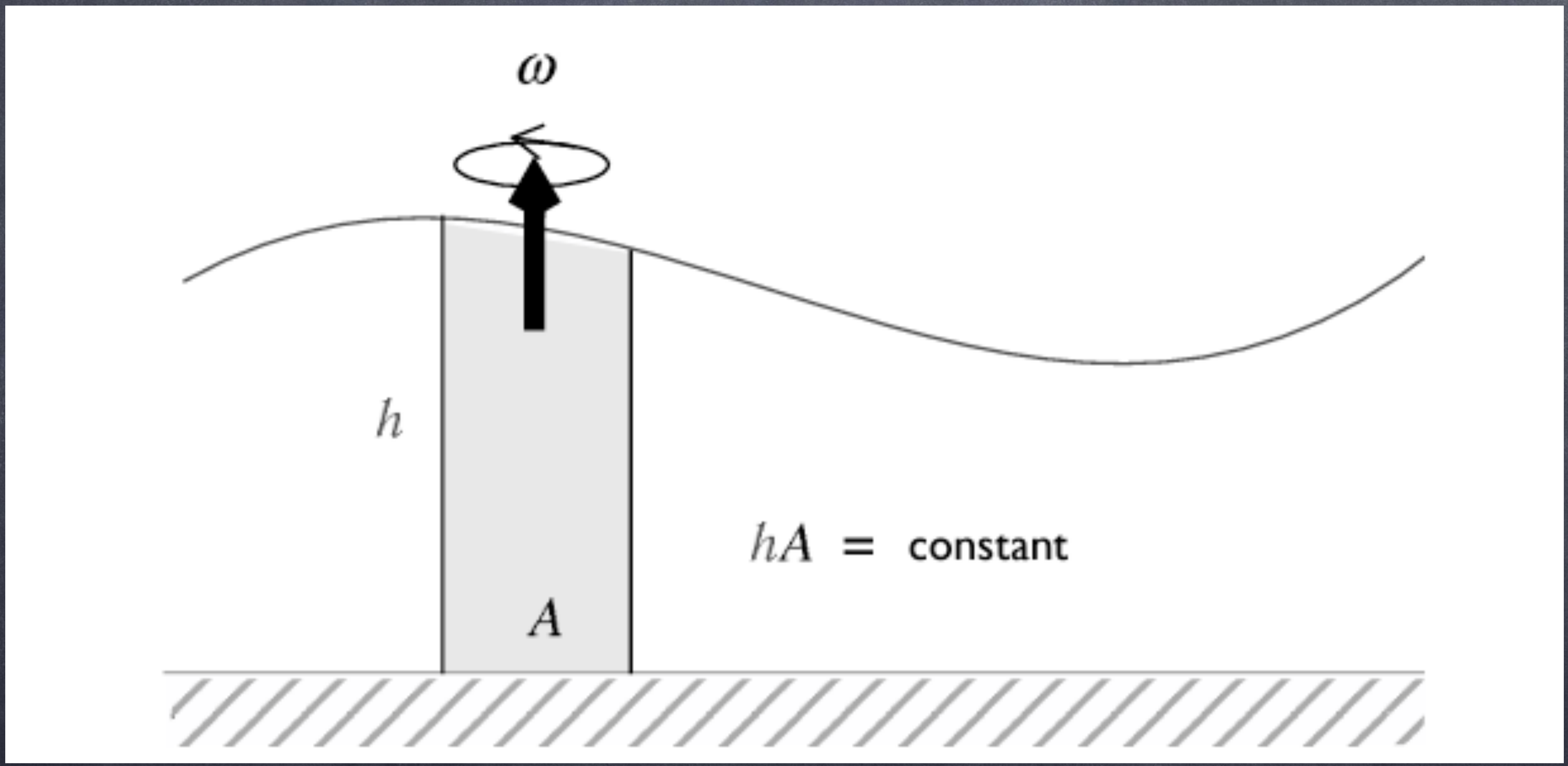
Yes:

- For a few eddy time-scales QG & 2D AGREE (Bracco et al. '04)
- Barotropic Flow--Obvious 2d analogue

No:

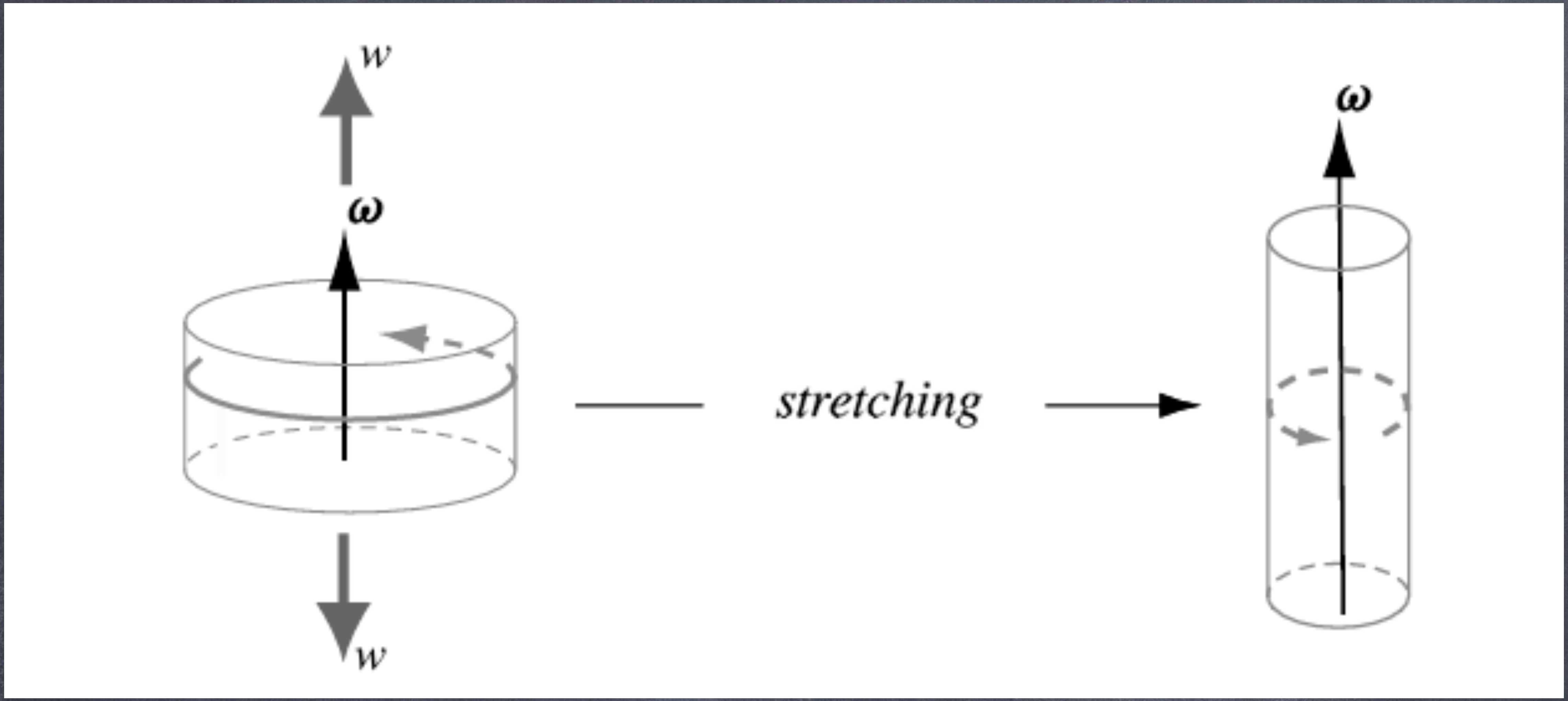
- Eddy Fluxes--Divergent 2d flow & advective fluxes
- Sloped, not horiz.
- Surface Effects?

Potential Vorticity in shallow water eqtns:



Vorticity: $\omega = \nabla \times \mathbf{v}$

Stretching & Squashing



Potential Vorticity: $\frac{\hat{k} \cdot \omega}{h} = \frac{\hat{k} \cdot \nabla \times \mathbf{v}}{h}$

QG Equations

$$\partial_t q_q + J(\psi, q_q) - D_{q_q} = O(\epsilon \mathrm{Ro}_*/\mathrm{M}_{R_*}, \epsilon \mathrm{Pl}_*/\mathrm{M}_{R_*}),$$

$$q_q = 1 + \mathrm{Ro}_* \left(\nabla_h^2 \psi + \mathrm{Bu}_*^{-1} \partial_z \frac{\partial_z \psi}{\partial_z \bar{b}} \right) + \frac{\mathrm{Pl}_* y}{\Delta y},$$

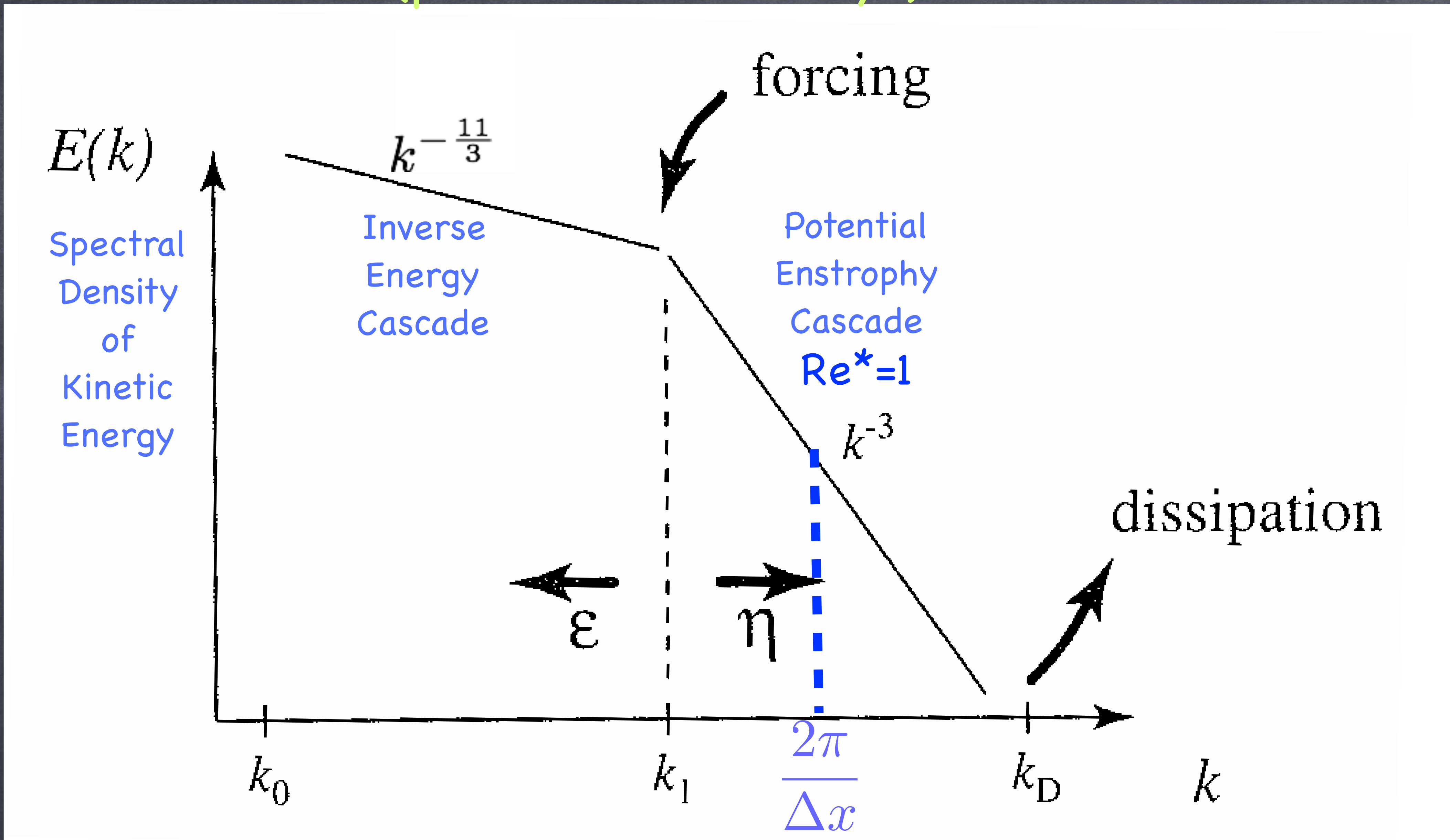
- Potential vorticity is the sole unknown (simplification from u,v,w,S,T,p)
- PV can be related to advection (streamfct) under specific conditions
- These equations are good asymptotic approximations to full Boussinesq for intermediate scales on Earth: (mesoscale—100km, weeks)

