# Syllabus for EEPS0350: GeoMath
# (a.k.a. Mathematical Methods of Fluid and Solid Geophysics and Climate Physics)

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## 1 Course Description

### 1.1 EEPS0350 Mathematics of Fluid and Solid Geophysics and Geology

Intended for undergraduates concentrating in geological and physical sciences or engineering, especially those interested in the quantitative study of Earth. Problem sets will cover common approaches to quantify the dynamics and chemistry of solids and fluids in nature. Mathematical topics to be introduced include linear algebra, vectors and tensors, differential equations, dynamical systems, eigenvalues and eigenvectors, empirical orthogonal functions, fractals, chaos, and statistics. Applications include waves in the oceans, atmosphere, and solid earth, convective and conductive heat flow, reaction rates, gravitational potential energy, Newton's laws on a rotating planet, measuring coastlines and ranges, and dating errors in stratigraphy. Prerequisites: MATH 0100, 0170, 0190, 0180, 0350 or 0200.

## 2 Contacts

The professor for this class is Baylor Fox-Kemper.
baylor@brown.edu, 401-863-3979, Office: Lincoln Field room 214
`http://fox-kemper.com/teaching`, `http://fox-kemper.com/geomath`

Portions of the website are password-protected to ensure that fair use and copyrights are correctly obeyed as I share images from books, etc. You can access these by using:

```
username: io
password: ocean
```

## 3 Getting Help!

We are usually available by email. Baylor's office hours will be Monday, Tuesday, Wednesday, and Thursday 8-9 and 5-6 by appointment (see my schedule at `http://fox-kemper.com/contact`). You can go to the webpage and pick a time to meet in my office (Lincoln Field 214) or by zoom (`https://brown.zoom.us/my/baylorfk`). Be sure to make a booking in advance so I know to look for you!

## 4 Meetings and Places

We will meet Monday, Wednesday, and Fridays from 9:00 to 9:50AM in Lincoln Field 117. You will also sign up for a lab/practicum/practice session with the UTA when their schedule is set.

## 5 Website and Canvas

The primary resource for this class is the webpage: `http://fox-kemper.com/0350`. The class webpage is where all of your assignments will be announced, questions posted, links to additional reading, lecture notes, etc. Assignments should be turned in as pdfs using canvas. Solution sets will be posted in canvas for your use in peer reviewing. The copiers in GeoChem and Lincoln Field can be used to scan handwritten assignments (for free).

You will want to familiarize yourself with Wolfram Alpha (`http://www.wolframalpha.com`), it is a great resource for looking up math definitions. Wikipedia is also handy in a pinch (due to the armies of math & physics grad students who apparently have so very few social commitments that they punch in all the details of their dissertation appendices). AI such as Claude or ChatGPT will give you answers, but be extremely skeptical of them–they are often wrong! Further, we will discuss in class ethical and unethical uses of AI.

## 6 Use of Artificial Intelligence

This course is mostly concerned with developing your human intelligence, but AI is in the world, and there are a wide range of classroom policies across campus. In some courses, it is absolutely forbidden, in others it is allowed in specific ways, in others you might even be learning to code your own AI!

In this class, we are learning about valuing and sharing each other's ideas and honing our mathematical prowess overall, not just when you have a computer handy. Using AI instead of working or discussing with a classmate is likely to be much less stimulating. Using AI instead of doing your own reading is surely wasting your opportunity to learn what *your* ideas are. However, this is a course about learning and exploring the mathematical and geophysical world, and learning where AI can be most useful in your mathematical and geophysical learning is part of our modern world. So, we will discuss the uses and misuses of AI, and how to acknowledge the use of AI in your class work.

All work that you submit during the course must be primarily your own original work and represent your own thoughts and ideas; for this reason **all assignments must be handwritten**. It is especially true that you must absorb the concepts even if AI is used in some way. It is your responsibility and required for you to document the *use and/or prompts* that you chose as examples of how its use still reflects your own thinking. Therefore, the use of AI-powered tools (such as ChatGPT, Claude, or GitHub's CoPilot code-completion tool) to complete significant portions of course assignments is discouraged, although its use in limited ways is not.

