

Assignment 3 for EEPS 1520: Ocean Circulation and Climate or, Notions for the Motions of the Oceans

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September 30, 2026

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2 Getting Help!

I am usually available by email. You can make an appointment other times. Just check my calendar at <http://fox-kemper.com/contact> and suggest a time that works for you.

The most important commands in matlab are ‘help’ and ‘lookfor’. The first one allows to to get a description of any matlab function, for example, ‘`>> help plot`’ tells you about the function named ‘plot’. The second one allows to to search for keywords within a function description (in case you don’t know or can’t remember the name of the function).

3 Introduction

As you know, our assignments are structured to resemble a scientific journal’s peer review process. So, when in doubt about procedures, try to understand the assignments in that context. I am listing together all of the pieces for the first paper at the same time here, so that you can see the full flow all at once (even though there are different due dates on each piece).

4 Reading

Chapter 10 of *Wunsch* (2015) is a good place to start thinking about this assignment, but sections 3.3 and 3.4 are also good to review.

5 Homework 3

Each of the subsections here is a homework problem that you should complete, make a pdf of the results, and turn in through canvas.brown.edu. Please include screenshots (as .pdf) or script files (as .m) if that is needed.

This homework and paper we will begin working with one particular simulation of the MIT General Circulation Model (MITgcm: *Adcroft et al.*, 2008). This particular simulation is the Estimating the Circulation and Climate of the Ocean (ECCO v4 release 2) simulation (ECCO4r2: *Forget et al.*, 2015). This kind of simulation has been built by a consortium of scientists worldwide (<http://www.ecco-group.org/>).

I have chosen this particular model for us to analyze because,

1. It is a state-of-the-art modeling framework.
2. Not just the vague variables, such as salinity and temperature, are saved. Also available are the advective & diffusive fluxes, winds, surface forcing, etc. Thus, unlike observations in the real world, we will have *everything we need to understand the behavior of the model*.
3. This model is data-assimilating. It is constrained to closely resemble a suite of satellite and in situ data (which you can access directly here: ftp://mit.ecco-group.org/ecco_for_las/version_4/release2/input_ecco/). Thus, as closely as possible, it resembles the real world (except without data gaps).
4. The model's numerical basis is finite volume discretization. This framework is identical to that we have been discussing in class, with fluxes into and out of the faces of gridded volumes, and each volume contains tracers and momentum.

5.0.1 ECCOv4r2 Version Details

This version is ECCO v4 release 2. It simulates the Earth over the 240 month period from 1992 to 2011. Data and readme files may be found at http://mit.ecco-group.org/opendap/ecco_for_las/version_4/release2.

There are three different temporal collections of data that are useful for our purposes. The monthly directory contains many of the model output variables, depending on time (240 months), latitude, longitude, and depth (50 levels). The files in this format are big (e.g., the salinity file is just over 3 gigabytes in latlon spatial gridding). Two dimensional variables, such as wind stress, are 50 times smaller. The directories ending with `_climatology` contain the average over all years of all variables for each month, that is, the typical January, the typical February, etc. For many projects, this climatological version will be adequate, and the data files are 20 times smaller. Finally, I have made you an average of the climatology, so it no longer has any seasonal information. This average of the whole model is in directories ending in `_avg`, and is 240 times smaller than the `_monthly` directory.

There are three versions of spatial grids that we use. The `ntiles_` spatial gridding is the native grid of the model, but it is confusing as it is laid out in 13 different files for each variable and the grids do not align with latitude and longitude easily. The `interp_` spatial griddings are interpolated versions of the `ntiles_` model onto a uniform global grid. The `interp_` gridding is useful, but some errors are introduced by interpolation. The `interp_monthly` and `ntiles_monthly` is available here: ftp://mit.ecco-group.org/ecco_for_las/version_4/release2/interp_monthly. The `latlon_` gridding is the best of both `interp_` (the grids are all lat-lon) and `ntiles_` (the grids presented are what the model uses), but the parts of the globe where `ntiles_` are not oriented along meridians and parallels is just omitted. The `latlon_monthly` is available here: http://fox-kemper.com/data/ecco_for_las/version_4/release2. As a starting point, I recommend using the `latlon_` version if possible, and then the `interp_` if you want to go polar, and finally the `ntiles_` only as a last resort.