Charney 46; Charney 71; Pedlosky 86

QG Turbulence: Pot'l Enstrophy cascade (potential vorticity²)

J. Charney, 1971 JAS

BFK and D. Menemenlis. Can large eddy simulation techniques improve mesoscale-rich ocean models? In M. Hecht and H. Hasumi, editors, Ocean Modeling in an Eddying Regime, volume 177, pages 319-338. AGU Geophysical Monograph Series, 2008.



$$\nu_{qg} = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^3 |\nabla q_{qg}| = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^3 \left|\nabla_h\left[\beta y + \nabla_h^2\psi + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial\psi}{\partial z}\right)\right]\right|.$$

$$q_{2d}^* = f + \hat{k}\cdot\nabla\times u^*$$

$$q_{qg}^* = f + \hat{k}\cdot\nabla\times u^* + \frac{\partial}{\partial z}\frac{f^2}{N^2}b^*$$

$$\nu_{qg} = \kappa_{Redi} = \kappa_{GM} = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^3 |\nabla q_{qg}|.$$

QG vs. 2D

$$\nu_{qg}^{*} = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^{3} |\nabla q_{qg}^{*}|$$

Different (Pot'l) Vorticity Gradients:

$$q_{2d}^{*} = f + \hat{k} \cdot \nabla \times u^{*}$$

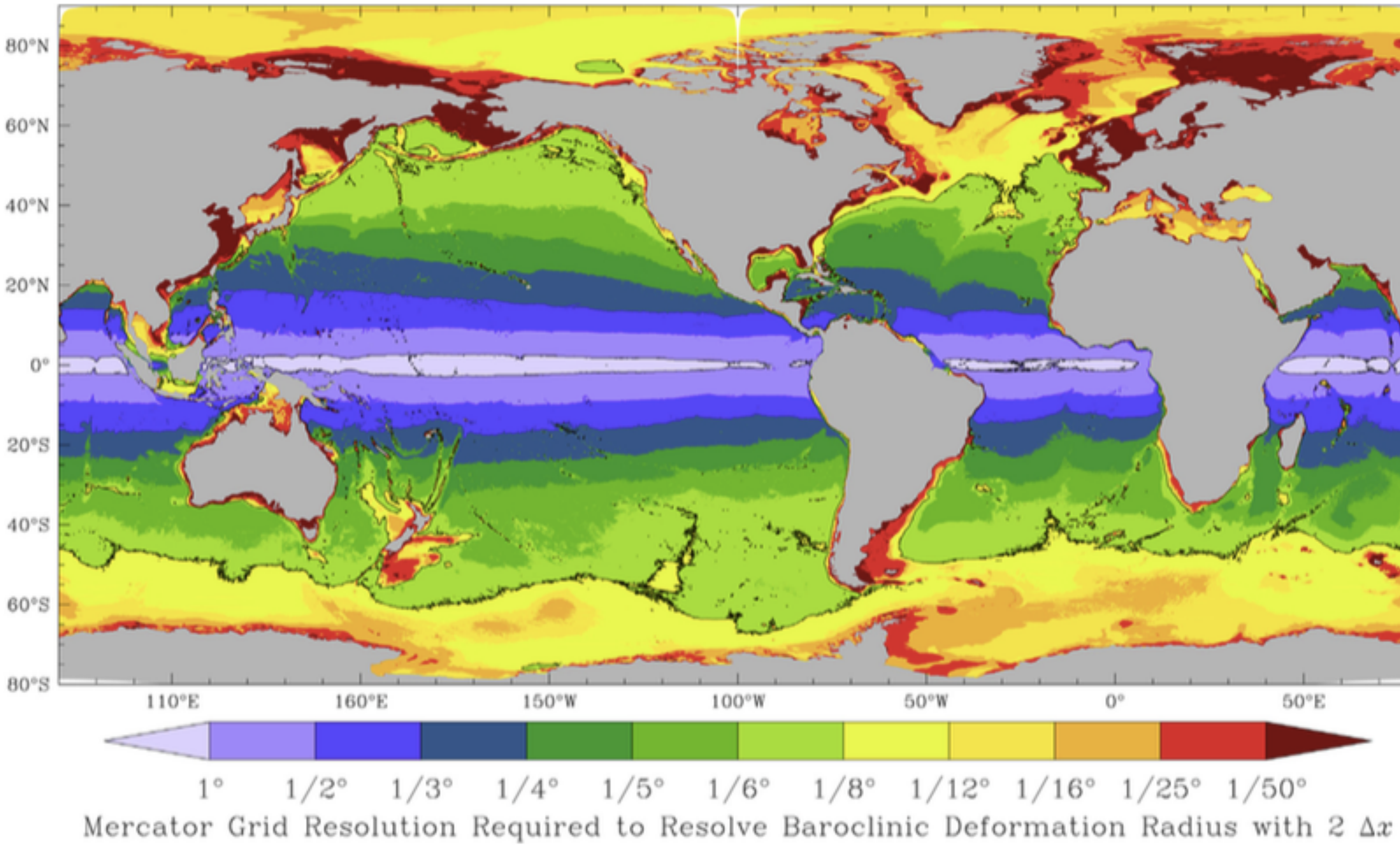
$$q_{qg}^{*} = f + \hat{k} \cdot \nabla \times u^{*} + \frac{\partial}{\partial z}\frac{f^2}{N^2}b^{*}$$

Also, different implications, because relative vorticity, buoyancy, T, S dissipation now must be consistent with PV:

$$\frac{Dq_{qg}^{*}}{Dt} = -\nabla \cdot \overline{u' q_{qg}'} \approx \nabla \cdot \left[\nu^{*}\nabla q_{2d} + \kappa_{gm}^{*}\nabla\left(q_{qg} - q_{2d}\right)\right] \to \kappa_{gm}^{*} = \nu^{*} = \kappa_{i}^{*}$$

S. D. Bachman, BFK, and B. Pearson. A scale-aware subgrid model for quasigeostrophic turbulence. Journal of Geophysical Research-Oceans, February 2017. In press.

R. Hallberg / Ocean Modelling 72 (2013) 92–103



- In most places, 0.1 degree resolves the largest deformation radius, plus a bit: Mesoscale Ocean Large Eddy Simulation

QG vs. 2D

$$\nu^*_{qg} = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^3 |\nabla q^*_{qg}|$$

Different Vorticity Gradients

$$q^*_{2d} = f + \hat{k}\cdot\nabla\times u^*$$

$$q^*_{qg} = f + \hat{k}\cdot\nabla\times u^* + \frac{\partial}{\partial z}\frac{f^2}{N^2}b^*$$

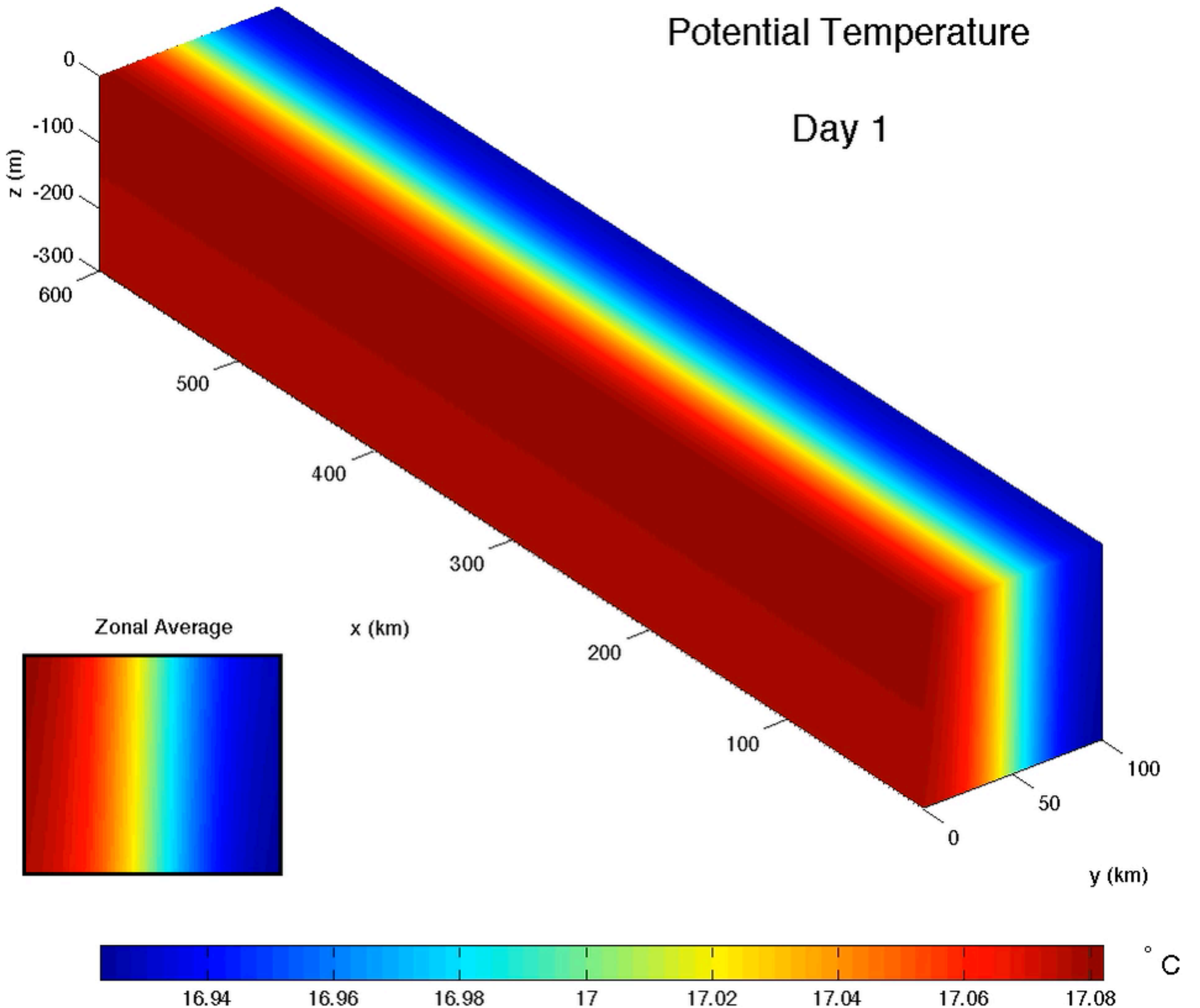
stretching–needs "taming" where QG is a bad approx (equator, boundary layers, etc.)

