

Effects of Ocean Surface Waves: on Turbulence, Climate, and Frontogenesis



Expanding on past work with:
Jim McWilliams (UCLA), Peter Hamlington
(CU-Boulder), Eric D'Asaro & Ramsey
Harcourt (UW), Luke Van Roekel (LANL),
Adrean Webb (TUMST), Keith Julien (CU-
APPM), Greg Chini (UNH), Peter Sullivan
(NCAR), Mark Hemer (CSIRO)

Friday, 9/14/18
10:30—11:30

Baylor Fox-Kemper
with Nobuhiro Suzuki
(Brown), Qing Li (Brown), Sean
Haney (UCSD)

URI GSO

Physical Oceanography Seminar Series

Sponsors:

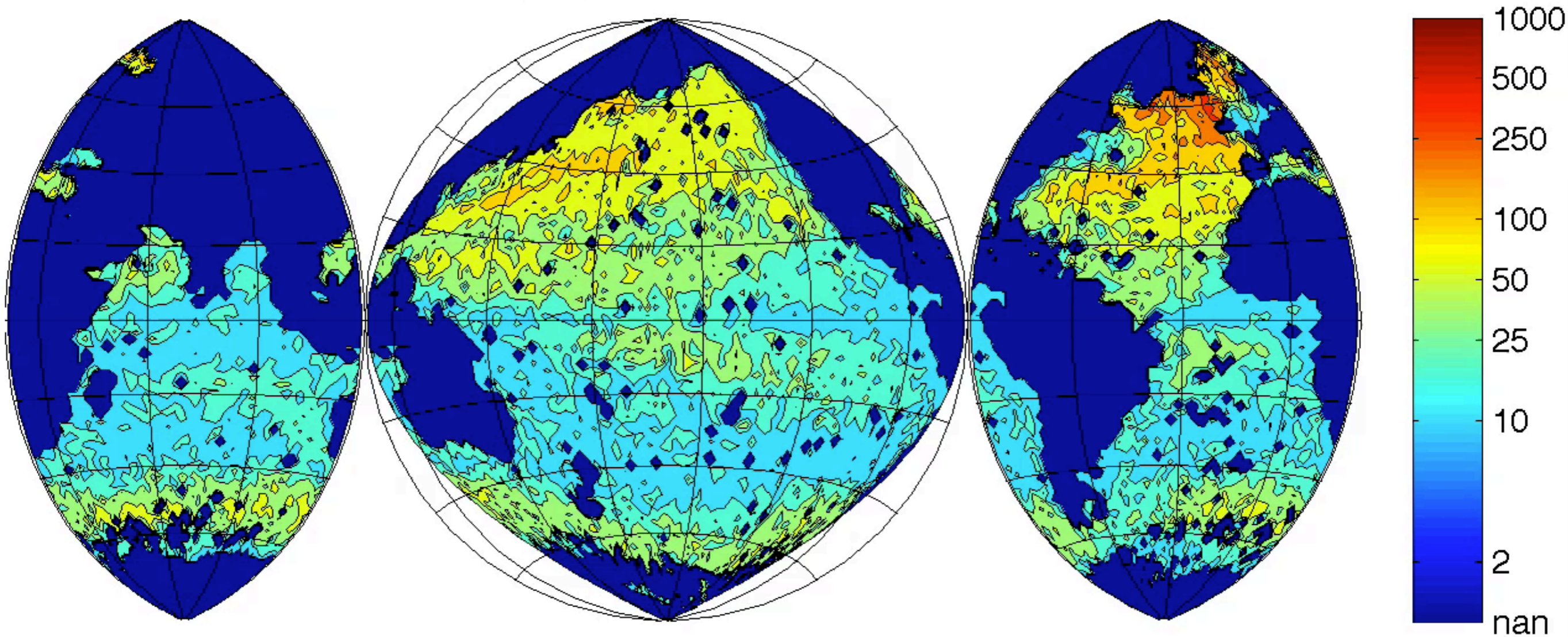
NSF (1350795, 1655221),
ONR (N00014-17-1-2393), National
Key Research Program of China



A photo of same in Narragansett Bay
(courtesy P. Cornillon)

The Ocean Mixed Layer

Mixed Layer Depth (Δ density = 0.03) in month 1



Stommel's Demon: ocean properties at depth set by deepest wintertime mixed layer & its properties
From Argo float data courtesy C. de Boyer-Montegut

We Will Examine the Effects of Surface Waves on:

- Boundary Layer Turbulence
(wave-driven or Langmuir Turbulence)
- Climate through Langmuir Turbulence
(via MLD changes)
- Submesoscale Fronts & Instabilities
within the Mixed Layer
(Stokes forces and Langmuir coupling)

3 Effects Dominate open ocean

“Wave-Averaged Equations”:

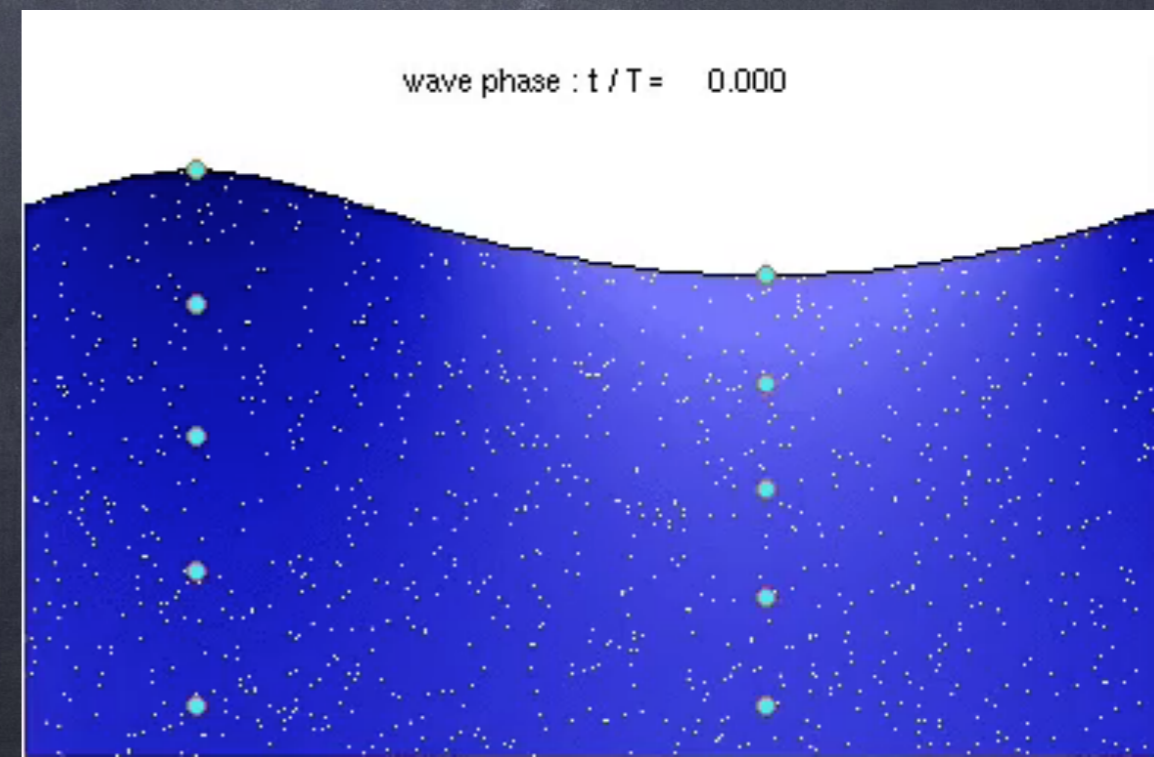
(Craik, Leibovich, McWilliams et al. 1997)

All rely only on Stokes drift of waves

1: Stokes Advection: parcels, tracers, momentum move with Lagrangian, not Eulerian flow

2: Stokes Coriolis: water parcels experience Coriolis force during this motion

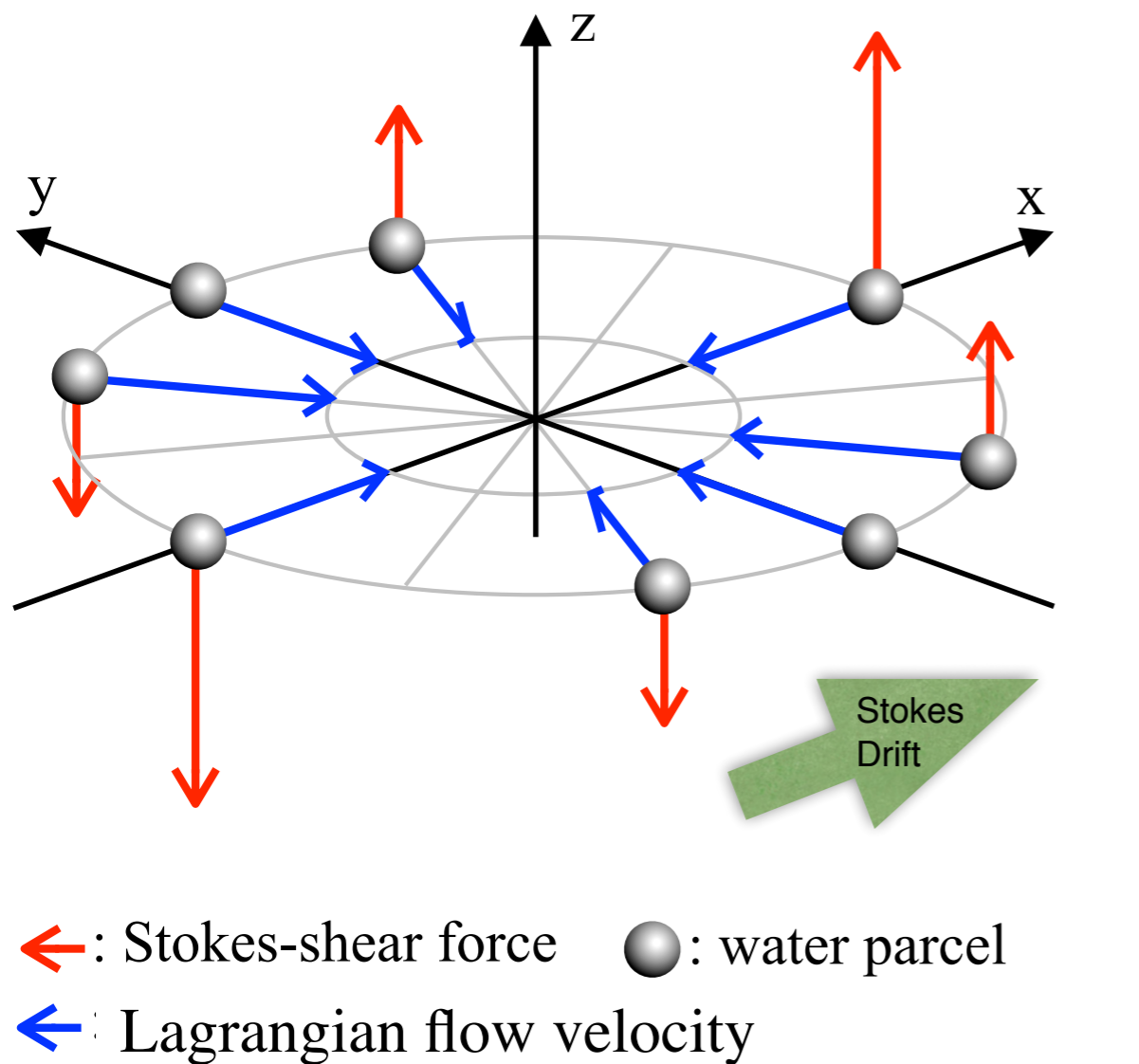
3: Stokes Shear Force



N. Suzuki and BFK. Understanding Stokes forces in the wave-averaged equations. *Journal of Geophysical Research-Oceans*, 121:1-18, 2016.

3 Wave Effects, 3: Stokes Shear Force and the CL2 mechanism for Langmuir circulations

Flow directed along Stokes shear=downward force



$$\epsilon = \frac{V^s H}{f L H_s}$$

"wavy hydrostatic" if

hydrostatic $\epsilon \gg 1$

$$\frac{\alpha^2}{Ri} \left[w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = \boxed{-\pi_{,z} + b} - \boxed{\epsilon v_j^L v_{j,z}^s} + \frac{\alpha^2}{Re Ri} w_{,jj}$$

The Character of Langmuir Turbulence

- Near-surface
- Langmuir Cells & Langmuir Turb.
- $Ro \gg 1$
- $Ri < 1$: Nonhydro
- 1–100m ($H=L$)
- 10s to 1hr
- $w, u = O(10\text{cm/s})$
- Stokes drift
- Eqtns: Wave-Averaged
- Params: McWilliams & Sullivan, 2000, Van Roekel et al. 2011
- Resolved routinely in 2170

Image: NPR.org,
Deep Water
Horizon Spill

What's plotted are
surfaces of large
vert. velocity,
colored by
temperature

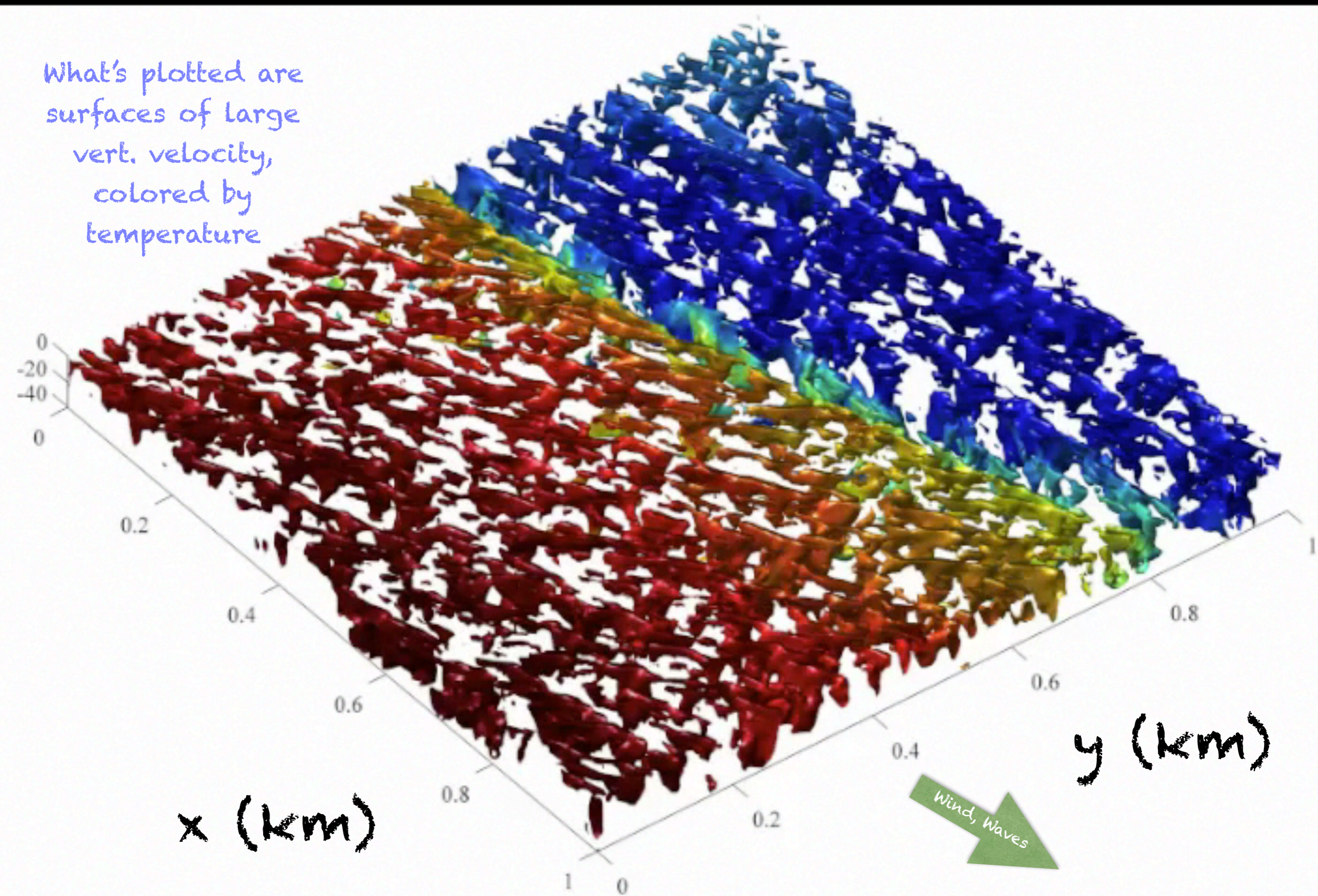
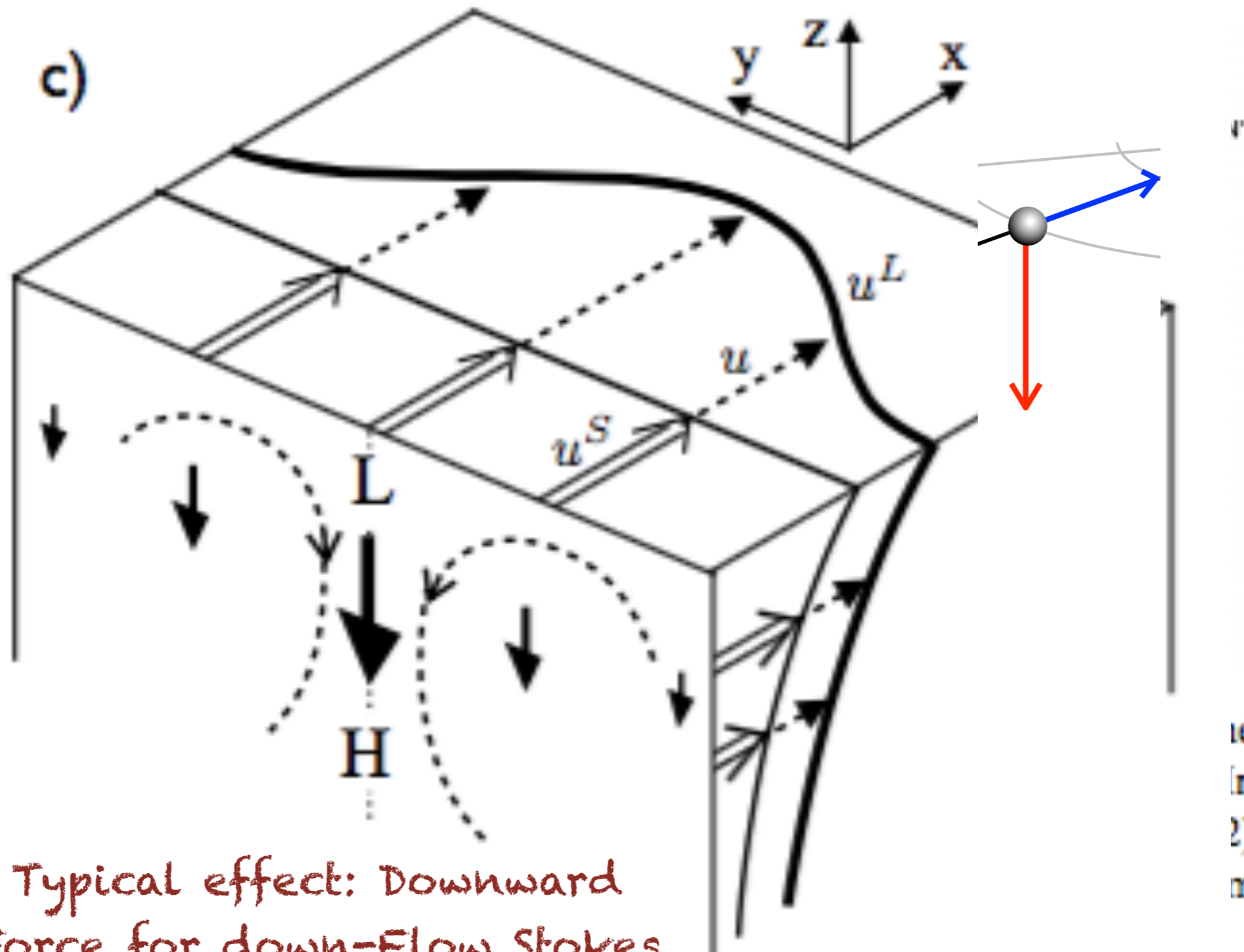


image:
Thorpe, 0



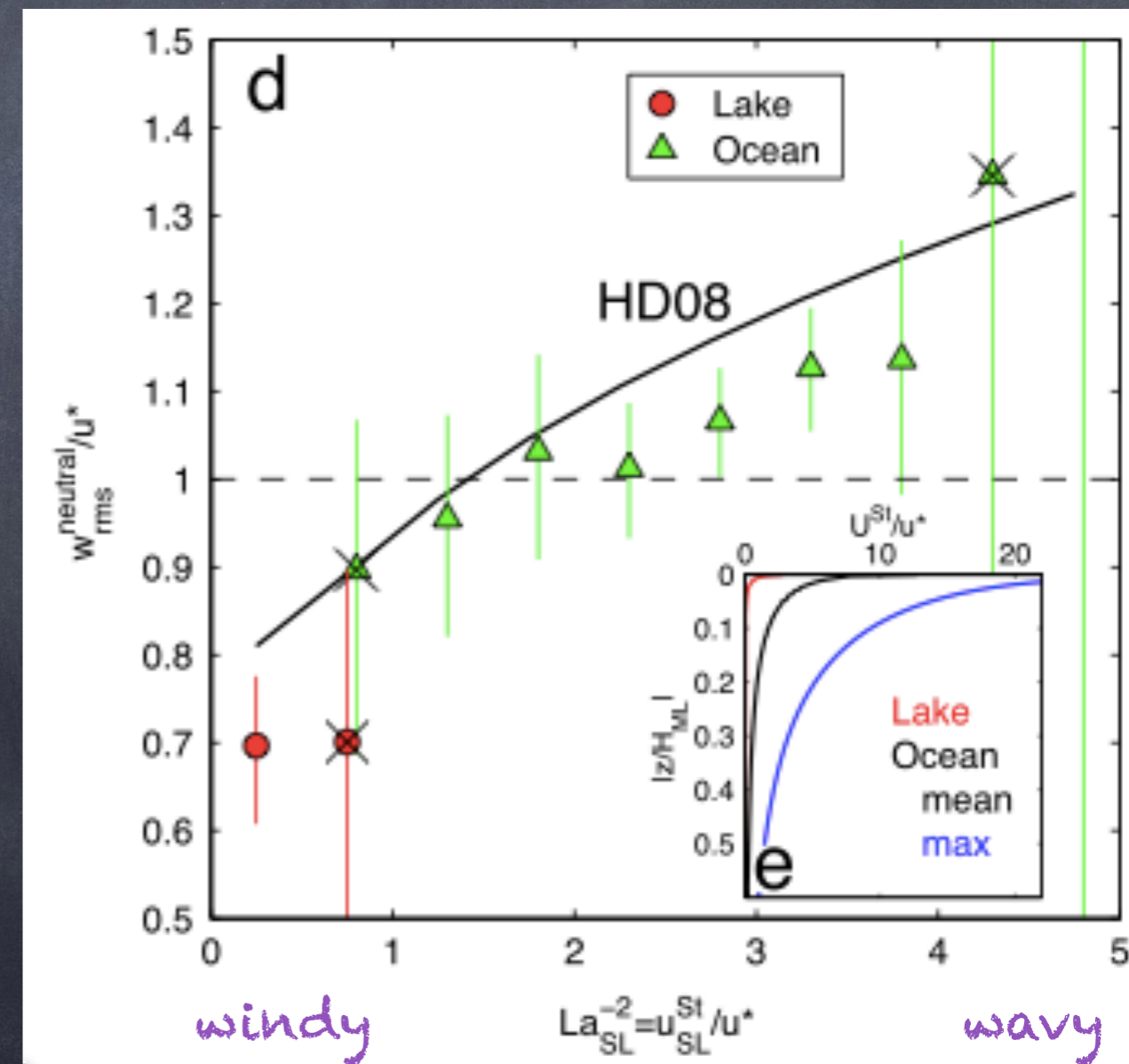
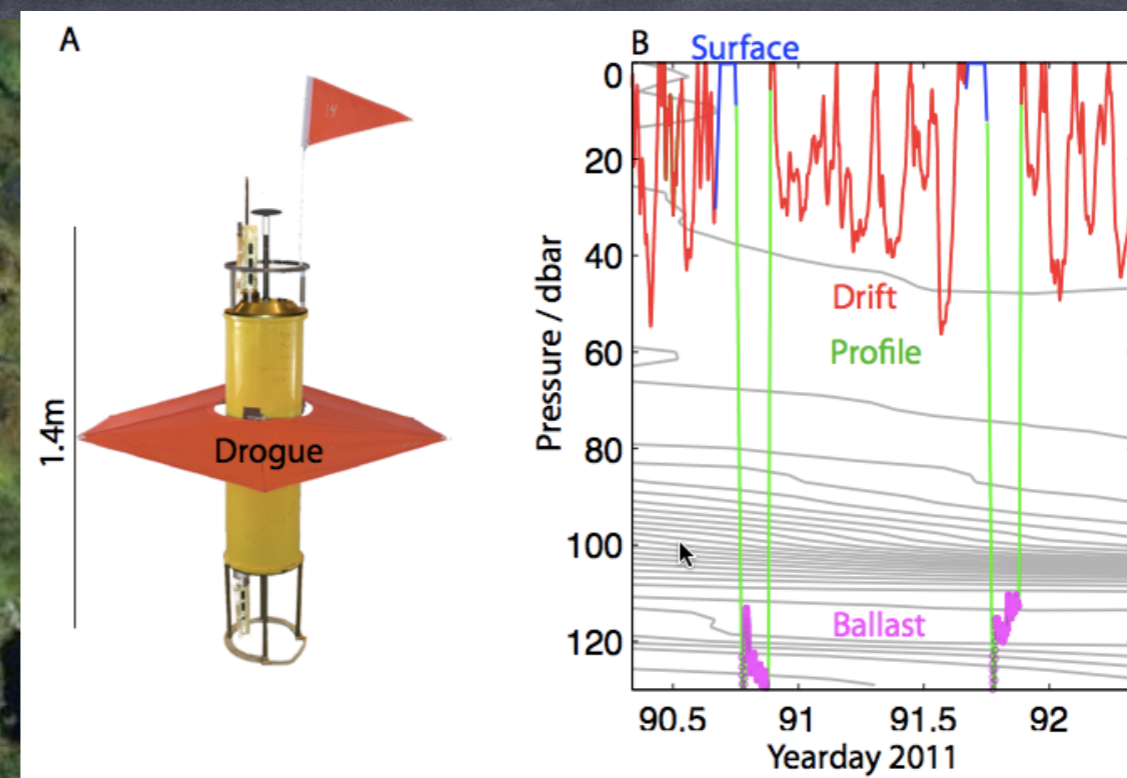
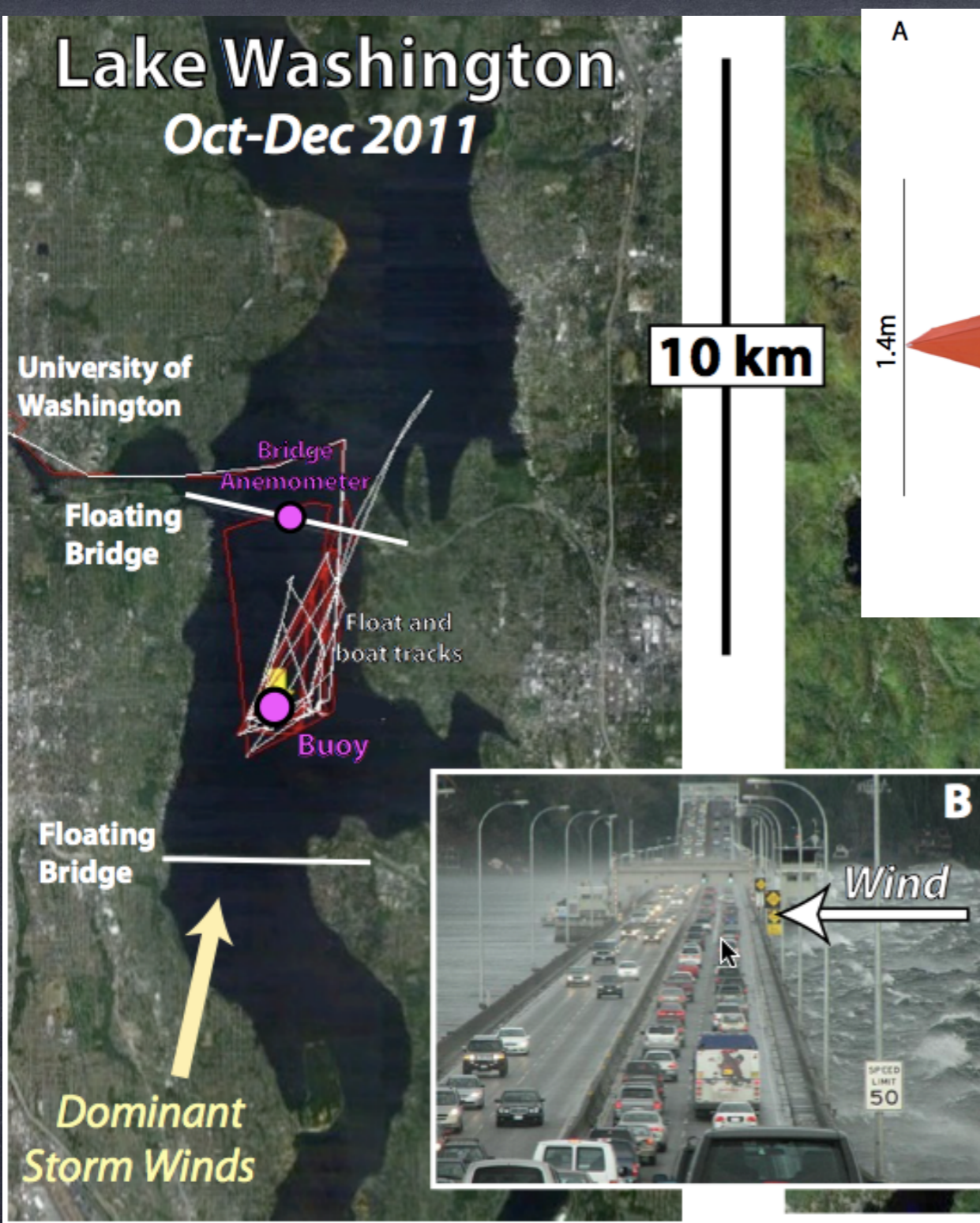
Figure 1
windrows
practice th
amalgama
within the



