Global Stokes Drift and Climate Wave Modeling

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Conclusions

Hierarchy of Stokes Drift Approximations:

1. *2D spectral data known:* Use first-order 2D Stokes drift

   $\downarrow$ Random Error $\sim$ 10%

2. *1D spectral data known:* Use 1D wave spread approximation

   - 1D Unidirectional approximation is not advised since it systematically overestimates the 2D Stokes drift by approximately 33%

3. *Third-spectral-moment known:* Same as 1D wave spread at the surface

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4. *Third-spectral-moment unknown:* Use the second moment to empirically approximate the third moment

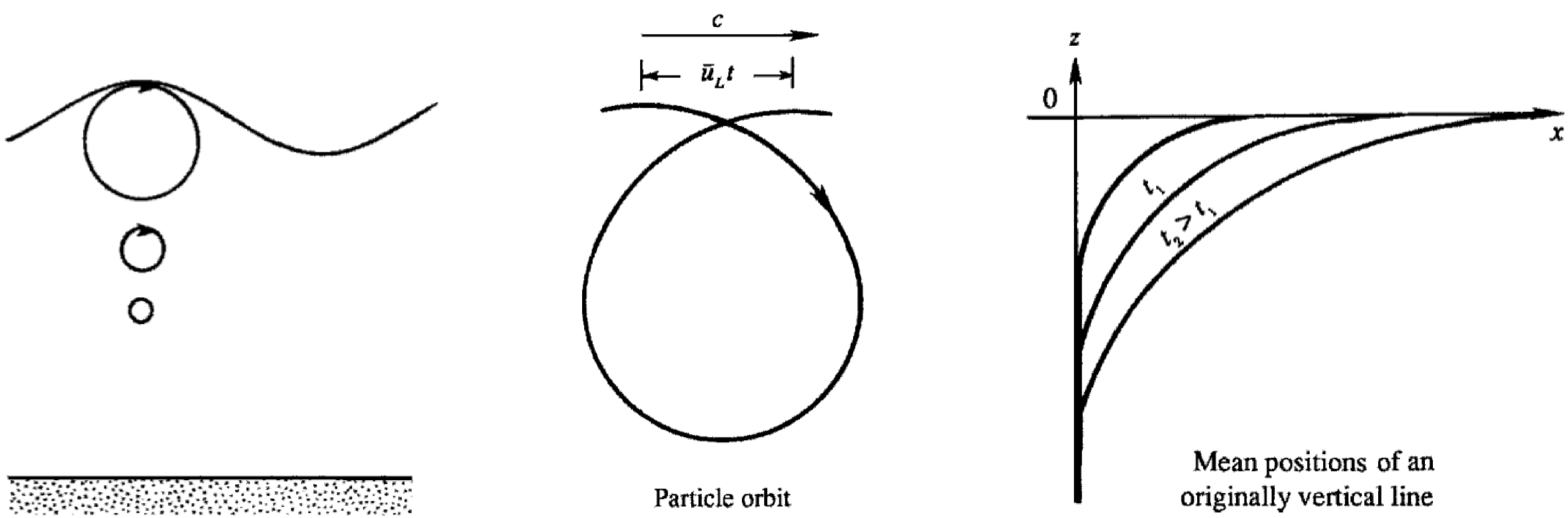
Climate Wave Model:

1. Unstructured node approach removes advective and directional singularities
2. Prototype model shows promise in great circle test case

Introduction: Stokes Drift Velocity

Stokes drift = mean(Lagrangian fluid velocity − Eularian current)

- Appears often in wave-averaged dynamics like Langmuir mixing
- Accuracy and data coverage remain challenges in global estimates
- Use of atmospheric data alone can be untrustworthy



(a) linear orbit (b) nonlinear orbit (c) $1D_h$ mean drift

Figure: 2D particle trajectories governed by the (a) linear and (b) nonlinear small-amplitude wave equations, and (c) the latter nonlinear mean drift over time (Kundu and Cohen, 2008)

Motivation: Importance for Climate Research

There is a persistent, shallow mixed layer bias in the Southern Ocean in global climate models (GCM): Langmuir mixing missing???

- Stokes drift plays a dominant role in determining the strength of Langmuir mixing
  - $1/La_t^2 \sim u^s(z=0)/u^*$

- Langmuir mixing is not currently in any GCM [2/2012]