Unattributed use of AI-powered tools (i.e., without proper citation or other disclaimers) will be considered academic misconduct. Brown's Academic Code for both undergraduate and graduate students states that:

> A student's name on any exercise (e.g., a theme, report, notebook, performance, computer program, course paper, quiz, or examination) is regarded as assurance that the exercise is the result of the student's own thoughts and study, stated in his or her own words, and produced without assistance, except as quotation marks, references, and footnotes acknowledge the use of printed sources or other outside help.

If you choose to use these tools for course assignments, *you must acknowledge and thoroughly document* your use of the tool. A note on the assignment, near your signature or within the document where the tool was used must:

1. cite the tool used,
2. include an explanation of how the tool was used for the assignment, and
3. fully document the student's own contribution as input to the tool vs. the contribution of the tool.

All assignments will be handwritten and graded based primarily your original ideas – you risk losing credit if documentation provided is insufficient to determine your original contributions. You risk more severe penalties if you are deceptive about your uses of technology.

### 6.1 Computation vs. Calculation

We will not be doing much on computers, except some basic statistics in the final section. So... We won't be doing numerical solutions of ODEs/PDEs (take EEPS0250 for that), we won't be doing machine learning (take EEPS1340 for that), we won't be doing climate modeling (take EEPS1400 and EEPS1700 for that), and we won't do much research data analysis (take EEPS1690 for that).

HOWEVER, we will be learning about concepts and basics that apply to these more computational classes. The mathematics of numerical methods, data science, machine learning, numerical weather prediction, climate modeling, and statistical modeling build on precisely the topics covered in this class. We will be doing calculations by hand, but you won't need a calculator–the numbers and digits will stay small most of the time!

## 7 Required Course Activities and Structure of Classtime

The regular class time will be presentation of new materials and discussion. This format requires buy-in from you, the student, however. You *must do the reading of the notes before class*, view any prerecorded lecture material, and preferably also *look ahead at coming homework problems* before class. If an individual student fails to do this, it will negatively influence their ability to follow, and if the class fails to do this I will have to expand the lecture mode–decreasing the problem-solving mode–which is not good for learning. In addition, you will visit the lab/practicum/practice sessions once a week (you sign up for one day with the UTA), where you can work on homework problems in small groups, with input from the TA. The TA will be scheduling students to come to discuss your understanding and help with homework problems during this time. You should provide feedback to the TA or the professors about these interactions, so the UTA can receive credit as due or so that changes can be made as needed.

Magdalene Lampert, a researcher in math education, has shown that learning and retention in mathematical methods is improved by inverting the common classroom presentation order. Lecture, followed by discussion, followed by individual homework is not as effective as individual effort, group effort, full discussion. We will use the discussions and the practicum sessions to adhere to the latter format as best as possible.

Your individual effort begins with reading the notes and skimming the chapter before lectures. Then you will be challenged with questions throughout the class and practicum. In the practicum sessions, you will work individually for a few minutes, and then discuss in a group for a few minutes, and ask the leader for guidance or clarification. Finally, the whole class will discuss approaches to problems and the correct solutions. You will then review these problems again as you review the chapter reading and finalize your (related) homework problems, and study for exams.

### 7.1 Assignments, Exams, Grading, and Expected Time for Activities

- Scheduled class meetings, which will be suspended in the Reading Period (3 hours/week; 38 hours) and practicum meetings, which will continue in the Reading Period (1 hours/week; 13 hours) [Grading: 10% Attendance and participation.]
- Reading and reviewing class work (2 hours/week; 26 hours)
- Weekly assignments (6 hours/week for 12 weeks; 72 hours) [Grading: 50% Weekly homework]
- Weekly peer reviews (1 hours/week for 12 weeks; 12 hours) [Grading: 10% Reviews of other students' homework assignments.]
- Preparation for Midterm and Final (16 hours) [Grading: 20% Final, 10% Midterm]
- Final Exam (3 hours)
- Total: 180 hours [Grading: 100%]

What can I do to get a good grade? Turn in all of the assignments on time. For the format of the course to work, ON TIME matters, so that we can get to the reviewing. If your assignment is late without receiving permission in advance, you may receive no credit for it. Also, *BONUS POINTS* are available on homework and exams for spotting typos in the notes, homework assignments, and exam problems. The more promptly you point them out (by email), and the more important they are, the more points you get!