Use gridscale nondims to determine when on the fly

$$Ro^* = \frac{U^*}{f\Delta x} \quad Bu^* = \frac{{N^*}^2\Delta z^2}{f^2\Delta x^2} = \frac{L_d^2}{\Delta x^2} \sim {Ro^*}^2 Ri^*$$

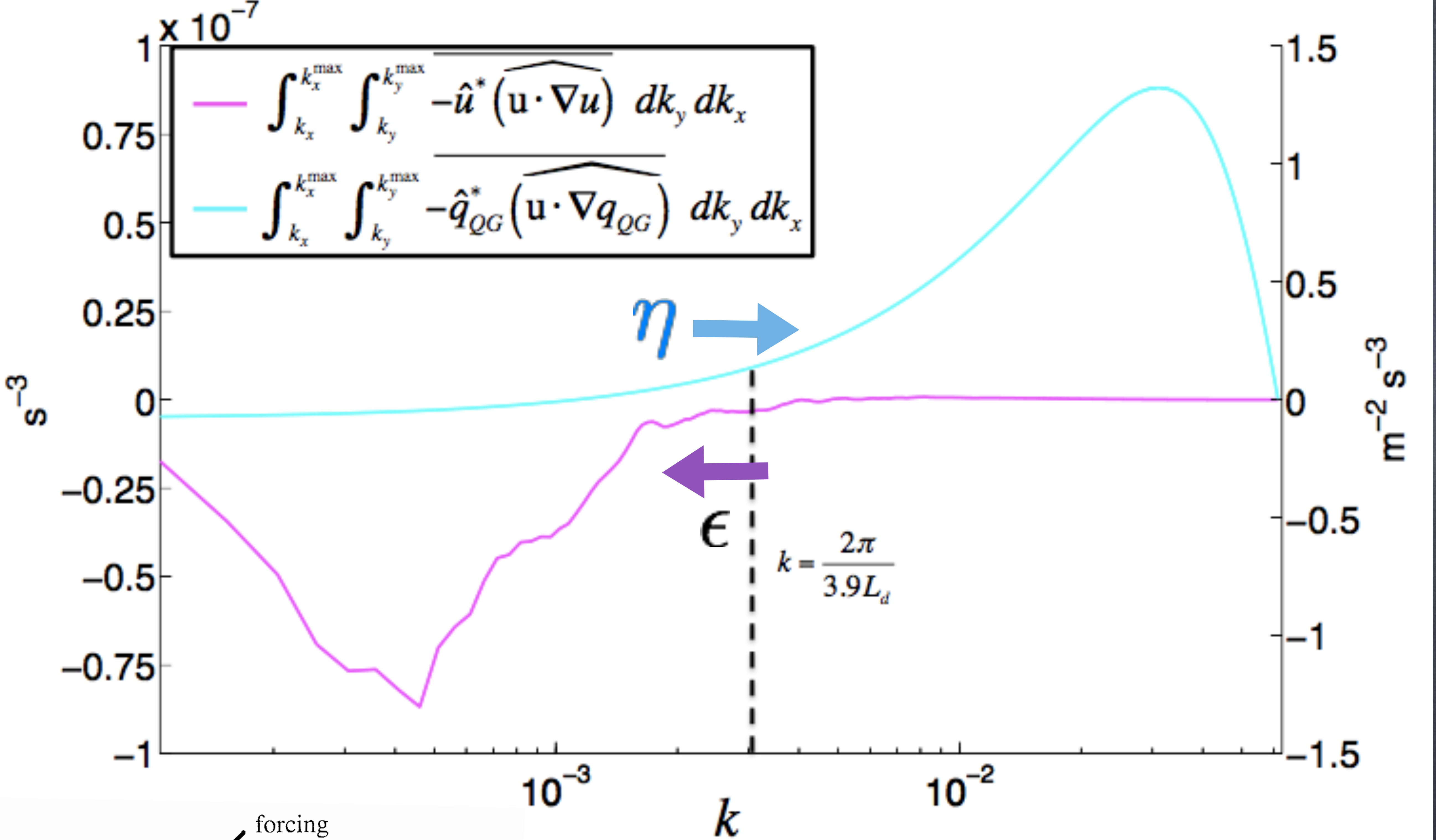
S. D. Bachman, BFK, and B. Pearson. A scale-aware subgrid model for quasigeostrophic turbulence. Journal of Geophysical Research-Oceans, February 2017.

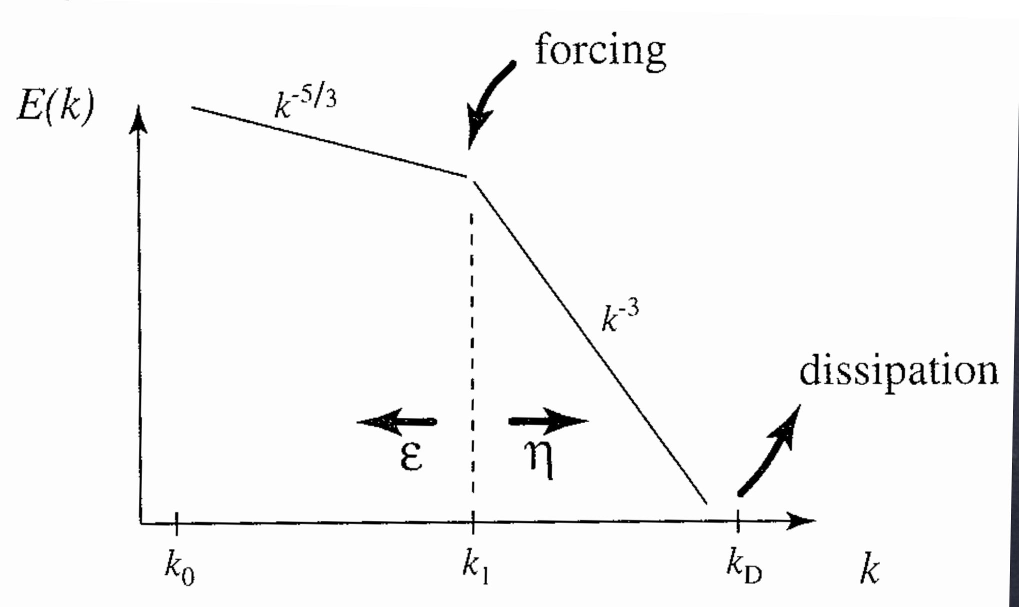
Movie: S. Bachman



S. Bachman and B. Fox-Kemper. Eddy parameterization challenge suite. I: Eady spindown. Ocean Modelling, 64:12-28, 2013.

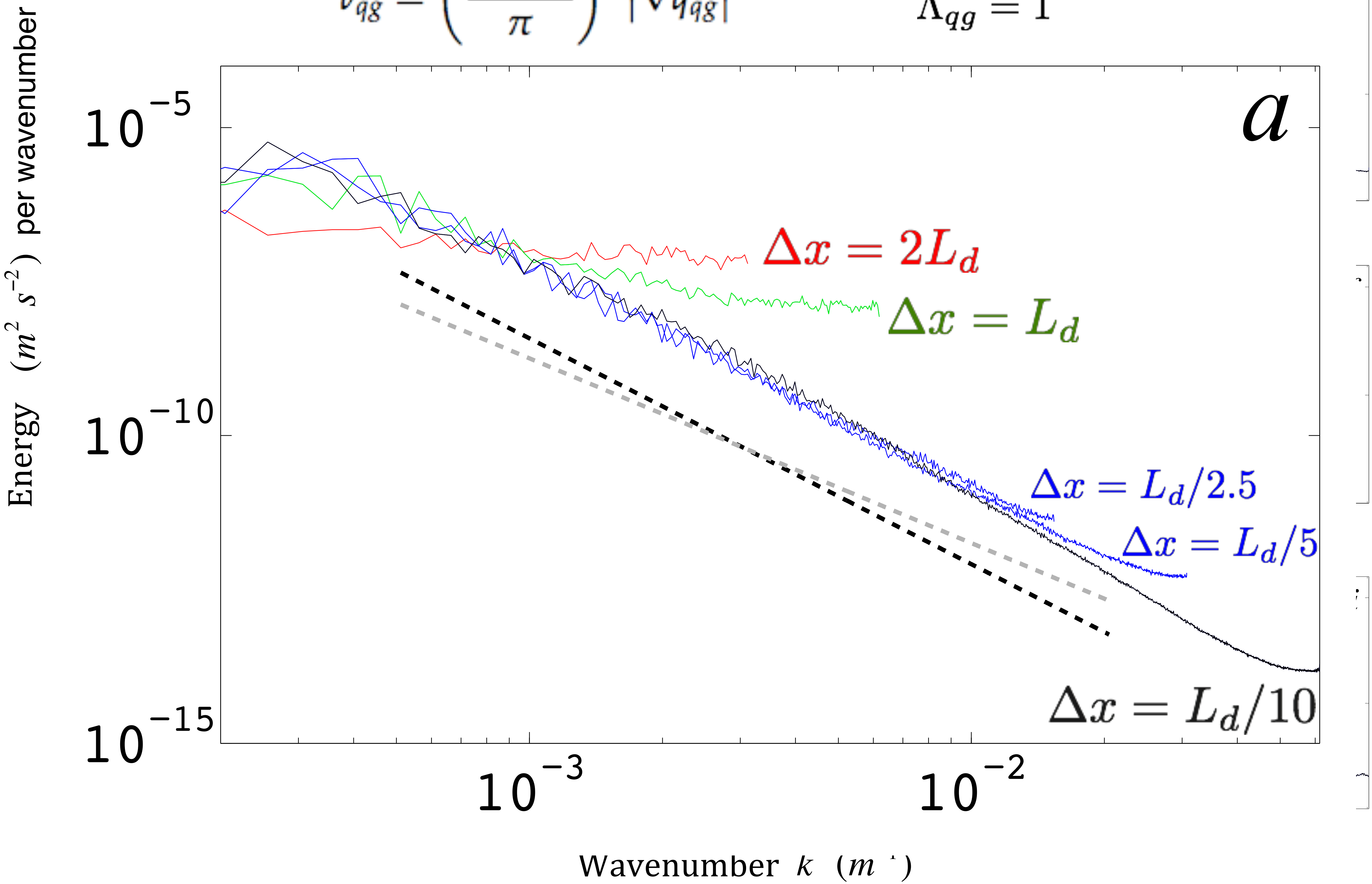
S. D. Bachman, BFK, and B. Pearson. A scale-aware subgrid model for quasigeostrophic turbulence. Journal of Geophysical Research-Oceans, February 2017. In press.