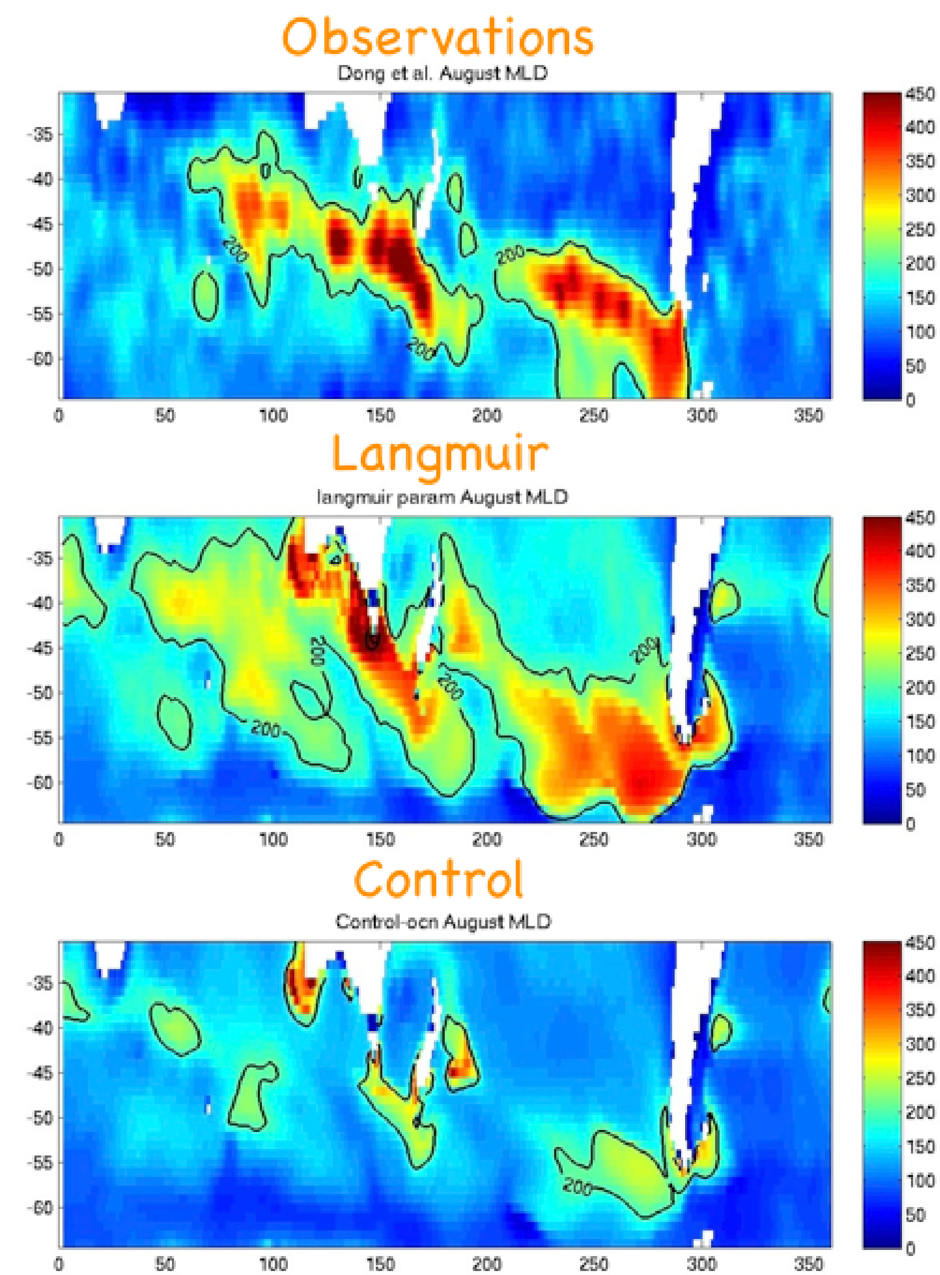


Figure: Mixed layer depth bias is reduced in CCSM 3.5 model runs

Lower First-order Stokes Drift Approximations

Overview:

- Survey and error analysis of lower first-order Stokes drift approximations (spectral moments)
- Comparison of surface Stokes drift estimates using different data products (e.g., satellites, buoys, models) = *Factor of 50% difference!*

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Wave spectral moments and Stokes drift estimation

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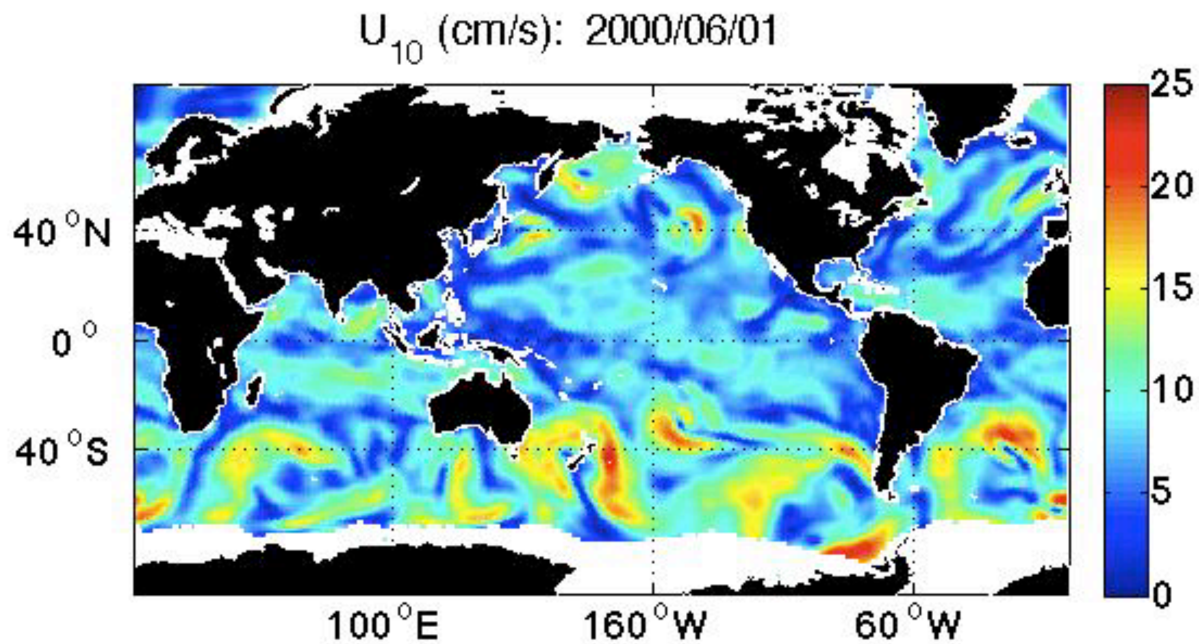
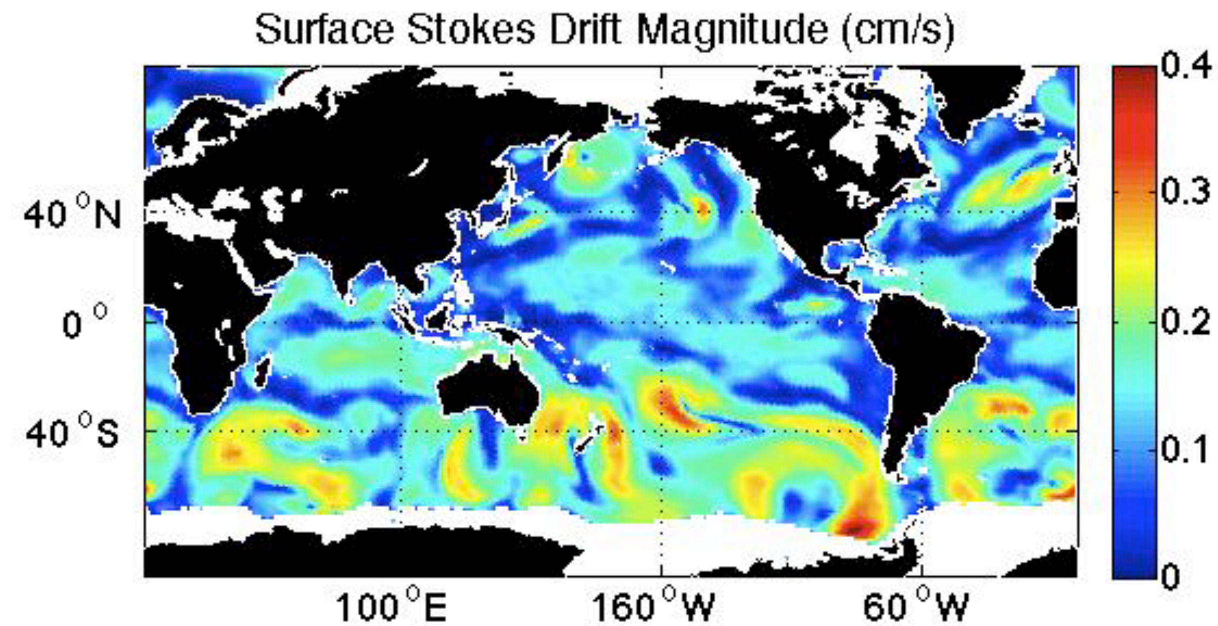
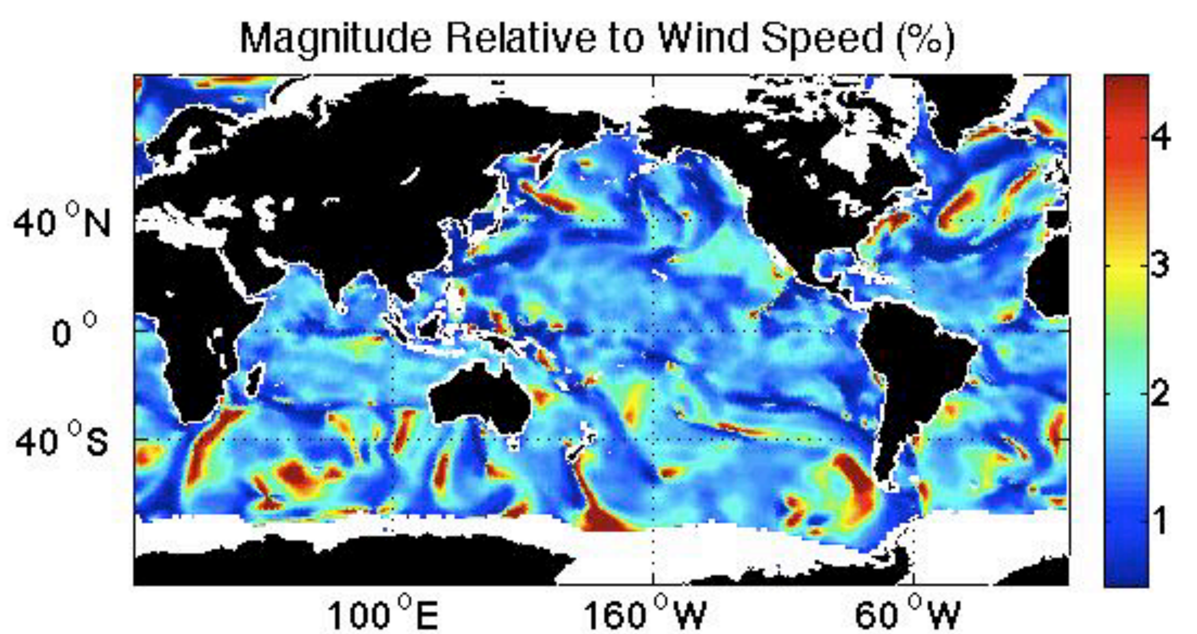
ABSTRACT

