A Mesoscale Eddy Parameterization Challenge Suite: Eady-like Model Results

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Introduction

Computational power limits the resolution of ocean models. Many results indicate that the oceanic mesoscale eddy field is important to the structure and sensitivity of the large-scale ocean, but resolving the eddy field is not routinely possible in oceanic general circulation models (OGCMs). Extrapolating current trends in computation predicts that eddy parameterizations will be needed for some decades into the future. As such, optimizing eddy parameterizations with high accuracy is desirable.

Ideally, eddy parameterizations would be validated directly against observations, but the sheer number of observations required makes this rare or unfeasible. The approach using numerical models would be to directly resolve mesoscale eddies at fine resolution, and to validate eddy parameterizations against what is observed from these models. This poster presents results from the first set of simulations in a test bed of such models, which will be referred to as an "eddy parameterization challenge suite". A full description of this research can be found in Bachman and Fox-Kemper (2012).

Background

It is standard practice to parameterize the effects of subgridscale eddies by including extra terms in the equations of motion. Consider the transport equation for a passive tracer, under Reynolds-averaging:

$$\mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'$$

$$\frac{\partial \bar{\tau}}{\partial t} + \bar{\mathbf{u}} \cdot \nabla \bar{\tau} + \overline{\mathbf{u}'\tau'} = 0$$

Present subgridscale parameterizations are typically represented as rules to approximate processes at fine resolution in terms of coarse resolution quantities. A flux-gradient relationship is often assumed to be of the form

$$\overline{\mathbf{u}'b'} = -\mathbf{R}\nabla\bar{b},$$

$$\overline{\mathbf{u}'\tau'} = -\mathbf{R}\nabla\bar{\tau},$$

which relates the subgridscale eddy fluxes with the resolved tracer gradients. This proportionality is governed by the eddy stirring tensor $\mathbf{R}$. In general, $\mathbf{R}$ is a 3×3 tensor, but in this problem it will be 2×2. Griffies (1998) demonstrates that the stirring tensor in the flux-gradient relationship can be interpreted as contributing both an advective and a diffusive component. This decomposition is uniquely equivalent to subdividing $\mathbf{R}$ into antisymmetric and symmetric parts, respectively,

$$R_{ji} = S_{ji} + A_{ji}$$

$$S_{ji} = \frac{R_{ji} + R_{ij}}{2}$$

$$A_{ji} = \frac{R_{ji} - R_{ij}}{2}$$

$$S_{ji}v_{ik} = v_{jn}\lambda_{nk}$$

$$A_{ji} = \psi,\ j \neq i$$

$$A_{ji} = 0,\ j = i$$

Fundamentally, we are interested in the magnitude of the elements of $\mathbf{R}$. By extension, this knowledge also will tell us about the structure of the tensors $\mathbf{S}$ and $\mathbf{A}$, the eigenvalues λ_1 and λ_2 of $\mathbf{S}$, and the overturning streamfunction ψ.

Solving for $\mathbf{R}$ with the Tracer Inversion Method

Often, models are diagnosed or theory is formulated where a special role has been given to a single tracer, usually potential temperature or buoyancy. A fundamental limitation of this approach is that for eddy transport in n dimensions, using one tracer in the flux-gradient relationship provides only n constraints on n^2 elements of the stirring tensor. It is thus appropriate to overdetermine $\mathbf{R}$ in the least-squares sense by initializing extra tracers. The flux-gradient equation for an overdetermined system is

$$-\overline{u_j'\tau_\pi'}\,[\bar{\tau}_{\pi,i}]^{-1} = R_{ji}$$

where in three spatial dimensions i and j run from 1 to 3 and π runs from 1 to the number of tracers used. We can thus invert to solve for $\mathbf{R}$ by initializing multiple passive tracers in a model.

Model Setup

The Massachusetts Institute of Technology general circulation model is used to simulate a zonally reentrant channel with a temperature front oriented in the along-channel direction. The background stratification is set to be linear in *z*, akin to the Eady (1949) model. Stratification, rotation, front width L_f, and velocity are set according to the desired nondimensional parameters Ro and Ri. The frontal spindown is meant to simulate the growth of baroclinic eddies, which dominate the mesoscale.