S. D. Bachman, BFK, and B. Pearson. A scale-aware subgrid model for quasigeostrophic turbulence. Journal of Geophysical Research-Oceans, February 2017.

$$\nu_{qg} = \left(\frac{\Lambda_{qg}\Delta x}{\pi}\right)^3 |\nabla q_{qg}| \qquad \Lambda_{qg} = 1$$



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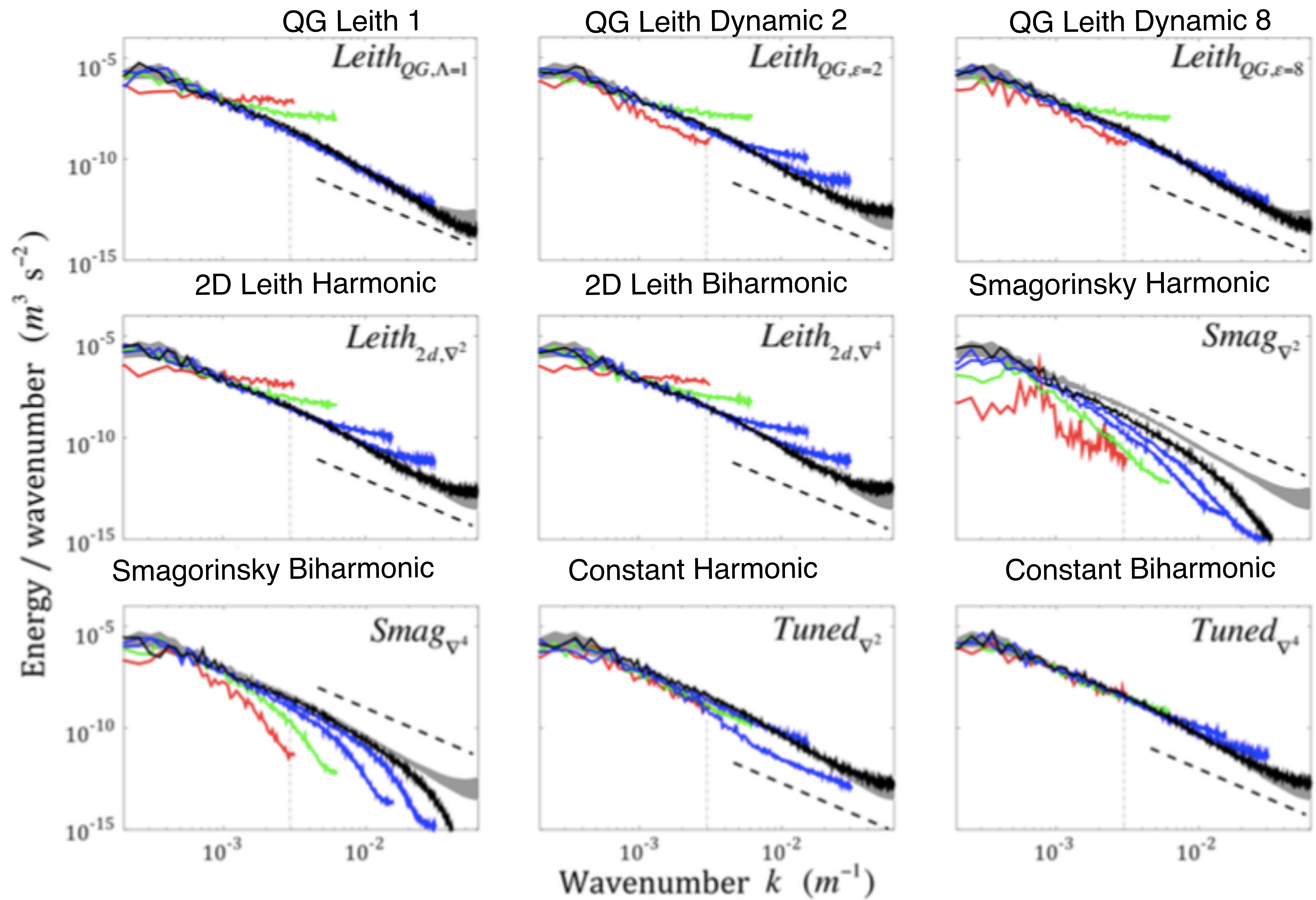

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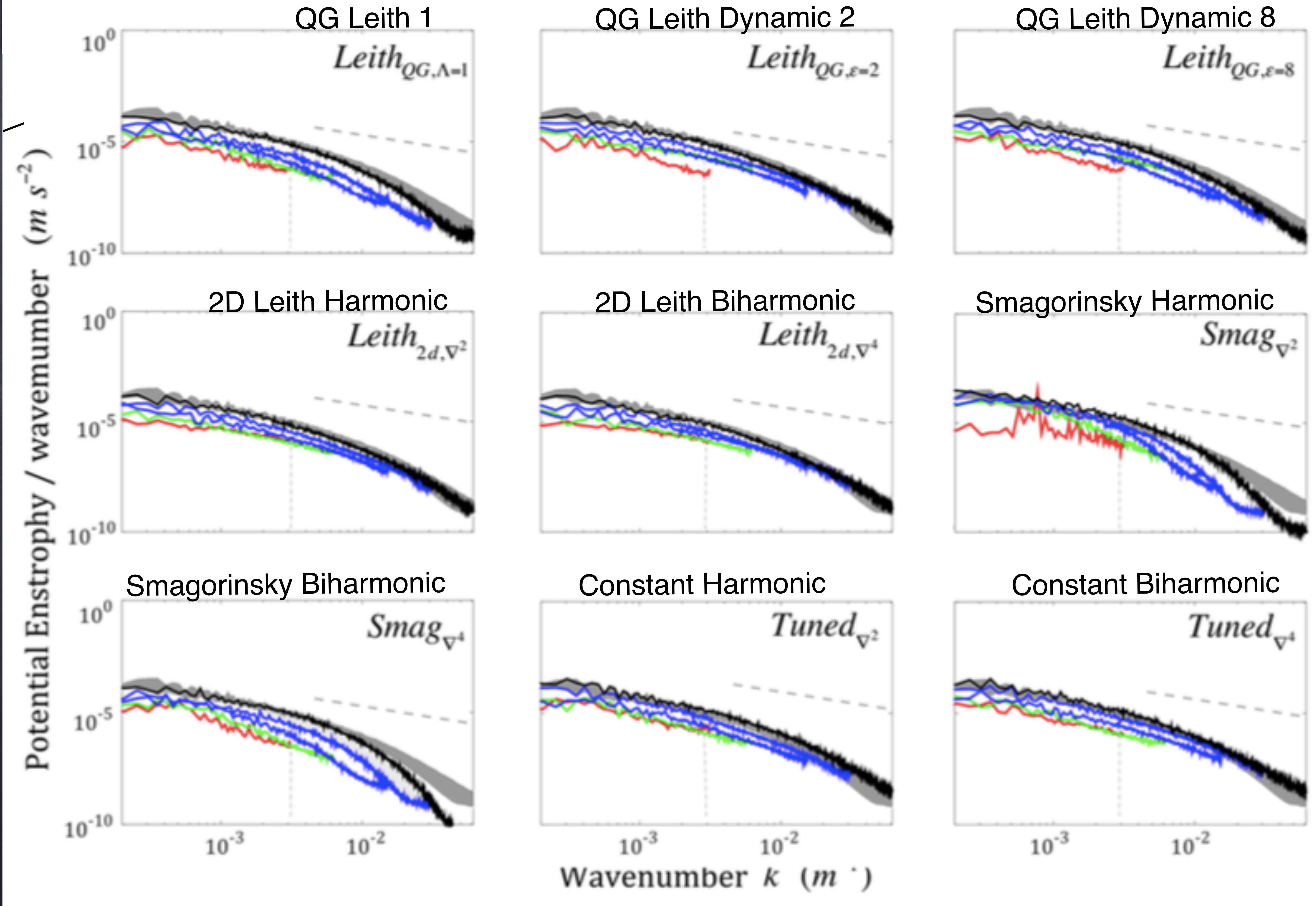
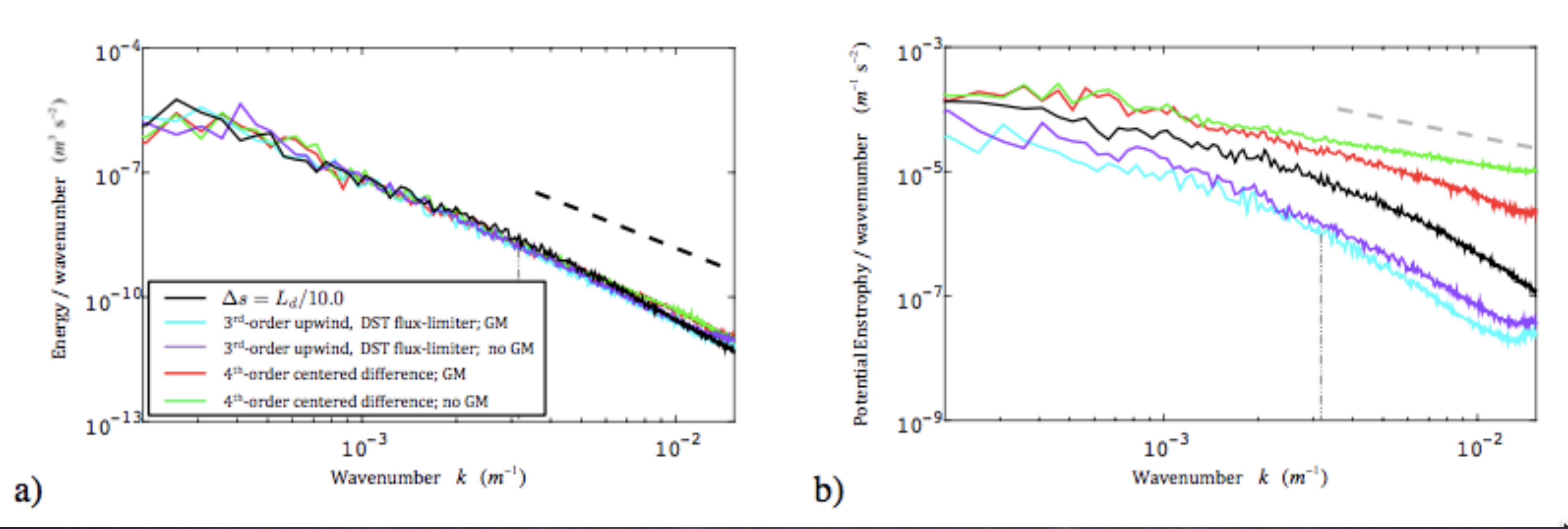


S. D. Bachman, BFK, and B. Pearson. A scale-aware subgrid model for quasigeostrophic turbulence. Journal of Geophysical Research-Oceans, February 2017. In press.

