$\boxed{\text{Concepts of linear transport theory (non-eq. thermo)}}$

The ideas comes from Ilya Prigogine, Lars Onsager--

From 2^{nd} law of thermodynamics, we know

$$dS \geq 0 \qquad (S \text{ is entropy})$$

However, it does not tell you anything about rate at which one goes back ~~at~~ to equilibrium.

σ_s = entropy production rate

$$\sigma_s = \sum_i J_i X_i \geq 0$$

J_i = thermodynamic fluxes

X_i = associated (driving) thermodynamic force

Under equilibrium conditions

$J_i = 0 \quad \forall i \Rightarrow \sigma_s = 0$, makes sense.

Onsager: J_i are caused (linear assumption) by forces X_j => Taylor expansion

$$J_i = J_i^{eq} + \sum_j \underbrace{\frac{\partial J_i}{\partial X_j}}_{L_{ij}} X_j + \underbrace{O(|X_j|^2)}_{\text{Small if we stay close to equilibrium}}$$

L_{ij} Onsager coefficients

$$L_{ij} = L_{ji}$$

$$J_i = \sum_j L_{ij} X_j$$

back in entropy production

$$\sigma_s \approx \sum_{i,j} L_{ij} X_i X_j \geq 0$$

In general, X_j are expressed in terms of gradients of potentials or pseudo-potentials

examples:

(a) Fourier's law $\qquad \vec{q}_H = -\underline{\underline{k}} \cdot \nabla T$

thermal conductivity

(b) Ohm's law $\qquad \vec{j} = -\underline{\underline{\sigma}} \cdot \nabla u$

↳ electric pot.

(c) Fick's law $\qquad \vec{q}_D = -\underline{\underline{D}} \cdot \nabla C$

(or better chemical pot μ)

(d) Darcy's law $\qquad \vec{q} = -\underline{\underline{K}} \cdot \nabla h$

$$\vec{J}_i = L_{ij} X_j$$

$L_{ij} = \{\underline{\underline{k}}, \underline{\underline{\sigma}}, \underline{\underline{D}}, \underline{\underline{K}}\}$; $X_j = -\nabla\{T, u, C, h\}$

What does it mean

(1) "conductivities" are symmetric tensors
(can be diagonalized)

(2) these constitutive laws (a-d) only work
close to equilibrium.

$\boxed{\text{Some principles of dimensional analysis}}$

Physical equations involve (generally) units / dimensions
they therefore need to obey dimensional homogeneity:

Each term of an equation should have identical dimensions
to the other one!

Let's define M for [mass]

L for [distance]

T for [time]

Buckingham-Π theorem:

If an equation is dimensionally homogeneous, then
it can be decomposed into relationship between 'dimensionless
products.

For a matter of time, I will not derive it here, but rather give 2 examples...

(A) Nuclear blast and propagation of shock wave

In the early 1940's G.I Taylor was put in charge of establishing the dynamics of the shock wave of the 1st nuclear explosion, he was not told the yield of the explosion (secret). His goal was to track radius (t) of blast and use photographs to infer dynamics

Taylor assumed that the parameters that controlled the physics of the explosion could be reduced to

E = energy of the blast
t = time
$r_f(t)$ = radius of shock wave
ρ_0 = density of atmosphere pre-blast

$$\left.\begin{aligned} [E] &= ML^2/T^2 \\ [t] &= T \\ [r_f] &= L \\ [\rho_0] &= M/L^3 \end{aligned}\right\}$$

3 dimensions M, L, T & 4 parameters

=> mean one parameter can be reconstructed (in terms of dimension) from the 3 others

=> 1 dimensionless # to express lack of linear independence, call it Π_1

$$\Pi_1 = \frac{r_f^5}{(E/\rho_0)\,t^2} = \text{Constant}$$

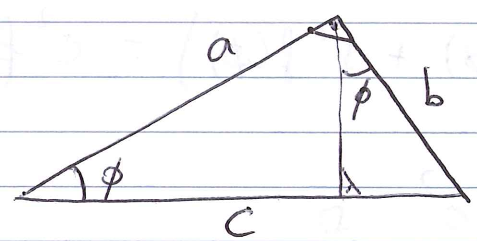
$$r_f(t) = (\text{Constant}) \left[\frac{E}{\rho_0} t^2 \right]^{1/5}$$

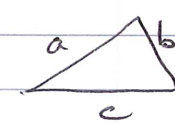
$$\log(r_f) = \frac{1}{5} \log\left(\frac{E}{\rho_0}\right) + \frac{2}{5} \log t + \text{Constant}$$

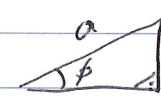
$\Rightarrow$ pefect match with visual data.

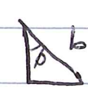
2nd example

(B) Pythagoras theorem



Let's call S_c surface area of this triangle 

S_a " " of 

S_b " " of 

they are all similar triangles

$[S_c] = L^2$, and if one knows ϕ & c

$\longrightarrow$ unique solution for S_c $\Rightarrow S_c = f(c, \phi)$

The angle is dimensionless so

$$\frac{S_c}{c^2} = f(\phi)$$

dimensionless ratio is function of angle alone

$$\Rightarrow \quad S'_c = c^2 f(\phi)$$

Now, the same can be done for

$$\frac{S'_a}{a^2} = f(\phi)$$, note the function is the same!

&

$$\frac{S'_b}{b^2} = f(\phi)$$

$$S'_c = S'_a + S'_b = \left(a^2 f(\phi) + b^2 f(\phi)\right) = c^2 f(\phi)$$

$$\Rightarrow \quad a^2 + b^2 = c^2 \quad \square$$