5.0.2 Earlier Versions

In previous versions of this course, I have used versions of this model that did not include the Arctic (v2.199 and v3.73 and v4r1). However, the newest version v4r2 is significantly better in many respects. So, we will use that one.

5.0.3 Where to Find

I have created copies of the most crucial files for your analyses. The files needed to do only the homework problems here can be downloaded from: http://fox-kemper.com/data/ecco_for_las/version_4/release2. These files are all averaged in time over the whole of the 240 months, to make for smaller files and easier analysis. Some of the original files are located at ftp://mit.ecco-group.org/ecco_for_las/version_4/release2, in case I have forgotten any files.

There are “normal year” climatologies (averages over all 20 januaries, then all 20 februarys, etc.) which are available for many of the files that can be downloaded. There are all time averages (avg) files, and there are the monthly mean “monthly” files, which are 240 times bigger than the avg files and 20 times bigger than the climatologies.

5.1 Opening Files

In HW2, you have already learned the basics of how to open and manipulate files and evaluate simple budgets. If you have forgotten, review these homeworks. You no longer have to focus so much on budgeting, as you now understand how to relate the sources and sinks of watermasses with their impacts on salinity, temperature, freshwater, and therefore climate.

5.2 Sverdrup and Western Boundary Current Exercise

This portion of the homework is to get you used to calculating the Sverdrup transport and comparing it to the depth-integrated model transport. For simplicity, we will choose to focus our analysis on one location, the line at 26N across the Atlantic. This is a good region to think about, as it is the location of the RAPID (<http://www.rapid.ac.uk>) meridional overturning circulation monitoring array. You should feel free to choose this same region for your paper, but be careful to expand upon the basic analyses presented here.

1. Write an equation that relates the wind stress to the meridional Sverdrup volume transport. Be clear about units and directions (Wunsch Chp. 10, esp. 10.10).

By integrating Wunsch’s formula (10.10), which includes both geostrophic and upper Ekman transport, as momentum equations (10.8a,b) indicate:

$$\beta \rho_0 \int_{-H+d}^0 v \, dx \, dz = \beta \rho_0 \int_{-H+d}^0 v_g + v_e \, dx \, dz = \int \hat{\mathbf{k}} \cdot \nabla \times \tau \, dx \approx (x_e - x_w) \hat{\mathbf{k}} \cdot \nabla \times \tau \quad (1)$$

Alternatively, we can examine the vorticity budget only below the Ekman layer, with just the geostrophic velocity, based on the Ekman pumping driving the vortex stretching:

$$\beta \rho_0 \int_{-H+d}^{-d} v_g \, dx \, dz = \int \rho_0 f w_e(-d) \, dx \approx \int f \nabla \times \frac{\tau}{f} \, dx \approx f(x_e - x_w) \nabla \times \frac{\tau}{f} \quad (2)$$

This is transport in m^3/s . Note that τ is assumed to be constant in x for a simple relationship (otherwise, the $x_e - x_w$ would be replaced with an integral).

2. Write an equation that relates the depth-integrated pressure P between two zonally distant locations x_e and x_w at a given latitude to the meridional geostrophic volume transport between these points. Be clear about units and direction (Wunsch 10.12).

$$\frac{V_g}{\rho_0} = \int_{-H+d}^{-d} v_g dx dz = \frac{P(x_e) - P(x_w)}{f \rho_0} \quad (3)$$

In principle this can be calculated from ETAN and PHIHYD in ECCO.

This is transport in m^3/s .