"wavy hydrostatic" if $\epsilon \gg 1$

$$\frac{\alpha^2}{Re} \left[w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{RoRe} w w_{,z} \right] = \boxed{-\pi_{,z} + b - \epsilon v_j^L v_{j,z}^S} + \frac{\alpha^2}{ReRe} w_{,jj}$$

N. Suzuki and BFK. Understanding Stokes forces in the wave-averaged equations. Journal of Geophysical Research-Oceans, 121:1-18, 2016.



1) Observations obey a particular scaling for $\langle w^2 \rangle$!

E. A. D'Asaro, J. Thomson, A. Y. Shcherbina, R. R. Harcourt, M. F. Cronin, M. A. Hemer, and BFK. Quantifying upper ocean turbulence driven by surface waves. *Geophysical Research Letters*, 41(1):102-107, January 2014.

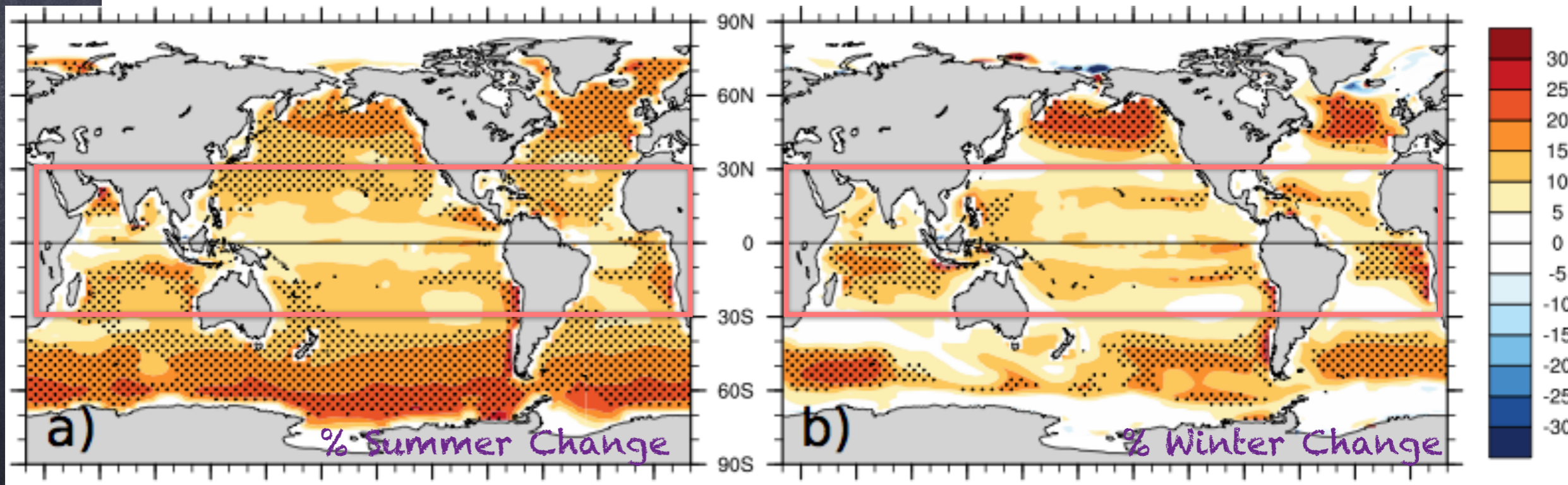
Langmuir Mixing in Climate: Boundary Layer Depth Improved

Case	Summer			Winter		
	Global	South of 30°S	30°S-30°N	Global	South of 30°S	30°S-30°N
CTRL	10.62±0.27 ^a (13.40±0.19) ^b	17.24±0.48 (21.73±0.32)	5.38±0.14 (6.71±0.09)	43.85±0.38 (45.50±0.40)	57.19±0.76 (56.53±0.59)	12.57±0.28 (16.16±0.29)
MS2K	15.37	15.47	17.03	119.91	171.92	40.31
SS02	36.79	63.83	7.54	99.32	164.34	17.39
VR12-AL	9.06	13.47	6.49	40.45	50.33	14.52
VR12-MA	8.73±0.30 (11.83±0.29)	12.65±0.47 (18.13±0.62)	6.61±0.22 (7.52±0.16)	40.99±0.37 (42.02±0.39)	51.78±0.65 (50.78±0.67)	14.23±0.30 (15.67±0.35)
VR12-EN	8.95	10.52	8.91	41.94	52.98	19.58

Control

Competition

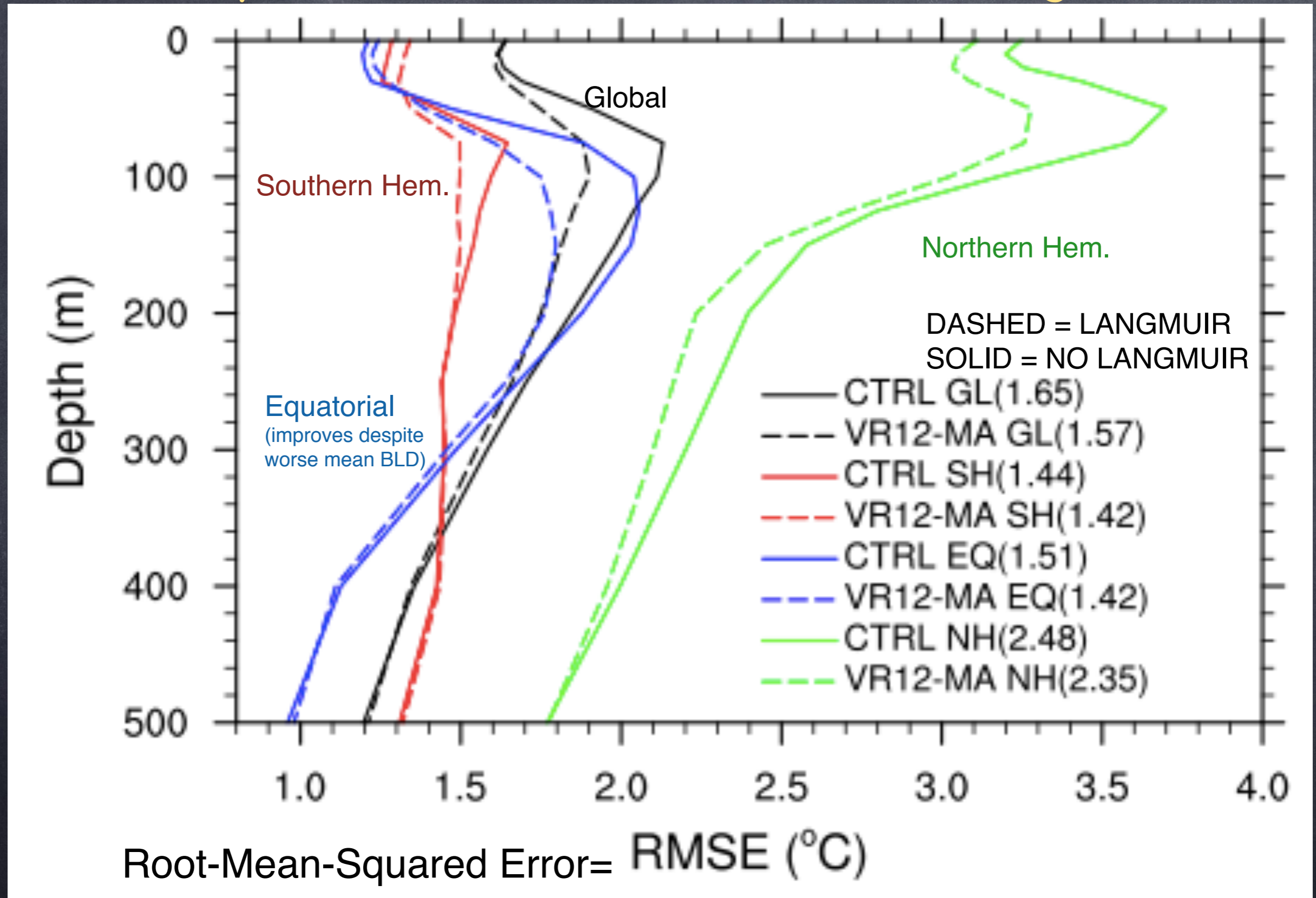
3 versions of
Van Roekel et
al



L. P. Van Roekel, BFK, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

Q. Li, A. Webb, BFK, A. Craig, G. Danabasoglu, W. G. Large, and M. Vertenstein. Langmuir mixing effects on global climate: WAVEWATCH III in CESM. *Ocean Modelling*, 103:145-160, July 2016.

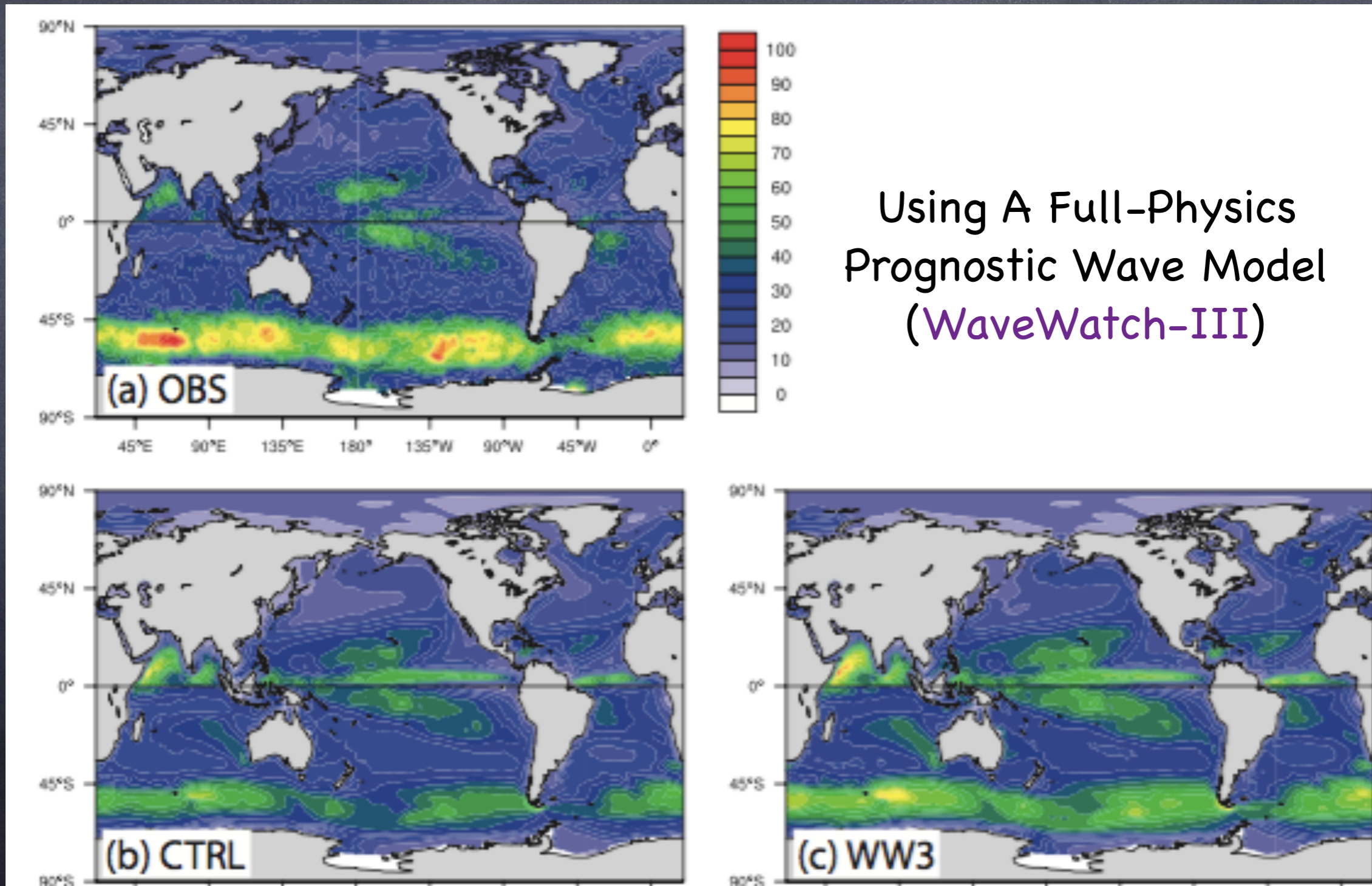
Stommel Demon, Subsurface Temperature (also CFCs, S, etc.): Improved vs. Observations with Langmuir



How accurate do we need the waves to be?

Langmuir Turbulence Parameterizations are robust to large approximations in wave modeling, e.g., replacing wave models with climatology, theoretical scalings

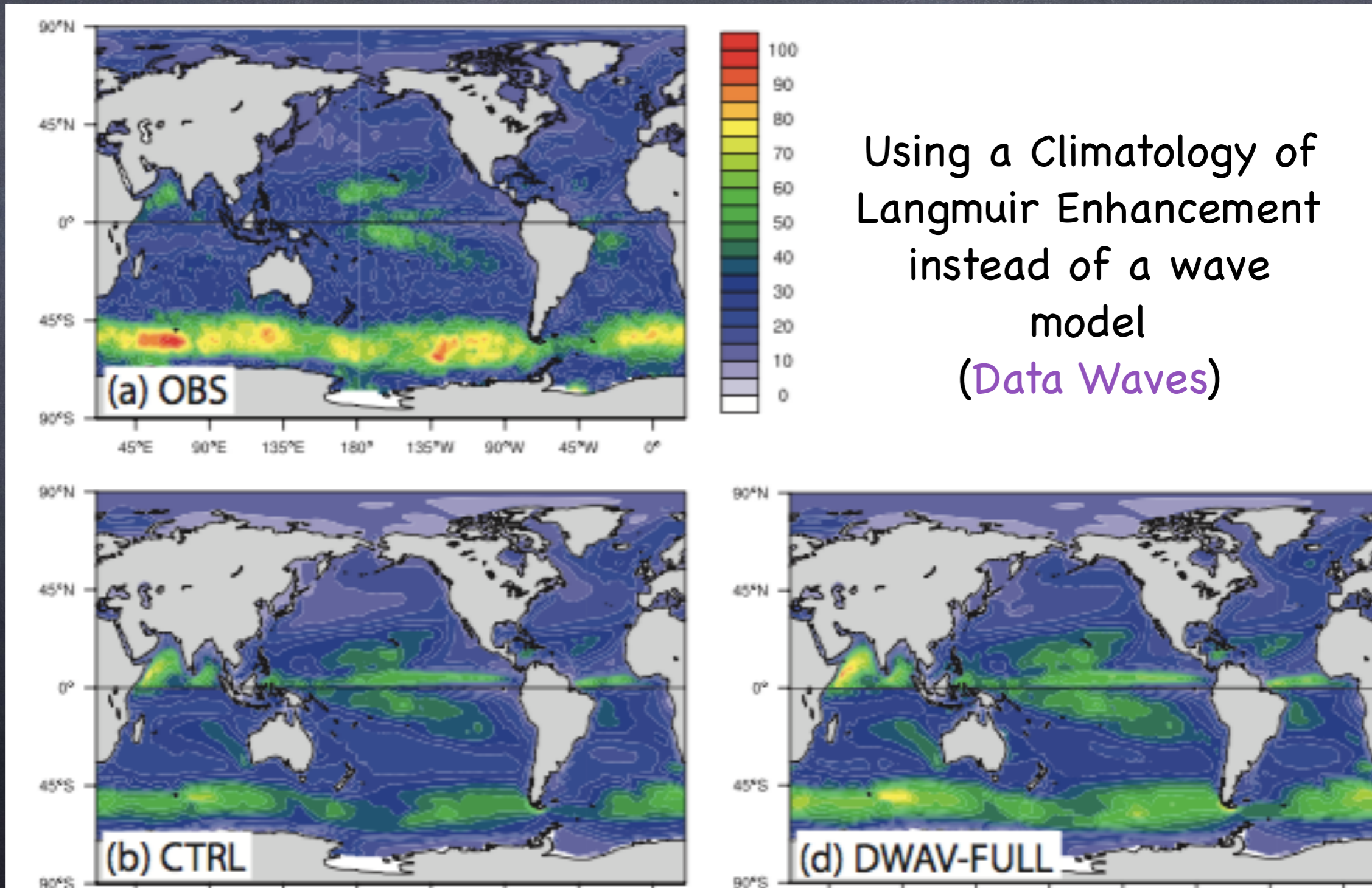
Q. Li, B. Fox-Kemper, O. Breivik, and A. Webb, 2016: Statistical modeling of global Langmuir mixing. *Ocean Modelling*. In press.



How accurate do we need the waves to be?

Langmuir Turbulence Parameterizations are robust to large approximations in wave modeling, e.g., replacing wave models with climatology, theoretical scalings

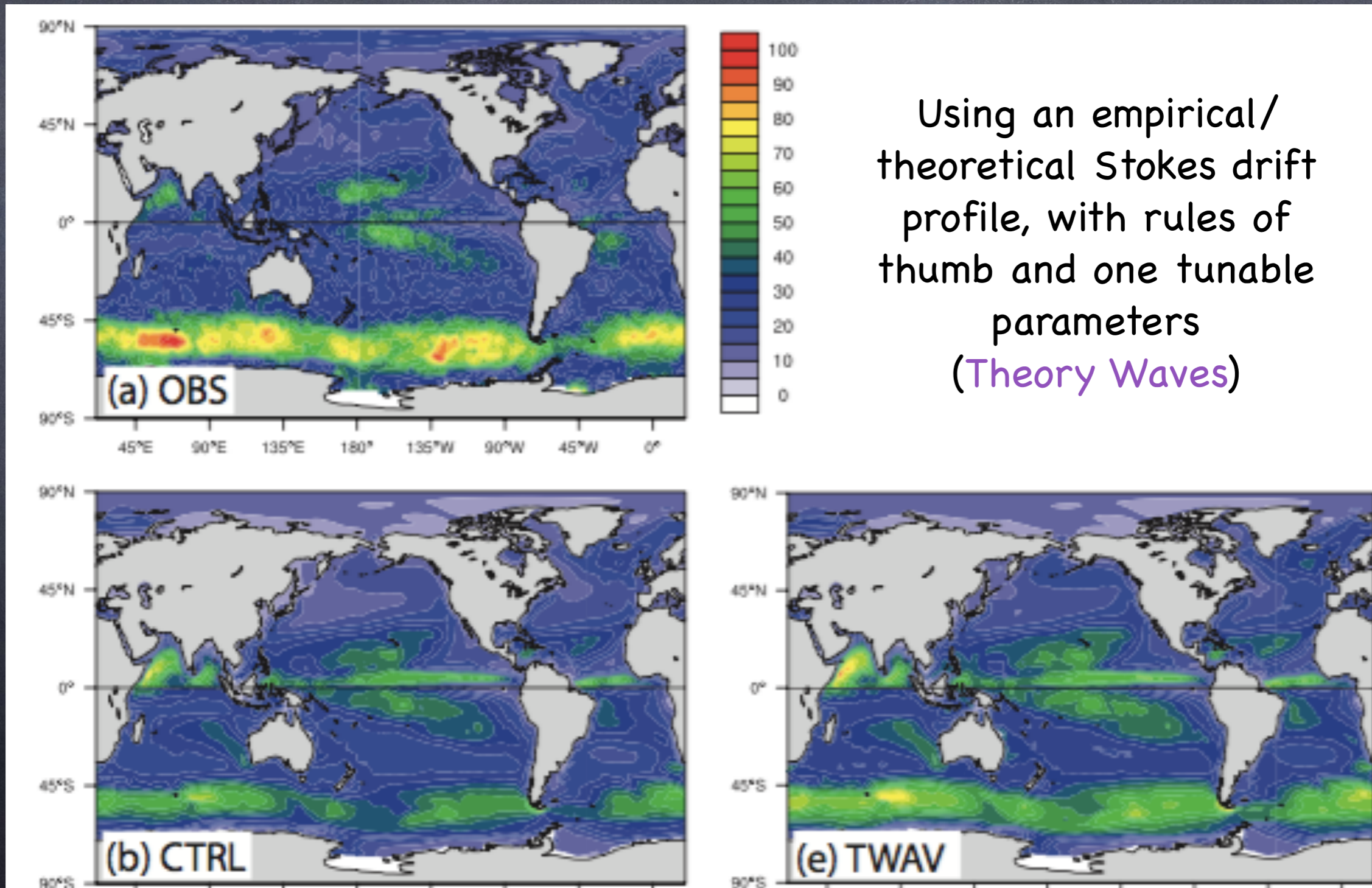
Q. Li, B. Fox-Kemper, O. Breivik, and A. Webb, 2016: Statistical modeling of global Langmuir mixing. Ocean Modelling. In press.