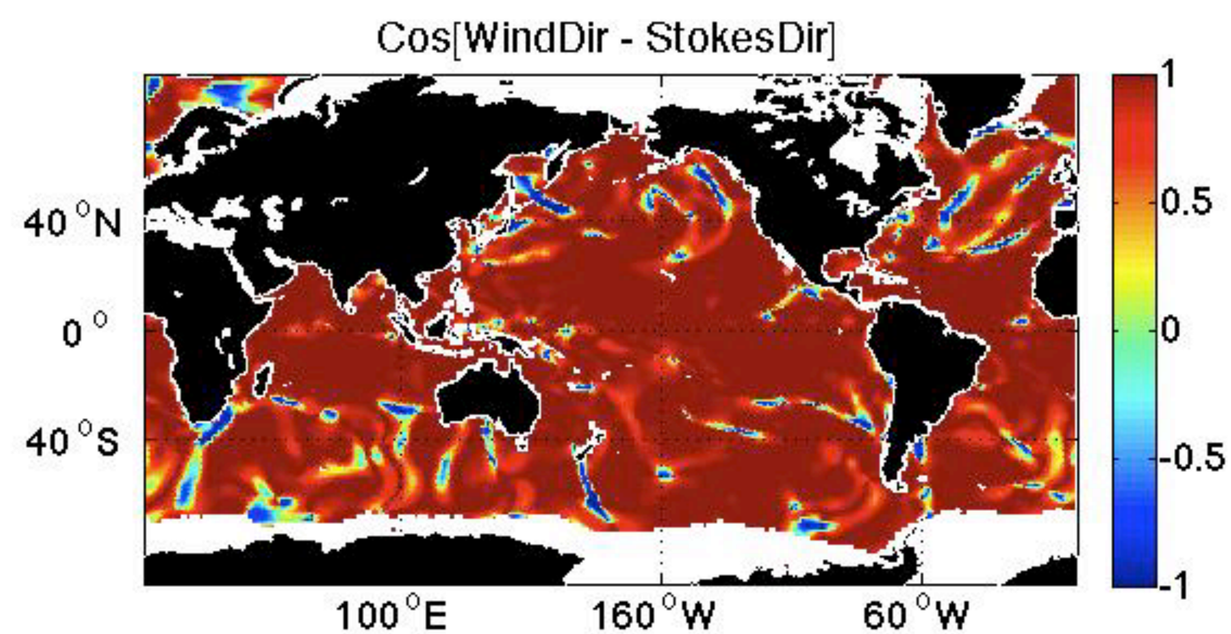
The relationships between the moments of wave spectra and Stokes drift velocity are calculated for empirical spectral shapes and a third-generation wave model. From an assumed spectral shape and only an estimate of wave period and significant wave height, one may determine: the leading-order Stokes