3. Assuming the depth integrated pressure anomalies result only from the sea surface height variation η , relate η to the meridional geostrophic volume transport. Be clear about units and direction (Wunsch 10.12).

$$\int_{-H+d}^{-d} v_g dx dz \approx \frac{gH}{f} (\eta(x_e) - \eta(x_w)) \quad (4)$$

This is transport in m^3/s .

4. Write an equation that relates the wind stress to the Ekman meridional volume transport (Wunsch 10.8). Be clear about units and direction.

$$\int_{-d}^0 v_e dx dz \approx \frac{-\tau_x}{\rho_0 f} (x_e - x_w) \quad (5)$$

Note that if τ is eastward ($\tau > 0$), the flow is equatorward as $x_e > x_w$, as expected for subtropics. This is transport in m^3/s .

5. Add together the previous results to write a result for the combination of the meridional Ekman and geostrophic transport. Note which portion will be detectable via η or pressure and which will not.

$$\int_{-H+d}^0 (v_g + v_e) dx dz = \int \left(\frac{f w_e(-d)}{\beta} - \frac{\tau_x}{\rho_0 f} \right) dx = \int \left(\frac{f}{\rho_0 \beta} \nabla \times \frac{\tau}{f} - \frac{\tau_x}{\rho_0 f} \right) dx \quad (6)$$

$$= \int \frac{1}{\rho_0 \beta} \nabla \times \tau dx \approx \frac{(x_e - x_w)}{\rho_0 \beta} \nabla \times \tau \quad (7)$$

Only the geostrophic portion will be detectable with η . Equals the integral of Wunsch's (10.10).

6. Write an equation that integrates model's volume transport (in terms of evelmass, nvelmass) over the same region as in 4. Please note that the reason why they are called "velmass" is that they are the mass-averaged velocity, so the small changes in model cell vertical dimensions is already accounted for (i.e., you can use a constant in time δz and arrive at the correct integral). Also note that as the model uses the Boussinesq equations, there is no meaningful distinction between mass budget and volume budget. Be clear about units and directions. Recall that the evelmass and nvelmass are oriented in the zonal and meridional directions.

The meridional velocity is:

$$v_m = vvelmass; \quad (8)$$

$$\int_{-H+d}^0 v_m dx dz \approx \int_{-H+d}^0 (v_g + v_e) dx dz \quad (9)$$

7. Thinking about the Stommel model and westward intensification, in what region do you expect the result for 1 and 2 to differ? In what region do you expect them to be similar? Compare the scaling for the vorticity term that depends on bottom drag and the vorticity term that depends on wind stress. Write a non-dimensional ratio that will explain the fraction of βV attributable to each.

Since the Sverdrup transport is useful only where bottom drag/friction is negligible, it will not apply within the western boundary current. Away from this current, the approximate equality above should hold from the eastern boundary toward the interior. When we examine the two terms in the vorticity equation in the Stommel model, we find:

$$\beta \int_{-H+d}^{-d} v_g dz \approx \frac{f}{\rho_0} \nabla \times \frac{\tau}{f} - \frac{d\mathbf{f}}{2} \cdot \nabla \times \mathbf{v} \quad (10)$$

$$\frac{\frac{d\mathbf{f}}{2} \cdot \nabla \times \mathbf{v}}{\frac{f}{\rho_0} \nabla \times \frac{\tau}{f}} \approx \frac{\rho_0 d\mathbf{f} \cdot \nabla \times \mathbf{v}}{2 \nabla \times \tau} \quad (11)$$

The ratio is a bit peculiar, but it relates the amount of vorticity being damped by bottom drag (Ekman drag) to the amount of vorticity being input by the wind (Ekman pumping). Note that if the sign of the vorticity ($\nabla \times v$) matches the sign of the wind stress ($\nabla \times \tau$), then the bottom drag term and the Sverdrup term are of opposite signs (e.g., equatorward in interior, poleward in WBC where drag dominates).

8. Finally, consider the thermal wind relation and the fact that the modeled depth-integrated velocity in the above relations will neglect baroclinic motions. Will baroclinic motions change the transport of depth integrated mass? Will they change the depth-integrated heat transport?

