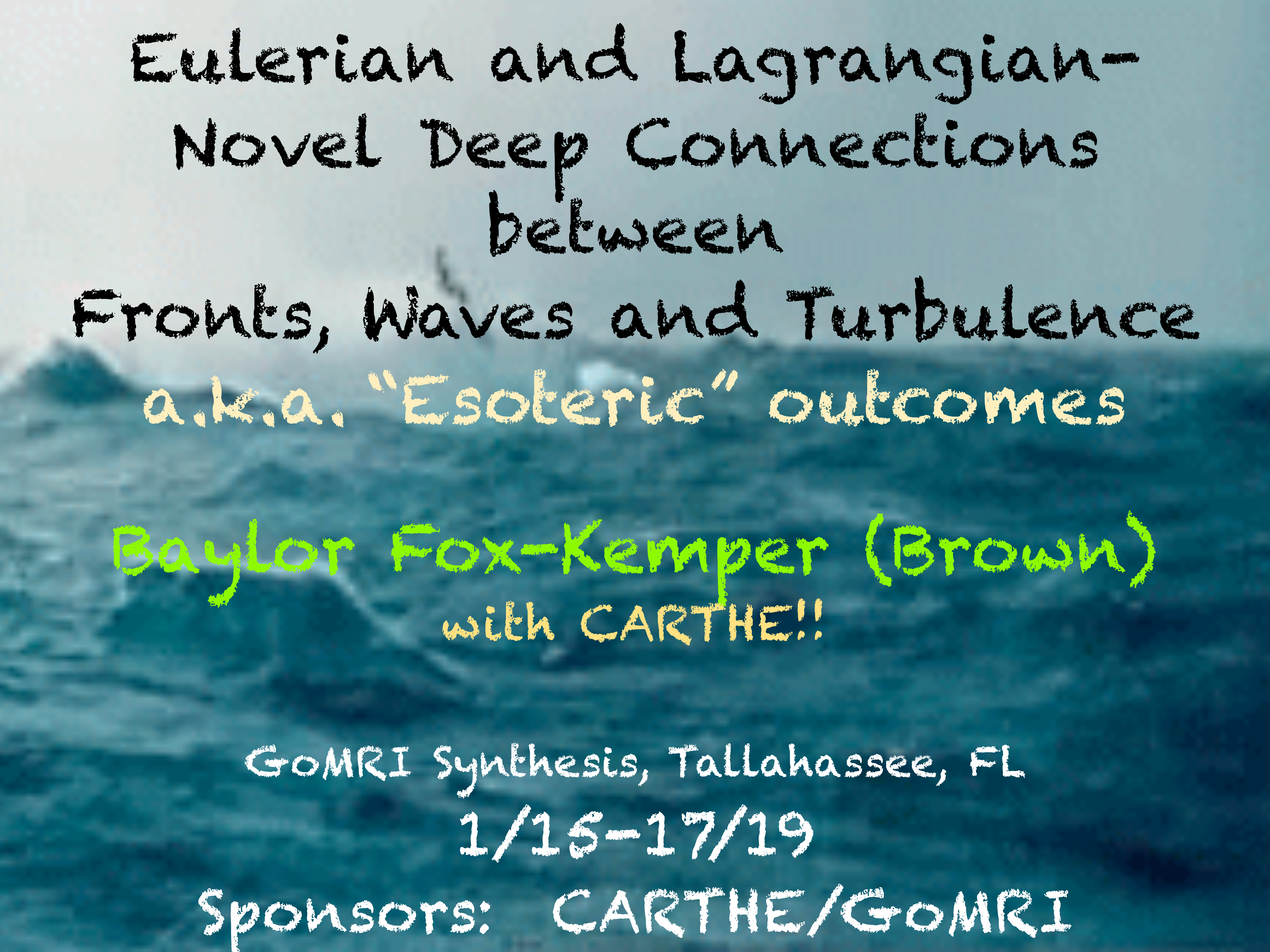


Eulerian and Lagrangian-Novel Deep Connections between Fronts, Waves and Turbulence

a.k.a. "Esoteric" outcomes

Baylor Fox-Kemper (Brown)

with CARTHE!!

GoMRI Synthesis, Tallahassee, FL

1/15-17/19

Sponsors: CARTHE/GoMRI

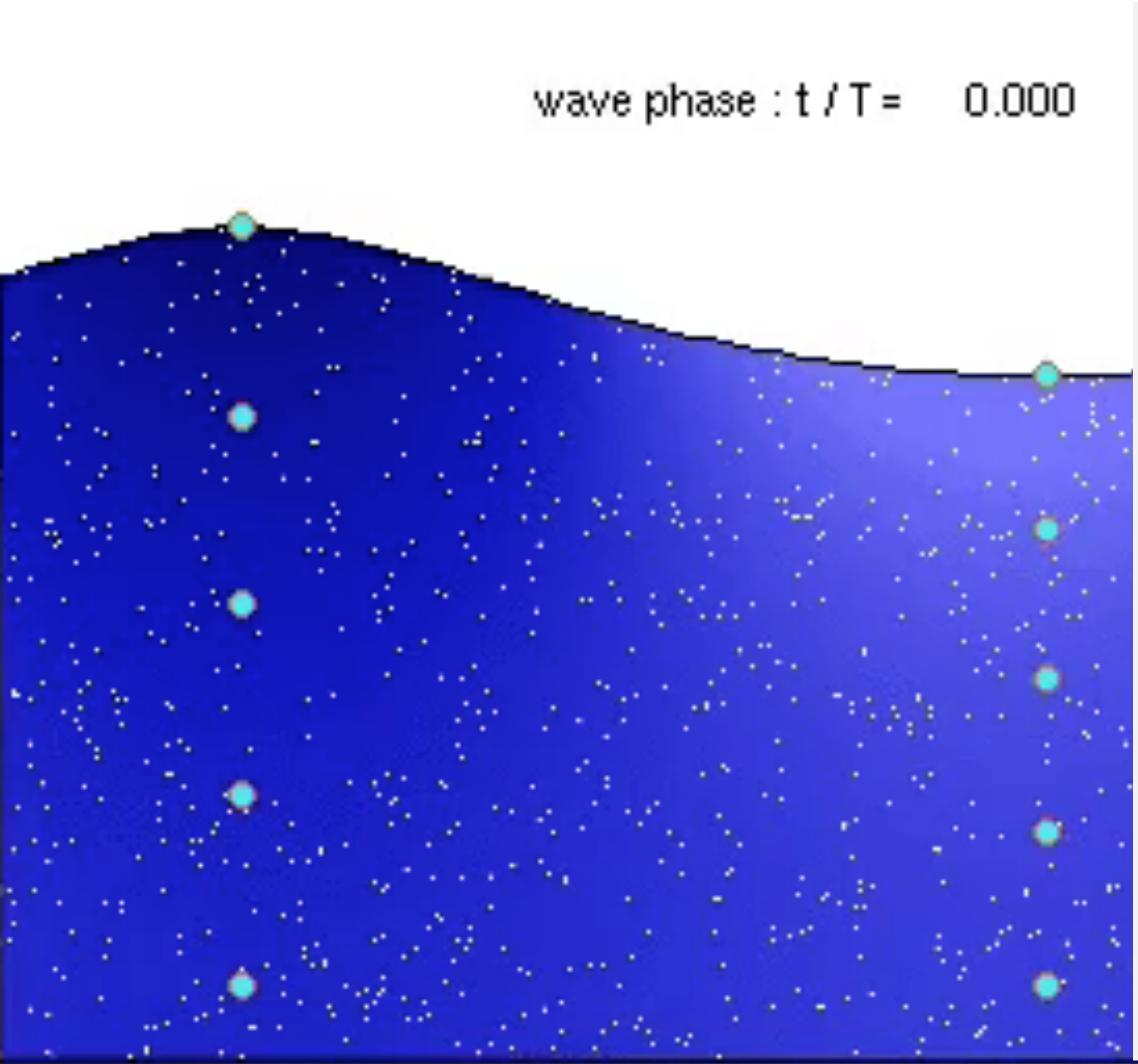
Oil Modeling (Old)

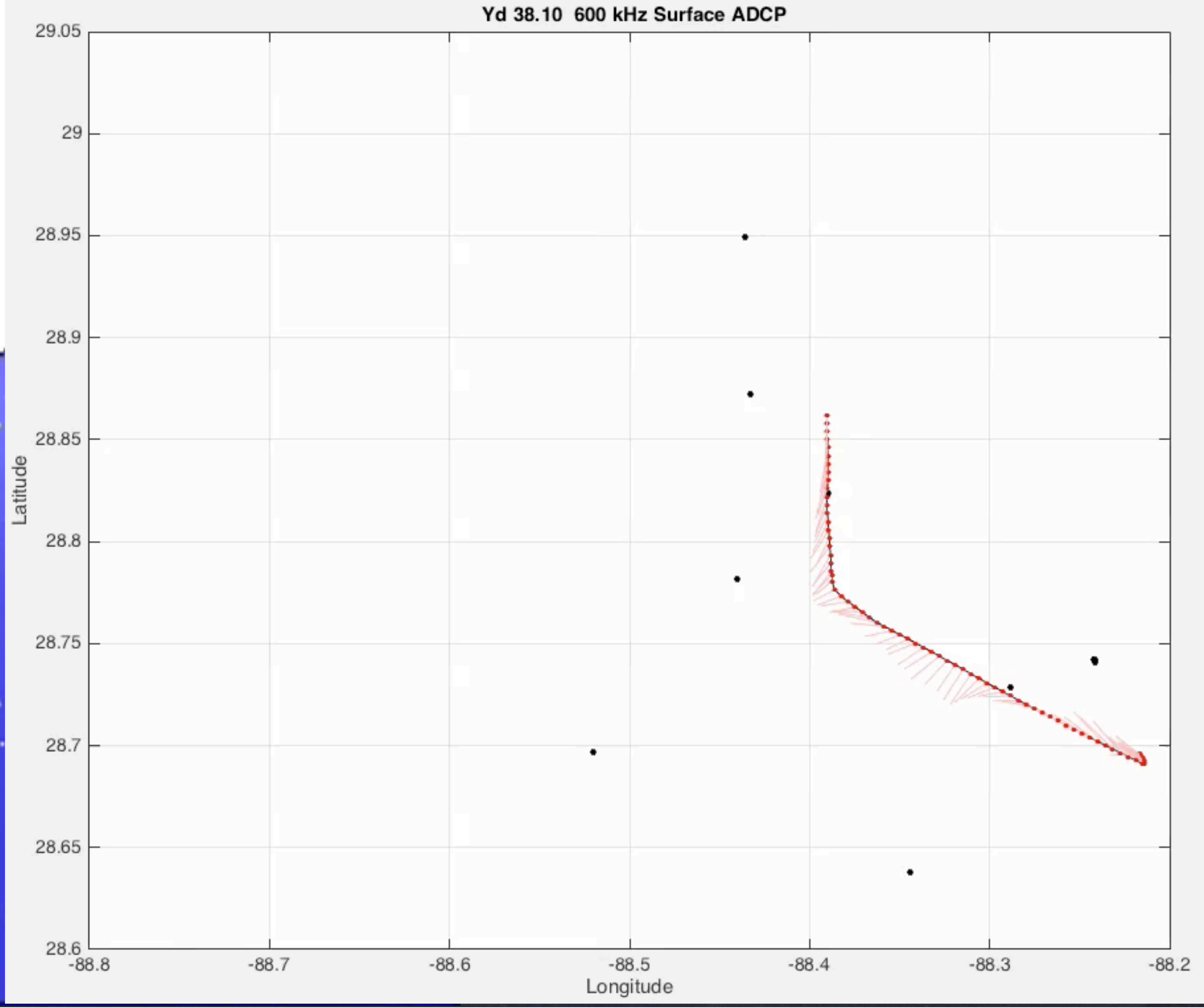
- Coastal models (hydrostatic, 5km to 100m), e.g., versions of ROMS, NCOM, ADCIRC, UWIN-CM, etc, are circulation models with simple 1D mixing & simple diffusion coupled to wave models
- Large Eddy Simulations (nonhydrostatic, 1m res.) show that accumulation at Langmuir cells & buoyant effects are important.
  Lots of detail lost in coastal models.
- Thus, the old approach combined empirical oil spreading & coastal models to get at the upper ocean transport & dispersion.