How accurate do we need the waves to be?

Langmuir Turbulence Parameterizations are robust to large approximations in wave modeling, e.g., replacing wave models with climatology, theoretical scalings

Q. Li, B. Fox-Kemper, O. Breivik, and A. Webb, 2016: Statistical modeling of global Langmuir mixing. Ocean Modelling. In press.



Do Details of Turbulence Matter Much?

- Our parameterization of Langmuir Turbulence comes in 2 parts:
 - Enhanced mixing within the boundary layer (based on Stokes parameters)
 - Enhanced entrainment (recasting the predicted boundary layer depth in terms of Stokes-dependent unresolved shear)

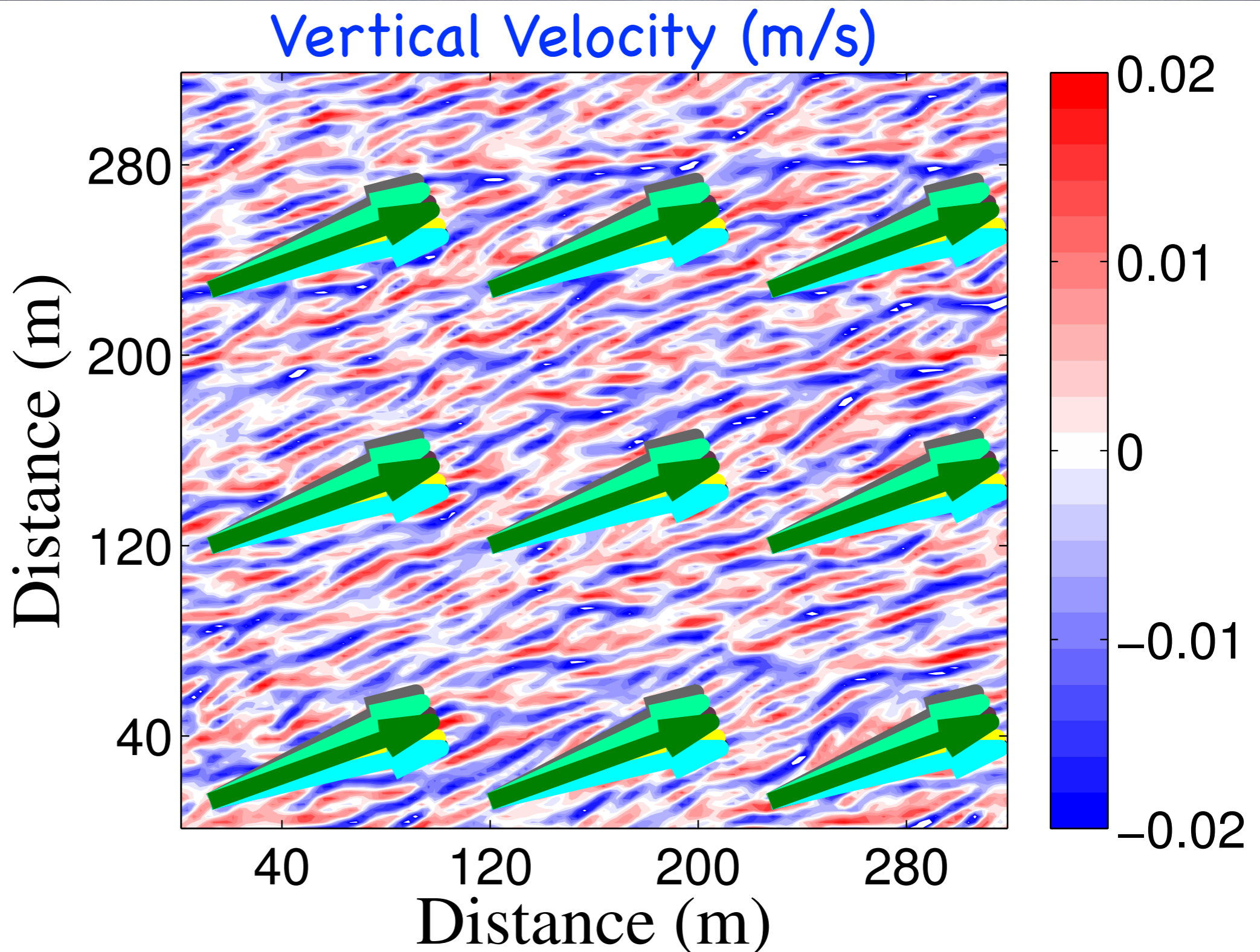
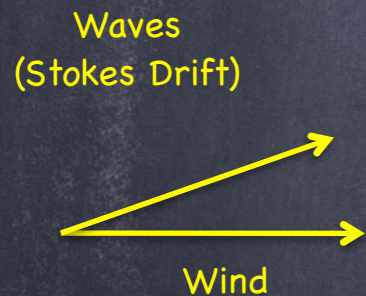
L. P. Van Roekel, BFK, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

Q. Li, A. Webb, BFK, A. Craig, G. Danabasoglu, W. G. Large, and M. Vertenstein. Langmuir mixing effects on global climate: WAVEWATCH III in CESM. *Ocean Modelling*, 103:145-160, July 2016.

Q. Li, B. Fox-Kemper, 2017: Assessing the effects of Langmuir turbulence on the entrainment buoyancy flux in the ocean surface boundary layer. *JPO*. In Preparation.

Something that happens often with waves: Tricky: Misaligned Wind & Waves

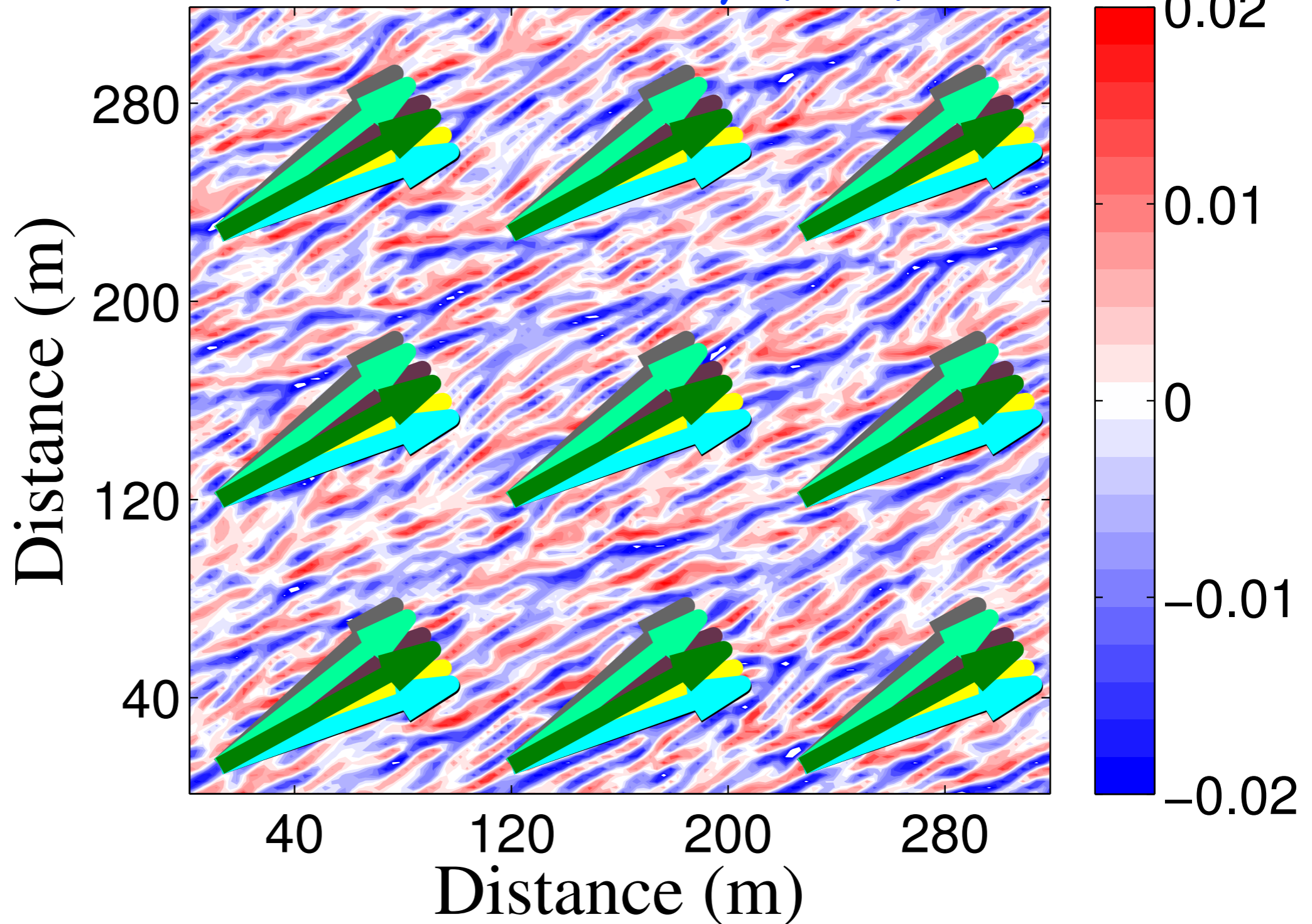
A. Webb and BFK. Impacts of wave spreading and multidirectional waves on estimating Stokes drift. *Ocean Modelling*, 96(1): 49-64, December 2015.



L. P. Van Roekel, BFK, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

Tricky: Misaligned Wind & Waves

Vertical Velocity (m/s)

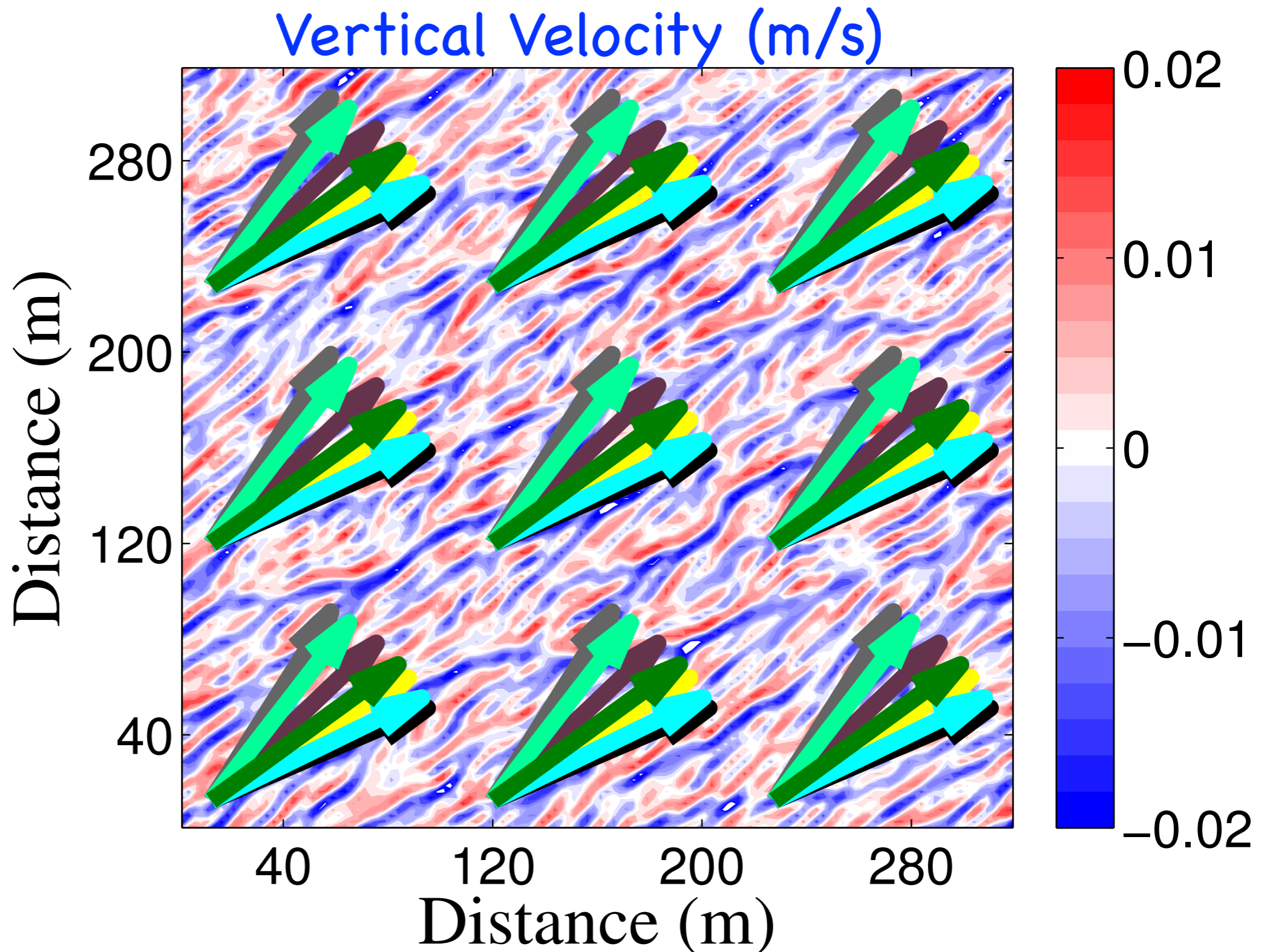
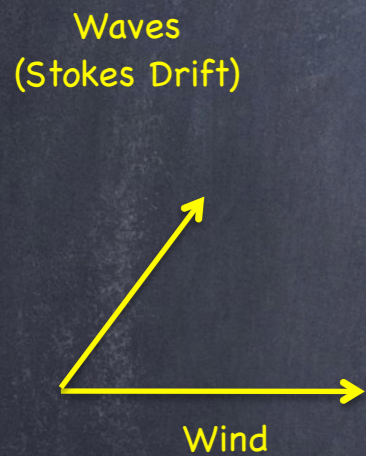


Waves
(Stokes Drift)



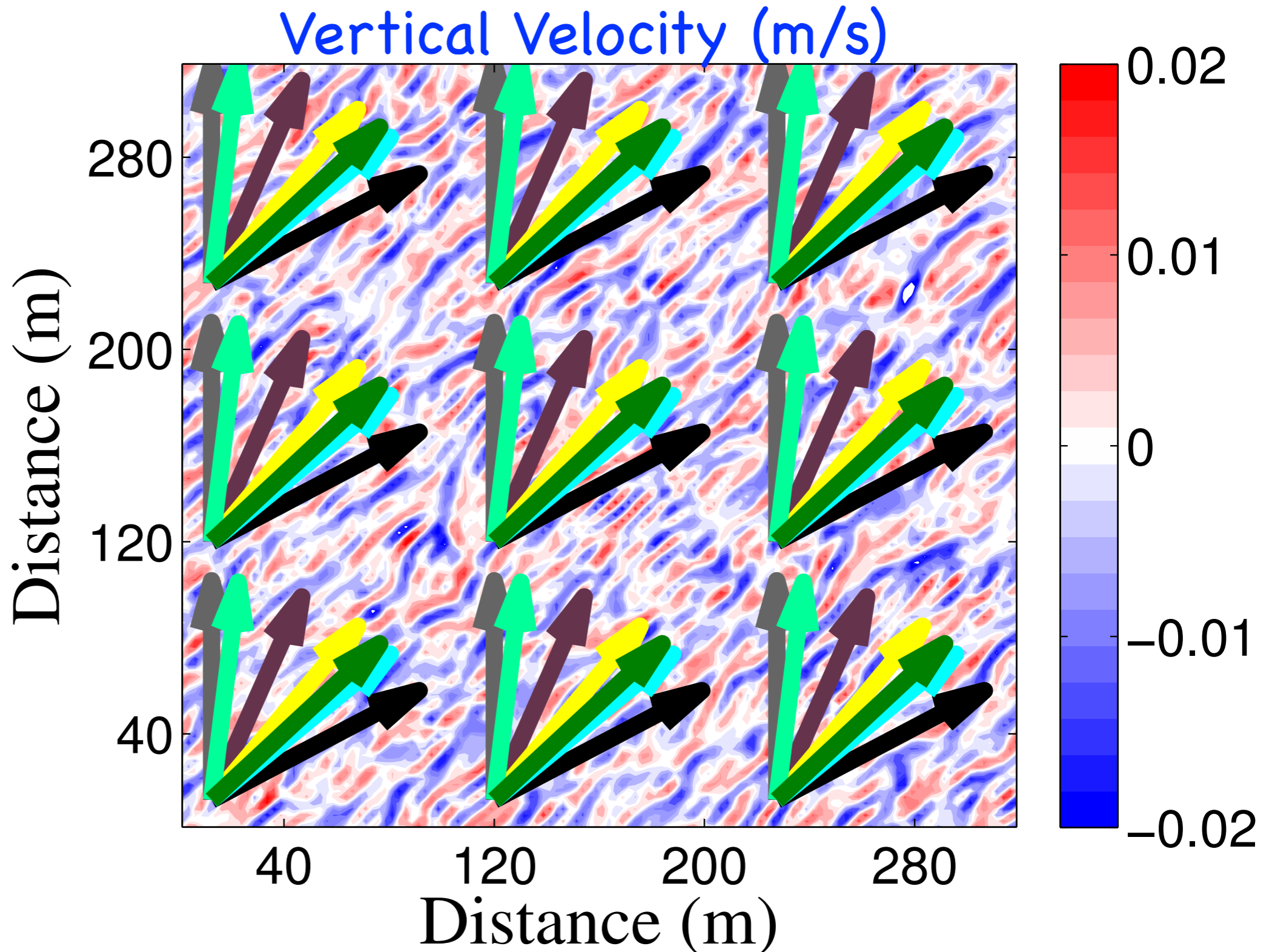
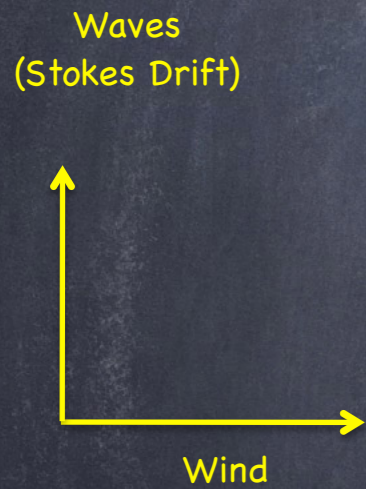
L. P. Van Roekel, BFK, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

Tricky: Misaligned Wind & Waves



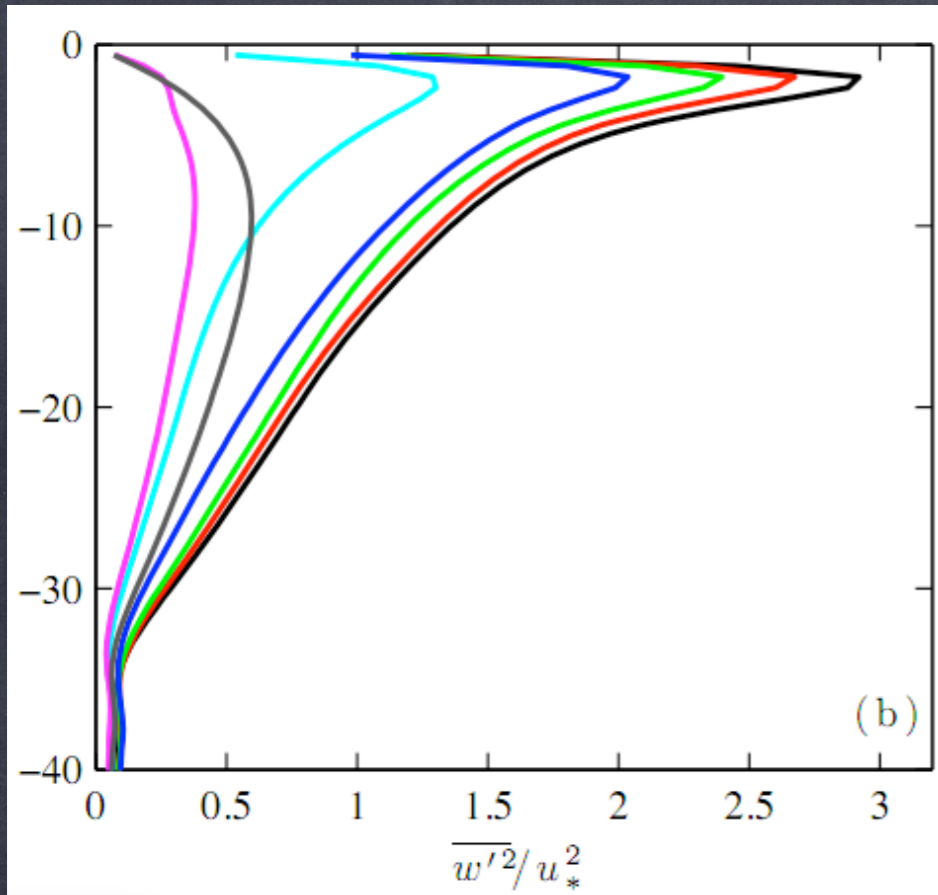
L. P. Van Roekel, BFK, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

Tricky: Misaligned Wind & Waves

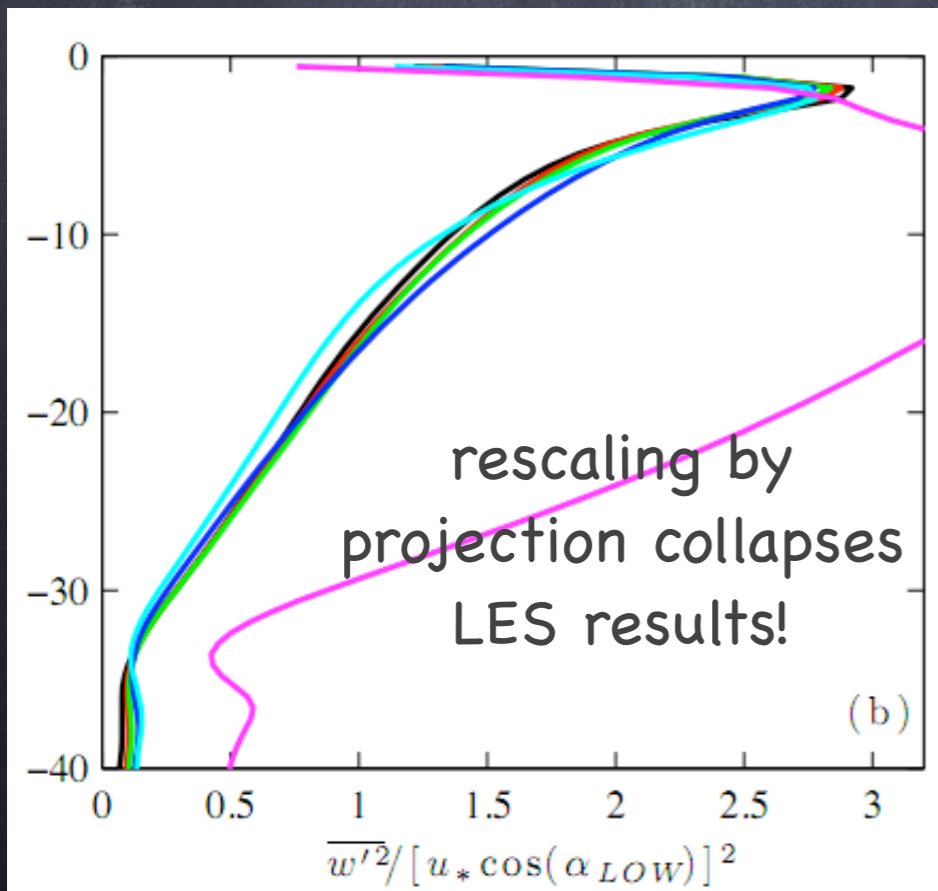


L. P. Van Roekel, BFK, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

$\langle w^2 \rangle$



rescaled $\langle w^2 \rangle$



Generalized Turbulent Parameter (Langmuir Number)

Projection of u^* , u_s into Langmuir
Direction

$$La_{proj}^2 = \frac{|u_*| \cos(\alpha_{LOW})}{|u_s| \cos(\theta_{ww} - \alpha_{LOW})}$$

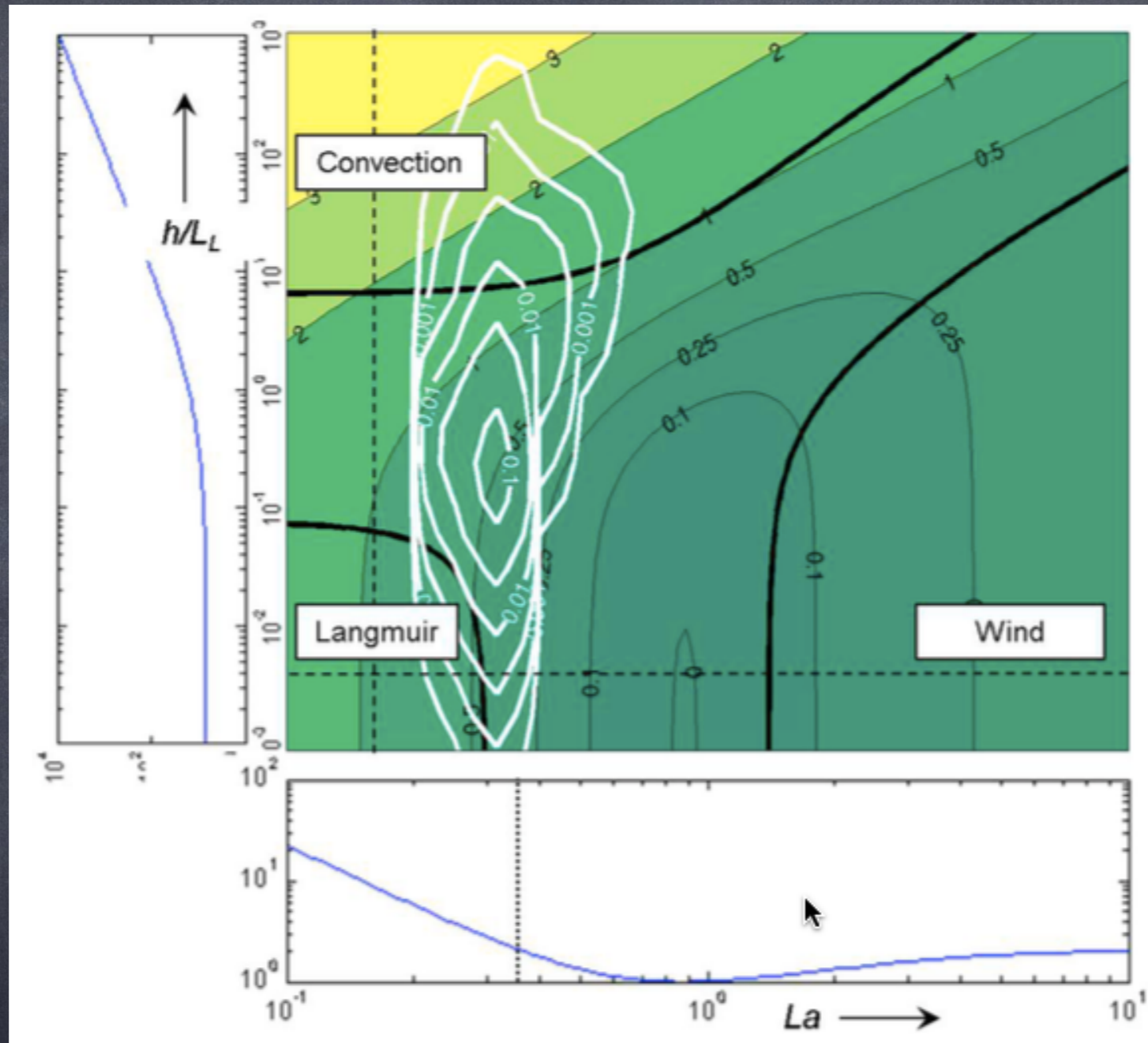
A scaling for LC strength & direction!
Enough for climate model application

Also, benefit from Harcourt & D'Asaro (2008) to
use a Surface Layer Average, rather than surface
La to be robust to wind waves vs. monochromatic

L. P. Van Roekel, BFK, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, 2012.

Do Details of Turbulence Matter Much?

