Fall 2019, GEOL2300- Homework 5

1. Conservation laws

1.1 The path to Navier-Stokes equations

In class we have seen that the stress tensor for a Newtonian homogeneous fluid

$T = (-p + \lambda \nabla \cdot \vec{v})\mathrm{I} + 2\mu\dot{\mathcal{E}}.$

- Using index notation, compute the divergence of the stress tensor for an incompressible fluid
- Use the expression you just obtained to derive Navier – Stokes equations (the momentum part) and take the curl of the equation, what do you get?

2.1 Non-inertial flows

Starting from Navier Stokes (N-S) equations for an incompressible fluid (neglect gravity here for simplicity) are

$$\rho\left[\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}\right] = \mu \nabla^2 \vec{v} - \nabla p$$
$$\nabla \cdot \vec{v} = 0$$

- Non-dimensionalize the equations using $\vec{v} = V\vec{v}^*, \vec{x} = L\vec{x}^*, t = \frac{D}{V}$.
- We have not played with pressure yet, in a non-inertial flow, what stress balances pressure? Use this knowledge to come up with a suitable choice to non-dimensionalize the pressure term.
- Write down the non-dimensionalized form of N-S equations and discuss the presence and magnitude of the dimensionless number that you find.
- Think about an experiment where the fluid chosen here is sheared in a coquette flow device (imposed displacement/velocity at the edges and D represents the spacing between the two boundaries with differential motion). The natural variables of this problem are the viscosity of the fluid and its density ρ, μ, the imposed velocity of the boundary (the other assumed at rest) V and the spacing between the two boundaries D. This represents 4 parameters and 3 dimensions (distance L, mass M and time T), so the 4 parameters cannot be entirely independent of each other. Use ρ, μ and D to get a quantity with units of velocity and take the ratio of V to that velocity, how does that compare to the dimensionless # you got above?

3.1 Flows where inertia matters

Repeat the same operations except that pressure now balances inertial stresses, find an appropriate dimensionless version of Navier-Stokes equations.