Eulerian & Lagrangian
2 Flavors in CARTHE

1: Waves

2: Drifters





The Character of the Langmuir Scale

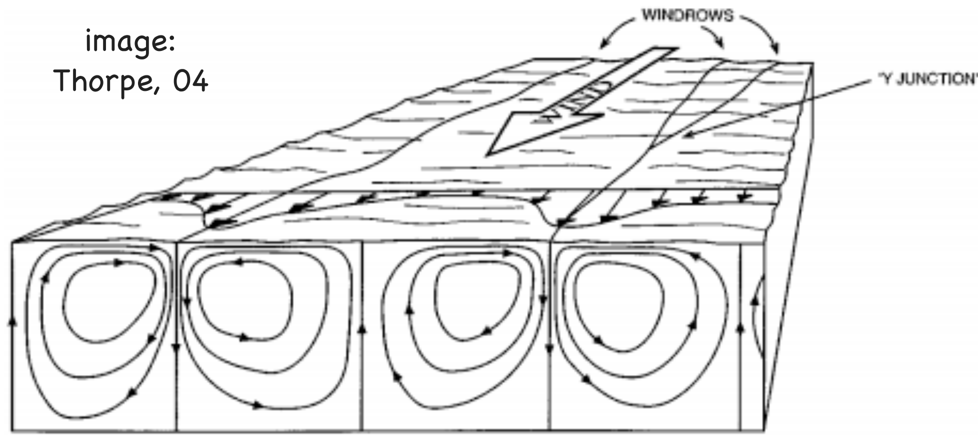


image:
Thorpe, 04

Figure 1 Sketch showing the pattern of mean flow in idealized Langmuir circulation. Th windows may be 2 m to 300 m apart, and the cell form is roughly square (as shown). I practice the flow is turbulent, especially near the water surface, and the windows (Figure 2 amalgamate and meander in space and time. Bands of bubbles or buoyant algae may for within the downward-going (or downwelling) flow (see Figure 3).

Carlson et al. 2018

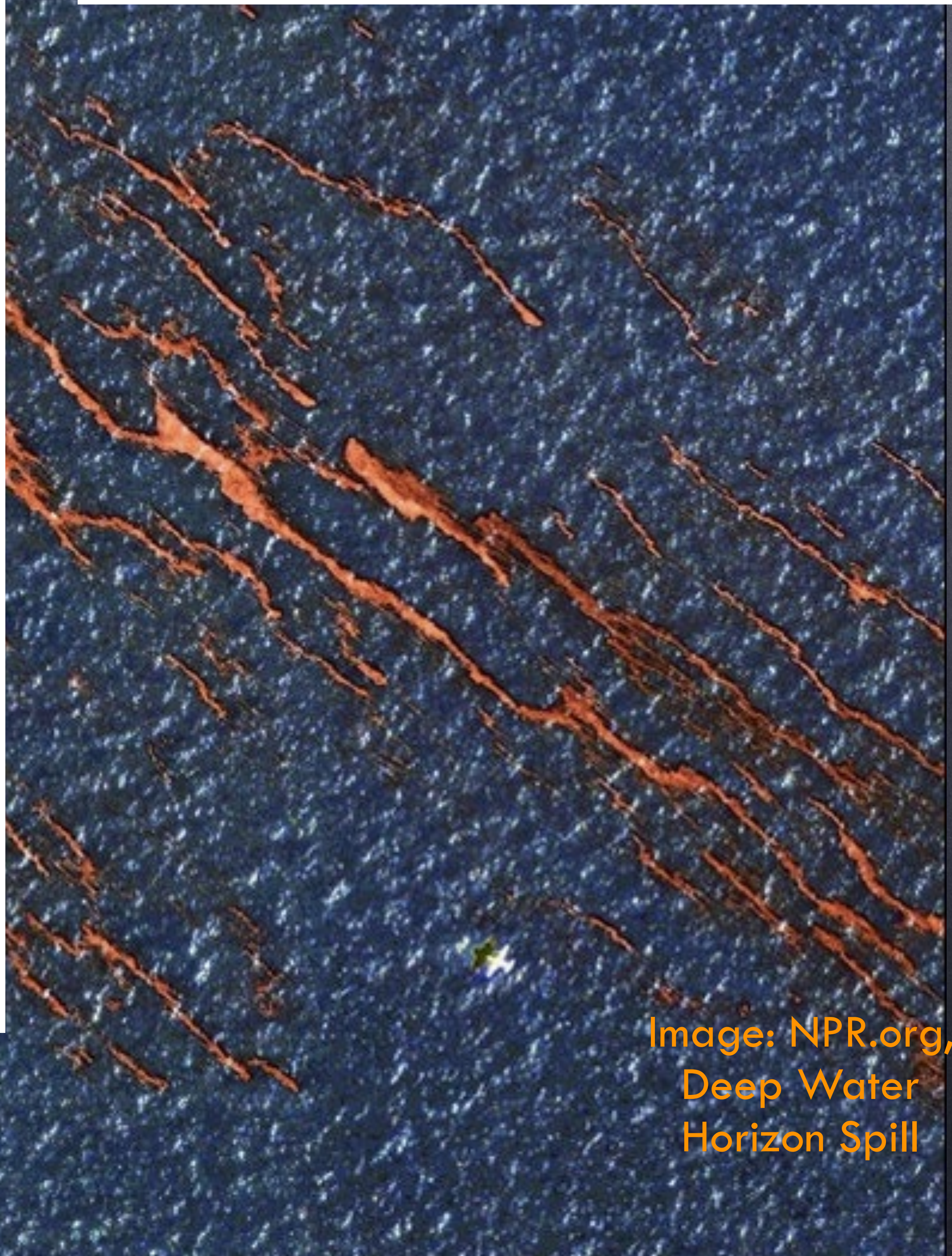


Image: NPR.org,
Deep Water
Horizon Spill

3 Stokes Forces appear in the "Wave-Averaged Equations": All rely only on Stokes drift of waves

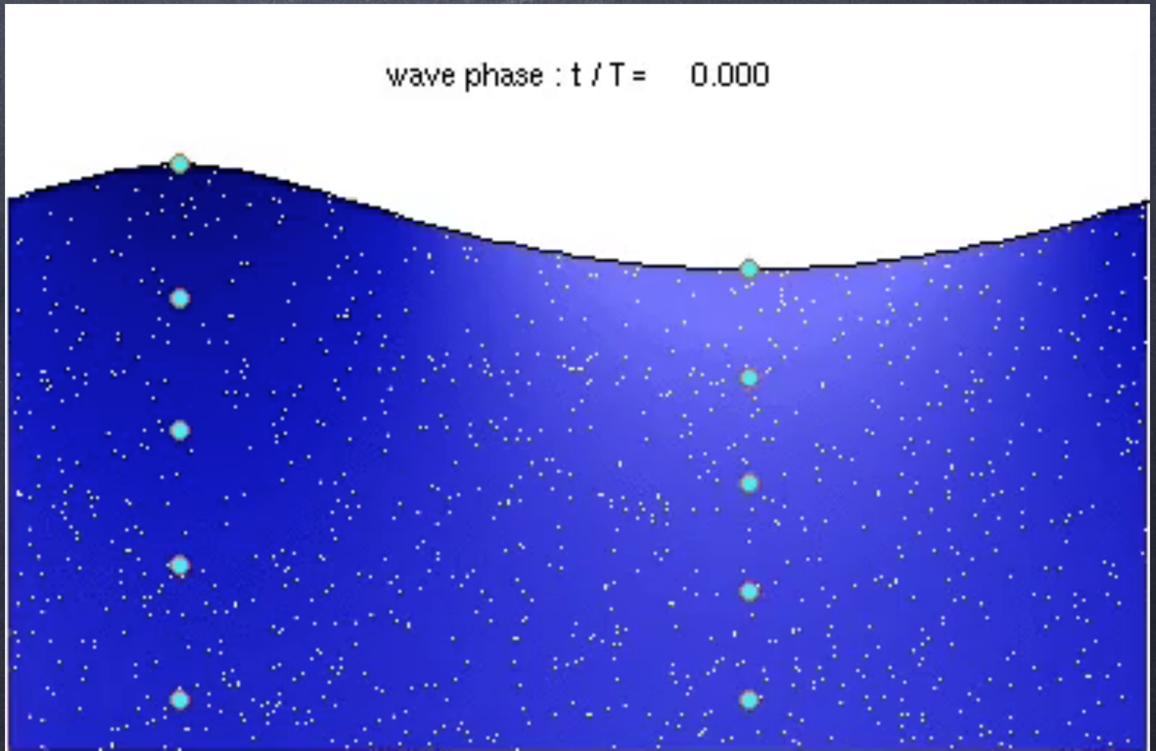
1: Stokes Advection: parcels, tracers, momentum move with Lagrangian, not Eulerian flow

2: Stokes Coriolis: water parcels experience Coriolis force during this motion

3: Stokes Shear Force

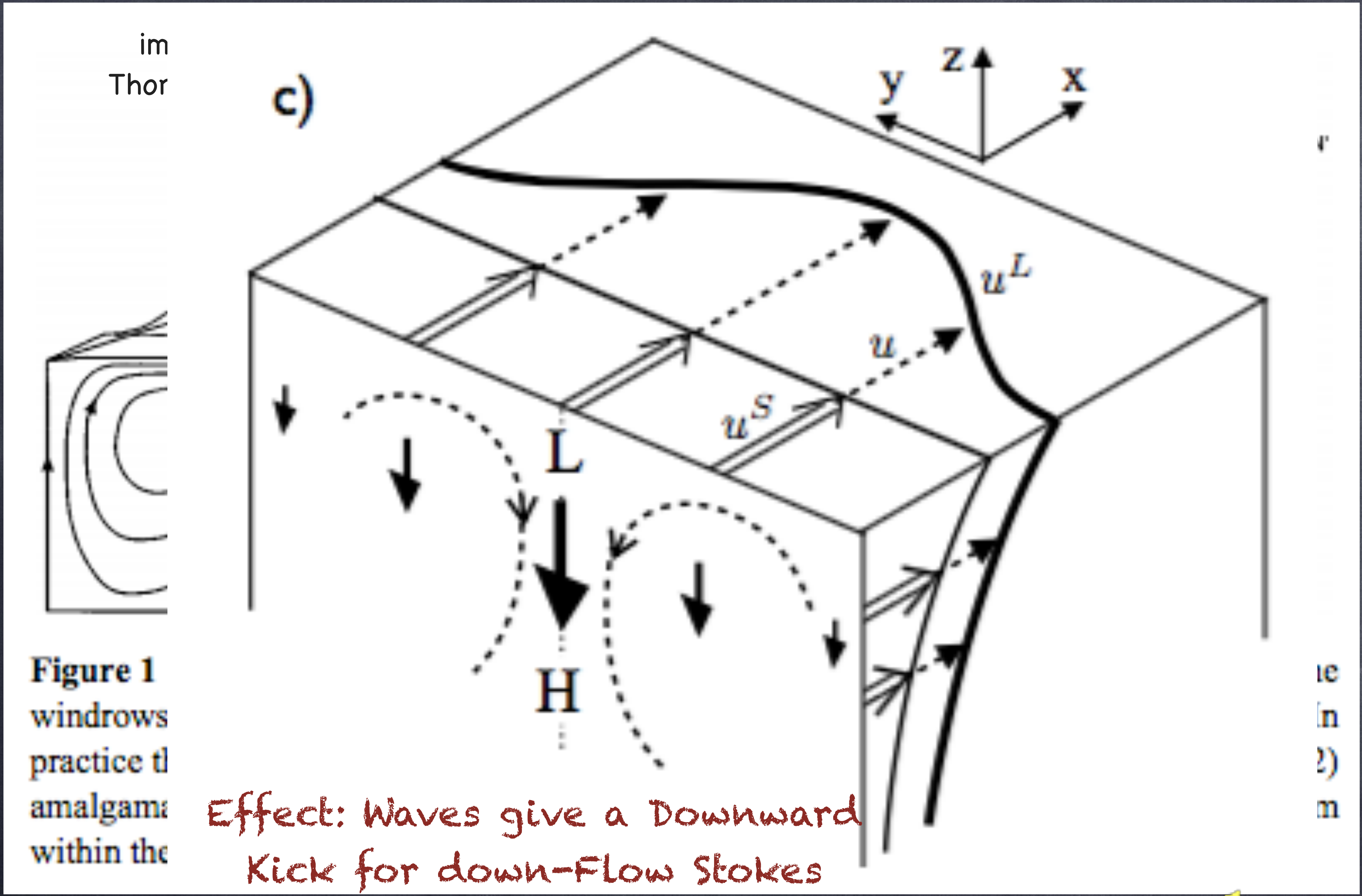
N. Suzuki and BFK. Understanding Stokes forces in the wave-averaged equations. Journal of Geophysical Research-Oceans, 121:1-18, 2016.