Beware the Numerical Artifacts!!

Changing the discrete approx. of advection matters.



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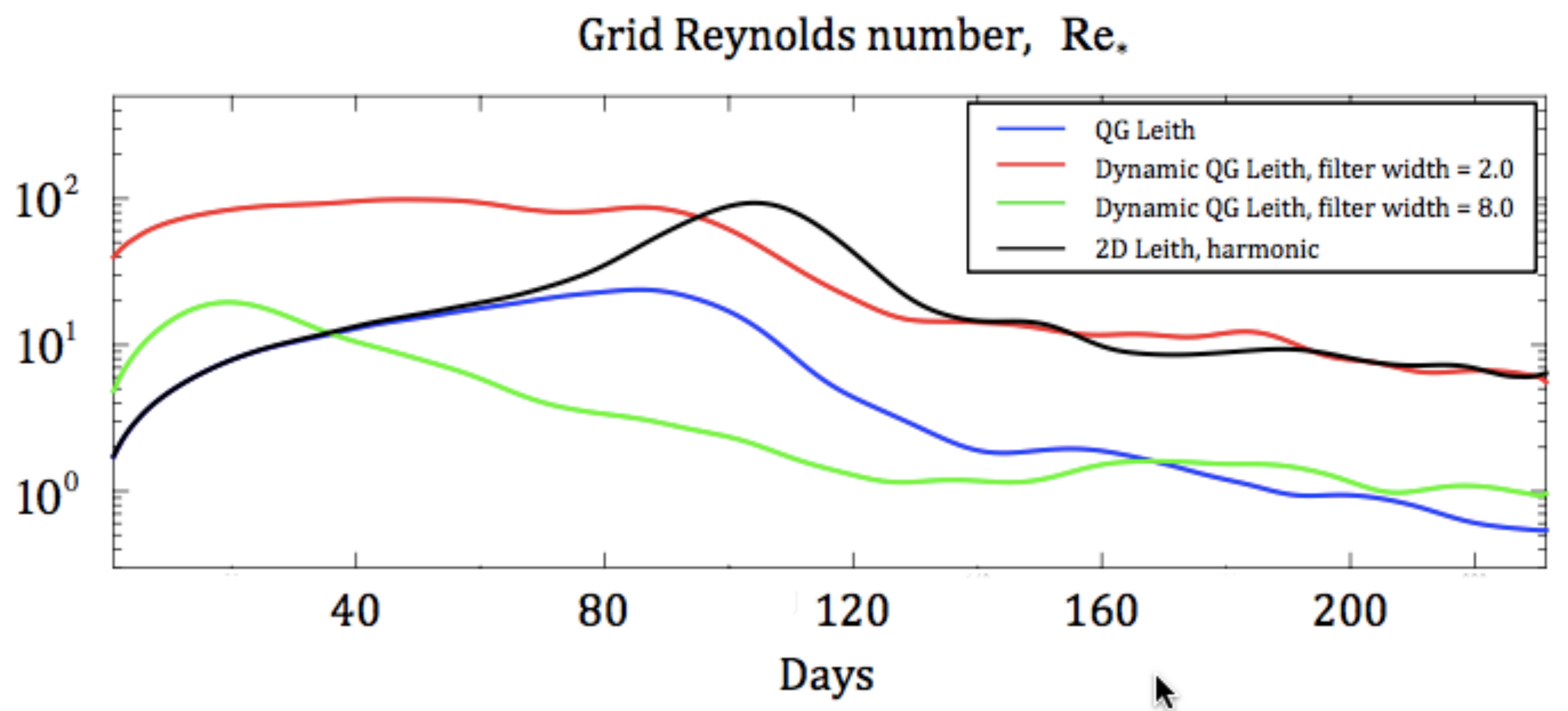
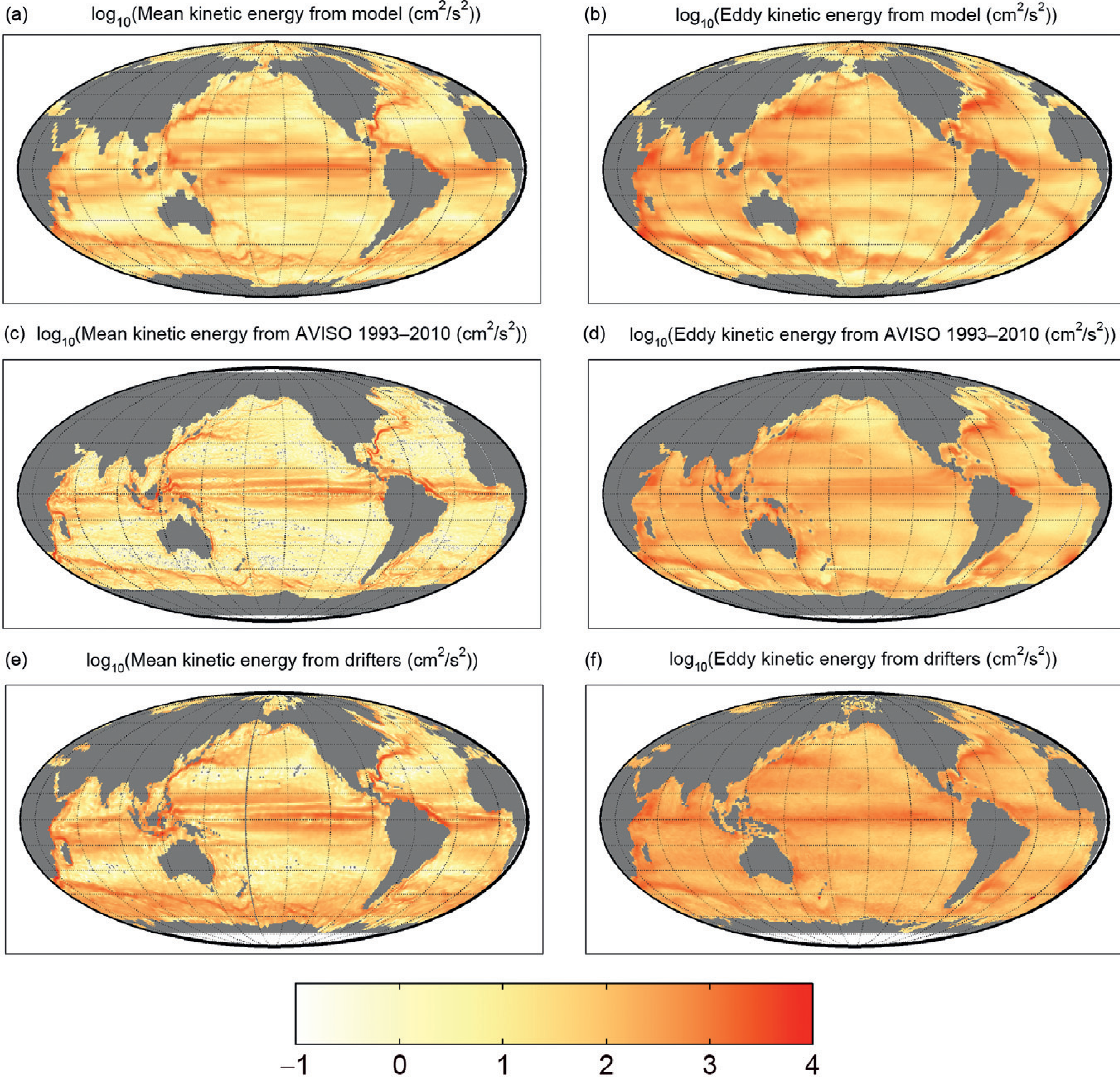


S. D. Bachman, BFK, and B. Pearson. A scale-aware subgrid model for quasigeostrophic turbulence. Journal of Geophysical Research-Oceans, February 2017. In press.

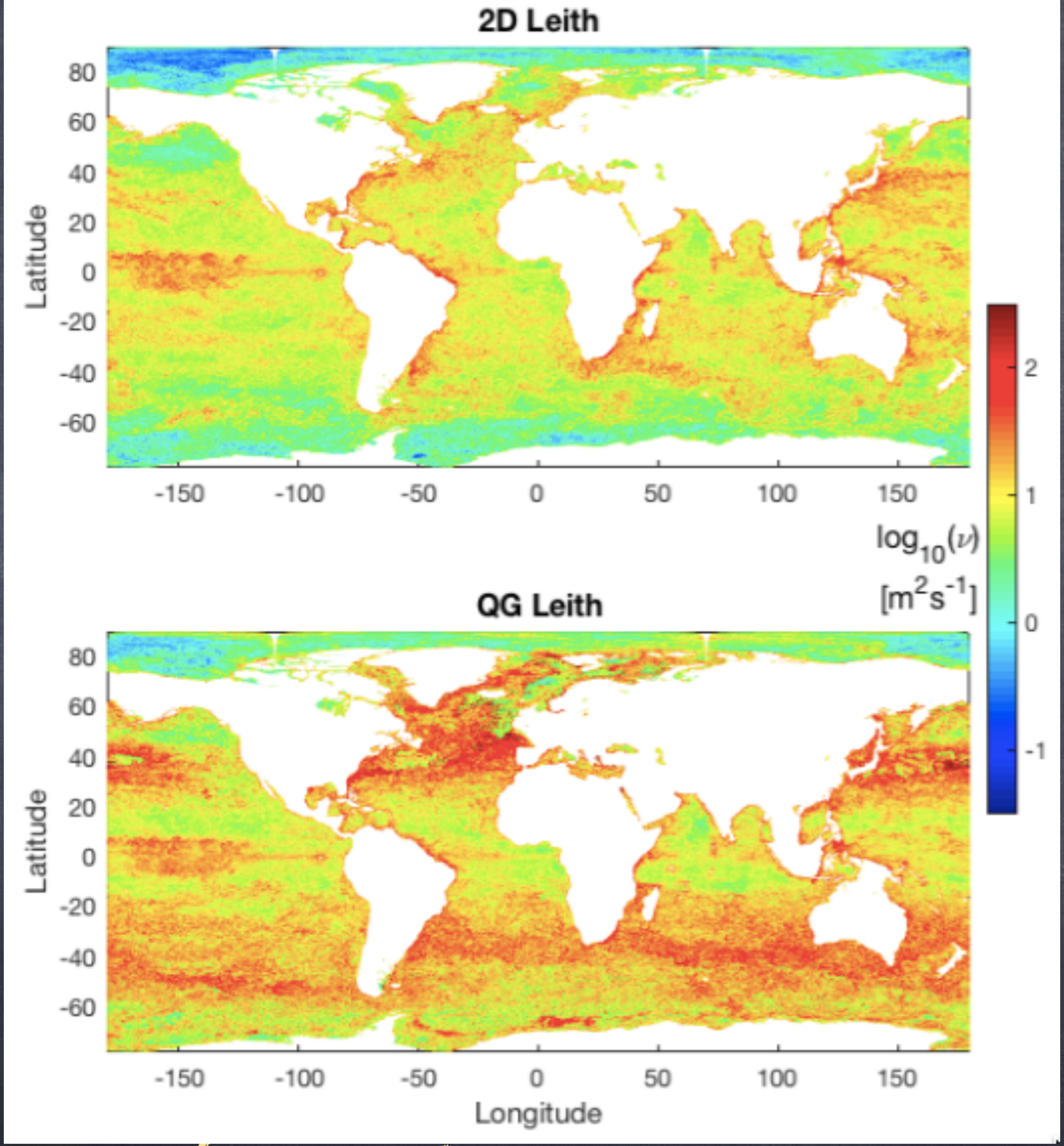
Now, for something more realistic—the global ocean!