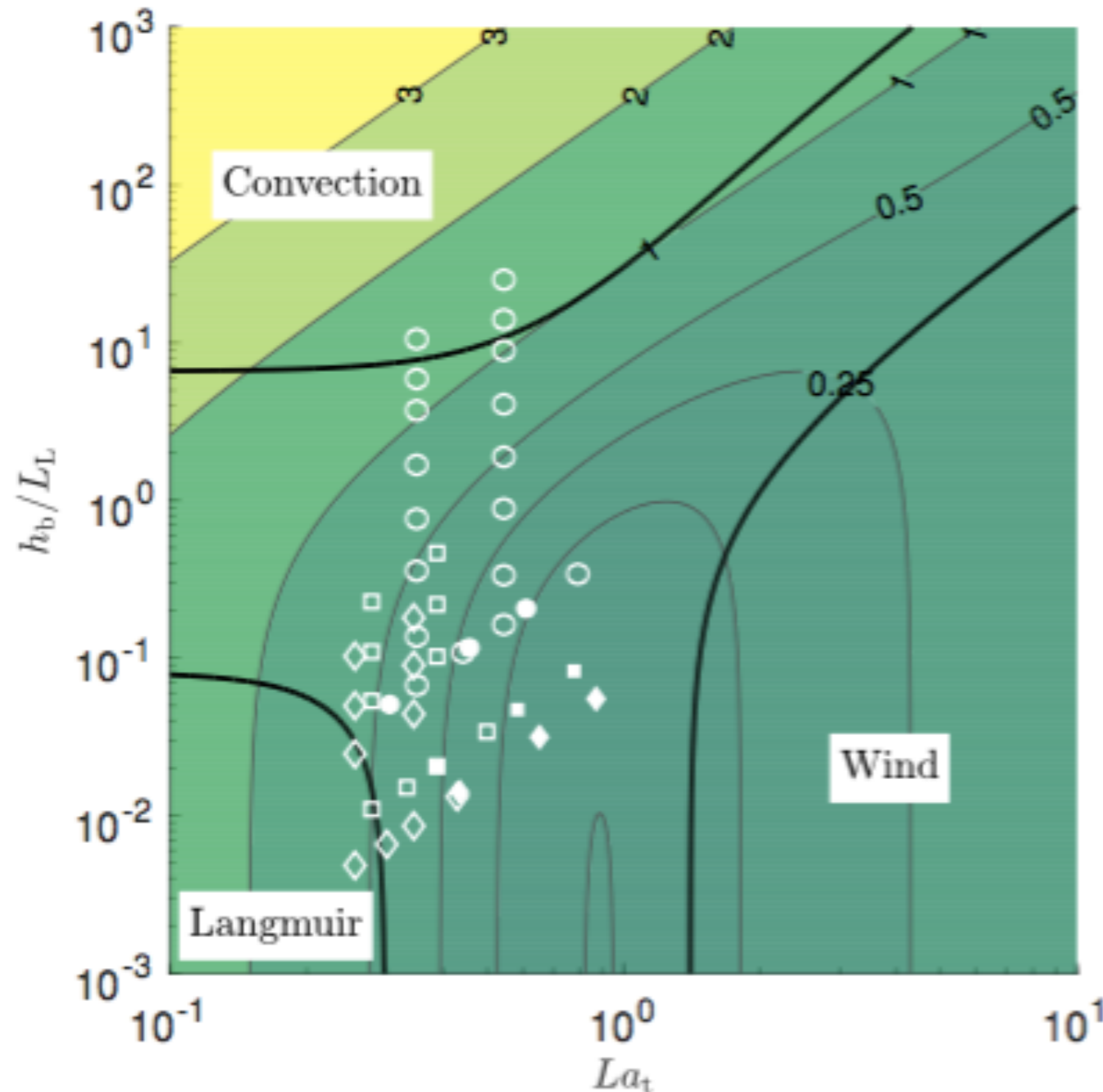
Dissipation Rate Regimes of S. Ocean



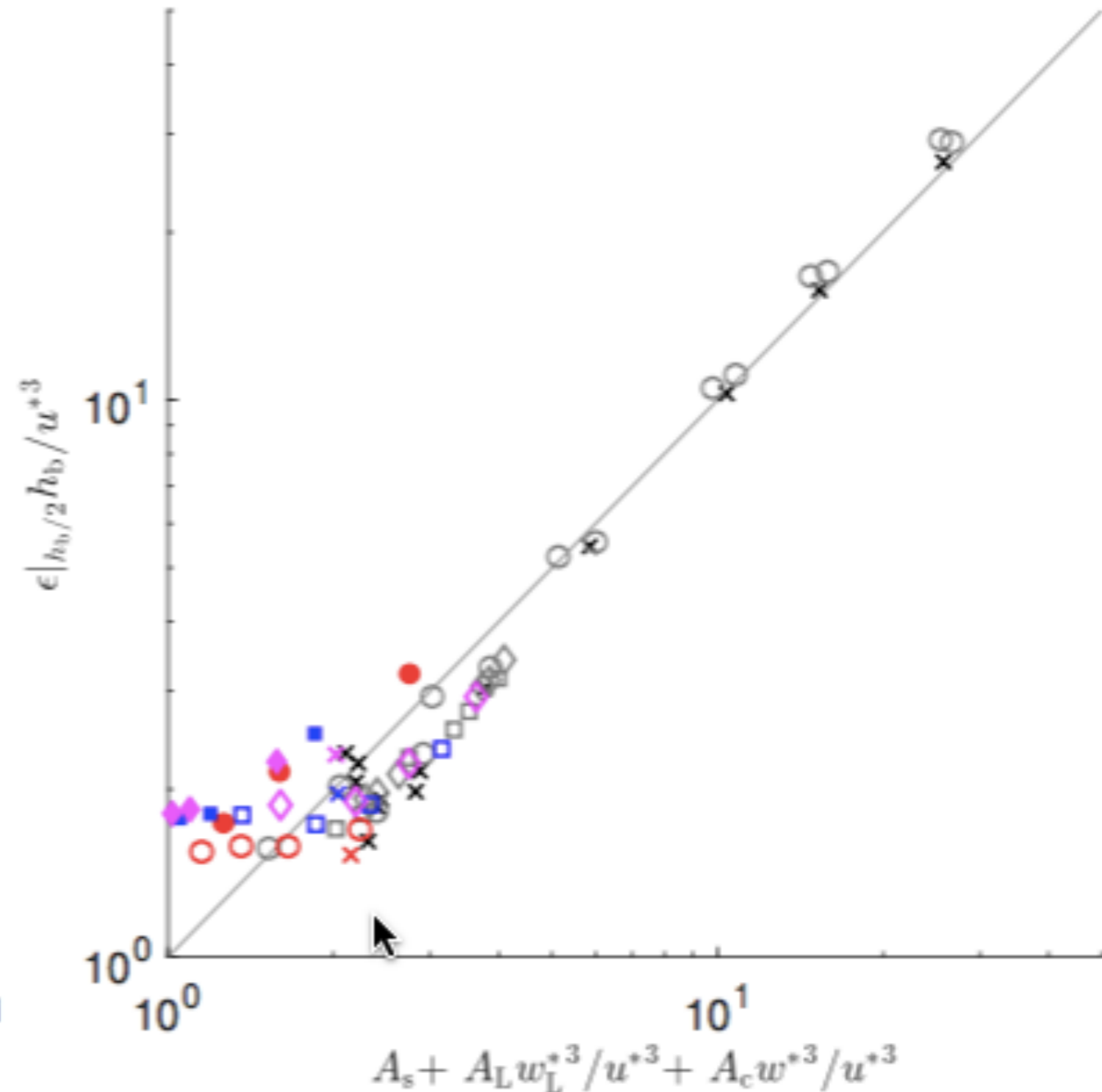
S. E. Belcher, A. A. L. M. Grant, K. E. Hanley, B. Fox-Kemper, L. Van Roekel, P. P. Sullivan, W. G. Large, A. Brown, A. Hines, D. Calvert, A. Rutgersson, H. Petterson, J. Bidlot, P. A. E. M. Janssen, and J. A. Polton, 2012: A global perspective on Langmuir turbulence in the ocean surface boundary layer. *Geophysical Research Letters*, 39(18):L18605

Do Details of Turbulence Matter Much?

Dissipation Rate Evaluation using LES



(a)

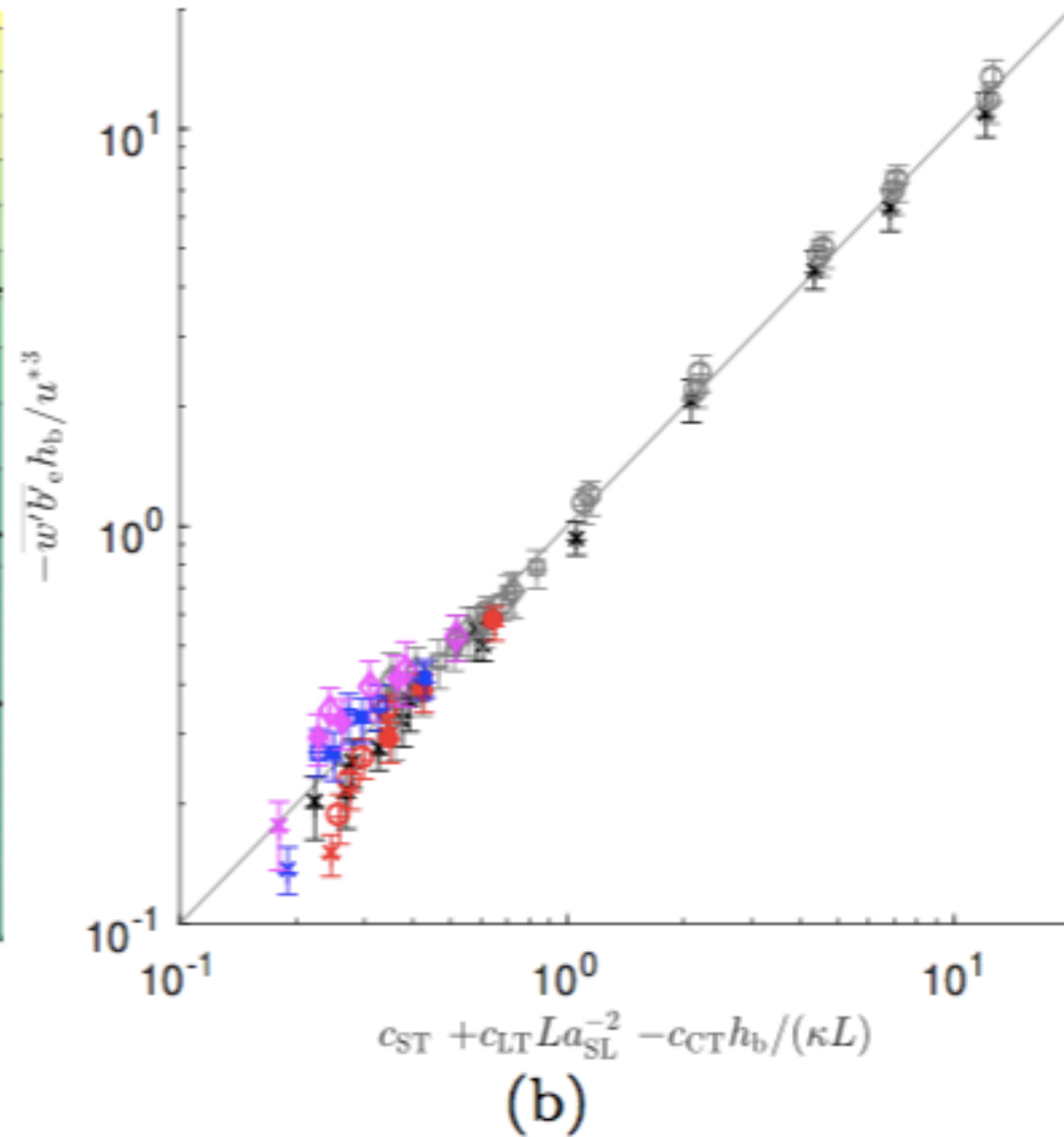
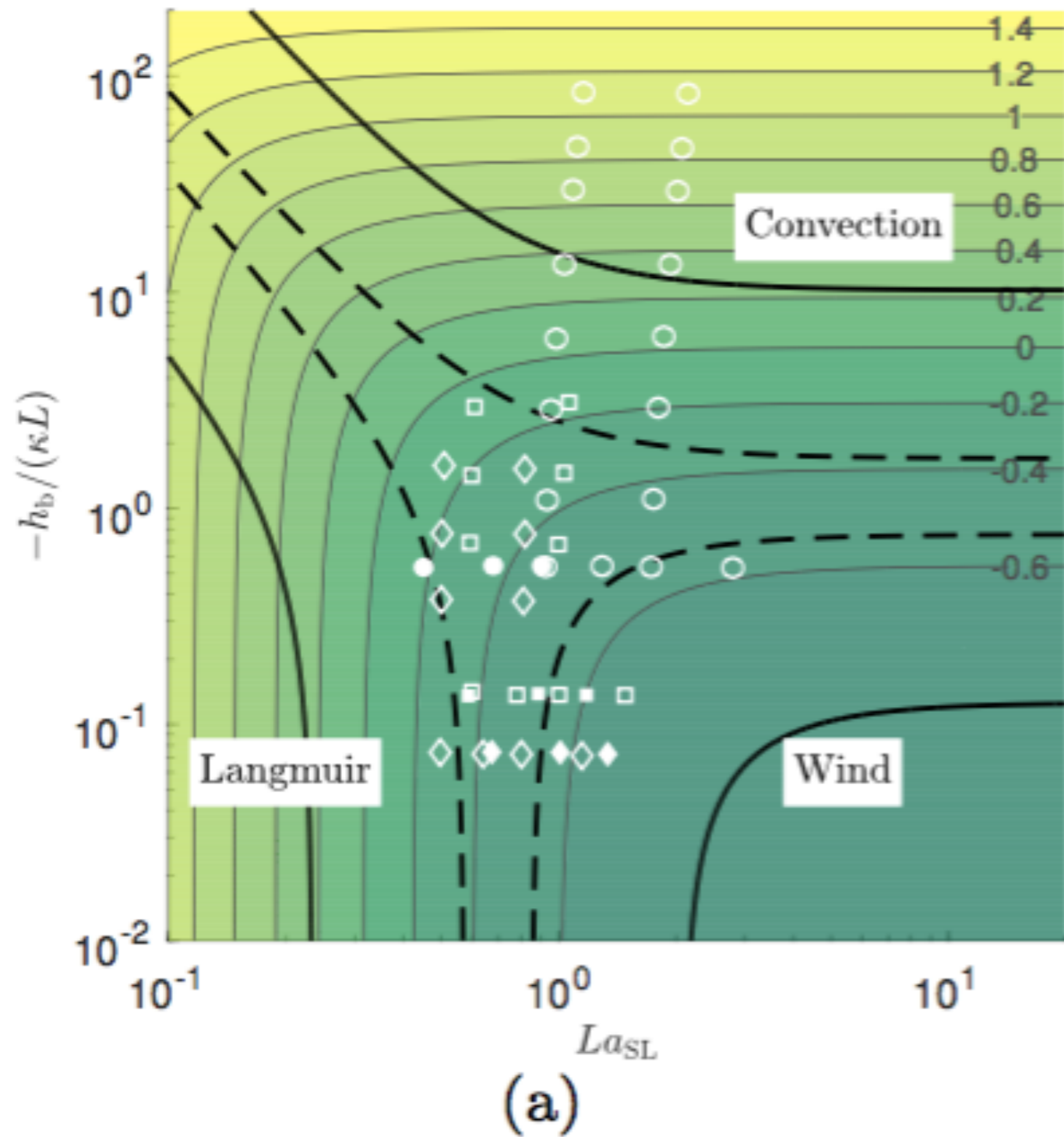


(b)

Q. Li, B. Fox-Kemper, 2017: Assessing the effects of Langmuir turbulence on the entrainment buoyancy flux in the ocean surface boundary layer. JPO. In preparation.

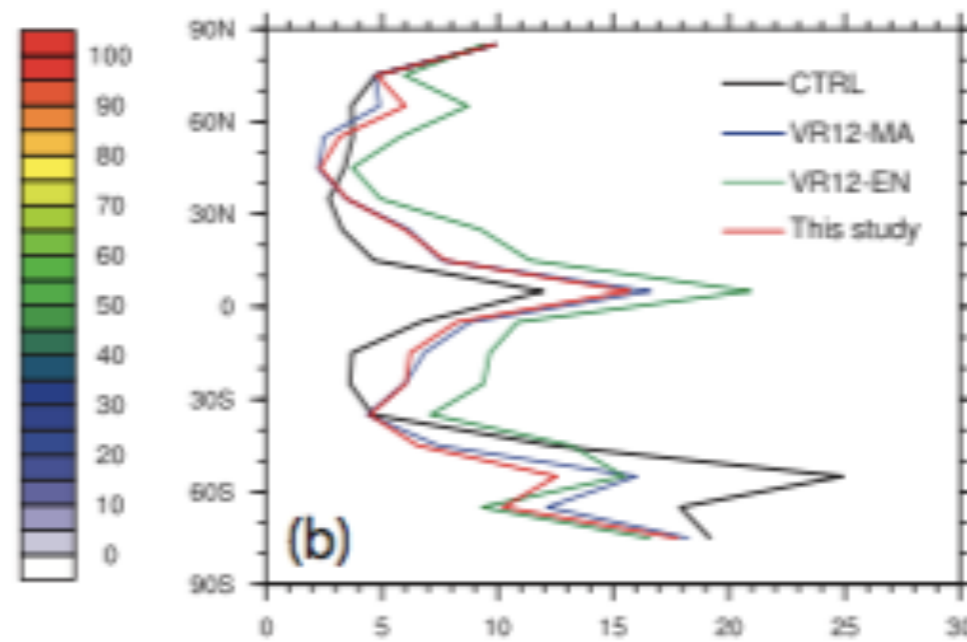
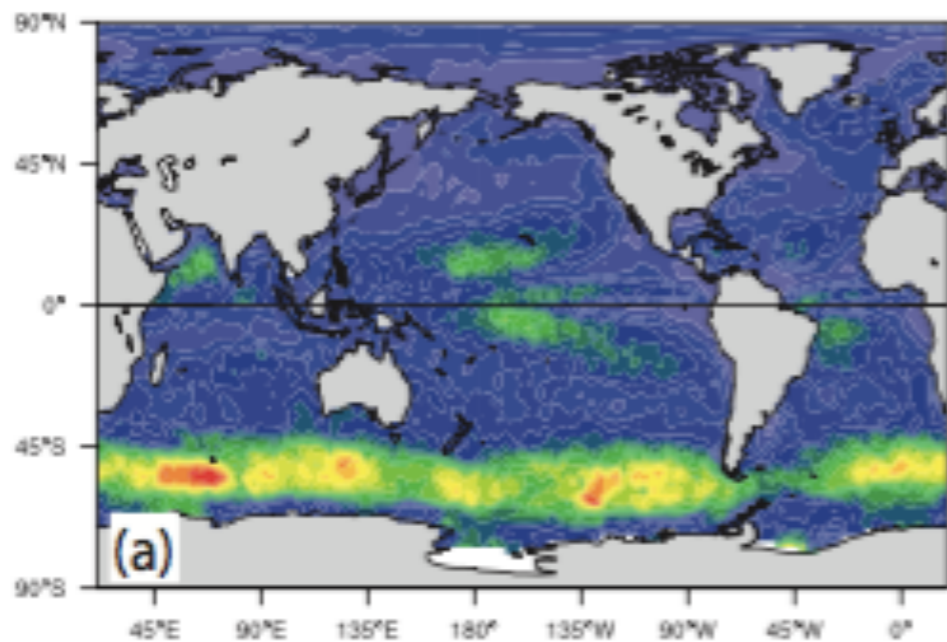
Do Details of Turbulence Matter Much?

Entrainment Rate Evaluation using LES

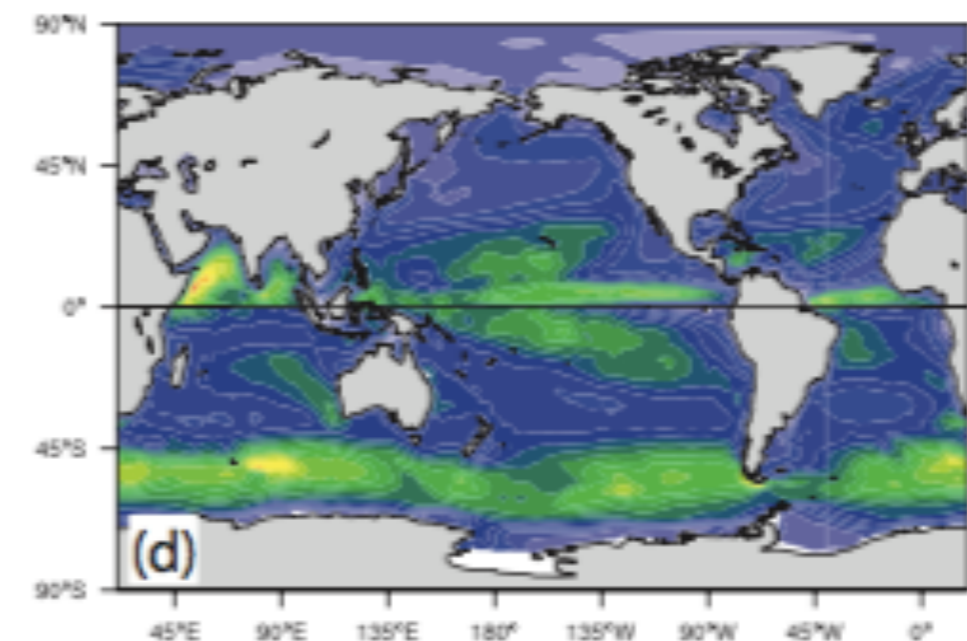
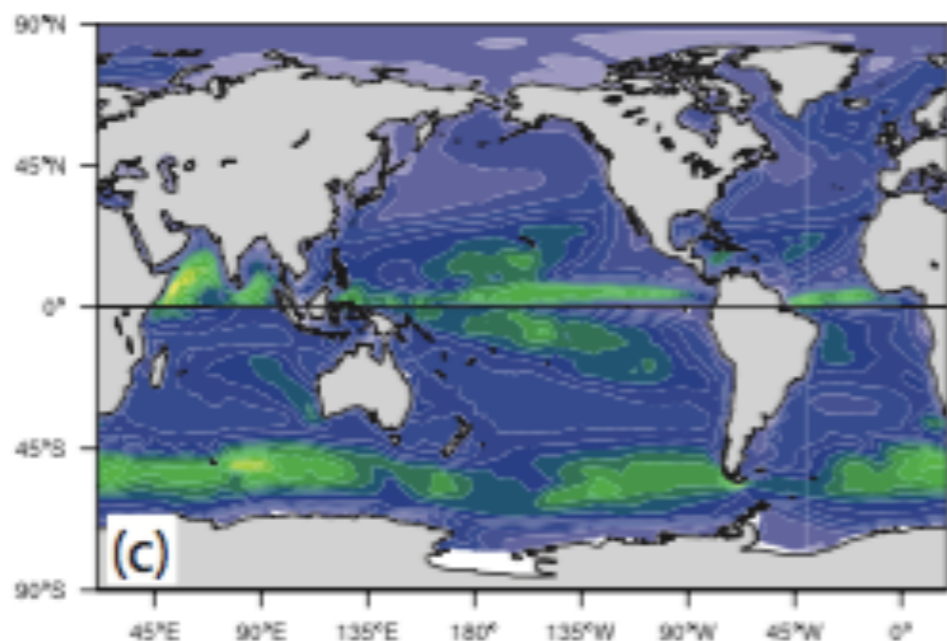


Q. Li, B. Fox-Kemper, 2017: Assessing the effects of Langmuir turbulence on the entrainment buoyancy flux in the ocean surface boundary layer. JPO. In preparation.