Notable: Wang & Ozgokmen (2017) examine these forces with wave-resolving models



Even before we get to the good bits...

- First guess: Stokes drift moves stuff, including oil, on the surface, e.g., Curcic et al. (2016), Judt et al. (2016).
- Problem: Anti-Stokes Eulerian response is critical, which only results if the Stokes forces are included in momentum equations, too.
- Outcome: should include Stokes forces in ocean models—our new formulation allows in hydrostatic models.



Effect: Waves give a Downward Kick for down-Flow Stokes

$M_{Ro} \equiv \max(1, Ro)$

$$Re = \frac{UL}{\nu}$$

$$Ro = \frac{U}{fL}$$

$$Ri = \frac{N^2}{(U_{,z})^2}$$

$\alpha = H/L$

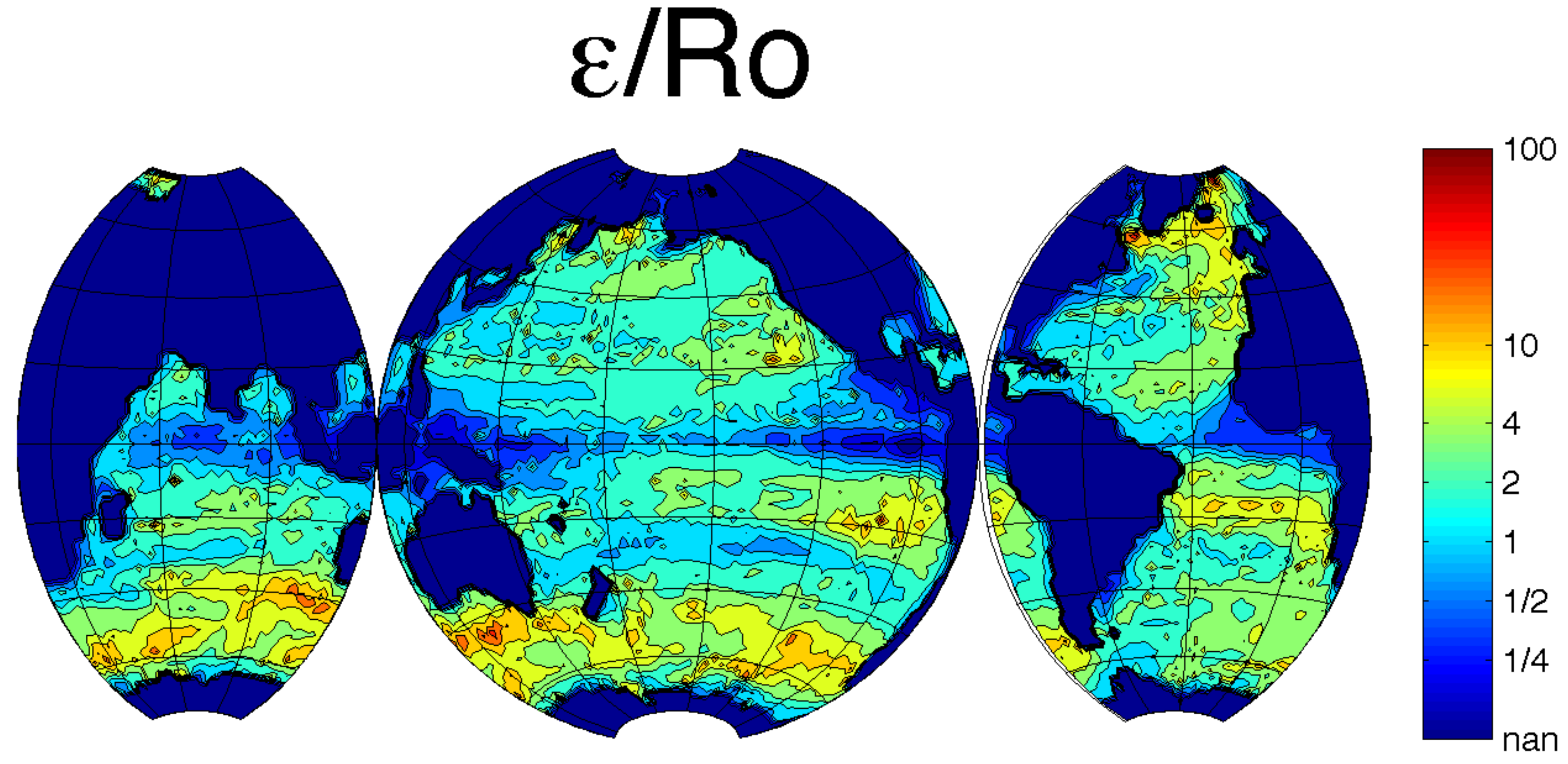
$$\varepsilon = \frac{V^s H}{f L H_s}$$

"wavy hydrostatic" if $\epsilon \gg 1$

$$\frac{\alpha^2}{Ri}\left[w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{RoRi} w w_{,z}\right] = \boxed{-\pi_{,z} + b - \varepsilon v_j^L v_{j,z}^s} + \frac{\alpha^2}{ReRi} w_{,jj}$$

N. Suzuki and BFK. Understanding Stokes forces in the wave-averaged equations. Journal of Geophysical Research-Oceans, 121:1-18, 2016.

Do Stokes force directly affect larger scales?



"wavy hydrostatic"