We did not have to assume that the fluid was barotropic to arrive at the Sverdrup relation. Thus, if the conditions of the Sverdrup relation hold (inviscid, low Rossby), the result will apply to the depth-integrated flow even if it includes a baroclinic component. The depth-integrated heat transport, however, might be affected by baroclinicity. Consider the following possibilities: 1) a barotropic (depth-independent) velocity advecting a stratified fluid versus 2) a vertically sheared flow, but with the same depth-averaged flow (governed by the Sverdrup relation), advecting a stratified fluid. Because 2) might differ in transport of surface versus bottom waters, the heat transport is unlikely to be the same in cases 1) and 2).

5.3 Sverdrup & WBC Budget Exercise

For this part, you'll examine the mass transports produced by Sverdrup transport, WBC transport, geostrophic transport, Ekman transport, and thermal wind transport. First of all, I have simplified the codes a bit. Place the `geostrophic_budget.m` files in the same location as the following data files: `NVELMASS`, . These files can be found at <http://fox-kemper.com/data>. Then

```
>> geostrophic_budget
```

Will activate the pathnames and load the variables and find the 26.5N section across the Atlantic.

1. Figure 1 shows the sea surface height, η from the time average. Label the western boundary current part of this figure and the interior Sverdrup flow part of the figure. How big is $\frac{\partial \eta}{\partial x}$ in each region?

The geostrophic relation tells us that the northward part of the current is beneath the $\frac{\partial \eta}{\partial x} > 0$ region and the southward (geostrophic, Sverdrup) part of the flow is where $\frac{\partial \eta}{\partial x} < 0$. The surface rises about 0.5m over 6000km in the interior and about 0.5 over 300km in the boundary current.

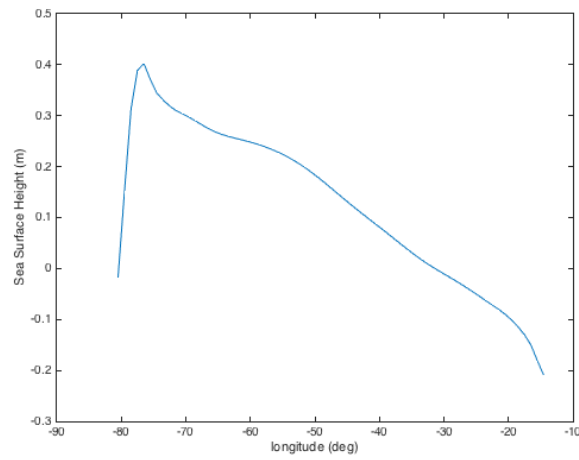


Figure 1: HW Figures.

- Figure 2 shows the meridional velocity as a function of depth and longitude. Label the western boundary current part of figure 2, and estimate its width, typical velocity, and depth. Also label where topography (the seafloor) is indicated. [See fig. 2.](#)

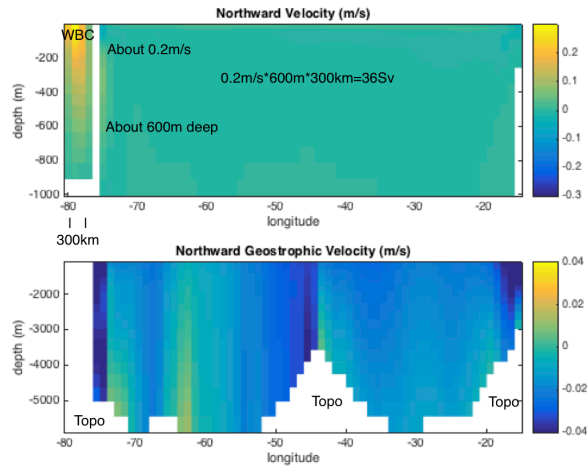


Figure 2: HW Figures.