Obs.

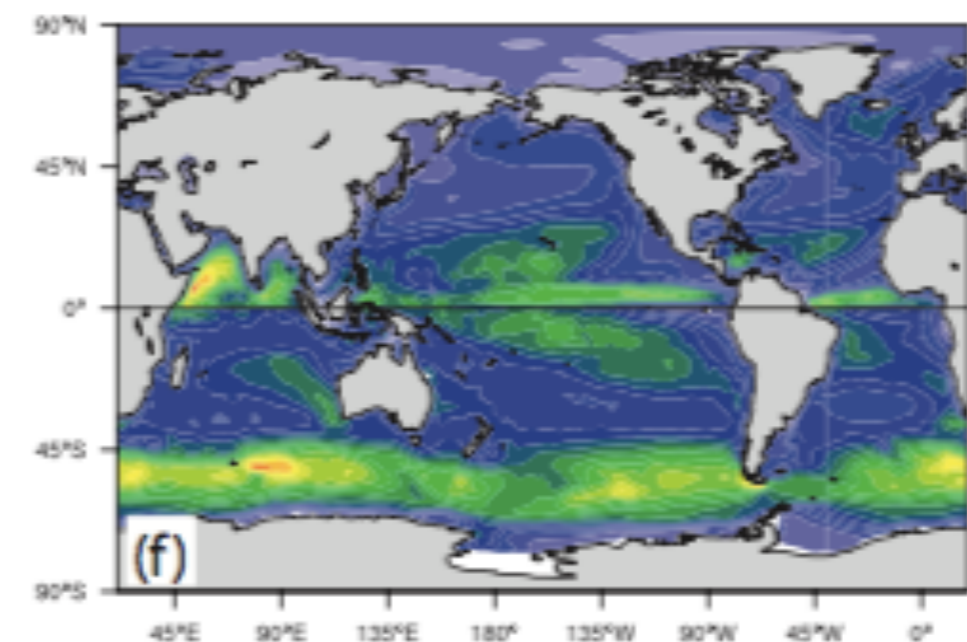
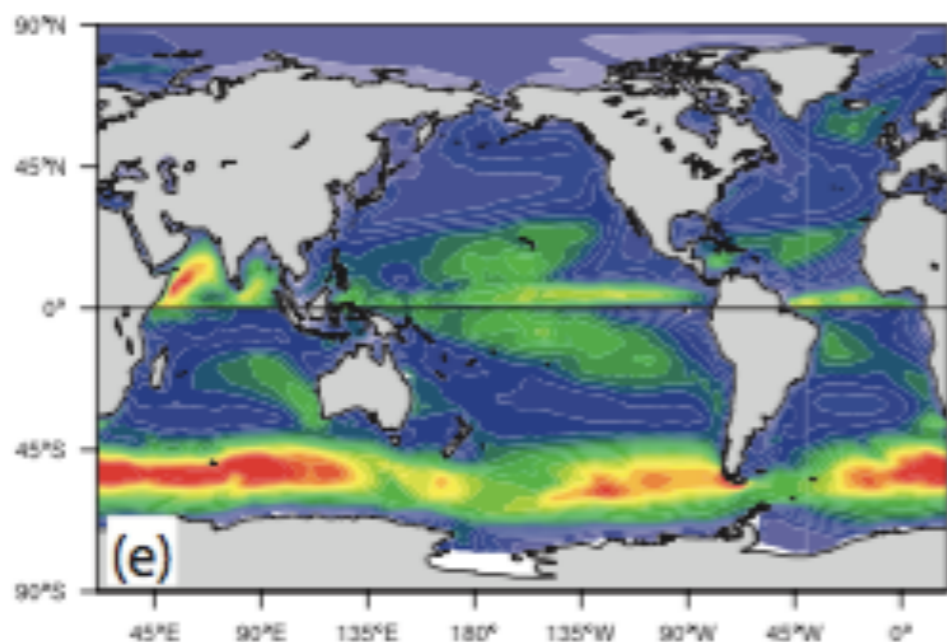


No
Lang.



Mixing
w/o
Entrain
Eval.

Early
Entrain
Guess.



Mixing
&
Refined
Entrain.

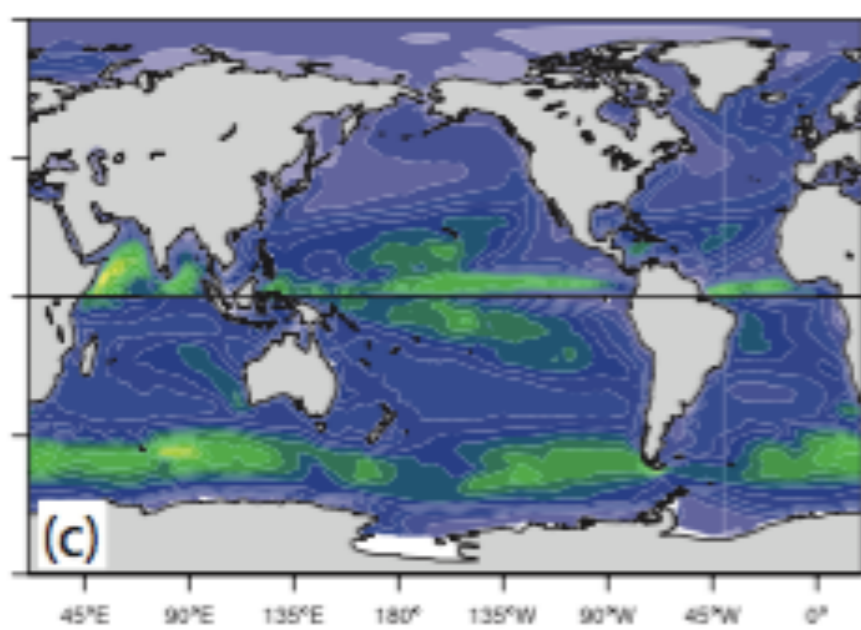
Langmuir Mixing in Climate: Boundary Layer Depth Improved

Case	Summer			Winter		
	Global	South of 30°S	30°S-30°N	Global	South of 30°S	30°S-30°N
CTRL	10.28 ± 0.29	16.00 ± 0.48	6.57 ± 0.23	50.24 ± 1.42	52.52 ± 0.54	15.89 ± 0.33
VR12-MA	9.31 ± 0.28	10.64 ± 0.49	9.60 ± 0.33	47.65 ± 1.15	48.47 ± 0.49	22.98 ± 0.42
VR12-EN	11.65 ± 0.29	11.91 ± 0.83	12.79 ± 0.39	56.85 ± 0.93	61.30 ± 1.21	33.60 ± 0.55
LF17	8.48 ± 0.24	8.92 ± 0.39	9.15 ± 0.30	47.78 ± 1.08	49.98 ± 0.77	22.43 ± 0.43

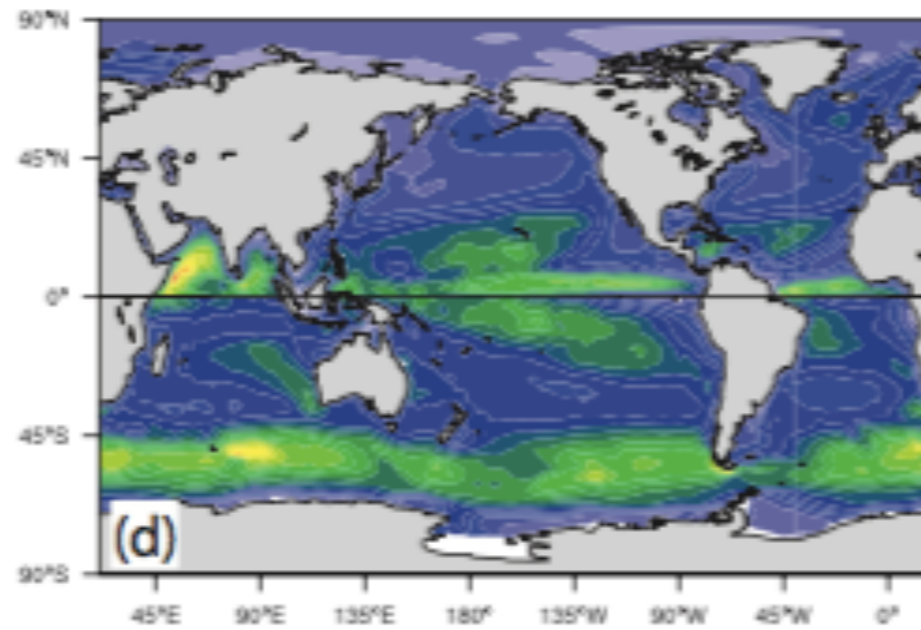
Control

3 versions of
Van Roekel et
al

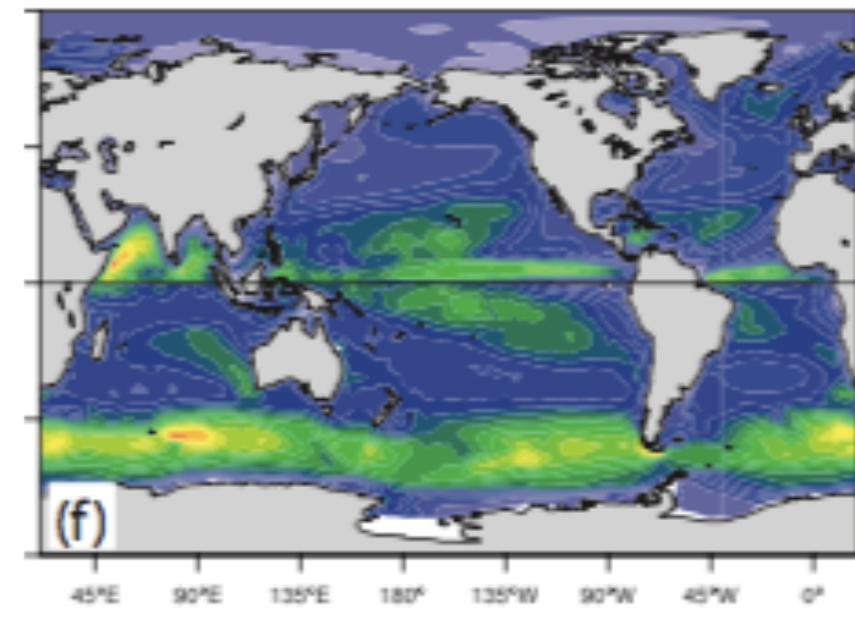
New Scheme



No Lang.



Mixing w/o Entrain Eval.



Mixing & Refined Entrainment

L. P. Van Roekel, BFK, P. P. Sullivan, P. E. Hamlington, and S. R. Haney. The form and orientation of Langmuir cells for misaligned winds and waves. *Journal of Geophysical Research-Oceans*, 117:C05001, 22pp, May 2012.

Q. Li, A. Webb, BFK, A. Craig, G. Danabasoglu, W. G. Large, and M. Vertenstein. Langmuir mixing effects on global climate: WAVEWATCH III in CESM. *Ocean Modelling*, 103:145-160, July 2016.

Q. Li, B. Fox-Kemper, 2017: Assessing the effects of Langmuir turbulence on the entrainment buoyancy flux in the ocean surface boundary layer. *JPO*. In preparation.

CONCLUSIONS:

Waves on Turbulence & Climate

- The inclusion of Langmuir (wave-driven) mixing is justified by obs., LES, and reduction of climate model bias.
- Generally, these schemes make mixed layer deeper— affecting air-sea, CFCs, carbon exchange, etc.
- The Data Waves and Theory Waves versions of our scheme are available through CVmix—no wave model required!
- Improvement of scalings vs. LES has worked very well to date, but as nearly all present schemes agree well with LES—returns are diminishing.

Do Stokes forces affect (sub)Meso-Scales?

Movie: P. Hamlington

LES of Langmuir turbulence with a submesoscale temperature front

Use NCAR LES model to solve Wave-Averaged Eqns.

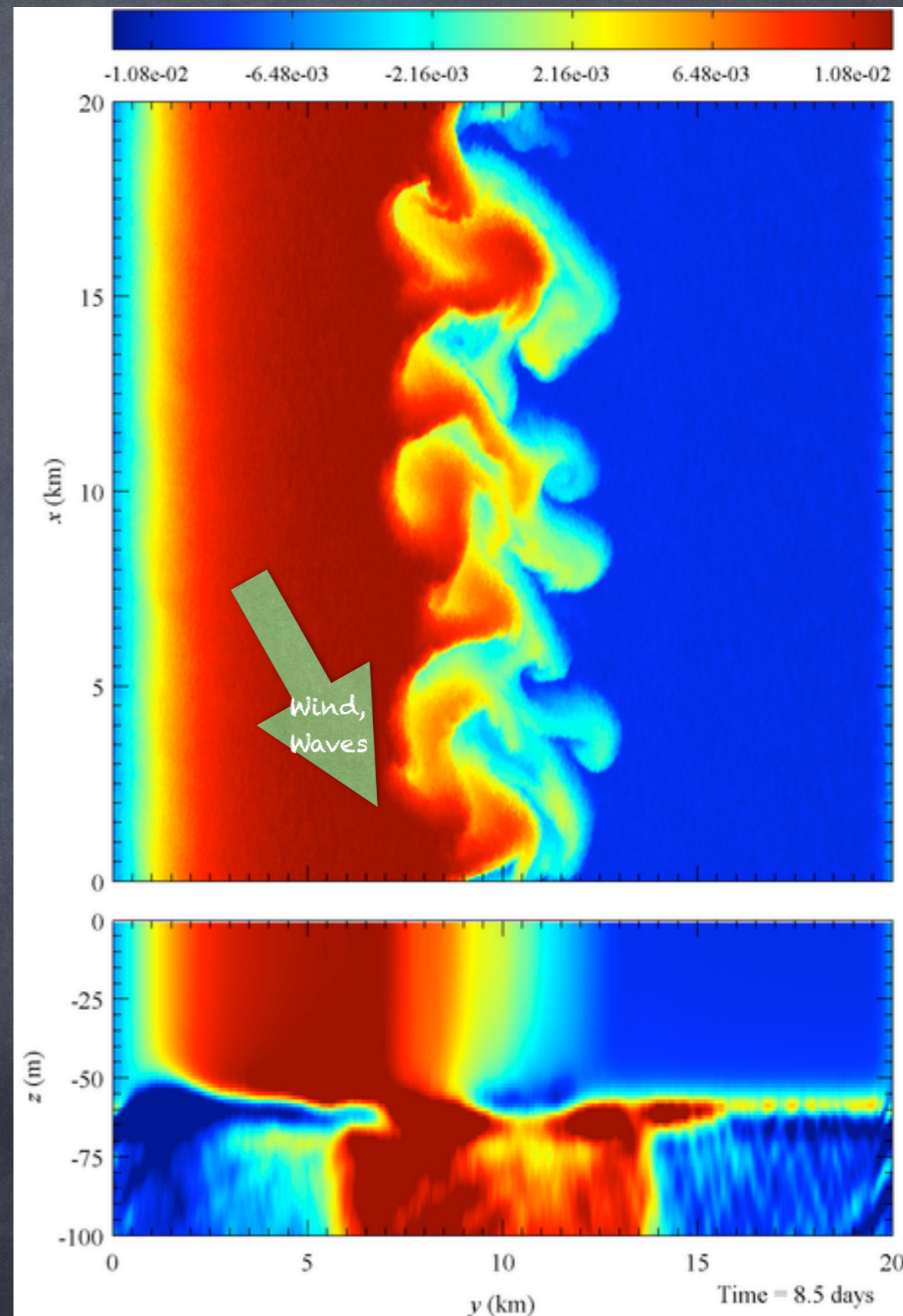
2 Versions: 1 With Waves & Winds
1 With only Winds

Computational parameters:

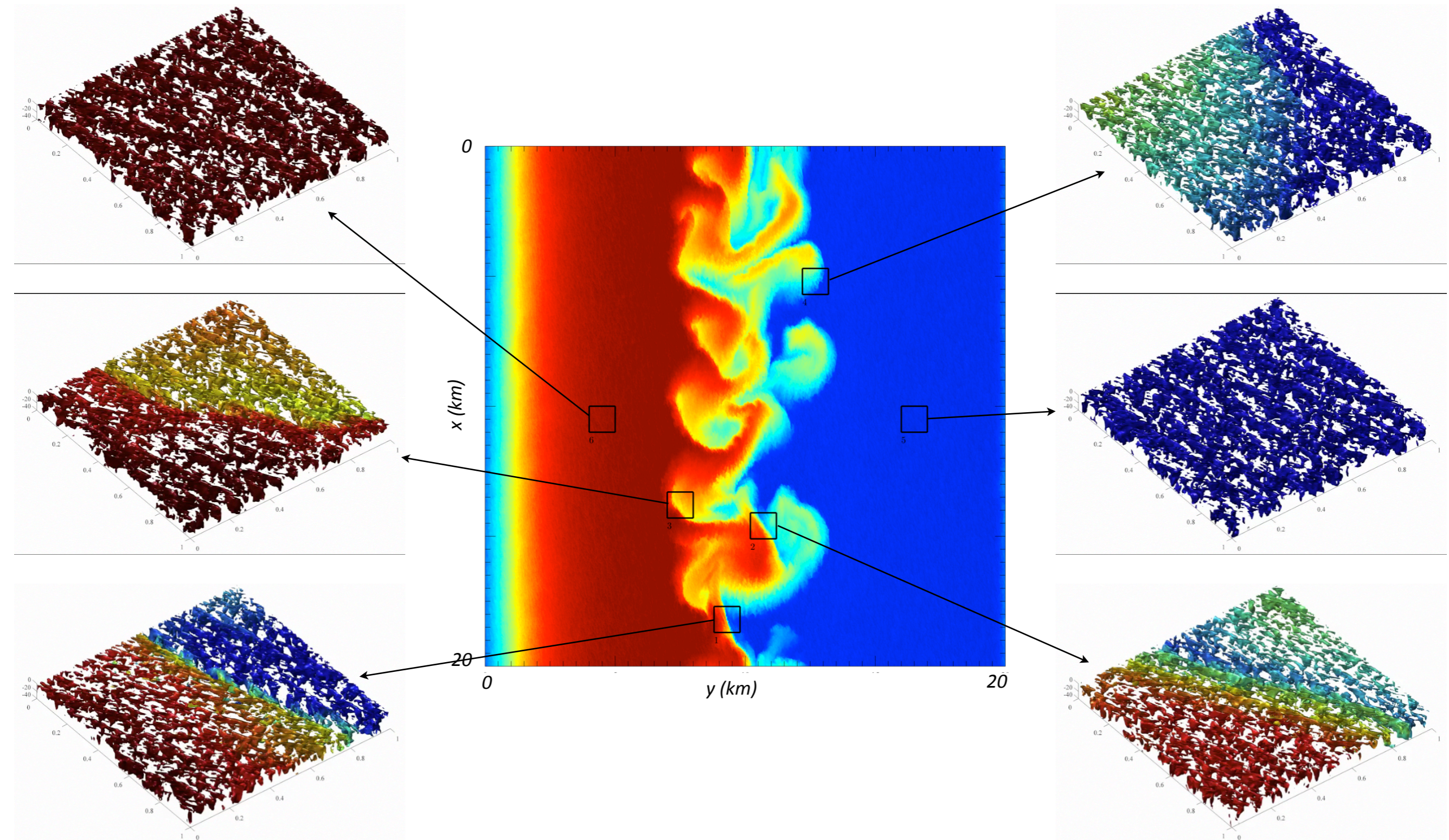
Domain size: 20km x 20km x -160m

Grid points: 4096 x 4096 x 128

Resolution: 5m x 5m x -1.25m

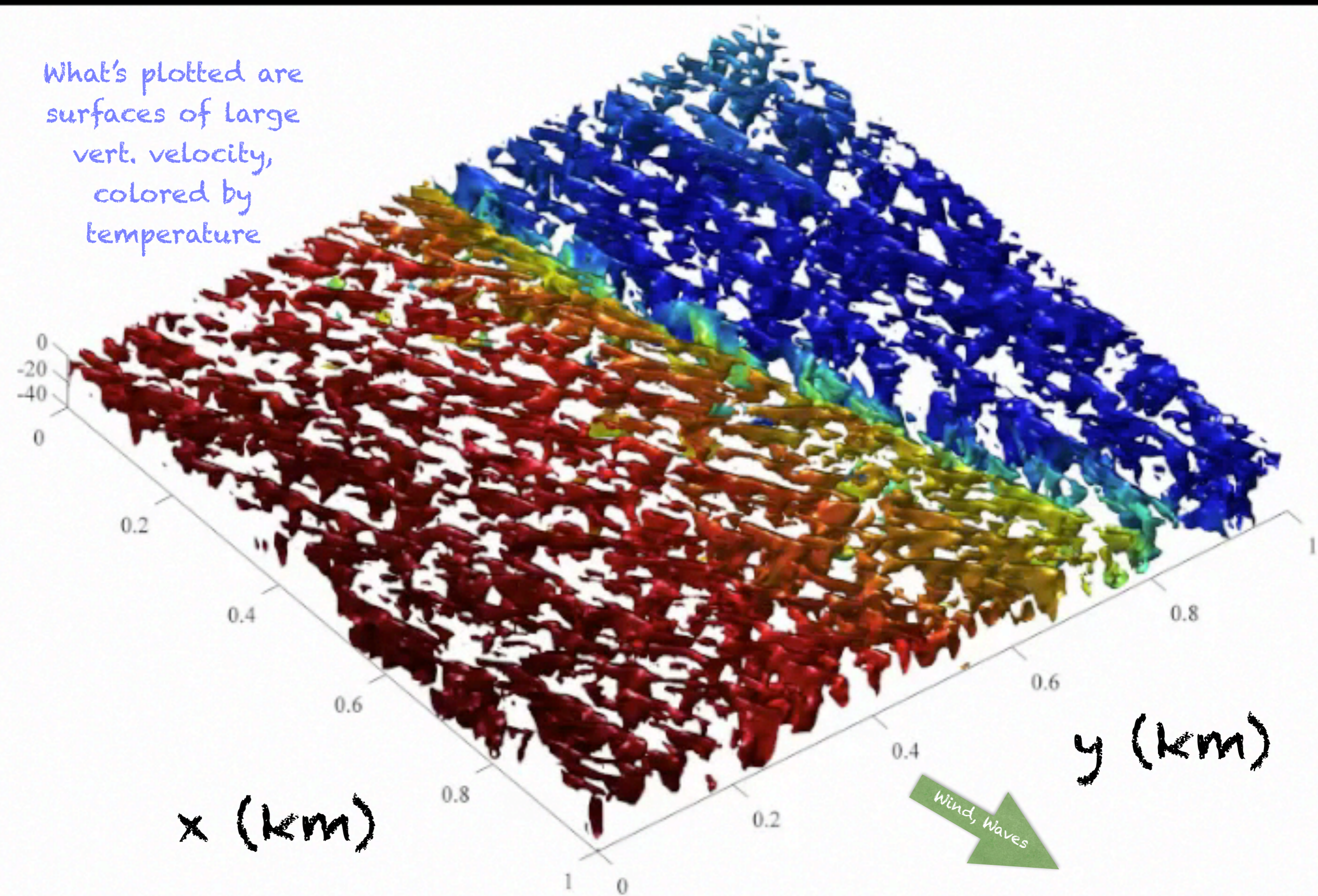


Diverse types of interaction: Stronger Langmuir (small) Turbulence

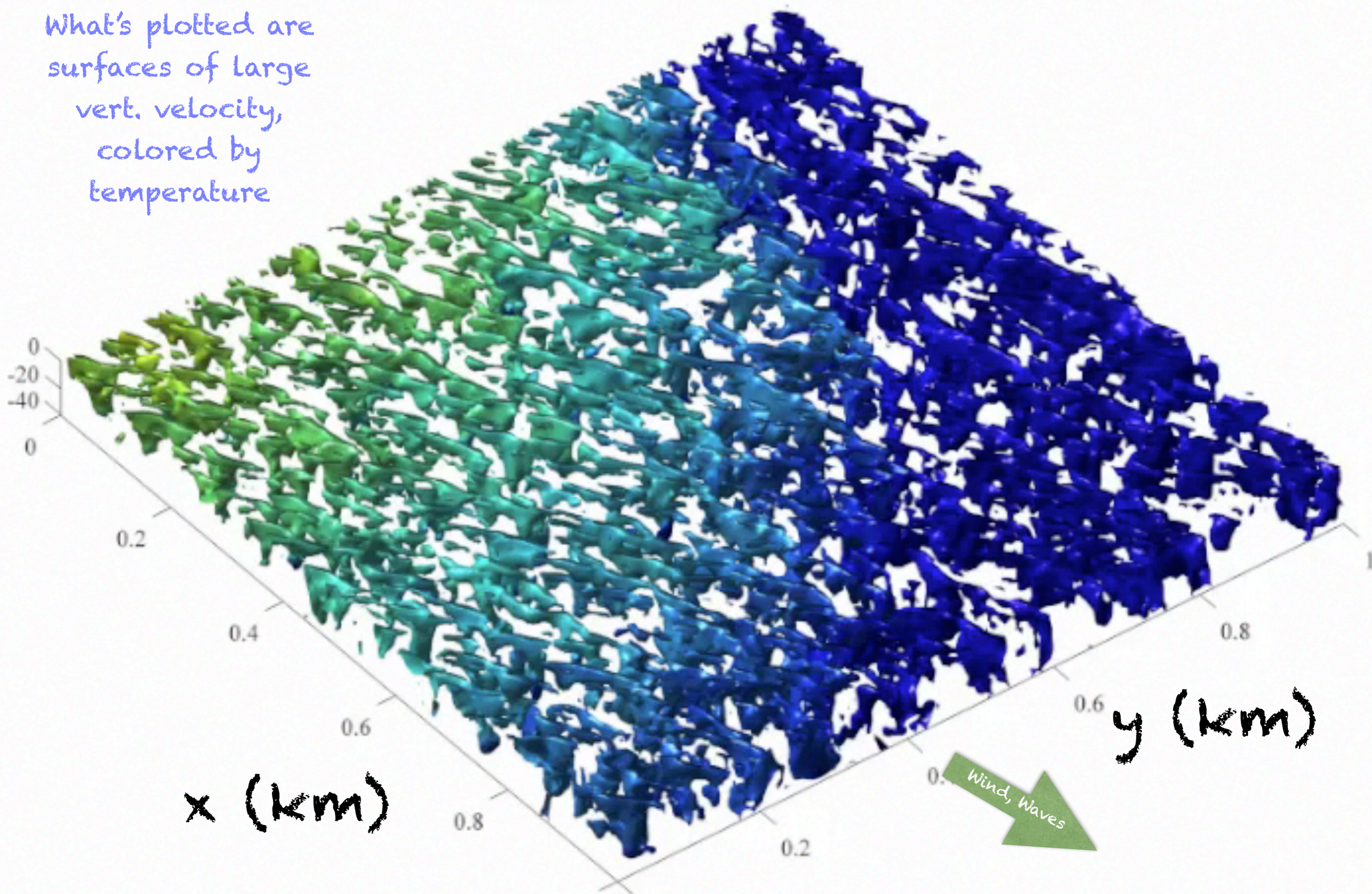


P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. *Journal of Physical Oceanography*, 44(9): 2249-2272, September 2014.