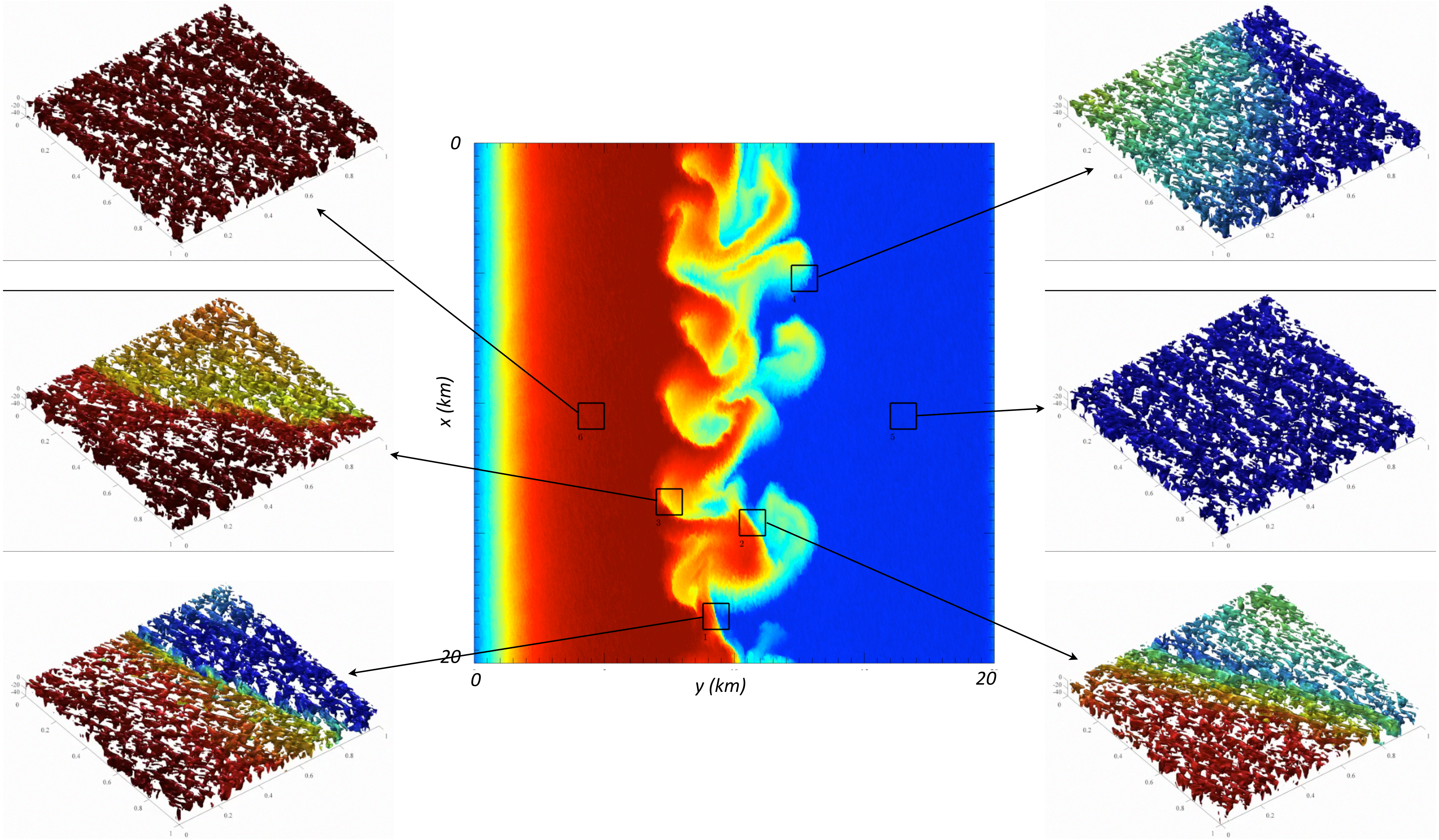
$$\epsilon \gg 1$$

$$\varepsilon = \frac{V^s H}{f L H_s}$$

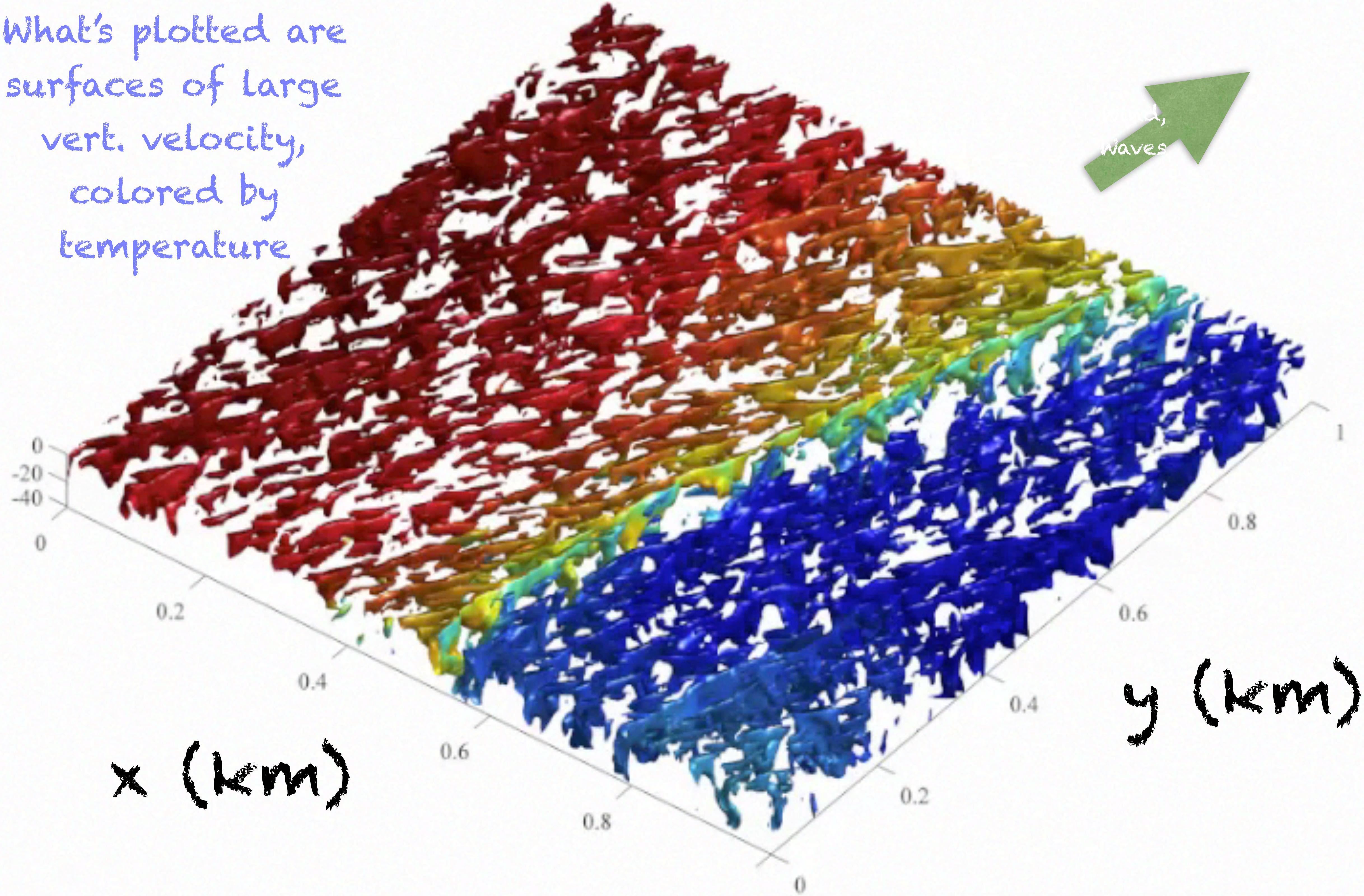
$$Ro = \frac{U}{f L}$$

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. Journal of Fluid Mechanics, 730:464-490, 2013.

A Large Eddy Simulation spanning the submesoscale (100m-10km) and boundary layer turbulence (5-50m)



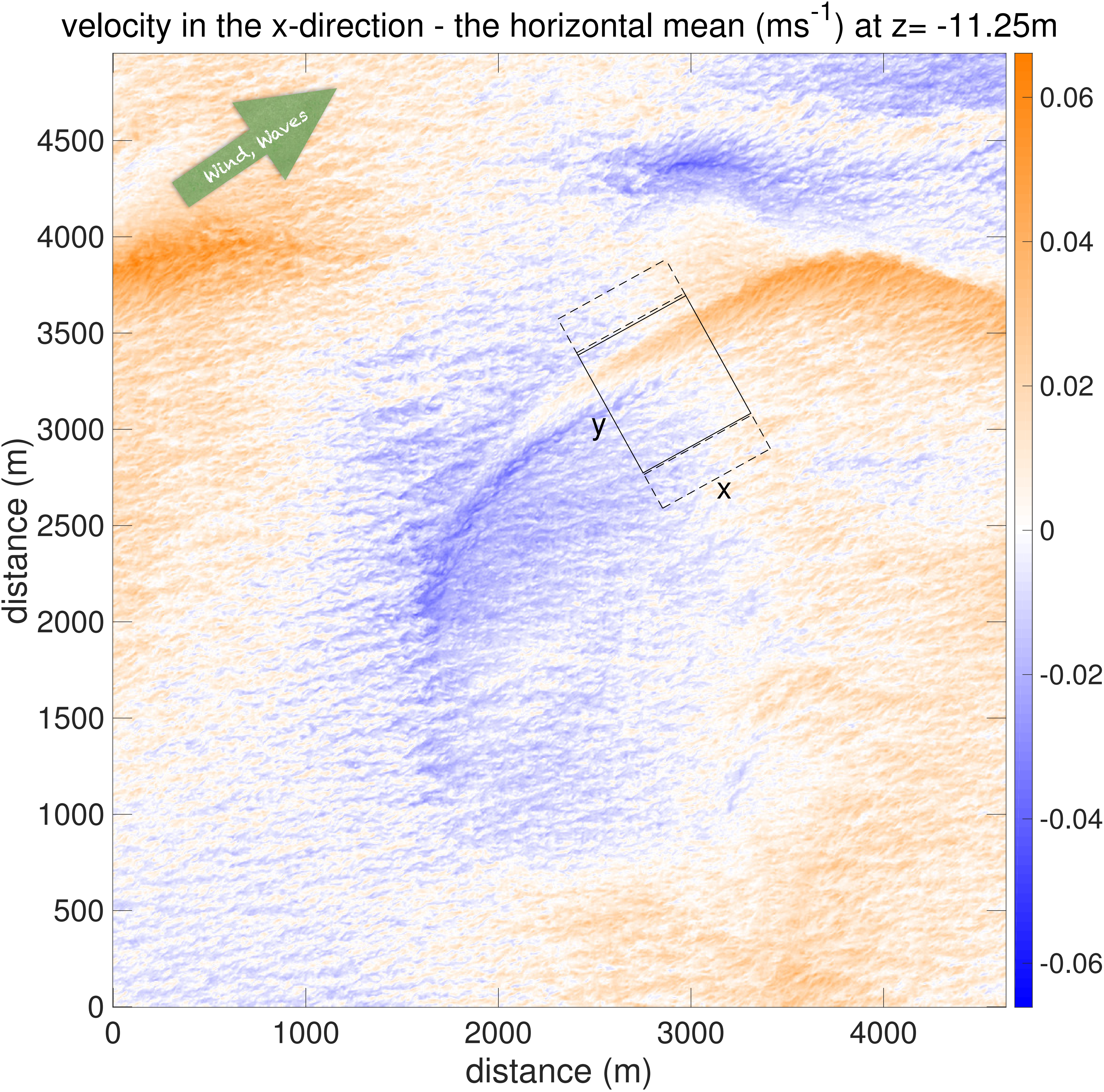
P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale frontal spin-down simulations. Journal of Physical Oceanography, 44(9): 2249-2272, September 2014.



What's plotted are surfaces of large vert. velocity, colored by temperature

P. E. Hamlington, L. P. Van Roekel, BFK, K. Julien, and G. P. Chini. Langmuir-submesoscale interactions: Descriptive analysis of multiscale fro down simulations. Journal of Physical Oceanography, 44(9):2249-2272, September 2014.

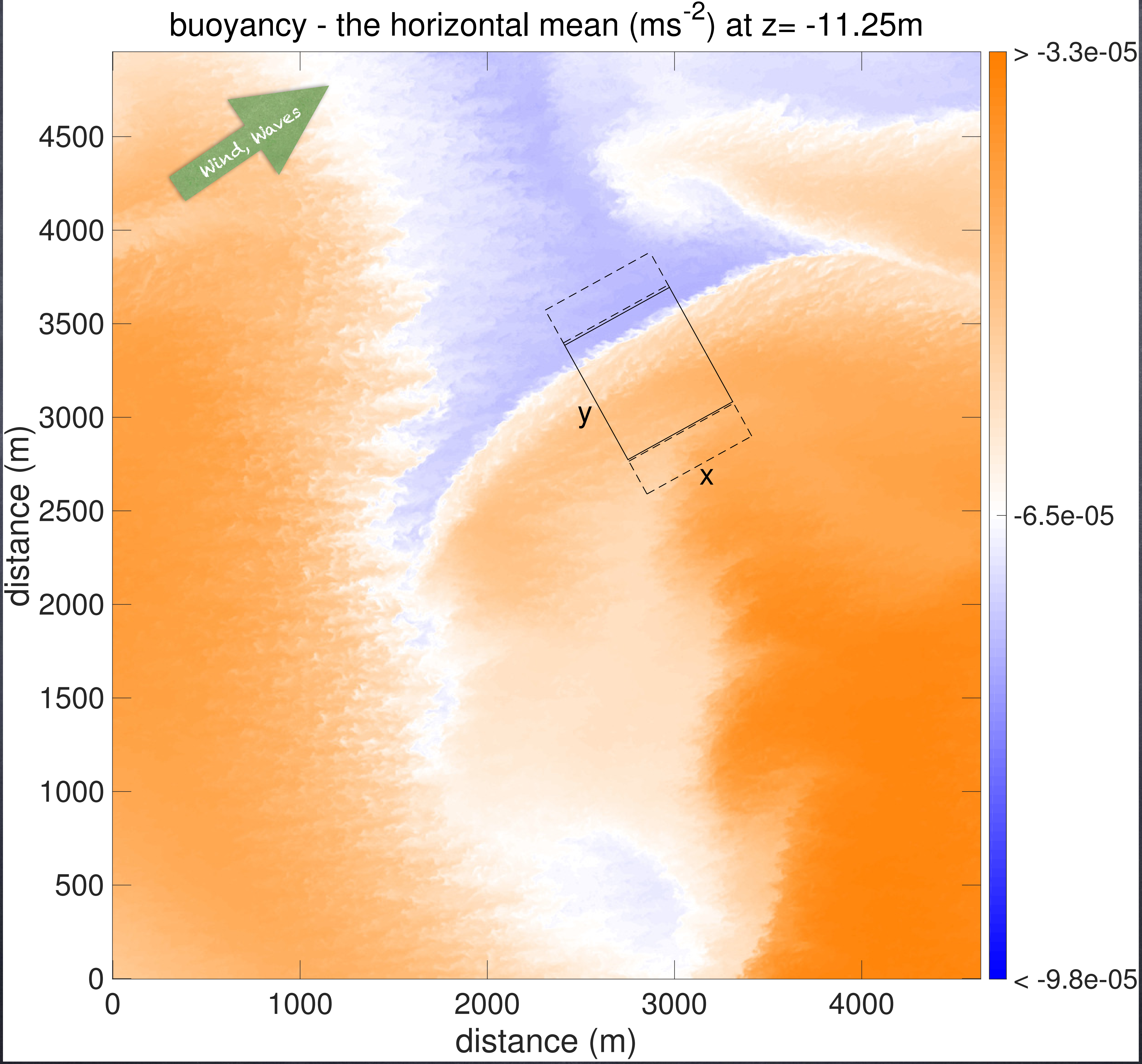
V



velocity in the x-direction - the horizontal mean (ms^{-1}) at z= -11.25m

N. Suzuki, B. Fox-Kemper, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. Journal of Geophysical Research-Oceans, 121:1-28, May 2016.

T


buoyancy - the horizontal mean (ms^{-2}) at z= -11.25m

N. Suzuki, B. Fox-Kemper, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. Journal of Geophysical Research-Oceans, 121:1-28, May 2016.