The scheduling of the assignments are listed on the webpage, and other than the exceptional weeks around holidays will be as follows.

- Weekly assignment due by class time on Monday, but if there are questions a short time for corrections is allowed.
- Solution sets distributed at the end of the day on Monday (assignments not accepted once it is distributed).
- Peer reviewing and grading due by Wednesday midnight.
- *Iterum usque ad finem*

All of this will be charted out on the calendar on the website and in canvas.

#### 7.1.1 Peer review

In addition to doing the problem sets, you will each be performing reviews of each others work. We will be using a rubric based on the AGU guidelines for review. A-F for presentation quality and 1-5 for science/math. Such a guide is useful to go by, and when you do reviews of your fellow students, I'll expect to get a A1 or B2 or B1 score, etc. An A1 will count for 100%, and presentation and accuracy will be equally weighted (an F5 will be 20%, a missed assignment or one turned in after the solution set is distributed will be 0%). There are a few lessons to be learned here, that will help you write your own papers and will help you provide effective and useful reviews in your career.

- Learning to spot unfounded claims
- Learning how to properly support claims

- Learning to distinguish poor writing/presentation from poor thinking
- Learning to label equations, graphs, and numerical information understandably
- Revisiting problems from a different perspective

You will have each of your homework assignments peer-reviewed by more than one person, and inconsistent results will be rechecked. The assignments for reviewers will rotate (ensuring fairness in grading by randomization). You should feel free to contact me with any concerns about the process or specific issues.

### 7.2 Calendar

The main webpage for the class `http://fox-kemper.com/0350` will have the calendar with all assignment deadlines, readings, etc. set up by the first class session. There will be weekly problem sets, one midterm, and a final exam.

## 8 Goals

In this class you will:

- Learn how to quantify some of the physical processes of the earth system.
- Get practice solving diverse geophysical and geological problems using new mathematical techniques.
- Learn how mathematical techniques are categorized, so that you may more effectively study mathematics relevant to your geophysical work.
- Gain a broader perspective and more practice by peer reviewing and collaborating.

This class cannot possibly provide a complete understanding of all of the mathematical topics presented, instead the goal is to introduce the most basic ideas and give geophysical and geological examples where the mathematical tools are useful. A key goal is to introduce the mathematical language, so that students can better choose later mathematics classes and look up mathematical concepts on their own (e.g., using Wolfram Alpha).

## 9 Textbooks and Software

We will work from the course notes. There is not a required textbook, although for reference you should familiarize yourself with a copy of something like Arfken and Weber (2005). You might also check out Wilks (2011), and Snieder (2004), which are in the library. Many of these are available electronically through the Brown Library–the links are provided in the bibliography and on the course webpage.

We will solve problems drawn from many geophysics and geology textbooks (LeBlond and Mysak, 1978; Turcotte, 2002; Schubert et al., 2001; Aki and Richards, 2002; Drazin and Reid, 2004; Holton, 2004; Snieder, 2004; McWilliams, 2006; Vallis and Press., 2017; Marshall and Plumb, 2008; Cushman-Roisin and Beckers, 2011; Fowler, 2011; Kaper and Engler, 2013; Bourguignon et al., 2015), but these books are not required for the course. If electronic copies of them are available at Brown, I have added an url to the bibliography here and on the course website. Sufficient background will be provided along with each problem so that no further reading will be required. You may want to use software, which is allowed for homework (although not required and you must still be able to explain your work without the program). I strongly recommend Matlab and Mathematica, but there are lots of others.

### 9.1 Applications

Geophysical and geological applications touched on in this class are:

- Global Energy Balance
  - Ice Ages
  - Energy Balance Models
- Data constrained models and maps
  - Climate Variability Patterns
  - Stochastic versus Deterministic Variability
- Waves and Oscillations
  - Ocean Waves, Tides, and Tsunamis
  - Earthquakes and Seismic Waves
  - Diurnal, Seasonal, and Orbital Variation Cycles
  - Dispersive Wave Kinematics: Phase & Group velocity
- Transport Budgets
  - Diffusion and Advection
  - Heat transfer
  - Tracers in Fluids
  - Rheology
- Boundary Layers
- Chemical Reactions, Rates, and Equilibria
- Mechanics
  - of Solids
  - of Fluids
- Gravity
  - Potential and Conservative Forces
- Stratigraphy
  - Dating and errors
  - Mapping