What's plotted are
surfaces of large
vert. velocity,
colored by
temperature



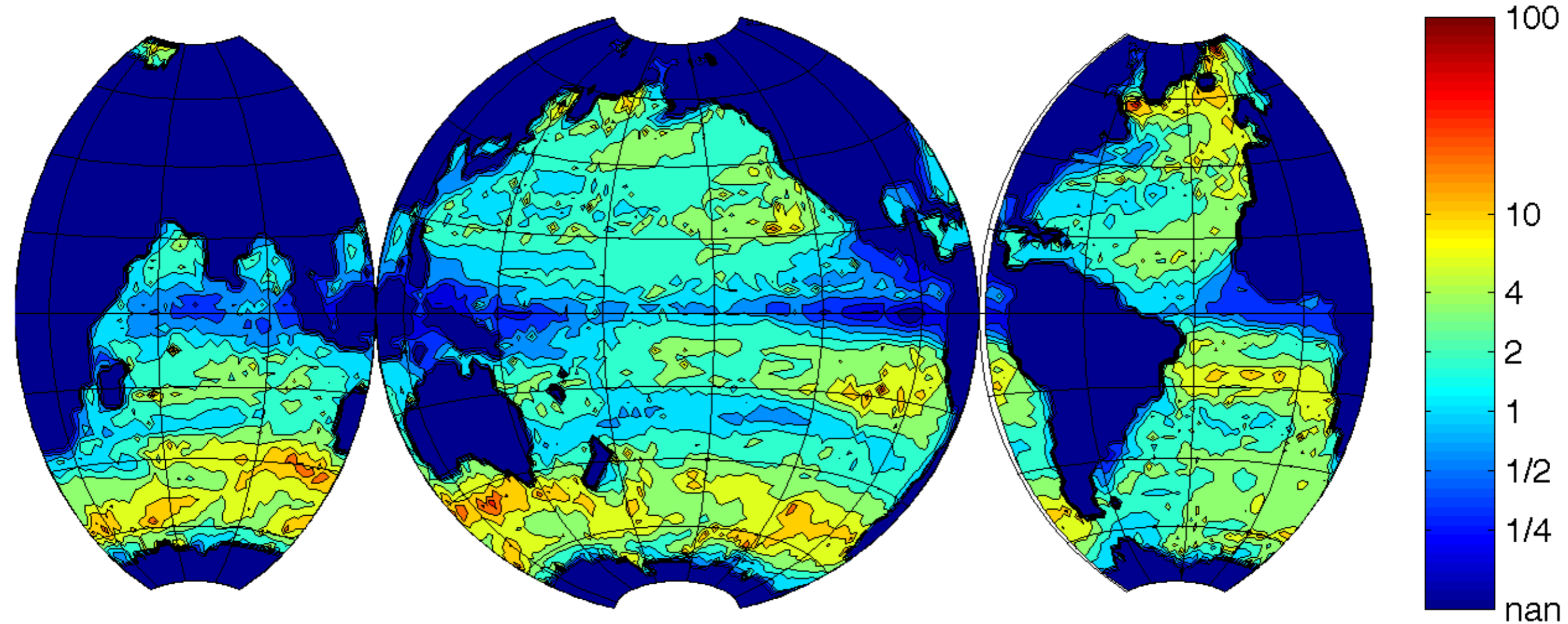
What's plotted are
surfaces of large
vert. velocity,
colored by
temperature



P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. JPO, 44(9):2249-2272, 2014.

Do Stokes force directly affect larger scales?

ϵ/Ro



“wavy hydrostatic” if

$$\epsilon \gg 1$$

$$\epsilon = \frac{V^s H}{f L H_s}$$

$$Ro = \frac{U}{f L}$$

~~Ri < 1 ⇒ SI~~

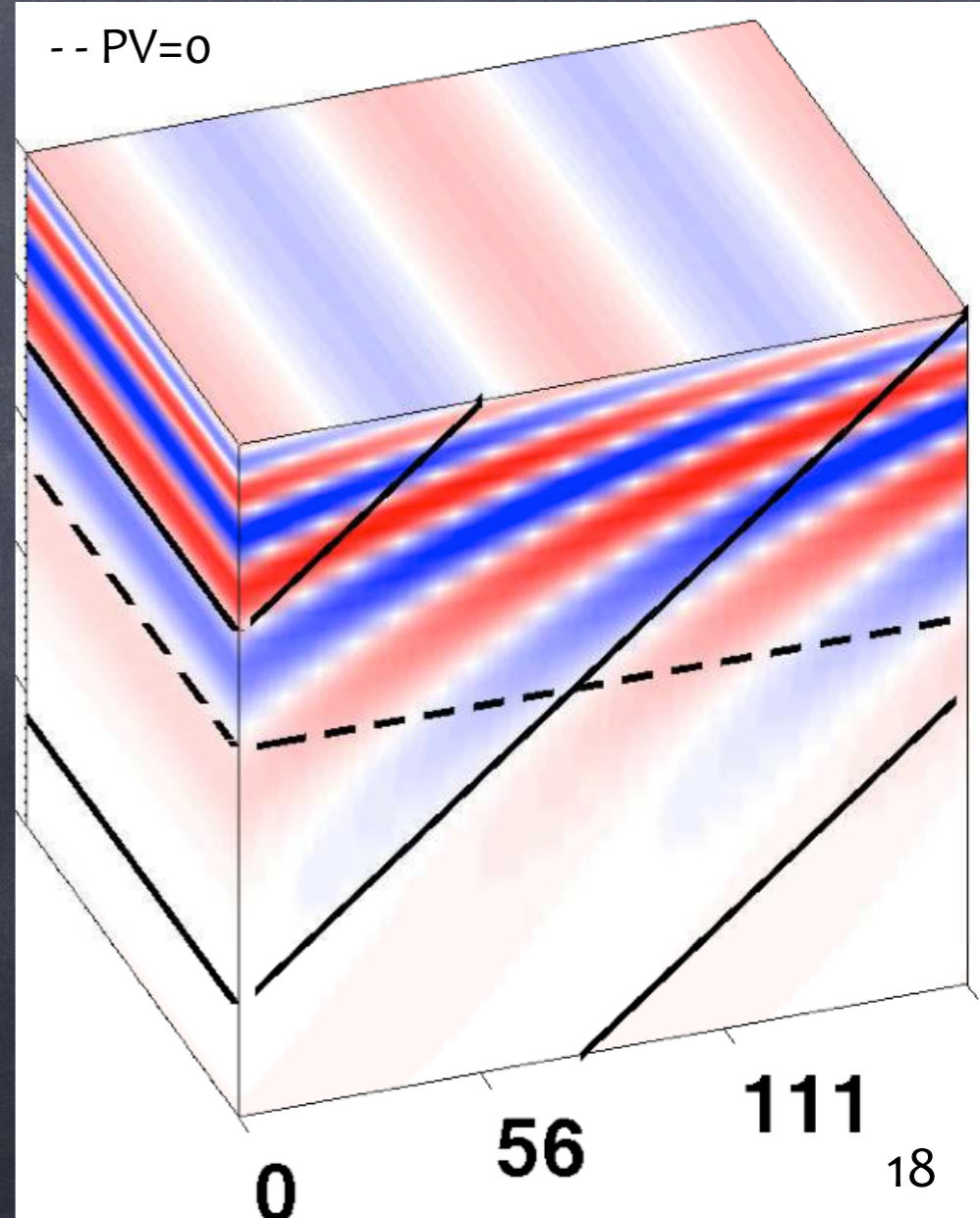
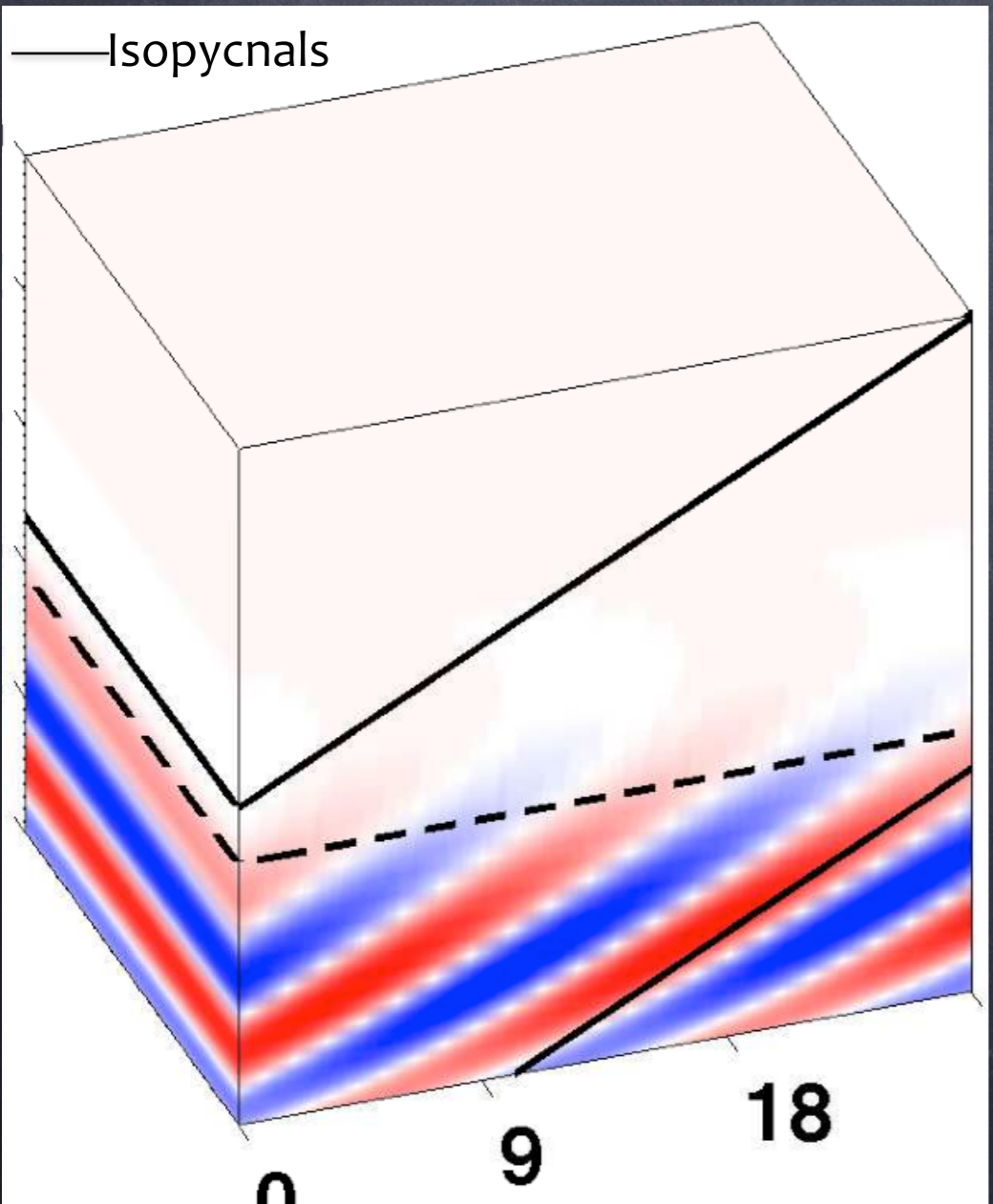
Wavy Submesoscale
Instability Different:
Symmetric Instability

★ $fQ < 0 \Rightarrow SI$

Ri = 0.5
Stokes Forces
Stabilize SI

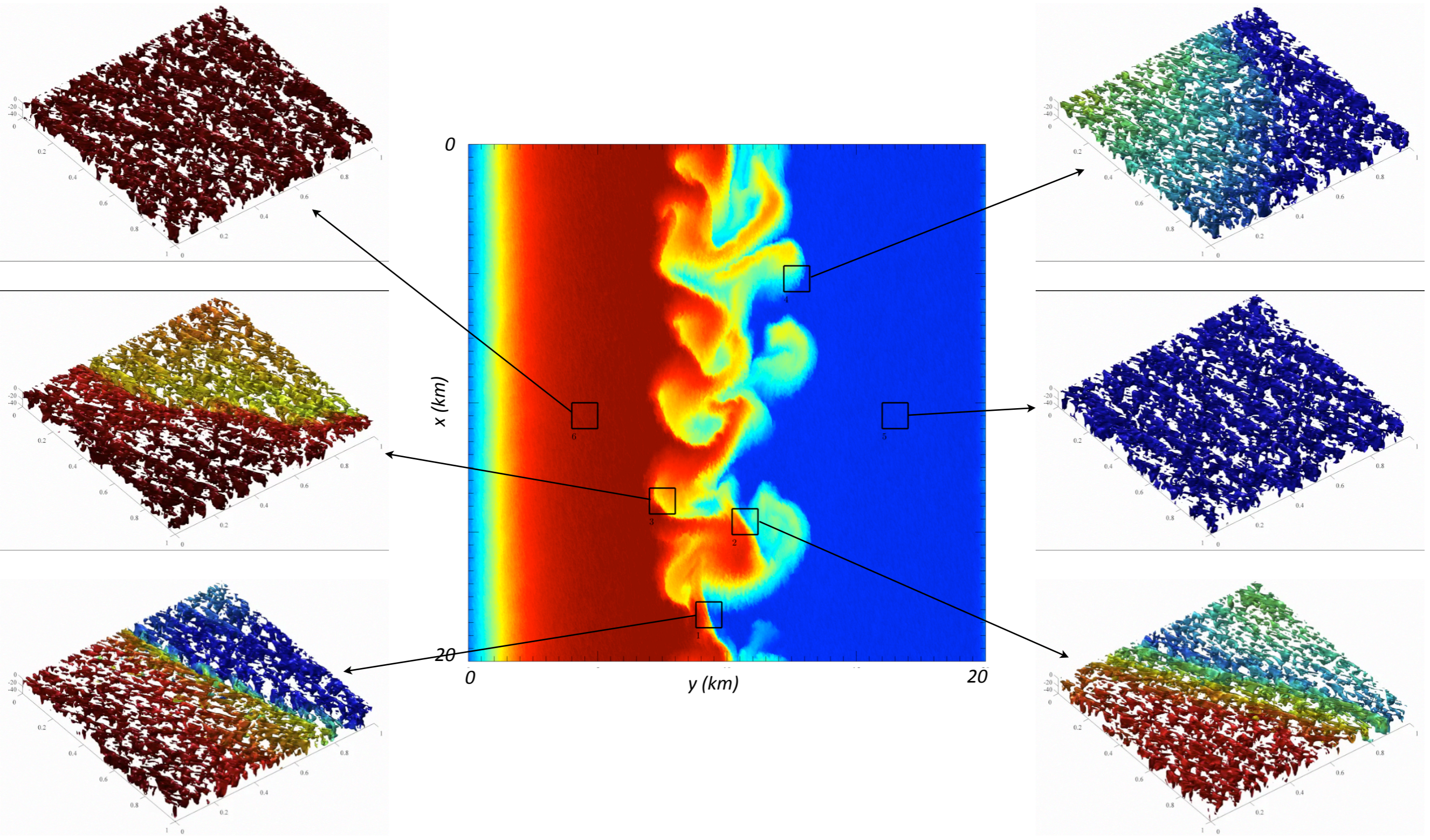
Cross front
velocity for
the fastest
growing
mode

Ri = 2
Stokes Forces
Destabilize SI



S. Haney, BFK,
K. Julien, and A.
Webb.
Symmetric and
geostrophic
instabilities in the
wave-forced
ocean mixed
layer. JPO
45:3033-3056,
2015.

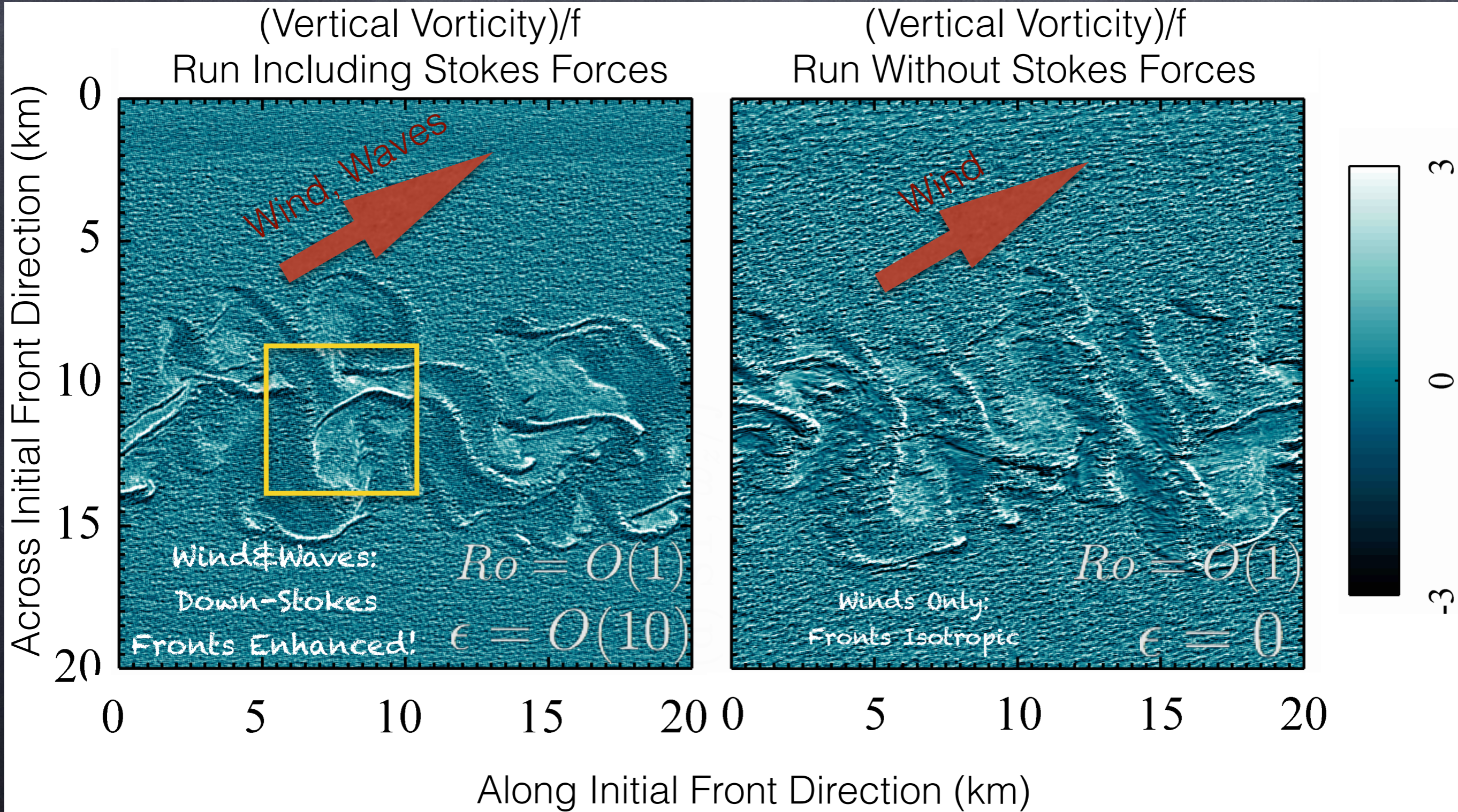
Diverse types of interaction



P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. JPO, 44(9):2249-2272, September 2014.

Are Fronts and Filaments different with Stokes shear force?

$$\frac{\alpha^2}{Ri} \left[w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = -\pi_{,z} + b - \epsilon v_j^L v_{j,z}^s + \frac{\alpha^2}{Re Ri} w_{,jj}$$



N. Suzuki, BFK, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. *Journal of Geophysical Research-Oceans*, 121:1-28, 2016.

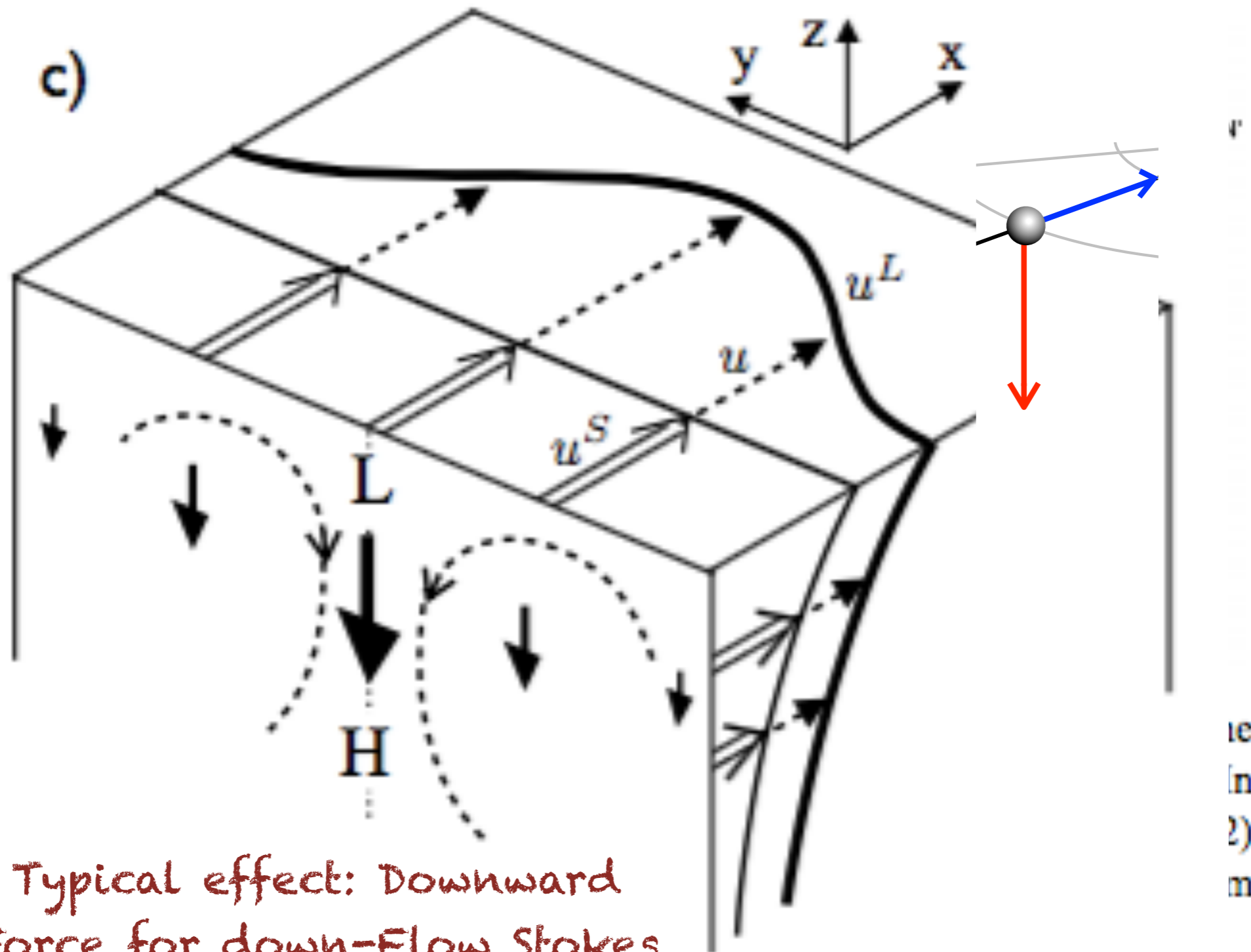
N. Suzuki and BFK. Understanding Stokes forces in the wave-averaged equations. *Journal of Geophysical Research-Oceans*, 121:1-18, 2016.