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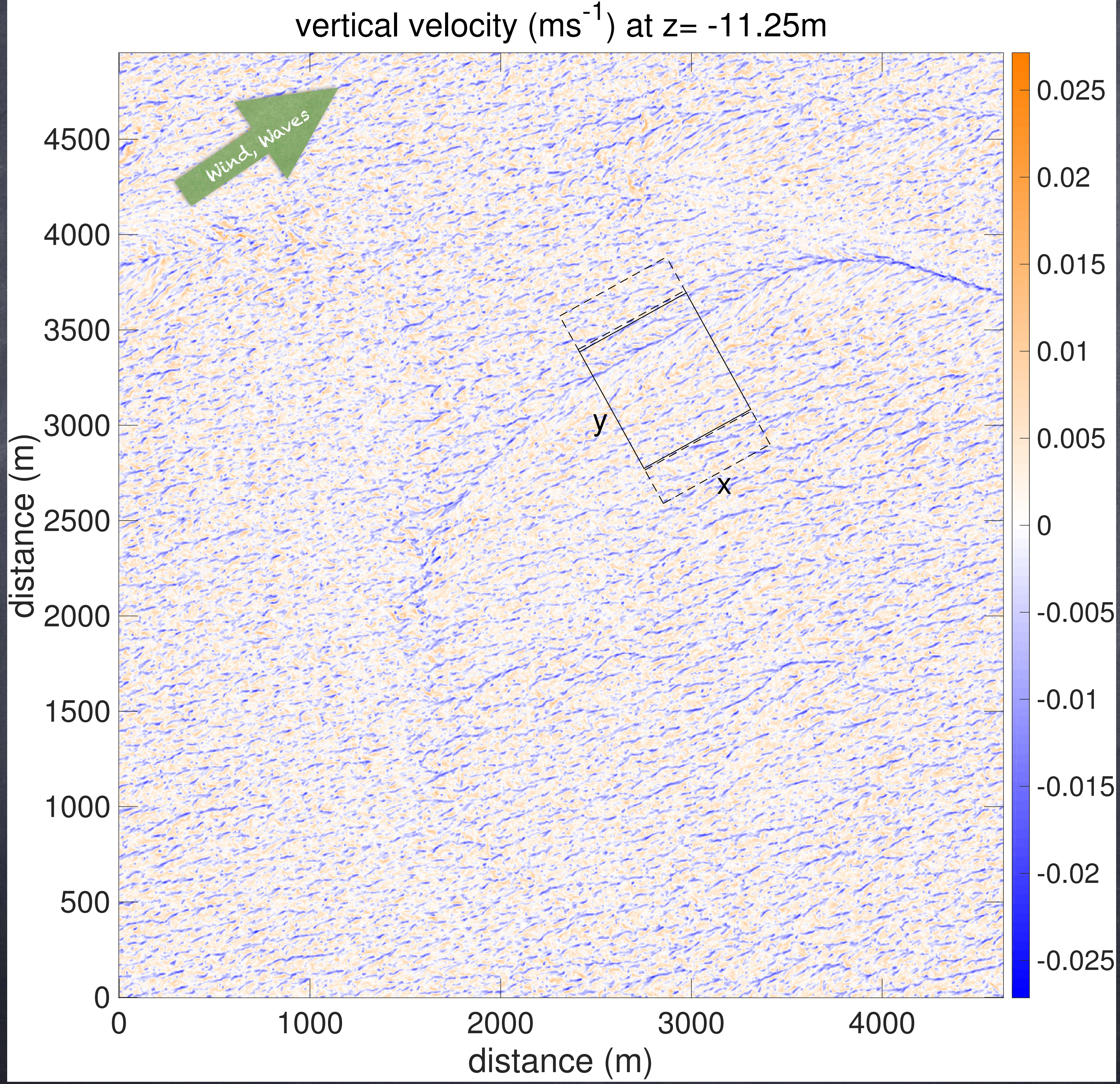
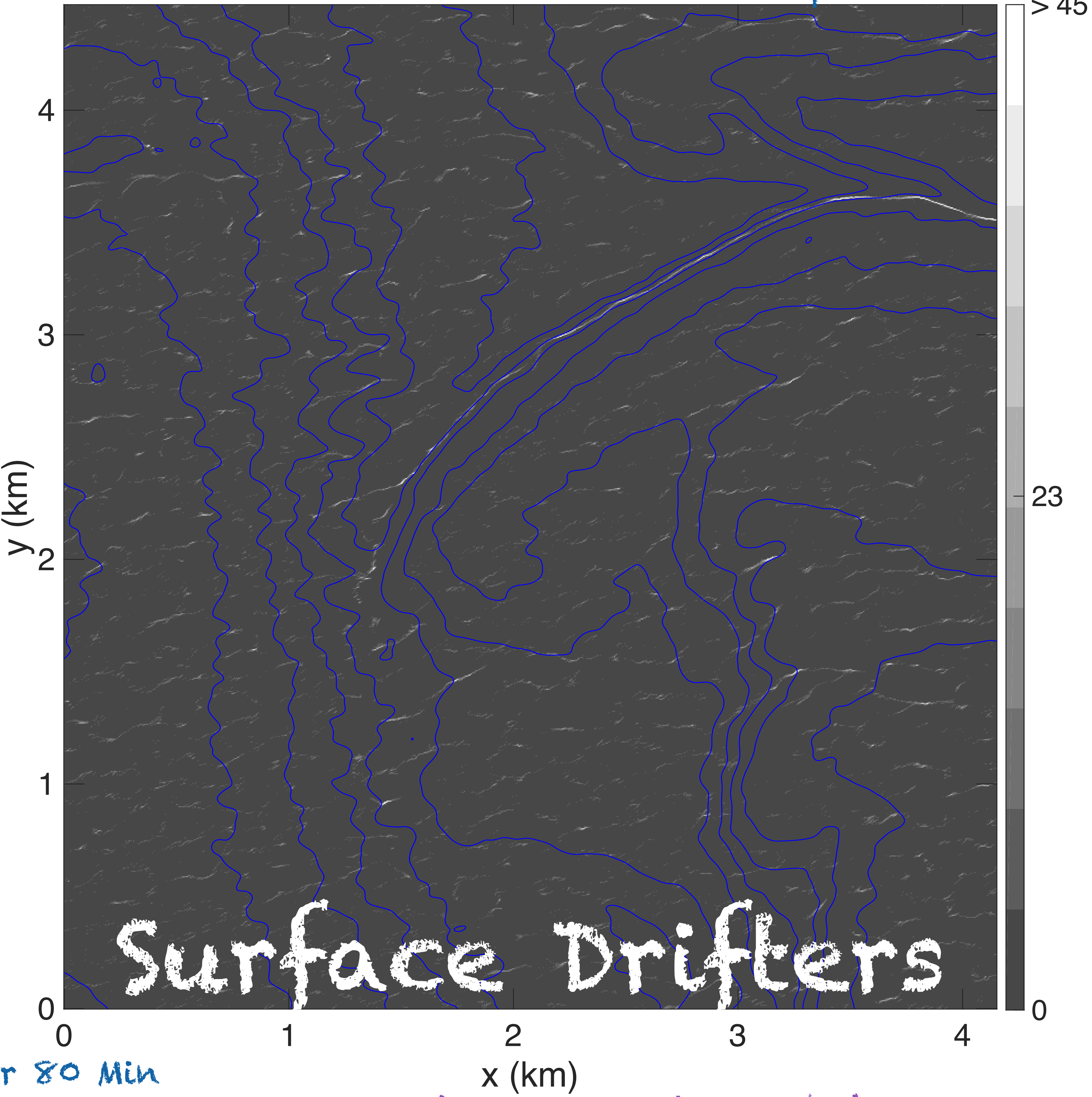


N. Suzuki, B. Fox-Kemper, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. Journal of Geophysical Research-Oceans, 121:1-28, May 2016.

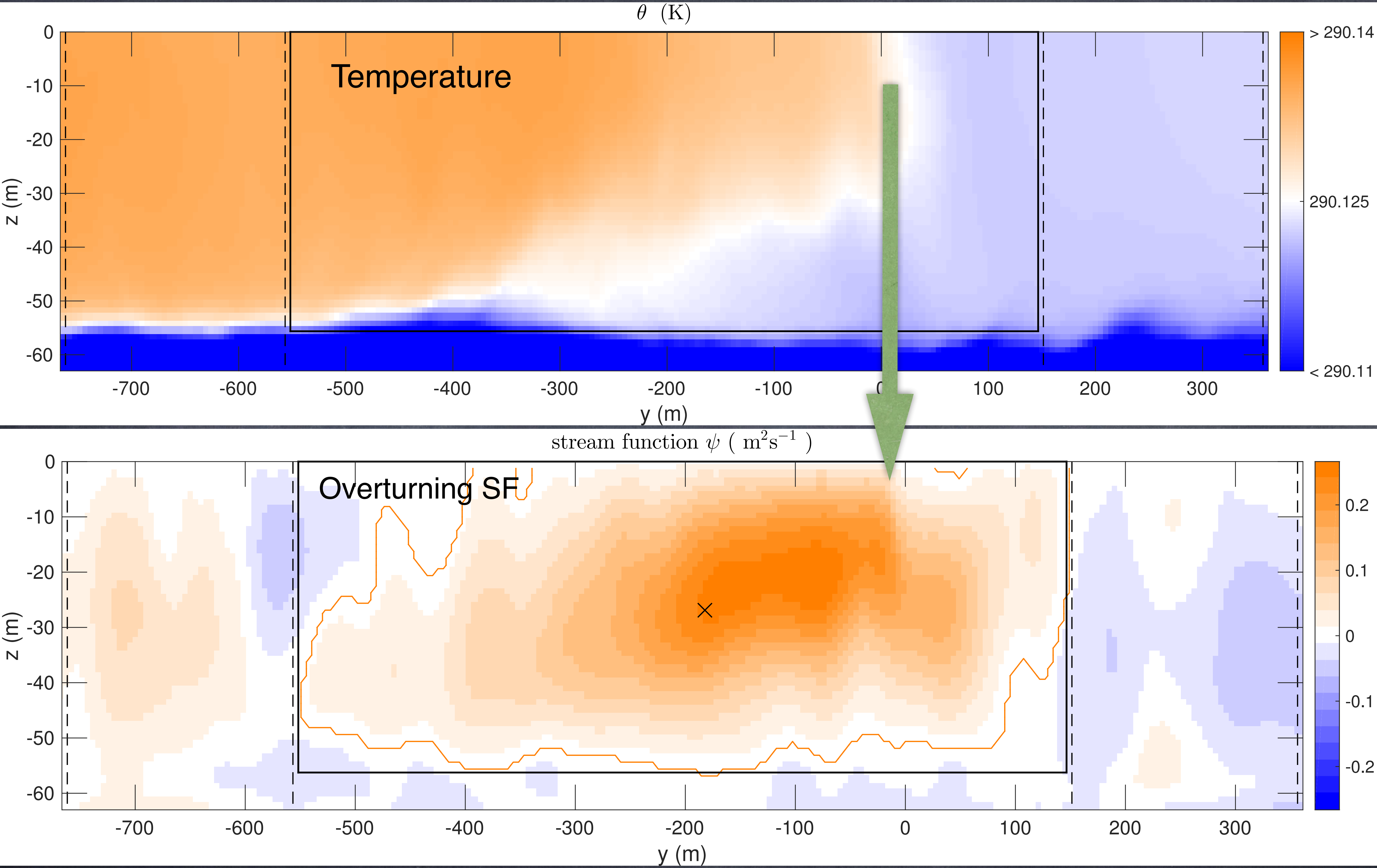
Initially every surface node has 1 drifter, so there are 851796 drifters in the picture