Global Wave Variable Examples





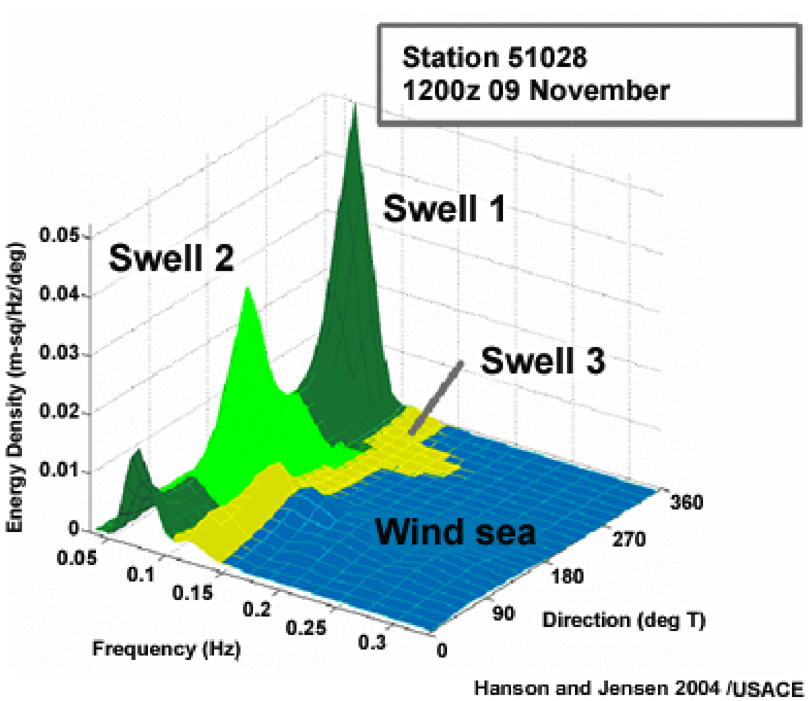


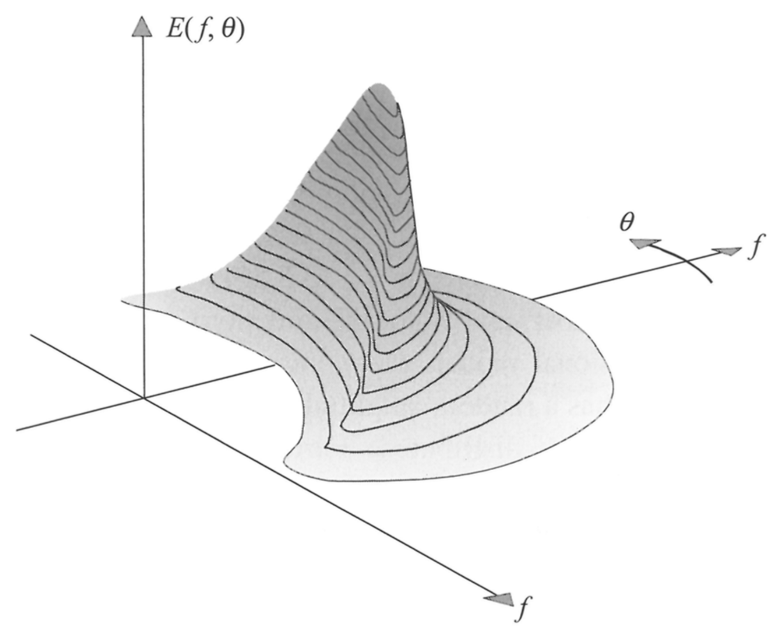


Stokes Drift and Wave Spectra Examples

The first-order Stokes drift magnitude depends both on the directional components of the wave field and the directional spread of wave energy

$$\mathbf{u}^S \approx \frac{16\pi^3}{g} \int_0^{\infty}\int_{-\pi}^{\pi} \Big(\cos\theta, \sin\theta, 0\Big)\, f^3 \mathcal{S}_{f\theta}(f,\theta)\, e^{\frac{8\pi^2 f^2}{g} z}\, d\theta df \quad (1)$$





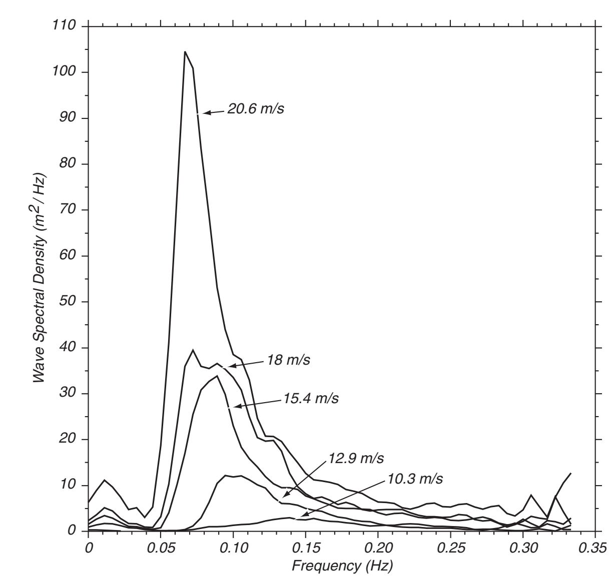


Figure: Examples of wave spectra: (a) 2D spectra generated by WAVEWATCH III, (b) idealized directional spread (Holthuijsen), and (c) 1D Pierson and Moskowitz observational spectra (Stewart)

Stokes Drift and Multidirectional Waves

Example: Consider a bichromatic spectrum with the same amplitude and peak frequency for each monochromatic wave but separated by an angle of incidence θ'.

$$\mathbf{u}^S \approx \frac{16\pi^3}{g} \int_0^{\infty}\int_{-\pi}^{\pi} \Big(\cos\theta, \sin\theta, 0\Big)\, f^3 \mathcal{S}_{f\theta}(f,\theta)\, e^{\frac{8\pi^2 f^2}{g} z}\, d\theta df$$

Then the following relation holds[1]:

$$\begin{aligned} \mathbf{u}^S_{bi} &\neq 2\,\mathbf{u}^S_{mono} \\ &= 2\left(\cos\theta', 0, 0\right)\mathbf{u}^S_{mono} \end{aligned}$$