- 0.1 degree (10km) resolution global ocean model (POP/CESM)
- Repeating Normal Year forcing
- Branches off of ``standard" simulation using biharmonic.
- Biharmonic, 2D Leith, QG Leith



On cursory analysis, 0.1 degree models do well vs. Satellites and Drifters

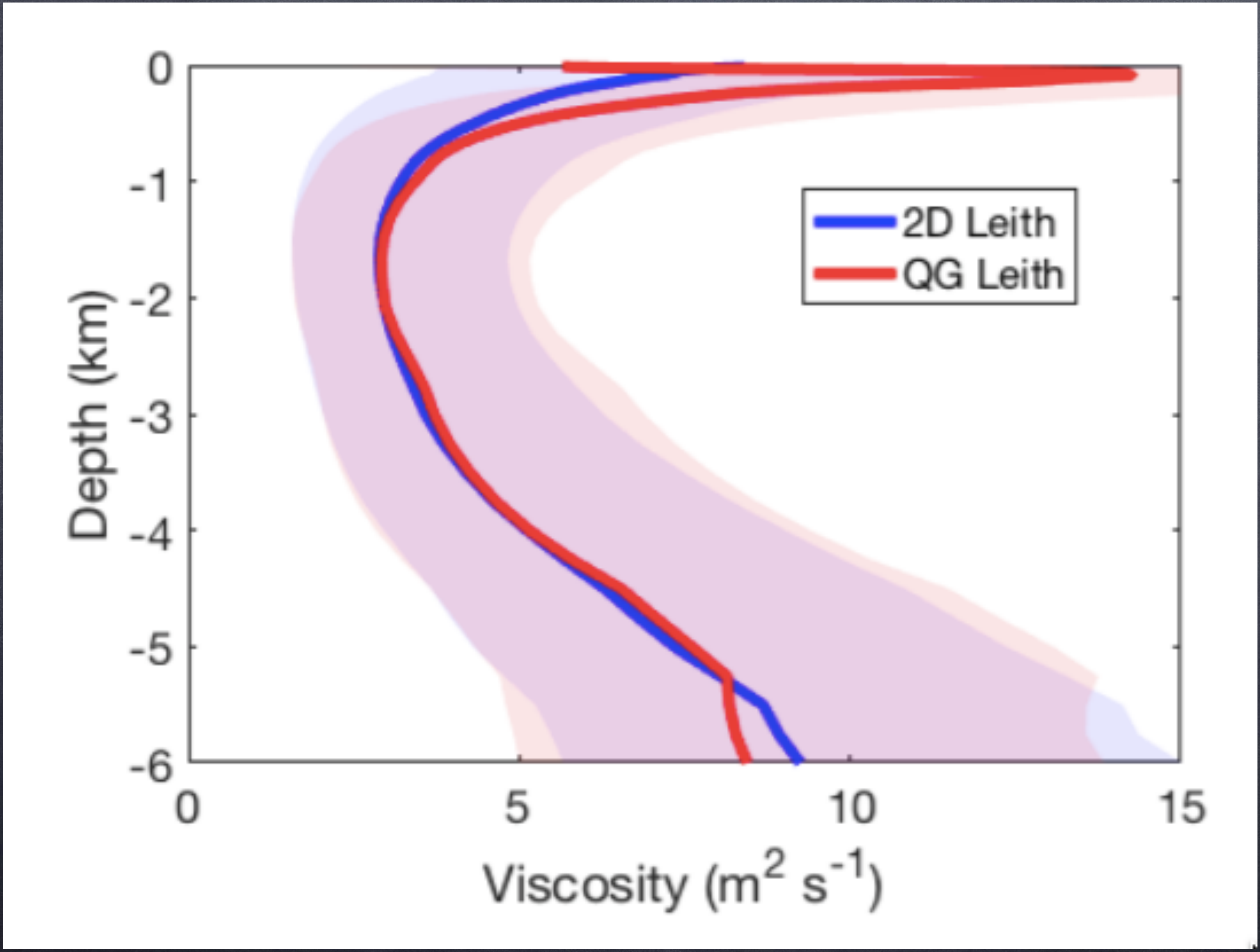
B. Fox-Kemper, R. Lumpkin, and F. O. Bryan. Lateral transport in the ocean interior. In G. Siedler, S. M. Griffies, J. Gould, and J. A. Church, editors, Ocean Circulation and Climate: A 21st century perspective, volume 103 of International Geophysics Series, chapter 8, pages 185-209. Academic Press (Elsevier Online), 2013.

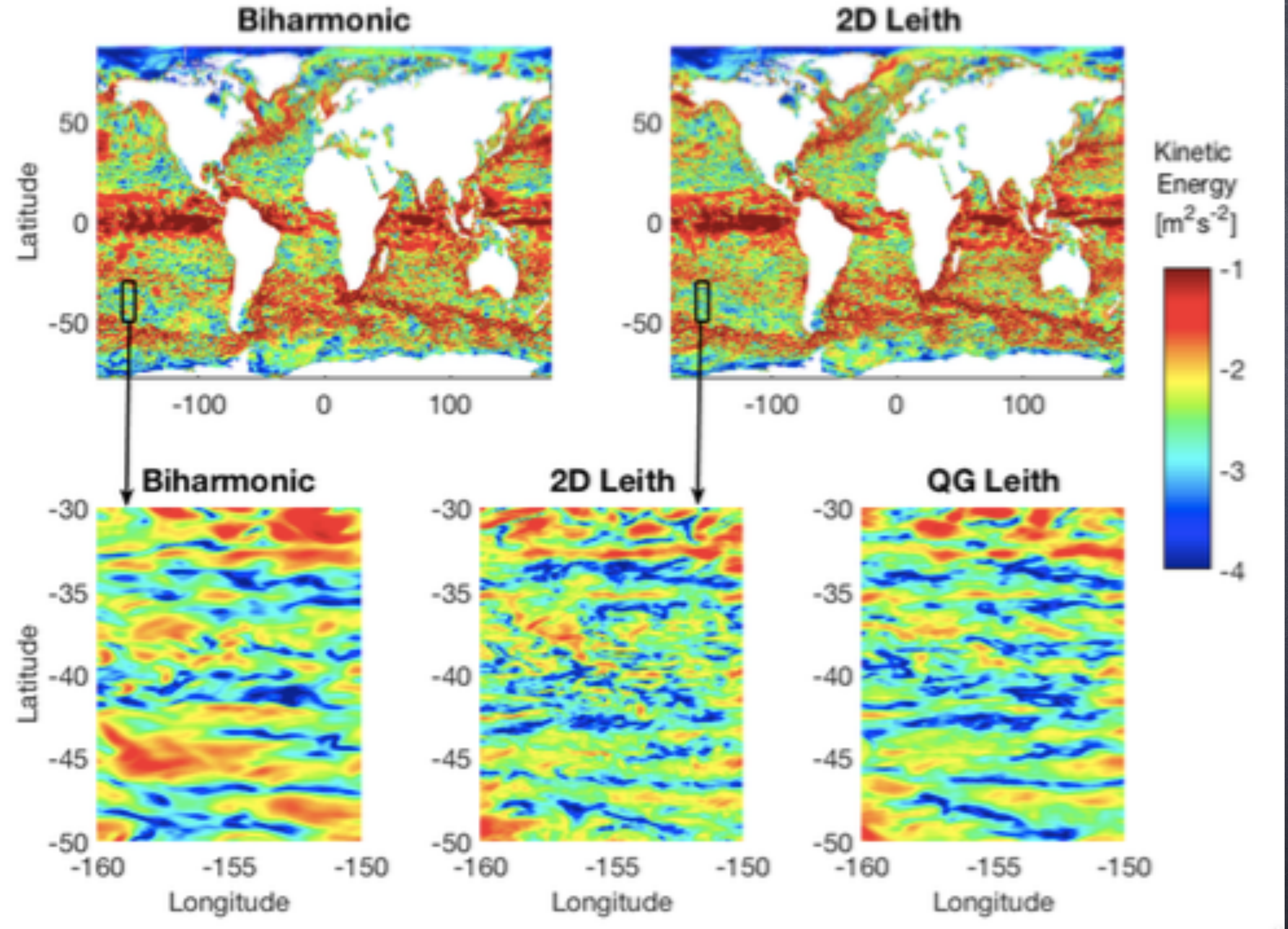


Viscosity in MOLES

B. Pearson, B. Fox-Kemper, and S. D. Bachman. Evaluation of scale-aware subgrid mesoscale eddy models in a global eddy-rich model. Ocean Modelling, December 2016. Submitted.