Surface Drifters

After 80 Min

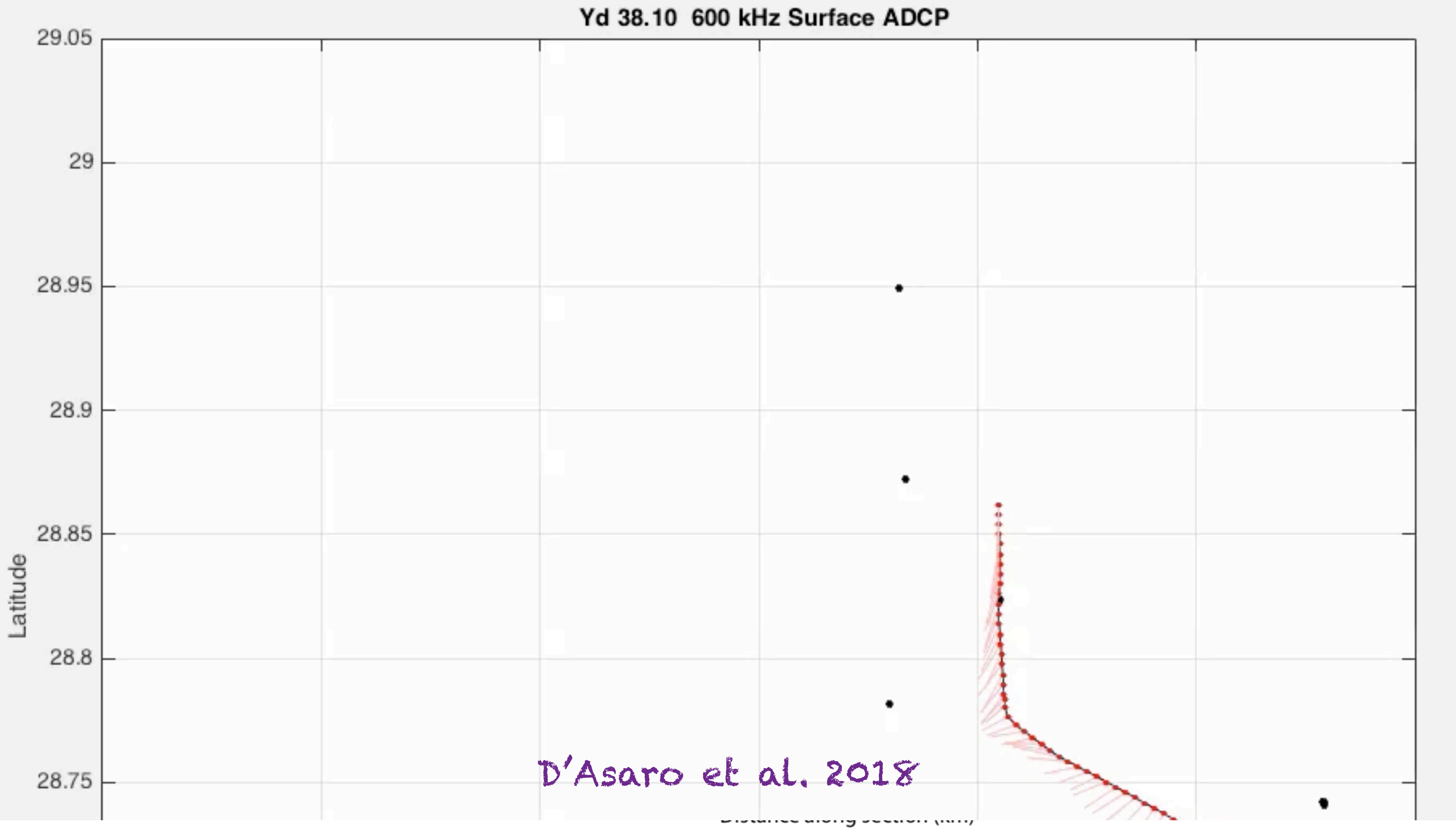
See also Harcourt & Taylor (2018) buoyant LES.



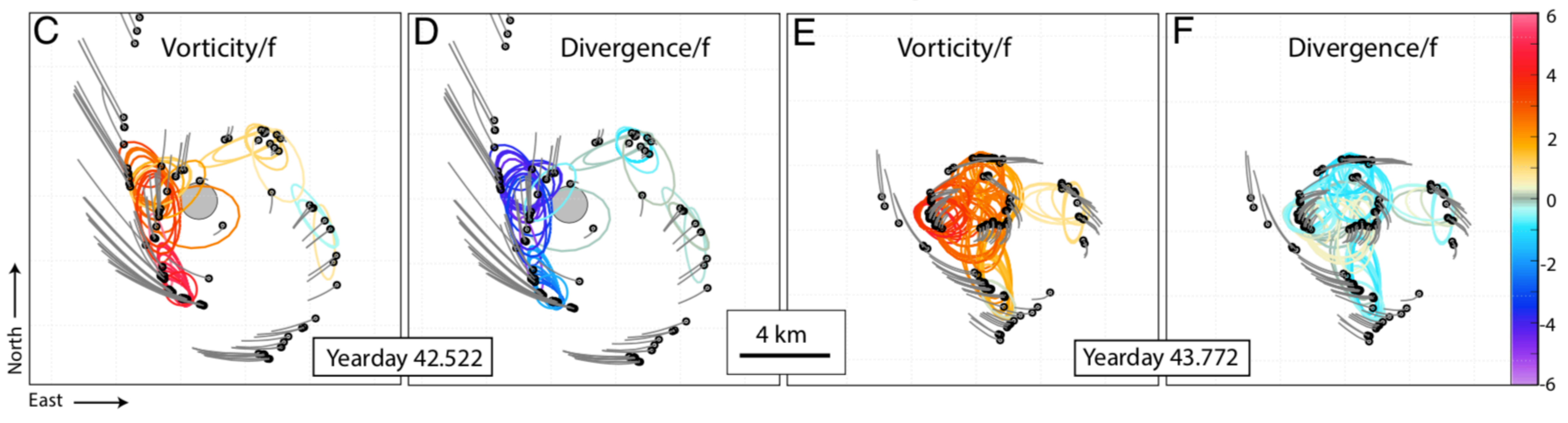
N. Suzuki, BFK, P. E. Hamlington, and L. P. Van Roekel. Surface waves affect frontogenesis. Journal of Geophysical Research-Oceans, 121:1-28, 2016.

Stokes Drift = Lagrangian-Eulerian, Energizes Turbulence (old) & Fronts (new)

- New: Tracer & drifter advection by Stokes drift can be included in coastal (nonhydrostatic) models; care is needed to preserve anti-Stokes flow—i.e., consistent application of Stokes forces. Future: wider application of these approaches
- New: Stokes Shear force energizes fronts Future: modeled, but not yet observed directly
- Improved: Turbulent mixing can be parameterized into Langmuir schemes for turbulence effects. Future: include buoyant tracer effects



D'Asaro et al. 2018



The ROMS version of same

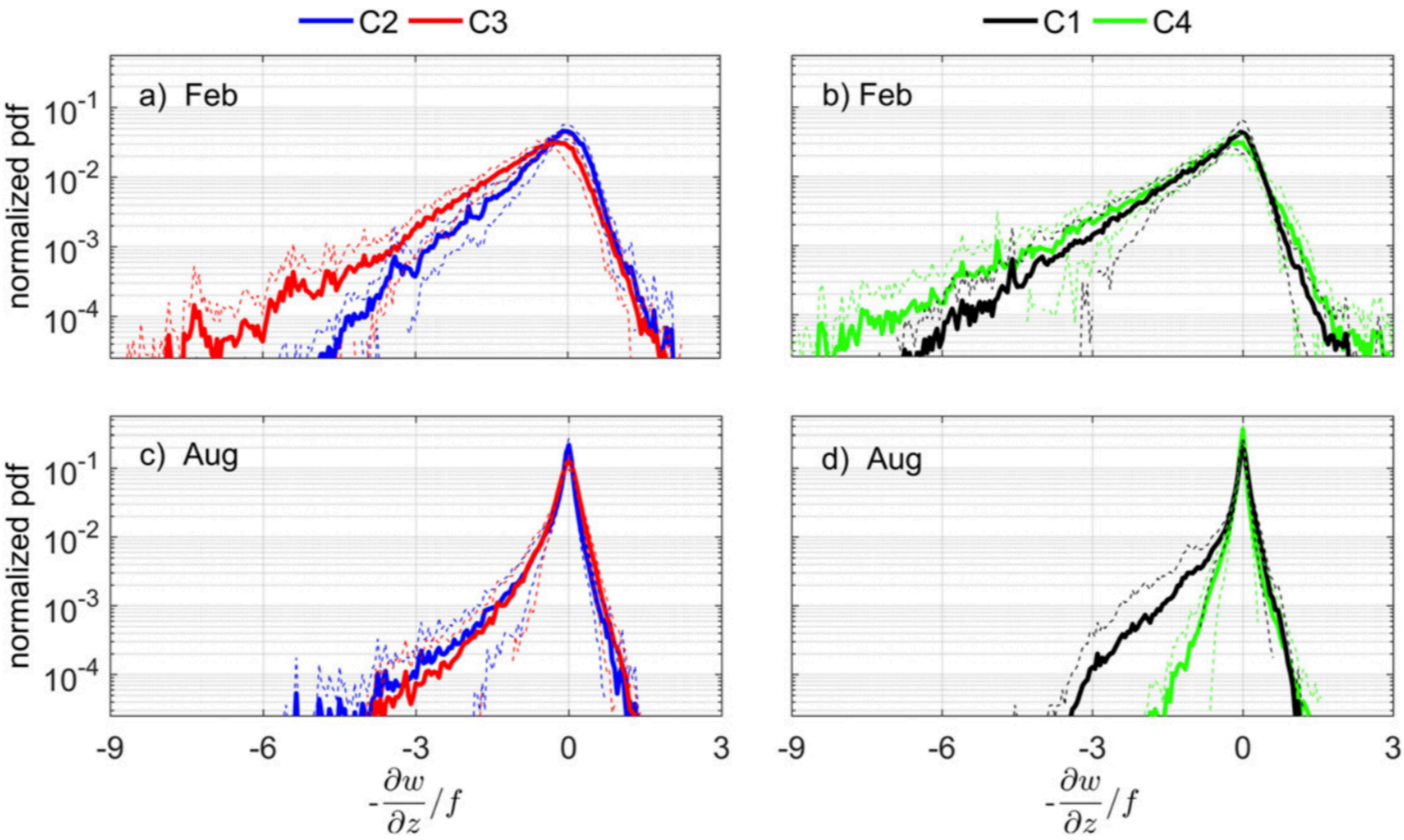


FIG. 12. Horizontal velocity divergence distributions for the four clusters in Fig. 1 in the HR With-River solution [(left) C2 (blue) and C3 (red); (right) C1 (black) and C4 (green)]. PDFs are averaged over eight deployments (thick lines) 3 days after release. The standard error, calculated as twice the standard deviation of the eight deployments, is also indicated (dashed lines). (a),(b) February and (c),(d) August.

GLAD: Isotropic Forward Cascade from 30km?