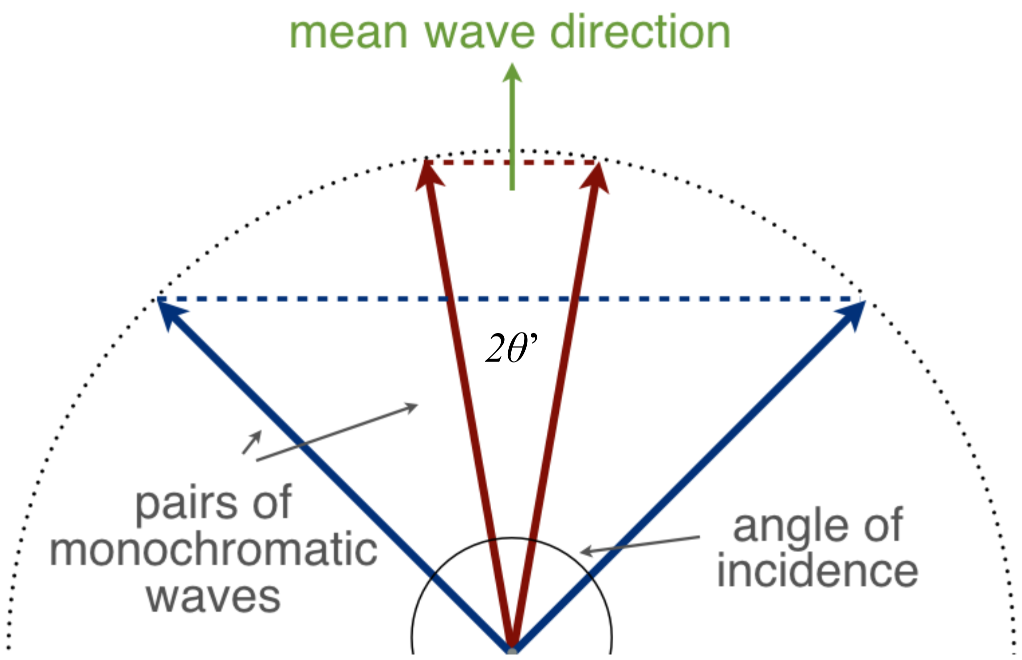


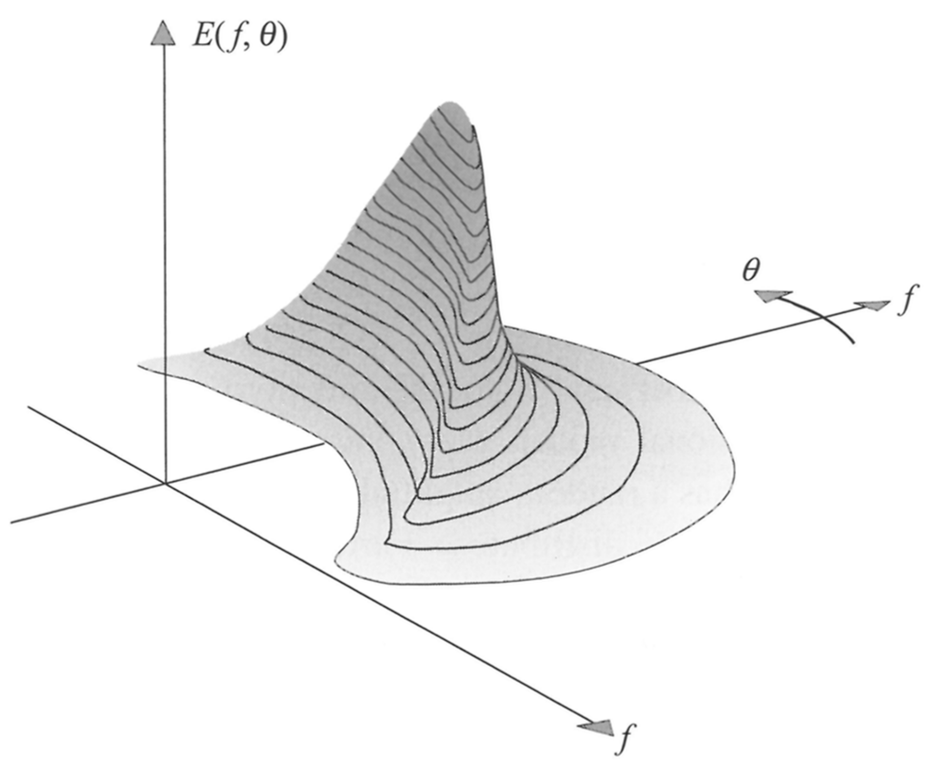
Figure: Example of how the directional components of a wave field affect the magnitude of Stokes drift

[1] The mean wave direction is along the x-axis

Stokes drift error due to wave spreading in 1D approximates can be minimized by first recreating the 2D wave spectrum

Idea: Use an *empirical* *directional distribution* (D_f) to recover the 2D spectrum

$$\int_0^\infty \int_{-\pi}^{\pi} \mathcal{S}_{f\theta}(f,\theta)\, d\theta df \;=\; \int_0^\infty \left[\int_{-\pi}^{\pi} D_f(f,\theta)\, d\theta \right] \mathcal{S}_f(f)\, df \;=\; \int_0^\infty \mathcal{S}_f(f)\, df$$