Viscosity in Vertical





More EKE and Small Structures in MOLES

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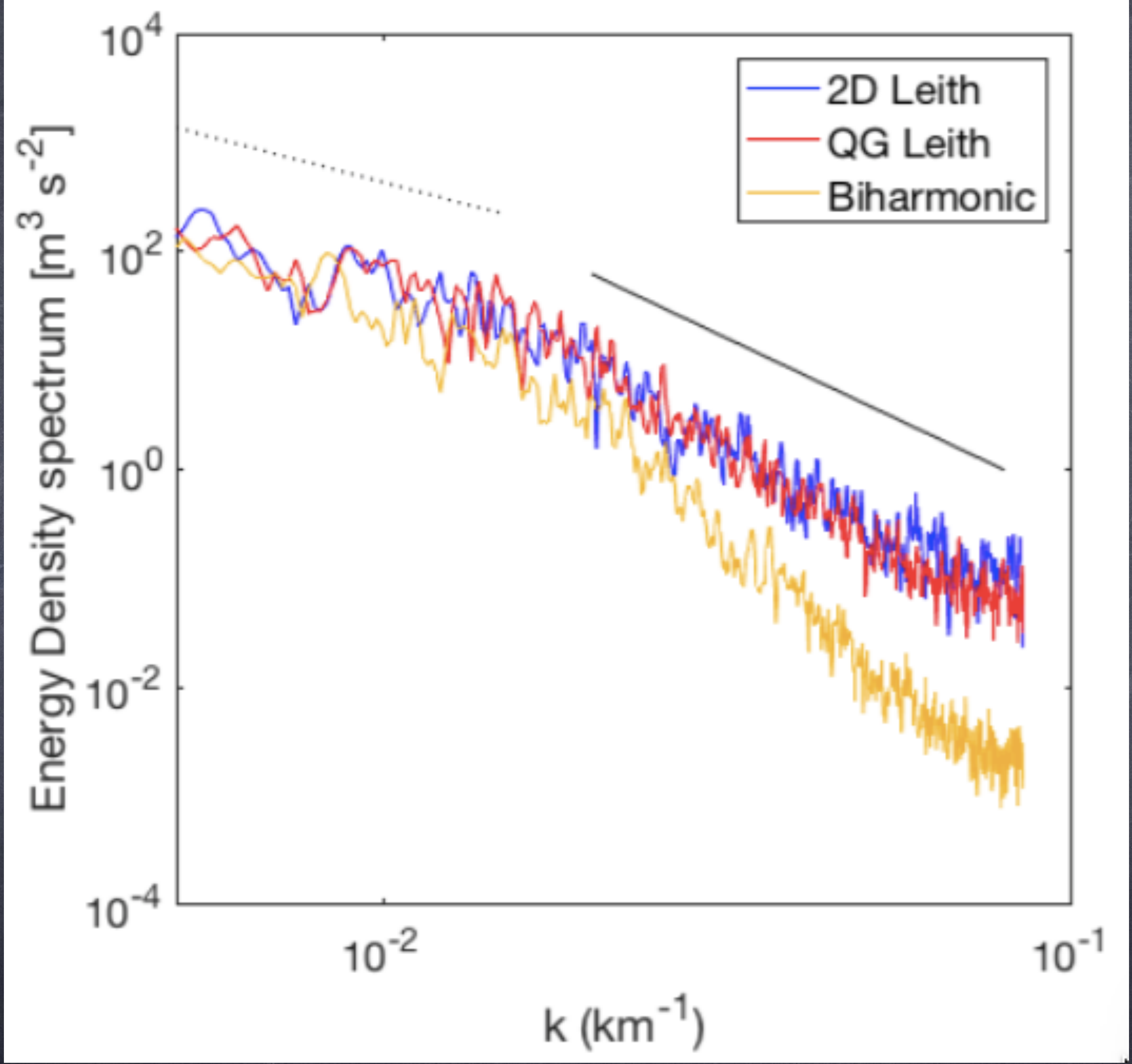

More EKE and Small Structures in MOLES

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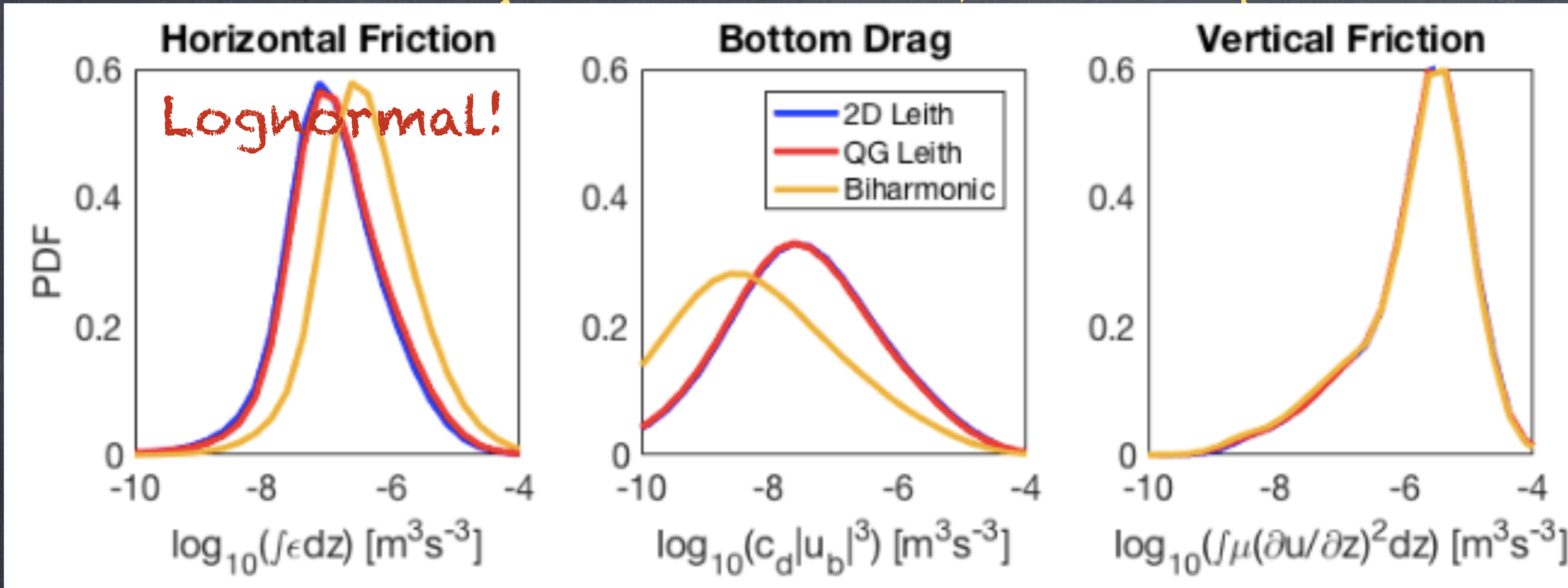
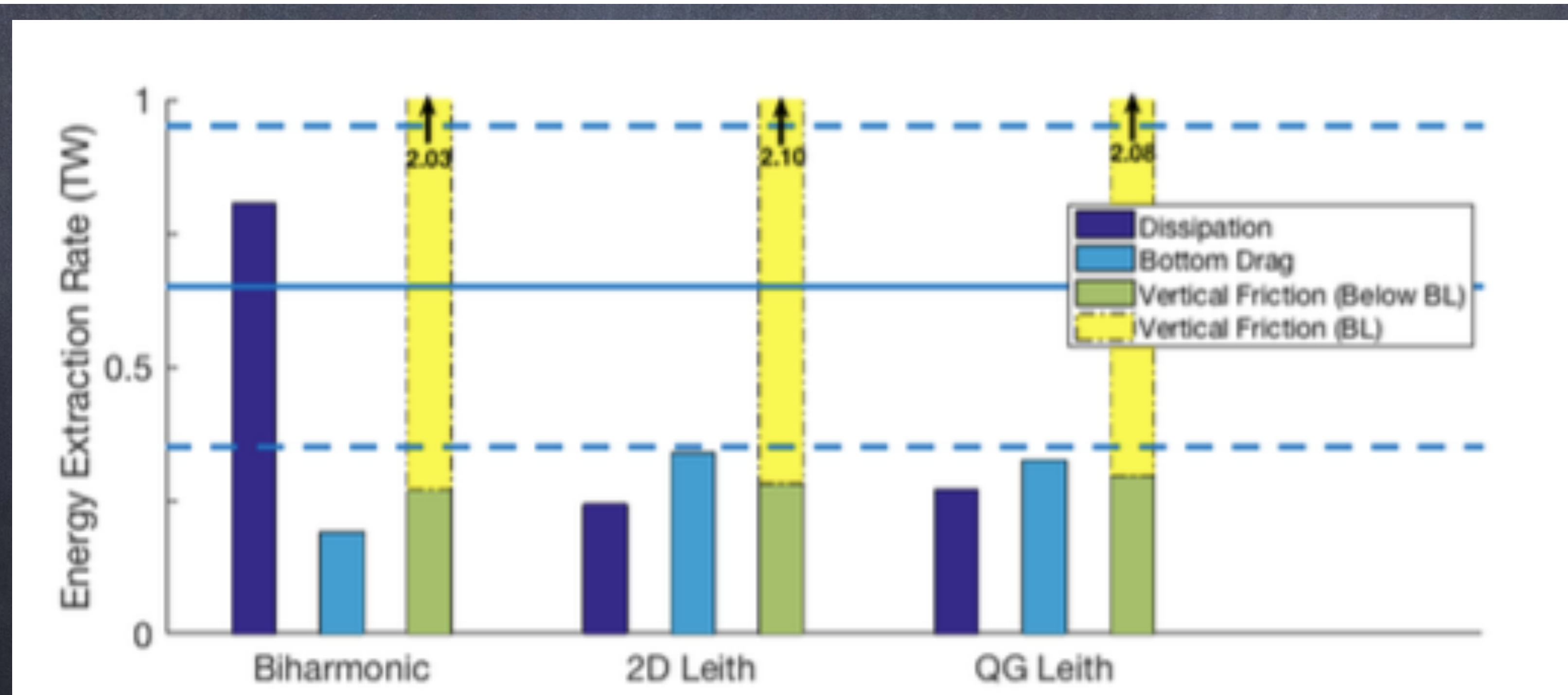
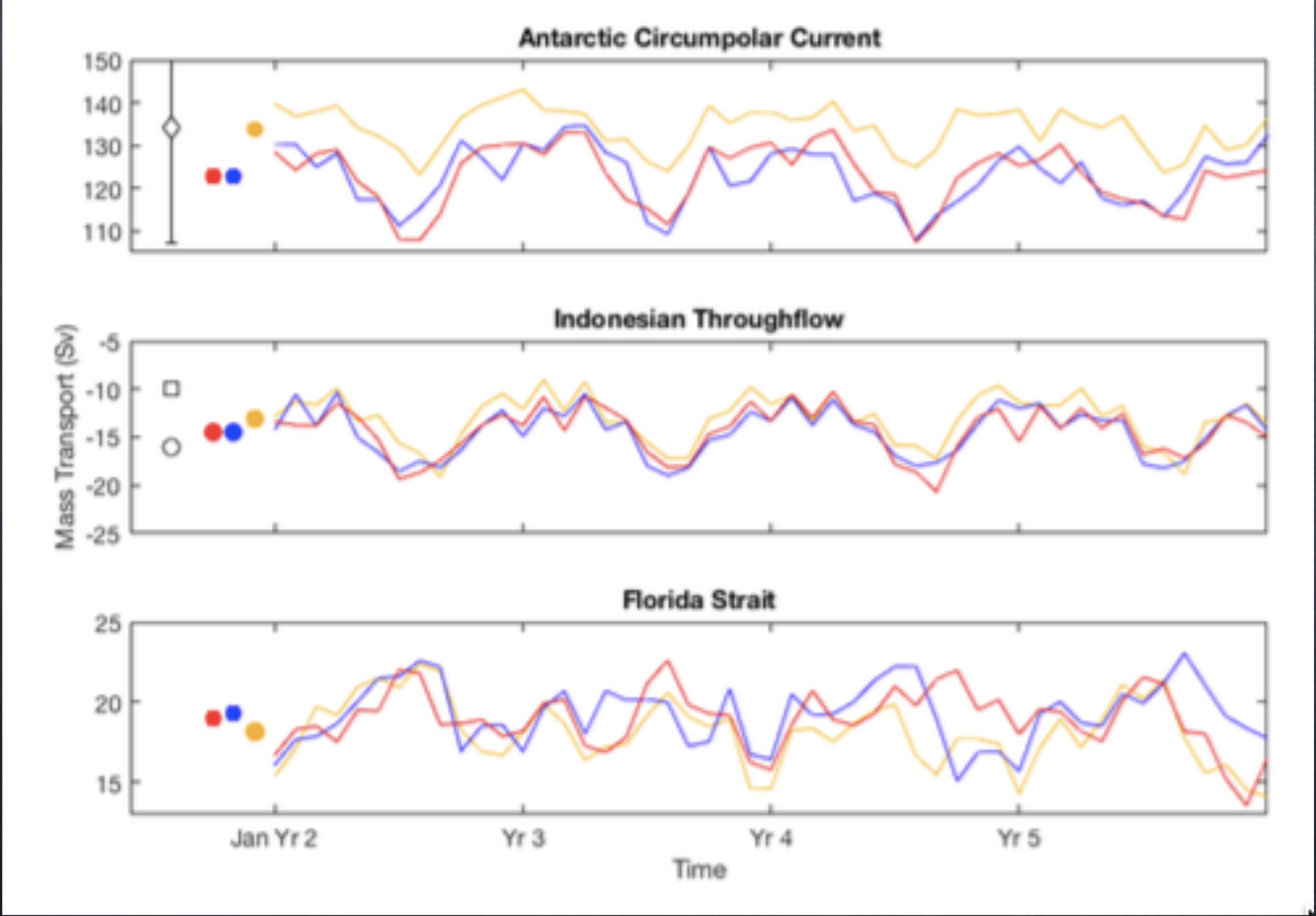
Probability Distribution of KE Dissipation