### 9.2 Math Tools & Critical Concepts

A list of the mathematical topics to be touched on in this class, and associated critical concepts:

- Review of Mathematical Preliminaries (1.5 Weeks)
  - Series and Sequences
  - Real, Imaginary, Complex

- Trigonometry
  - Exponentials and Logarithms
  - Units and Dimensions
  - Derivatives and Integrals

- Linear Algebra (2 Weeks)
  - Vector Spaces
  - Matrices and Linear Equations
  - Bases and Orthogonality; Rank; Null Space and Span
  - Inverse Methods
  - Eigenvalues and Eigenvectors; Singular Value Decomposition/Empirical Orthogonal Functions/Principal Component Analysis

- Multivariate Calculus and Differential Geometry (2 Weeks)
  - Vectors
    - Coordinate Transformations
    - Rotation & Reflection, Angular Momentum, and Vorticity
    - Vector Differentiation and Integration: Div, Grad, Curl; Gauss, Green & Stokes
  - Tensors
    - Cartesian Tensors
    - Inner and Outer Products versus Matrix Multiplication
    - Symmetries: Principle of Tensor Covariance, Tensor Invariants, & Anisotropy
    - Curvilinear Coordinates and Transforms, especially Spherical Coordinates
    - Deriving Calculus Identities from Tensor Symmetries

- Differential Equations
  - Linear Ordinary Differential Equations and Dynamical Systems (2 Weeks)
    - Rate equations
    - First and Second Order Equations
    - Homogeneous and Inhomogeneous Equations
    - Linear and Nonlinear Equations
    - Series Solutions: Perturbation Analysis, Asymptotics, and Linearization
    - Sturm-Liouville Problems: Free Modes of Oscillation, Superposition
    - Time Series and Fourier Analysis
  - Linear Partial Differential Equations and Dynamical Systems (2 Weeks)
    - Boundary and Initial Value Problems
    - Separation of Variables
    - Laplace and Poisson Equations (applications of Elliptic PDEs)
    - Heat Flow and Wave Equations (applications of Elliptic and Hyberbolic PDEs)

Separation of variables in linear wave problems: Cramer's Rule and Oscillation Modes

Decompositions, e.g.: Helmholtz Streamfunction and Potential, Toroidal and Polloidal, Polarization

- Chaos and Nonlinear Dynamics (1 Week)
- Probability and Statistics (1.5 Weeks)

# 10 Policies

## 10.1 Deadlines

Because of the reviewing process, the scheduling of assignments is tight. Thus, I will have to insist that all problem sets be turned in on time. If they are late, they will drop a letter grade. If they are really late (so that they mess up the next step in the reviewing process) they will be counted as missed and can not be made up. If you foresee that there are big problems coming up (medical, family, etc.) let me know *before* an assignment is due and we can figure something out.

## 10.2 Handwriting

You will be turning in handwritten homework assignments, to absorb some of the feel of the equations under your fingertips. You should aspire to write neatly, as I am visually impaired and your peers will be reviewing your work for exposition and clarity.

## 10.3 Collaboration

I encourage you to work together, and I do not mind at all if you have similar problem sets or share figures or computer code. However, when working together or using AI, I want you to list all of your study group (including Claude or ChatGPT) on each homework assignment. That way I can avoid you peer-reviewing your group and check to ensure you are using AI appropriately. You are all required to submit a version of each assignment as first author (that is, one that you wrote yourself in your own handwriting), so don't submit identical versions of a problem. You need to be careful to cite your colleagues or the textbooks, websites, or papers you might be working from if you are quoting or using solutions you found elsewhere.

## 10.4 Miscellany

- Attendance is expected. If you will miss a class, please let me know when and why so I can be sure you'll get any announcements, etc. Prerecorded segments and post-class recordings or lectures and zoom access to the classroom will be provided.
- Clothing and behavior (e.g., cell & laptop use) should be appropriate for a learning environment.
- Discrimination and harassment will not be tolerated.
- Please contact me if you have any disabilities that require accommodation.

# References

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