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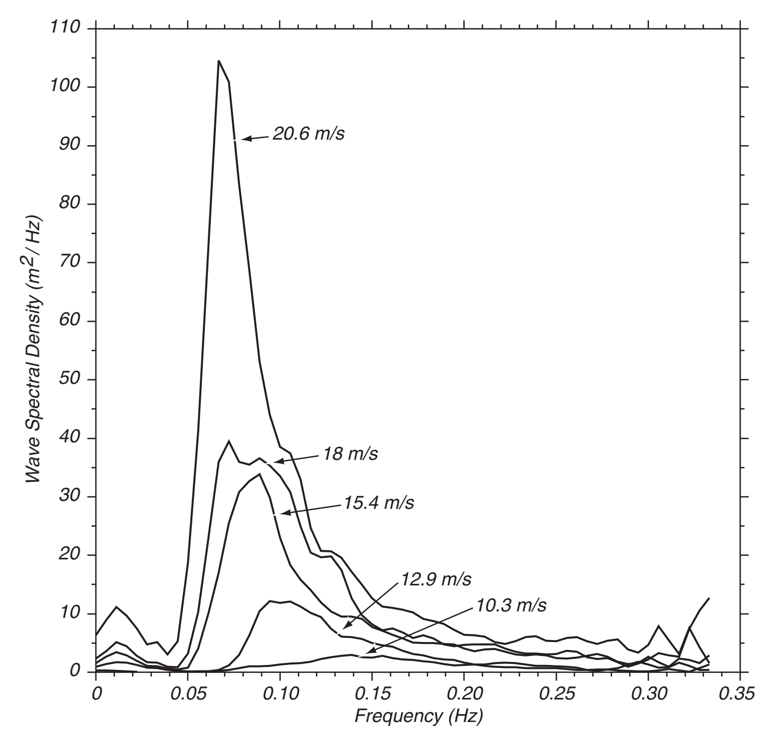
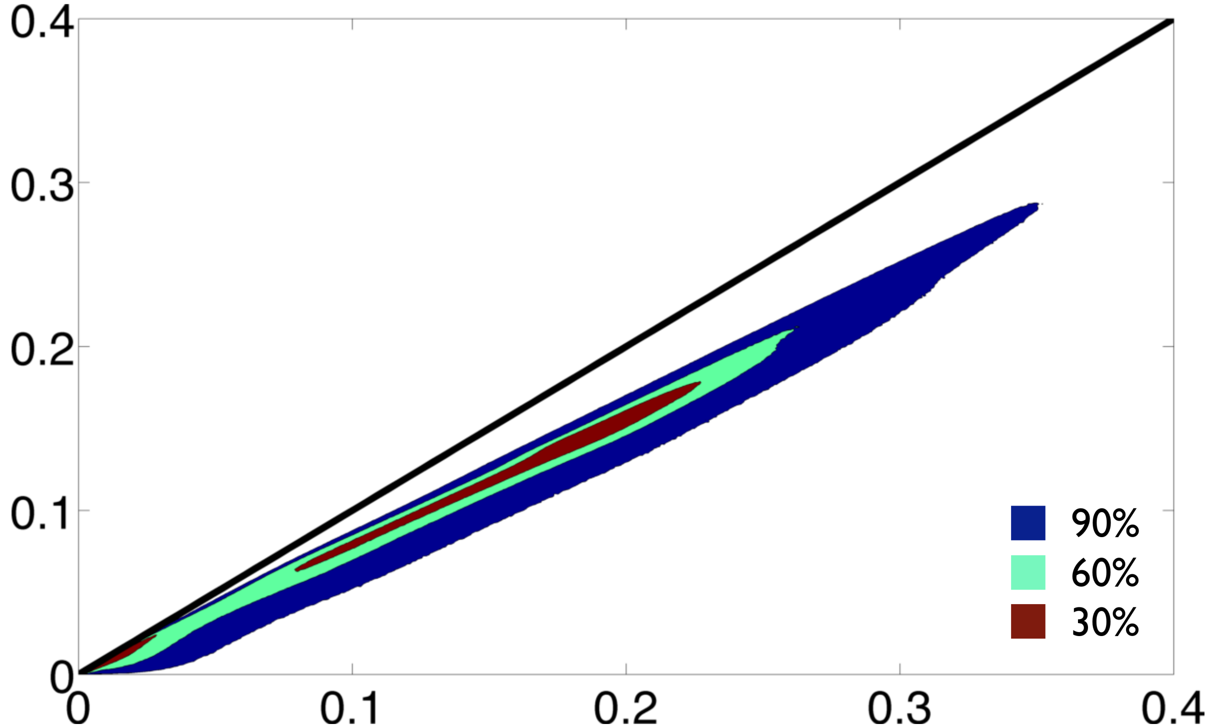
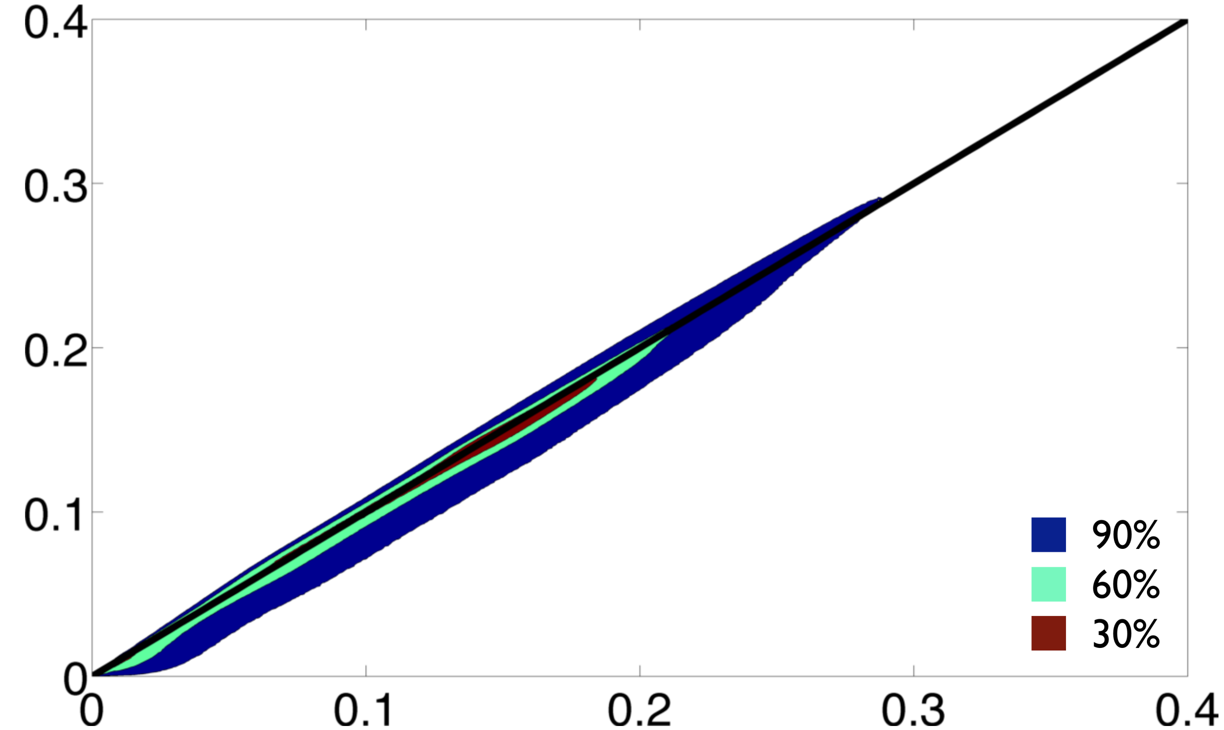


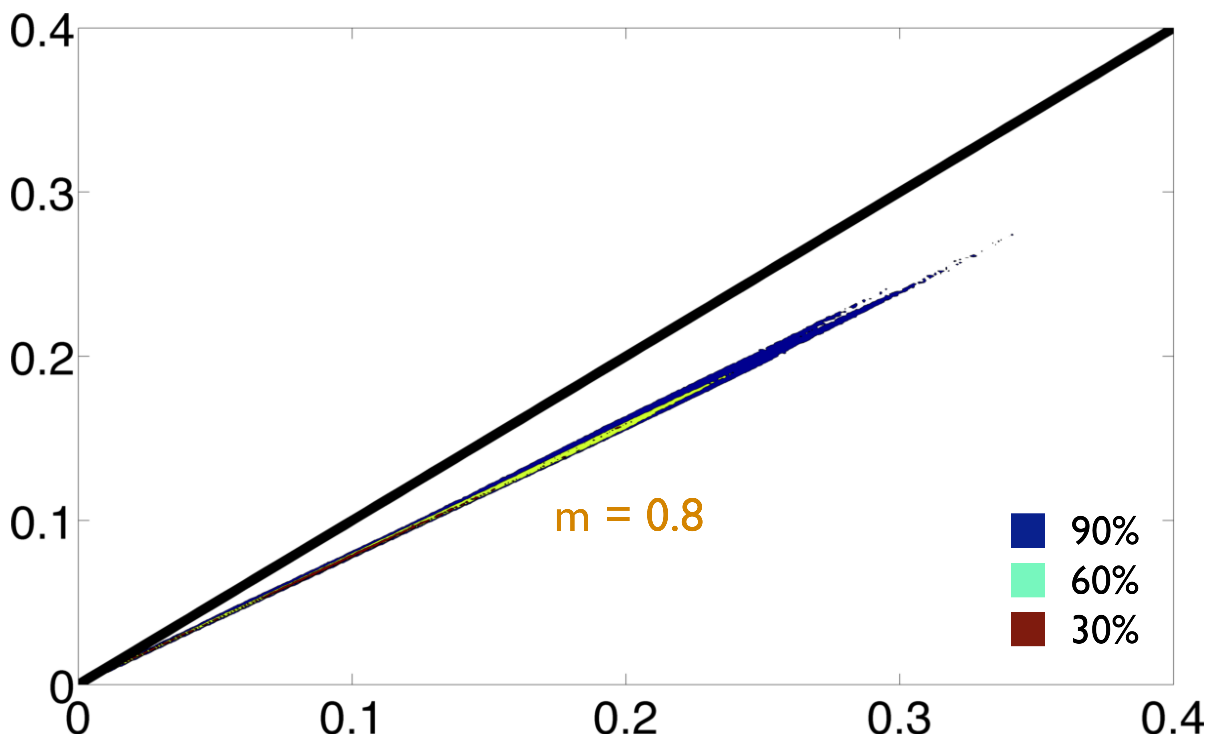
Higher Order: Comparison of 2D and 1D Estimates