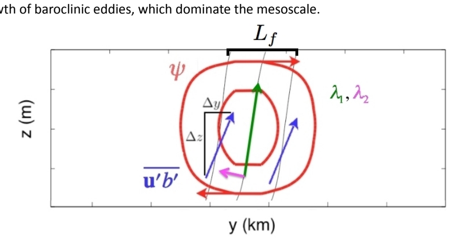

Figure 1. Schematic of the overturning of the temperature front.

Six passive tracers are initialized sinusoidally in the *y* and *z* directions. The validity of the tensor inversion method is verified by comparing the diagnosed buoyancy fluxes with a set of "reproduced" fluxes derived by multiplying $\mathbf{R}$ by the buoyancy gradient.

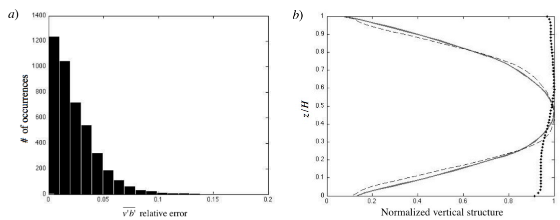

Figure 2. *a)* The relative error of the horizontal buoyancy flux reconstruction, $\frac{|\overline{v'b'} - \mathbf{R}\nabla b|}{|\overline{v'b'}|}$, averages less than 3%. The relative error for the vertical flux reconstruction averages less than 5% (not shown). *b)* Vertical structures of ψ (solid line), λ_1 (dotted line), and λ_2 (dashed line). The scaling theory developed below assumes that the vertical structures are multiplicatively separable from the dimensional scalings themselves.

Scaling by Fluid Parcel Exchange

The high accuracy of the buoyancy flux reconstruction suggests that a scaling for the inverted tensor components can follow the dimensional scaling of the stirring tensor for buoyancy. Consider an exchange of fluid parcels over a decorrelation distance in time , so that

$$\overline{w'b'} \propto \frac{-\Delta z(\Delta y M^2 + \Delta z N^2)}{\Delta t} \qquad \overline{v'b'} \propto \frac{-\Delta y(\Delta y M^2 + \Delta z N^2)}{\Delta t}$$

If we assume that time scales advectively and the Stokes Drift is small, the Eulerian RMS eddy velocities $\sqrt{v'^2}$ and $\sqrt{w'^2}$ can be used to approximate $\Delta y/\Delta t$ and $\Delta z/\Delta t$.

If we take the horizontal parcel excursion to be approximately the time-evolving front width N^2H/M^2, then dimensional scalings emerge for the buoyancy fluxes and tensor elements:

$$\overline{v'b'} \propto \frac{N^2H}{M^2}\left(\sqrt{\overline{v'^2}}M^2 + \sqrt{\overline{w'^2}}N^2\right)$$

$$\overline{w'b'} \propto \frac{N^2H}{M^2}\frac{\sqrt{\overline{w'^2}}}{\sqrt{\overline{v'^2}}}\left(\sqrt{\overline{v'^2}}M^2 + \sqrt{\overline{w'^2}}N^2\right)$$

$$R_{yy} \propto \frac{N^2H}{M^2}\frac{1}{M^2}\left(\sqrt{\overline{v'^2}}M^2 + \sqrt{\overline{w'^2}}N^2\right)$$

$$R_{zy} \propto \frac{N^2H}{M^2}\frac{1}{M^2}\frac{\sqrt{\overline{w'^2}}}{\sqrt{\overline{v'^2}}}\left(\sqrt{\overline{v'^2}}M^2 + \sqrt{\overline{w'^2}}N^2\right)$$

$$R_{zz} \propto \frac{N^2H}{M^2}\frac{1}{N^2}\frac{\sqrt{\overline{w'^2}}}{\sqrt{\overline{v'^2}}}\left(\sqrt{\overline{v'^2}}M^2 + \sqrt{\overline{w'^2}}N^2\right)$$

These scalings do not rule out dependencies on nondimensional parameters Ro and Ri, or on nondimensional constants. We investigate the possibility of such dependencies using the model results.