- Read through the geostrophic_budget code. Use the **legend**, **label**, **title** commands to properly label the Sverdrup, geostrophic, Ekman, and Sverdrup+Ekman transports on the two subplots in figure 3, including units. [See fig. 3. Code is available here: This Link.](#)
- Baroclinic Shear. Figures 4 and 5 (shown together in Fig. 4 here) show the vertical shear from the model and from thermal wind relation. Are they consistent? Where do they disagree and why?
[See Fig. 4. They are consistent everywhere except at the near surface, where the Ekman shear dominates the geostrophic/thermal wind shear.](#)

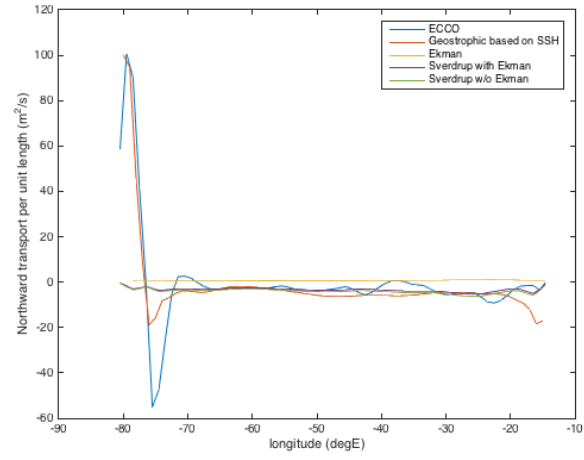


Figure 3: HW Figures.

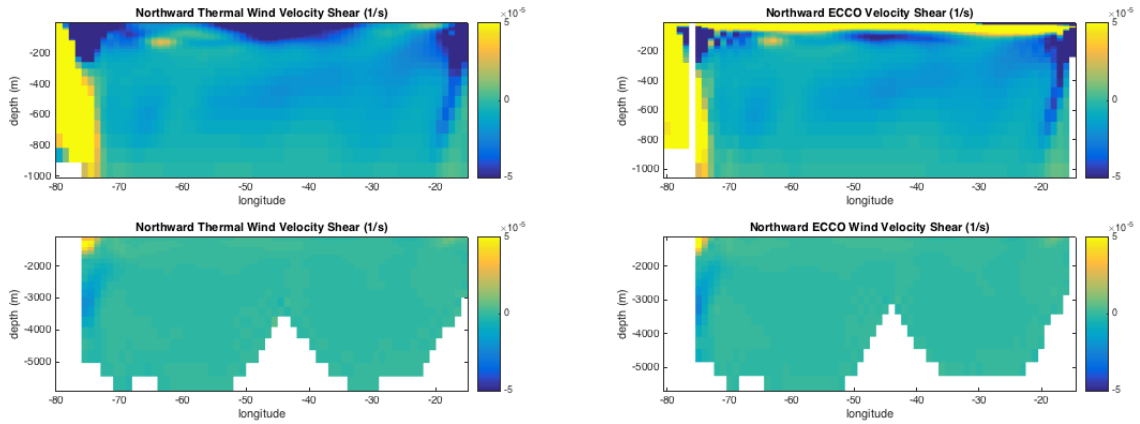


Figure 4: HW Figures 4 and 5.

I have also provided one extra figure here (Figure 5) and in the `geostrophic_budget_key.m` which is useful for considering the meridional mass balance. It is the zonal integral of the meridional transports per unit length.

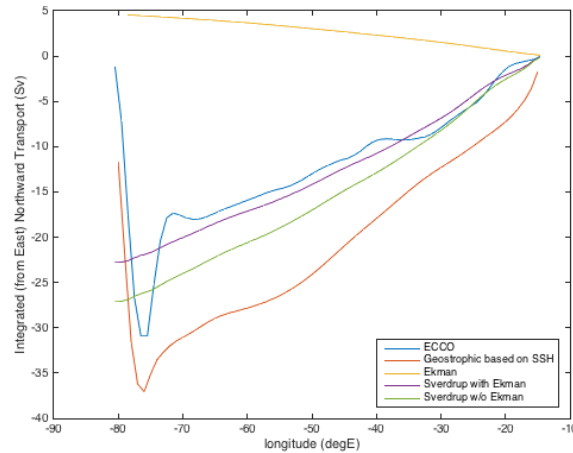


Figure 5: This is the zonal indefinite integral of Figure 5, showing the meridional transport between the eastern boundary and each location in Sverdrups. It is generated with commands similar to the following: `plot(lon,cumsum(NVELINT*dx,'reverse')/1e6)` . See the full answers at [This Link](#).