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. *Journal of Fluid Mechanics*, 730:464-490, 2013.

image:
Thorpe, 0



Figure 1
windrows
practice th
amalgama
within the

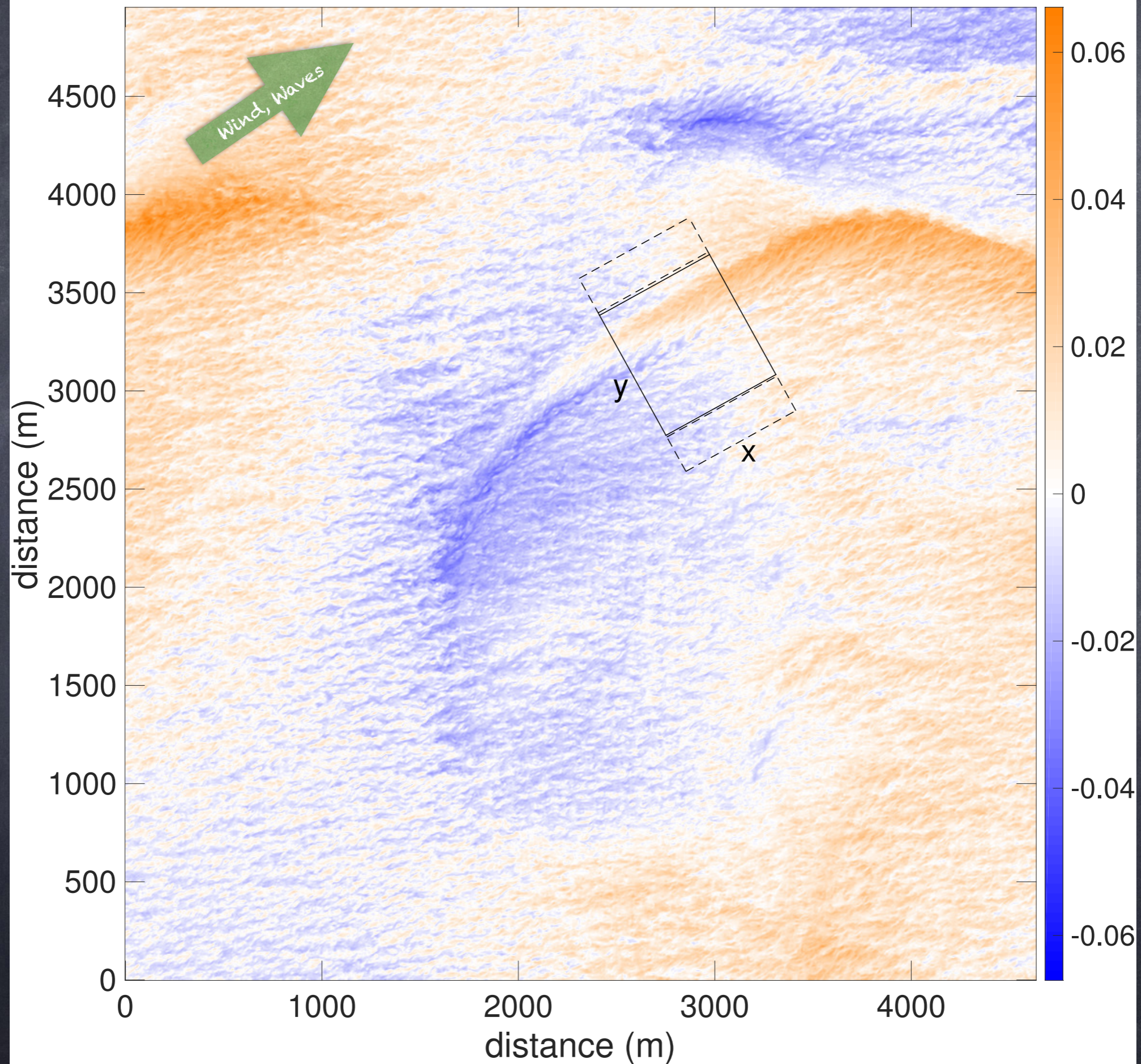


"wavy hydrostatic" if $\epsilon \gg 1$

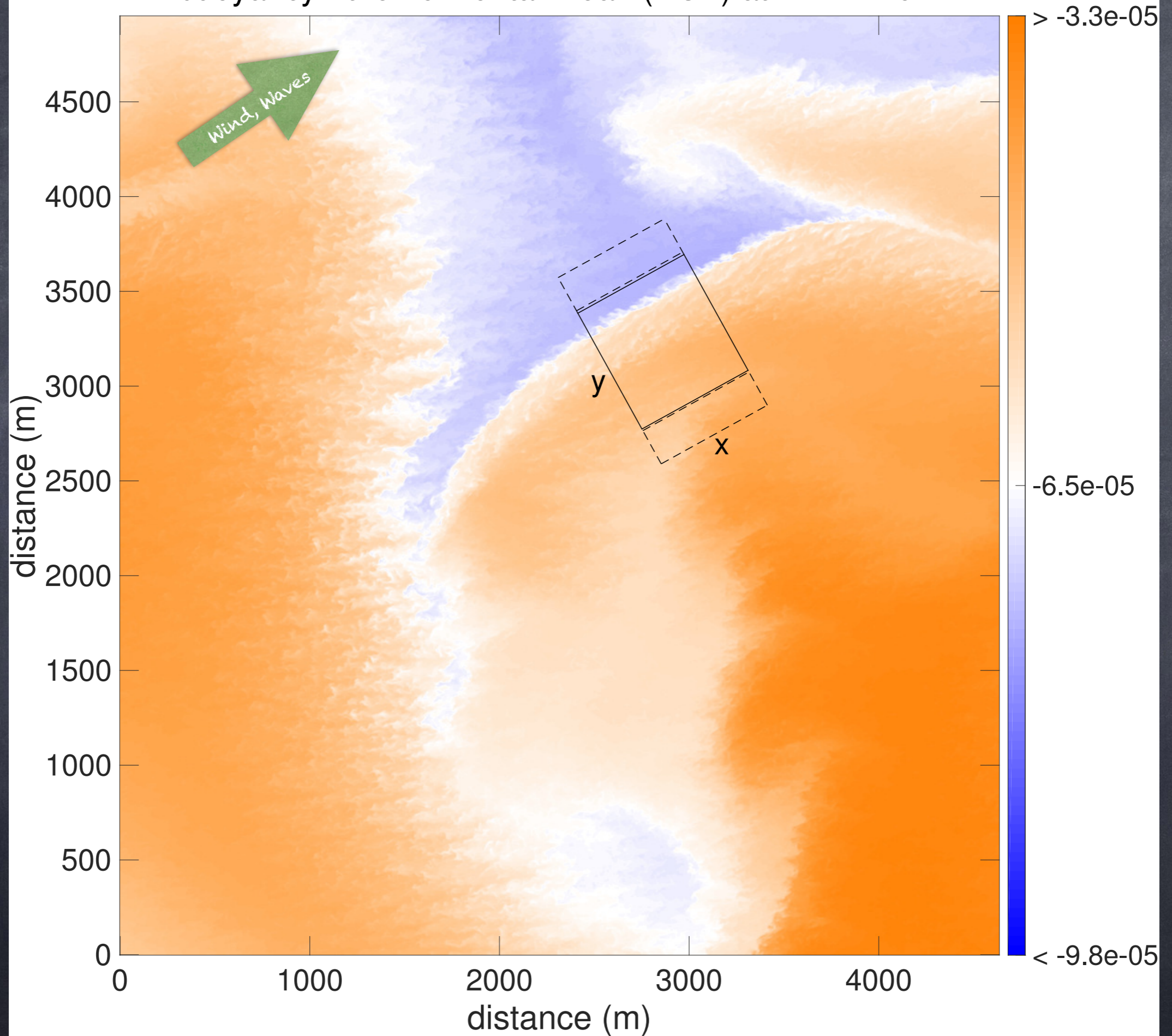
$$\frac{\alpha^2}{Re} \left[w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{RoRe} w w_{,z} \right] = \boxed{-\pi_{,z} + b - \epsilon v_j^L v_{j,z}^s} + \frac{\alpha^2}{ReRe} w_{,jj}$$

N. Suzuki and BFK. Understanding Stokes forces in the wave-averaged equations. Journal of Geophysical Research-Oceans, 121:1-18, 2016.

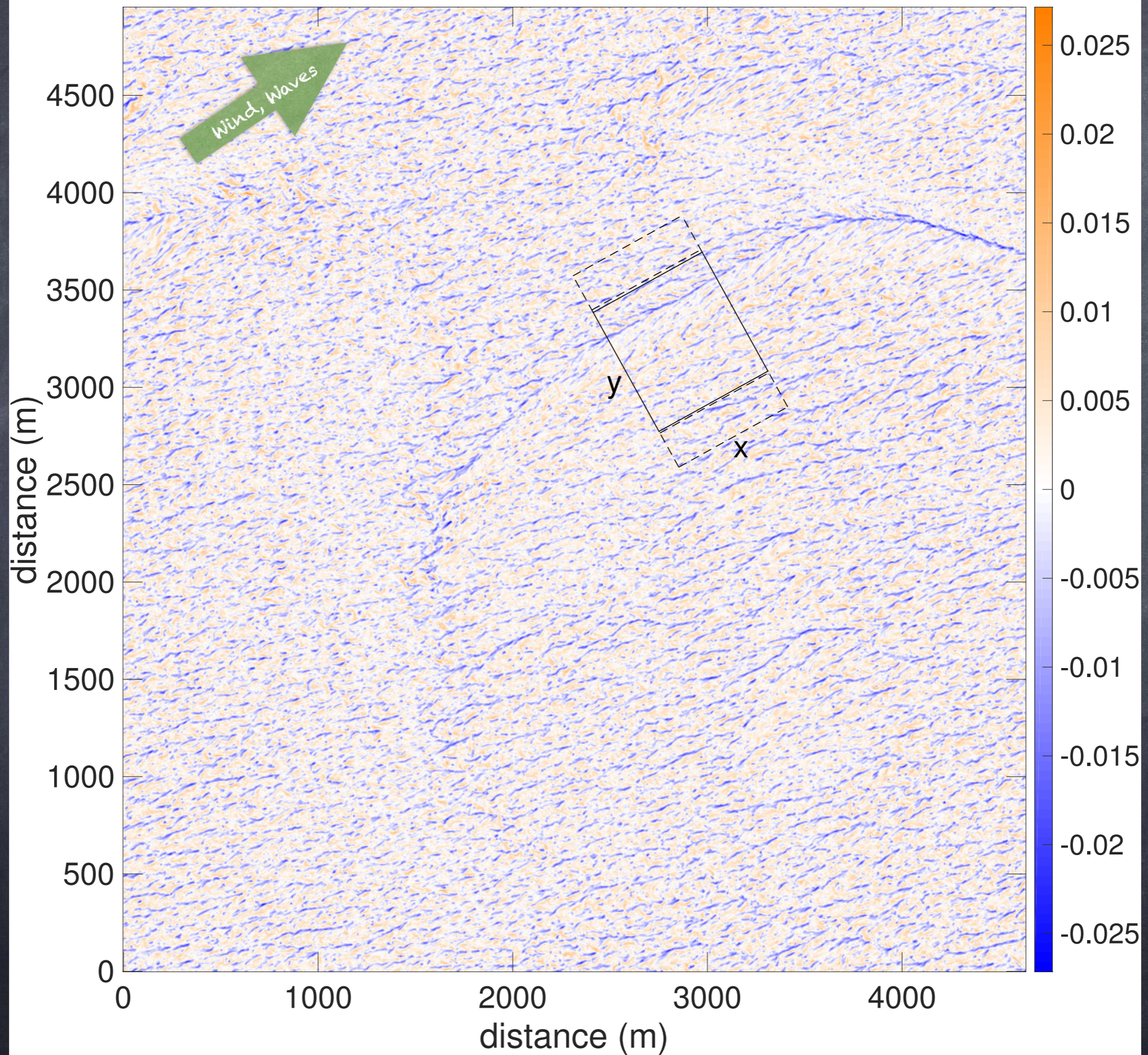
velocity in the x-direction - the horizontal mean (ms^{-1}) at $z = -11.25\text{m}$



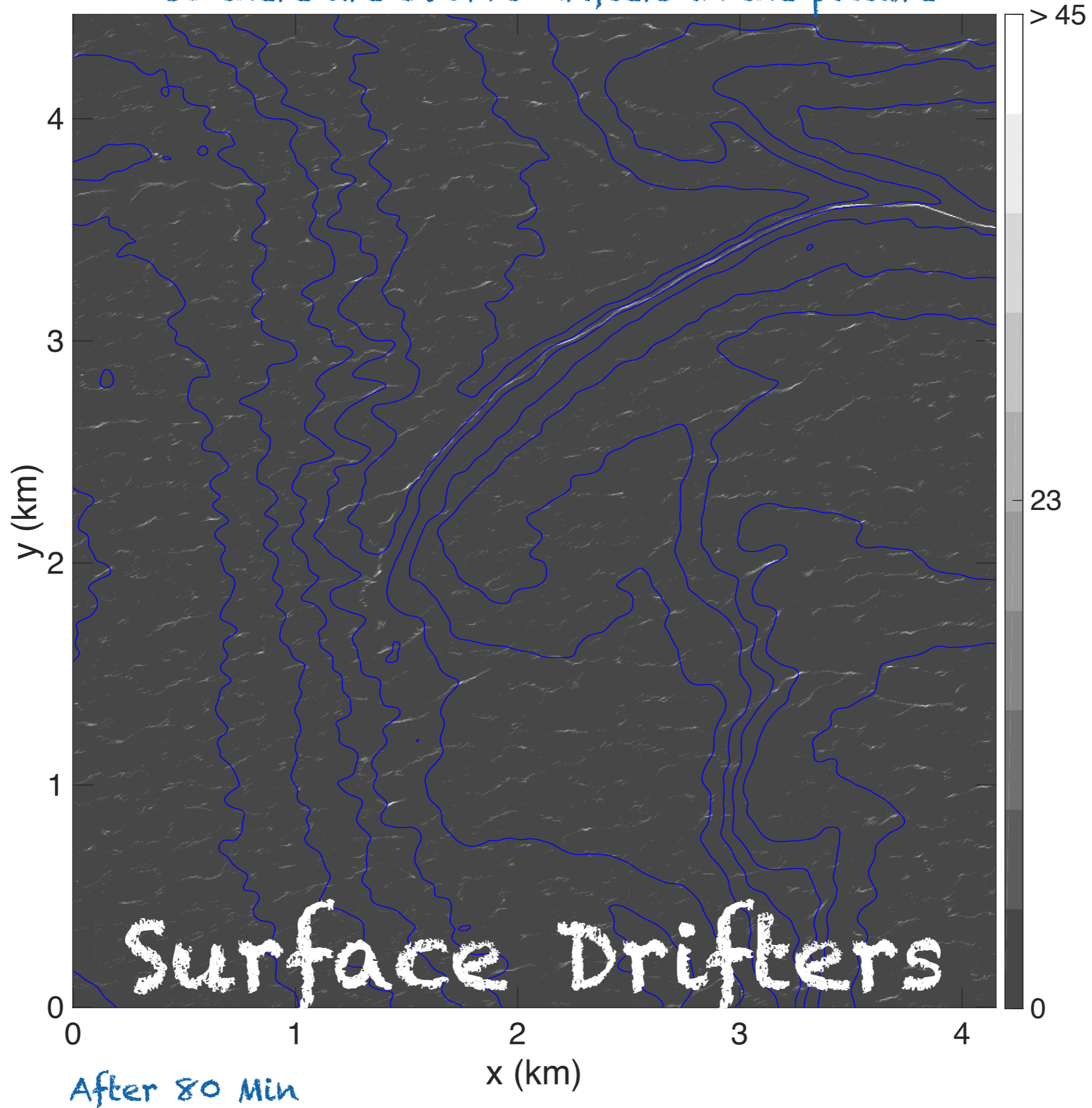
buoyancy - the horizontal mean (ms^{-2}) at $z = -11.25\text{m}$



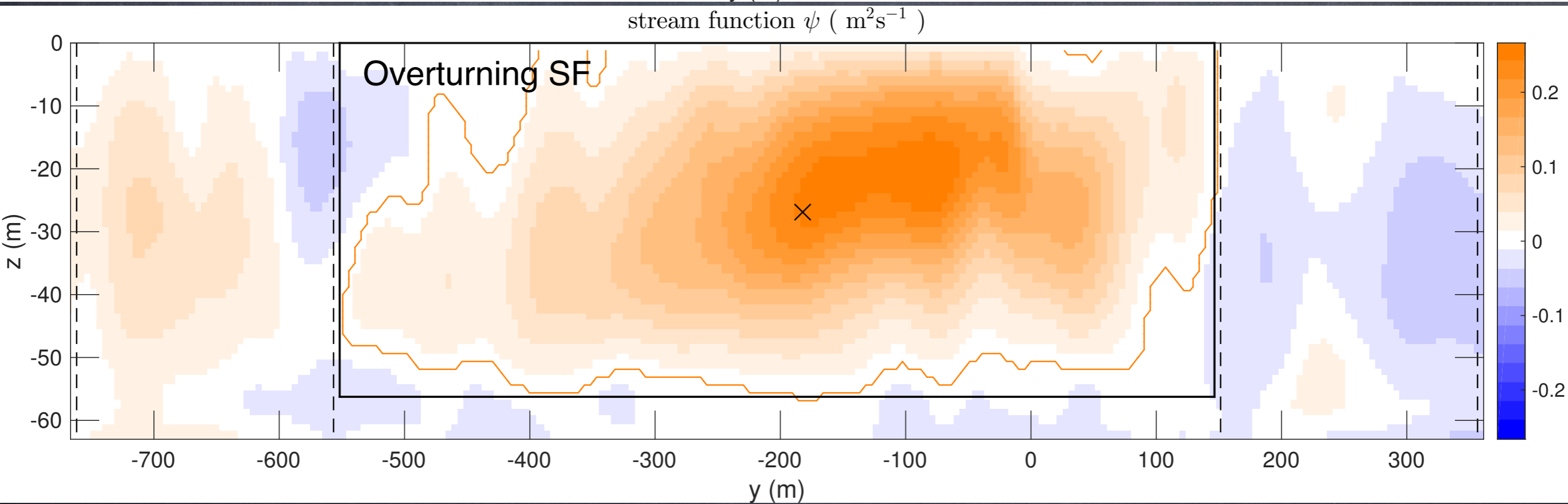
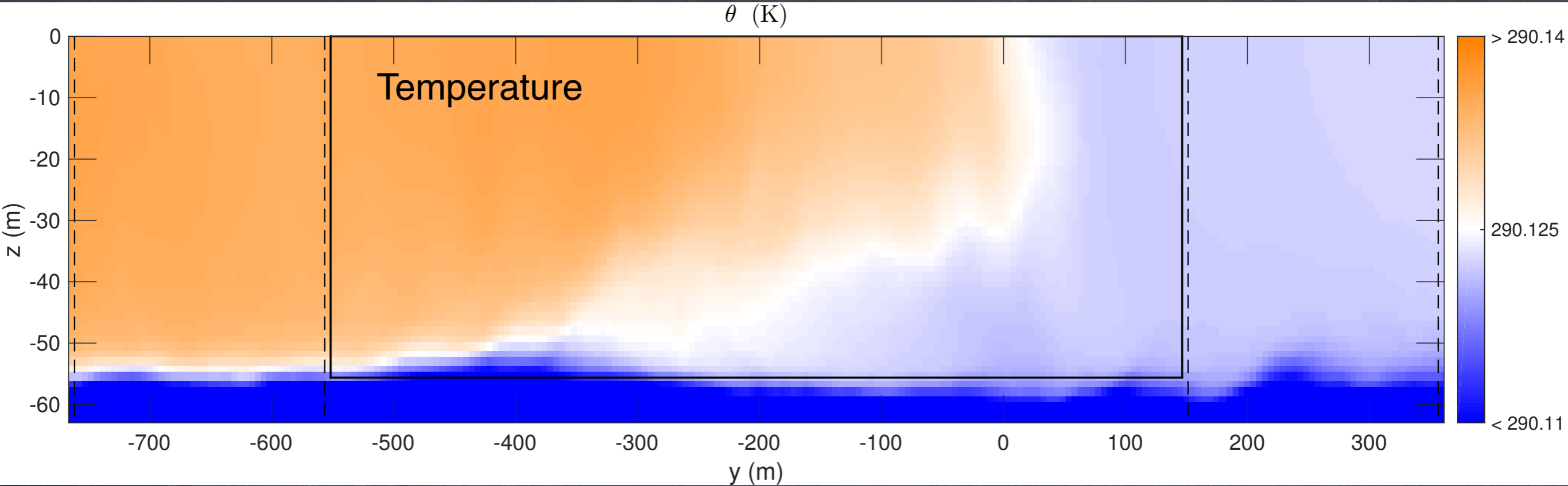
vertical velocity (ms^{-1}) at $z = -11.25\text{m}$



Initially every surface node has 1 drifter,
so there are 851796 drifters in the picture



After 80 Min



N. Suzuki, BFK, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. *Journal of Geophysical Research-Oceans*, 121:1-28, 2016.

Do (wavy hydrostatic) Stokes Forces Matter?

Yes! At Leading Order (in LES)

Table 3. Integrated Budget for Overturning Vorticity^a

Responsible Force	Relative Value
<i>Relative Tendency of Overturning Circulation along the Cell Boundary</i>	
Net tendency	11 ± 8%
Sources	
Buoyancy anomaly	100%
Stokes shear force anomaly	44 ± 4%
Interaction with v^H	44 ± 8%
Frontal anomaly in pressure gradient	6 ± 9%
Nonlinear interaction with v^B :	2 ± 1%
Sinks	
Frontal turbulence anomaly (mostly, imbalance in wavy Ekman relation)	-82 ± 11%
Coriolis on along-front jet	-66 ± 2%
Lagrangian advection of (v^ψ, w^ψ)	-36 ± 7%



N. Suzuki and BFK. Understanding Stokes forces in the wave-averaged equations. *Journal of Geophysical Research-Oceans*, 121:1-18, April 2016.

N. Suzuki, BFK, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. *Journal of Geophysical Research-Oceans*, 121:1-28, May 2016.

Conclusions

- Langmuir mixing scalings consistent with LES & observations, reduce climate model biases in MLD, T, CFCs vs. observations by 5-25%.
- Stokes forces, as treated here, can be included in hydrostatic models like GCMs (wavy hydrostatic)
- Stokes forces affect Langmuir turbulence, but also (sub)mesoscale fronts (more energy, anisotropy) and submesoscale instabilities.
Need to assess climate & environmental impact!
- All papers at: fox-kemper.com/pubs

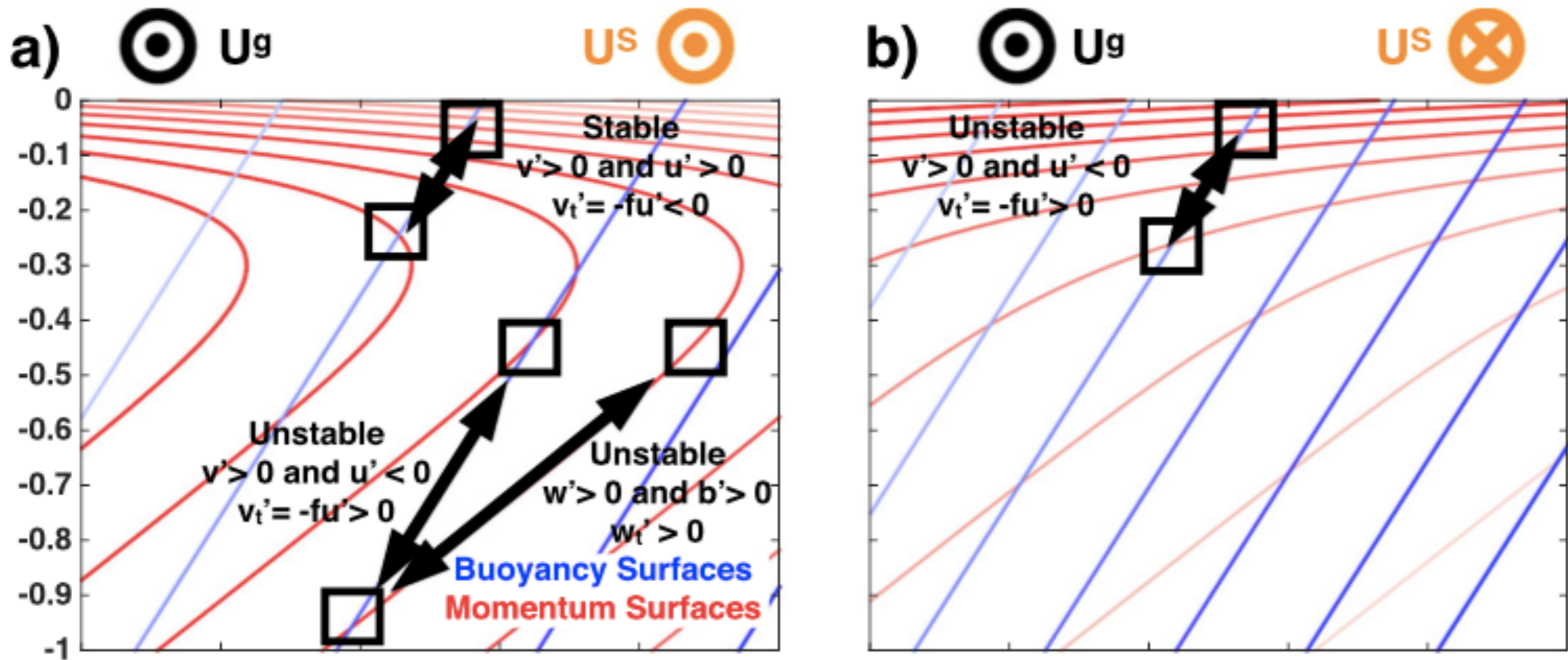



FIG. 1. A schematic of the (a) downfront and (b) upfront Stokes drift scenarios. The blue lines show isopycnals, with darker blue indicating denser water. The red lines show surfaces of constant downfront absolute Eulerian momentum, with darker red indicating greater momentum. The perturbation equations are written from the perspective of the lower of the two parcels. A change of all signs would be from the perspective of the upper parcel and have the same stability. For example, in (b) the lower parcel moves to the right ($v' > 0$) along an isopycnal and brings with it lower downfront momentum than its surroundings ($u' < 0$). This exerts an acceleration in the cross-front v' direction due to the Coriolis force that further enhances the initial perturbation ($v' > 0$). In both cases, $Ri = 0.5$. Lines of constant buoyancy and absolute momentum are only parallel when $Ri = 1$.