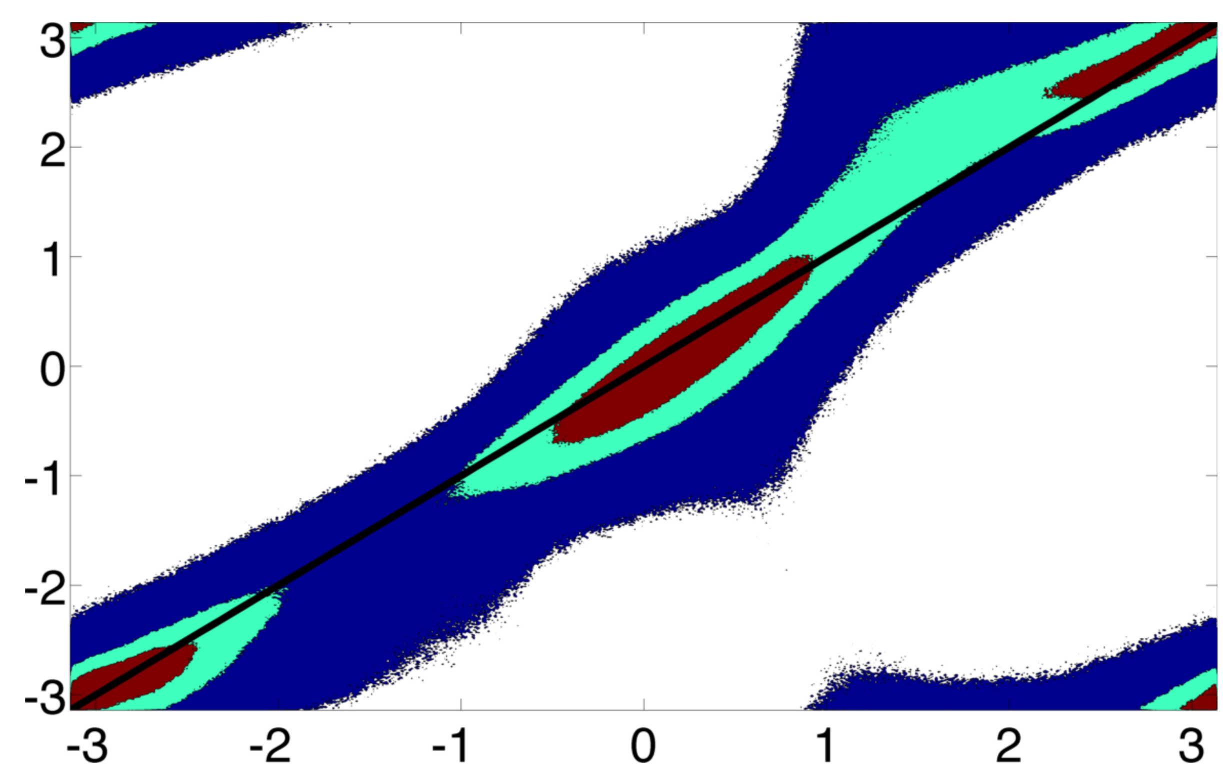
(a) Mag: 1D Unidirectional (x) vs 2D (y) (cm/s)



(b) Mag: 1D Spread (x) vs 2D (y) (cm/s)



(c) Mag: 1D Unidirectional (x) vs 1D Spread (y) (cm/s)



(d) Dir: Mean Wave (x) vs Stokes Drift (y) (rad)

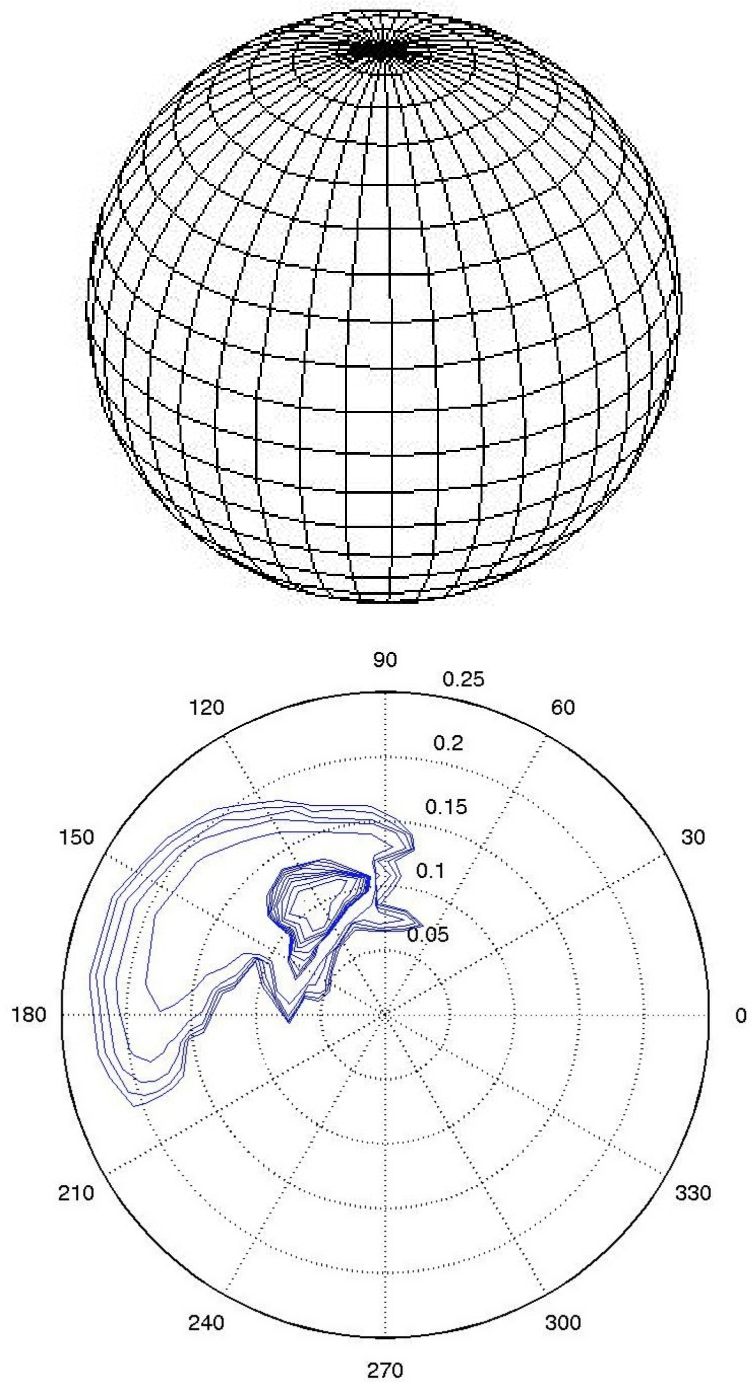


Figure: Spatial and spectral grid examples

Current Model Basics:

- Uses structured grids (lat-lon, polar)
- Includes extensive physics and parameterizations

Current Model Deficiencies:

- Spatial and spectral singularities near the poles
- Performance declines as N/S boundaries are moved higher (presently $\pm 75^\circ$)
- Designed to forecast weather not climate

Lat-lon grids:

G3: 2.4×3
G4: 3.2×4

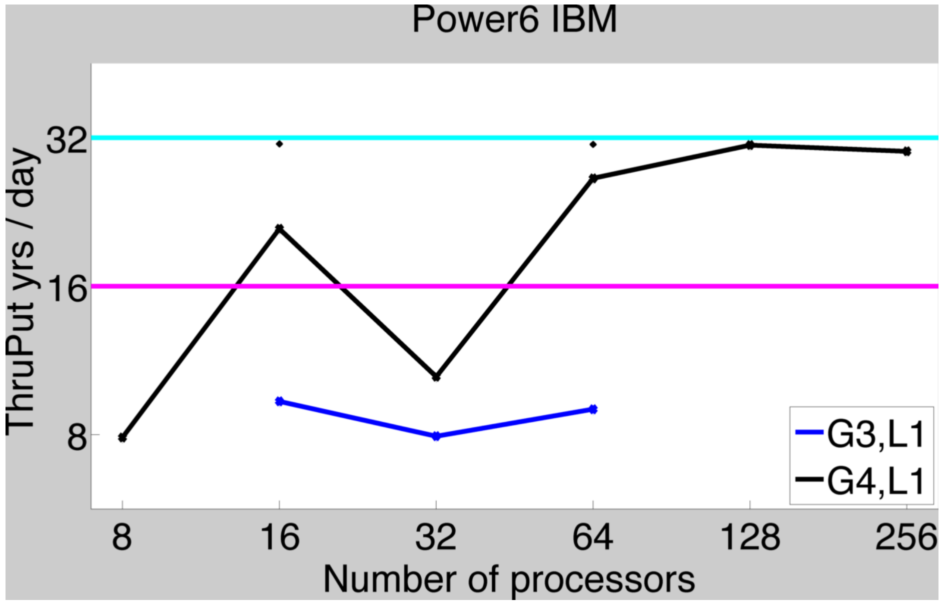


Figure: WAVEWATCH III grid performance with benchmarking targets for coupling to NCAR CESM

Unstructured Approach: RBF-Generated Finite Differences

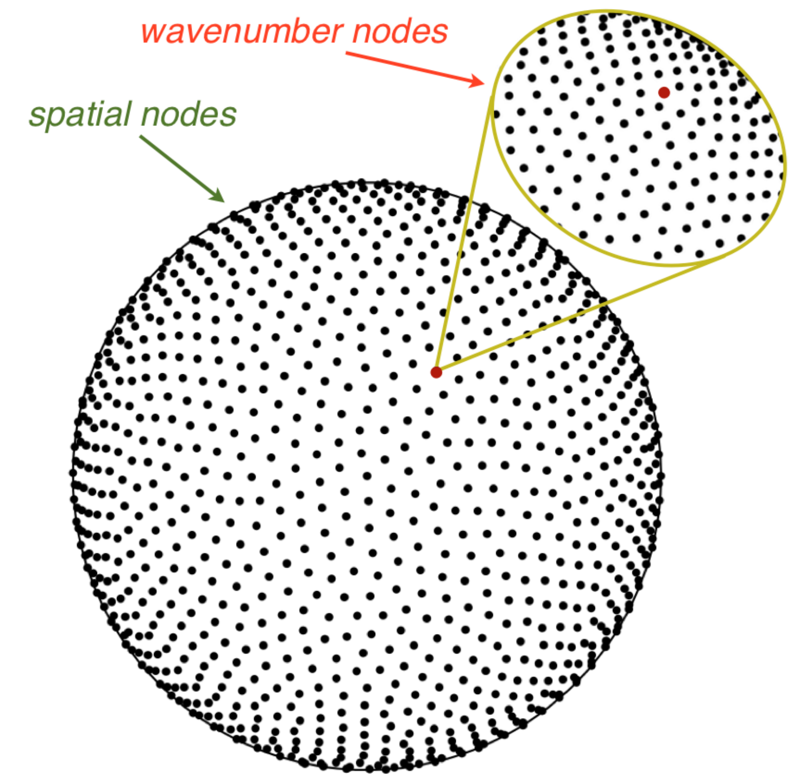


Figure: Possible node layout

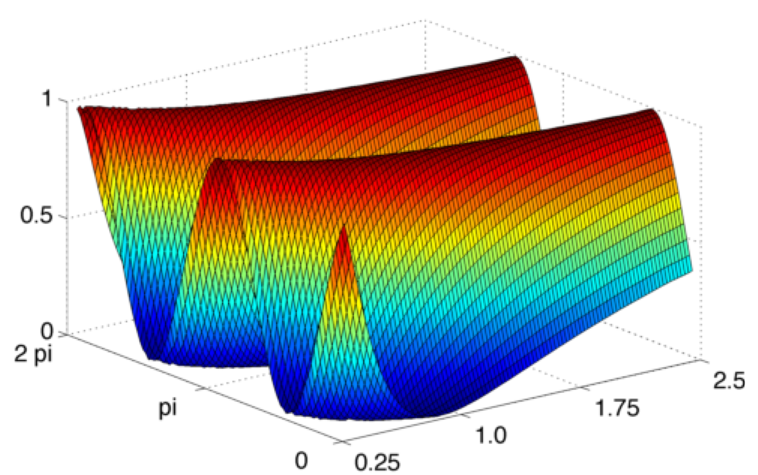


Figure: Great circle propagation

RBF-Generated Finite Difference Method (RBF-FD):

- Solves advective problems with near spectral accuracy
- Uses an unstructured node layout
- Allows geometric flexibility and local node refinement
- No advective and directional singularities
- Computational costs are spread equally
- Possibly well-suited for parallelization

Great Circle Propagation Test Case:

- 20 spatial x 10 spectral nodes
- Dissipation and dispersion error after 0.5 cycles
  - Third-order upwind ~ 0.2
  - Radial Basis Functions $\sim 0.5\text{x}10^{-4}$

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Thank You!

References

Stokes Drift:

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