6 Plans 3

You now have some experience with making a plans document. Again, you will produce one. However, this time there are some specific constraints on what you will calculate (see below), so take that into account as you prepare your plans.

For the plans document, you will outline what plans you have for the project, including as many details as you have collected by this time. Here are some critical points to make in the plans document, which will help me to better advise you in moving to the paper stage.

- Describe your region (i.e., a zonal section in one ocean or another. Choose one that intersects a western boundary current and name which one) and timeframe of interest.
- Give a working title.
- Make a list of 1-3 hypotheses you plan to address.
- Identify specific journal articles, papers, or subsections of the book that are relevant. Briefly summarize what they say that is relevant to your planned paper, and what missing hypotheses to test that you will try to test. Look back at comments on previous plans to help in the selection.
- Here the only relevant datasets are those coming from ECCOv4r2. However, some of you may be planning to look at regions that are seasonal or inter-annually varying. For these, you will need to adapt the code I have provided for the time average fields to the time varying fields. Indicate whether you have been successful in doing so (i.e., have you mastered the file format?), locating the relevant dates/locations (have you mastered the data subselection/data layout?), and which figures you intend to create (have you designed the most important metrics for your problem?).

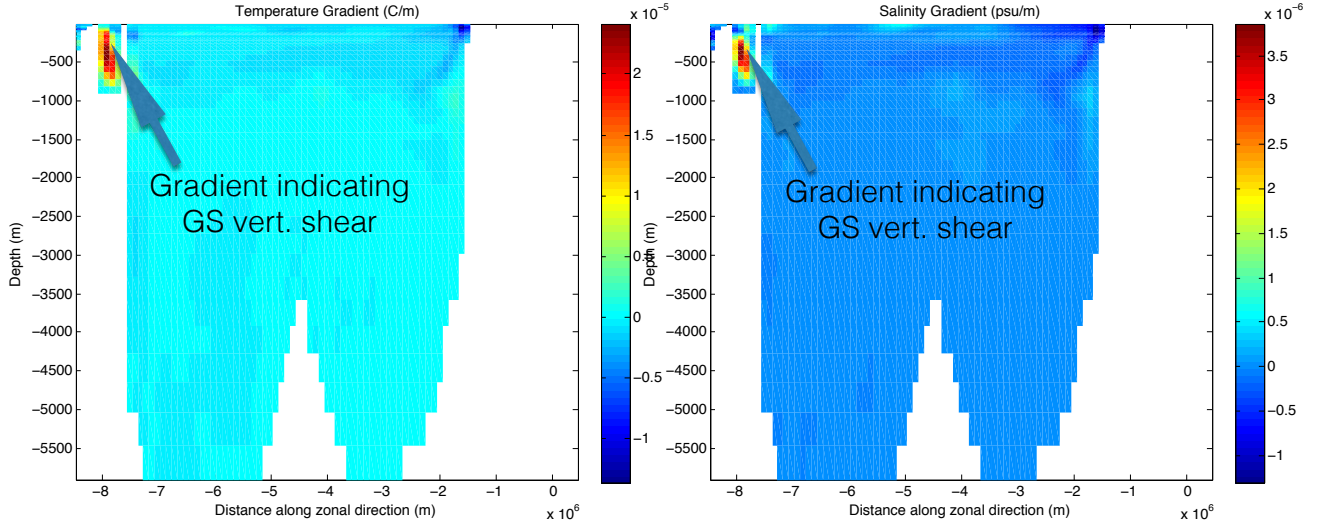


Figure 6: HW Figures of T & S.