Model Results and Scaling Comparison

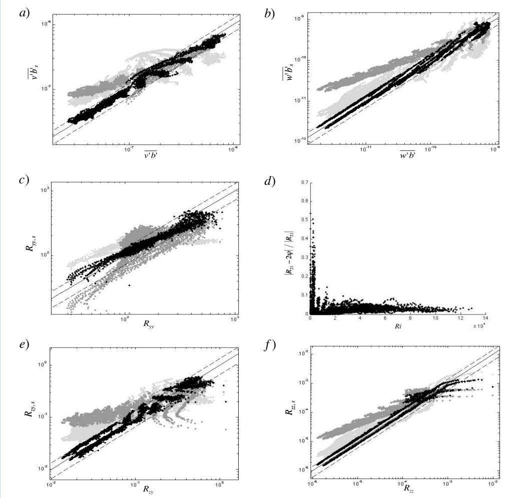

Figure 3. (a-c, e-f) Comparison between true fluxes and diffusivities, and their respective scalings (subscript). Black dots indicate model data, dark grey indicates scalings without term, and light grey indicates the scaling method of Fox-Kemper et al. (2008). d) A special scaling applies to R_{yz}, which Grifffies (1998) notes is often set equal to 2ψ in OGCMs. In this figure the relative error between R_{yz} and 2ψ averages less than 5%, confirming this idea for cases with Eady-like stratification.

From the model results, the scalings that best approximate the true fluxes are:

a) $\overline{v'b'}_s = (0.25 \pm 0.06)\ Ri^{-0.25\pm0.06}\ \frac{N^2H}{M^2}\left(\sqrt{v'^2}M^2 + \sqrt{w'^2}N^2\right)$

b) $\overline{w'b'}_s = (0.54 \pm 0.17)\ Ri^{-0.48\pm0.15}\ \frac{N^2H}{M^2}\frac{\sqrt{w'^2}}{\sqrt{v'^2}}\left(\sqrt{v'^2}M^2 + \sqrt{w'^2}N^2\right)$

c) $R_{yy,s} = (0.24 \pm 0.07)\ Ri^{-0.25\pm0.08}\ \frac{N^2H}{M^2}\frac{1}{M^2}\left(\sqrt{v'^2}M^2 + \sqrt{w'^2}N^2\right)$

d) $R_{zy,s} = (0.59 \pm 0.20)\ Ri^{-0.45\pm0.14}\ \frac{N^2H}{M^2}\frac{1}{M^2}\frac{\sqrt{w'^2}}{\sqrt{v'^2}}\left(\sqrt{v'^2}M^2 + \sqrt{w'^2}N^2\right)$

e) $R_{zz,s} = (0.23 \pm 0.09)\ Ri^{-0.44\pm0.18}\ \frac{N^2H}{M^2}\frac{1}{N^2}\frac{\sqrt{w'^2}}{\sqrt{v'^2}}\left(\sqrt{v'^2}M^2 + \sqrt{w'^2}N^2\right)$

Conclusion

A new diagnostic method for diagnosing the eddy diffusivity tensor $\mathbf{R}$ has been developed and implemented. In this method, a set of passive tracers is initialized in a series of frontal spindown simulations, whose fluxes and gradients are inverted to obtain $\mathbf{R}$. Scalings have been derived for the advective and diffusive components of $\mathbf{R}$, and the vertical structure of each of these parts has been found. The GM/Redi framework (Griffies 1998) for eddy fluxes is validated, with only one spatiotemporally varying coefficient/diffusivity given by R_{yy}, whose scaling in this problem is given in c) above.

The models used in this project are part of a bigger test bed meant to calibrate and refine mesoscale eddy parameterizations. The scalings found using such a test bed will act as "rules" for each flow regime that any new parameterization must obey. This work will provide an improvement on extant mesoscale eddy parameterizations in the form of enhanced accuracy of tracer fluxes at mesoscale resolution, and will render OGCMs more sensitive to the buoyancy and velocity profiles at each level.

References

Bachman, S. and Fox-Kemper, B., 2012. Eddy Parameterization Challenge Suite. I: Eady Spindown. *Ocean Modelling*, in preparation.

Eady, E. T., 1949. Long waves and cyclone waves. *Tellus* **1**, 33–52.

Fox-Kemper, B., Ferrari, R., Hallberg, R., 2008. Parameterization of mixed layer eddies. Part I: Theory and diagnosis. *Journal of Physical Oceanography* **38** (6), 1145–1165.

Griffies, S. M., 1998. The Gent-McWilliams skew flux. *Journal of Physical Oceanography* **28** (5), 831–841.