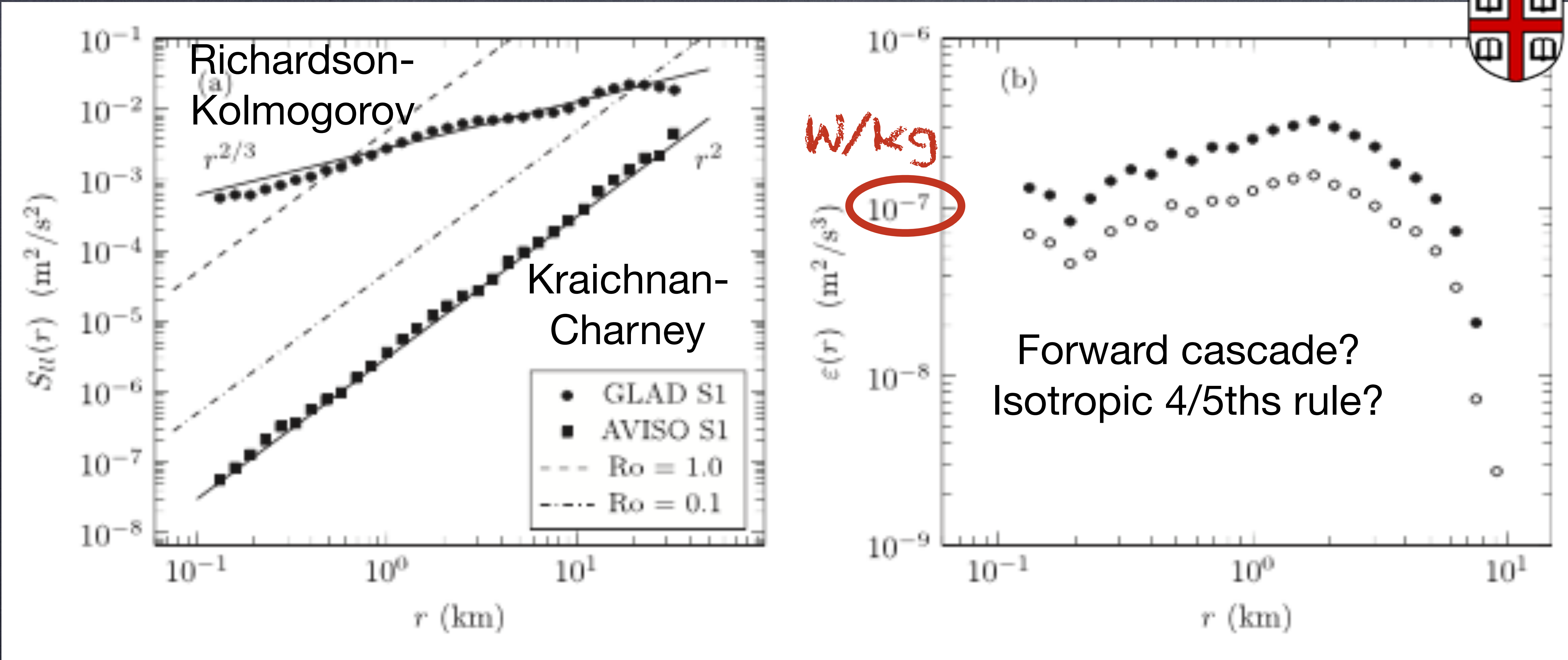


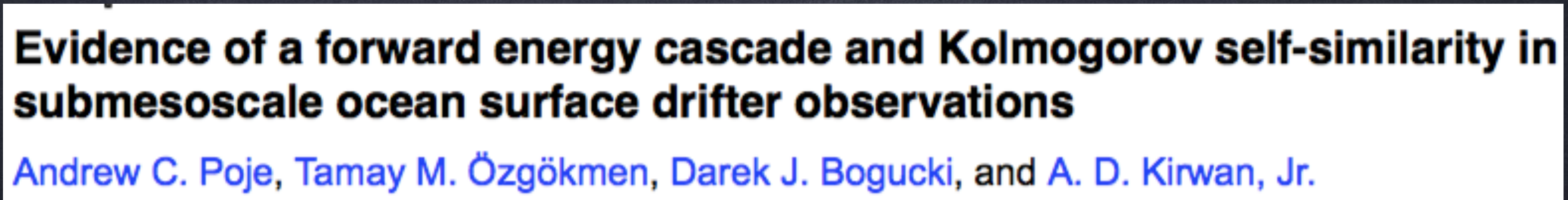
FIG. 7. (a) Second order longitudinal structure function versus separation distance showing Richardson-Kolmogorov, $r^{2/3}$, energy cascade scaling for GLAD data and Kraichnan, r^2, enstrophy cascade scaling for AVISO-based synthetic trajectories. (b) Sign-reversed third order longitudinal structure function scaled by r for the GLAD observations.

140TW (global)/(1.4 10^{21} kg)=10^{-7} W/kg

Winds: ~20TW global

Tides: 3.5TW global

D'Asaro et al (2011): Enhanced @ Fronts: 10^{-5} to 10^{-6} W/kg



Evidence of a forward energy cascade and Kolmogorov self-similarity in submesoscale ocean surface drifter observations

Andrew C. Poje, Tamay M. Özgökmen, Darek J. Bogucki, and A. D. Kirwan, Jr.

“Cascade” Scalings

3D: Richardson/Kolmogorov/Smagorinsky/Corrsin

$$E \propto \epsilon^{2/3}\ell^{5/3}, \quad S_2 \propto \epsilon^{2/3} r^{2/3}, \quad \epsilon \propto \nu\alpha^2, \quad \nu = \mathrm{Pr}\,\kappa \propto \Delta x^2|\alpha| \propto \epsilon^{1/3}\ell^{4/3}$$

Drifters?

2D/QG: Barnier/Kraichnan/Charney/Leith

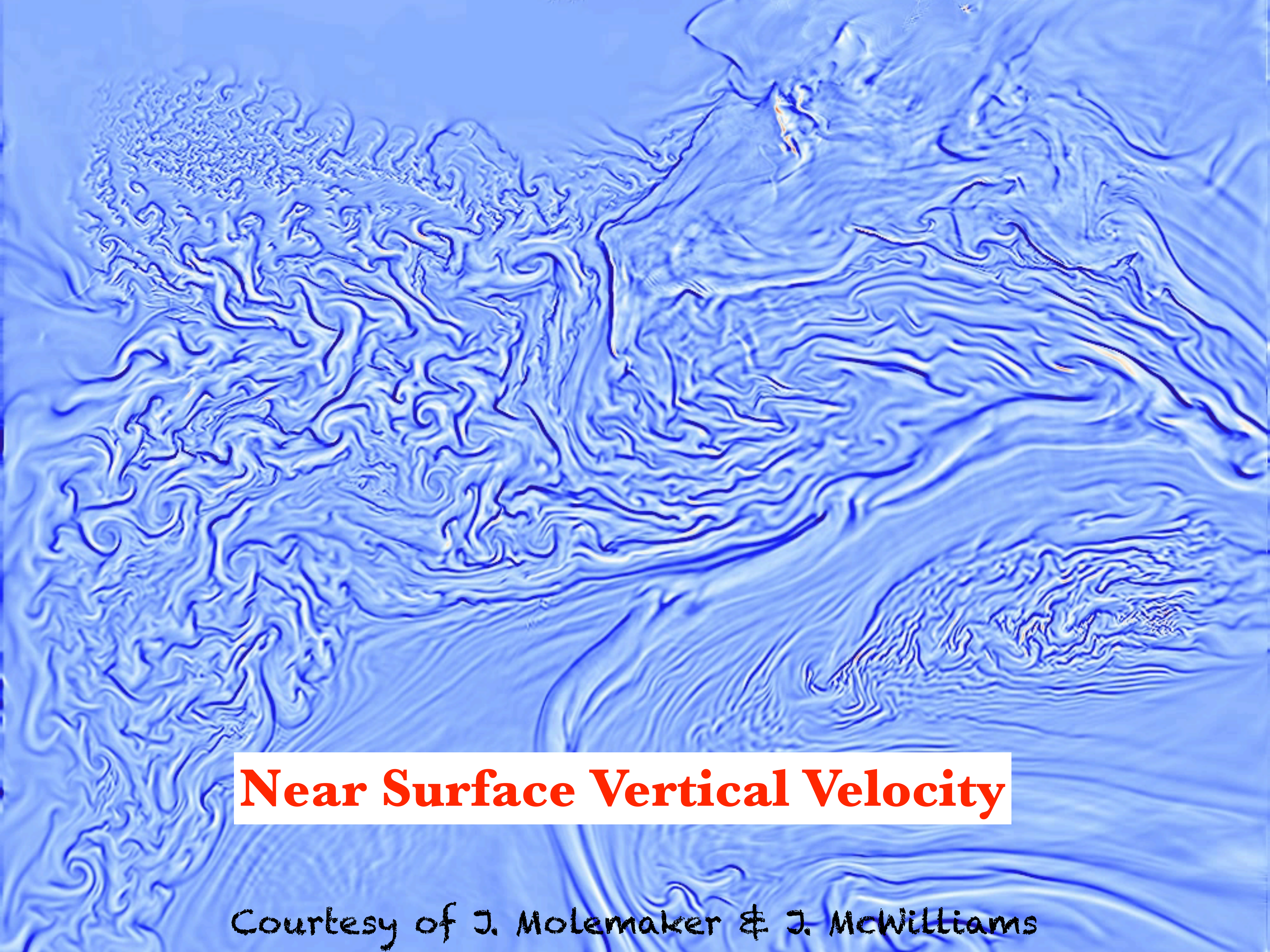
$$E \propto \eta^{2/3}\ell^{3}, \quad S_2 \propto \eta^{2/3} r^{2}, \quad \eta \propto \nu(\nabla\omega)^2, \quad \nu =\propto \Delta x^3|\nabla\omega| \propto \eta^{1/3}\ell^{2}, \quad \kappa \propto ?$$

AVISO?

Submesoscale: McWilliams/?/?F-K?

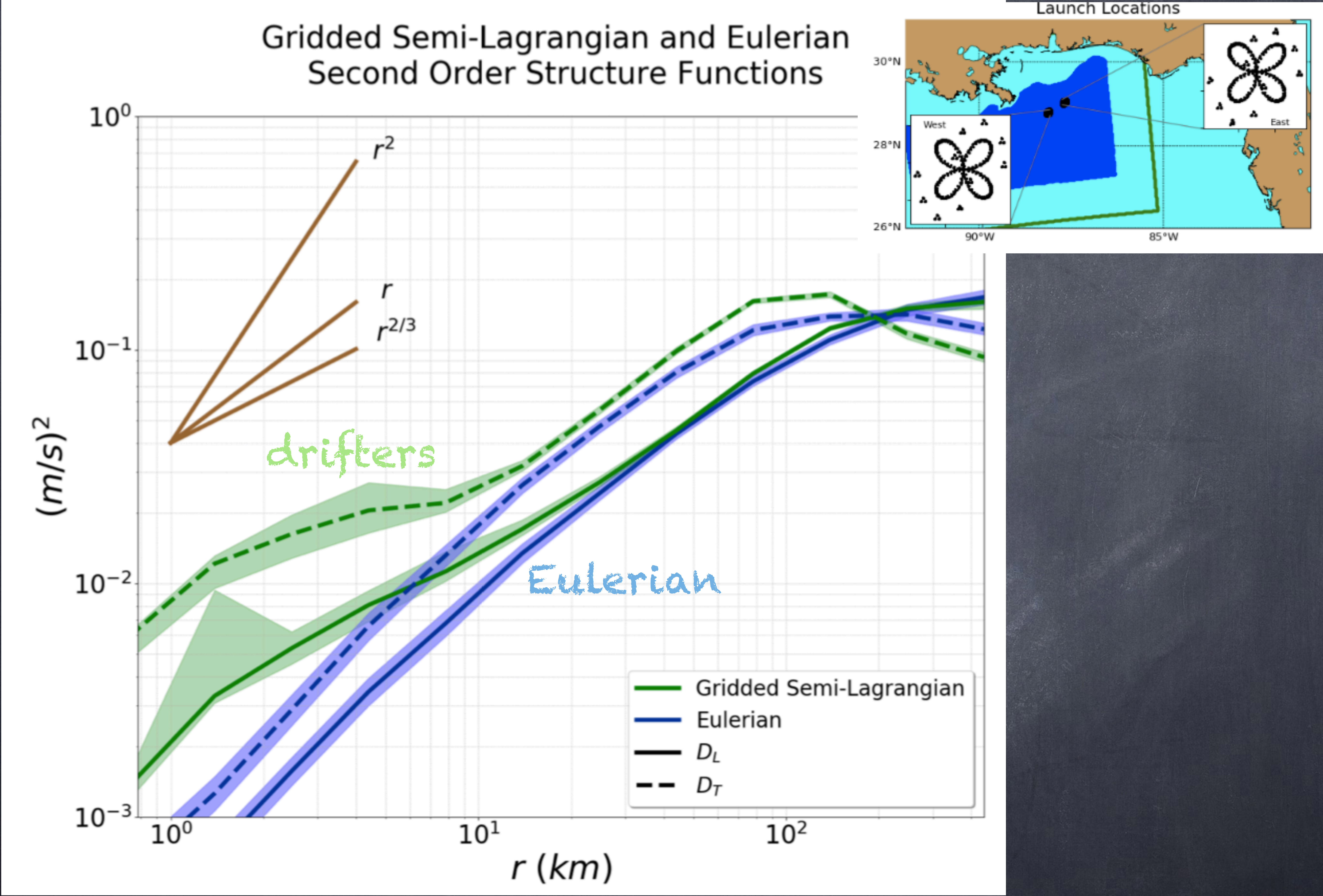
$$E \propto \ell^{2}, \quad S_2 \propto r^{1}, \quad d/dt(PE + KE) = ??, \quad \nu = ?, \quad \kappa = ?$$

ROMS?