- **MOST IMPORTANT.** If possible, include draft versions of the figures you plan to use in your paper, based on the real data and the real metrics you plan to calculate. Normally, if you can get to this stage, completing the paper on time is no problem.
- If you are unable to get to the preceding step, give your fallback plans if all else fails. Because of the reviewing process and timeframe, you will *not* be able to get an extension on the paper deadline if things don't go smoothly. You can include more figures in the revised version of the paper if you figure it out after you turn in the first version of the paper.

7 Paper 3

As you plan your paper, go through my paper primer document, available on the webpage.

7.1 Topic: Swimming Upstream

The goal here is to choose a zonal transect across one of the ocean basins that is extratropical and continentally bounded (i.e., not the Southern, Arctic, or Equatorial oceans) and timeframe. You will relate the Sverdrup, Ekman, and geostrophic transports across your transect.

- Find a region of interest based on the existence of a western boundary current and its consequences. Locate a review paper or an observational campaign to ground your choice of location.
- Using the code from the HW, and the ECCOV4r2 dataset, find the following for your transect:
 1. The depth-integrated mass transport in the model as a function of longitude.
 2. The depth-integrated (barotropic) geostrophic mass transport in the model as a function of longitude.
 3. The depth-integrated Ekman transport from the model's winds as a function of longitude.
 4. The depth-integrated Sverdrup transport from the model's winds as a function of longitude.
 5. The vertical shear from the model as a function of longitude and depth.

6. The vertical shear from the thermal wind as a function of longitude and depth.

- You will then discuss where these estimates agree and disagree, and whether the agreements are expected or unexpected. Depending on your region, you may find interesting features such as seasonality, monsoons, recirculation gyres, or separated boundary currents that may complicate the interpretation. It is up to you to recognize these features in the data and address them.
- These figures should show whether the western boundary current acts as a return flow for the Sverdrup + Ekman transport in your region, and whether there is a net mass transport across your section (if so, you should quantify it). Note that your region *must include a western boundary current*.
- Depending on your region, you may choose to expand your exploration into heat fluxes (northward, southward, equatorial, poleward, deep, shallow?), salt fluxes, or variability by month, season, or interannual.

Don't be afraid to piggyback off of other published work. Use Google Scholar or Web of Science to locate articles by keyword, and then see what they say. Feel free to use the matlab references, and refer to similar projects.

8 Reviews 3

As you plan your reviews, go through my review primer document, available on the webpage.

You will read and review two of your colleagues work. You can look at the peer reviewing primer for more details as to how to proceed. Be sure that you are actually critical where you see room to improve ("Everything looks great! Good work!" is not a useful review unless it is *totally* true), but do not be heedless of feelings or unfriendly. Make sure you comment on the *science* as well as the presentation and writing. For example, it is more important to catch unfounded claims than to fix spelling errors. Do you believe the metrics used? Do you understand the fundamentals of the dataset and measurement technique? Do you suspect that the data is cherry-picked? Etc.

9 Revisions 3

Learning how to respond to reviews, both positive and negative, is a crucial part of any scientist's (or just person's) training. You've done a couple now, so it should be pretty automatic. Don't forget the response primer document!

References

- Adcroft, A., J. M. Campin, S. Dutkiewicz, C. Evangelinos, D. Ferreira, G. Forget, B. Fox-Kemper, P. Heimbach, C. Hill, E. Hill, et al., The mitgcm user manual, 2008.
- Forget, G., J.-M. Campin, P. Heimbach, C. N. Hill, R. M. Ponte, and C. Wunsch, ECCO version 4: an integrated framework for non-linear inverse modeling and global ocean state estimation, *Geoscientific Model Development*, 8, 3071–3104, doi:10.5194/gmd-8-3071-2015, 2015.
- Wunsch, C., *Modern observational physical oceanography: understanding the global ocean*, Princeton University Press, 2015.