B. Pearson, B. Fox-Kemper, and S. D. Bachman. Evaluation of scale-aware subgrid mesoscale eddy models in a global eddy-rich model. Ocean Modelling, December 2016. Submitted.

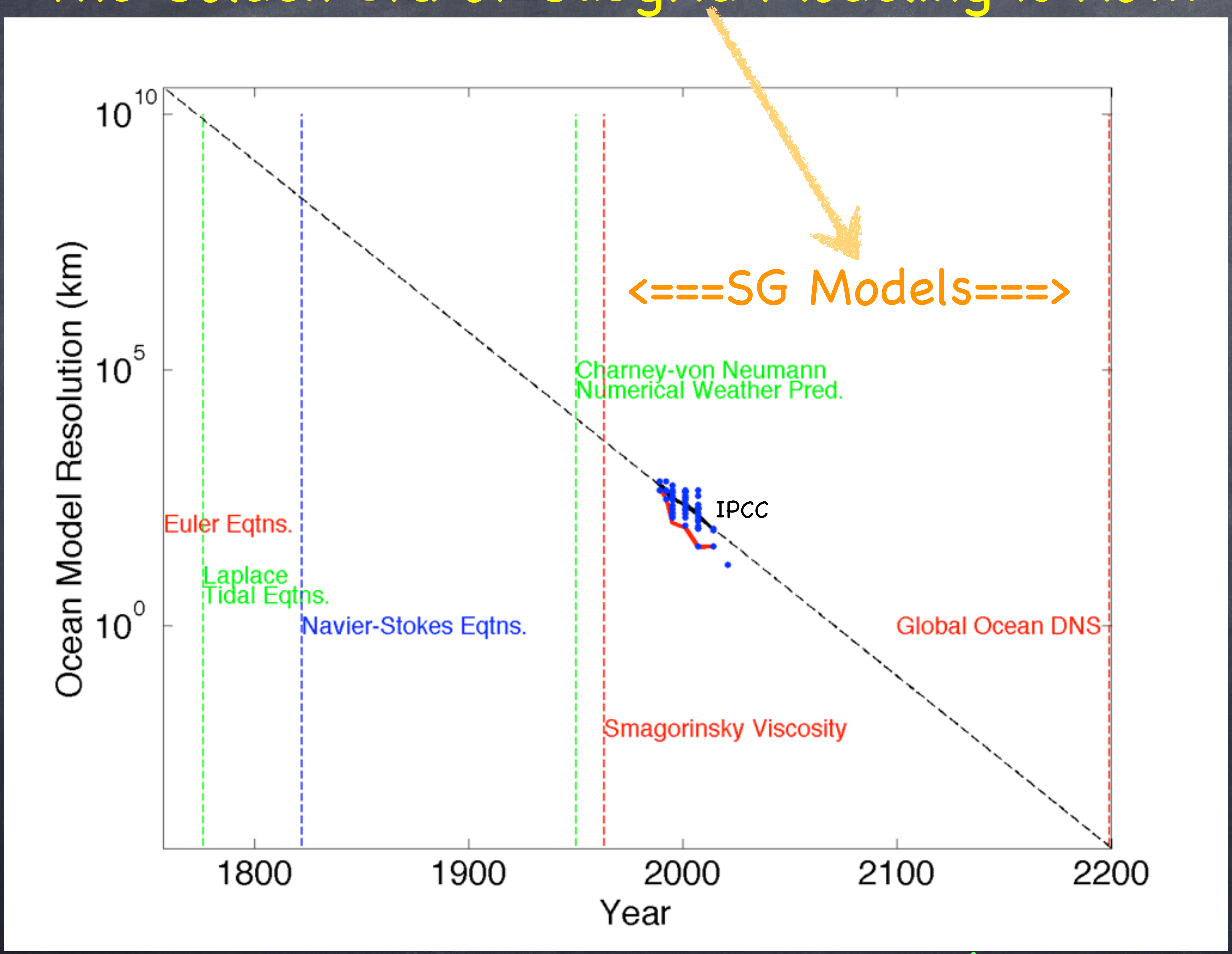
Major Currents Affected



Conclusions

- It is best to think of high-res ocean simulations as "large eddy simulations"—need eqtns for large scale!
- Take advantage of resolved flow and scaling for physically-based subgrid schemes.
- QG theory has provided such a scheme for mesoscale-permitting to resolving simulations.
- 10x less dissipative than traditional viscosity and dissipates where theory suggests it should.
- Small scales more energetic, salinity variance doubled even at 1000km scales, major currents affected, dominant ordering of energy budget affected.

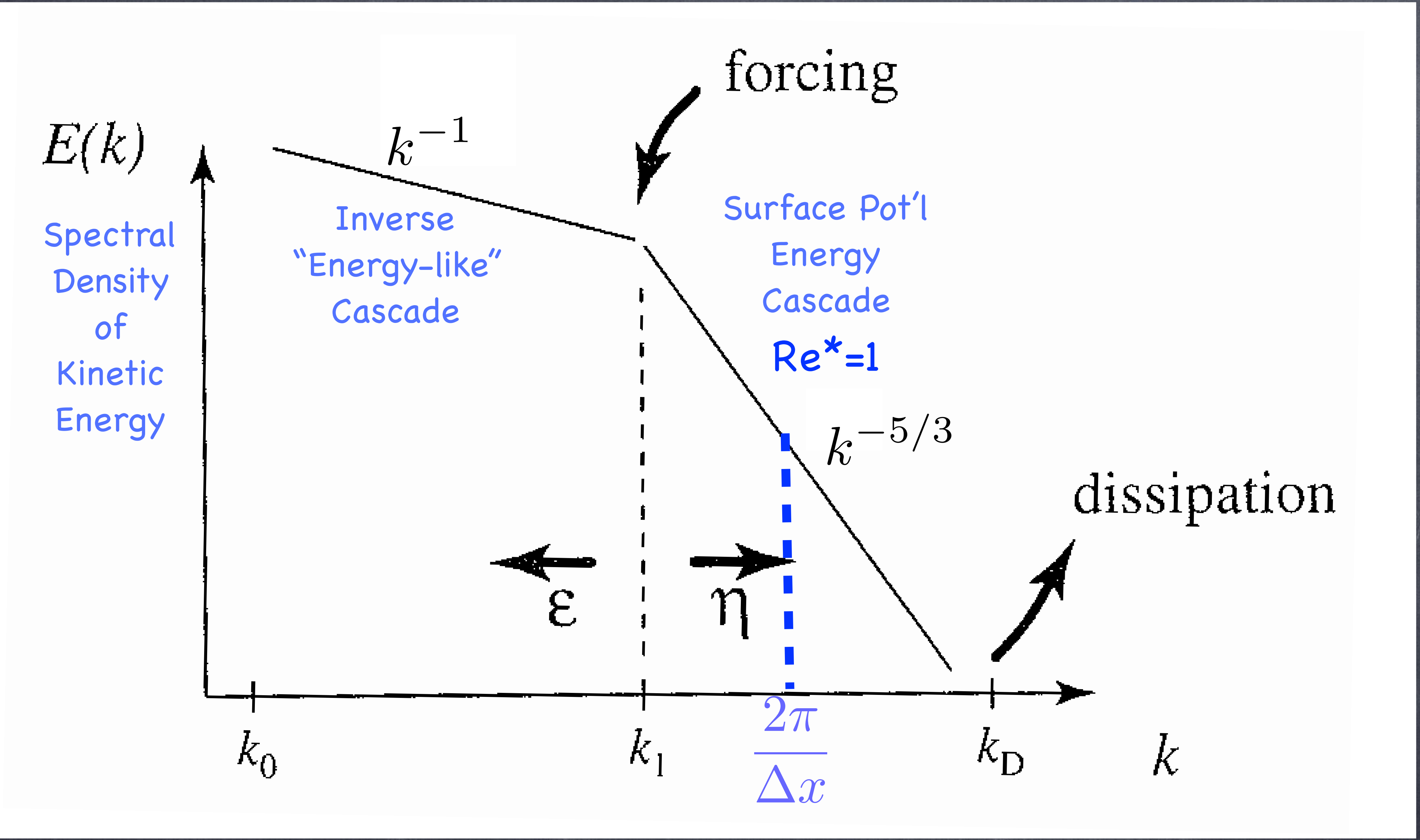
A Moore's-law-like historical perspective: The Golden Era of Subgrid Modeling is Now!



All papers at: fox-kemper.com/research

SQG Turbulence: Surface Buoyancy & Velocity cascade--scales surface horiz. diffusivity only

W. Blumen, 1978 JAS
Held et al 1995, JFM.
Smith et al. 2002, JFM



Smag-Like (Inverse):

$$\kappa_* = \left(\frac{\Upsilon \Delta x}{\pi}\right)^{4/3} \left|\frac{1}{f}\nabla_h b\right|^{2/3}$$

Leith-Like (Direct):

$$\kappa_* = \left(\frac{\Lambda \Delta x}{2\pi}\right)^{3/2} \left[-\frac{\partial}{\partial z}|\nabla_h \psi|^2\right]^{1/2}$$