Analytic & Numerical Wavy Submesoscale Stability: Geostrophic Instabilities

Charney, Stern, Pedlosky criteria (appropriately generalized) apply:

• Instability allowed if:

- 1) Q_Y^L changes sign in the interior of the domain;
- 2) Q_Y^L is the opposite sign as U_z^L at $z = 0$;
- 3) Q_Y^L is the same sign as U_z^L at $z = -H$;
- 4) U_z^L has the same sign at $z = -H$ and $z = 0$.

$$Q^L = \nabla_H^2 \Psi + \beta Y + \partial_z \left(\frac{f_0^2}{N^2} \frac{\Psi_z^L}{B_z} \right)$$


$$U + U^S = -\Psi_y^L, \quad \text{such that} \quad U = -\Psi_y, \quad \text{and}$$
$$V + V^S = \Psi_x^L = 0.$$

Streamfunctions
with and w/o Stokes

Analytic & Numerical Wavy Submesoscale Stability: Symmetric Instabilities

- Hoskins (1974) showed that if a front in thermal wind balance is symmetrically unstable, the PV must be anticyclonic.
- Haney et al extend Hoskins' analysis to flows in Lagrangian thermal wind balance in the special case that the Stokes shear is constant.

Vert.
Density
Gradient

Horiz.
Density
Gradient

Anti-Stokes
Shear

$$fQ = \underbrace{f^2 N^2 - M^4}_{\text{geostrophic } fQ} - \underbrace{fM^2 U_z^S}_{\text{Stokes-modified } fQ} < 0.$$

- In the absence of Stokes drift, this is equivalent to the familiar criteria on Richardson Number, with Stokes drift is distinct.

Do Stokes forces affect Larger Scales?

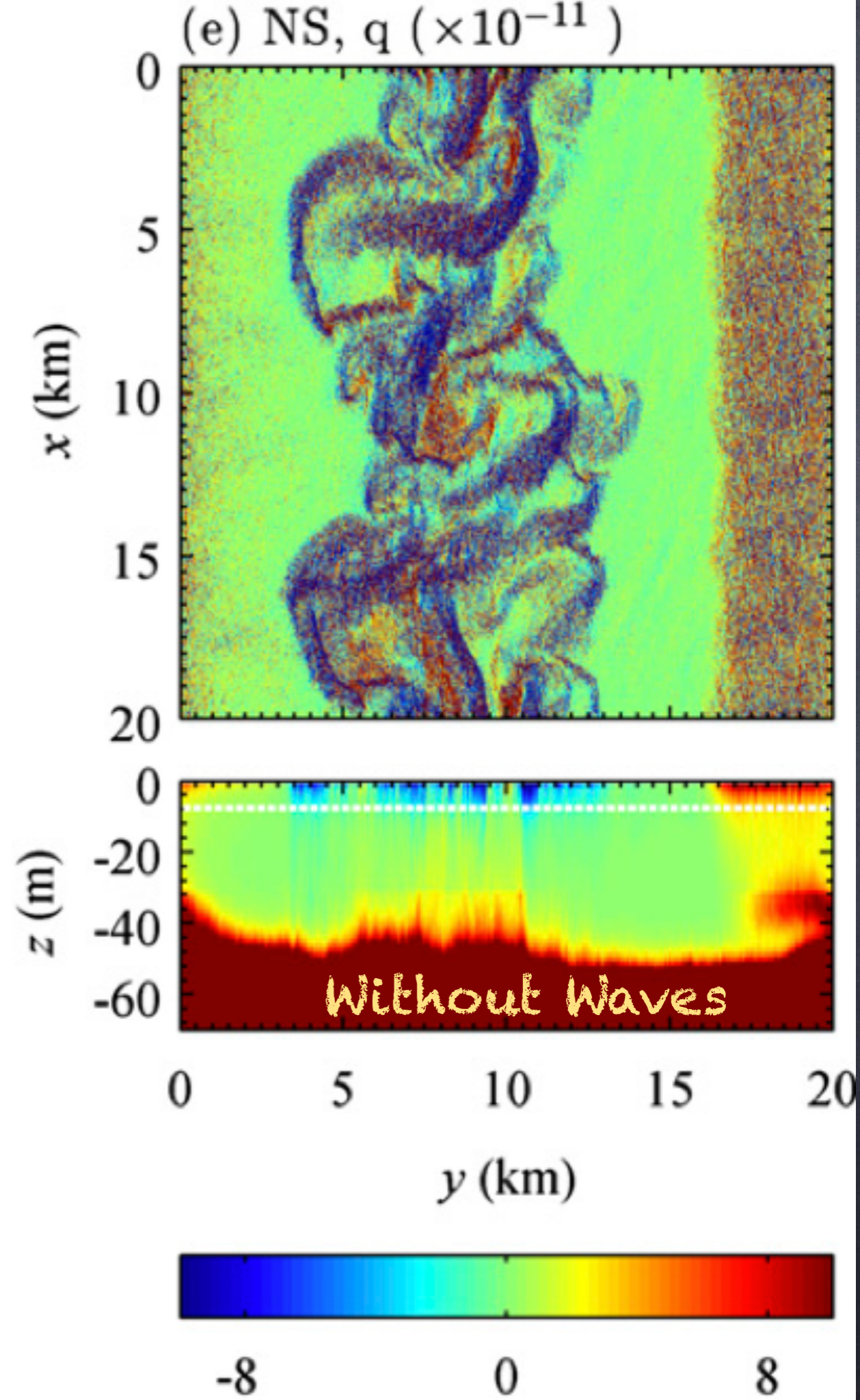
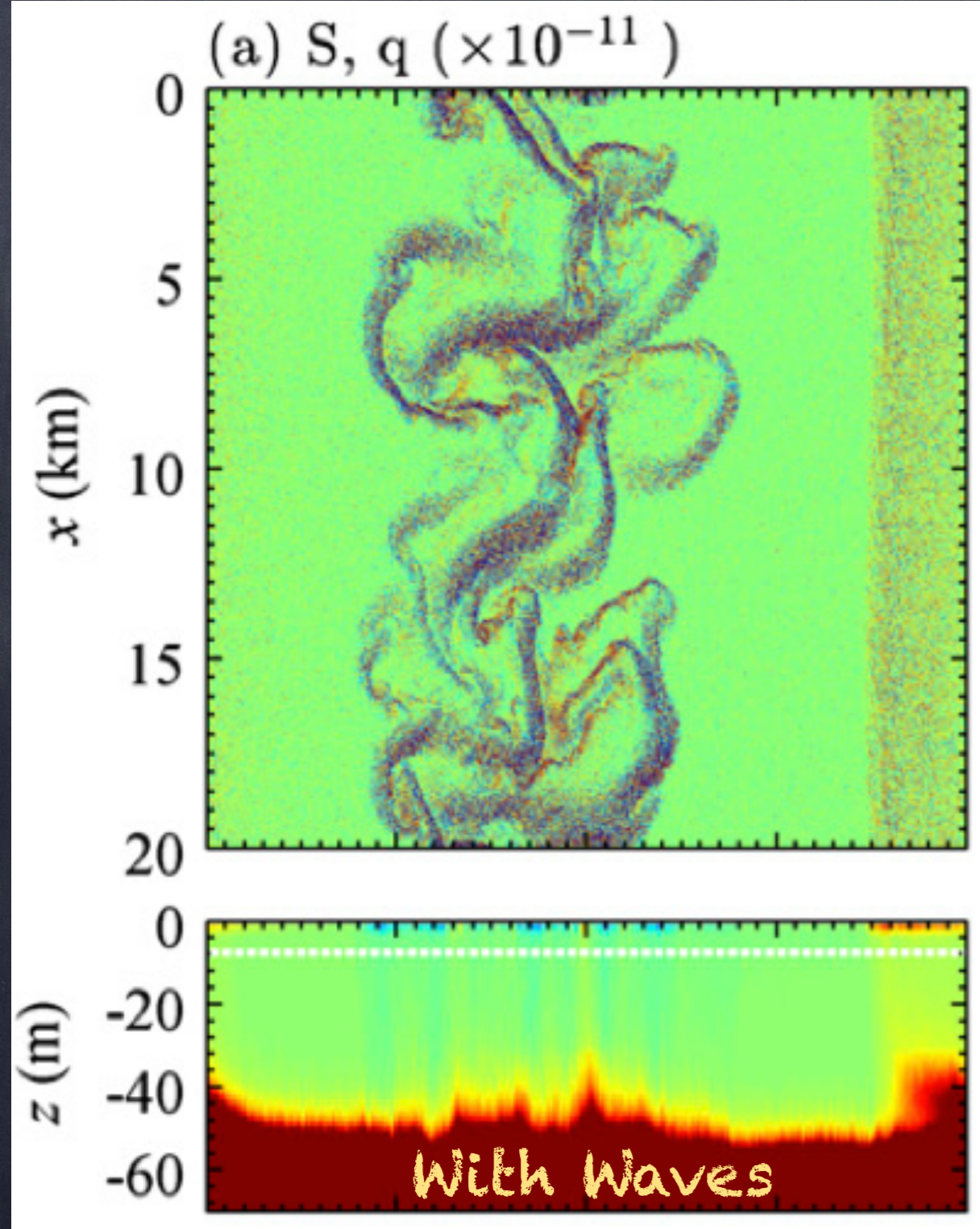
$$\mathbf{f} \times \frac{\partial \mathbf{v}}{\partial z} = -\nabla b$$

Becomes Lagrangian Thermal Wind Balance

$$\mathbf{f} \times \frac{\partial}{\partial z} (\mathbf{v} + \mathbf{v}_s) = \mathbf{f} \times \frac{\partial \mathbf{v}_L}{\partial z} = -\nabla b$$

Now the temperature gradients govern the
Lagrangian flow, not the Eulerian!

The Eulerian response to Stokes is often to cancel it out!
(Anti-Stokes flow, Lab: Monismith et al., Obs: Lentz et al.)



So, if $f_Q < 0$ indicates likely regions of symmetric instability—Surface Waves STRONGLY affect SI!

Lagrangian Thermal Wind Linear Stability

Like Eady, but
with Lagrangian
Thermal Wind
Background
State

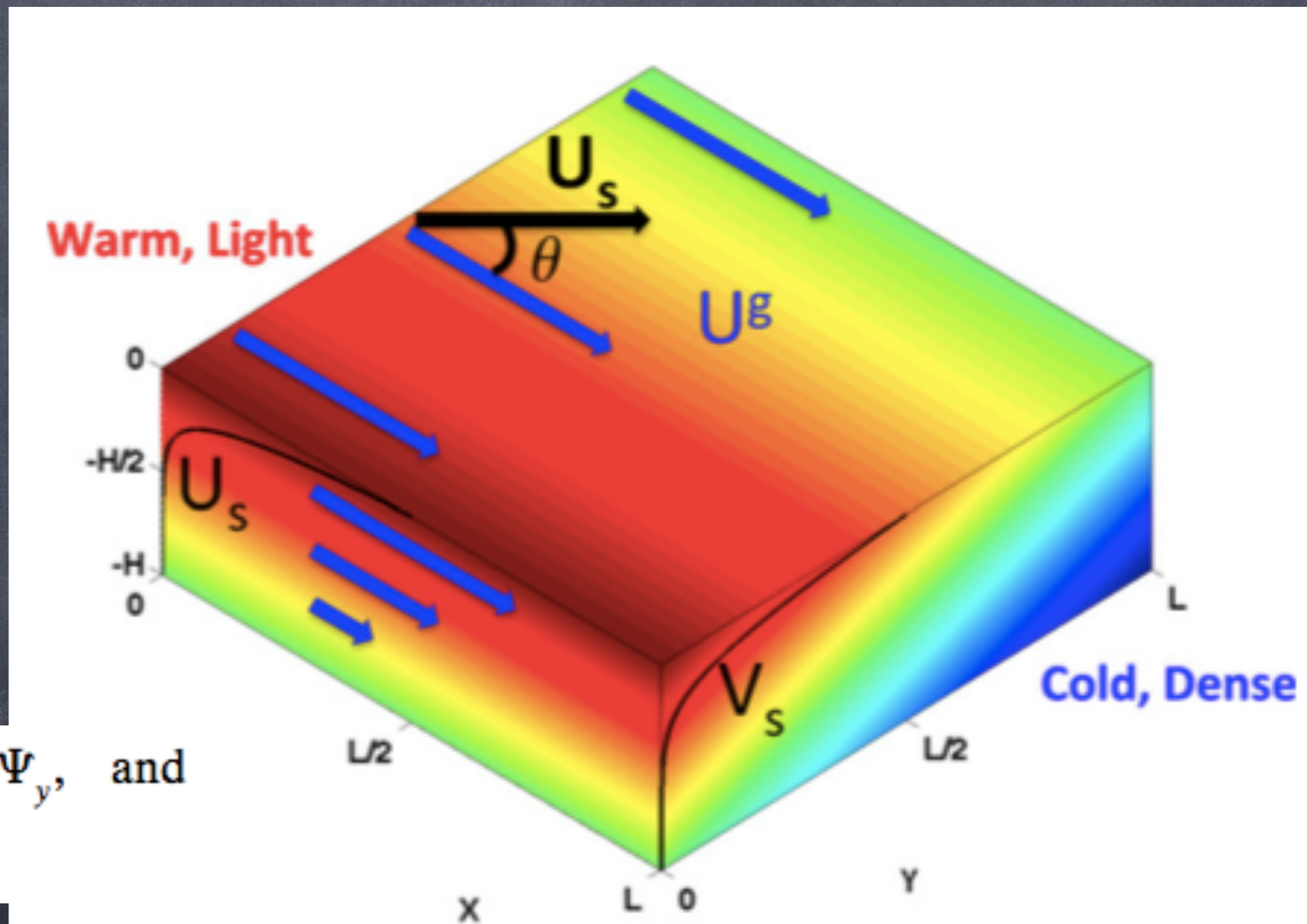


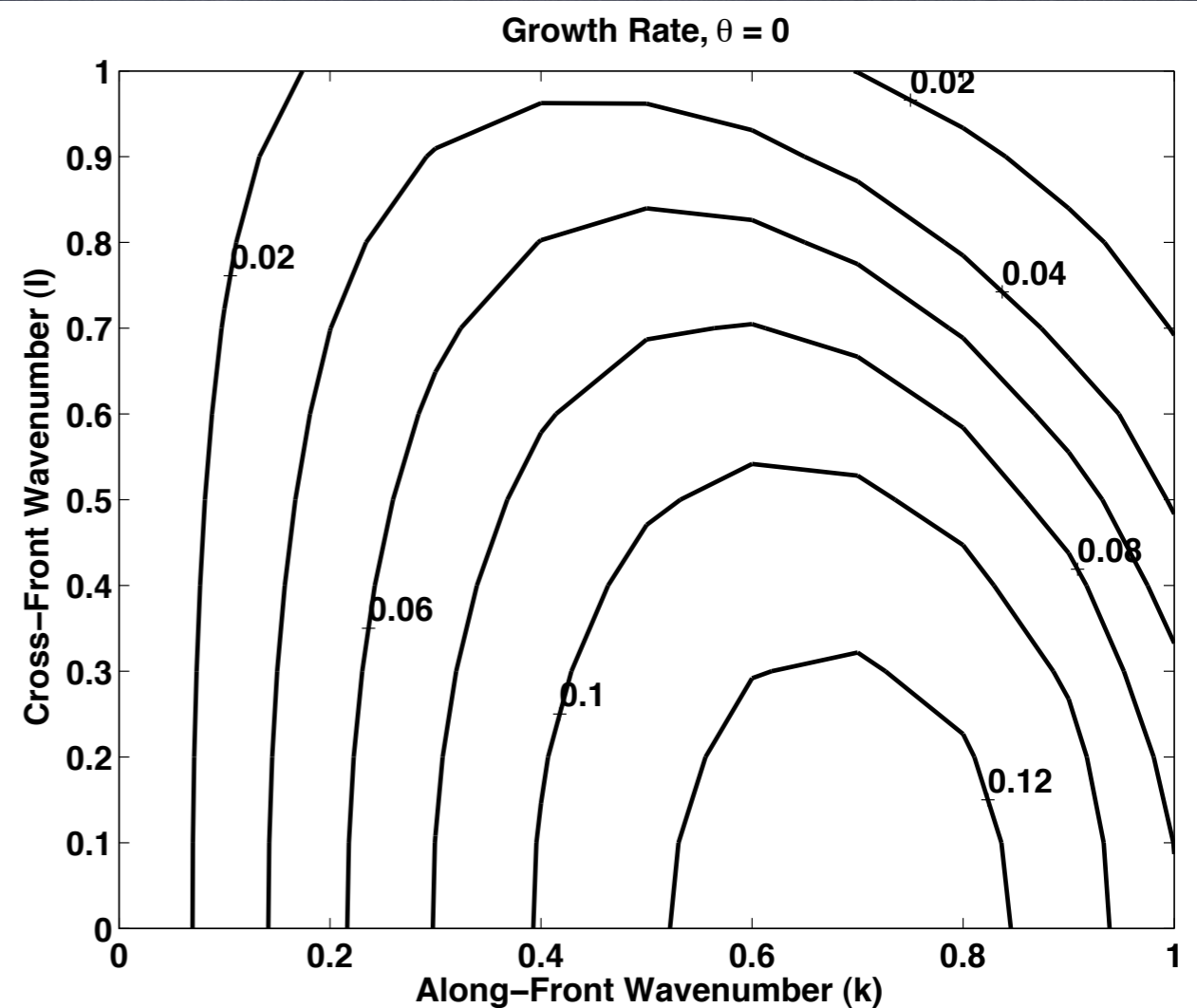
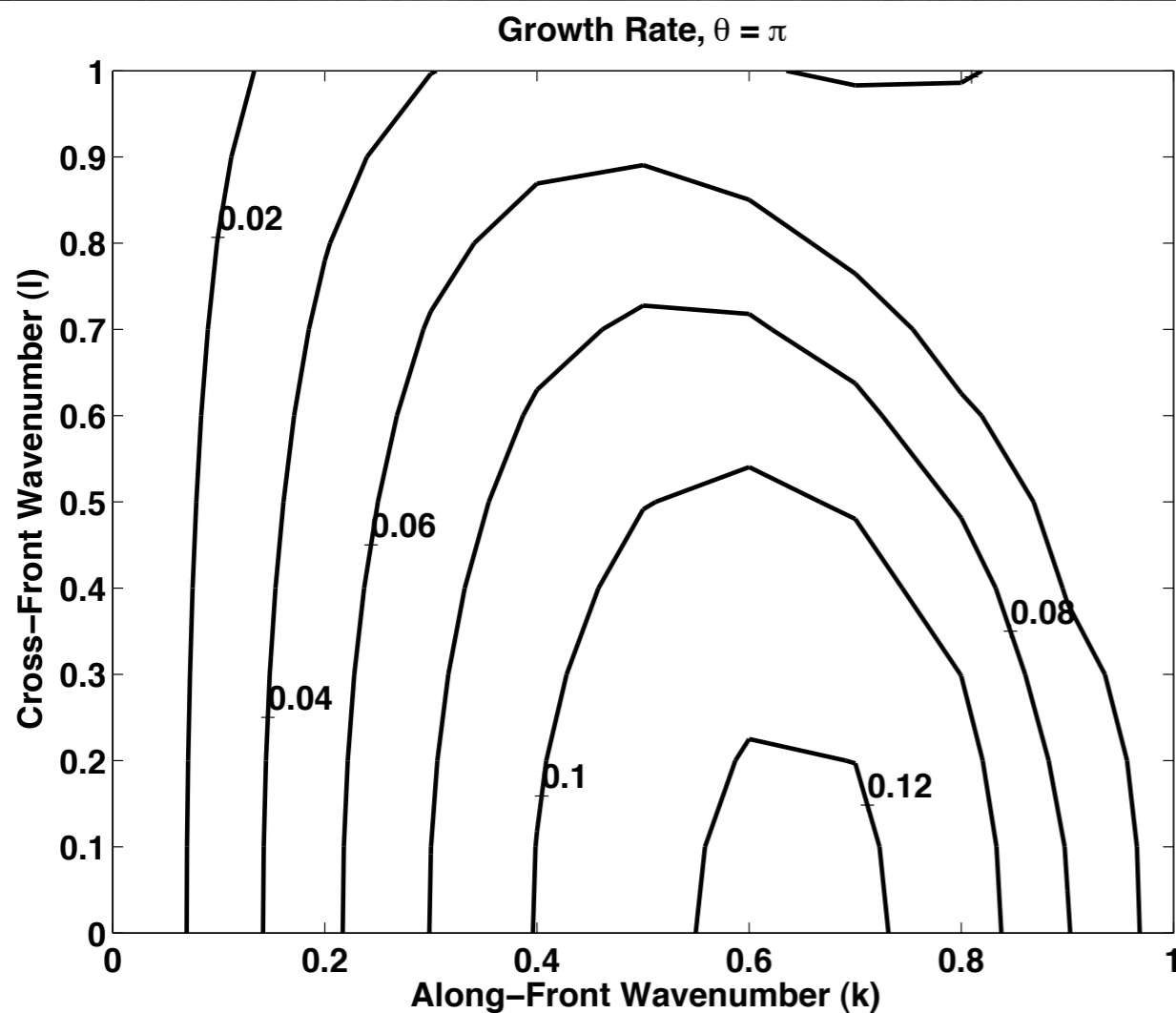
FIG. 2. The background flow with arbitrary θ (the angle between the Stokes drift and the geostrophic flow) and a prescribed exponential Stokes drift U^S , V^S profile. The geostrophic flow U^G , corresponding to the imposed buoyancy gradient, is shown with blue arrows.

$$U + U^S = -\Psi_y^L, \text{ such that } U = -\Psi_y, \text{ and}$$

$$V + V^S = \Psi_x^L = 0.$$

Analytic & Numerical Wavy Submesoscale Stability: Geostrophic Instabilities

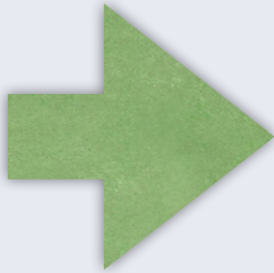
- For typical conditions, the Stokes effect amounts to a small change in geostrophic instability (mixed layer eddy) growth rates.



Do (wavy hydrostatic) Stokes Forces Matter? Yes! At Leading Order (in LES)

Table 2. Integrated Budget for the Kinetic Energy of the Overturning Circulation^a

Name	Term	Relative Value
<i>Rate of Change of Overturning Circulation KE</i>		
Total	$(\partial_t + u_j^L \partial_j) \frac{v^\psi v^\psi + w^\psi w^\psi}{2}$	45 ± 6%
Sources		
Buoyancy production	$w^\psi b'$	100%
Energy increase due to interaction with v^H	$v^\psi (-F^h)$	49 ± 5%
Stokes shear force work	$w^\psi (-u_j^L \partial_z u_j^S)$	24 ± 1%
Energy increase due to nonlinear interaction with v^B	$v^\psi (-F^v)$	7 ± 1%
Sinks		
Generation of along-front jet by Coriolis turning of v^ψ	$-f v^\psi u^H$	-69 ± 3%
Work done against Coriolis of background flows	$v^\psi (\overline{\partial_y p'^B} + \overline{\partial_j L_{2j}^B})$	-45 ± 3%
Generation of shear turbulence	$L_{kj} \partial_j u_k^\psi$	-16 ± 1%
Turbulent transport through the cell boundary	$-\partial_j (u_k^\psi L_{kj})$	-2 ± 0.4%



$$\frac{\alpha^2}{Re Ri} \left[w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{Ro Ri} w w_{,z} \right] = -\pi_{,z} + b - \varepsilon v_j^L v_{j,z}^S + \frac{\alpha^2}{Re Ri} w_{,jj}$$

N. Suzuki and B. Fox-Kemper. Understanding Stokes forces in the wave-averaged equations. *Journal of Geophysical Research-Oceans*, 121:1-18, April 2016.

N. Suzuki, B. Fox-Kemper, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. *Journal of Geophysical Research-Oceans*, 121:1-28, May 2016.