Near Surface Vertical Velocity

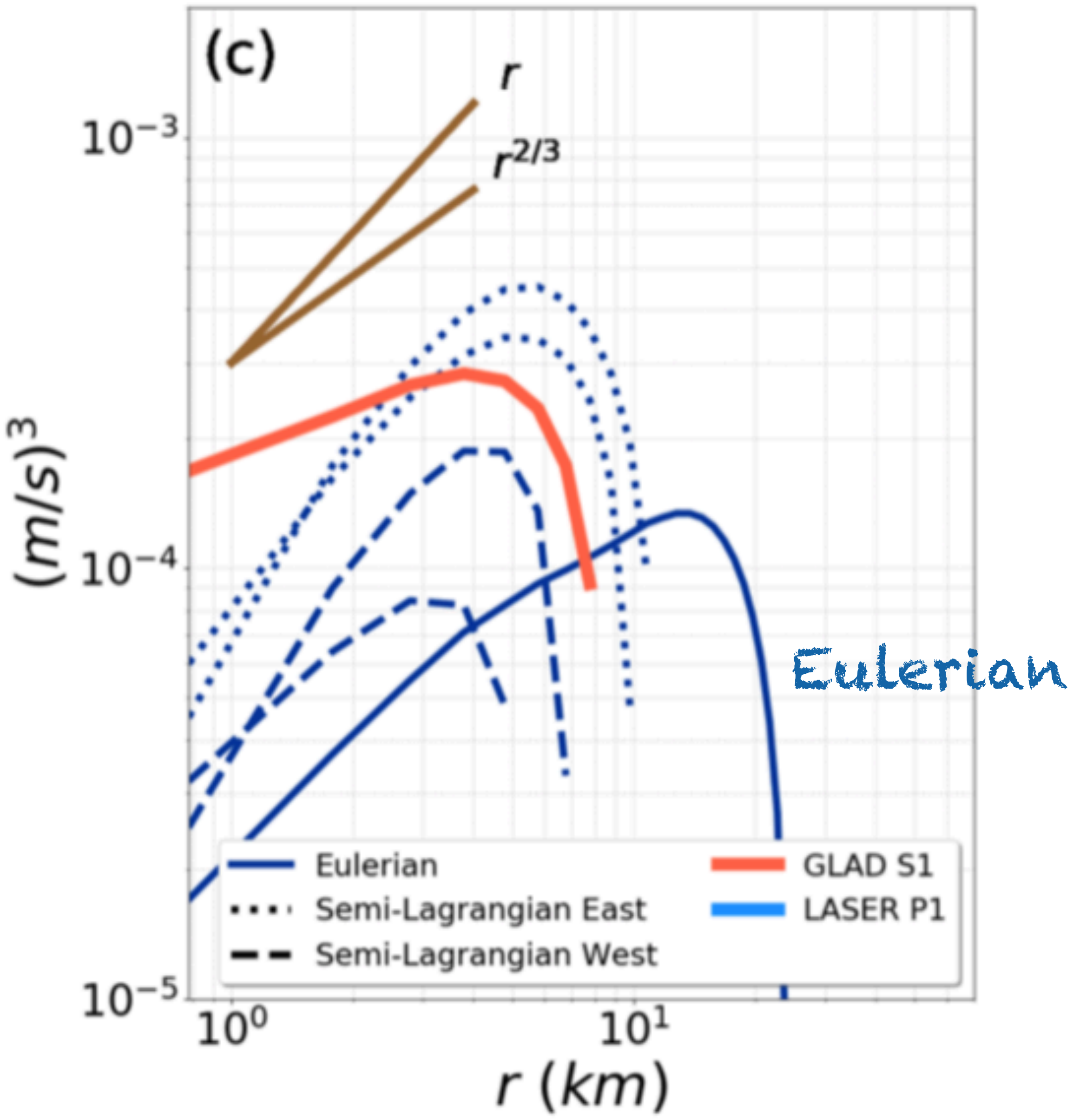
Courtesy of J. Molemaker & J. McWilliams



J. Pearson, B. Fox-Kemper, R. Barkan, J. Choi, A. Bracco, and J. C. McWilliams. Impacts of convergence on Lagrangian statistics in the Gulf of Mexico. Journal of Physical Oceacnography, 2019. In press.

Eulerian vs. Lagrangian: Lagrangian is biased toward sampling convergent, energetic fronts &tc.

$D_L^3 < 0$ "forward cascade"



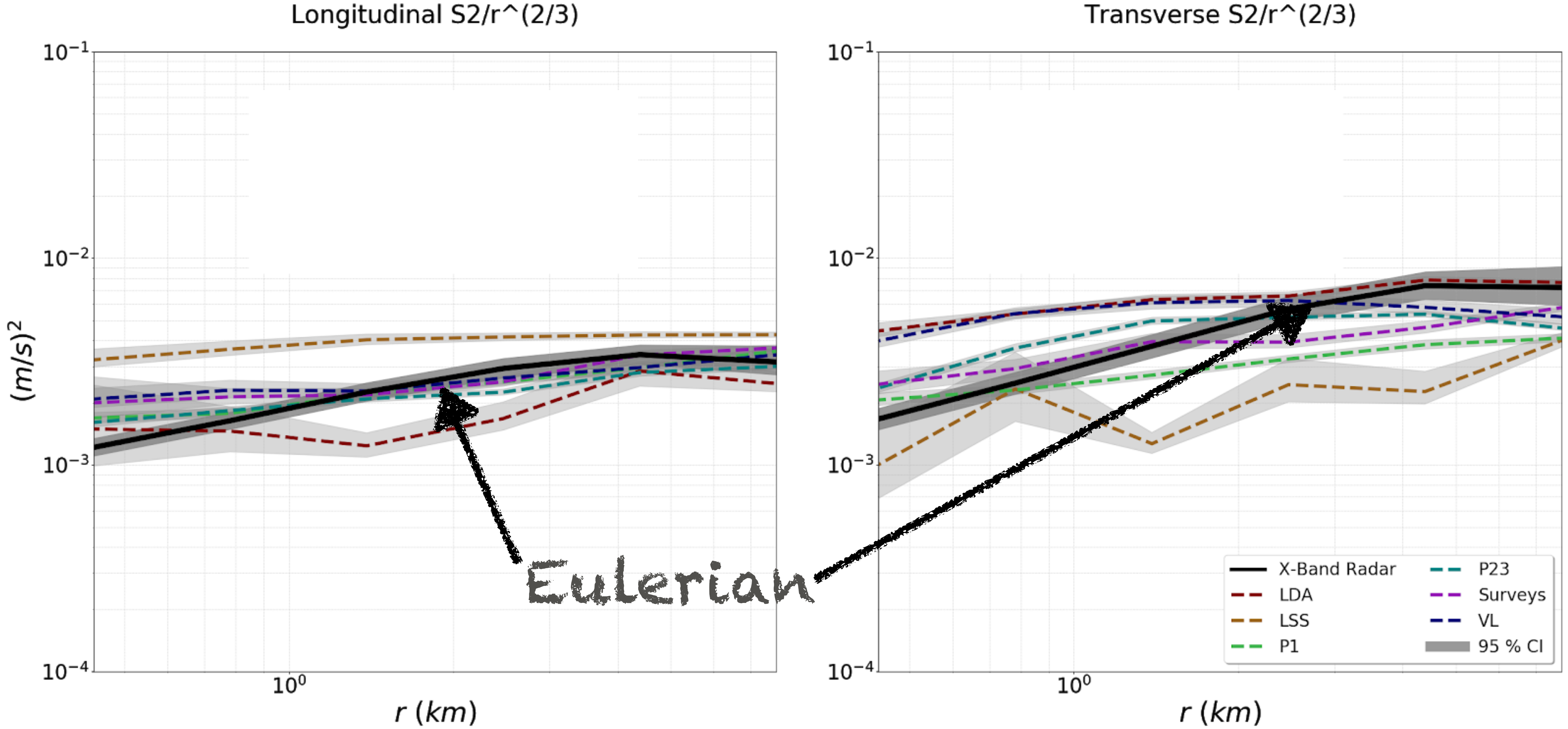
$$\epsilon \not\propto \langle([\mathbf{u}(\mathbf{x}+\mathbf{r}) - \mathbf{u}(\mathbf{x})] \cdot \mathbf{r}/|\mathbf{r}|)^3\rangle/|r|$$

Lagrangian ϵ overestimates by factor of 2 to 8 on 1km to 10km

J. Pearson, B. Fox-Kemper, R. Barkan, J. Choi, A. Bracco, and J. C. McWilliams. Impacts of convergence on Lagrangian statistics in the Gulf of Mexico. Journal of Physical Oceacnography, 2019. In press.

Same Story in LASER Observations (w/ UDel.)

(Note: compensated w/ 3D slope)



Conclusions

- Old & Improved: Eulerian and Lagrangian flows, and the difference between them, have major implications for modeling, observation, and analysis.

- Old: Stokes drift advects, but (New:) Stokes forces affect Eulerian flows. Stokes forces energize Langmuir turbulence & down-Stokes fronts.
  New and Future: coastal model improvements

- New: Turbulence statistical scalings from drifters & Eulerian measurements differ on the submesoscales in models & obs, because drifters sample fronts more.
  Future: drifter theory & assimilation techniques including bias.

- Conjecture: the dynamics governing the drifters seems to resemble 3D turbulence, so perhaps energy dissipation rate at fronts is a key to open the scaling factor challenge.

Wave-Averaged Equations

following Lane et al. (07), McWilliams & F-K (13) and Suzuki & F-K (15)

(for horizontally uniform Stokes drift)

(Lagrangian) geostrophic

$$Ro\left[v_{i,t} + v_j^L v_{i,j}\right] + \frac{M_{Ro}}{Ri} w v_{i,z} + \boxed{\epsilon_{izj} v_j^L = -M_{Ro}\pi_{,i}} + \frac{Ro}{Re} v_{i,jj}$$

$$\frac{\alpha^2}{Ri}\left[w_{,t} + v_j^L w_{,j} + \frac{M_{Ro}}{RoRi} w w_{,z}\right] = \boxed{-\pi_{,z} + b} - \varepsilon v_j^L v^s_{j,z} + \frac{\alpha^2}{ReRi} w_{,jj}$$

hydrostatic

$$b_t + v_j^L b_{,j} + \frac{M_{Ro}}{RoRi} w b_z = \frac{1}{Pe} b_{,jj}$$

$$v_{j,j} + \frac{M_{Ro}}{RoRi} w_z = 0$$

Plus boundary condition

$$M_{Ro} \equiv \max(1, Ro)$$

$$Re = \frac{UL}{\nu} \quad Ro = \frac{U}{fL} \quad Ri = \frac{N^2}{(U_{,z})^2}$$

$$\alpha = H/L \quad \varepsilon = \frac{V^s H}{f L H_s}$$

J. C. McWilliams and BFK. Oceanic wave-balanced surface fronts and filaments. Journal of Fluid Mechanics, 730:464-490, 2013.

N. Suzuki and BFK. Understanding Stokes Forces in the Wave-Averaged Equations, JGR, in prep, 2015.